Sophisticated Intermediation
and Aggregate Volatility

Luigi Iovino*
Massachusetts Institute of Technology
Job Market Paper
18 November 2011

Abstract

I consider an economy where investors delegate their investment decisions to financial institutions that choose across multiple investment opportunities featuring different levels of idiosyncratic risk and different degrees of correlation with the aggregate of the economy. Investors solve an optimal contracting problem to induce financial institutions to allocate their investment optimally. I then study how investment decisions are affected when financial securities are introduced that allow agents to trade their risks. Investors do not have the necessary information to understand these securities, but give incentives to financial institutions to hedge certain risks. I show that hedging idiosyncratic risks ameliorates the agency problem between investors and financial institutions and reduces aggregate volatility. On the contrary, when aggregate risk can be hedged the agency problem worsens and aggregate volatility increases. Finally, I study the efficiency properties of the equilibrium and the potential role for financial regulation.

*email: liovino@mit.edu. I am extremely grateful to my advisors George-Marios Angeletos, Guido Lorenzoni, and Mikhail Golosov for their invaluable guidance and feedback. I also thank Ruchir Agarwal, Joaquin Blaun, Marco Di Maggio, Stefano Giglio, Bengt Holmstrom, Felipe Iachan, Leonid Kogan, Plamen Nenov, Michael Peters, Giovanni Reggiani, Bruno Strulovici, Jean Tirole, and Iván Werning and participants at the MIT macro workshop. The contents of the paper are solely the responsibility of the author.
1 Introduction

The last two decades have witnessed an enormous expansion of markets for financial securities. Derivative instruments, often customized to the specific needs of their users, have become very popular and have enabled firms and financial institutions to manage their risks more efficiently. The over-the-counter (OTC) derivatives market, where contracts are traded directly between two parties (without going through an exchange), has become the largest market for derivatives. The notional amount outstanding of OTC derivatives has increased from 94 trillion dollars in 2000 to 601 trillion dollars by the end of 2010. Commercial banks, in particular, hold large exposures to derivative contracts. The notional value of derivatives held by US commercial banks was estimated to be 244 trillion dollars by the end of 2010. The typical derivatives are interest rate products (such as interest rate swaps) which comprise 82% of the total. Credit derivatives represent 6.1% of the total with their share increasing over time (OCC Quarterly Report, First Quarter 2011). Even though these are notional amounts that can include double counting, it seems that net exposures have increased at a similar pace.

In a perfect world, financial markets allow households and firms to share risks more efficiently. Idiosyncratic risks can be pooled and eliminated with great benefits for risk-averse agents. Aggregate risks – which cannot be eliminated – can be transferred to the agents better equipped to bear them. However, following the dramatic events of recent years, markets for derivatives have come under increased scrutiny. Lack of regulation in derivatives has been partly blamed for the turmoil in financial markets. Many observers have pointed out the “opaqueness” of derivative markets and the “complexity” of the positions held on and off the balance sheets of big financial institutions. Complex financial securities, the argument goes, may actually pose a threat to the system by increasing and concentrating the risks of some institutions.

These observations have motivated a growing literature. Caballero and Simsek (2011) study the possibility that complex financial arrangements may make banks susceptible to contagion; Arora et al. (2011) show how computational complexity amplifies the costs of asymmetric information; Gennaioli et al. (2011) study the possibility that investors neglect certain unlikely events; Farhi and Tirole (2011) show that banks’ choices are distorted by the anticipation of collective bail outs; and Stein (2011) studies the “special” demand for riskless assets.

This paper contributes to this literature by focusing on the agency problem between investors and bankers. In particular, I start from the idea that banks perform two types of activities. First, they have the expertise to select projects and monitor their performance. This expertise, however,
comes with agency problems: investors have to induce bankers to exert effort and select projects with larger expected returns and lower correlation with the aggregate economy.

Second, banks combine the cash flows of these projects with a rich set of financial securities to hedge risks and produce a final payoff. The way I capture the idea of “complexity” is by assuming that investors do not have the ability to fully understand the banks’ balance sheets. This means that in designing the optimal contract investors can only punish and reward the bankers based on their total payoff, i.e., on the sum of the cash flows from real projects and security trading.

The key questions that I address are: (i) why it may be optimal to expose banks to aggregate risk; (ii) whether access to security trading ameliorates or worsens the bankers’ incentive problem; (iii) whether there is room for government intervention.

First, I study the benchmark case in which security trading is not allowed. In this case, I show that investors provide incentives by conditioning the payment of the contract on both idiosyncratic and aggregate risks. In particular, they punish financial institutions for generating profits that are very correlated with the rest of the economy, tilting the choice of the bank away from aggregate risk. Even when optimal incentives are in place, however, the agency problem is never fully resolved and, relative to the first best, investors are exposed to excessive aggregate volatility.

Next, I allow banks to trade securities, both securities contingent on idiosyncratic risks and securities contingent on aggregate risk. Investors cannot observe the trading activity of the banks, but can design the optimal contract to influence it. Security trading interacts with the agency problem in different ways. In particular, when banks can trade securities on idiosyncratic risks, this mitigates the agency problem and lowers aggregate volatility. On the contrary, trading of securities on aggregate risk exacerbates the agency problem and increases aggregate volatility. In summary, the effects of the complexity of banks’ hedging activity are ambiguous and depend on the relative importance of the two types of risks. Complexity by itself does not necessarily lead to worse economic outcomes and in some cases may reduce volatility in the economy and can be beneficial for investors.

Finally, I derive the normative implications of the interaction between agency problems and security trading. First, I show that, when banks cannot trade financial securities, the equilibrium with the agency problem is constrained efficient. Thus, the higher exposure of investors to aggregate risk relative to the first-best does not by itself open the door to government intervention. When financial securities can be traded, government intervention may be desirable. Inefficiencies originate since investors suffer from a coordination failure. They do not internalize how the activity of trading
securities interacts with the agency problem of financial institutions. Therefore, in equilibrium it is too easy for financial institutions to trade securities contingent on aggregate risk. Government intervention can fix this coordination failure and restore efficiency. The government can reduce aggregate volatility (and increase welfare) in different ways. The most effective policy tool is regulation of the issuers of aggregate risk securities. An alternative and less effective policy is to tax transactions in financial markets.

2 Related Literature

The core of this paper is a principal-agent model where the principal delegates an investment choice to the agent. The principal provides incentives by exposing the agent to some risk. The seminal contribution of Holmstrom (1979) shows under what conditions more information should be incorporated in the contract. He studies a moral hazard problem with many agents and correlated signals and derives the general principle that observable, correlated signals which are not affected by the agent’s effort should not be included in the optimal contract. Contrary to Holmstrom (1979), in this paper the effort of the agents determines the correlation of the projects in the economy and, thus, the optimal contract exposes the agent to the common noise.

The seminal contribution to the literature on delegated portfolio management is Bhattacharya and Pfeiderer (1985) who propose a model where an informed agent has to reveal his information to the principal. The agency problem in this paper arises because managers have access to better information than investors, but they have to be incentivized to collect this information. This is similar to the model of delegated expertise developed by Demski and Sappington (1987) (see also Allen (1990)) and to the delegated portfolio problem with hidden actions (Admati and Pfeiderer (1997), Stoughton (1993)).

The principal-agent model can also be interpreted as a two-tier incentive problem whereby investors lend money to managers who then monitor entrepreneurs who run the projects and choose in what type of risk to invest. The classical paper on delegated management is Diamond (1984) who shows that it is optimal for banks to fully diversify their portfolios. However, Diamond (1984) considers a model where there is no trade-off between different types of risks as in this model.

In the basic version of the model, complexity is modelled by assuming that trades are unobservable (Allen (1985), Arnott and Stiglitz (1993), Hellwig (1983), Bisin and Guaitoli (2004), Cole and Kocherlakota (2001), Bizer and DeMarzo (1999)). Unobservable trades limit risk-sharing in Jacklin (1987) who shows that financial markets can reduce welfare. Farhi et al. (2009) show how regulation can correct the externality generated by the unobservable trades (see also Allen and
In corporate finance, several papers have focused on how hedging opportunities affect incentives when the effort of the managers increases the expected return of the firm (Li (2002), Garvey and Milbourn (2003), Ozerturk (2006), Bisin et al. (2008)). An important difference is Acharya and Bisin (2009) who study a model where firms make investment decisions and can choose the loading on the aggregate state of the economy. They also allow the manager to transfer (aggregate) risk. They focus on the optimal ownership share of the manager and show that a manager who is too risk-averse should own a smaller part of the firm’s capital. Acharya and Bisin (2009) do not make the distinction between different types of securities, which is central in this paper, and do not allow investors to write the optimal contract to managers. Also, they study a partial equilibrium model and, thus, they don’t consider policy implications.

One key difference between the model of this paper and the literature on agency problems with unobservable trades is that in these models agents trade on their own account and undo the incentives provided by the principals. On the contrary, this paper proposes a new way to look at how hidden trades can affect the incentives of the managers. Managers select projects and trade financial securities that affect the balance sheets of the financial institution they manage. Investors observe only the combined payoff of these two activities and, thus, they can provide incentives based only on this payoff. I find this assumption more realistic since complex securities are mostly held on the balance sheets of financial institutions.

A more recent literature studies how the complexity of financial securities and opacity of OTC markets can pose threats to the financial system. Caballero and Simsek (2011) show how complexity (modelled as limited information about the network of counterparties) can potentially cause a cascade of bank failures. Dang et al. (2009) study how some securities, such as debt, that are usually informational insensitive can lose much of their value in bad states of the world because of asymmetric information. Brunnermeier and Oehmke (2011) focus on the definition of complexity when agents are boundedly rational and observe that disclosing more information can lead to information overload, which has important implications for designing disclosure requirements and consumer protection. Their reason for regulation is not driven by the agency problem combined with the general equilibrium effects as in this paper.

The paper is organized as follows. Section 3 introduces the model and defines the equilibrium. Section 4 solves the model for the special case where securities markets are absent. The solution of

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1See Acharya (2009) for a model where firms strategically coordinate their actions and increase the systemic risk in the economy.
the model with securities is derived in section 5, where I consider the different types of securities separately. In section 6, I allow agents to trade both securities and extend the model to include trading costs. The efficiency properties of the equilibrium are studied in section 7 and some optimal policy prescriptions are discussed. Finally, section 8 contains the concluding remarks.

3 The model

The economy lasts for two periods, \( t = 0, 1 \) and there is one consumption good. There are two types of agents: investors (the principals) and fund managers (the agents). There is a large number of identical investors. They are born with an endowment of one unit of capital which can be invested to produce consumption goods in period 1. Each investor has access to a continuum of managers, to which he delegates the investment of his capital. The managers of the representative investor form a continuum of measure 1. They are indexed by \( i \in [0, 1] \) and have no endowment. They receive capital from the representative investor in period 0 and select projects on his behalf. Investors and managers value consumption only in period 1 according, respectively, to the utility functions \( v(\cdot) \) and \( u(\cdot) \), which are assumed to be differentiable, increasing and concave.

The economy is characterized by a continuum of sectors, denoted by \( j \in [0, 1] \). At time 0, each manager \( i \) is randomly matched to a sector \( j \), his area of expertise. The assignment of managers to sectors is one-to-one. Let \( \mathcal{F} \) the set of all possible realizations of this matching process. An element \( F \in \mathcal{F} \) is a full description of the assignment of managers to sectors.

The investment technology is modelled to capture the idea that managerial effort determines both the expected returns of the selected projects and their correlation with the aggregate economy.

A project requires 1 unit of capital at time 0 and produces a random return at time 1. Each manager \( i \) has access to a continuum of potential projects in sector \( j \), some of them are “specialized” projects and have a random payoff

\[
    r_{i,j} = \bar{r} + \varepsilon_j + u_i
\]

some of them are “standard” projects and have a random payoff

\[
    R_i = \bar{R} + \omega + u_i.
\]

\( \bar{r} \) and \( \bar{R} \) are the expected payoffs of the two projects, \( \varepsilon_j \) is a sector-specific shock, \( \omega \) is an aggregate shock, and \( u_i \) is a manager-specific shock.

Let \( K_i \) denote the units of capital that the manager receives from the investors. After receiving
the manager chooses how much effort to spend in selecting specialized projects. His effort level is denoted by $k_i$. By exerting effort $k_i$ the manager is able to select exactly $k_i$ specialized projects. A manager exerting effort $k_i$ incurs a utility cost $C(k_i)$, which is assumed to be differentiable, increasing and convex. This cost is meant to capture resources required to screen more innovative ideas in financing startups, more reliable borrowers in loan markets, or unexploited arbitrage opportunities in asset markets.

After selecting $k_i$ specialized projects, the manager can allocate $k_i$ units of capital to these projects and the remaining $K_i - k_i$ to standard projects, generating a total payoff

$$\pi_i = r_{i,j} k_i + R_i (K_i - k_i)$$

$$= \bar{R}K + (r + \varepsilon_i) k_i + \omega (K - k_i) + u_i K,$$

where $r \equiv \bar{r} - \bar{R}$. A higher $k_i$ implies that the total payoff is more sensitive to the sector-specific shock and less to the aggregate shock. I assume that $\omega$ has cdf $F_{\omega}$, with mean 0 and variance $\sigma_\omega^2$, and the other random variables are Gaussian, $\varepsilon_i \sim N (0, \sigma_\varepsilon^2), u_i \sim N (0, \sigma_u^2), \forall i$. All random variables are assumed to be independent of each other.

I assume that $\bar{r} > \bar{R}$ so specialized projects have a higher expected payoff and are uncorrelated with the aggregate shock $\omega$. So investors will strictly prefer specialized projects. However, selecting specialized projects requires costly, unobservable effort by the manager. This is the source of the agency problem in the model.

At time 0, after the contract is signed, the manager has access to a Walrasian market where he can trade securities contingent on the sector-specific shocks $\varepsilon_j$ and on the aggregate shock $\omega$. Denote by $z_{j,\varepsilon}$ the Arrow security that pays one unit of consumption at time 1 when the realization of the idiosyncratic shock $\varepsilon_j$ is $\varepsilon$. Similarly, $z_{\omega}$ denotes the security that pays one unit of consumption at time 1 when the realized aggregate state is $\omega$. Let $Z^\varepsilon$ and $Z^\omega$ be the space of Arrow securities contingent on $\varepsilon$-risk and $\omega$-risk, respectively, and let $Z = Z^\varepsilon \cup Z^\omega$. Let $p : Z \rightarrow \mathbb{R}_+$ be the price schedule of these securities. Denote by $d_i : Z \times \mathbb{R}_+ \rightarrow \mathbb{R}$ the demand of Arrow securities $z \in Z$ at price $p$ by agent $i$.

To simplify notation, without loss of generality, I will assume that manager $i$ is matched with sector $i$.

Payoffs. Securities are traded in a competitive market at the equilibrium price $p(\cdot)$. Managers decide the amount $k_i$ to invest in the specialized projects and the quantities of Arrow securities to trade.

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2In the appendix, I show how the process of project selection can be modelled explicitly.
The final profits generated by manager $i$ with demand $d_i$ of Arrow securities are

$$\Pi_i^m = \pi_i + \int (z_{j,\hat{z}} - p_{j,\hat{z}}) d_{i,j,\hat{z}} \, d\hat{z} \, dj + \int (z_{\omega} - p_{\omega}) d_{i,\omega} \, d\omega$$

(2)

These profits are delivered to investors who then make a payment $\xi_i$ to manager $i$. This payment depends on the what investors can observe as stated in Assumption 1. Each manager chooses an investment fraction $k_i$ and a demand schedule $d_i$ so as to maximize the expected utility

$$\max_{k_i,d_i} \mathbb{E}[u(\xi_i)] - C(k_i),$$

where the expectation is taken over the realizations of $\xi_i$.

