Countercyclical Restructuring and Jobless Recoveries

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Abstract

In the past three recessions, two major features of the business cycle have changed. First, employment now lags output growth, leading to jobless recoveries. Second, average labor productivity (ALP) has become acyclical or even countercyclical. This paper proposes a joint explanation for both facts. I develop a quantitative model in which firms streamline and restructure during recessions. The model captures the idea that firms grow "fat" during booms but then quickly "restructure" during recessions by laying off their unproductive workers. Firms then enter the recovery with a greater ability to meet expanding demand without hiring additional workers. This model explains 55% of the decline in the procyclicality of ALP observed in the data and generates a 4 quarters long jobless recovery after the Great Recession.

JEL Classification: E10, E24, E32, D21, J23, J24

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1 Introduction

"[Productivity growth] has enabled firms to meet the demand for their output without hiring workers.... Although other explanations for the jobless recoveries have played a role, in my view the productivity explanation is, quantitatively, probably the most important." - Ben Bernanke (2003)

What explains jobless recoveries? Interest in this question has been revived by the current slow recovery in U.S. labor markets. A full two years after the end of the Great Recession, employment is still 5% below its pre-recession level. Average labor productivity (ALP), however, rebounded quickly: by the end of the recession it was already back above trend. These twin facts present a puzzle, since standard models would predict that firms would start hiring again at such productivity levels. That recoveries can be jobless is widely recognized. What is less well known is that rising labor productivity during recessions is also a pervasive feature of recent business cycles. Since the mid-1980s, ALP has become markedly less cyclical at the same time as employment recoveries have become markedly more jobless. In this paper, I formalize a single explanation for both facts.

I develop a quantitative model where firms take advantage of recessions as an opportunity to streamline and restructure. The model captures the idea that firms grow "fat" during economic expansions, but then quickly trim this fat by aggressively cutting costs in recessions. During booms, firms employ unproductive workers because learning about match quality takes time and because adjustment is costly. In recessions, firms shed unproductive workers, entering the recovery with a greater ability to meet expanding demand without hiring additional workers. I call this mechanism "restructuring" because it formalizes the conventional wisdom that firms survive recessions by focusing on cost cutting and boosting productivity. The model can quantitatively match both facts: it explains 55% of the decline in the procyclicality of ALP and generates a four quarters long jobless recovery after the Great Recession.

Standard RBC models cannot explain why ALP is acyclical. If the model is driven by technology shocks, then labor productivity will be procyclical almost by construction. Even with other types of shocks, it is difficult to generate acyclical ALP because with a standard Cobb-Douglas production function and a competitive labor market, ALP is proportional to the wage. Thus, ALP can only be acyclical if real wages are too. Empirically, however, real wages are procyclical (Barsky, Parker and Solon, 1994; Haefke and van Rens, 2010).
The difficulty standard models have generating acyclical ALP motivates examining other classes of models. A natural starting place is a competitive industry model (Hopenhayn 1992). This model has the potential to induce countercyclical ALP movements through composition effects driven by endogenous entry and exit. If low productivity establishments are disproportionately "cleansed" during recessions, then qualitatively, the model will be able to generate less cyclical ALP.\footnote{Caballero and Hammour (1994, 1996) formalised the notion of cleansing in a business cycle context which goes back to Schumpeter (1939) and appears in a large number of models, among others Mortensen and Pissarides (1994); Campbell (1998).} Berger (2011) shows that these composition effects are quantitatively unimportant because general equilibrium effects are quite strong in this class of models due to free entry: wages adjust enough so that the marginal firm, which is indifferent between continuing and exiting, does not vary much over the cycle. Furthermore, at business cycle frequencies almost 80% of gross job creation and destruction occurs at continuing establishments rather than through establishment entry and exit. This motivates examining mechanisms which operate on the within establishment/firm margin.

In this paper, I base my model on the standard competitive industry model, Hopenhayn (1992), but departs significantly from it in three major respects. First, workers are heterogeneous and employers are able to selectively dismiss their least productive employees. Worker heterogeneity and selective firing create the within establishment "restructuring" margin that is central to my model. Second, I add employment adjustment costs. These costs, generate a trade-off between living with a low quality workforce and foregoing production while adjusting employment. This creates opportunities for intertemporal substitution and helps ensure that restructuring will be concentrated in recessions. Third, I include aggregate shocks, which are essential to investigating the business cycle implications of the model.

