Optimal Monetary Policy
with Informational Frictions

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July 2017
How should fiscal and monetary policy respond to business cycles when firms have imperfect information about the world?
What is the relevant informational friction?

is it uncertainty about **fundamentals**?

- representative agent models, single-agent decision problem
- can feature rich first-order beliefs about future fundamentals
What is the relevant informational friction?

is it uncertainty about **fundamentals**?

- representative agent models, single-agent decision problem
- can feature rich first-order beliefs about future fundamentals
  

... or incomplete info about the **actions of others**?

- beauty contests with strategic complementarity
  
  → info friction impedes coordination among agents
  

- Movements in Higher-order beliefs → Sentiment-driven Fluctuations
  
What is the relevant informational friction?

does informational frictions affect nominal choices?

- info friction may be the source of nominal rigidity
  - sluggish price adjustment & monetary non-neutrality
- Mankiw Reis (2003), Woodford (2003), Mackowiak Wiederholt (2008)
  Paciello Wiederholt (2014)
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... or real quantity decisions?

- info friction may impede firms’ real choices
  → generate inertia to fundamentals,
  → amplify aggregate response to noise or common errors

- beliefs- or noise-driven aggregate fluctuations
  Lorenzoni (2009), Angeletos La’O (2009, 2013)
What is the relevant informational friction?

what type of signals do agents receive?

- sticky info (Mankiw and Reis 2003)
- Gaussian dispersed info (Woodford 2003, Angeletos La’O 2009)
- binary signals, non-Gaussian signals, fat-tailed posteriors, etc.
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is there endogenous information acquisition?

- given some cost, agents optimally choose their information
- rational inattention
  (Sims 2003, Mackowiak Wiederholt 2008, Paciello Wiederholt 2014)
- what is the exact shape of the cost function?
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- what is the exact shape of the cost function?

Informational constraint or cognitive limitations?
- limits on cognitive capacity (Woodford 2016, Gabaix 2014, Tirole 2015)
What we do

We study Optimal Fiscal and Monetary Policy when firms face both nominal and real informational frictions.
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Micro-founded business cycle model with the following features:

1. Nominal and real decisions subject to informational frictions
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2. Flexible, General Information structure
   - remain agnostic about informational frictions (baseline: exogenous)
   - extension: endogenous information/rational inattention
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3. Multiple sources of aggregate fluctuations
   - technology, government spending shocks
   - news, noise, higher-order beliefs, sentiments
Methodological Contribution

• The Ramsey Problem
  • Optimal Policy without Informational Frictions:
    Lucas and Stokey (1983), Chari, Christiano, Kehoe (1994)
  • with Sticky Prices: Correia, Nicolini, Teles (2008)

• The Primal Approach
  • characterize set of allocations implementable as equilibria
  • identify welfare-maximizing allocation within that set
  • back-out policies that implement the Ramsey optimum

• We extend primal approach to heterogeneous info. environments
  • study normative properties while completely bypassing an explicit solution for the equilibrium
What we show

1. **Flexible-price** allocations remain **optimal**, despite info frictions
   - optimal taxes as in Lucas Stokey; Chari, Christiano, Kehoe
   - tax final goods and labor, zero taxation of capital
   - tax smoothing (constant taxes if utility is homothetic)
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2. Despite nominal frictions, **Price Stability is Suboptimal**

3. Optimal Policy: **Negative Correlation between Prices and GDP**
The Model
The Model

• continuum of monopolistic firms, $i \in I$

• managers make decisions under incomplete info
  
  • nominal pricing decision
  
  • real intermediate good and investment decision
The Model

- continuum of monopolistic firms, \( i \in I \)
- managers make decisions under *incomplete info*
  - *nominal* pricing decision
  - *real* intermediate good and investment decision
- representative household
  - continuum of workers
  - continuum of managers
  - representative consumer
Intermediate Good Firms

\[ y_{it} = A_t F (k_{it}, h_{it}, \ell_{it}) \]

\[ k_{i,t+1} = (1 - \delta) k_{i,t} + x_{it} \]

