Global Imbalances in a Monetary Model of Liquidity Constraints

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Abstract

I construct a monetary model in which liquidity constraints emerge in equilibrium. For fast-growing economies, financial integration is associated with lower consumption and increased consumption volatility. Foreigners extract rents from supplying liquidity to the constrained productive sector. Input prices appreciate, but equilibrium output remains unchanged. This results in lower consumption, as some of the output is used as payment to liquidity suppliers. The magnitude of the flows implied by the model are roughly 2%-6% of GDP. In the presence of sticky input prices, the reliance on foreign liquidity supply is a potential source of instability, as contractions in foreign liquidity supply lead to drops in employment, output and consumption.

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1 Introduction

When firms are liquidity constrained, there are opportunities for liquidity suppliers to extract rents. In this paper I explore the implications of this in a global equilibrium context, in which some fast-growing regions are liquidity constrained while other regions are at their steady state and are able to supply liquidity. The

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model predicts an increasing flow of goods from the constrained region to the unconstrained region, consistent with “global imbalances”. From a normative perspective, this model suggests that financial integration is welfare-reducing for the constrained regions, but beneficial for the unconstrained regions.

I consider a monetary model in which there is a cash in advance constraint on employing production inputs, which I refer to as a liquidity constraint. In the closed economy, a binding liquidity constraint offers a positive rate of return on holding cash, guaranteeing that firms will be willing to carry cash from one period to the next. A feature of this model is that faster-growing economies will be more constrained in equilibrium, in the sense that the marginal product of inputs is higher relative to their price.

I study the effects of financial integration between a fast growing constrained economy and a steady state rest of the world. Financial integration is assumed to be limited in the sense that only short term borrowing and lending from abroad is permitted: firms can borrow within a period to bridge the gap between production expenses and sales, but households cannot borrow across periods to smooth consumption. In the integrated environment, the returns to holding cash are pinned down by the rest of the world at a relatively low level. Given their increasing consumption path, domestic shareholders are no longer willing to hold cash at the low rate of return. Thus, in equilibrium, the financing of production expenses is done entirely through foreign liquidity. The rents paid to foreign liquidity suppliers reduce equilibrium consumption compared to the closed economy equilibrium. Further, the economy’s production becomes entirely reliant on foreign liquidity supply, which may be source of instability.

The mechanics of this model are close to Eden [2011b]. In Eden [2011b], I consider a closed economy with liquidity constraints in which the price of production inputs in terms of liquidity is determined in equilibrium. The equilibrium determination of this price is the key ingredient in both models. There are two main differences between this paper and Eden [2011b]: first, the focus of the two papers is different. Here, I focus on the welfare implications of financial integration and on global imbalances. In Eden [2011b] I focus on the general equilibrium costs of financial intermediation in a closed economy context. Second, this paper extends Eden [2011b] by endogenizing the liquidity supply in the context of a dynamic, monetary model. In Eden [2011b], I consider a single-period model in which liquidity is given exogenously. Here, I demonstrate how a binding liquidity constraint can arise in equilibrium, and how it changes following financial integration.

This model implies an increasing flow of goods from constrained economies to the liquidity supplying regions. This model of “global imbalances” is most closely related to the equilibrium view in Caballero et al. [2008], in which financial frictions lead to a permanent flow of savings towards economies with better savings facilities. While the essence of the financial friction is similar - inability to pledge
future output - the implications are quite distinct. In Caballero et al. [2008], the
driving force behind the global imbalances is the demand for savings from emerging
economies. In contrast, in this model, while the central bank ends up accumulating
foreign reserves, there is no demand for savings. In fact, if they had access to long
term international borrowing and lending, domestic agents would choose to borrow
to smooth consumption. The accumulation of foreign reserves is just an artifact
of the increasing demand for converting foreign into domestic currency in order to
be able to supply liquidity to domestic producers.

There are other papers in which agents face endogenous borrowing constraints,
such as Holmstrom and Tirole [1997] and, more recently in the context of financial
integration, Korinek [2010]. In these papers, borrowing constraints result from
a moral hazard problem and incomplete enforcement. Here, while it is assumed
that future output is not pledgable and inputs must be purchased with cash, en-
forcement does not play an important role. Rather, the binding cash in advance
constraint is necessary in order for people to be willing to hold cash across periods.
In contrast to this literature, a binding liquidity constraint does not necessarily im-
ply a loss of welfare. This paper provides an alternative model for binding liquidity
constraints, that is immune to the critique in Aiyagari [1994] that, in a dynamic
model, liquidity constraints vanish in the long run as firms accumulate capital.

This paper is also related to the literature on cash in advance constraints in the
productive sector, such as Stockman [1981] or Abel [1985]. Most closely related
to this paper is Neumeyer and Perri [2005], who consider an open economy with
a “working capital” constraint which requires that part of the wage bill must be
financed with foreign credit. This paper endogenizes this restriction in a model in
which there is a cash in advance constraint on hiring labor. In equilibrium firms
must rely on foreign liquidity suppliers for cash.

Finally, this paper is broadly related to the literature on the distributional
implications of opening to trade and the welfare implications of liberalization.
Bhagwati and Brecher [1980] present the idea that in the presence of foreign-
owned inputs of production, trade liberalization can decrease domestic welfare if
it increases the relative wage of the inputs that are disproportionately owned by
foreigners. In this model, the distributional implications of liberalization go in the
opposite direction: financial integration increases the return to domestic labor,
while it lowers the return to liquidity that can be supplied by foreigners. However,
welfare decreases as the economy imports liquidity in equilibrium, despite the fact
that the economy’s liquidity needs can be satisfied in autarky at no aggregate
cost. The distributional implications of financial integration are in the spirit of
Antras and Caballero [2009]: the returns to liquidity decline, as liquidity can be
imported more cheaply from abroad, allowing labor to absorb a higher share of
output. However, in contrast to Antras and Caballero [2009], the net surplus of
the economy declines. The key difference is that liquidity is not a real factor of
production, in the sense that importing liquidity does not allow the economy to expand its production.

The rest of the paper is organized as follows. In section 2, I illustrate the main insights using a single period real model, where liquidity constraints are assumed exogenous. In section 3, I endogenize the liquidity constraints in a dynamic monetary model. In section 4, I solve for the closed economy equilibrium. In section 5, I consider the implication of financial integration. I show that financial integration is associated with lower consumption. In section 6, I explore the implications for the current account. In section 7, I show that the integrated equilibrium is one in which the domestic economy is vulnerable to shocks to foreigner’s willingness to supply liquidity, and demonstrate that the central bank has an incentive to depreciate the currency in such an event. In section 8, I conclude.

2 Single Period Real Model

I begin by illustrating the main insights using a simplified single period real model. There is a unit measure of households and a unit measure of firms. Each household supplies $L$ units of labor inelastically. Each household owns a share of the productive sector. The technology is given by $\rho F(L^e)$, where $L^e$ is employed labor and $F(\cdot)$ is an increasing function with diminishing returns ($F'(\cdot) > 0$, $F''(\cdot) < 0$).

At the beginning of the period, each firm is endowed with $Q$ units of current output which will be referred to as liquidity. Firms can trade in liquidity. The amount of liquidity demanded by firm $i$ (in addition to its endowment) is denoted $Q^d_i$, where $Q^d_i$ can be positive or negative. The total amount of liquidity held by the firm at the beginning of the period is therefore $Q + Q^d_i$.

The price of liquidity is set in terms of output at the end of the period: one unit of liquidity at the beginning of the period costs $1 + r$ units of the final good, to be delivered after production takes place. Under autarky, $r^{aut}$ is set such that the market for liquidity clears; in the open economy, it is assumed that $r$ is exogenously fixed by the rate of return to liquidity in global markets.

Denote by $L^e_i$ the amount of labor employed by firm $i$, and let $w$ be the equilibrium wage.

