

03/29/2001

Adjusting One's Standard of Living: Two-Period Models

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Foreword by PD

In the summer of 1967, Jim and I began collaborating on optimal taxation. Our paper was essentially finished that summer, although it was a number of years before it appeared in print. Since then, we have collaborated continuously and have published eight more papers plus one forthcoming. At one point, we contemplated writing a graduate text. We got as far as a table of contents and an allocation of who would write the first version of each chapter – for our working style, after initial results, was to alternate writing complete drafts. But, neither of us was really interested in taking on a task of the magnitude needed to write a book we would be satisfied with, and the project had a mercifully quick ending.

Collaboration with Jim has been a source of great pleasure for me, as well as a way to write papers that, I believe, are different from what either of us would have done alone. I can also attest to the positive impact on papers that I wrote by myself. Whether in Cambridge, Oxford, Cambridge, or other locales, such as the Mull of Kintyre, I always relished being with Jim, with work on the papers being both an end and a justification for getting together. As much as possible, these meetings were family affairs, made far better by the presence of Gill and Kate, not to mention Catriona, Fiona, Matt and Andy.

Almost all of the papers we started we have finished. However, there are a few incomplete papers in our file drawers. I have resurrected one that was last revised in March, 1982 and completed it as my contribution for this occasion – I hope Jim doesn't object to the revisions and additions. While there has been relevant literature in the interim, the issue addressed still seems to me important, indeed pressing, as both research and legislation address social security reform. I have not attempted to bring the references in the paper fully up-to-date.

1. Introduction

¹ We are grateful to Mike Whinston, Saku Aura and Tom Davidoff for research assistance and to the Social Security Administration and the National Science Foundation (under grant SBR-9618698) for research support.

Because of the tractability it provides, use of an intertemporally additive utility function is widespread. It is used for abstract theoretical work, and it is used for simulations. Yet, we know that preferences are not intertemporally additive; tastes are affected by experience. We know this for individual products, where experience with a good can add to or subtract from later enjoyment of the same good. And we know this for consumption in the aggregate. The way in which savings provide for consumption late in life should accordingly allow for the influence of earlier consumption levels on later “needs.” Thus, we explore simple forms of intertemporal connection that would capture some of the intertemporal interaction and yet remain tractable. We draw a distinction between two simple forms of interaction, and then ask straightforwardly with what further assumptions some results that assumed additivity of preferences carry over to these simple forms. The results examined are both about individual savings and the design of an optimal social security system for a representative agent in the presence of asymmetric information about the ability to continue work in the range of ages we consider early retirement.

In addition to assuming that preferences are intertemporally additive, the common formulation also assumes that preferences are the same in every period. Like additivity, this assumption, is clearly wrong. However, we chose to continue to use this assumption, not examining how to formulate plausible age-varying preferences.

Although we do not explore it in detail, the models suggest a route into modeling individual savings where people are not successfully optimizing. Since widespread inadequate savings for retirement is seen by some as a justification for the existence of mandatory social security systems, modeling savings is an important question. Simulations of the effects of social security that omit justifications for the existence of the program are bound to find it costly and without benefit. But this is an inadequate basis for simulations. The approach we took to nonoptimizing behavior was simply to assume that people ignored the intertemporal link between current consumption and the future marginal utility of consumption, even though they correctly forecast future marginal utility. We have not explored any justification for this assumption, but merely followed convenient mathematics. This is a less satisfactory approach than building on psychological insights about intertemporal decisions, such as the work on hyperbolic and quasi-hyperbolic discounting (Ainslie 1992, Laibson, 1997), an approach which I am applying to retirement issues together with Botond Koszegi (1999).

We introduce a simple model of savings, in which consumption affects a variable we call the “standard of living”, which in turn affects the utility of future consumption. This approach fits with the common vocabulary of thinking about pensions in terms of replacement rates. Using a two period certainty model,

we examine two different simple ways that the standard of living could enter the instantaneous utility function, and then compare savings behavior of naïve and sophisticated savers, differing in their perception of the link between present consumption and future instantaneous utility (Section 2). In Section 3, we examine savings under certainty. In Section 4 we examine savings under uncertainty to relate this model to the analysis of H. Leland (1968). In Section 5, the analysis is applied to earnings uncertainty. In Section 6, we summarize our earlier work on social insurance with uncertainty only in the second period (1978, forthcoming) and relate the standard-of-living models to the assumptions used there. In Section 7, we consider an alternative model where individuals face uncertainty in both periods, examining the optimal wage path assuming that both moral hazard constraints are binding. In Section 8, we consider the question of when both moral hazard constraints do bind. In section 9, concluding remarks consider further research opportunities.

Although it has been common to assume an additive intertemporal utility function, there are previous analyses that have also sought simple generalizations, as we do. G. Heal and H. Ryder (1973) have examined an optimal growth model using the assumptions we describe as the addiction model. P. Samuelson (1971) has extended the turnpike theorem to a similar case. H. Houthakker and L. Taylor (1970) have done similar analysis for the demand for durables. In alternative formulations, M. Kurz (1968) included wealth in the instantaneous utility function while S. Chakravarty and A. Manne (1968) included the rate of growth of consumption. Our treatment of naïve savers is related to the analysis of changing preferences of C. Weizsacker (1971) and R. Pollak (1970). There is now a sizable literature directly addressing addiction, both with and without quasi-hyperbolic discounting (Becker and Murphy, 1988, Gruber and Koszegi, 1999).