- for today
  \[ y_{it} = A_t g (k_{it}, h_{it}) \ell_{it}^\alpha \]

- firm faces a revenue tax and a capital income tax
  \[ \frac{\Pi_{it}}{P_t} = \left(1 - \tau_t^k\right) \left[(1 - \tau_t^r) \frac{p_i y_{it}}{P_t} - (h_{it} + W_t \ell_{it})\right] - x_{it} \]
Final Good Firm and the Household

- final good firm
  \[ Y_t = \left[ \int y_{it}^{\frac{\rho - 1}{\rho}} \, di \right]^{\frac{\rho}{\rho - 1}} \]

- household
  \[ \mathbb{E} \sum_{t=0}^{\infty} \beta^t [U(C_t) - V(L_t)] \]
  \[ (1 + \tau^C_t) P_t C_t + B_t \leq (1 - \tau^\ell_t) P_t W_t L_t + R_t B_{t-1} \]

- labor market clearing
  \[ \int \ell_{it} \, di = L_t \]
Government and Resource Constraints

- government budget constraint
  - exogenous government spending shocks, no lump sum taxes
  - must finance expenditure with proportional taxes and nominal debt
  - debt has a one-period maturity and a state-contingent return

\[
R_t B_{t-1} + P_t G_t \leq \tau^r_t P_t Y_t + \tau^c_t P_t C_t + \tau^\ell_t P_t W_t L_t \\
+ \tau^k_t \int \eta_{it} di + \int \Pi_{it} di + B_t
\]

- resource constraints

\[
C_t + H_t + X_t + G_t = Y_t
\]

\[
H_t = \int h_{it} di \quad \text{and} \quad X_t = \int x_{it} di
\]
Shocks and Information Structure
Shocks and Information

1. Nature draws $s_t \in S_t$ according to $s_t \sim \mu(s_t)$
Shocks and Information

1. Nature draws $s_t \in S_t$ according to $s_t \sim \mu(s_t)$
   - aggregate “real” shocks $A_t, G_t$
   - cross-sectional distribution of information sets $\Omega_t$
   - thereby contains shocks to beliefs (noise, sentiments)
   - history: $s^t = (s_t, s_{t-1}, \ldots)$
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   - history: $s^t = (s_t, s_{t-1}, \ldots)$

2. Nature draws $\omega_{it} \in \Omega^t$, $\omega_{it} \sim \mu(\omega_i^t | s^t)$, $\forall i \in I$

3. Information of manager $i$ is $\omega_i^t = (\omega_{it}, \omega_{i,t-1}, \ldots)$
   - $\omega_i^t$ is manager’s “Harsanyi type”
Examples of Info Structures

• sticky info (Mankiw Reis 2003)

\[ \omega_{it} = \begin{cases} 
  s^t & \text{with prob } \mu \\
  \omega_{i}^{t-1} & \text{with prob } 1 - \mu 
\end{cases} \]

• noisy info (Woodford 2003, Angeletos La’O 2009)

\[ \omega_{it} = (x_{it}, z_t) = \begin{cases} 
  x_{it} = \log A_t + \nu_{it} \\
  z_t = \log A_t + \epsilon_t 
\end{cases} \]

• may also construct examples with “sentiments”

(Angeletos La’O 2013)
1. Managers make nominal and real decisions with incomplete info

   thus $p_{it}, h_{it}, x_{it}$ contingent on $\omega_i^t$
Informational Frictions and Market Clearing

1. Managers make nominal and real decisions with incomplete info
   
   thus $p_{it}, h_{it}, x_{it}$ contingent on $\omega_i^t$

2. All other market outcomes/choices/wages adjust to aggregate state
   
   ◇ given prices, household chooses consumption
   ◇ thus hours $\ell_{it}, y_{it}$ are contingent on $(\omega_i^t, s^t)$
   must adjust so that supply = demand
   ◇ govt policy, household consumption, savings contingent on $s^t$
Info Friction is both Nominal and Real