2.1 The Unconstrained Benchmark

Before adding the liquidity constraint, it is useful to consider the benchmark of the unconstrained economy. In the unconstrained economy, the firm’s problem is:

$$\max_{L^e_i, Q^d_i} \rho F(L^e_i) + Q + Q^d_i - (1 + r^{aut})Q^d_i - wL^e_i$$  (1)
In equilibrium, firms choose labor such that the marginal product of labor is equal to the wage. By symmetry, labor market clearing implies that $L_i^e = L$ for all $i$. The wage is therefore given by:

$$w = \rho F'(L_i^e) = \rho F'(L) \tag{2}$$

Under autarky, liquidity market clearing implies that $r^{aut} = 0$: otherwise, if $r^{aut} < 0$, firms demand an infinite amount of liquidity; if $r^{aut} > 0$ firms want to supply an infinite amount of liquidity. Having $r^{aut} = 0$ guarantees that each firm demands 0 additional units of liquidity ($Q_i^d = 0$).

As the entire labor force is employed in equilibrium, and the marginal product of labor is equated across firms, output is equal to $\rho F(L)$. Output is distributed both as wages and as dividends and is consumed in its entirety.

Dividends are given by:

$$d = \rho F(L) + Q - wL \tag{3}$$

The first best level of consumption is therefore given by:

$$c^{FB} = wL + d = wL + \rho F(L) + Q - wL = \rho F(L) + Q \tag{4}$$

Note that the unconstrained economy necessarily gains from financial integration. To see this, note that the derivative of the firm’s problem with respect to $Q_i^d$ is $-r$. If $r > 0$, firms would export liquidity at an arbitrarily large amount, resulting in large profits. If $r < 0$, firms would import liquidity at an arbitrarily large amount. Profits would be arbitrarily large in either case. If $r = 0$, prices remain at their autarkic level; the economy therefore (weakly) gains from financial integration, regardless of the international rate of return to liquidity.

### 2.2 The Constrained Economy

I continue by describing the equilibrium in the constrained economy, in which labor inputs must be purchased in advance of production.

Firms must purchase labor with liquidity; the amount of labor employed by firm $i$ must satisfy:

$$wL_i^e \leq Q + Q_i^d \tag{5}$$

Firms therefore face the following optimization problem:

$$\max_{L_i^e, Q_i^d} \rho F(L_i^e) + Q + Q_i^d - (1 + r)Q_i^d - wL_i^e \tag{6}$$

s.t.

$$wL_i^e \leq Q + Q_i^d \tag{7}$$
The equilibrium under autarky. It turns out that for $\rho$ sufficiently large, the liquidity constraint (equation 7) is binding in equilibrium. The following lemma will be useful in deriving this result:

**Lemma 1** Under autarky, the liquidity constraint (equation 7) is binding if and only if $r^{aut} > 0$. If the liquidity constraint is not binding, $r^{aut} = 0$.

The proof of this lemma, together with other omitted proofs, is in the appendix.

**Claim 1** For $\rho$ sufficiently large, the liquidity constraint is binding in equilibrium.

To prove this claim, I will show that a 0 rate of return on liquidity ($r^{aut} = 0$) will generate excess demand for liquidity. As, by Lemma 1, $r^{aut} = 0$ is the only rate of return consistent with an unconstrained equilibrium, it will follow that the liquidity constraint must bind in equilibrium.

Consider the firm’s problem above, holding the wage $w$ constant, and assuming that $r^{aut} = 0$. Ignoring the liquidity constraint, the firm would like to set the marginal product of labor equal to the wage. Note that the marginal product of labor is increasing in $\rho$ - thus, the higher $\rho$, the more labor the firm wants to hire. For $\rho$ sufficiently large the liquidity constraint is binding for $Q^l_i = 0$, that is:

$$\rho F'(L^e_i = \frac{Q}{w}) \geq w$$

The amount of labor $L^e_i = \frac{Q}{w}$ is the maximum amount of labor that the firm can hire given its liquidity endowment. Thus, for any given $w$, for a sufficiently high $\rho$ the rate of return $r^{aut} = 0$ generates excess demand for liquidity. Thus, in equilibrium, it must be the case that $r^{aut} > 0$ and that the liquidity constraint is binding.

Consider next the equilibrium determination of the wage $w$. Whenever the firm’s liquidity constraint is binding, the wage $w$ is determined by the aggregate liquidity constraint, that is:

$$wL = Q \Rightarrow w = \frac{Q}{L}$$

From the analysis of the firm’s problem, for any given $w$ the liquidity constraint is binding for a sufficiently large $\rho$. Thus, for $\rho$ sufficiently large, the liquidity constraint binds for $w = \frac{Q}{L}$. In this case, the liquidity constraint is binding in equilibrium, as the wage is determined by the aggregate liquidity constraint and - given that wage - all firms use their entire liquidity endowment to purchase labor.

The price of liquidity is determined in equilibrium to assure that the market for liquidity clears. The price of liquidity is equal to the additional revenue that
it generates. An additional unit of liquidity can buy $\frac{1}{w}$ units of labor; each unit of labor produces $\rho F'(L)$ units of output. The price of liquidity is therefore given by:

$$1 + r_{aut} = \frac{\rho F'(L)}{\omega_{aut}} = \frac{\rho F'(L)L}{Q}$$ (10)

It turns out that despite the fact that the liquidity constraint is binding, output is still at its first best level. To see this, consider the problem of a planner trying to maximize output:

$$Y^{FB} = \max_{L_i} \int_0^1 \rho F(L_i) \, di$$ (11)

subject to:

$$\int_0^1 L_i \, di \leq L$$ (12)

Given the assumption that $F(\cdot)$ is increasing and concave, the solution to this problem is that output is maximized when the economy is at full employment and the marginal product of labor is equated across firms. These conditions are satisfied in the constrained economy. As labor is supplied inelastically, the entire labor force is employed in equilibrium. The fact that there is a market for liquidity guarantees that the marginal product of labor is equated across firms: all firms equate the return to liquidity with its price:

$$\frac{F'(L_i^\epsilon)}{w} = 1 + r_{aut} \Rightarrow F'(L_i^\epsilon) = F'(L_j^\epsilon)$$ (13)

In this model, output is at its first best level, and consumption is therefore also at its first best level. To see this, denote dividend income by $d$. In the autarkic equilibrium, $d$ is given by aggregate output plus the initial liquidity endowment, net of the wage bill:

$$d = \rho F(L) + Q - wL$$ (14)

Consumption is given by:

$$c_{aut} = wL + d = wL + (\rho F(L) + Q - wL) = \rho F(L) + Q = c^{FB}$$ (15)

To conclude, under autarky the presence of the liquidity constraint has no effect on welfare. Consumption is at its first best level, as inflated dividend income exactly offsets the effect of depressed labor income.

\(^1\)The assumption that firms have identical technologies and identical liquidity endowments is not important for this result. It is easy to show that the result generalizes to settings in which the liquidity endowments $Q_i$ are bounded away from 0 ($Q_i > \epsilon > 0$ for all $i$) and $F_i(\cdot)$ changes continuously across $i$ in the bounded set $i \in [0, 1]$.  

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Financial integration. In contrast to the unconstrained case, when the economy is constrained financial integration can decrease equilibrium welfare. Consider the implications of having the constrained economy open to international liquidity flows. Assume that the return to liquidity on global markets is pinned down by $1 + r$, where $1 + r$ is small compared to the return to liquidity in the closed economy but still greater than $1^2$.

$$\frac{\rho F'(L)}{w^{\text{aut}}} = 1 + r^{\text{aut}} > 1 + r > 1$$ (16)

Claim 2 Financial integration reduces domestic consumption.