2. Modeling the Standard-of-Living Effect

We start by considering individual savers with lifetime utility functions, U , defined over first and second period consumptions (c_1 and c_2). (For the present we ignore the question of labor supply.)

$$(1) \quad U(c_1, c_2) = u(c_1, s_0) + \delta u(c_2, s_1),$$

where δ is the utility discount factor and s_i the standard of living to which the individual has become accustomed by the end of period i . We assume that s_0 is given and think of it as the standard of living at

age 40, with each period covering 15 to 20 years.² We assume that the accustomed standard of living adjusts according to the equation ($\alpha > 0$).

$$(2) \quad s_1 = \frac{s_0 + \alpha c_1}{1 + \alpha}.$$

For a well-behaved choice problem once we extend the model to take expected values, we assume that U is concave in both variables. A sufficient condition for this would be the concavity of u in both variables, which we also assume. We naturally assume a positive instantaneous marginal utility of consumption, $u_c > 0$.

To reflect on additional assumptions, we distinguish two separate cases. One incorporates the view that it is "relative consumption" that matters. In this case the greater the standard of living to which the individual has become accustomed, the lower the level of instantaneous utility and the greater the marginal utility of consumption:

$$(3) \quad u_s < 0, \quad u_{cs} \geq 0.$$

Pure examples of a relative consumption view (with v concave and increasing) are:

$$u = v(c-s)$$

$$u = v(c/s)$$

We refer to the assumptions in (3) as the "addiction" model.

As an alternative approach, we can consider individuals who adapt their habits (spending patterns) to the expenditure level they have become accustomed to. Such individuals have greater instantaneous utility from long-run adjustments to a changed expenditure level than from short-run changes. This idea is shown in pure form in Figure 1, depicting the short-run and long-run utility functions. In this case, the plausible assumptions are

$$(4) \quad (s - c)u_s \leq 0, u_{cs} \geq 0, u_s(c, c) = 0.$$

² We do not incorporate any pure age effects in the analysis.

Pure examples satisfying these assumptions are:

$$u = v(c - s) + sv'(0)$$

$$u = v(c/s) + v'(1) \log s$$

where v' is the derivative. We refer to this model as the “habit” model.

With both models it seems reasonable to add the condition that consumption is more important than standard of living in determining marginal utility.

$$(5) \quad u_{cc} + u_{cs} \leq 0$$

3. Savings under Certainty

Consumer choice for a two-period model can be stated as

$$(6) \quad \text{Max } u(c_1, s_0) + \mathbf{d}u\left(c_2, \frac{s_0 + \mathbf{a}c_1}{1 + \mathbf{a}}\right)$$

s.t. $c_1 + c_2 / r = A$,

where r is one plus the interest rate.

The first order condition for individual choice is

$$(7) \quad u_c(1) + \frac{\mathbf{d}\mathbf{a}}{1 + \mathbf{a}} u_s(2) = \mathbf{d}r u_c(2),$$

where $u_c(1)$ means the instantaneous marginal utility of consumption evaluated at period 1 values (c_1, s_0) .

Differentiating the first order condition, we can express the income derivative of consumption as

$$(8) \quad \frac{dc_1}{dA} = - \frac{\left(\frac{rda}{1+a} \right) u_{cs}(2) - dr^2 u_{cc}(2)}{u_{cc}(1) + dr^2 u_{cc}(2) - \frac{2dar}{(1+a)} u_{cs}(2) + \frac{da^2}{(1+a)^2} u_{ss}(2)}.$$

With u concave and $u_{cs} \geq 0$, as is assumed in both formulations, present and future consumption are both normal goods. It is interesting to compare sophisticated choice as given in (7) with the particular version of naïve choice given by a correct perception of the marginal utility of consumption, but a failure to recognize the connection between c_1 and $u_c(2)$. That is, consider a naïve equilibrium given by

$$(9) \quad u_c(1) = dru_c(2).$$

This would be the case if an individual looked at the marginal utility of consumption of a similarly situated (in terms of consumption) older individual to form an estimate of $u_c(2)$, but did not look at how a change in consumption in period 1 affected marginal utility in the later period. With the addiction model, $u_s < 0$ and sophisticated choice has more savings than naïve choice. With the habit model, the analysis depends on the level of s_0 . When s_0 equals the sophisticated choice level of c , then sophisticated choice is closer to the constant consumption path than is the naïve choice equilibrium.

4. Savings under Uncertainty

We now assume that individuals want to maximize the expected value of lifetime utility. We assume that lifetime resources, A , are a random variable, whose value becomes known after the choice of c_1 . With this modification of the choice problem given in (6), the first order condition becomes

$$(10) \quad u_c(1) = E \left[dru_c(2) - \frac{da}{1+a} u_s(2) \right].$$

We want to examine the response of first period consumption to an increase in risk in lifetime income, a question posed by H. Leland (1968). In Table 1 we state sufficient conditions for savings to increase with riskiness using the definitions of mean preserving increase in risk of Rothschild and Stiglitz (1970) and mean utility preserving increase in risk of Diamond and Stiglitz (1974). The Table states the conditions

for a general 2-period utility function and then specializes them for the additive and standard-of-living special cases.