- standard in the literature: info friction = nominal friction

\[ p \text{ contingent on } \omega^t_i \]

but all real choices adjust to \( s^t \)

- Ball, Mankiw, Reis (2005), Adam (2007), Lorenzoni (2010), Paciello Wiederholt (2014)
Info Friction is both Nominal and Real

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\[ p \] contingent on \( \omega_i^t \)

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- our generalization: info friction = both nominal and real

\[ p \text{ and } h, x \text{ contingent on } \omega_i^t \]

\[ \ell \text{ adjusts to } s^t \]

- info friction still relevant even under flexible prices
Feasibility

Let $\zeta$ denote an allocation

$$\zeta(s^t) \equiv \left\{ Y(s^t), C(s^t), L(s^t), (x(\omega_i^t), k(\omega_i^t), h(\omega_i^t), \ell(\omega_i^t, s^t), y(\omega_i^t, s^t))_{i \in I} \right\}$$

Definition
An allocation $\zeta$ is feasible if and only if it satisfies the following:

$$C(s^t) + \int_I h(\omega_i^t) \, di + \int_I x(\omega_i^t) \, di + G(s^t) = Y(s^t) = \left[ \int_I (y(\omega_i^t, s^t))^\frac{\rho-1}{\rho} \, di \right]^\frac{\rho}{\rho-1}$$

$$y(\omega_i^t, s^t) = A(s^t) F\left( k\left(\omega_i^{t-1}\right), h(\omega_i^t), \ell(\omega_i^t, s^t) \right),$$

$$k(\omega_i^t) = (1 - \delta) k\left(\omega_i^{t-1}\right) + x(\omega_i^t)$$
Equilibrium
We Analyze Two Scenarios

1. **sticky-price equilibrium.** Firm chooses

   \[ p(\omega_t^i), h(\omega_t^i), x(\omega_t^i) \quad \text{conditional on } \omega_t^i \]

   both real and nominal informational friction

2. **flexible-price equilibrium.** Firm chooses

   \[ h(\omega_t^i), x(\omega_t^i) \quad \text{conditional on } \omega_t^i, \]

   but \[ p(\omega_t^i, s_t) \quad \text{adjusts to realized } s_t \]

   only the real informational friction
Equilibrium Definitions

Let $\theta$ denote a government policy

$$\theta (s^t) \equiv \{ \tau^r (s^t), \tau^c (s^t), \tau^l (s^t), \tau^k (s^t), R (s^t) \}$$

Definition

A **sticky-price equilibrium** is a policy $\theta$, an allocation $\xi$, and prices

$$\{ p (\omega_i^t) \}_{i \in I}$$

such that

(i) the household and firms are at their respective optima
(ii) the government’s budget constraint is satisfied, and
(iii) markets clear.

Definition

A **flexible-price equilibrium** is a policy $\theta$, an allocation $\xi$, and prices

$$\{ p (\omega_i^t, s^t) \}_{i \in I}$$

such that (i)-(iii) hold.
Flexible-Price Equilibrium
Household Optimization

\[ V_\ell (s^t) = U_c (s^t) \frac{\left( 1 - \tau_\ell (s^t) \right)}{\left( 1 + \tau_c (s^t) \right)} W (s^t) \]

\[ \frac{U_c (s^t)}{(1 + \tau_c (s^t)) P (s^t)} = \beta \mathbb{E} \left[ \frac{U_c (s^{t+1})}{(1 + \tau_c (s^{t+1})) P (s^{t+1})} R (s^{t+1}) \bigg| s^t \right] \]
Intermediate Firm’s Problem