As long as $\frac{\rho F'(L)}{w} > 1 + r$, there are rents from supplying liquidity to constrained firms. Thus, the wage appreciates until $\frac{\rho F'(L)}{w} = 1 + r$. The higher wage bill is financed by foreign liquidity supply. Denote the equilibrium supply of foreign liquidity by $Q^*$. The wage bill now satisfies:

$$wL = Q + Q^*$$ (17)

Similarly to the autarkic equilibrium, output is still at its first best level. However, welfare declines as equilibrium consumption is lower. This is because some of the output has to be paid as rents to foreign liquidity suppliers:

$$d = \rho F(L) + Q + Q^* - wL - (1 + r)Q^*$$ (18)

$$c = wL + d = wL + (\rho F(L) + Q + Q^* - wL - (1 + r)Q^*)$$ (19)

$$= \rho F(L) + Q - rQ^* < c^{\text{aut}}$$ (20)

Financial integration reduces equilibrium consumption because constrained firms fail to internalize the effect of their borrowing on the equilibrium wage. From the perspective of each firm faced with a tight liquidity constraint, the prospect of borrowing cheaply from abroad to finance the purchase of more labor is an attractive one. However, as all labor is already employed, this activity only bids up the price of labor. As the wage bill becomes unaffordable to the domestic productive sector, some of the returns to labor must be paid as rents to foreign liquidity suppliers.

This model also admits to the following features:

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$^2$When this condition is violated, the constrained economy will gain from financial integration. However, in section 3 it will be shown that this condition will be satisfied in equilibrium for relatively fast-growing economies.
• **Global imbalances.** Note that this model predicts a net flow of goods from constrained economies to foreign liquidity suppliers of the amount of \((1 + r)Q^* - Q^* = rQ^*.\) This “global imbalance” is a result of equilibrium rents from supplying liquidity to constrained economies.

From the government’s perspective, there is an incentive to issue new liquidity to crowd out foreign liquidity supply. In the monetary economy, it will be shown that if the central bank can issue currency that is not backed by foreign reserves, it can improve welfare by distributing liquidity directly to firms. However, this policy will not be sufficient to eliminate global imbalances: the rents paid to foreigners will still be positive even under the optimal policy.

• **Instability associated with financial integration.** In the integrated equilibrium, the productive sector relies on foreign liquidity supply in order to finance the wage bill. This implies that, if wages are sticky, a sudden reluctance of foreigners to supply liquidity will induce a drop in employment. To illustrate this, assume that the interest rate at which foreigners are willing to lend jumps to \(r' > r\), and that \(w = \hat{w}\) is constant at its “no shock” equilibrium level. Employment must drop so that \(L^e\) satisfies \(\rho F(L^e) \hat{w} = 1 + r'\). Drops in output and consumption will follow as well.

Given this shock, the government has an incentive to increase employment by depreciating the currency, thereby making domestic labor effectively cheaper to foreigners. The price of one unit of labor in terms of foreign currency is \(\hat{w}e\), where \(e\) is the exchange rate. The domestic rate of return is therefore \(\frac{\rho F(L^e)e}{\hat{w}} = 1 + r'\). By increasing \(e\), the government can induce higher employment.

### 3 A Dynamic Monetary Model of Liquidity Constraints

The key novel ingredient in this model is the equilibrium determination of the price of production inputs in terms of liquidity. The relevance of this mechanism therefore depends on the equilibrium determination of liquidity and how it responds to changes in the environment. In this section I construct a monetary model in which liquidity is the amount of money held by firms at the production stage. Both in the closed economy and in the integrated equilibrium, binding liquidity constraints are a necessary feature of the environment.

I consider a discrete time infinite horizon model, where time periods are indexed by \(t = 1, 2, \ldots\).
Labor supply. There is a unit measure of households that supply $L$ units of labor inelastically each period. I assume for simplicity that labor is the only input of production.

Technology. The production technology in period 0 is given by $F_0(L)$, where $F'_0 > 0$ and $F''_0 < 0$. I assume constant productivity growth, governed by $\rho > 1$:

$$F_t(\cdot) = \rho^t F_0(\cdot) \quad (21)$$

Productive sector. There is a competitive productive sector, composed of a unit measure of identical firms. Firms hire workers to produce final consumption goods.

At $t = 0$, each firm is endowed with $M_0$ units of domestic currency. At time $t + 1$, the money supply is given by some $M_{t+1}$ that is determined by the central bank. At the beginning of each period, the additional units of money are equally distributed among firms by a helicopter. Each firm gets $T_t$ units of money at time $t$, where:

$$T_t = M_t - M_{t-1} \quad (22)$$

There is a market for cash. The price of 1 unit of cash at the beginning of period $t$ is set at $1 + r_t$ units of output at the end of period $t$. Denote by $Q_{i,t}$ the money holding of firm $i$ carried over from time $t - 1$, and by $Q_{i,t}^d$ the amount of liquidity demanded by firm $i$ at the beginning of period $t$ (given the price $r_t$).

At the beginning of period $t$ (for $t \geq 1$), firm $i$’s money holdings are:

$$M_{i,t} = Q_{i,t} + Q_{i,t}^d + T_t \quad (23)$$

There is a cash in advance constraint on hiring labor. In other words, the amount of labor employed by firm $i$ must satisfy the following condition:

$$w_t L_{i,t}^e \leq M_{i,t} \quad (24)$$

Where $L_{i,t}^e$ denotes the labor employed by firm $i$ at time $t$, $M_{i,t}$ is the money holdings of firm $i$ at time $t$ and $w_t$ denotes the nominal wage at time $t$.

The nominal price of goods at time $t$, denoted $p_t$, is taken as given by the firms. Consumers purchase goods with cash at the equilibrium price (implicitly, there is a cash in advance constraint on consumption as well).

As revenues are generated, firms decide whether to hold on to cash in order to be able to make use of it in the next period, or whether to distribute the cash as dividends. In equilibrium, within a period, money will be transferred back and forth between firms and households multiple times: first, firms will transfer their money to households as wage bills. Households will use their wage bills to buy
products; this will generate some revenue, which, given the initial high marginal utility of consumption, share holders will decide to distribute as dividends. At the end of the period, when the marginal utility of consumption of shareholders is sufficiently low, firms will decide not to distribute dividends and rather use the revenues as cash reserves.

**Menu costs.** I assume that changing prices is socially costly. In period $t+1$, the output cost of changing the equilibrium price from $p_t$ to $p_{t+1}$ is given by $\kappa(p_t, p_{t+1})$, where $\kappa(p_t, p_{t+1}) = \epsilon > 0$ for $p_t \neq p_{t+1}$ and $\kappa(p_t, p_{t+1}) = 0$ for $p_t = p_{t+1}$. I assume for simplicity that this cost is incurred by consumers in a lump-sum fashion at time $t$.

Assuming some form of menu costs is necessary in order to guarantee that the central bank’s optimal policy is to insure price stability. Otherwise, there are multiple equilibria.\[3\]

**Households.** The utility of households is given by:

$$U(c_0, c_1, c_2...) = \sum_{t=0}^{\infty} \beta^t u(c_t)$$  \hspace{1cm} (25)

Where $u(\cdot)$ takes the form:

$$u(c) = \frac{c^{1-\theta}}{1-\theta}$$  \hspace{1cm} (26)

I assume that $\beta \rho^{1-\theta} < 1$. This assumption will be necessary to assure that utility takes a finite value in the autarkic equilibrium (in which consumption equals output and hence grows at a rate $\rho$).

Each household owns one productive firm. I assume that the agent indexed $i$ owns the firm indexed $i$. Households consume their wage bill and their dividends, minus the cost of the price change:

$$p_t c_{i,t} = w_t L + d_{i,t} - \kappa(p_t, p_{t+1})$$  \hspace{1cm} (27)

Implicitly, it is assumed that household $i$ cannot supply its own labor to firm $i$. Otherwise, the cash in advance constraint would be nonsensical, as households could supply their own labor to the firms that they own and “pay themselves” later with dividends. The cash in advance constraint captures a reality in which production inputs are differentiated, and production requires the purchase of a variety of production inputs sold in a market setting.

\[3\]In the absence of a menu cost assumption, any sequence of money supply can achieve the first best level. This result changes if we assume that labor supply is elastic. In this case, in the absence of menu costs, the optimal sequence of money supplies implements the Friedman rule (Friedman [1969]).
The central bank. The central bank pre-commits to a sequence of money supplies, \((M_1, M_2, \ldots)\) (I assume that \(M_0\) is given). The aim of the central bank is to maximize household utility.