5. Earnings Uncertainty

We now consider the situation where the source of uncertainty about lifetime utility is uncertainty about second-period earnings. We write utility, U , as a function of first-period consumption, second-period consumption, and the number of periods worked. The uncertainty for the individual is about the ability to work in the second period, with the probability of being able to work being given by q . Thus, for an individual planning on two periods of work if able, expected utility can be written as³

$$(11) \quad \text{Max}_c [qU(c, W + w - rc, 2) + (1 - q)U(c, W - rc, 1)],$$

where w is the second period wage and W is nonrandom income measured in second period units. The first order condition for this consumption choice problem is

$$(12) \quad q(U_1(2) - rU_2(2)) + (1 - q)(U_1(1) - rU_2(1)) = 0$$

where $U_i(j)$ is the i^{th} partial derivative of U evaluated where j periods of work are done.

Now let us consider similar questions to that asked in the previous section - what happens to c in response to changes in w or W that keep expected income or expected utility constant? The first step in the analysis is the calculation of the response of c to changes in w and W separately. To calculate these derivatives, let us write the second order condition as

$$(13) \quad D = q(U_{11}(2) - 2rU_{12}(2) + r^2U_{22}(2)) + (1 - q)(U_{11}(1) - 2rU_{12}(1) + r^2U_{22}(1)) < 0.$$

Then, by differentiating the first order condition, we have the following comparative statics:

³ Adding an additive disutility of being unable to work would make no change in the analysis.

$$(14) \quad \frac{\partial c}{\partial w} = -D^{-1}[\mathbf{q}(U_{12}(2) - rU_{22}(2))] \\ \frac{\partial c}{\partial W} = -D^{-1}[\mathbf{q}(U_{12}(2) - rU_{22}(2)) + (1 - \mathbf{q})(U_{12}(1) - rU_{22}(1))]$$

To calculate the change in consumption for an increase in risk, expected income held constant, we need to increase w while decreasing W , recognizing that w is only received with probability \mathbf{q} . Thus, we have:

$$(15) \quad \frac{\partial c}{\partial w} - \mathbf{q} \frac{\partial c}{\partial W} = -D^{-1}\{[\mathbf{q}(U_{12}(2) - rU_{22}(2))] - \mathbf{q}[\mathbf{q}(U_{12}(2) - rU_{22}(2)) + (1 - \mathbf{q})(U_{12}(1) - rU_{22}(1))]\} \\ = -D^{-1}\mathbf{q}(1 - \mathbf{q})[(U_{12}(2) - rU_{22}(2)) - (U_{12}(1) - rU_{22}(1))]$$

if the disutility of labor is additive, this expression is signed by the conditions in Table 1. This follows abstractly from this being a special case of the analysis behind the Table. It can also be seen directly by noting that the difference between terms in (18) is signed by the condition in the Table. Without additive disutility of labor, there is an additional term, the sign of which depends on whether $U_{12} - rU_{22}$ is raised or lowered by work.

Similarly, to consider the mean utility preserving spread, we need to weight the changes in w and W by their impacts on expected utility. Thus the relevant derivative is:

$$(16) \quad \frac{\partial c}{\partial w} - \mathbf{q}U_2(2) \frac{\partial c}{\partial W} / (\mathbf{q}U_2(2) + (1 - \mathbf{q})U_2(1)) = \\ \frac{\partial c}{\partial w} - \mathbf{q} \frac{\partial c}{\partial W} + \frac{\partial c}{\partial W} \mathbf{q}(1 - \mathbf{q})(U_2(2) - U_2(1)) / (\mathbf{q}U_2(2) + (1 - \mathbf{q})U_2(1))$$

We have not examined a direct argument from the condition in Table 1.

In the additive model, U_{12} is zero and the effect of labor in the second period on $U_{12} - rU_{22}$, needed in (15) above, depends on the direct impact of labor on U_{22} and the sign of U_{222} , which depends on u'' .

In the standard-of-living model, however, we need to consider more carefully the instantaneous utility function, $u(c,s,h)$ (where h is hours of work). We shall consider the special cases considered above. With the addiction model we considered the two special cases

$$(17) \quad \begin{aligned} U(c_1, c_2, h_1, h_2) &= u(c_1 - s_o, h_1) + \mathbf{d}u(c_2 - s_1, h_2) \\ U(c_1, c_2, h_1, h_2) &= u(c_1 / s_o, h_1) + \mathbf{d}u(c_2 / s_1, h_2) \end{aligned}$$

Differentiating we have

$$(18) \quad \begin{aligned} U_2 = \mathbf{d}u_1(2), U_{12} - rU_{22} &= -\mathbf{d}u_{11} \frac{\mathbf{a}}{1 + \mathbf{a}} - r\mathbf{d}u_{11} = -\mathbf{d}u_{11} \left(\frac{\mathbf{a}}{1 + \mathbf{a}} + r \right) \\ U_2 = \mathbf{d}u_1(2)s_1^{-1}, U_{12} - rU_{22} &= -\mathbf{d}u_{11}s_1^{-2} \frac{\mathbf{a}}{1 + \mathbf{a}} - \mathbf{d}u_{11}c_2s_1^{-3} \frac{\mathbf{a}}{1 + \mathbf{a}} - r\mathbf{d}u_{11}s_1^{-2} \end{aligned}$$

Thus, with the difference addiction model we need to inquire how u_{11} varies with labor supply. With the ratio addiction model, we also need to ask how u_1 varies with labor supply.