Choose functions \((h, x, \ell)\) so as to maximize expected profits

\[
\max \mathbb{E} \left[ \mathcal{M}(s^t) \frac{\Pi(\omega_t^t, s^t)}{P(s^t)} \mid \omega_i \right]
\]

subject to

\[
\frac{p(\omega_i^t)}{P(s^t)} = \left( \frac{y(\omega_i^t, s^t)}{Y(s^t)} \right)^{-\frac{1}{\rho}} \quad \forall \omega_i^t, s^t
\]

\[
k(\omega_i^t) = (1 - \delta)k(\omega_i^{t-1}) + x(\omega_i^t) \quad \forall \omega_i^t
\]

\[
y(\omega_i^t, s^t) = A(s^t) \frac{F\left(k\left(\omega_i^{t-1}\right), h(\omega_i^t), \ell(\omega_i^t, s^t)\right)}{A(s^t)} \quad \forall \omega_i^t, s^t
\]

where

\[
\mathcal{M}(s^t) = \frac{U_c(s^t)}{1 + \tau^c(s^t)}
\]
Firm FOCs

intermediate goods demand optimality:

\[ \mathbb{E} \left[ M(s^t) \left( (1 - \tau^r(s^t)) \frac{\rho - 1}{\rho} MP_h(\omega_i^t, s^t) - 1 \right) \bigg| \omega_i^t \right] = 0 \quad \forall \omega_i^t \]

labor demand optimality:

\[ (1 - \tau^r(s^t)) \frac{\rho - 1}{\rho} MP_\ell(\omega_i^t, s^t) - W(s^t) = 0 \quad \forall \omega_i^t, s^t \]

where \( MP_z(\omega_i^t, s^t) \equiv \left( \frac{y(\omega_i^t, s^t)}{Y(s^t)} \right)^{-\frac{1}{\rho}} A(s^t) f_z(\omega_i^t, s^t) \) for any \( z \in \{k, h, \ell\} \).
Proposition

A feasible allocation is implementable as a flexible-price equilibrium iff

\[ \exists \text{ functions } \phi^r, \phi^c, \phi^\ell, \phi^k : S^t \to \mathbb{R}_+, \text{ such that} \]

(i) equil. labor condition

\[
\mathcal{M} (s^t) \phi^\ell (s^t) \phi^r (s^t) MP_\ell (\omega_i^t, s^t) - V_\ell (s^t) = 0 \quad \forall \omega_i^t, s^t
\]

with \[ \mathcal{M} (s^t) = U_c (s^t) / \phi^c (s^t) \]

(ii) equil. intermediate goods condition

\[
\mathbb{E} [ \mathcal{M} (s^t) (\phi^r (s^t) MP_h (\omega_i^t, s^t) - 1) | \omega_i^t ] = 0 \quad \forall \omega_i^t
\]
Flexible Price Equilibrium Allocations

Proposition

(iii) equil. capital investment condition

\[ E \left[ \mathcal{M}(s^t) - \beta \mathcal{M}(s^{t+1}) \left\{ 1 - \delta + \phi^r(s^{t+1})\phi^k(s^{t+1}) MP_k(\omega_i, s^{t+1}) \right\} \bigg| \omega_i^t \right] = 0 \]

and (iv) implementability condition for govt solvency:

\[ \sum_{t,s^t} \beta^t \mu(s^t) \left[ U_c(s^t) C(s^t) - V_\ell(s^t) L(s^t) \right] = \mathcal{M}(s^0) R_b(s^0) B_{-1} \]
Tax Wedges

- wedges result from taxes and markups

\[
\phi^c(s^t) \equiv 1 + \tau^c(s^t), \quad \phi^\ell(s^t) \equiv 1 - \tau^\ell(s^t), \quad \phi^k(s^t) \equiv 1 - \tau^k(s^t)
\]

\[
\phi^r(s^t) \equiv (1 - \tau^r(s^t)) \left( \frac{\rho - 1}{\rho} \right)
\]
Sticky-Price Equilibrium
Intermediate Firm’s Problem