4 Closed Economy Equilibrium

In the closed economy equilibrium, liquidity constraints are binding in the sense that the marginal product of labor is higher than the real wage. This is because firms will agree to carry cash reserves only if there is a high enough return to money. The liquidity constraints will be more binding for economies that are growing at a faster rate. However, despite the fact that liquidity constraints may be very binding, output and consumption are still at their first best level.

Definition 1 An equilibrium of the closed economy is a sequence of prices \((p_0, p_1, \ldots)\), a sequence of rates of return on liquidity \((r_0, r_1, \ldots)\), a sequence of nominal wages \((w_0, w_1, \ldots)\), a sequence of employed labor \(\{(L^e_{i,0}, L^e_{i,1}, \ldots)\}_{i \in [0,1]}\)

\[\text{a sequence of outputs } \{Y_0, Y_1, \ldots\},\]

\[\text{a sequence of cash holdings } \{(Q^i_0, Q^i_1, \ldots)\}_{i \in [0,1]},\]

\[\text{a sequence of liquidity demands } \{(Q^d_0, Q^d_1, \ldots)\}_{i \in [0,1]},\]

\[\text{a sequence of firms’ initial money holdings } \{(M^i_0, M^i_1, \ldots)\}_{i \in [0,1]} \]

\[\text{a sequence of money supplies } (M_0, M_1, \ldots)\]

and a sequence of consumptions \(\{(c_{i,0}, c_{i,1}, \ldots)\}_{i \in [0,1]}\) that jointly satisfy the following conditions:

1. Given the wage and the price sequences, the consumption sequence, the labor sequence, and the cash sequence solve the optimization problem of agent \(i\):

\[
\max_{c_{i,t},L^e_{i,t},Q^i_{t+1},Q^d_{i,t}} \sum_{t=0}^{\infty} \beta^t u(c_{i,t})
\]

\[\text{s.t.} \quad p_t c_{i,t} = w_t L_i + d_{i,t} - \frac{\kappa(p_t, p_{t+1})}{p_t} \]

\[d_{i,t} = p_t F_i(L^e_{i,t}) + M_{i,t} - w_t L^e_{i,t} - Q_{i,t+1} - (1 + r_t)Q^d_{i,t} \]

\[w_t L^e_{i,t} \leq M_{i,t} \]

\[M_{i,t+1} = Q_{i,t+1} + Q^d_{i,t+1} + T_{t+1} \]

Where \(T_t = M_t - M_{t-1}\), and \(M_{0,i} = M_0\).

2. Goods market clearing condition:

\[Y_t = \int_0^1 \rho F_i(L^e_{i,t}) di = \int_0^1 c_{i,t} di + \kappa(p_t, p_{t+1}) \]
3. Labor market clearing condition:

\[ \int_{0}^{1} L_{e,i}^t \, di = L \tag{34} \]

4. Money market clearing condition:

\[ \int_{0}^{1} M_{i,t} \, di = M_t \tag{35} \]

5. The sequence \((M_1, M_2, \ldots)\) maximizes household utility.

It will be useful to compare the properties of this equilibrium to the first best of this economy, where the “first best” refers to the unconstrained benchmark without the liquidity constraint and without menu costs.

**Lemma 2** Consumption is at its first best level if and only if prices are stable.

**Proof:** If the liquidity constraint is not binding, each firm sets the marginal product of labor to be equal to the wage. If the liquidity constraint is binding, then each firm sets \( F_t^i(L_{i-1}^t) = 1 + r_t \). Thus, output is at its first best level, as the economy is at full employment and the marginal product of labor is equated across firms. If \( p_t = p_{t-1} \), then \( \kappa(p_t, p_{t+1}) = 0 \) and consumption is equal output; in this case, consumption is at its first best level. If \( p_t \neq p_{t-1} \), consumers have to incur some menu costs, so consumption is less than output, and hence less than the first best level.

A corollary of this lemma is that the central bank would like to choose \((M_1, M_2, \ldots)\) in a way that achieves price stability.

The following proposition characterizes the closed economy equilibrium:

**Proposition 1** There exists a unique equilibrium. In equilibrium, the liquidity constraint is binding for all \( t \): \( 1 + r^\text{aut}_t = \frac{p_t F_t^i(L)}{w_t} = \frac{\rho^\text{c}}{\beta} \).

The intuition is as follows. The consumer-shareholder’s Euler equation is given by:

\[ \frac{1}{p_t} u'(c_t) = \frac{\beta}{p_{t+1}} u'(c_{t+1}) \frac{p_{t+1} F_{t+1}^i(L)}{w_{t+1}} \tag{36} \]

The term \( \frac{p_{t+1} F_{t+1}^i(L)}{w_{t+1}} \) is the additional revenue generated in period \( t+1 \) from carrying over one more unit of currency. This unit of currency can hire \( \frac{1}{w_{t+1}} \) units of labor; each unit of labor produces at the margin a revenue of \( p_{t+1} F_{t+1}^i(L) \).
In equilibrium, the central bank chooses a sequence of money supplies that assures price stability. Thus, \( p_t = p_{t+1} \), and, by Lemma 2, \( c_t = F_t(L) \). Thus, the Euler equation can be rewritten as:

\[
u'(F_t(L)) = \beta u'(F_{t+1}(L)) \frac{p_{t+1}F'_{t+1}(L)}{w_{t+1}} (37)\]

From this, it is straightforward to conclude that firms are constrained in equilibrium, in the sense that the marginal product of labor is higher than the wage:

\[
\frac{p_{t+1}F'_{t+1}(L)}{w_{t+1}} = \frac{u'(F_t(L))}{\beta u'(F_{t+1}(L))} > 1
\]

(38)

Thus, firms are constrained in equilibrium and use their entire money holdings to hire labor.

Similar to the model in section 2, there is a positive relation between the growth rate of the economy and the equilibrium rate of return on liquidity. From equation 38:

\[
1 + r^{aut} = \frac{p_{t+1}F'_{t+1}(L)}{w_{t+1}} = \frac{u'(F_t(L))}{\beta u'(\rho F_t(L))} = \frac{F_t(L)^{-\theta}}{\beta(\rho F_t(L))^{-\theta}} = \frac{\rho^\theta}{\beta}
\]

(39)

Faster growing economies (higher \( \rho \)) are more liquidity constrained in the sense that the ratio of the marginal product of labor and the wage is higher. This is because higher output growth leads to higher consumption growth; the marginal utility of consumption therefore declines more rapidly, and firms require a higher rate of return on money to be willing to carry over money from one period to the next.

5 Integrated Equilibrium

Consider the following global equilibrium environment. There is a large “developed” economy that is at its steady state level of output (\( \rho = 1 \)). The central bank implements the optimal policy and chooses a constant price level. In this economy, the within period return to holding cash is given by (by equation 39 with \( \rho = 1 \)):

\[
1 + r = \frac{p_tF_t'(L)}{w_t} = \frac{1}{\beta}
\]

(40)

I normalize the price of goods in terms of international currency to be \( p = 1 \).

In this global environment, I assume a small open economy that grows at rate \( \rho > 1 \). The exchange rate is fixed at 1 and the central bank follows a 100% reserve ratio. Later I will show that abandoning the reserve requirement can increase equilibrium welfare. The initial supply of domestic currency at time 0 is \( M_0 \). At
time \( t \), the amount of domestic currency is determined by the demand for domestic currency given domestic wages and the fixed exchange rate. The price of goods in terms of the foreign currency is constant at \( p_t = 1 \). This implies that the economy incurs no menu costs due to price changes.