Let $y : \mathbb{Z} \times \mathbb{R}_+ \to \mathbb{R}$ be the quantity of security $z$ supplied by the representative investor at price $p(z)$. The profits from selling securities are given by

$$\Pi^f = \int (p_{j,\hat{z}} - z_{j,\hat{z}}) y_{j,\hat{z}} \, d\hat{z} \, dj + \int (p_{\omega} - z_{\omega}) y_{\omega} \, d\omega$$

The representative investor receives profits from managers and makes payments $\xi_i$ to each manager. Therefore, in period 1 his consumption is $c(\omega) = \int (\Pi_i^m - \xi_i (\Pi_i^m, \omega)) d\omega + \Pi^f$ and his marginal utility of consumption in state $\omega$ is $m(\omega) \equiv v'(c(\omega))$. In equilibrium, the representative investor is fully diversified across managers. Diversification across managers also implies that it is optimal for the representative investor to write a contract with each manager separately and solve:

$$\max_{y,\xi_i} (\mathbb{E}[m(\omega)(\Pi_i^m - \xi_i)] + \mathbb{E}[m(\omega) \Pi^f]).$$

**Information.** To complete the description of the model, I need to make assumptions on the information sets of the different agents.

**Assumption 1**

(a) The effort $k_i$, the matching $F$, the demand $d_i$, and the shock $u_i$ are observed only by the managers and not by the investors.

(b) The incentive contract cannot be a function of $\varepsilon_j$.

(c) The random variables $\varepsilon_i$, $\forall i$, and $\omega$ are realized at time 1 and observed by every agent.

The fact that effort $k_i$ is not observable (part (a)) is the key moral hazard problem: without the right incentives, a manager will avoid paying the non-monetary cost by investing all the capital in standard projects.
The complexity of the balance sheets of banks and their trading activity is captured with two assumptions. First, part (a) implies that investors cannot condition their incentives on the trading activity of the managers. Second, part (b) restricts the space of contracts available to investors. Even if the realizations of the shocks $\varepsilon_j$ are observable (part (c)), investors cannot condition their payments on these shocks. This prevents them from designing a contract that elicits information about the sector $j$ to which each manager is matched. The first two assumptions together imply that the payment $\xi_i$ can be conditioned only on the profits of the managers $\Pi_i^m$, $\forall i$, and the aggregate shock $\omega$. These assumptions will imply that securities will have ambiguous effects on the quantity of aggregate risk and welfare in the economy. Anticipating some results, if investors could observe the trades of securities, they would always forbid trading of aggregate risk. Securities contingent on aggregate risk distort the incentives of the managers away from the desired solution and act as a constraint on the incentives that can be provided to managers.

In section 6, I replace part (a) with the assumption that investors can observe the quantity of securities traded by managers, but not the type of securities. This alternative assumption is motivated by the idea that, while investors can often observe whether financial institutions are trading securities, they may not have the expertise to understand what risks are being hedged with these complex securities.

I make the following assumptions on the utility and cost functions.

**Assumption 2**

(a) The utility function $u(\cdot)$ is such that $\tilde{u}(x) \equiv (u')^{-1}(1/x)$ is increasing and concave.

(b) The utility $u(\cdot)$ and cost $C(\cdot)$ functions are such that $u(\lambda x) - C(\lambda x) = h(\lambda)(\tilde{u}(x) - C(x))$, for some positive function $h(\cdot)$.

Part (a) of assumption 2 is basically an assumption on the curvature of the utility function of the agent which is discussed in Jewitt (1988)\(^3\). This assumption is used to justify the first-order approach. Also, as it will become clear in section 5.1, this assumption implies that the agent will want to buy full insurance against the idiosyncratic risk.

Part (b) is a homogeneity property that serves an important purpose. This assumption and the fact that the profits (1) are proportional to the capital invested imply that the incentives problem for each manager will be invariant to the quantity of capital invested. In other words, under this assumption, incentives are invariant on how managers distribute capital across managers. Thus,

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\(^3\)Rogerson (1985), Sinclair-Desgagne (1994), and Conlon (2009) study other conditions for the first-order approach to be valid.
in equilibrium investors will give the same amount of capital to each manager and fully diversify their investment. Finally, if investors are diversified across managers, I can simplify the problem by solving for the optimal contract of each manager separately.

Managers and investors meet in a Walrasian market to trade Arrow securities. The assumption of a Walrasian market deserves some comments. Most complex financial securities are traded on OTC markets where the seller and the buyer trade in a decentralized fashion. In this sense, the choice of a Walrasian environment is not very realistic and there is a growing literature that dispenses with the Walrasian assumption and focuses on decentralized markets (Duffie et al. (2005)). However, while models of decentralized trading would describe the functioning of OTC markets more realistically, they would also greatly complicate the analysis without changing the main message of the model. The focus of this paper is on the effects of complex securities on investment choices and not on the specific features of the market where these securities are exchanged. Also, the conclusions of this paper are likely to hold under different trading arrangements as long as the different types of risks are hedgeable and some trades cannot be observed.

Finally, the intermediation role of financial institutions is only implicit: managers borrow money from investors and run the projects themselves. There is, however, an alternative interpretation which leads to similar conclusions. Investors lend money to financial institutions which then channel this money to entrepreneurs who can select and run projects. In this more general setting there is room for two layers of moral hazard. Financial intermediaries have the expertise to monitor the entrepreneurs (Diamond (1984)) who, in turn, need incentives to make the right investment choice (that is, projects with higher return and lower correlation)\(^4\).

**Equilibrium**

The equilibrium of the model is a combination of a standard Walrasian equilibrium and an optimal contracting problem between principals and agents.

Since the problem of every agent is perfectly symmetric, I restrict attention to a symmetric equilibrium where all the investors and managers make the same choice. Under assumption 2, in equilibrium investors will fully diversify their investments by lending an equal share of their endowment to each manager. Thus, investors will care only about the mean return and the aggregate risk of their portfolio and consume \(c(\omega)\), which is a function of only the aggregate state.

\(^4\)See, for example, Holmstrom and Tirole (1997) and Brunnermeier and Sannikov (2011).
Definition 1 (Contract) Given a price schedule $p(\cdot)$, a contract between a principal and a manager $i$ is a tuple $(k_i, d_i, \xi_i)$ where $k_i$ is the suggested level of investment in the specialized projects, $d_i$ is the suggested demand schedule of the different securities, and $\xi_i : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is the payment made to the manager when $(\pi_i, \omega)$ is observed.

An agent who behaves as specified by a contract $(k_i, x_i, \xi_i)$ receives utility

$$\int u(\xi_i(\pi_i, \omega)) dF_{\pi_i, \omega}(\pi_i, \omega|k_i, d_i, p(\cdot)) - C(k_i),$$

where $F_{\pi_i, \omega}(\pi_i, \omega|k_i, d_i, p(\cdot))$ is the cdf of the joint distribution of $(\pi_i, \omega)$ when the specified allocations are $(k_i, d_i)$ and securities are priced according to $p(\cdot)$. A contract $(k_i, d_i, \xi_i)$ is individually rational (IR) if it delivers utility at least equal to $\bar{u}$ to agent $i$, that is,

$$\int u(\xi_i(\pi_i, \omega)) dF_{\pi_i, \omega}(\pi_i, \omega|k_i, d_i, p(\cdot)) - C(k_i) \geq \bar{u}.$$

A contract is incentive compatible (IC) if the agent doesn’t want to deviate by choosing different quantities $(\hat{k}_i, \hat{d}_i)$, formally

$$(k_i, d_i) \in \arg\max_{(k_i, d_i)} \int u(\xi_i(\pi_i, \omega)) dF_{\pi_i, \omega}(\pi_i, \omega|\hat{k}_i, \hat{d}_i, p(\cdot)) - C(\hat{k}_i).$$

Denote by $\mathcal{C}(p(\cdot))$ the set of contracts which are IR and IC when the equilibrium price schedule is $p(\cdot)$.

Definition 2 (Equilibrium) An equilibrium is a price schedule $p(\cdot)$, contracts $(k_i, d_i, \xi_i) \in \mathcal{C}(p(\cdot))$, $\forall i$, and supply schedule $y$ such that:

(a) Given prices $p(\cdot)$, $(k_i, d_i, \xi_i, y)$ is optimal for the investors;
(b) Prices $p(\cdot)$ are such that securities markets clear: $\int d_i(z, p(z)) \, dz = y(z, p(z)), \forall z$.

The definition of equilibrium essentially requires that every agent optimizes by taking the pricing function $p(\cdot)$ as given and markets clear.

4 No contingent securities

This section studies a simpler version of the model where markets for securities are absent. The goal of this section is twofold: it helps gain intuition before solving the full model and it represents an important benchmark for the full model’s solution.
With no trades of securities, the only frictions in the economy are the fact that only managers observe the fraction of wealth invested in specialized projects. I show that the agency problem reduces the level of investment in the specialized projects that is implemented in equilibrium. In turn, this increases aggregate volatility in the economy and lowers welfare. However, this is not a reason for policy intervention as the equilibrium without trading of securities is constrained efficient.

When securities markets are absent the model and equilibrium definition are as in section 3, except that the demand and supply schedules \( d_i, y \), and the price \( p(\cdot) \) are absent.

Let \( k_i \) be the level of specialized investment that the principal wants to implement in equilibrium. It is convenient to rewrite the problem by considering the following transformation of \( x_i \)

\[
x_i = \frac{\pi_i - \bar{R} - r k_i - \omega (1 - k_i)}{\sigma_x},
\]

where \( \sigma_x = \sqrt{k_i^2 \sigma_e^2 + \sigma_u^2} \) is the idiosyncratic volatility of \( \pi_i \) if the manager chooses \( k_i \). Suppose now that the investor recommends an investment level \( k_i \) to manager \( i \), but the latter deviates to a fraction \( \hat{k}_i \neq k_i \). Then, the distribution of \( x_i \) will in general depend on both \( k_i \) and \( \hat{k}_i \). Also, when \( \hat{k}_i = k_i \), \( x_i \) is the linear projection of \( \pi_i \) on the space orthogonal to \( \omega \) and, hence, \( x_i \) is uncorrelated with \( \omega \). Moreover, by the Gaussian assumption, \( x_i \) turns out to be the best predictor of the idiosyncratic part of \( \pi_i \).

The distribution of \( x_i \) conditional on \( \omega \) when the recommended fraction is \( k_i \) but the manager deviates to \( \hat{k}_i \) is also Gaussian with with mean and variance given by

\[
\mu_{x|\omega} = \frac{r - \omega}{\sigma_x} \left( \hat{k}_i - k_i \right), \quad \sigma_{x|\omega}^2 = \frac{1}{\sigma_x^2} \left( \hat{k}_i \sigma_e^2 + \sigma_u^2 \right).
\] (3)

In equilibrium where \( \hat{k}_i = k_i \), we have \( \mu_{x|\omega} = 0 \) and \( \sigma_{x|\omega}^2 = 1 \). This is the reason why this transformation makes it easier to solve the problem.

If we let \( F_{x|\omega}(x_i|\omega; k_i, \hat{k}_i) \) be the cdf of this conditional distribution, then in equilibrium \( F_{x|\omega}(x_i|\omega; k_i, \hat{k}_i) \) doesn’t depend on \( \omega \) nor on \( k_i \) and \( F_{x|\omega}(x_i|\omega; k_i, \hat{k}_i) = \Phi(x) \), where \( \Phi(x) \) is the cdf of a standard Gaussian distribution.

With this transformation, the contracting problem with no securities solves

\[
\max_{\xi, k} \int m(\omega) \left( x + \bar{R} + r k + (1 - k) \omega - \xi(x, \omega) \right) d\Phi(x) dF_{\omega}(\omega) \quad (P(NS))
\]
subject to:

\[ k \in \arg \max_k \int u(\xi(x, \omega)) dF_{x|\omega}(x|\omega; k, \hat{k}) dF_\omega(\omega) - C(\hat{k}), \quad \text{(IC)} \]

\[ \int u(\xi(x, \omega)) d\Phi(x) dF_\omega(\omega) - C(k) \geq \bar{u}. \quad \text{(IR)} \]

The investor maximizes the final payoff of the investment weighted by his marginal utility of consumption subject to two constraints. The first constraint requires that the compensation scheme and the recommended efforts are such that the manager finds it optimal to comply with the recommendation. The second constraint is the usual IR constraint.

The common strategy in the moral hazard literature is to relax problem P(NS) by replacing the IC constraint with its first-order condition. Formally, I can replace IC by

\[ \frac{\partial}{\partial k} \int u(\xi(x, \omega)) dF_{x|\omega}(x|\omega; k, \hat{k}) dF_\omega(\omega) \bigg|_{k=\hat{k}} - C'(k) = 0. \quad \text{(IC') } \]

The big advantage is that we can now use Lagrangian methods and solve P(NS) by taking first-order conditions. Let \( \lambda \) and \( \mu \) be the Lagrange multipliers on the IR and IC’ constraints, respectively. The following proposition characterizes the optimal contract.

**Proposition 1** Let \( k \) be the investment fraction that the principal wants to implement. The optimal contract for the model with no contingent securities solves

\[ \frac{m(\omega)}{u'(\xi(x, \omega))} = \mu + \lambda \left[ \frac{1}{\sigma_x} (r - \omega) x + \frac{k \sigma_x^2}{\sigma_z^2} (x^2 - 1) \right]. \quad (4) \]

To gain some intuition on the optimal contract (4) we can compare it to the case with no agency problem where investors are allowed to observe also \( k_i \) (I refer to this case as the "first-best"). If investors can observe \( k_i \), they will severely punish the manager who doesn’t comply with the recommendation. Assuming that the punishment can be made severe enough that we can drop the IC constraint from the problem we have that the first-best contract will be given by (4) with \( \lambda = 0 \).\(^5\) Thus, the first-best optimal contract simply allocates aggregate risk between the two risk-averse agents and the optimal payment schedule does not depend on the realization of \( \pi \) (Stoughton (1993)). However, since the manager incurs in the cost \( C(k) \), even in the first-best the optimal choice of \( k \) may be different from 1 and there may be some aggregate risk in the economy.

The form and the interpretation of (4) is made easy by the assumption of Gaussian random

\(^5\)Note that the first-best Lagrange multiplier \( \mu_{FB} \) will differ from \( \mu \) in (4).
variables. The contract has two main components. First, the usual risk-sharing component given by the left-hand side of (4). This term determines how aggregate risk is shared between the investor and the manager depending on the curvature of their utility functions. This term was the only piece in the first-best contract which completely insulated the agent from the idiosyncratic risk. However, to provide the manager with the right incentives, the payment schedule has to be a function of the new variable $x$. Incentives are provided through the right-hand side of (4).

The optimal contract has the same structure as that obtained by Holmstrom (1979), who shows that the best way to incentivize the agent is to make his payment conditional on the likelihood ratio of his action. The Gaussian assumption for $\varepsilon_i$ and $u_i$ delivers the simple expression for the likelihood ratio given by the the term that multiplies $\lambda$ in (4). From (3) we know that, if the agent invests in the specialized projects an amount $\hat{k}_i$ that is slightly lower than the suggested $k_i$, this will have three effects on the distribution of $x_i$. First, the mean of $x_i$ will be lower. Thus, when observing a lower realization of $x_i$ the principal should infer a deviation by the agent and punish him accordingly. This explains the term $r x_i$ in the right-hand side of (4). Secondly, when a lower $\hat{k}_i$ is selected, the distribution of $x_i$ will be correlated with $\omega$. Thus, a comovement between $x_i$ and $\omega$ is a signal of a possible deviation and, thus, the optimal contract punishes the agent (this explains the term $\omega x$ in the contract). Finally, a lower choice of $\hat{k}_i$ also reduces the volatility of $x_i$ and so the contract rewards the agent for realizations of $x_i$ which are far from its mean. This is the reason why the convex term $x^2$ enters the contract. From (3) we know that in equilibrium the variance of $x$ is 1, hence the optimal contract rewards the agent for realizations of $x^2$ relative to this value. In equilibrium the term that multiplies $\lambda$ in (4) has mean 0 (this is a general property of likelihood ratios).