Choose functions \((p, h, x, \ell)\) so as to maximize expected profits

\[
\max E \left[ \mathcal{M}(s^t) \frac{\Pi(\omega^t_i, s^t)}{P(s^t)} \bigg| \omega^t_i \right]
\]

s.t. same technological constraints in flexible-price firm problem,
but faces one additional constraint when choosing nominal price:

\[
A(s^t) F \left( k\left(\omega^t_i^{t-1}\right), h(\omega^t_i), \ell(\omega^t_i, s^t) \right) = \left( \frac{p(\omega^t_i)}{P(s^t)} \right)^{-\rho} Y(s^t) \quad \forall \omega^t_i, s^t
\]
Proposition

A feasible allocation is implementable as a sticky-price equilibrium iff

\[ \exists \text{ functions } \phi^r, \phi^c, \phi^\ell, \phi^k : S^t \to \mathbb{R}_+ \text{ and } \chi : \Omega^t \times S^t \to \mathbb{R}_+ \text{ such that} \]

(i) equil. labor condition

\[ \mathcal{M} (s^t) \phi^\ell (s^t) \phi^r (s^t) \chi (\omega^t_i, s^t) \mathcal{M} \ell (\omega^t_i, s^t) - \mathcal{V} \ell (s^t) = 0 \quad \forall \omega^t_i, s^t \]

(ii) equil. intermediate goods condition

\[ \mathbb{E} \left[ \mathcal{M} (s^t) (\phi^r (s^t) \chi (\omega^t_i, s^t) \mathcal{M} \ell (\omega^t_i, s^t) - 1) \right| \omega^t_i] = 0 \quad \forall \omega^t_i \]

(iii) equil. capital investment condition

\[ \mathbb{E} \left[ \mathcal{M} (s^t) - \beta \mathcal{M} (s^{t+1}) \left( 1 - \delta + \phi^r (s) \chi (\omega_i, s) \phi^k (s) \mathcal{M} \ell (\omega_i, s) \right) \right| \omega^t_i] = 0 \]
Sticky Price Equilibrium Allocations

Proposition

(iv) firm optimality condition for the nominal price

\[ \mathbb{E} \left[ \mathcal{M}(s^t) Y(s^t)^{1/\rho} y(\omega^t, s^t)^{1-1/\rho} \phi^r(s^t) \{ \chi(\omega^t, s^t) - 1 \} \bigg| \omega^t \right] = 0 \quad \forall \ \omega^t \]

and (v) implementability condition for govt solvency exactly the same as in flex-price equilibrium.
Comparing Flexible and Sticky Allocations

- In sticky price equilibrium allocations we have the new wedge:
  \[ \chi(\omega_i^t, s^t) = \text{realized markup due to monetary policy & sticky prices} \]

- In any flexible price equilibrium,
  \[ \chi(\omega_i^t, s^t) = 1 \quad \text{for all } \omega_i^t, s^t \]
Comparing Flexible and Sticky Allocations

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- In any flexible price equilibrium,
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- Let \( \Phi^f \) denote the set of implementable allocations under flexible prices

- Let \( \Phi^s \) denote the set of implementable allocations under sticky prices.

- Then
  \[ \Phi^f \subset \Phi^s. \]
The Ramsey Problem
Definition

The Ramsey Planner’s Problem is to maximize welfare over $\Phi^s$, the set of sticky-price allocations.

A Ramsey Optimal allocation is a solution to this problem.
The Relaxed Set

**Definition**

The Relaxed set $\Phi^R$ is the set of all feasible allocations in which the implementability condition for govt solvency holds.

**Definition**

A Relaxed Ramsey Optimal allocation is an allocation $\zeta^*$ which maximizes household ex-ante utility subject to

$$\zeta^* \in \Phi^R$$

- Note that the relaxed planner still respects informational feasibility
  - measurability constraints = technological constraints
- relaxed planner also respects government solvency constraint
Why look at the Relaxed Ramsey Problem?