I assume that borrowing is limited in the following sense. Agents can borrow and lend only within a period; if agents borrow, they must repay whatever they borrow at the end of the period. This structure will imply that domestic firms will borrow for liquidity purposes, but households will not be able to borrow to smooth consumption across time.

Given the assumption that the price of goods is constant, the only feasible equilibrium is the constrained equilibrium, in which the between-periods return on holding cash is 0, and the within-period return on holding cash is \( 1 + r \):

\[
\frac{F'(L)}{w_t} = 1 + r
\]  

(41)

**Definition 2** An equilibrium of the integrated economy is a sequence of nominal wages \((w_0, w_1, \ldots)\), a sequence of employed labor \(\{(L_{i,0}, L_{i,1}, \ldots)\}_{i \in [0,1]}\), a sequence of outputs \((Y_0, Y_1, \ldots)\), a sequence of domestic cash holdings \(\{(Q_{i,0}, Q_{i,1}, \ldots)\}_{i \in [0,1]}\), a sequence of liquidity demands \(\{(Q^d_{i,0}, Q^d_{i,1}, \ldots)\}_{i \in [0,1]}\), a sequence of foreign liquidity supply \((Q^*_0, Q^*_1, \ldots)\) and a sequence of consumptions \(\{(c_{i,0}, c_{i,1}, \ldots)\}_{i \in [0,1]}\) that jointly satisfy the following conditions:

1. Given the wage sequence, the consumption sequence, the labor sequence, and the cash sequence solve:

\[
\max_{c_{i,t}, L_{i,t}, Q_{i,t+1}, Q^d_{i,t}} \sum_{t=0}^{\infty} \beta^t u(c_{i,t})
\]

s.t.

\[
c_{i,t} = w_t L_{i,t} + d_{i,t}
\]

(43)

\[
d_{i,t} = F_t(L_{i,t}^e) + Q_{i,t} + Q^d_{i,t} - w_t L_{i,t}^e - Q_{i,t+1} + (1 + r)Q^d_{i,t}
\]

(44)

\[
w_t L_{i,t}^e \leq Q_{i,t} + Q^d_{i,t}
\]

(45)

\( Q_{i,0} = M_0 \) is given.

2. Goods market clearing condition:

\[
Y_t = \int_0^1 \rho F_t(L^e_{i,t}) di = \int_0^1 c_{i,t} di + r Q^*_t
\]

(46)
3. Labor market clearing condition:

\[ \int_{0}^{1} L_{i,t}^e di = L \]

(47)

4. Liquidity market clearing condition:

\[ \int_{0}^{1} Q_{i,t}^d di = Q_t^* \]

(48)

**Proposition 2** For \( M_0 \) sufficiently small:

1. In the financially integrated economy, firms always hold 0 cash reserves.

2. Financial integration reduces equilibrium consumption: \( c_t < c_t^{aut} \) for all \( t \).

Note that firms will agree to hold cash only as long as:

\[ u'(c_t) \leq \beta u'(c_{t+1})(1 + r) = u'(c_{t+1}) \]

(49)

It turns out that this condition is violated in equilibrium, as the economy experiences positive consumption growth which it is unable to smooth. Firms therefore choose to hold 0 cash reserves, and are always constrained in equilibrium. As in the simple example in section 2, financial integration creates a dependence on foreign liquidity supply as the wage bill appreciates. In this dynamic setting, there is an additional effect: not only are wages higher, but the equilibrium choice of cash reserves is lower, further increasing the reliance of foreign liquidity supply and the equilibrium rents absorbed by foreign liquidity suppliers.

As in the simple example, the equilibrium inefficiency results from the fact that constrained entrepreneurs do not internalize the effects of their borrowing decisions on the equilibrium wage. From the perspective of each firm, the wage is depressed so borrowing from foreigners is profitable. However, collectively, firms would be better off if borrowing from foreigners was restricted. Dividends would be higher, both because wages would be lower and because there would be no payment to foreign liquidity suppliers.

From the worker’s perspective, financial integration is beneficial, as it increases the real wage. In this model, the negative effect on dividends more than offsets this effect, resulting in overall lower consumption. This model therefore suggests that financial integration is associated with a redistribution of surplus from capital owners (firms) to workers, that cannot be corrected by redistribution.
6 Global Imbalances

In the integrated equilibrium, despite the fact that agents would like to borrow, the central bank accumulates foreign reserves over time, and the economy runs a trade deficit.

The current account is given by:

\[ CA_t = (1 + r)Q_t^* - Q_t^* = rQ_t^* \]  

(50)

Note that:

\[ \frac{F_t'(L)}{w_t} = 1 + r \Rightarrow \frac{F_t'(L)}{1 + r} = w_t \Rightarrow \frac{F_t'(L)L}{1 + r} = w_tL \]  

(51)

As foreign liquidity supply finances the entire wage bill, by equation 51,

\[ w_tL = Q_t^* \Rightarrow \frac{F_t'(L)L}{1 + r} = w_tL = Q_t^* \]  

(52)

The current account in period \( t \) is therefore:

\[ CA_t = \frac{rF_t'(L)L}{1 + r} \]  

(53)

Assuming the functional form \( F_t = AL^\alpha \), we have that \( F_t'(L)L = \alpha F_t(L) \).

Rewriting the equation above,

\[ CA_t = \frac{r\alpha F_t(L)}{1 + r} \Rightarrow \frac{CA_t}{F_t(L)} = \frac{r\alpha}{1 + r} \]  

(54)

To calibrate the current account surplus implied by this equation, I choose \( r \) to be equal to the US prime rate (taken from the St. Luis Fed’s FRED database) plus 3%. This choice is roughly consistent with the return on risky productive lending. The choice of \( \alpha \) is more tricky: \( \alpha \) need not be interpreted necessarily as the labor share, but rather as the share of inputs that must be purchased in advance of production. I choose \( \alpha = 0.6 \), based on estimates of the credit share in production in Evans et al. [2002].

Figure 1 plots the current account predicted by the model and the actual Chinese current account between the years 1990 and 2010 (both as a percent of GDP). The model predicts a current account surplus of between 2%-6%. This range falls well within the range of the actual Chinese current account, which is between -2%

\[^4\text{Evans et al. 2002 use a panel of 82 countries covering 21 years to estimate a translog production function, and find the share of credit to be around 0.6. An alternative estimate can be found in Khan and Ahmad 1985, that estimates the share of money in the production function to be 0.43 in the manufacturing sector in Pakistan.}\]
Figure 1: The Chinese current account as a percent of GDP.
Figure 2: The Chinese current account as a percent of GDP: first differences on an annual frequency.
and 10%. It is possible that the larger range can be accounted for by time varying risk premia on Chinese short-term borrowing.\footnote{See Broner et al. [Forthcoming] for evidence that time varying risk premia plays an important role in determining emerging market borrowing rates. This mechanism may potentially help account for the large discrepancy between the model and the data during the recent crisis, in which interest rates were held very low, but firms experienced difficulty acquiring credit. The short term borrowing rate faced by Chinese firms may be badly proxied by the US prime lending rate.} Note further that, on a yearly frequency, the increases in the current account predicted by the model roughly coincide with those observed in the data (see figure 2). Of course, on a lower frequency, the relationship seems to be failing as the Chinese current account is increasing while the current account predicted by the model is slightly downward trending.

6.1 Optimal Policy

Interestingly, the central bank accumulates foreign reserves, despite the fact that domestic consumers would like to borrow. To see this, note that the demand for domestic currency is increasing over time: the demand for domestic currency at time $t$ is given by $Q_t^r$, which, from the analysis above, is increasing at a rate $\rho$. The central bank therefore has to print new currency every period, and accumulates foreign reserves.