The optimal contract uses the aggregate state $\omega$ to provide the agent with incentives. In equilibrium, the investor conditions the payment of each manager to the average performance of the other managers in the economy. This type of benchmarking, however, is different from the result stressed in the moral hazard literature with common noise following Holmstrom (1979). The latter also considers a principal-agent problem with multiple agents and correlated risk. He assumes that $\pi_i = (r + \varepsilon_i) k_i + \omega + u_i$, that is, the choice of the agent doesn’t affect the amount of aggregate risk in the project. With this alternative payoff structure the model of this paper would reproduce the classical result that, when the aggregate state is known, the optimal contract should not be conditioned upon it. Intuitively, more risk that is not related to the agent’s effort only makes it
harder to incentivize a risk-averse agent\textsuperscript{6}.

The agency problem makes it more expensive for the principal to implement a certain value of $k$. Thus, it is natural to expect a lower value of $k$ to be implemented in equilibrium.

**Proposition 2** When the choice of $k$ is not observable, equilibrium $k$ is lower (and aggregate volatility is higher).

The agency problem, therefore, cause the volatility of the economy to increase. However, the higher volatility is not a symptom of inefficiency. A social planner who has access to the same information as the investors (that is, the planner also faces the same agency problem) cannot improve on the equilibrium.

**Proposition 3** The equilibrium outcome of the economy is efficient.

The agency problem causes the economy to be more volatile and yet there is no room for policy intervention. This conclusion will change radically when agents will be allowed to trade securities.

## 5 Trades of Securities

In this section, I consider the full model where managers can trade securities contingent on the different risks in the economy. Some assumptions on the distributions of the shocks guarantee that managers will find it optimal to trade contingent securities and hedge their risks. One of the main conclusions of this model is that, under certain conditions, trading of securities has dramatically different implications for aggregate volatility and welfare depending on whether idiosyncratic or aggregate risk is traded. More specifically, I show that investors are better off when managers pool and eliminate their exposure to idiosyncratic risks.

These conclusions change substantially when aggregate risk is considered. By definition, aggregate risk cannot be pooled and eliminated, but some agents have to ultimately bear it. Also, in a symmetric equilibrium, managers receive the same contract and, thus, bear the same amount

\textsuperscript{6}We can see this in my model by observing that with this new definition of $\pi_i$ (4) becomes:

$$\frac{m(\omega)}{w^\iota(\xi(x,\omega))} = \mu + \lambda \left[ \frac{1}{\sigma_x} x + \frac{k}{\sigma^2_x} (x^2 - 1) \right],$$

and the aggregate state disappears from the incentives component of the contract ($\omega$ appears only through the risk-sharing component). As expected, the principal doesn’t uses aggregate risk to incentivize the agent. If in addition the principal was risk-neutral, then he would completely insulate the agent from aggregate risk.
of aggregate risk. Thus, the only gains from trading securities on aggregate risk are possible only if investors participate in the market. The fact that investors are willing to take some aggregate risk might seem unreasonable. After all, investors are those who design the contracts that exposes managers to some aggregate risk to incentivize them. Indeed, if investors could affect the amount of \( \omega \)-securities that are traded by managers, they would forbid these trades. However, by assumption 1, investors do not observe and, hence, cannot contract upon the trades investors make. Even if investors cannot observe the trades of securities, they will design a new contract that takes these trades into account. In particular, they will change the optimal contract so that it will not be optimal for managers to trade \( \omega \)-securities in equilibrium. Thus, while \( \omega \)-securities will not be traded on equilibrium, the possibility of trading these securities will act as a constraint on the optimal contracting problem. This stands in contrast to the case with \( \varepsilon \)-securities, which are traded on equilibrium.

The amount of insurance bought by managers depends on its equilibrium price. In the case of \( \varepsilon \)-securities, the possibility of eliminating risks by pooling them together allows for insurance to trade at an actuarially fair price\(^7\). If actuarially fair insurance is at least possible for securities contingent on idiosyncratic risk, this is no longer true when aggregate risk is traded. Intuitively, this risk has to be shared between investors and managers and the price of insurance will depend, among other things, on their marginal utility of consumption.

Since they have potentially different effects on the principal-agent problem, it is helpful to first analyze \( \varepsilon \)-securities and \( \omega \)-securities separately.

### 5.1 Securities on idiosyncratic risk

I first focus on securities contingent on idiosyncratic risk and forbid trades of securities contingent on the aggregate state. By assumption 1, investors cannot observe the trades made by managers and, thus, they can’t condition the payment schedule on this information. I start with the characterization of the equilibrium price \( p(\cdot) \) and then solve for the optimal contract. Markets are assumed to be competitive and all the agents take the price schedule \( p(\cdot) \) as given.

When \( \omega \)-securities are not allowed, the final payoff (2) of manager \( i \) who invests \( k_i \) in the specialized projects and buys a quantity \( d_{i,j,z} \) of security \( z_{j,z} \) at price \( p_{j,z} \) is

\[
\Pi_i^{\omega} = \pi_i + \int (z_{j,z} - p_{j,z}) \ d_{i,j,z} \ dj
\]

\(^7\)Of course, in the presence of a cost to trade securities (as in sections 6 and 7), market power, or other frictions, the price of insurance would deviate from the actuarially fair price.
Let $F_{\Pi_i, \omega}(\Pi_i, \omega|k_i, d_i, p(\cdot))$ be the distribution of the pair $(\Pi_i, \omega)$ for given choice of $k_i$, demand schedule $d_i$, and price schedule $p(\cdot)$.

The agent now chooses both the investment $k_i$ and the demand schedule $d_i$ for given price schedule $p(\cdot)$. The IC constraint for the contracting problem becomes:

$$\left(k_i, d_i\right) \in \arg\max_{k, d_i} \int u\left(\xi_i(\Pi_i, \omega)\right) dF_{\Pi_i, \omega}(\Pi_i, \omega|\hat{k}_i, \hat{d}_i, p(\cdot)) - C\left(\hat{k}_i\right)$$

(5)

As for the analysis of section (4), it is convenient to consider the linear projection of $\Pi_i^n$ onto the space orthogonal to $\omega$. Formally, define $x_i$ as follows:

$$x_i = \frac{\Pi_i^n - \bar{R} - r \ k_i - \omega (1 - k_i) + \int p_{j, \xi} \ d_{i, j, \xi} \ d\xi \ dj}{\sigma_{i,x}},$$

(6)

where $\sigma_{i,x}^2 \equiv Var\left(x_i\right)$ is the equilibrium variance of $x_i$ (that is, when $\hat{k}_i = k_i$).

Let $\Phi_\varepsilon$ be the cdf of a Gaussian distribution with mean 0 and variances $\sigma_\varepsilon^2$. As lemma 1 shows, the equilibrium price is such that the idiosyncratic risk is traded at an actuarially fair price.

**Lemma 1** The equilibrium price of an Arrow security $z_{i, \hat{\xi}}$ is $p_{i, \hat{\xi}} = \phi_\varepsilon\left(\hat{\xi}\right)$.

The problem is now whether the principal wants the agent to buy full insurance at the price of lemma 1. The answer is complicated by the fact that the idiosyncratic shock $\varepsilon_i$ multiplies $k_i$ in the final payoff of the manager. Thus, a more volatile $\varepsilon_i$ can potentially convey some information about the actual choice of $k_i$. Thus, it may be optimal for the principal if the manager traded securities so as to increase the variance of $\varepsilon_i$. This reasoning applies only to the securities contingent on the shock of the sector that is matched to the manager. If a manager traded securities conditional on the shocks of other sectors, this would only add noise to his profits and worsen the agency problem.

The main complication with the fact that a more volatile $\varepsilon_i$ contains information about $k_i$ is that we are allowing any $\varepsilon$-security to be traded. By trading in the securities market, the manager can buy $\varepsilon$-securities that change the distribution of $\varepsilon$. The distribution of (6) will not be necessarily Gaussian, since the agent can potentially demand any quantity $d_{i, j, \xi}$ of any security $z_{j, \xi}$. The principal has to incentivize manager $i$ to choose a demand schedule $d_i$ and, thus, a whole distribution $F_{\Pi_i, \omega}(\Pi_i, \omega|k_i, d_i, p(\cdot))$. Thus, the quantity of securities demanded by the agent depends on the optimal contract which, in turn, has to be chosen by the principal so that the agent demands the right amount of securities.

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When full insurance is optimal for the principal the problem becomes simpler. To see this in a more formal way, note that an agent wants to buy full insurance whenever his payment schedule $\xi$ makes his problem concave in $x$. Conjecture now that the agent buys full insurance and solve for the optimal $\xi$. If this payment schedule makes the problem of the agent concave, then the conjecture is verified and we have found the solution to original the problem.

The problem is then to find conditions under which the principal wants the agent to buy full insurance. It is easy to see that full insurance would be optimal in the absence of the error term $u_i$ in the payoff of the manager. If $u_i$ was absent, full insurance would make the choice of $k_i$ perfectly observable and the agency problem would disappear. This would lead to the first-best outcome which, by definition, is the best outcome for the principal. On the other side, suppose that the volatility of $\omega$ is close to 0 and so is the mean $r$. In this case, it is harder for the principal to identify the value of $k_i$ chosen by the agent. A more volatile $\varepsilon_i$ makes the distribution of $\Pi^m_i$ more sensitive to $k_i$ and helps the principal.

In the appendix, I derive a condition for full insurance to be optimal. This condition is related to the volatility of the likelihood ratio of the distribution $F_{x_i|\omega}$. Intuitively, the likelihood ratio can be seen as a measure of how informative are the signals about the choice of $k_i$. Signals are more informative when the likelihood ratio is more volatile and, therefore, a more volatile likelihood ratio leads to a better outcome for the principal. The condition in the appendix requires that the likelihood ratio is most informative when the variance of $\sigma^2_{\varepsilon}$ is 0, that is, when the agent is fully insured. This condition is more likely to be satisfied for higher values of $r$, for lower values of $\sigma^2_u$, and for higher values of $\sigma^2_{\omega}$. This confirms the intuition in the discussion above.

The problem would be simpler if the agent was allowed to trade only linear securities, that is, securities with a payoff $q \varepsilon_i$, for some scalar $q$. These securities, in fact, preserve the normality of the $F_{x_i|\omega}$ and I can derive intuitive sufficient conditions on the parameters of the model for which full insurance is optimal.

**Lemma 2** Assume that only linear $\varepsilon$-securities can be traded. If $0 \leq \sigma^2_u \leq (r - \omega)^2$, $\forall \omega$, then full insurance is optimal.

The condition is easy to interpret. Take for example $r = 0$. Then the condition says that full insurance is optimal whenever the realizations of the random variable $\omega$ are "big" enough$^8$. This

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$^8$Remember that $\omega$ is not restricted to be continuous, but it can be a discrete random variable with mean 0. For example, $\omega_H > 0 > \omega_L = -\omega_H$. 

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sufficient condition may seem restrictive because it has to hold for any problem (of course, under the assumption of linear securities). For each specific problem, however, it is possible to weaken this assumption (for example, to substitute it with some appropriate average of $\omega$).

Let’s conjecture that it is optimal for the agent to buy full insurance. Formally, this means that an agent who invest $k_i$ in the specialized project will demand $-\hat{\varepsilon}_i k_i$ units of the Arrow securities $z_i, \forall \hat{\varepsilon}$, and zero units of all the other securities. Also, from lemma 1, we know that the cost of this insurance is

$$\int p_{i,\hat{\varepsilon}} d_{i,j,\hat{\varepsilon}} d\hat{\varepsilon} = -k_i \int \hat{\varepsilon} d\Phi_\varepsilon (\hat{\varepsilon}) = 0$$

Combining these two results implies that the profits of the agent are given by

$$\Pi_i^m = \pi_i - k_i \varepsilon_i$$

Similarly, (6) becomes

$$x_i = \frac{\Pi_i^m - \bar{R} - r k_i - \omega (1 - k_i)}{\sigma_u}.$$  

As usual, in equilibrium where the agent trades $-k_i \varepsilon_i$ the random variables $x_i$ and $\omega$ are uncorrelated.

With a slight abuse of notation, let $F_{x_i|\omega}(x_i|\omega; k_i, \hat{k}_i, -k_i \varepsilon_i, p(\cdot))$ be the cdf of the Gaussian random variable $x_i$ conditional on $\omega$, when the agent chooses $\hat{k}_i$ and trades $-k_i \varepsilon_i$. Note that when the agent buys full insurance, the distribution $F_{x_i|\omega}(x_i|\omega; \hat{k}_i, -k_i \varepsilon_i, p(\cdot))$ will not depend on $p(\cdot)$. However, out of the equilibrium, if the agent deviates to a different portfolio allocation, then this distribution will depend on the price schedule $p(\cdot)$.

The moments of $F_{x_i|\omega}(x_i|\omega; k_i, \hat{k}_i, -k_i \varepsilon_i, p(\cdot))$ are given by

$$\mu_{x|\omega} = \frac{r - \omega}{\sigma_u} \left( \hat{k}_i - k_i \right), \quad \sigma_{x|\omega}^2 = \left( \hat{k}_i - k_i \right)^2 \frac{\sigma_u^2}{\sigma_u^2} + 1 \quad (7)$$

In equilibrium, $\hat{k}_i = k_i$ and the agent trades $-k_i \varepsilon_i$, so the moments are $\mu_{x|\omega} = 0$ and $\sigma_{x|\omega}^2 = 1$. Thus, once again we have that $F_{x|\omega}(x|\omega; k_i, k_i, -k_i \varepsilon_i, p(\cdot)) = \Phi (x)$.

We are now ready to solve the optimal contracting problem where the agent buys full insurance and both the principal and the agent take the price of $\varepsilon$-securities as given. As shown in (5), the agent now faces two choices. First, he has to decide what fraction $k_i$ to invest in the specialized projects. Second, he has to decide the quantity of Arrow securities to trade.
Formally, the optimal contract solves (omitting subscripts \( i \) for convenience):

\[
\max_{\xi,k,d} \int m(\omega) \left( x + \bar{R} + r k + (1 - k) \omega - \xi(x,\omega) \right) d\Phi(x) dF(\omega)
\]  

(P(\varepsilon))

subject to:

\[
(k, d) \in \arg \max_{\hat{k}, \hat{d}} \int u(\xi(x,\omega)) F_{x|\omega}(x|\omega; k, \hat{k}, \hat{d}) - C(\hat{k}), \quad (IC)
\]

\[
\int u(\xi(x,\omega)) d\Phi(x) dF(\omega) - C(k) \geq \bar{u}. \quad (IR)
\]

This problem is similar to P(NS) in the case with no securities, except that now the IC constraint takes into account the two choices of the agent. Under the assumptions that make full insurance optimal, we can considerably simplify this problem. To see this, relax problem P(\varepsilon) by dropping the constraint IC on the choice of \( d_i \). If the contract that solves the relaxed problem is such that the agent wants to buy full insurance then this contract must also solve the original problem with the full IC constraints. Formally, we look for the values of \( \xi \) and \( k \) that solve

\[
\max_{\xi,k} \int m(\omega) \left( x + \bar{R} + r k + (1 - k) \omega - \xi(x,\omega) \right) d\Phi(x) dF(\omega)
\]

subject to:

\[
k \in \arg \max_{\hat{k}} \int u(\xi(x,\omega)) F_{x|\omega}(x|\omega; k, \hat{k}, -k, \varepsilon, p(\cdot)) - C(\hat{k}),
\]

\[
\int u(\xi(x,\omega)) d\Phi(x) dF(\omega) - C(k) \geq \bar{u}.
\]

Once again, I conjecture that the FOA is valid for this problem, relax the IC constraint by replacing it with its first-order condition, and then verify that this conjecture is valid at the optimal contract. The FOA allows us to solve for the optimal contract using Lagrangian methods as shown in the next proposition.