- Clearly the relaxed set is a larger set

\[ \Phi^f \subset \Phi^s \subset \Phi^R \]
Why look at the Relaxed Ramsey Problem?

- Clearly the relaxed set is a larger set
  \[ \Phi^f \subset \Phi^s \subset \Phi^R \]

- We show the following:
  \[ \xi^* \in \Phi^f \]
  which further implies,
  \[ \xi^* \in \Phi^s \]

- Therefore \( \xi^* \) solves the (non-relaxed) Ramsey problem!
Proposition

The Relaxed Ramsey optimal allocation satisfies

\[
\tilde{U}_c (s^t) MP_\ell (\omega_i^t, s^t) - \tilde{V}_\ell (s^t) = 0 \quad \forall \omega_i^t, s^t
\]

\[
\mathbb{E} \left[ \tilde{U}_c (s^t) (MP_h (\omega_i^t, s^t) - 1) \middle| \omega_i^t \right] = 0 \quad \forall \omega_i^t
\]

\[
\mathbb{E} \left[ \tilde{U}_c (s^t) - \beta \tilde{U}_c (s^{t+1}) \left\{ 1 - \delta + MP_k (\omega_i^{t+1}, s^{t+1}) \right\} \middle| \omega_i^t \right] = 0 \quad \forall \omega_i^t
\]

with

\[
\tilde{U}(C (s^t)) \equiv U(C (s^t)) + \Gamma U_c (s^t) C (s^t)
\]

\[
\tilde{V}(L (s^t)) \equiv V(L (s^t)) + \Gamma V_\ell (s^t) L (s^t)
\]

and \(\Gamma\) is the Lagrange-multiplier on the implementability condition
The Relaxed Ramsey Optimum

Proposition

There exists a set of state-contingent taxes

\[
\phi^c(s^t) = \frac{U_c(s^t)}{\bar{U}_c(s^t)}, \quad \phi^\ell(s^t) = \frac{V_\ell(s^t)}{\bar{V}_\ell(s^t)}, \quad \phi^k(s^t) = 1, \quad \text{and} \quad \phi^r(s^t) = 1, \quad \text{for all} \quad s^t
\]

such that the Relaxed Ramsey optimum is implemented under flexible prices.

\[
\xi^* \in \Phi^f
\]
The Relaxed Ramsey Optimum

Proposition

There exists a set of state-contingent taxes

\[
\phi^c(s^t) = \frac{U_c(s^t)}{\tilde{U}_c(s^t)}, \quad \phi^\ell(s^t) = \frac{V_\ell(s^t)}{\tilde{V}_\ell(s^t)}, \quad \phi^k(s^t) = 1, \quad \text{and} \quad \phi^r(s^t) = 1, \quad \text{for all} \quad s^t
\]

such that the Relaxed Ramsey optimum is implemented under flexible prices.

\[
\xi^* \in \Phi^f
\]

Corollary

\[
\xi^* \in \Phi^s
\]

The Relaxed Ramsey optimum is implemented under sticky prices with the same taxes as above and

\[
\chi(\omega_i^t, s^t) = 1, \quad \text{for all} \quad \omega_i^t, s^t.
\]
Optimal Policy
Theorem

ξ* is implemented as part of sticky-price equilibrium with

(i) a monetary policy that replicates flexible prices; and

(ii) a tax policy that satisfies the following:

\[ 1 + \tau^c (s^t) = \frac{U_c (s^t)}{U_c (s^t)}, \quad 1 - \tau^\ell (s^t) = \frac{V_\ell (s^t)}{V_\ell (s^t)}, \quad 1 - \tau^k (s^t) = 1, \]

\[ 1 - \tau^r (s^t) = \left( \frac{\rho - 1}{\rho} \right)^{-1} \]
Lemma

Suppose preferences are homothetic

\[ U(C) = \frac{C^{1-\gamma}}{1-\gamma} \quad \text{and} \quad V(L) = \frac{L^{1+\epsilon}}{1+\epsilon} \]