Note that the analysis above was done under the assumption that the central bank must follow a fixed exchange rate regime with a reserve ratio of 1. A central bank which lacks credibility and is vulnerable to attacks will therefore be forced to accumulate cash reserves. The central bank may choose to use these cash reserves to buy liquid low-risk assets such as treasuries; the interest on treasuries can be distributed to domestic agents immediately without compromising the reserve ratio. Will the treasuries ever be redeemed? This depends on whether the central bank continues to be vulnerable to attacks forever. It is possible that over time, the central bank will acquire credibility and will no longer need to back its currency with the international currency. At that point, the central bank will find it optimal to stop accumulating bonds. In this sense, the US current account deficit can be seen as “unsustainable”. If, at some point, the Chinese government acquires sufficient credibility to have an unbacked currency, the demand for US treasuries will decline.

Two questions arise: first, can the central bank increase welfare by abandoning the commitment to a 100% reserve ratio? Second, can the central bank further improve welfare by moving to a flexible exchange rate regime?

The answer to the first question is yes: the central bank can increase welfare if it is able to commit to a fixed exchange rate without a 100% reserve ratio. To
see this, note that given a fixed exchange rate, the amount of domestic currency at time \( t \) must be equal to the demand for domestic currency in the 100% reserve ratio case analyzed above, denoted \( \hat{Q}_t^* \):

\[
\hat{Q}_t^* = \frac{F_t'(L)L}{1+r}
\]

(55)

The amount of domestic currency therefore must grow at a rate \( \rho \). Assume that the only restriction on the central bank is that it must have a positive reserve ratio (it cannot hold a negative amount of foreign currency). At the end of period \( t \), foreigners hold \( \hat{Q}_t^* \) units of domestic currency. At the beginning of time \( t + 1 \), the central bank can improve welfare by distributing the new units of currency directly to domestic firms, without backing it by foreign reserves.

Firms will receive \( \hat{Q}_{t+1}^* - \hat{Q}_t^* \) units of currency, thereby absorbing a fraction \( \frac{\rho - 1}{\rho} \) of the rents to liquidity supply at time \( t + 1 \). By following this policy every period, the government can transfer \( \frac{\rho - 1}{\rho} \) of the rents to liquidity supply to domestic agents. Note that this policy is consistent with a fixed exchange rate, as all equilibrium prices and rates of return replicate the 100% reserve ratio case analyzed in the previous section. Foreigners are therefore still willing to trade one unit of domestic currency for one unit of foreign currency.

Note that this policy is consistent with a fixed exchange rate, as all equilibrium prices and rates of return replicate the 100% reserve ratio case analyzed in the previous section. Foreigners are therefore still willing to trade one unit of domestic currency for one unit of foreign currency.

Note that under this policy, the reserve ratio will approach 0 as \( t \to \infty \). To see this, note that if the central bank begins to implement this policy at time 1, the foreign reserves at any time \( t \) remain at \( M_0 \), so the reserve ratio is:

\[
RR = \frac{M_0}{\hat{Q}_t^*} = \frac{M_0}{\frac{F_t'(L)L}{1+r}} = \frac{M_0}{\frac{\rho F_t'(L)L}{1+r}} \to_{t \to \infty} 0
\]

(56)

Thus, a central bank trying to improve welfare by following this policy must be sufficiently credible to be able to sustain a fixed exchange rate with effectively a 0 reserve ratio.

Can the central bank improve welfare further by abandoning the fixed exchange rate regime and moving to a flexible exchange rate regime? Unfortunately, no. The reason is as follows. For foreigners to be willing to hold a finite amount of domestic currency between the end of period \( t \) and the beginning of period \( t + 1 \), it must be the case that the exchange rate remains the same between these two dates. If it is expected that the central bank will depreciate the currency, nobody will hold domestic currency, as it is expected that it will be cheaper to purchase domestic currency at the beginning of the next period. Similarly, if it is expected that the central bank will appreciate the currency, everyone will want to hold domestic currency. The exchange rate at the beginning of period \( t \) therefore must be equal to the exchange rate at the end of the previous period.

Similarly, it must be the case that the within-period return on each unit of domestic currency is \( 1 + r \), by the international no arbitrage condition. Thus, if
foreigners hold $Q^*_t$ units of domestic currency at the beginning of time $t + 1$, by the end of period $t + 1$ the economy must pay at least $(1 + r)Q^*_t$ units of output as rents to foreigners. The best the central bank can do is therefore replicate the allocation of the fixed exchange rate regime with no reserve requirement, and distribute the new units of domestic currency directly to firms.

### 7 Instability

As the wage bill increases following financial integration, the economy becomes increasingly reliant on foreign liquidity supply. Domestic firms are no longer able to afford the wage bill with their own money holdings. In the presence of some wage stickiness, this means that the economy is particularly vulnerable to shocks to foreign liquidity supply.

I augment the model to allow for wage stickiness. Assume that at the time of financial integration, employers and workers contract on a sequence of wages, $\hat{w}_t, \hat{w}_{t+1}, \ldots$. The contracted wages are determined by the “no shock” equilibrium described in section [5]. Thus, the wage $\hat{w}_t$ is characterized by the following international no-arbitrage condition, that takes into account full employment of labor:

$$\frac{F_t'(L)}{\hat{w}_t} = 1 + r$$  \hspace{1cm} (57)

Assume that there is a single period shock to the foreigners’ willingness to supply liquidity that leads the within-period borrowing rate to jump from $r$ to $r' > r$.

As wages are sticky, this increase in the interest rate necessarily implies a drop in production. The international no arbitrage condition is now given by:

$$\frac{F_t'(L_e)}{\hat{w}_t} = 1 + r' > 1 + r = \frac{F_t'(L)}{\hat{w}_t} \Rightarrow L_e < L$$  \hspace{1cm} (58)

Thus, unemployment increases and output drops. Consumption at time $t$ drops as well:

**Lemma 3** *Consumption drops following an increase in $r$.*

It is easy to see that $Q_{i,t+1} = 0$, as borrowing conditions at $t + 1$ are expected to go back to normal; firms therefore have no incentive to save, and will choose to hold 0 cash reserves. Thus, consumption returns back to the no-shock equilibrium benchmark. As consumption decreases in period $t$ and returns to normal thereafter, it can be concluded that the shock to the foreigners’ willingness to lend reduces equilibrium welfare.

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6This is potentially a realistic concern, as the literature suggests that foreign liquidity supply to emerging economies is highly volatile (see Fostel and Geanakoplos [2008] and Eden [2011a] for models).
“Beggar thy neighbor” policies. Given a shock to the foreigners willingness to lend, the government may have an incentive to depreciate the currency in order to boost employment. To see this, note that the domestic return to a unit of foreign currency is determined by the exchange rate $e_t$. A unit of labor costs $\frac{w_t}{e_t}$ units of foreign currency. The return to foreign currency is therefore equal to:

$$\frac{F'_{t}(L^e_t)}{\frac{w_t}{e_t}} = e_t \frac{F'_{t}(L^e_t)}{w_t}$$

(59)

Consider a government that sets the exchange rate at $e_t$. Employment is then given by the following international no arbitrage condition:

$$1 + r' = e_t \frac{F'_{t}(L^e_t)}{\hat{w}_t}$$

(60)

Lemma 4 Consumption at time $t$ is maximized when $e_t$ is chosen such that full employment is restored.

Given a crisis, the central bank faces a tradeoff: on the one hand, a depreciation may compromise the central bank’s credibility. This is problematic as there are welfare gains from the ability to institute a currency that is not backed by foreign reserves. On the other hand, during a crisis, there is an incentive to renege on the promise to hold the exchange rate fixed, as a depreciation can help restore full employment.

8 Conclusion

The presence of binding liquidity constraints implies a transfer of surplus to liquidity suppliers. In the closed economy, liquidity is supplied domestically so the presence of liquidity constraints is welfare-neutral. However, in the integrated equilibrium, binding liquidity constraints in a fast-growing economy imply a transfer of surplus to foreign liquidity suppliers, thereby reducing domestic equilibrium welfare.

