Problem Set 1

Due: Wednesday September 28th

These questions are from: Acemoglu, Daron (2008) "Introduction to Modern Economic Growth"

1. 13.13
2. 14.15

3. Optimal State Dependent IPR in a Partial Equilibrium Framework:

Let assume there are only two firms \((i \in \{1, 2\})\) in one industry, maximizing their expected present discounted value of net profits. Moreover we assume:

- maximum number of gaps between two firms is two \((n \in \{-2, -1, 0, 1, 2\})\)
- the profit of each firm (without taking into account expenditure on R&D) is only a function of the number of the gaps in the quality of the firm and its rival. \(\pi^i = \pi_n \in \{\pi_{-2}, \pi_{-1}, \pi_0, \pi_1, \pi_2\}\)
- R&D results to step by step innovation and the cost of R&D is linear in the arrival rate of innovation. \(\Phi(x^i) = \phi x^i\)
- interest rate is constant. \(r = r_0\)

In this framework, intellectual property right policy is modeled as the rate at which a technology becomes available for the rival firm for free and it is only a function of number of gaps between the leader and follower.

\[
\eta = \begin{cases} 
\eta_1 & n = 1 \\
\eta_2 & n = 2 \\
\to \infty & n > 2
\end{cases}
\]

Throughout this question we are focusing on the Markov Perfect Equilibrium in which strategies of firms are only function of the number of gaps between the firm and its rival. \(x^i = x_n \in \{x_{-2}, x_{-1}, x_0, x_1, x_2\}\)

(a) Write down the value function for a firm that is \(n\) step ahead (behind) its rival.

(b) Solve the system of the equations for the optimal R&D decisions: \(x_n\)
(c) Let assume we are restricted to the policy $\eta_1 = \eta_2 = \eta$. Can $\eta > 0$ increase R&D? Interpret the result.

(d) If we relax the above restriction, can we have IPR policy which results to more R&D than the case $\eta_1 = \eta_2 = 0$? Interpret the result and give an intuition why $\eta_1 > 0$ can increase R&D.