Then the optimal consumption and labor tax rates are constant:

\[ 1 + \tau^c = \frac{1}{1 + \Gamma (1 - \gamma)}, \quad 1 - \tau^\ell = \frac{1}{1 + \Gamma (1 + \epsilon)}, \quad \tau^k = 0, \]

\[ 1 - \tau^r (s^t) = \left( \frac{\rho - 1}{\rho} \right)^{-1} \]

Lemma

There exist functions $\Psi^\omega, \Psi^s$ such that in any sticky-price equilibrium, firm output is log-separable

$$y(\omega^t_i, s^t) = \Psi^\omega(\omega^t_i) \Psi^s(s^t),$$

where $\Psi^\omega(\omega) = g(k(\omega), h(\omega))$.
Monetary Policy

Lemma
There exist functions $\Psi_\omega, \Psi^s$ such that in any sticky-price equilibrium, firm output is log-separable

$$ y(\omega^t_i, s^t) = \Psi_\omega(\omega^t_i) \Psi^s(s^t), \text{ where } \Psi_\omega(\omega) = g(k(\omega), h(\omega))^\zeta $$

- stickiness implies relative prices must be independent of $s^t$

$$ \frac{p(\omega^t_i)}{p(\omega^t_j)} = \left[ \frac{y(\omega^t_i, s^t)}{y(\omega^t_j, s^t)} \right]^{-1/\rho} = \left[ \frac{\Psi_\omega(\omega^t_i)}{\Psi_\omega(\omega^t_j)} \right]^{-1/\rho} $$

- further implies relative output must be independent of $s^t$

- a sticky-price allocation may be implemented with nominal prices

$$ p(\omega^t_i) = \Psi_\omega(\omega^t_i)^{-1/\rho} $$
Optimal Monetary Policy

let $\mathcal{B}(s^t) \equiv \left[ \int \psi^\omega (\omega_i^t)^{\frac{\rho-1}{\rho}} d(\omega_i^t|s^t) \right]^{\frac{\rho}{\rho-1}}$

**Theorem**

Along any equilibrium that implements the Ramsey optimal allocation, 

$$\log P(s) - \log P(s') = -\frac{1}{\rho} \left[ \log \mathcal{B}(s) - \log \mathcal{B}(s') \right] \quad \forall s, s' \in \mathcal{S}^t, \forall t$$
What does this theorem mean?

\[
\mathcal{B}(s^t) = \left[ \int \Psi^\omega \left( \omega_i^t \right)^{\frac{\rho - 1}{\rho}} d\omega_i^t | s^t \right]^{\frac{\rho}{\rho - 1}} \quad \text{where} \quad \Psi^\omega (\omega) = g(k(\omega), h(\omega))^\zeta
\]

**Proposition**

*Along any implementable allocation,*

\[
Y(s^t) = A(s^t) \mathcal{B}(s^t)^{1-\alpha} L(s^t)^\alpha
\]

*where, up to a first-order log-linear approximation,*

\[
\log \mathcal{B}(s^t) = \zeta_K \log K(s^t) + \zeta_H \log H(s^t),
\]

- \(\mathcal{B}\) is a proxy for aggregate beliefs
- variation in \(\mathcal{B}\) related to variation in aggregate labor productivity
- inherits the cyclical properties of capital and intermediate goods
Corollary

Suppose that capital and intermediate goods investment are procyclical along the Ramsey optimal allocation. Then, the optimal monetary policy targets a countercyclical price level.
Intuition for Countercyclical Price

• consider two firms: \( \omega \) and \( \omega' \). Efficiency requires that

\[
\frac{y(\omega, s)}{y(\omega', s)} \text{ increases in belief } \omega
\]
Intuition for Countercyclical Price