With the habit models, we have the same structure since the additive terms in standard of living do not show up in $U_{12} - rU_{22}$.

6. Social Insurance

In our forthcoming paper we consider a social insurance plan to ease the wage uncertainty problem depicted in Section 4. That is, we consider a social security system that is providing insurance relative to the length of working life. The government selects w and W to maximize expected utility as given in (11) subject to two constraints. One constraint is the resource constraint. The second constraint is a moral hazard constraint, that expected utility for a worker planning on two periods of work, if able, be at least as large as expected utility with a plan of no work in the second period.

We showed that the moral hazard constraint was binding provided the following plausible condition was satisfied

$$(19) \quad \begin{aligned} \text{Max}_c U(c_2, W + w - rc_2, 2) &= \text{Max}_c U(c_1, W - rc_1, 1) \\ \text{implies } U_2(2) &< U_2(1) \end{aligned}$$

In addition, we considered the further tool of wealth taxation. We showed that wealth should be taxed or subsidized as a worker planning on two periods of work saved less or more than a worker planning on one period of work who had the same expected utility. In this section, we examine conditions on the additive

and standard-of-living models that are sufficient to satisfy the moral hazard assumption (19) and the wealth taxation condition.

If utility is additive, then we can write instantaneous utilities as $u(c,1)$ or $u(c,0)$ as individuals are working or not. A sufficient condition to imply that the moral hazard constraint is binding is then

$$(20) \quad \begin{aligned} u(c,1) &= u(c',0) \\ \text{implies } u_c(c,1) &< u_c(c',0) \end{aligned}$$

That is, compensating people sufficiently to just induce work results in a lower marginal utility of consumption for workers. Thus there would be a gain from redistributing to nonworkers, which must violate the moral hazard constraint if we are to have an optimum. Conditions (19) and (20) can also be reversed in the sense that reversal of the inequalities in both cases implies that the first-best solution is feasible.

Turning to the standard-of-living model we can consider the analogous one period condition

$$(21) \quad \begin{aligned} u(c, s, 1) &= u(c', s, 0) \\ \text{implies } u_c(c, s, 1) &< u_c(c', s, 0) \end{aligned}$$

As with (20), (21) is sufficient to imply that the moral hazard condition is binding.

It seems to us that the plausible case of the savings condition is that someone planning on 2 periods of work if able would consume more than if planning on only 1 period of work, $c_2^* > c_1^*$. In turn, this implies the desirability of taxing wealth. We explore this condition first in the general model by exhibiting a sufficient condition to imply $c_2^* > c_1^*$.

$$(22) \quad \begin{aligned} \text{Max}_c U(c_1, W - rc_1, 1) &= \text{Max}_c U(c_2', W + w - rc_2', 2) \\ \text{implies } c_1^* &< c_2'^* \end{aligned}$$

That is, we consider two certainty problems with 1 and 2 periods of work and equal levels of utility. The one-period certainty problem is the same as the uncertainty problem for a worker planning on only one period of work. However, the two-period certainty problem is different from the uncertainty problem with a plan of two-periods of work if able. Thus the distinction in the notation. The assumption in (22) is

that comparing the two certainty problems, the individual with greater lifetime income then consumes more in the first period. We depict (22) in Figure 2, where we have drawn utility as a function of consumption, with w chosen to equate optimized utility, as in (22), showing the condition that $c_1^* < c_2'^*$. If we also consider the level of consumption that maximizes expected utility for this wage conditional on planning on two periods of work if able, c_2^* , we would have $c_1^* < c_2^* < c_2'^*$. However, at this wage, expected utility for someone planning on two periods of work if able (but subject to disability risk) would be less than utility of a one period worker. To equate expected utilities we need to raise w , but not to a point where $U(c_1^*, W + w' - rc_1^*, 2)$ exceeds \bar{U} . Thus the expected utility equating wage w' must give a picture like the dashed curve in Figure 2. It is clear from the Figure (using normality) that c_2^* is greater than c_1^* .

We turn now to examining analogs to (22) in the context of additive and standard-of-living models. In the additive model, (23) (which is the familiar moral hazard condition) is sufficient to imply (22):

$$(23) \quad \begin{aligned} &u(c,1) = u(c',0) \\ &\text{implies } u_c(c,1) < u_c(c',0) \end{aligned}$$

The argument proceeds in a similar fashion. Consider the wage, w , which just equates second period utilities, first period consumption held constant at the optimal level given no work in the second period, c_1^* . That is, the wage w is chosen so that utility with this wage and an extra period of work would be the same if first-period consumption did not change. Allowing first-period consumption to vary implies that optimized utility must be higher; while (23) implies that the slope of the utility function is positive at c_1^* , as shown in Figure 3. Lowering the wage lowers the utility curve at all levels of first period consumption, resulting in a utility equating wage, w' , as shown by the dashed curve. With the standard-of-living model the argument is identical to that with the additive model and the sufficient condition for (22) is

$$(24) \quad \begin{aligned} &u(c, s,1) = u(c', s,0) \\ &\text{implies } u_c(c, s,1) < u_c(c', s,0) \end{aligned}$$