Proposition 4 Let \( \lambda \) and \( \mu \) be the Lagrange multipliers on the IC and IR constraints, respectively, and suppose full insurance is optimal. The optimal payment schedule \( \xi(x,\omega) \) satisfies:

\[
\frac{m(\omega)}{u'(\xi(x,\omega))} = \mu + \frac{1}{\sigma_u} (r - \omega) \ x. \quad (8)
\]

Proposition 4 immediately implies that the approach of relaxing the IC constraint and then verify that the agent wants to buy full insurance is valid. This follows from assumption 2 which
guarantees that under the optimal contract (8) the agent’s problem is concave. In turn, concavity implies that the agent wants to buy full insurance at the actuarially fair price of lemma 1. Concavity of the agent’s problem also implies that the FOA is valid for this problem (Jewitt (1988)).

The shape of the new optimal contract is similar to (4) and, not surprisingly, the main difference is the absence of the convex term $x^2$. In equilibrium, the agent buys full insurance against his idiosyncratic risk and, thus, the principal does not reward him if his profits are volatile.

To gain more intuition about proposition 4 and about how $\varepsilon$-securities affect the equilibrium, I allow the principal to use $\varepsilon_i$ when providing incentives to the agent. Formally, I relax part (a) and part (b) of assumption 1 and allow the principal to write a contract on the shock of the specific sector to which the manager is matched. Thus, the payment $\xi_i$ can be conditioned on both shocks $\omega$ and $\varepsilon_i$. Similarly to the case with $\omega$-securities analyzed in the following section, if the principal can observe the realization of $\varepsilon_i$ then it is easy to see that there are no gains from allowing trades of $\varepsilon$-securities. Instead, the presence of $\varepsilon$-securities can only hurt investors to the extent that they cannot limit these trades. However, the scope of this exercise is to compare the equilibrium of the model where the principal provides incentives through the financial markets to case where the principal can contract on $\varepsilon_i$ directly. Therefore, I also forbid trades of securities.

Formally, the contracting problem is similar to P(NS) of section 4 with the difference that now $\xi$ can also be a function of $\varepsilon$. I can then define $x$ as follows:

$$x_i = \pi_i - \bar{R} - r \ k_i - \omega (1 - k_i) - \varepsilon_i k_i \sigma_u,$$

so that, in equilibrium where $\hat{k}_i = k_i$, I have that $\pi_i = u_i / \sigma_u$. The next lemma describes the optimal contract under these new assumptions.

**Lemma 3** Let $\lambda$ and $\mu$ be the Lagrange multipliers on the IC and IR constraints, respectively. The optimal payment schedule $\xi(x, \omega, \varepsilon)$ satisfies:

$$\frac{m(\omega)}{w'(\xi(x, \omega, \varepsilon))} = \mu + \lambda \frac{1}{\sigma_u} (r - \omega + \varepsilon) \ x. \quad (9)$$

The contract (9) treats the two shocks $\omega$ and $\varepsilon_i$ symmetrically. The agent is punished if $x_i$ is correlated with the aggregate state $\omega$ and is *rewarded* if $x_i$ is correlated with the idiosyncratic shock $\varepsilon_i$. In fact, a correlation between $x_i$ and $\varepsilon_i$ is a sign that the agent selected $\hat{k}_i > k_i$. Lemma 3 shows that, when the principal can contract on $\varepsilon_i$, he will choose a contract that differs from (8).
The principal does not fully insure the manager against the idiosyncratic risk, but exposes him to some $\varepsilon$-risk. Of course, under the assumptions of lemma 3 the principal is better off relative to the case of proposition 4.

Lemma 3 is interesting also for another reason. Suppose that the principal has access to some information about $\varepsilon_i$. For example, suppose that the principal receives a partially informative signal about the identity $i$ of the specialized investment and he can use this signal in the contract. How will the principal use this information? Lemma 3 suggests that the principal will expose the agent to some $\varepsilon$-risk by conditioning the optimal contract to this signal and the optimal payment will resemble (9).

I can now state the main results of this section. The key question is what happens to the aggregate volatility of the economy and to welfare when securities contingent on $\varepsilon$-risk are traded in the market. Since managers face a binding IR constraint, welfare here is simply the utility of the representative investor. As the next proposition shows, under the conditions that make full insurance of the $\varepsilon$-risk optimal for the principal, these securities reduce aggregate volatility and increase welfare.

**Proposition 5** Securities contingent on $\varepsilon$-risk increase equilibrium $k$ (and, thus, lower aggregate volatility) and increase welfare in the economy.

As discussed above, securities on $\varepsilon$-risk make it easier for a principal to identify whether the agent has deviated or not. This lowers the cost of implementing higher values of $k$ and, thus, lowers aggregate volatility. Finally, a principal who can implement higher values of $k$ more cheaply can also achieve higher levels of welfare.

**5.2 Securities on aggregate risk**

In this section, I only allow trades of securities contingent on the aggregate state $\omega$. This case is very different from the previous section where only the idiosyncratic risk was hedgeable. In the symmetric equilibrium considered in this paper, all the managers are perfectly symmetric and, hence, share the same quantity of aggregate risk. Thus, the only way to hedge the aggregate risk is to transfer it to investors.

From a mathematical point of view, the main difference between these two types of risks is that investors can always condition the payment schedule on aggregate risk if they find it optimal to do so. Thus, the fact that managers can trade away some of their aggregate risk should only act as
a constraint on the contracting problem. I show that the possibility for managers to trade away some of the aggregate risk through $\omega$-securities weakens the incentives that investors can provide in equilibrium. Therefore, contrary to the case where only idiosyncratic risk can be hedged, the fact that managers can transfer some risk to investors, makes the agency problem worse. A direct implication is that the existence of $\omega$-securities reduces welfare. Again, the intuition for this result is very simple: if transferring some aggregate risk was optimal for the principal, the optimal contract would already take this into account. The key friction, therefore, is that trades are not observable. Intuitively, if investors could contract upon the quantities of $\omega$-securities purchased by managers, they would provide incentives for the latter to stay out of this market.

Securities contingent on aggregate risk differ from those contingent on idiosyncratic risk also because the former type of risk cannot be eliminated and some agent in the economy has to ultimately bear it. In turn, this implies that the price to hedge aggregate risk cannot be actuarially fair as it was for $\varepsilon$-securities. In equilibrium, aggregate risk will be transferred back to the principals who are risk-averse and, thus, demand a compensation to take this risk.

The fact that the principal can always condition the payment to the aggregate state and replicate any portfolio of $\omega$-securities chosen by the agent implies that I can focus on the case where there is no trading of $\omega$-securities. To see this, suppose that manager $i$ demands a quantity $d_{i,\hat{\omega}}$ of security $z_{\hat{\omega}}$ at price $p_{\hat{\omega}}$, generates profits equal to $\Pi_i^m = \pi_i + \int (z_{\hat{\omega}} - p_{\hat{\omega}}) d_{i,\hat{\omega}} \, d\hat{\omega}$ and obtains a payment $\xi_i (\Pi_i^m, \omega)$. The principal can always define a new payment $\tilde{\xi}_i (\pi_i, \omega) = \xi_i (\Pi_i^m, \omega), \forall \pi_i, \omega, \Pi_i^m$, such that the agent finds it optimal not to trade $\omega$-securities. Clearly, this is suboptimal for the principal who had chosen $\xi_i$ over $\tilde{\xi}_i$ in the first place.

**Proposition 6** The optimal contract is such that on the equilibrium path the manager does not trade $\omega$-securities, that is, $d_{i,\hat{\omega}} = 0, \forall \hat{\omega}, i$.

Thus, there are no trades in the securities market when only the $\omega$-risk can be traded. However, even in the absence of trades, prices are still determined by the assumption of competition. The representative investor is risk-averse with stochastic discount factor $m (\omega)$. He is then willing to trade $\omega$-securities (in fact, an infinite amount of them) whenever the price of these securities is above their marginal utility of consumption. Therefore, in equilibrium the price of these securities has to be such that the representative investor is indifferent on the quantity of securities to trade.

**Lemma 4** The equilibrium price of an Arrow security $z_{\hat{\omega}}$ is $p_{\hat{\omega}} = m (\hat{\omega}) f_\omega (\hat{\omega}) / \mathbb{E} [m (\omega)]$.

The price of insurance against state $\hat{\omega}$, $m (\hat{\omega}) f_\omega (\hat{\omega}) / \mathbb{E} [m (\omega)]$, is a combination of the probability density that $\hat{\omega}$ is realized (this is the same as in the equilibrium with $\varepsilon$-securities) and the
principal’s marginal utility of consumption. The more valuable is consumption for the principal in
the state of the world \( \omega \), that is, the higher is \( m(\omega) / \mathbb{E}[m(\omega)] \), the higher will be the price of a
security that pays in that state.

The contracting problem with \( \omega \)-securities is different from \( P(\varepsilon) \) in an important way. Contrary
to the case with \( \varepsilon \)-securities, here we cannot conjecture that the agent doesn’t want to trade the \( \omega \)-
risk and then verify this conjecture. In fact, if we were to relax the problem by assuming no trades of
\( \omega \)-securities and derive the optimal contract, this contract would not satisfy the initial conjecture:
an agent receiving the payment schedule that solves the relaxed problem has an incentive to deviate
and trade \( \omega \)-securities. This implies that we have to explicitly incorporate the portfolio choice of
the agent into the optimal contracting problem.

In the case with only \( \omega \)-securities, the profits of a manager are

\[
\Pi^m_i = \pi_i + \int (z_\omega - p_\omega) d_i, \omega d\omega,
\]

and in equilibrium where no securities are traded we have \( \Pi^m_i = \pi_i \). Similarly, (6) becomes

\[
x_i = \frac{\Pi^m_i - \tilde{R} - r \ k_i - \omega (1 - k_i)}{\sigma_x}.
\]

Let \( F_{x_i|\omega} \left( x|\omega; k_i, \hat{k}_i, \hat{d}_i, p(\cdot) \right) \) be the conditional distribution of \( x_i \) when the agent chooses \( \hat{k}_i \) and
instead of \( k_i \) and demands \( \hat{d}_i \). In equilibrium, the principal wants the agent to choose \( \hat{d}_i, \omega = 0 \), \( \forall \omega \),
and of course \( \hat{k}_i = k_i \). Again, if the agent follows the optimal contract, the equilibrium distribution
becomes \( F_{x_i|\omega} \left( x|\omega; k_i, \hat{k}_i, \hat{d}_i, p(\cdot) \right) = \Phi(x) \).

Let \( \bar{U}_\omega (k_i, p(\cdot)) \) be the value of the best deviation available to the manager:

\[
\bar{U}_\omega (k_i, p(\cdot)) = \max_{\hat{k}, \hat{d}} \int u(\xi(x, \omega)) dF_{x_i|\omega} \left( x|\omega; k_i, \hat{k}, \hat{d}, p(\cdot) \right) dF_\omega (\omega) - C(\hat{k})
\]

So \( \bar{U}_\omega (k_i, p(\cdot)) \) is the value for a manager of deviating to a different \( \hat{k} \) and a different demand
schedule \( \hat{d} \). Thus, to prevent the manager from deviating, the optimal contract has to be such that

\[
\int u(\xi(x, \omega)) d\Phi(x) dF_\omega (\omega) \geq \bar{U}_\omega (k_i, p(\cdot)).
\]
subscripts $i$ for convenience):

$$\max_{\xi, k, d} \int m(\omega) \left( x + \bar{R} + r k + (1 - k) \omega - \xi(x, \omega) \right) d\Phi(x) dF_\omega(\omega) \quad (P(\omega))$$

subject to:

$$\frac{\partial}{\partial k} \int u(\xi(x, \omega)) dF_{x|\omega}(x|\omega; k, \hat{k}, d = 0, p(\cdot)) dF_\omega(\omega) \bigg|_{k=k} - C'(k) = 0, \quad (IC_k)$$

$$\frac{\partial}{\partial \hat{d}} \int u(\xi(x, \omega)) dF_{x|\omega}(x|\omega; k, \hat{k}, \hat{d}, p(\cdot)) dF_\omega(\omega) \bigg|_{\hat{d}=0} = 0, \quad (IC_\omega)$$

$$\int u(\xi(x, \omega)) d\Phi(x) dF_\omega(\omega) - C(k) \geq \bar{u}. \quad (IR)$$

The next lemma characterizes the optimal contract with $\omega$-securities.

**Lemma 5** Let $\lambda$, $\nu_\omega$, and $\mu$ be the Lagrange multipliers on the $IC_k$, $IC_\omega$, and $IR$ constraints, respectively. The optimal payment $\xi(x, \omega)$ satisfies:

$$\frac{m(\omega)}{u'(\xi(x, \omega))} = \mu + \lambda \frac{1}{\sigma_x} (r - h(\omega)) x + \lambda \frac{k}{\sigma_x^2} (x^2 - 1) \quad (11)$$

where $h(\omega) = \omega - \nu_\omega (1 - \mathbb{E}[m(\omega)])$ and $\nu_\omega \geq 0$.

Problem $P(\omega)$ shows that the presence of these securities acts as an extra constraint on the contracting problem. Intuitively, this should lead to lower welfare. Also, the presence of $\omega$-securities makes it more costly for the principal to implement higher levels of $k$ and, thus, aggregate volatility in the economy increases. The proof of these results is not immediate since the marginal utility of the principals $m(\omega)$ is endogenous. The next proposition confirms our original intuition and represents the main result of this section.

**Proposition 7** Securities contingent on $\omega$-risk decrease equilibrium $k$ (and, thus, increase aggregate volatility) and reduce welfare in the economy.

6 Full model

In this section I allow both types of securities to be traded. The previous analysis showed that the two types of securities tend to have opposite effects on aggregate volatility and welfare. It is natural
to expect that when we introduce both types of securities the overall effect will be ambiguous. More specifically, we can expect welfare to increase when it is possible to hedge the \( \varepsilon \)-risk and the opposite result to hold for the \( \omega \)-risk.

To derive further results, I generalize the model in two ways. First, I assume that the economy is populated by a firm (which I refer to as “issuer”) which creates securities. There are \( N \) issuers in the economy, denoted by \( \ell \), which are owned by investors. The securities created can then be traded in the Walrasian market by paying a fixed cost per trade. In general, the portfolio problem with \( \varepsilon \)-securities can become very hard to solve with most assumptions on the costs of trading securities. An easy departure from the basic model of the previous sections is to assume that, every time an issuer sells a security to a manager, the former has to pay a fixed cost\(^9\). This assumption has the advantage to allow for some flexibility in the cost of securities without making the model intractable. In equilibrium, prices of securities will reflect the presence of these costs. The key feature of having a fixed cost per trade is that equilibrium prices will resemble a two-part tariff, that is, the price of a security will be given by a fee (which is independent of the specific security and is high enough to cover the fixed cost) plus a term which is the same as those in lemmas 1 and 4\(^{10}\).