• consider two firms: \( \omega \) and \( \omega' \). efficiency requires that

\[
\frac{y(\omega, s)}{y(\omega', s)} \quad \text{increases in belief } \omega
\]

• implementability: demand implies

\[
\frac{p(\omega)}{p(\omega')} = \left[ \frac{y(\omega, s)}{y(\omega', s)} \right]^{-1/\rho} = \left[ \frac{\Psi^\omega(\omega)}{\Psi^\omega(\omega')} \right]^{-1/\rho}
\]

• relative price must fall in belief \( \omega \)
Intuition for Countercyclical Price

• consider two firms: $\omega$ and $\omega'$. efficiency requires that

$$\frac{y(\omega, s)}{y(\omega', s)} \text{ increases in belief } \omega$$

• implementability: demand implies

$$\frac{p(\omega)}{p(\omega')} = \left[ \frac{y(\omega, s)}{y(\omega', s)} \right]^{-1/\rho} = \left[ \frac{\Psi^\omega(\omega)}{\Psi^\omega(\omega')} \right]^{-1/\rho}$$

• relative price must fall in belief $\omega$

• relative price falls iff

$$p(\omega) \text{ falls with belief } \omega$$

$$P(s^t) \text{ falls in aggregate belief } B(s^t)$$
Simple Example
Simple Example

\[ U(C) = \frac{C^{1-\gamma}}{1-\gamma} \quad \text{and} \quad V(L) = \frac{L^{1+\epsilon}}{1+\epsilon} \]

- assume capital is fixed at 1 for all firms
- no government spending shocks
- variant with aggregate and idiosyncratic productivity shocks

\[ y_{it} = A_{it} \left( h_{it}^{\eta} \right)^{1-\alpha} \ell_{it}^{\alpha}, \]

\[ A_{it} = A_t \exp v_{it} \]
Gaussian Information Structure

\[ \omega_{it} = (x_{it}, z_t) \]

\[ x_{it} = \log A_{it} = a_t + v_{it}, \quad v_{it} \sim \mathcal{N}(0, 1/\kappa_v) \text{ iid} \]

\[ z_t = a_t + u_t, \quad u_t \sim \mathcal{N}(0, 1/\kappa_u) \]

- \( u_t \) introduces correlated noise in beliefs
  - common shock orthogonal to aggregate productivity
  - source of beliefs-driven aggregate fluctuations
The Power of Tax Instruments

\[ \log (1 - \tau^r (A_t, Y_t)) = \hat{\tau}_0 + \hat{\tau}_A \log A_t + \hat{\tau}_Y \log Y_t \]

Proposition

Under flexible prices, equilibrium GDP satisfies

\[ \log GDP (s^t) = \gamma_0 + \gamma_a \log A_t + \gamma_u u_t \]

for some scalars

\[ \gamma_0, \gamma_Z, \gamma_z \in \mathbb{R} \]

which are determined by the tax contingencies

\[ \hat{\tau}_0, \hat{\tau}_A, \hat{\tau}_Y \in \mathbb{R} \]
Proposition

In any equilibrium that implements the Ramsey optimal allocation,

\[ \log C(s^t) = \Delta_{ca} \log A(s^t) + \Delta_{cu} u_t, \]

\[ \log P(s^t) = -\Delta_{pa} \log A(s^t) - \Delta_{pu} u_t, \]

where

\[ \frac{\Delta_{pa}}{\Delta_{ca}} > 0 \quad \text{and} \quad \frac{\Delta_{pu}}{\Delta_{cu}} > 0. \]
Conclusion: Policy Lessons

Despite informational frictions and beliefs-driven fluctuations,

- Flexible-price allocations remain optimal
  - optimal taxes as in Lucas Stokey (1983)

- In order to implement Flex-price allocations:
  
  **Negative Correlation** between prices and output