7. Wage Profile

In the model described above, there is uncertainty in only one period. Thus one cannot draw conclusions on the optimal time shape of the net financial return to working. In our previous paper, we considered an alternative model where there was uncertainty in both periods. There are then two moral hazard constraints - that a two-period work plan be at least as good as one- and zero-period work plans. That is, since there will now be some people unable to work at all, there is a need to provide income for someone who does no work at all. But this should not be so large as to induce everyone to stop working. Considering the additive model with the one-period moral hazard assumption, we reached two conclusions. Both moral-hazard constraints were binding and the second-period wage exceeded the first period wage.

In this section we will assume that both moral hazard constraints are binding and examine sufficient conditions for a rising wage. In the next section we will examine sufficient conditions for both constraints to bind.

We continue to assume that the impact of disability on utility is additive and that individuals know their disability before choosing current consumption. We assume that someone doing no work is given a lifetime wealth level of W . Someone working in the first period receives an additional amount w_1 . Someone who also works in the second period receives a further increment w_2 . Then, we have three possible individual plans for an individual who is able to work in the first period, based on plans to work 0, 1 or 2 periods if able.

$$\begin{aligned}
 V_0(W) &\equiv \text{Max}_{c_0} U(c_0, W - rc_0, 0) \\
 (25) \quad V_1(W, w_1) &\equiv \text{Max}_{c_1} U(c_1, W + w_1 - rc_1, 1) \\
 V_2(W, w_1, w_2) &\equiv \text{Max}_{c_2} (\mathbf{q}U(c_2, W + w_1 + w_2 - rc_2, 2) + (1 - \mathbf{q})U(c_2, W + w_1 - rc_2, 1))
 \end{aligned}$$

We now examine the implications of equal expected utility with all three plans,

$$(26) \quad V_0(W) = V_1(W, w_1) = V_2(W, w_1, w_2).$$

First, we note that concavity would imply a rising wage if individuals were facing a certainty problem. Denoting the wages that solve equal utility in the certainty problems by w'_i , we have:

$$\begin{aligned}
(27) \quad & \text{Max}_{c_0} U(c_0, W - rc_0, 0) = \text{Max}_{c_1} U(c_1, W + w'_1 - rc_1, 1) \\
& = \text{Max}_{c_2} U(c_2, W + w'_1 + w'_2 - rc_2, 2) \\
& \text{implies } w'_1 < w'_2
\end{aligned}$$

Since a worker planning on a single period is not subject to uncertainty, the equal utility conditions with and without uncertainty are the same. Therefore, (26) and (27) give us $w_1 = w'_1$. To complete the proof we argue that $w_2 \geq w'_2$. From the definition of V we have:

$$\begin{aligned}
(28) \quad & V_2(W, w_1, w'_2) = \text{Max}_{c_2} (\mathbf{q}U(c_2, W + w_1 + w'_2 - rc_2, 2) + (1 - \mathbf{q})U(c_2, W + w_1 - rc_2, 1)) \\
& \leq \mathbf{q} \text{Max}_c U(c, W + w_1 + w'_2 - rc, 2) + (1 - \mathbf{q}) \text{Max}_c U(c, W + w_1 - rc, 1) = V_1(W, w_1)
\end{aligned}$$

We note that we would have a strict inequality in (28) if first-period consumptions are different in the two certainty problems.

We also note that $w_1 > 0$ if labor is disliked and w_2 is not greater than the marginal product of labor. The latter conclusion follows from considering a small decrease in w_2 . This leads workers to plan on only one period of work, while having no effect on expected utility, by the equal expected utility condition. For w_2 to be optimal, therefore, work in the second period must not lose revenue for the government.

This result is stated in terms of wages measured in second period units of account. With a positive discount rate we have the further result that wages rise in current units of account.

8. Moral Hazard Constraints

We now examine sufficient conditions for both moral hazard constraints to be binding. Recognizing that the fraction $(1 - \mathbf{q}_1)$ of the population is disabled at the start of period one, while the remainder of the population is induced to plan on 2 periods of work if able, the social choice problem can be stated as

$$\begin{aligned}
(29) \quad & \text{Max}_{w, w_1, w_2} (1 - \mathbf{q}_1)V_0(W) + \mathbf{q}_1V_2(W, w_1, w_2) \\
& \text{subject to } W + \mathbf{q}_1(w_1 - mr) + \mathbf{q}_2(w_2 - m) = A \\
& V_2 \geq V_1, V_2 \geq V_0
\end{aligned}$$

where m is the marginal product. We solve this problem in two steps. For a given W , we do the suboptimization in w_1 and w_2 , ignoring the constraint $V_2 \geq V_0$. Then, we consider the choice of the optimal W . This sequence is permissible since V_0 does not depend on w_1 and w_2 and w_1 and W are perfect substitutes as control variables in the suboptimization if the income of those disabled at the start is held constant. The suboptimization is

$$(30) \quad \begin{aligned} & \text{Max}_{w_1, w_2} \mathbf{q}_1 V_2(W, w_1, w_2) \\ & \text{subject to } \mathbf{q}_1(w_1 - mr) + \mathbf{q}_2(w_2 - m) = A - W \\ & \quad \quad \quad V_2 \geq V_1 \end{aligned}$$

The suboptimization problem is the same one considered in Section 5 above. Thus, with the moral hazard condition, we know that at the optimum $V_2 = V_1$. Thus there are two types of solutions to (29) depending on whether the remaining moral hazard constraint is binding. If it is, $V_0 = V_2$. Otherwise, $V_0'(W) = \mathbf{I}$ where \mathbf{I} is the Lagrangian on the resource constraint in (30). We look for conditions to rule out the latter type of solution.