Secondly, I assume that transactions in the financial market are observable, that is, I relax part (a) of assumption 1. As I show later, the presence of the fixed cost implies that each manager will trade at most once and with only one issuer. Thus, a transaction has to be interpreted as any trade between a manager and an issuer, independently of how many securities are exchanged.

The fact that now managers have to pay a fixed fee per trade to buy securities implies that it is optimal for them to trade with only one issuer (and, thus, pay the fee only once). For example, a manager who in the previous sections was buying two Arrow securities from two issuers (or even the same issuer), now will prefer to combine the two Arrow securities and make only one trade. Thus, I let agents trade insurance contracts which are general functions of the underlying Arrow securities. A manager will find it optimal to buy this insurance contracts instead of the Arrow securities to save on the trading costs.

Every insurance contract can be contingent on idiosyncratic risks or aggregate risk. Let
\[
J^\varepsilon = \{ s : \mathbb{R}^{[0,1]} \to \mathbb{R} \text{ such that } \int \int s (\{ \varepsilon_i \}) d\Phi_\varepsilon (\varepsilon_i) \, d\varepsilon_i = 0 \},
\]
where \( \mathbb{R}^{[0,1]} \) is the set of functions from the unit interval to \( \mathbb{R} \). Similarly, for the case of \( \omega \)-securities, let

\(^9\)This assumption in the context of a model with endogenous creation of securities was first proposed by Pesendorfer (1995) (see Allen and Gale (1988), Allen and Gale (1991), Bisin (1998) for alternative assumptions).

\(^{10}\)Makowski (1979) first derives the result that the price schedule in a competitive equilibrium with fixed costs of trading can be represented as a two-part tariffs.
\[ J^\omega = \{ s \in \mathbb{R} \to \mathbb{R} \text{ such that } \int s(\omega) \, dF_\omega(\omega) = 0 \}. \] Finally, let \( J = J^\varepsilon \cup J^\omega \) be the space of all securities.

Let \( s_{\ell,m}^\varepsilon \in J^\varepsilon \) be the \( m \)-th insurance contract issued by issuer \( \ell \) that is in principle contingent on all the idiosyncratic shocks of the economy. Similarly, \( s_{\ell,m}^\omega \in J^\omega \) the corresponding insurance contract contingent on aggregate risk. Each contract can be represented as a function of the Arrow securities defined above. Let \( p : J \to \mathbb{R}_+ \) be the price schedule of these contracts. As it will be clear after introducing the costs of trading securities, in equilibrium \( p(\cdot) \) is not a linear function over \( J \). To denote that agent \( i \) is not participating in the market for \( \varepsilon \)-risk (\( \omega \)-risk) I will simply write \( s_i^\varepsilon = \emptyset \) (\( s_i^\omega = \emptyset \)). I assume that only issuers can create and sell securities and, thus, a trade can occur only between a manager and an issuer. Marketing insurance contracts to potential buyers is a costly process. I assume that an issuer who sells an insurance contract conditional on \( \varepsilon \)-risk (\( \omega \)-risk) has to pay a fixed cost \( c_\varepsilon > 0 \) (\( c_\omega > 0 \)).

For simplicity, the trading costs are common across issuers and do not depend on the state where the security pays-off nor on the identity \( i \) of the project on which they are contingent. In other words, costs will differ only on whether the securities depend on idiosyncratic or aggregate risk. As it will be clear in following sections, an interesting comparative static exercise will be to vary the costs \( c_\varepsilon \) and \( c_\omega \) and study the implications for the managers’ investment decisions. This analysis will be central in section (7), where I introduce taxes to fix the inefficiency of the equilibrium. Indeed, in this model taxes rise the costs of issuing and selling securities and, thus, are isomorphic to a particular increase of the trading costs.

**Payoffs.** The fixed cost for every trade immediately implies that each manager will buy at most one insurance contract of each type (idiosyncratic or aggregate) from at most one issuer. This also implies that we can identify each insurance contract with the index of the manager \( i \) who buys it. Thus, \( s_{\ell,i}^\varepsilon \) and \( s_{\ell,i}^\omega \) will be the \( \varepsilon \)-contract and \( \omega \)-contract created by issuer \( \ell \) and customized to manager \( i \), respectively. I will simply write \( s_{\ell,i} \) to denote any insurance contract, idiosyncratic or aggregate, sold by issuer \( \ell \) to manager \( i \). Given that in equilibrium issuers make zero profits, it is without loss of generality assume that each issuer will face an equal mass of measure \( 1/N \) of agents. Without loss of generality, I assume that issuer \( \ell \) trades with all the managers with index in \( [\lfloor (\ell - 1)/N \rfloor, \ell/N] \).

The final profits generated by manager \( i \) who buys contracts \( s_{\ell,i}^\varepsilon \) and \( s_{\ell,i}^\omega \) from issuer \( \ell \) are

\[
\Pi_i^m = \pi + s_{\ell,i}^\varepsilon - p(s_{\ell,i}^\varepsilon) + s_{\ell,i}^\omega - p(s_{\ell,i}^\omega) .
\] (12)
Similarly, the profits of issuer $t^{11}$ are

$$\Pi'_t = \int_{(\ell-1)/N}^{\ell/N} (p(s^ {\xi}_{t,i}) - s^ {\xi}_{t,i}) \, di - \frac{c_{\xi}}{N}. $$

The issuer is owned by investors, so it maximizes $E \left[ m(\omega) \Pi'_t \right]$. As already discussed above, the equilibrium price of an insurance contract will now contain a fee to cover the fixed cost. Since $\omega$-contracts are not traded on equilibrium, all prices at which issuers do not want to trade $\omega$-contracts can be consistent with the equilibrium. Here, I am going to select the equilibrium price that leaves issuers indifferent between trading or not.

**Lemma 6** The equilibrium price of an insurance contract $s^ {\xi}_{t,i}$ is given by a two-part tariff:

$$p^ {\xi} + \int s^ {\xi}_{t,i} \left( \hat{\xi} \right) d\Phi_x \left( \hat{\xi} \right) d\xi \left( \hat{\xi} \right) + \int s^ {\omega}_{t,i} \left( \hat{\omega} \right) m(\omega) dF_\omega \left( \hat{\omega} \right) / E \left[ m(\omega) \right],$$

where $p^ {\xi} = c_{\xi}$ and $p^ {\omega} = c_{\omega}$.

Except for the costs of trading contracts, the model of this section resembles the particular cases studied in sections 5.1 and 5.2. In particular, it is still true that the principal wants the manager not to trade $\omega$-contracts. Finally, I am going to assume that the cost of trading $\varepsilon$-contracts is small enough that it is optimal for the principal to pay the fee $p^ {\xi}$ and have the agent trade $\varepsilon$-contracts.

Assume now that trading costs are zero (so that $c_{\varepsilon} = c_{\omega} = 0$) and, as we did in section 5.1, and assume that in equilibrium it is optimal that the agent buys full insurance. The principal wants the agent to trade only the $\varepsilon$-risk and observes the transactions made by the agent. However, the principal does not observe the type of insurance contract that the agent is trading, that is, whether this contract is contingent on $\varepsilon$-risk or $\omega$-risk. The agent is constrained by the principal to make only one transaction, thus, the only feasible deviation is to stop trading $\varepsilon$-securities and trade only $\omega$-contracts. This double-deviation cannot be detected by the principal who will still observe that only one transaction has occurred.

Formally, let $\tilde{U}_{\varepsilon,\omega} (k, p(\cdot))$ be the value of the double-deviation for the agent, that is,

$$\tilde{U}_{\varepsilon,\omega} (k, p(\cdot)) = \max_{k, s^ {\varepsilon} \neq \emptyset} \int u\left( \xi (x, \omega) \right) dF_{x|\omega} \left( x|\omega; k, \hat{k}, s^ {\varepsilon}, p(\cdot) \right) dF_\omega (\omega),$$

where $F_{x|\omega} \left( x|\omega; k, \hat{k}, s^ {\varepsilon}, s^ {\omega}, p(\cdot) \right)$ is the conditional distribution of $x$ when the agent trades only the $\omega$-contract $s^ {\omega}$.

---

$^{11}$To simplify notation, I am writing profits of issuers using the fact that $\omega$-contracts will not be traded in equilibrium.
The contracting problem becomes:

$$\max_{\xi, \kappa} \int m(\omega) \left( x + \bar{R} + r \kappa + (1 - \kappa) \omega - p^\kappa - \xi(x, \omega) \right) \Phi(x) dF_\omega(\omega)$$  \hspace{1cm} (P(full))

subject to:

$$\frac{\partial}{\partial \kappa} \int u(\xi(x, \omega)) dF_{x|\omega}(x|\omega; \kappa, \hat{k}, s^\epsilon = -k \epsilon, s^\omega = \varnothing) dF_\omega(\omega) \bigg|_{\hat{k} = \kappa} - C'(\kappa) = 0, \quad \text{(IC}_k\text{)}$$

$$\bar{U}_{\epsilon \omega}(k, p(\cdot)) \leq \bar{u}, \quad \text{(IC}_{\epsilon \omega}\text{)}$$

$$\int u(\xi(x, \omega)) dF(x) dF_\omega(\omega) - C(k) = \bar{u}. \quad \text{(IR)}$$

Here, $F_{x|\omega}(x|\omega; \kappa, \hat{k}, s^\epsilon = -k \epsilon, s^\omega = \varnothing)$ is the conditional distribution of $x$ when the agent buys full insurance against the $\epsilon$-risk and doesn’t trade $\omega$-contracts. The optimal contract is a combination of 8 and 11, so I will not repeat it here.

The next proposition contains the effects on equilibrium $k$ and welfare of changing the trading costs of the two types of insurance contracts.

**Proposition 8** For low enough prices $c_\epsilon$ and $c_\omega$, when both types of insurance contracts are traded, equilibrium $k$ and welfare decrease with the cost of $\epsilon$-contracts, $c_\epsilon$, and increase with the cost of $\omega$-contracts, $c_\omega$.

Proposition 8 shows that changing the price of the two types of securities has opposite effects on equilibrium volatility and welfare. These comparative static results extend the conclusions derived separately in sections 5.1 and 5.2 to the case with fixed costs of trading. These effects will be the source of the trade-off faced by the social planner, which I consider in section 7, who has the power to tax transactions in the securities markets.

## 7 Efficiency and optimal policy

### 7.1 Taxation

This section studies the efficiency properties of the equilibrium derived in section 6. Of course, whether the equilibrium is socially optimal will depend on the powers we grant to the social planner. In particular, different conclusions on the efficiency of the equilibrium – and, thus, different policy prescriptions – follow from different assumptions on the information available to the social planner.
A stark way to see this is by going back to the intuition behind the welfare implications of cheaper \( \omega \)-risk insurance in section 5.2. There, I proved that, since the principal could always replicate the market allocation, he could only suffer from trades contingent on the \( \omega \)-risk. It follows immediately that a social planner, who maximizes the welfare of investors and who can observe the trades of the different securities, could easily improve on the equilibrium allocation by forbidding the trades of \( \omega \)-contracts. For this reason, in what follows I will restrict the social planner’s information set by assuming that he doesn’t have access to more information than the representative investors.

The inefficiency of the equilibrium follows from the fact that investors fail to coordinate the contract they design for the managers with the incentives faced by the issuers. This coordination failure is related to the conclusions in agency problems with multiple agents and common principal (Holmstrom and Milgrom (1990), Itoh (1993), Mookherjee (1984)). In most of these models, both agents face an agency problem and can potentially interact with each other. The principal has to design the contract by taking into account this interaction. In this model, the two agents are the managers and the issuers who interact through the securities market. Issuers are owned by the principal and they don’t face any agency problem. However, the principal fails to understand how their activity affects prices and, thus, the incentives of the managers. The planner, then, can restore efficiency by fixing this coordination problem.

In this section, I consider two separate cases. First, I show that if the planner can observe the total number of transactions in the economy, but investors cannot observe the trading activity of managers (as in section 3), then a small positive on transactions can increase welfare in the economy. This tax makes it harder for managers to deviate and trade \( \omega \)-contracts. Secondly, I show that when investors can observe the transactions made by each manager (as in section 6), then they can do better than the social planner and the transaction tax is redundant.

Suppose for now that the planner cannot observe the total quantity of transactions, but the investors do not observe any trading activity made by the managers (as in the model of section 3).

Let \( e (\tau) \) be the equilibrium for a given value of the tax \( \tau \)\(^{12}\) and \( W (e (\tau)) \) the equilibrium welfare of the investors. It is immediate to derive that \( e (\tau) \) resembles the equilibrium derived in section 5, except that the insurance fees are now \( p^e + \tau \) and \( p^\omega + \tau, \tau > 0 \). The social planner chooses \( \tau \) so as to maximize welfare for investors subject to allocations and prices being an equilibrium. Formally, the social planner solves

\[
\max_\tau W (e (\tau))
\]

\(^{12}\)If we choose the price of \( \omega \)-securities so that the fee \( p^\omega \) equals the trading cost, the equilibrium is unique.
For given $\tau$, this is exactly the same contracting problem as in section 6.

**Lemma 7** A small enough positive tax $\tau$ increases welfare in the economy.

When choosing $\tau$, the social planner optimally weighs the benefits and the costs of changing insurance prices $p^\varepsilon$ and $p^\omega$. A higher $p^\varepsilon$ makes it more profitable for the agent to deviate by refusing to trade in the $\varepsilon$-risk market. On the other hand, a higher $p^\omega$ has the effect of lowering the value for the agent of trading $\omega$-contracts and, hence, relaxes the contracting problem. When $\tau$ is small enough, the latter effect dominates since the former effect is only second order.

Suppose now that investors can observe the number of transactions made by the manager in the securities market as in section 6. The question is whether the planner can still improve on equilibrium welfare by using the transaction tax $\tau$. A transaction is a trade between a manager and an issuer. We know that the principal wants the agent to trade only $\varepsilon$-contracts, thus, to execute only one transaction in the financial market. If the number of transactions is observable, then the only deviation for the manager that cannot be detected by the principal is when the manager stops trading $\varepsilon$-contracts and trades only $\omega$-contracts (double-deviation).

To gain some intuition, suppose for a moment that only linear securities can be traded and there is a constant marginal cost for each unit of security. This case is easier to analyze since the choice variables are continuous. Suppose that the principal can observe the total number of units bought by the manager, call it $\bar{q}$, but not the type of security traded. The principal can then use the extra choice variable $\bar{q}$ to control the trades of the managers. As usual, the principal will design a contract so that in equilibrium the manager will buy a quantity $q^\varepsilon$ of $\varepsilon$-contracts and a quantity $q^\omega = 0$ of $\omega$-contracts. Thus, the principal sets $\bar{q} = q^\varepsilon$. Now, when $\bar{q}$ is observable, at the margin the only deviation available to the manager is to reduce $q^\varepsilon$ by $dq^\varepsilon$ and increase $q^\omega$ by $dq^\omega = dq^\varepsilon$ so as to leave the total quantity $\bar{q}$ unchanged. On the contrary, when transactions are not observable, the manager has three possible deviations: decrease $q^\varepsilon$, increase $q^\omega$, and do both. In the latter case, since the total quantity $\bar{q}$ is unchanged, a tax on transactions would have no effect on the value of the deviation. The following diagram illustrates the different possibilities: the first two deviations lead to points A and B, respectively, while the double-deviation leaves the total quantity of trades unchanged.

Note that the tax raises revenues from the transactions of $\varepsilon$-securities (these securities are traded in equilibrium), but these revenues are rebated to investors.
unaffect ed at point E.