From a policy perspective, this model suggests that emerging economies have an incentive to discourage foreign liquidity supply, even if domestic entrepreneurs are heavily constrained. Opening to capital flows will increase the supply of liquidity, but this will only bid up the domestic input prices such as labor and land. This, in turn, will make the domestic firms less liquidity constrained but more heavily reliant on foreign liquidity supply. In a dynamic equilibrium, the lower return on holding liquidity will imply that firms choose not to hold sufficient cash reserves to finance their expenses, and rely heavily on foreigners to supply liquidity.
The integrated economy is therefore highly vulnerable to shocks to foreign liquidity supply. In the presence of sticky input prices, a sudden shock to foreigner’s willingness to supply liquidity will result in an increase in unemployment and a drop in output. The government therefore has an incentive to depreciate the domestic currency, thereby providing a higher dollar rate of return and encouraging foreign liquidity supply.

This temptation to depreciate the currency creates a certain tension. A central bank that is sufficiently credible to maintain a fixed exchange rate that is not vulnerable to attacks can increase domestic welfare, as it can distribute new currency directly to domestic firms without backing it by foreign reserves. As long as this is done in a manner consistent with the fixed exchange rate, it allows domestic firms to absorb some of the rents to liquidity supply. A shock to foreigner’s willingness to supply liquidity therefore creates a tension between the short term gains of a depreciation and the long term gains of central bank credibility.

An important thing to note is that, in this model, the driver of global imbalances is the differences in growth rates: the fast-growing economy is liquidity constrained in equilibrium, and the steady-state developed world is the equilibrium liquidity supplier. This suggests that global imbalances are a temporary phenomenon: once the emerging markets converge to the steady state growth level, we should see a balanced current account. This suggests that, from the developed market’s perspective, the rents from supplying liquidity to emerging economies should be viewed as a “temporary” source of income.

References


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A Proof of Lemma 1

If the liquidity constraint is not binding, firms set the marginal product of labor equal to the wage. As in the unconstrained benchmark, the only rate of return on liquidity that is consistent with market clearing is \( r_{aut} = 0 \).

Assume next that the liquidity constraint is binding, that is, the weak inequality in equation 7 holds with equality:

\[
wl_i = Q + Q^d_i \tag{61}
\]
The firm’s optimization problem (equation 6) can be rewritten as:

\[
\max_{Q_i^d} \rho F\left(\frac{Q + Q_i^d}{w}\right) - (1 + r)Q_i^d
\]  

(62)

The first order condition with respect to \(Q_i^d\) is:

\[
\frac{F'(L_i)}{w} = 1 + r
\]  

(63)

It follows that \(r > 0\): if \(r = 0\), the marginal product of labor is equated with the wage and the liquidity constraint is not binding. If \(r < 0\), the marginal product of labor exceeds the wage and this is never optimal.

It is left to show that when \(r^{aut} > 0\), the liquidity constraint is binding. This follows from the fact that if the liquidity constraint is not binding, \(r^{aut} = 0\) is the only return consistent with market clearing in the liquidity market.

B Proof of Proposition 1

In equilibrium, firms must be indifferent, given the price level, between holding on to cash and and distributing cash as dividends. This can imply one of two equations, depending on whether the liquidity constraint is binding at \(t + 1\). I will consider first the situation in which the liquidity constraint is binding. Substituting in \(w_{t+1}L_{t+1} = Q_{t+1} + T_{t+1}\), the Euler equation implies:

\[
\frac{1}{p_t} u'(c_t) = \beta \frac{1}{p_{t+1}} u'(c_{t+1}) \frac{p_{t+1}F'_{t+1}(L_t)}{w_{t+1}}
\]  

(64)

The left hand side is the marginal utility of households (share holders) generated from receiving an additional unit of currency in this period. An additional unit of currency can purchase \(\frac{1}{p_t}\) consumption goods, and the marginal utility of consumption is \(u'(c_t)\). The right hand side represents the marginal utility of households generated from having firms hold an additional unit of cash as reserves. With an additional unit of cash, firms can hire \(\frac{1}{w_{t+1}}\) more workers. Each worker generates an additional nominal revenue of \(p_{t+1}F'_{t+1}(L_t)\). Each unit of cash in period \(t + 1\) can finance \(\frac{1}{p_{t+1}}\) units of consumption, and the marginal utility of consumption is \(u'(c_{t+1})\). The marginal utility of consumption next period is discounted by \(\beta\).

I conjecture that there exists a sequence \((M_1, M_2, ... )\) that guarantees price stability. Under this conjecture, by Lemma 2, output is equal consumption and is at its first best level. Substituting in the market clearing condition, \(c_t = Y_t\), and the condition that output is at its first best level,
\[
\frac{1}{p_t} u'(Y_t) = \beta u'(Y_{t+1}) \frac{F'_{t+1}(L)}{w_{t+1}} = \beta u'(\rho Y_t) \frac{\rho F'(L)}{w_{t+1}}
\]  
(65)

Substituting in the functional form for \( u(\cdot) \), after some simple algebraic manipulations:
\[
p_t F'_{t+1}(L) = \rho \theta 
\]  
(66)

Note that in this environment, \( w_{t+1} = \frac{M_{t+1}}{L} \). Thus,
\[
p_t = \frac{\rho^\theta M_{t+1}}{\beta F'_{t+1}(L)L} = \frac{\rho^\theta M_{t+1}}{\beta \rho F'(L)L}
\]  
(67)

Thus, for price stability, \( p_t = p_{t+1} \), and \( M_{t+1} = \rho M_t \). Thus, the unique price sequence \( M_t = \rho^t M_0 \) guarantees price stability, and, from Lemma 2, this will be the central bank’s choice.

I have proved therefore that there is a unique equilibrium in which the liquidity constraint is binding. It is left to show that there are no additional equilibria in which the liquidity constraint does not bind. If the liquidity constraint is not binding at \( t + 1 \), the Euler equation implies:
\[
\frac{1}{p_t} u'(c_t) = \frac{1}{p_{t+1}} \beta u'(c_{t+1})
\]  
(68)

First, I show that given the price sequence \( M_t = \rho^t M_0 \), the unique equilibrium is the constrained equilibrium. Then, I show that any unconstrained market equilibrium is one in which welfare is suboptimal. I conclude that only way that the central bank can implement the first best outcome is by choosing the money supply sequence \( M_t = \rho^t M_0 \). In this case, there is a unique equilibrium in which consumption is at its first best.

Assume therefore that \( M_t = \rho^t \). Assume in the way of contradiction that there exists an unconstrained equilibrium in which there is price stability. Substituting in the market clearing condition \( c_t = Y_t \), and using the expressions for utility and output growth, the following relation emerges:
\[
\frac{p_{t+1}}{p_t} = \frac{\beta}{\rho^\theta}
\]  
(69)

Thus, price stability must be violated. It is left to show that there is no market equilibrium in which there is no price stability. Assuming no price stability, the market clearing condition implies that:
\[
c_t = Y_t - \kappa(p_t, p_{t-1}) = Y_t - \epsilon
\]  
(70)
Thus, from the Euler equation,

\[ \frac{1}{p_t} u'(\rho^t F_0(L) - \epsilon) = \frac{\beta}{p_{t+1}} u'(\rho^{t+1} F_0(L) - \epsilon) \]  

(71)

\[ \frac{p_{t+1}}{p_t} = \beta \frac{u'(\rho^{t+1} F_0(L) - \epsilon)}{u'(\rho^t F_0(L) - \epsilon)} \]  

(72)

Denote:

\[ \theta = \beta \frac{u'(\rho^{t+1} F_0(L) - \epsilon)}{u'(\rho^t F_0(L) - \epsilon)} \]  

(73)

Note that \( \theta < 1 \) and \( p_{t+1} = \theta p_t \). Thus, real money balances \( (\frac{M^t}{p^t}) \) grow at rate \( \frac{\rho}{\theta} \), whereas equilibrium consumption, for \( t \) large, grows at a rate equal approximately to \( \rho \). This is in violation of the transversality condition: agents accumulate money reserves that they never use. From an individual standpoint, they can do better by consuming more in every period and holding less money reserves. Thus, an unconstrained equilibrium in which money grows at rate \( \rho \) does not exist.