Setting up (30) as a Lagrangian problem, and differentiating with respect to w_1 , we have

$$(31) \quad \mathbf{q}_1 \frac{\partial V_2}{\partial w_1} - \mathbf{I} \mathbf{q}_1 - \mathbf{m} \left(\frac{\partial V_2}{\partial w_1} - \frac{\partial V_1}{\partial w_1} \right) = 0$$

Thus a sufficient condition to rule out the solution with $V_2 > V_0$ is to have the following two conditions:

$$(32) \quad V_1 = V_2 \text{ implies } \frac{\partial V_1}{\partial w_1} - \frac{\partial V_2}{\partial w_1} > 0$$

$$(33) \quad V_0 = V_1 \text{ implies } \frac{\partial V_0}{\partial W} > \frac{\partial V_1}{\partial w_1}$$

Condition (33) is the familiar moral hazard condition applied to first period work and is a reasonable additional assumption (along with (19)) in the general case. It is satisfied by the one-period assumptions (20) and (21) in the additive and standard-of-living models.

In our earlier paper, we showed that (32) was satisfied in the additive model by the condition $c_1^* < c_2^*$, which, in turn, was implied by the moral hazard condition. That is, as summarized in Table 2, the assumption in (20) was sufficient for the two results that the wage would rise if both moral hazard constraints were binding and that both constraints would bind. We now complete the argument that in the standard-of-living model (21) is sufficient for both results by considering sufficient conditions for the general case and showing that they are implied by (21).

While (32) is similar to the moral hazard assumptions we have made above, it differs in that it involves choice under uncertainty rather than comparing two choices under certainty. In the additive model, this problem is avoided since marginal utility in the first period is not random. In the standard-of-living and general models, marginal utility of first period consumption depends on the level of second-period consumption, and so is a random variable.

For the general case, a set of sufficient conditions⁴ for (32) is (34), (35), and (19)

$$(34) \quad U_{11} \leq rU_{12}$$

$$(35) \quad U_{12} \geq rU_{22}$$

We note that (34) and (35) are normality conditions for present and future consumption and are satisfied in the standard-of-living model.

Bringing together the definitions and conditions, and notationally combining $W + w$, into W , we have

$$(36) \quad \begin{aligned} & c_1^* \text{ maximizes } U(c_1, W - rc_1, 1) \\ & c_2^* \text{ maximizes } [\mathbf{q}U(c_2, W + w_2 - rc_2, 2) + (1 - \mathbf{q})U(c_2, W - rc_2, 1)] \\ & V_1(W) \equiv U(c_1^*, W - rc_1^*, 1) = \mathbf{q}U(c_2^*, W + w_2 - rc_2^*, 2) + (1 - \mathbf{q})U(c_2^*, W - rc_2^*, 1) \equiv V_2(W, w_2) \end{aligned}$$

We are proving the equivalent statement of (32):

$$(37) \quad U_2(c_1^*, W - rc_1^*, 1) > \mathbf{q}U_2(c_2^*, W + w_2 - rc_2^*, 2) + (1 - \mathbf{q})U_2(c_2^*, W - rc_2^*, 1).$$

Define w'_2 by

$$(38) \quad \text{Max}_{c_2} U(c_2, W + w'_2 - rc_2, 2) = V_1(W).$$

Denote the optimizing level of c_2 as c'_2 . From the definition, we see that $w'_2 \leq w_2$ with a strict inequality if $c_1^* \neq c_2^*$. From the moral hazard condition, (19), and the fact that c'_2 and c_1^* are optimizing values, we have

$$(39) \quad \begin{aligned} r^{-1}U_1(c'_2, W + w'_2 - rc'_2, 2) &= U_2(c'_2, W + w'_2 - rc'_2, 2) < U_2(c_1^*, W - rc_1^*, 1) \\ &= r^{-1}U_1(c_1^*, W - rc_1^*, 1) \end{aligned}$$

Since c'_2 and c_2^* are both optimizing values and U is concave we have

$$(40) \quad \begin{aligned} r^{-1}U_1(c_2^*, W + w_2 - rc_2^*, 2) &= U_2(c_2^*, W + w_2 - rc_2^*, 2) \\ &\leq U_2(c'_2, W + w'_2 - rc'_2, 2) = r^{-1}U_1(c'_2, W + w'_2 - rc'_2, 2) \end{aligned}$$

with a strict inequality if $w'_2 < w_2$. Combining (39) and (40) we have

$$(41) \quad U_2(c_2^*, W + w_2 - rc_2^*, 2) < U_2(c_1^*, W - rc_1^*, 1)$$

and

$$(42) \quad U_1(c_2^*, W + w_2 - rc_2^*, 2) < U_1(c_1^*, W - rc_1^*, 1)$$

⁴ This result is due to M. Whinston.