Intuitively, the constraint on the quantity traded is at least weakly preferred to a tax on the transactions in the setting with linear securities. In ongoing work, I am exploring the consequences of restricting the space of securities to linear securities, but to allow investors to observe a signal on the total amount of resources that managers invest in the trading activity. However, managers can still benefit from a deviation that leaves the value of trades unaffected. The interpretation is that investors have access to the balance sheets of financial institutions and can infer the amount of resources spent in trading activities. However, they don’t have the expertise to understand the type of securities that are being traded.

A similar result holds in the case considered here as the following proposition shows.

**Proposition 9** When transactions are observable, the transaction tax cannot improve on equilibrium welfare.

### 7.2 Regulation

When transactions are observable the planner cannot help investors by taxing them. However, managers have still access to a double-deviation that allows them to trade insurance contracts contingent on \( \omega \). Thus, \( \omega \)-contracts can still be traded off the equilibrium.

Taxing transactions in derivatives markets is not the only possible way to increase welfare in this economy. Thus, that even maintaining the assumption that the social planner has no superior information over the other agents, the social planner can do much better by \textit{regulating} the issuers of financial securities. By regulation I mean giving the planner the power to write a contract that incentivizes issuers to maximize welfare in the economy. Regulation goes to the heart of the coordination problem of the investors: the planner takes the place of investors and realizes that incentives for managers have to be coordinated with incentives for issuers.
To see this, remember that trades of securities contingent on the aggregate risk can only undo the incentives set up by investors and these securities are not traded in equilibrium. As shown in the analysis of section 5.2, the presence of issuers selling \( \omega \)-contracts matters to the extent that it constrains the contracts space of the principal. In other words, \( \omega \)-contracts matter only as they can represent a profitable deviation for the agents. Hence, issuers trade only \( \varepsilon \)-contracts and make constant (zero) profits in equilibrium. In contrast, off the equilibrium, issuers sell \( \omega \)-contracts to managers and, thus, take some aggregate risk on their balance sheets.

Profits of issuers are assumed to be observable, hence, a social planner can always increase welfare in the economy by punishing any volatility of these profits. Therefore, the optimal regulation in this model is to forbid issuers to ever take aggregate risk on their balance-sheets and to punish them in case of deviation. This policy limits (and, in the extreme, eliminates) the incentives to trade aggregate risk out of the equilibrium and, therefore, it relaxes the investors’ problem.

Formally, I am going to assume that the social planner can regulate the trading activity of financial issuers by choosing a function

\[
\eta^*_\ell : \mathbb{R} \times \mathbb{R} \to \mathbb{R}
\]

that maps pairs \((\Pi^*_\ell, \omega)\) into a payment to each issuer.

**Proposition 10** Let \( \Pi^*_\ell \) be the equilibrium profits of issuer \( \ell \) (\( \Pi^*_\ell \) is zero since \( \omega \)-contracts are not traded in equilibrium). Then, the optimal policy \( \eta^*_\ell \) is given by

\[
\eta^*_\ell (\Pi^*_\ell, \omega) = \Pi^*_\ell \quad \text{and} \quad \eta^*_\ell (\Pi^*_\ell, \omega) = -\infty \quad \text{if} \quad \Pi^*_\ell \neq \Pi^*_\ell.
\]

If this policy is implemented then equilibrium welfare coincides with that of proposition 5.

The intuition for proposition 10 has already been discussed. The result on welfare is also quite intuitive. If the optimal policy \( \eta^*_\ell \) is implemented, then issuers will never find it optimal to sell \( \omega \)-contracts. Thus, the IC constraint that precludes deviations with \( \omega \)-contracts drops from the contracting problem and we are back to the case of section 5.1.

The reason why regulation is so effective is not related to the fact that \( \omega \)-contracts are not traded in equilibrium and profits of issuers are constant. If there was some trading of aggregate risk in equilibrium (say, due to some unmodelled demand for hedging), then profits would vary with the aggregate state of the economy. However, as long as the planner can determine the right amount of aggregate risk that issuers can hold on their balance sheets, then the optimal policy would still have the same power as in proposition 10. Of course, this is possible because the social
planner is assumed to observe the aggregate state $\omega$. The optimal regulation is then given by a limit on how risky the balance sheets of the issuers of financial securities can be.

The optimal policy still requires a great deal of information to be implemented since the social planner has to understand what is the optimal amount of aggregate risk that should be traded. In particular, the planner has to realize what part of the $\omega$-risk is traded by those portfolio managers who determine the quantity of risk in the economy through their investment decisions. While this is easy in this abstract model, it may be less so in real financial markets.

8 Discussion

Unlike the securities of this model – which are contingent either on idiosyncratic or aggregate risk – derivative contracts traded in financial markets are often contingent on many different risks, both aggregate and idiosyncratic. Thus, while the model highlights a fundamental difference between securities contingent on idiosyncratic and aggregate risks, this distinction is much less clear in real financial markets.

Nonetheless, we can argue that some financial securities are more sensitive to idiosyncratic risks while others are used to hedge risks that are more aggregate. For example, a Credit Default Swap (CDS)\textsuperscript{14} that insures the buyer against the default of a firm that is independent from the rest of the economy is a derivative that is relatively more sensitive to idiosyncratic risks. On the contrary, a CDS written on a bond issued by a big firm (say, GE or Walmart) is likely to be relatively more sensitive to the aggregate state of the economy.

Another example of a security contingent on aggregate risk is an Interest Rate Swap that banks use to hedge the interest rate risk of their loan portfolios.

Finally, a more involved example is given by a tranche of a Funded Synthetic CDO\textsuperscript{15}. This derivative allows an investors to take a position on the credit risk of a portfolio of loans and, as the number of loans in the underlying portfolio increases, idiosyncratic risks wash away and the CDO will be relatively more sensitive to common risks.

Recent work in empirical finance focuses on how trading in derivatives markets affects the risk of financial institutions. Ideally, to see whether the predictions of the model are consistent

\textsuperscript{14}A CDS is a credit derivative which obliges the seller to compensate the buyer in the event of a loan default. The buyer pays a premium to the seller for this insurance.

\textsuperscript{15}This is a derivative contract that allows investors with different appetite for risk buy tranches of a Special Purpose Vehicle (SPV). The SPV then buys Treasuries and sells a portfolio of CDS. The buyers of the CDS pay a periodic premium to the SPV which transfers it to the investors. However, if a loan in the portfolio defaults, then the Treasuries are sold to pay the buyer of the protection.
with the data, we would need to make a distinction between securities contingent on idiosyncratic or aggregate risks. In the former case, the model predicts that banks’ balance sheets become less correlated with each other while the latter case leads to the opposite conclusion. This ideal experiment assumes that we can distinguish financial securities depending on the type of risk they hedge. In practice, however, this distinction is not as sharp.

A less demanding exercise would be to ask what happens to a financial institution’s balance sheet after it start trading in the derivatives markets. In a recent work, Nijskens and Wagner (2011) study two separate datasets of banks which include information on various types of securitization around the world. In particular, one dataset contains data on Credit Default Swaps (CDS) and the other on Collateralized Loan Obligation (CLO)\textsuperscript{16}.

Both datasets allow Nijskens and Wagner (2011) to observe the date on which each bank start trading each of these financial products. They then look at the effect on the returns of each bank after the date of the first trade of CDS or CLO. They find that a bank that trades CDS or CLO experiences a permanent increase in its beta, which is a measure of the systematic risk of a bank. Also, the magnitude of such effect is bigger in the case of CLO. Remember that in our model the profits of a bank are given by

$$\pi_i = \bar{R} + (r + \varepsilon_i) k_i + \omega (1 - k_i) + u_i.$$  

If we take the average across different sectors, $\int \pi_i \, di$, then the market return is given by $\bar{R} + \omega (1 - k)$ (where $k$ is the equilibrium choice of all managers). If we let $\sigma_i$ be the standard deviation of the return of bank $i$, the correlation of bank $i$ with the market is $\rho_i = (1 - k) \sigma_i / \sigma$. Nijskens and Wagner (2011) find that, after the first CLO or CDS trade, the value of $\rho_i$ increases while the relative volatility $\sigma_i / (1 - k) \sigma$ decreases. This is consistent with the prediction of this model that derivatives tend to decrease the value of $k_i$. It is also tempting to speculate that the bigger effect of CLO trades on the beta of the bank relative to CDS trades is related to the fact that CLOs, which are pools of loans, are more similar to the $\omega$-securities of this paper.

Similar evidence is found by Haensel and Krahnen (2007). They use a dataset of CDOs issued by European financial institutions. They also find that banks engaging in these transactions tend to increase their exposure to the market. Of course, while these results are consistent with the conclusions of this paper, they are certainly not conclusive evidence. There may be many reasons for why banks increase their systematic risk after trading some types of derivatives.

\textsuperscript{16}A CLO is a form of securitization through which banks transfer pools of loans to the buyers of these securities. The payoff of this derivative resembles, to a first approximation, the payoff of a funded synthetic CDO.
On the information side, this model implies that an easy way to improve welfare is by requiring more information disclosure. Formally, this would be equivalent to modify part (d) of Assumption 1 and assume perfect observability of trades and types of security. Once investors have the ability to contract on the different securities traded by managers, they will forbid trading of $\omega$-risk (and allow trading of $\varepsilon$-risk). In fact, we can conjecture that the equilibrium when part (d) of assumption 1 is removed will resemble that of section 5.1.

While information disclosure is a strong and interesting implication of this model, it derives from the mathematical way I chose to model complex securities and OTC markets. In general, it is realistic to assume that even if big financial institutions were required to disclose all their trading activities to outside investors, it would probably be a daunting task for many investors to process this information (Brunnermeier and Oehmke (2011)).

For simplicity, in this model I have assumed that agents and issuers trade in a Walrasian market by paying a fixed cost per trade. These costs can be interpreted as a reduced-form way to capture the effects of imperfect competition in the securities market or the liquidity of these markets. Remember that the financial market in this model is an abstraction of OTC markets where typically market makers provide liquidity by posting a price and trading securities at that price. The creation of new financial products and the growth of OTC markets have stimulated important research on decentralized markets. These papers explore the main features of these markets like price determination, liquidity and diffusion of information. Duffie et al. (2005), for example, provide a theory of asset pricing in decentralized markets\textsuperscript{17}. The focus of this paper, however, is not about the specific trading environment, but on how complex securities can affect the portfolio choice of investors and, thus, the aggregate volatility of the economy.

A Walrasian market for securities is also the typical assumption in the literature on markets with endogenous securities creation (Allen and Gale (1991), Pesendorfer (1995), Bisin (1998)). These papers depart from the standard assumption that traded securities are exogenously given and, instead, assume that they are issued by optimizing issuers. Some of these papers, in particular, assume that issuers have market power (Allen and Gale (1988) and Bisin (1998)). This alternative assumption is not explored in this paper, but from the social planner’s problem we can conjecture that, by increasing the price of the insurance contracts contingent on $\omega$, some market power may actually be beneficial for welfare. Similarly, if we interpret the trading cost as the liquidity of these markets, then it may be that case that less liquid markets are beneficial for welfare.

The stark conclusion about the welfare effects of $\omega$-securities depend on some strong assump-

\textsuperscript{17}Other important contributions are Duffie et al. (2005), Duffie et al. (2007), Duffie and Manso (2007), Lagos (2010), Lagos and Rocheteau (2009), Lagos et al. (2007), Vayanos (1998), Vayanos and Weill (2008), and Weill (2008).
tions of the model. First, the assumption of symmetric preferences, technology, and equilibrium eliminates any gains from trading aggregate risk among managers. Also, I have assumed that investors can perfectly condition their contracts on aggregate states, but they cannot do the same for idiosyncratic states. Financial markets help allocate aggregate risk to the agents who are better prepared to hold it. However, as long as investors cannot fully control the risks traded by their managers, then trading of \( \omega \)-risk has the potential to reduce welfare. Also, the assumptions of this model help me isolate this particular mechanism and analyze its (negative) implications. In a more general model, different effects of aggregate risk trading would coexist and the optimal policy would be characterized by a richer set of actions.
Appendix

Project Selection

In this appendix, I model the project selection process of section 3 more explicitly. In the economy there is a continuum of sectors denoted by \( j \in [0, 1] \). In each sector \( j \) there is a two-dimensional continuum of potential projects indexed by \( (x, y) \in [0, 1]^2 \). A one-dimensional subset \( S \subset [0, 1]^2 \) of these projects are specialized projects, the rest are standard projects. In particular, for each \( x \in [0, 1] \) there is a unique \( y \) such that \( (x, y) \in S \). This value of \( y \) is randomly drawn from a uniform distribution on \([0, 1]\) for each \( x \). The set of projects is depicted in the figure below. Every manager is an expert of a particular sector and has access to information about projects in that sector. More precisely, if a manager \( i \) is an expert of sector \( j \), then he can exert one unit of effort and inspect all the projects with a given index \( x \) in his sector and find out the specialized project \( (x, y) \in S \). If no effort is exerted, the manager has zero probability of choosing the specialized project.

All the specialized projects in sector \( j \) have the same random payoff \( r_{i,j} = \bar{r} + \varepsilon_j + u_i \) and all the standard projects have the same random payoff \( R_i = \bar{R} + \omega + u_i \). Thus, a manager who wants to invest an amount \( k \) in specialized projects has to exert \( k \) units of effort and suffer a loss of utility equal to \( C(k) \).

A Proofs and additional lemmas

Proof of proposition 1. Assume for now that the FOA is valid, the maximization problem P(NS) is

\[
\max_{\xi, k} \int m(\omega) (x + \bar{R} + rk + (1 - k)\omega - \xi(x, \omega)) \, d\Phi(x) \, dF_\omega(\omega)
\]
subject to:
\[
\frac{\partial}{\partial k} \int u(\xi(x, \omega)) \, dF_{x|\omega}(x|\omega; k, \hat{k}) \, dF_\omega(\omega) \Bigg|_{k=\hat{k}} - C'(k) = 0,
\]
\[
\int u(\xi(x, \omega)) \, d\Phi(x) \, dF_\omega(\omega) - C(k) \geq 0.
\]
By using the properties of Gaussian distributions together with the moments (3), I can rewrite the IC constraint as:
\[
\int u(\xi(x, \omega)) \left( \frac{1}{\sigma_x} (r - \omega) x + \frac{k \sigma_x^2}{\sigma_x^2} (x^2 - 1) \right) \, d\Phi(x) \, dF_\omega(\omega) - C'(k) = 0.
\]
Here, I have used the fact that in equilibrium \( x \) and \( \omega \) are independent by construction. Let \( \lambda \) and \( \mu \) be the Lagrange multipliers on the two constraints, respectively, and define the Lagrangian:
\[
\Lambda = \int m(\omega) \left( x + \bar{R} + rk + (1 - k) \omega - \xi(x, \omega) \right) \, d\Phi(x) \, dF_\omega(\omega)
+ \lambda \left( \int u(\xi(x, \omega)) \left( \frac{1}{\sigma_x} (r - \omega) x + \frac{k \sigma_x^2}{\sigma_x^2} (x^2 - 1) \right) \, d\Phi(x) \, dF_\omega(\omega) - C'(k) \right)
+ \mu \left( \int u(\xi(x, \omega)) \, d\Phi(x) \, dF_\omega(\omega) - C(k) \right)
\]
If we differentiate \( \Lambda \) pointwise w.r.t. \( \xi(x, \omega) \), we get that the optimal contract solves:
\[
\frac{m(\omega)}{V'(\xi(x, \omega))} = \mu + \lambda \left[ \frac{1}{\sigma_x} (r - \omega) x + \frac{k \sigma_x^2}{\sigma_x^2} (x^2 - 1) \right]
\]
which is the expression in proposition 1.