It is left to show that no unconstrained equilibrium can achieve the first best level of consumption. To see this, note that by equation 69, any unconstrained equilibrium in which output is at its first best would require price deflation. By Lemma 2, an economy with price deflation cannot be at its first best.

\section*{C Proof of Proposition 2}

Whether or not firms are borrowers or lenders at \( t = 0 \) depends on their liquidity endowment. Specifically, firms are lenders if the following condition holds:

\[ \frac{F'_0(L)}{(w_0 = \frac{M_0}{L})} > 1 + r \]  

(74)

Rewriting this condition,

\[ \frac{F'_0(L)L}{M_0} > 1 + r \]  

(75)

This condition will be true for \( M_0 \) sufficiently low. Note that here the liquidity endowment \( M_0 \) has a real interpretation: the liquidity endowment is the amount of foreign goods that domestic agents can purchase at time 0. For the firms to be net borrowers, this must be small in comparison to the value of international goods that they can produce. I am going to assume throughout that this is the case. Note that this condition holds in the autarkic constrained equilibrium\(^7\).

\(^7\)In the autarkic constrained equilibrium, by equation \( 39 \), \( \frac{p_t F'_0(L)}{w_t} = \frac{\rho^t}{\rho^t} \). As \( 1 + r = \frac{1}{\theta} \), \( \frac{p_t F'_0(L)}{w_t} = \rho^t(1 + r) > 1 + r \)
Denote by $Q^*_t$ the amount of foreign borrowing of firms at time $t$. Consumption at time $t$ is given by:

$$c_t = w_tL + d_t = w_tL + F_t(L) + Q_t - w_tL - rQ^*_t - Q_{t+1}$$  \hspace{1cm} (76)

$$c_t = F_t(L) + Q_t - rQ^*_t - Q_{t+1}$$  \hspace{1cm} (77)

What is $Q^*_t$? If the liquidity constraint binding, $Q^*_t + Q_t = w_tL$. Thus, $Q^*_t = w_tL - Q_t$. Consumption can be rewritten as:

$$c_t = F_t(L) + (1 + r)Q_t - rw_tL - Q_{t+1}$$  \hspace{1cm} (78)

Note that:

$$\frac{F'(L)}{w_t} = (1 + r) \Rightarrow \frac{F'(L)}{1 + r} = w_t \Rightarrow \frac{F'(L)L}{1 + r} = w_tL$$  \hspace{1cm} (79)

Thus,

$$c_t = F_t(L) + (1 + r)Q_t - \frac{rF'(L)L}{1 + r} - Q_{t+1}$$  \hspace{1cm} (80)

For $t = 0$, consumption is maximized for $Q_1 = 0$. I conjecture that $Q_t = 0$ for every $t$ is an optimal choice whenever $Q_0 = M_0$ is sufficiently low. Under this conjecture,

$$c_1 = F_1(L) - \frac{rF'(L)L}{1 + r} = \rho(F_0(L) - \frac{rF'(L)L}{1 + r})$$  \hspace{1cm} (81)

For $M_0$ sufficiently small,

$$c_0 = F_0(L) + (1 + r)M_0 - \frac{rF'(L)L}{1 + r} \leq \rho(F_0(L) - \frac{rF'(L)L}{1 + r}) = c_1$$  \hspace{1cm} (82)

Thus, $u'(c_0) > u'(c_1)$, so firms are choosing optimally not to hold any cash and to distribute all sales revenues as dividends. For periods $t \geq 1$, not holding cash continues to be optimal, as consumption is given by:

$$c_t = F_t(L) - \frac{rF'(L)L}{1 + r} < \rho(F_t(L) - \frac{rF'(L)L}{1 + r}) = c_{t+1}$$  \hspace{1cm} (83)

Thus, in equilibrium, firms hold 0 cash reserves in all period, so the second part of the proposition is proven.

The first part of the proposition, that consumption is lower for all $t$, is trivial for $t \geq 1$, as:

$$c_t = F_t(L) - \frac{rF'(L)L}{1 + r} < F_t(L) = c^\text{aut}_t$$  \hspace{1cm} (84)

For $t = 0$, this is true for a sufficiently small $Q_0$:

$$c_0 = F_0(L) + (1 + r)Q_0 - \frac{rF'(L)L}{1 + r} \leq F_0(L) = c^\text{aut}_0$$  \hspace{1cm} (85)
D Proof of Lemma 3

Assume that $r' > r$, and that the wage is fixed at its “no shock” level $\hat{w}_t$ such that \( \frac{F'_t(L)}{\hat{w}_t} = 1 + r \). Consumption after the shock is given by:

\[
c_t = w_tL_t^e + F_t(L_t^e) - w_tL^e - Q_{t+1} - r'Q_t^e \leq F_t(L_t^e) - r'Q_t^e
\]

\[
= F_t(L_t^e) - r'\hat{w}_tL_t^e < F_t(L_t^e) - r\hat{w}_tL_t^e
\]

Consider the function:

\[
\phi(l) = F_t(l) - r\hat{w}_t l
\]

Thus, we have that:

\[
c_t < \phi(L_t^e)
\]

The maximum value that the function $\phi(\cdot)$ takes is at $l^{\text{max}}$, that is determined by the first order condition:

\[
\phi'(l) = 0 \implies F'_t(l^{\text{max}}) = r\hat{w}_t
\]

Note further that $\phi''(l^{\text{max}})$ is negative:

\[
\phi''(l^{\text{max}}) = F''_t(l^{\text{max}}) - r\hat{w}_t < 0
\]

Thus, for $l < l^{\text{max}}$, $\phi(\cdot)$ is increasing in $l$. Note that $L < l^{\text{max}}$, as:

\[
F'_t(L) = (1 + r)\hat{w} > r\hat{w} = F'_t(l^{\text{max}}) \implies L < l^{\text{max}}
\]

As $L > L_t^e$, both $L$ and $L_t^e$ are less than $l^{\text{max}}$. As $\phi(\cdot)$ is increasing in this range,

\[
c_t < \phi(L_t^e) < \phi(L) = F_t(L) - r\hat{w}L = \hat{c}_t
\]

Where $\hat{c}_t$ is the level of consumption that domestic consumers would have in the absence of the shock (that is, if the $r$ was at its expected level).

E Proof of Lemma 4

Recall that the international no arbitrage condition is given by:

\[
1 + r' = e_t \frac{F'_t(L_t^e)}{\hat{w}_t}
\]

Note that a higher value of $e_t$ corresponds to higher employment. Let $\bar{r}$ be given by:

\[
1 + \bar{r} = \frac{1 + r'}{e_t}
\]
By choosing $e_t$, the government can control $\tilde{r}$. Note that this is the real rate of return that firms have to pay liquidity suppliers in equilibrium:

$$1 + \tilde{r} = \frac{F_t'(L_t^e)}{\hat{w}_t} \quad (96)$$

Consumption is given by:

$$c_t = F_t(L_t^e) - \tilde{r}Q_t^* = F_t(L_t^e) - \tilde{r}\hat{w}_t L_t^e$$

$$= F_t(L_t^e) - \left(\frac{F_t'(L_t^e)}{\hat{w}_t} - 1\right)\hat{w}_t L_t^e = F_t(L_t^e) - F_t'(L_t^e)L_t^e + \hat{w}_t L_t^e \quad (97)$$

Define the function $\psi(\cdot)$ as:

$$\psi(l) = F_t(l) - F_t'(l)l + \hat{w}_t l$$

The derivative of $\psi(\cdot)$ with respect to $l$ is always positive:

$$\psi'(l) = F_t''(l)l - F_t''(l)l - F_t''(l) + \hat{w}_t = -F_t''(l)l + \hat{w}_t > 0 \quad (100)$$

Thus, it is optimal for the government to set $e_t$ such that the economy is at full employment.