We proceed by considering separately the different cases as $c_1^* \leq / \geq c_2^*$. If $c_1^* = c_2^*$, (41) implies (37).

If $c_1^* > c_2^*$, then, by (35) we have

$$(43) \quad U_2(c_1^*, W - rc_1^*, 1) > U_2(c_2^*, W - rc_2^*, 1).$$

(41) and (43) imply (37). If $c_1^* < c_2^*$, then, by (34) we have

$$(44) \quad U_1(c_1^*, W - rc_1^*, 1) > U_1(c_2^*, W - rc_2^*, 1).$$

(42) and (44) imply (37) since c_1^* and c_2^* are both optimizing values. This completes the proof.

Summarizing the extensions of our earlier work, we have the results in Table 2.

9. Concluding Remarks

Exploring non-additive preferences is both tractable and important. Habit formation implies a different degree of risk aversion to resource changes that are learned about early in life relative to those learned about late in life (beyond what would be present anyway with additive preferences). It gives added importance to errors in planning that result in large drops in consumption at and after retirement. It gives a starting place for an analytical underpinning for thinking about pensions in terms of replacement rates. It can be used as a way to think about the averaging period used for defining benefits in defined-benefit plans. Moreover, it will change evaluations of the risks associated with different types of pension plans – such as a comparison of defined benefit and defined contribution plans. With social security reform on the agenda of so many countries and with more economists thinking about these programs, it is important to avoid taking over-simple models too seriously, even if an over-simple description of preferences is embedded in a very complex dynamic simulation. One way to combat this natural tendency is by having more general models. While this paper does not get very far in examining a generalization, it is a start that may be a useful jumping off place.

References

Ainslie, George, 1992, Picoeconomics: The Strategic Interaction of Successive Motivational States within the Person, Cambridge: Cambridge University Press.

- Becker, G., and K. Murphy, 1988, A theory of rational addiction, *Journal of Political Economy*,
- Chakravarty, S., and A. Manne (1968), "Optimum Growth when the Instantaneous Utility Function Depends upon the Rate of Change in Consumption," *American Economic Review* 58, 1351-54.
- Diamond, P., and B. Koszegi (1999), Quasi-hyperbolic Discounting and Retirement, unpublished.
- Diamond, P. and J. Mirrlees (1978), "A Model of Social Insurance with Variable Retirement," *Journal of Public Economics*, 295-336.
- Diamond, P., and J. Mirrlees (forthcoming), "Social Insurance with Variable Retirement and Private Savings," *Journal of Public Economics*.
- Diamond, P. and J. Stiglitz (1974) Increases in risk and in risk aversion, *Journal of Economic Theory* 8:337-360.
- Gruber, J. and B. Koszegi, 1999, Time-inconsistent Theories of Addiction, unpublished, MIT.
- Heal, G. and H. Ryder (1973), "Optimum Growth with Intertemporally Dependent Preferences," *Review of Economic Studies* 60, 1-32.
- Houthakker, H., and L. Taylor (1970), "Consumer Demand in the United States: Analyses and Projections," Harvard University Press.
- Kurz, M. (1968), "Optimal Economic Growth with Wealth Effects," *International Economic Review* 9, 348-357.
- Laibson, David, 1997, Golden Eggs and Hyperbolic Discounting, *Quarterly Journal of Economics*.
- Leland, H. (1968), "Savings and Uncertainty: The Precautionary Demand for Saving," *Quarterly Journal of Economics* 82, 465-73.
- Pollak, R. (1970), "Habit Formation and Dynamic Demand Functions," *Journal of Political Economy* 78, 745-763.
- Rothschild, M. and J. Stiglitz (1970) Increasing Risk: I A definition, *Journal of Economic Theory*, 2: 225-243.
- Samuelson, P.A. (1971), "Turnpike Theorems Even Though Tastes are Intertemporally Dependent," *Western Economic Journal* 9, 21-25.
- Weiszacker, C.C. von (1971), "Notes on Endogenous Changes of Tastes," *Journal of Economic Theory* 3, 345-72.

Table 1

Sufficient Conditions for Savings to Increase with Risk

<i>Utility Function</i>	<i>Mean Preserving Spread</i>	<i>Mean Utility Preserving Spread</i>
General Function $U(c_1, c_2)$	$rU_{122} - U_{222} < 0$	$U_2(rU_{122} - U_{222}) - U_{22}(rU_{12} - U_{22}) < 0$
Additive Function $u(c_1) + \mathbf{d}u(c_2)$	$u''' > 0$	$u'u''' - u''u'' > 0$
Standard of Living $u(c_1, s_0) + \mathbf{d}u(c_2, s_1)$	$ru_{ccc} - \frac{\mathbf{a}}{1+\mathbf{a}}u_{ccs} > 0$	$u_c \left(ru_{ccc} - \frac{\mathbf{a}}{1+\mathbf{a}}u_{ccs} \right) - u_{cc} \left(ru_{cc} - \frac{\mathbf{a}}{1+\mathbf{a}}u_{cs} \right) > 0$

Table 2

Summary of Results

General Model	Additive	Standard of Living
Model with uncertainty in second period only		
(19) → Moral hazard constraint binds	(20) → (19)	(21) → (19)
(22) → Optimal to tax wealth	(20) → (22)	(21) → (22)
Model with uncertainty in both periods		
(19), (32), (33) → Rising wage	(20) → (19), (32), (33)	(21) → (19), (32), (33)
(19), (34), (35) → Both constraints bind, (33)	(20) → (33)	(21) → (19), (34), (35)

Figure 1.