**Proof of proposition 3.** To see that this equilibrium is efficient we can set up the social planner’s problem. The only difference between the planner’s problem and the equilibrium is that the former takes into account how aggregate consumption \( c(\omega) \) depends on the profits of all the managers. Formally, the planner solves:
\[
\max_{\xi, k, c} \int v(c(\omega)) \, d\Phi(x) \, dF_\omega(\omega)
\]
subject to:
\[
\frac{\partial}{\partial k} \int u(\xi(x, \omega)) \, dF_{x|\omega}(x|\omega; k, \hat{k}) \, dF_\omega(\omega) \Bigg|_{k=\hat{k}} - C'(k) = 0,
\]
\[
\int u(\xi(x, \omega)) \, d\Phi(x) \, dF_\omega(\omega) - C(k) \geq 0,
\]
\[ c(\omega) + \int \xi(x,\omega) \, d\Phi(x) = \int \left[ x + \bar{R} + r \, k + (1 - k) \, \omega \right] \, d\Phi(x), \quad \forall \omega. \]

The last constraint is the resource constraint of the economy. Note that only this constraint depends on \( c(\omega) \). So, if we let \( \varphi(\omega) f_\omega(\omega) \) denote the Lagrange multiplier on the resource constraint, we can separate the problem by first solving for \( c \):

\[
\max_c \int v(c(\omega)) \, d\Phi(x) \, dF_\omega(\omega)
\]

subject to:

\[
c(\omega) + \int \xi(x,\omega) \, d\Phi(x) = \int \left[ x + \bar{R} + r \, k + (1 - k) \, \omega \right] \, d\Phi(x), \quad \forall \omega.
\]

Setting up the Lagrangian and taking the first-order condition w.r.t. \( c(\omega) \) gives:

\[ m(\omega) \equiv v'(c(\omega)) = \varphi(\omega). \]

Now, for given \( \varphi(\omega) \), we can solve the dual problem of maximizing total resources, that is,

\[
\max_{\xi,k} \int m(\omega) \left[ x + \bar{R} + r \, k + (1 - k) \, \omega - \xi(x,\omega) \right] \, d\Phi(x) \, dF_\omega(\omega),
\]

subject to the IR and IC constraint. This is the same problem as \( \text{P(NS)} \). This proves that the equilibrium is a solution to the planner’s problem. □

### A.1 Only \( \varepsilon \)-securities

**Proof of lemma 1.** Consider the portfolio problem of the representative investor:

\[
\max_y \mathbb{E} \left[ m(\omega) \Pi^f \right],
\]

where

\[
\Pi^f = \int (p_{i,\hat{\varepsilon}} - z_{i,\hat{\varepsilon}}) \, y_{i,\hat{\varepsilon}} \, d\hat{\varepsilon} \, di.
\]

Take the first-order condition pointwise w.r.t. \( y \)

\[ p_{i,\hat{\varepsilon}} - \phi_\varepsilon(\hat{\varepsilon}) \geq 0. \]

If \( p_{i,\hat{\varepsilon}} \) was different from \( \phi_\varepsilon(\hat{\varepsilon}) \), then the investor would buy or sell an infinite amount of this security. This cannot be an equilibrium, so \( p_{i,\hat{\varepsilon}} = \phi_\varepsilon(\hat{\varepsilon}) \). □
Proof of lemma 2. The optimality of full insurance can be proved using the results in Kim (1995) and Jewitt (2007).

These papers show how to rank different information systems in the principal-agent framework. They consider a principal-agent model in which the principal can choose among different set of signals about the agent’s action. An information system will be preferred to another if the former can implement every action at a lower cost for the principal. This criterion is more general than the informativeness criterion in Holmstrom (1979) who considers only information systems which are inclusive in the sense that one system contains more signals than the other (Holmstrom (1979) defines the notion of informativeness in terms of a sufficiency criterion).

This is, however, restrictive in our context since systems are not inclusive. Kim (1995) and Jewitt (2007) show how to rank information systems which are not inclusive in terms of their likelihood ratios. More specifically, they show that, given two information systems, one of them will implement any action at a lower cost for the principal if and only if its likelihood ratio is a mean preserving spread of the other.

The result in Kim (1995) requires the principal to be risk-neutral, but in this paper there is the stochastic discount factor \( m(\omega) \). However, the argument easily generalizes to our case if we consider the conditional likelihood ratio. Thus, I will use the result that given two signals \( \tilde{x} \) and \( \tilde{x} \), it is cheaper for a principal to implement a given action \( k \) with \( \tilde{x} \) if the conditional likelihood ratio

\[
L_{\tilde{x}|\omega}(x|\omega; k, \tilde{x}, p(\cdot)) = \frac{\partial f_{\tilde{x}|\omega}(x|\omega; k, k, d^{\tilde{x}}, p(\cdot))}{\partial \tilde{x}}
\]

is riskier than the likelihood ratio obtained with \( \tilde{x} \). Intuitively, since different actions lead to different realizations of the signal with higher probability, the higher the volatility of the likelihood ratio, the more informative the signal.

Also, when \( \tilde{k} = k \) the distribution \( F_{\tilde{x}x} \) does not depend on \( \omega \), so I will simply write \( F_{\tilde{x}x} \). The
mean of $L_{x|\omega}$ is given by:

$$
\int \left( L_{x|\omega} (x|\omega; k, d^\varepsilon, p(\cdot)) \right) dF_x (x; k, d^\varepsilon, p(\cdot)) dF_\omega (\omega)
$$

$$
= \int \frac{\partial}{\partial k} \int f_{x|\omega,\varepsilon} (x|\omega; k, d^\varepsilon, p(\cdot)) d\Phi_\varepsilon (\varepsilon) \ dx
$$

$$
= \int \int \left( \frac{(r - \omega + \varepsilon)(x - k \varepsilon)}{\sigma_u^2} \right) f_{x|\omega,\varepsilon} (x|\omega; \varepsilon; k, d^\varepsilon, p(\cdot)) d\Phi_\varepsilon (\varepsilon) \ dx
$$

$$
= \int \int \left( \frac{(r - \omega + \varepsilon)(x - k \varepsilon)}{\sigma_u^2} \right) dF_{x|\omega,\varepsilon} (\varepsilon|\omega, x; k, d^\varepsilon, p(\cdot)) dF_x (x; k, d^\varepsilon, p(\cdot))
$$

$$
= \frac{1}{\sigma_u^2} \int \left( (r - \omega + \mathbb{E} [\varepsilon|\omega, x]) x - k \mathbb{E} [(r - \omega + \varepsilon)|\omega, x] \right) dF_x (x; k, d^\varepsilon, p(\cdot))
$$

$$
= \frac{1}{(\sigma_u^2)^2} \int \left( (r - \omega) \mathbb{E} [u|x] + \mathbb{E} [\varepsilon u|x] \right) dF_x (x; k, d^\varepsilon, p(\cdot)) = 0,
$$

since the distribution of $x$ does not depend on $\omega$.

Thus, the variance of $L_{x|\omega}$ is:

$$
\int \left( L_{x|\omega} (x|\omega; k, d^\varepsilon, p(\cdot)) \right)^2 dF_x (x; k, d^\varepsilon, p(\cdot)) dF_\omega (\omega)
$$

$$
= \int \frac{1}{f_x (x; k, d^\varepsilon, p(\cdot))} \left( \frac{\partial}{\partial k} \int f_{x|\omega,\varepsilon} (x|\omega; k, d^\varepsilon, p(\cdot)) d\Phi_\varepsilon (\varepsilon) \right)^2 \ dx
$$

$$
= \int \frac{1}{f_x (x; k, d^\varepsilon, p(\cdot))} \int \left[ \frac{(r - \omega + \varepsilon)(x - k \varepsilon)}{\sigma_u^2} \right] f_{x|\omega,\varepsilon} (x|\omega, \varepsilon; k, d^\varepsilon, p(\cdot)) d\Phi_\varepsilon (\varepsilon) \ dx
$$

$$
= \int \int \left( \frac{(r - \omega + \varepsilon)(x - k \varepsilon)}{\sigma_u^2} \right) dF_{x|\omega,\varepsilon} (\varepsilon|\omega, x; k, d^\varepsilon, p(\cdot)) dF_x (x; k, d^\varepsilon, p(\cdot))
$$

$$
= \frac{1}{(\sigma_u^2)^2} \int \left( (r - \omega + \mathbb{E} [\varepsilon|\omega, x]) x - k \mathbb{E} [(r - \omega + \varepsilon)|\omega, x] \right)^2 dF_x (x; k, d^\varepsilon, p(\cdot))
$$

$$
= \frac{1}{(\sigma_u^2)^2} \int \left( (r - \omega) \mathbb{E} [u|x] + \mathbb{E} [\varepsilon u|x] \right)^2 dF_x (x; k, d^\varepsilon, p(\cdot))
$$

Now, for given distribution for $\varepsilon$, if the variance $\sigma^2_\varepsilon$ of this distribution increases, then $\mathbb{E} [u|x] \to 0 \ \forall x$ ($x$ becomes a worse predictor for $u$). On the contrary, as $\sigma^2_\varepsilon$ increases, the cross moment $\mathbb{E} [\varepsilon u|x]$ also increases.

The condition for full insurance to be optimal is that the first effect dominates the second. Formally, the function

$$
G (\omega, d^\varepsilon, k) = \frac{1}{(\sigma_u^2)^2} \int \left( (r - \omega) \mathbb{E} [u|x] + \mathbb{E} [\varepsilon u|x] \right)^2 dF_x (x; k, d^\varepsilon, p(\cdot))
$$

is maximized at $d^\varepsilon = -k \varepsilon$ for any value of $\omega$ and $k$. This condition is implicit because different choices of $d^\varepsilon$ affect the distribution of $x$. 
However, if we restrict attention to linear securities, then $x$ follows a Gaussian distribution and, in particular,
\[ \mathbb{E}[\varepsilon|x] = \frac{k\sigma_{\varepsilon}^2}{\sigma_x^2} x, \]
and
\[ \mathbb{E}[\varepsilon^2|x] = \frac{k\sigma_{\varepsilon}^2}{\sigma_x^2} x = \frac{\sigma_u^2 \sigma_{\varepsilon}^2}{\sigma_x^2} + \left( \frac{k\sigma_{\varepsilon}^2}{\sigma_x^2} \right)^2 x^2. \]
Therefore,
\[ \int (L_{x|\omega}(x|\omega; k, d^\varepsilon))^2 dF_x(x; k, d^\varepsilon) = \frac{(r - \omega)^2 \sigma_x^2}{\sigma_{\varepsilon}^2} + \frac{k^2 (\sigma_{\varepsilon}^2)^2}{\sigma_x^2}. \]
A sufficient condition for this expression to be maximized at $\sigma_{\varepsilon}^2 = 0$ for every value of $\omega$ and $k$ is:
\[ 0 \leq \sigma_u^2 \leq (r - \omega)^2, \forall \omega. \]
This is the condition in lemma 2. 

Proof of proposition 8. Assume that he conditions for full insurance to be optimal are met.
Under the assumption that the FOA is valid, we can rewrite $P(\varepsilon)$ assuming that the distribution of $\varepsilon$ is degenerate at 0. By taking the first-order condition w.r.t. $\xi$, we obtain:
\[ \frac{m(\omega)}{V'(\xi(x, \omega))} = \mu + \lambda \frac{1}{\sigma_x} (r - \omega) x. \]
With this contract the problem of the manager is concave in $x$. This implies two things. First, the agent will buy full insurance at the actuarially fair price of lemma 1. In turn, this means that this contract is optimal for the principal. Secondly, the concavity of the problem implies that the FOA is valid (Jewitt (1988)).

Proof of proposition 5.
Let $C^*(k)$ and $C_{\varepsilon}^*(k, p(\cdot))$ be the minimum costs of implementing $k$ when no securities are available and when the agent fully hedges his idiosyncratic risk, respectively. I consider the equilibrium where $p(\cdot)$ is given by lemma 1. To prove that the optimal $k$ increases when the agent fully insures his risk, I can show that $C^*(k) - C_{\varepsilon}^*(k, p(\cdot))$ is increasing in $k$ and use a monotone comparative static argument. First, note that under the assumption that full insurance is optimal, $C^*(k) - C_{\varepsilon}^*(k, p(\cdot)) > 0$.

Remember that in equilibrium the utility of the agent is an average of $ar{u}((\mu + \lambda L_{x|\omega}(x|\omega; k, d^\varepsilon, p(\cdot)))/m(\omega)).$
By the envelope theorem, differentiating \( C^* (k) - C^*_\varepsilon (k, p (\cdot)) \) w.r.t. \( k \) gives:

\[
\frac{\partial}{\partial k} \left( C^* (k) - C^*_\varepsilon (k, p (\cdot)) \right) = -\hat{\lambda} \int \tilde{u} \left( \frac{\hat{\mu} + \hat{\lambda} L_{x|\omega} (x|\omega; k, 0, p (\cdot))}{m (\omega)} \right) \frac{\partial}{\partial k} L_{x|\omega} (x|\omega; k, 0, p (\cdot)) d\Phi (x) dF_\omega (\omega)
\]

\[+ \lambda \int \tilde{u} \left( \frac{\hat{\mu} + \hat{\lambda} L_{x|\omega} (x|\omega; k, -k, \varepsilon, p (\cdot))}{m (\omega)} \right) \frac{\partial}{\partial k} L_{x|\omega} (x|\omega; k, -k, \varepsilon, p (\cdot)) d\Phi (x) dF_\omega (\omega) - \lambda C'' (k)
\]

\[- \frac{\partial}{\partial k} \hat{\mu} \int \tilde{u} \left( \frac{\hat{\mu} + \hat{\lambda} L_{x|\omega} (x|\omega; k, 0, p (\cdot))}{m (\omega)} \right) d\Phi (x) dF_\omega (\omega) + \hat{\mu} C' (k)
\]

\[+ \frac{\partial}{\partial k} \mu \int \tilde{u} \left( \frac{\hat{\mu} + \hat{\lambda} L_{x|\omega} (x|\omega; k, -k, \varepsilon, p (\cdot))}{m (\omega)} \right) d\Phi (x) dF_\omega (\omega) - \mu C' (k),
\]

where a "hat" denotes the Lagrange multipliers for the case with no insurance. Therefore,

\[
\frac{\partial}{\partial k} \left( C^* (k) - C^*_\varepsilon (k, p (\cdot)) \right) = -\hat{\lambda} \int \tilde{u} \left( \frac{\hat{\mu} + \hat{\lambda} L_{x|\omega} (x|\omega; k, 0, p (\cdot))}{m (\omega)} \right) \frac{\partial}{\partial k} L_{x|\omega} (x|\omega; k, 0, p (\cdot)) d\Phi (x) dF_\omega (\omega)
\]

\[+ (\hat{\lambda} - \lambda) C'' (k) + (\hat{\mu} - \mu) C' (k)
\]

\[= -\hat{\lambda} \int \tilde{u} \left( \frac{\hat{\mu} + \hat{\lambda} L_{x|\omega} (x|\omega; k, 0, p (\cdot))}{m (\omega)} \right) \frac{\partial}{\partial k} L_{x|\omega} (x|\omega; k, 0, p (\cdot)) d\Phi (x) dF_\omega (\omega) +
\]

\[\left( \hat{\lambda} - \lambda \right) C'' (k) + (\hat{\mu} - \mu) C' (k) > 0,
\]

where \( \hat{\lambda} > \lambda \) and \( \hat{\mu} > \mu \) are implied by the assumption that insuring the idiosyncratic risk is optimal for the principal and \( \int \tilde{u} \left( \frac{\hat{\mu} + \hat{\lambda} L_{x|\omega} (x|\omega; k, 0, p (\cdot))}{m (\omega)} \right) \frac{\partial}{\partial k} L_{x|\omega} (x|\omega; k, 0, p (\cdot)) d\Phi (x) dF_\omega (\omega) < 0 \) since the variance of \( L_{x|\omega} \) is lower when \( k \) is higher.