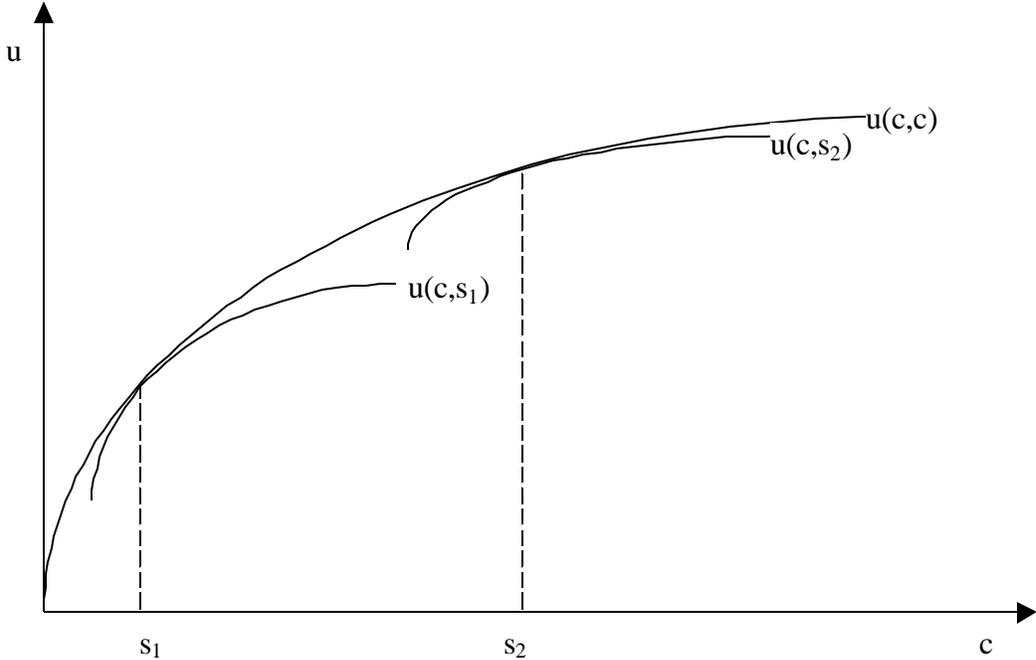


Figure 2.

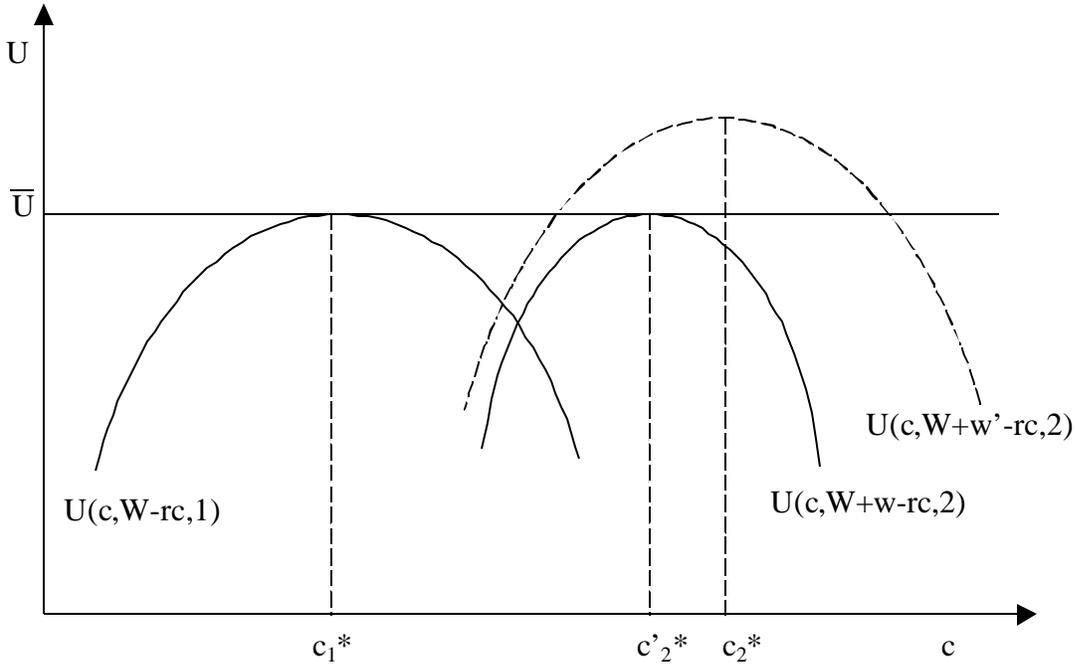


Figure 3.