The proof that welfare is higher when \( \varepsilon \)-securities are traded follows from similar steps to the proof of proposition 3. I first define a social planner that chooses \( \xi, k, \) and \( c \) so as to maximize the welfare of investors. Then I show that, for given aggregate consumption, the planner’s problem is equivalent to finding the optimal contract that maximizes the value of resources produced by each single manager. Under the assumption that full insurance is optimal, it follows that the value of resources when \( \varepsilon \)-securities are available is higher. This proves the claim that welfare in the economy is higher. ■

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A.2 Only $\omega$-securities

**Proof of lemma 4.** Again, consider the portfolio problem of the representative investor:

$$\max_y \mathbb{E} \left[ m(\omega) \Pi^I \right],$$

where

$$\Pi^I = \int (p_\omega - z_\omega) y_\omega \, d\bar{\omega},$$

and differentiate the expression pointwise w.r.t. $y$

$$\mathbb{E}[m(\omega)] p_\omega - m(\bar{\omega}) f_\omega(\bar{\omega}) \geq 0.$$  

If $p_\omega$ was different from $m(\bar{\omega}) f_\omega(\bar{\omega}) / \mathbb{E}[m(\omega)]$, then the representative investor would buy or sell an infinite amount of this security. This cannot be an equilibrium, so $p_\omega = m(\bar{\omega}) f_\omega(\bar{\omega}) / \mathbb{E}[m(\omega)]$.

\[\blacksquare\]

**Proof of lemma 11.** When only $\omega$-securities are traded, the optimal contract solves

$$\max_{\xi,k} \int m(\omega) \left( x + \bar{R} + r \ k + (1 - k) \omega - \xi(x,\omega) \right) d\Phi(x) dF_\omega(\omega)$$

subject to:

$$\frac{\partial}{\partial k} \int u(\xi(x,\omega)) dF_{x|\omega}(x|\omega,k,\hat{k},0,p(\cdot)) dF_\omega(\omega) \bigg|_{k=\hat{k}} - C'(k) = 0,$$

$$\frac{\partial}{\partial \tilde{d}} \int u(\xi(x,\omega)) dF_{x|\omega}(x|\omega,k,\hat{d},p(\cdot)) dF_\omega(\omega) \bigg|_{\tilde{d}=0} = 0,$$

$$\int u(\xi(x,\omega)) d\Phi(x) dF_\omega(\omega) - C(k) = \bar{u}.$$  

This is program P($\omega$) in the main text. I am assuming here that the FOA is valid. We can now define the Lagrangian and maximize it pointwise w.r.t. $\xi$. The optimal contract solves:

$$\frac{m(\omega)}{V'(\xi(x,\omega))} = \mu + \lambda \frac{1}{\sigma_x} (r - h(\omega)) \ x + \lambda \frac{k \sigma^2_x}{\sigma^2_x} (x^2 - 1)$$

where $h(\omega) = \omega - \nu_\omega (1 - \mathbb{E}[m(\omega)])$ and $\nu_\omega \geq 0$ is the Lagrange multiplier on the IC constraints that determine the choice of $\tilde{d}$. This is the expression in proposition 5. \(\blacksquare\)

**Proof of proposition 7.** First, I will show that, when $\omega$-securities are available, the agent has a profitable deviation. Take the special case of section 4 and let $\xi$ denote the payment schedule
that solves (4). Consider the deviation where the agent sells some risk by buying a portfolio of \(\omega\)-securities that pays off \(-\kappa\omega/m(\omega)\), for a small \(\kappa > 0\). From lemma 4, the price of this security is \(-\kappa E[\omega]/E[m(\omega)] = 0\). If the agent buys this portfolio, the mean of \(x\) will be unaffected, but \(x\) becomes less correlated with \(\omega\). With a slight abuse of notation, let \(F_{x|\omega}(x|\omega; k, k, -\kappa\omega/m(\omega), p(\cdot))\) be the conditional distribution of \(x\), when the portfolio \(-\kappa\omega/m(\omega)\) is selected. Differentiating the agent’s utility w.r.t. \(\kappa\) around \(\kappa = 0\) (that is, around the point where the agent doesn’t deviate) yields

\[
\frac{\partial}{\partial \kappa} \left( \int u(\xi(x, \omega)) dF_{x|\omega}(x|\omega; k, k, -\kappa \frac{\omega}{m(\omega)}, p(\cdot)) dF_{\omega}(\omega) - C(k) \right)
\]

\[
= \int u(\xi(x, \omega)) \left( \frac{\partial}{\partial \kappa} dF_{x|\omega}(x|\omega; k, k, -\kappa \frac{\omega}{m(\omega)}, p(\cdot)) \right) dF_{\omega}(\omega)
\]

\[
= \int u(\xi(x, \omega)) \left( -\frac{1}{\sigma_x m(\omega)} \frac{\omega x}{\omega} \right) d\Phi(x) dF_{\omega}(\omega) > 0
\]

The last inequality comes from the optimal contract (4). Thus, \(\bar{U}_{\omega}(k, p(\cdot)) \geq \int u(\xi(x, \omega)) d\Phi(x) dF_{\omega}(\omega) - C(k)\) holds with a strict inequality.

I have to prove that \(\omega\)-securities reduce the equilibrium level of \(k\). A quick way to prove this result is to rewrite the contracting problem without using the transformation \(x\), that is, I let the payment \(\xi\) be conditional on \((\Pi, \omega)\) instead of \((x, \omega)\). Let

\[
L_{\Pi|\omega}(\Pi|\omega; \hat{k}, d = 0, p(\cdot)) = \frac{\partial F_{\Pi|\omega}((\Pi|\omega; \hat{k}, d = 0, p(\cdot))}{\partial \hat{k}} / F_{\Pi|\omega}(\Pi|\omega; \hat{k}, d = 0, p(\cdot))
\]

be the likelihood ratio of \(\Pi\) conditional on \(\omega\). Note that \(L_{\Pi|\omega}\) depends only on the actual choice of the agent, \(\hat{k}\), but not on the level suggested by the principal, \(k\). Without restating the problem, from Holmstrom (1979), we know that the optimal payment will be such that the agent’s utility is the average of \(\bar{u} \left( \left( \mu + \lambda L_{\Pi|\omega}(\Pi|\omega; \hat{k}, 0, p(\cdot)) \right) / m(\omega) \right)\). The reason why using \((\Pi, \omega)\) instead of \((x, \omega)\) simplifies the proof is that now the outside option

\[
\bar{U}_{\omega}(p(\cdot)) = \max_{\hat{k}, \hat{d}} \int u(\xi(x, \omega)) dF_{\Pi|\omega}(x|\omega; \hat{k}, \hat{d}, p(\cdot)) dF_{\omega}(\omega) - C(\hat{k})
\]

depends on the recommended \(k\) only through the contract \(\xi\). In the contract of proposition 4, \(\mu\) and \(\lambda\) are both increasing functions of \(k\). Therefore, if a deviation is profitable for some \(k\), that is, \(\bar{U}_{\omega}(k, p(\cdot)) \geq \bar{u}\), then the same deviation must be profitable for a higher \(k\). Formally, \(\bar{U}_{\omega}(k', p(\cdot)) \geq \bar{U}_{\omega}(k, p(\cdot)) \geq \bar{u}\) for \(k' \geq k\).
Finally, we have to prove that welfare decreases when $\omega$-securities are available. The proof is analogous to the proof of proposition 3. The social planner solves the problem of choosing $\xi$, $k$, and $c$ so as to maximize welfare in the economy. Formally, the social planner solves:

$$\max_{\xi, k, c} \int v(c(\omega)) \, dF_\omega(\omega)$$

subject to:

$$\frac{\partial}{\partial k} \int u(\xi(x, \omega)) \, dF_{x|\omega}(x|\omega; k, \hat{k}, 0, p(\cdot)) \, dF_\omega(\omega)_{k=k} - C(k) = 0,$$

$$U_\omega(k, p(\cdot)) \leq \bar{u}$$

$$\int u(\xi(x, \omega)) \, d\Phi(x) \, dF_\omega(\omega) - C(k) = \bar{u},$$

$$c(\omega) + \int \xi(x, \omega) \, d\Phi(x) = \int \left[ x + \bar{R} + r k + (1 - k) \omega \right] \, d\Phi(x), \quad \forall \omega.$$  

Compared to problem $P(\omega)$, the social planner has one extra control variable, $c(\omega)$, but he has to satisfy the resource constraint of the economy. Note that $c(\omega)$ appears only in the objective function and in the resource constraint. So, we can separate the problem by first choosing $c(\omega)$ to solve

$$\max_{c} \int v(c(\omega)) \, dF_\omega(\omega)$$

subject to

$$c(\omega) + \int \xi(x, \omega) \, d\Phi(x) = \int \left[ x + \bar{R} + r k + (1 - k) \omega \right] \, d\Phi(x), \quad \forall \omega.$$  

Let $\varphi(\omega)$ be the Lagrange multiplier on the resource constraint. The first-order condition w.r.t. $c$ is

$$m(\omega) \equiv v'(c(\omega)) = \varphi(\omega).$$

Now, conditional on $\varphi(\omega)$, we can solve the dual problem of choosing $\xi$ and $k$ to maximize the value of resources in the economy. This problem is the same as $P(\omega)$. Thus, the equilibrium of the economy is a solution to the planner’s problem and $\tilde{U}_\omega(k, p(\cdot)) \leq \bar{u}$ is a constraint on the planner’s problem. Therefore, welfare is lower when $\omega$-securities are available. ■

### A.3 Both types of securities

#### Proof of Lemma 6.

The proof of this result is similar to those of lemmas 1 and 4 combined with the analysis in Makowski (1979). ■
Proof of proposition 8. Assume that we are in the environment of section 5.1. The optimal contract is given in proposition 4. Assume now that the double-deviation is possible and its value is given by \( \tilde{U}_{\varepsilon \omega}(k, p(\cdot)) \). Consider first a small increase of \( c_\varepsilon \) and, thus, \( p^\varepsilon \) (the case with \( c_\omega \) is analogous). From proposition 4 we know that the optimal contract is increasing in the mean of \( x \). This implies
\[
\frac{\partial}{\partial p^\varepsilon} \tilde{U}_{\varepsilon \omega}(k, p(\cdot)) \geq 0,
\]
which tightens the constraint \( \tilde{U}_{\varepsilon \omega}(k, p(\cdot)) \leq \bar{u} \). Now, for higher values of \( k \), the Lagrange multipliers \( \mu \) and \( \lambda \) in 8 increase to satisfy the IC and IR constraint. From
\[
\tilde{U}_{\varepsilon \omega}(k, p(\cdot)) = \max_{k, x^\omega} \int \bar{u} \left( \frac{\mu + \lambda (r - \omega) x^u}{m(\omega)} \right) dF_{x|\omega}(x|\omega; k, \bar{k}, s^\varepsilon = \emptyset, s^\omega \neq \emptyset, p(\cdot)) dF_\omega(\omega)
\]
we see that if a deviation was profitable for a certain \( k \), \( \tilde{U}_{\varepsilon \omega}(k, p(\cdot)) > \bar{u} \), then it will be profitable also for \( k + dk \). Formally,
\[
\frac{\partial^2}{\partial k \partial p^\varepsilon} \tilde{U}_{\varepsilon \omega}(k, p(\cdot)) \geq 0.
\]
The latter proves that higher values of \( p^\varepsilon \) have a bigger effect on the cost of implementing a certain action when \( k \) is higher. In turn, this implies that the optimal level of \( k \) decreases with \( p^\varepsilon \).

Proof of lemma 7. Consider a tax on the transactions in the securities market. A transaction in this context has to be interpreted as a trade between an issuer and a manager. In other words, I assume that issuers and managers will pay the tax any time they trade something, independently of the quantity of securities exchanged.

When transactions are not observable, the principal has to consider three possible deviations: not trading any security, trading both securities, and trading only \( \omega \)-securities (double-deviation). Formally, let \( \tilde{U}_\varepsilon(k, p(\cdot)), \tilde{U}_\omega(k, p(\cdot)), \) and \( \tilde{U}_{\varepsilon \omega}(k, p(\cdot)) \) denote the values of the three deviations, respectively. Also, let \( \tau \) be the tax per transaction, then in equilibrium the fixed cost of transaction increases from \( c_\varepsilon \) to \( c_\varepsilon + \tau \) (and from \( c_\omega \) to \( c_\omega + \tau \)).

The social planner’s problem is:
\[
\max_{\tau} \max_{\varepsilon, k} \int \nu'(c(\omega))(x + \tilde{R} + \tau k + (1 - k) \omega - \xi(x, \omega) + p^\varepsilon) d\Phi(x) dF_\omega(\omega)
\]
subject to:
\[
\frac{\partial}{\partial k} \int u(\xi(x, \omega)) dF_{x|\omega}(x|\omega; k, \tilde{k}, d^\varepsilon = -k\varepsilon, d_i = \emptyset) dF_\omega(\omega) \bigg|_{k=k} - C'(k) = 0,
\]
47
\[ \tilde{U}_\varepsilon (k, p (\cdot)) \leq \bar{u}, \]
\[ \tilde{U}_\omega (k, p (\cdot)) \leq \bar{u}, \]
\[ \tilde{U}_{\varepsilon\omega} (k, p (\cdot)) \leq \bar{u}, \]
\[ \int u (\xi (x, \omega)) d\Phi (x) dF_\omega (\omega) - C (k) \geq \bar{u}. \]

Here, the tax \( \tau \) affects prices both directly (the tax is imposed on each transaction) and through its effect on \( c (\omega) \) and, thus, \( m (\omega) \). However, note that in equilibrium the proceeds from tax are rebated to the representative investor. Thus, aggregate consumption \( c (\omega) \) is not affected directly by \( \tau \), but only through the effect on \( p (\cdot) \). Now, if we relax the problem by dropping the constraints on the three deviations, the problem is the same as \( P (\varepsilon) \). In section 5.1, I proved that the optimal contract of this problem is such that the agent wants to buy full insurance. Thus, at \( \tau = 0 \), we have that \( \tilde{U}_\varepsilon (k, p (\cdot)) = \bar{u} \). However, when \( \tau = 0 \), \( \omega \)-securities represent a profitable deviation for the agent, that is, \( \tilde{U}_\omega (k, p (\cdot)) < \bar{u} \). This implies that a small positive \( \tau > 0 \) has a second order effect on \( \tilde{U}_\varepsilon (k, p (\cdot)) \) but a first order effect on \( \tilde{U}_\omega (k, p (\cdot)) \). Thus, a small positive \( \tau > 0 \) increase welfare in the economy. ■

**Proof of proposition 9.** The proof of this result follows from the intuition given in the text. When transactions are observable, then the principal can limit the manager to make only one transaction by punishing him (by setting \( \xi_i = -\infty \)) for deviating. Thus, the manager will always trade one and only one security. Now, except for choosing a different investment fraction \( k \), the only other possible deviation is the double-deviation of trading only \( \omega \)-securities. Thus, the only constraint on the optimal contracting problem is \( \tilde{U}_{\varepsilon\omega} (k, p (\cdot)) \leq \bar{u} \). Also, note that the value of \( \tau \) doesn’t directly affect \( \tilde{U}_{\varepsilon\omega} (k, p (\cdot)) \) since the agent is trading only one security. In turn, this implies that this problem is a relaxed version of the problem of lemma 7 and the result follows. ■
References