I calibrate the model to match a variety of cross-sectional facts, including the size and employment distribution of all U.S. nonfarm establishments as well as the average level of job creation and destruction. I estimate the level of employment adjustment costs by using simulated method of moments (SMM) applied to moments of the employment change distribution derived from the micro data of the Longitudinal Business Database, a comprehensive annual panel of all US formal employers maintained at the U.S. Census Bureau. The model replicates key empirical facts at the establishment level, such as a positive correlation between productivity and age and a negative correlation between employment growth and size.
After calibrating the model, I add aggregate productivity shocks and perform several computational experiments. First, I show that the model without worker heterogeneity and labor adjustment costs cannot match the post-1984 labor market facts: ALP is strongly procyclical and there are no jobless recoveries. In this sense, an industry model a la Hopenhayn (1992) is not different than a RBC model with flexible labor. It can replicate the pre-1984 business cycle dynamics, but not the post-1984 ones. Second, I show that adding adjustment costs to the model can generate moderate jobless recoveries, but makes ALP strongly procyclical due to a standard labor hoarding effect. Third, I add worker heterogeneity and selective firing and show that this model generates jobless recoveries that average four quarters in duration and a large decline in ALP cyclicality. In other words, a model with adjustment costs, worker heterogeneity and selective firing can explain the post-1984 labor market facts. Fourth, I explore whether the employment recovery in 2009 was jobless because the corresponding output recovery was anemic relative to previous recoveries. To do so, I examine what the employment recovery after Great Recession would have been had output growth been as strong as it was after the 1982 recession, which was a much more robust recovery. I find that the jobless recovery would have only been about two quarters in duration, about half of what we have observed.

Finally, I discuss what changed in the 1980s. I provide suggestive evidence that the structural change was the result of a large decline in union power in the 1980s. This led to a sharp reduction in the restrictions firms faced when adjusting employment, which lowered firing costs and made it easier for firms to fire selectively. I test this hypothesis using variation from U.S. states and industries. I show that states and industries that had larger percentage declines in union coverage rates had larger declines in the in the cyclicality of ALP, consistent with my hypothesis. The union power hypothesis is also consistent with evidence from detailed industry studies. A recent paper by Dunne, Klimek and Schmitz (2010) shows that there were dramatic changes in the structure of union contracts in the U.S. cement industry in the early-1980s, which gave establishments much more scope to fire workers based on performance rather than tenure. They show that immediately after these

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2 Closely related to the union power hypothesis is the "disposable worker hypothesis" of Gordon (2010). He argues that there was an increase in managerial power and a decline in labor power that contributed to both the rise in inequality and the increasingly cyclically sensitive labor market that we have observed. Another related explanation is that there was a dramatic shift in corporate philosophy towards maximizing shareholder value, especially since 1990 (Sinai 2010).

3 Gali and van Reus (2010) also suggest that the decline in union power can help explain the decline in the procyclicality of ALP. In their model, less union power means lower adjustment costs which leads to decreased labor hoarding. However, this explanation does not explain why the recoveries have become jobless. Thus it must be the case that a decline in union power led the prevalence of selective firing to increase and not just to a decline in firing costs.

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workplace restrictions were lifted, ALP and TFP in the industry increased significantly.

This paper contributes to several literatures. It first relates to the literature on "pit-stop" theories of the business cycle (Davis and Haltiwanger, 1990; Aghion and Saint-Paul, 1993; Hall, 2000), the idea that recessions are times when productivity-improving activities are undertaken because of relatively low opportunity costs. The paper that is most similar to my own is Koenders and Rogerson (2005). In their model, organizational inefficiencies build up within a firm during booms because managers focus on growth, then get eliminated during recessions when managers switch their focus to removing inefficiencies. My research is importantly different in that it is in general equilibrium and it explores the quantitative implications of this class of models.

This paper also relates to the growing literature on jobless recoveries and the changing cyclical dynamics of ALP. Typically, this literature has examined these facts in isolation from each other. Moreover, candidate theories tend to be inconsistent with the other post-1984 fact. For example, Bachmann (2011) argues that labor hoarding can help explain the 1991 and 2001 jobless recoveries. The problem with this explanation is that labor hoarding tends to make ALP more rather than less procyclical, in contrast to what we have observed in the data. Gali and van Rens (2010) present a model where the decline in the procyclicality of ALP is due to lower labor adjustment costs. In their model, lower adjustment costs leads to less labor hoarding, which makes ALP less procyclical. The difficulty with this explanation is that a decline in labor adjustment costs is inconsistent with jobless recoveries.

To my knowledge, there is only one other paper (Garin, Pries and Sims 2011) that also presents a unifying explanation for both jobless recoveries and the declining cyclicality of ALP. That paper’s hypothesis is that, because of the Great Moderation, reallocation shocks have become relatively more important than aggregate shocks, so that now reallocative shocks play an increasingly important role during downturns. Increased reallocation from less productive to more productive sectors during recessions leads aggregate productivity to rise, helping to explain the decline in the cyclicality of ALP. If this reallocation takes time, it will take longer for employment to recover after a downturn providing an explanation for jobless recoveries. However, there are two potential issues with their explanation. First, the evidence that reallocation and mismatch have become relatively important in recent recessions is mixed. Recent work by Valetta and Kuang (2010) and Barnichon and Figura

\footnote{Also see Aaronson et. al., 2004; Groshen and Potter, 2003; Shimer, 2010.}
(2010) suggests that while there is evidence of mismatch during recessions, mismatch has not significantly increased since 1980s. Second, the Great Moderation was a global phenomenon: it occurred in every G-7 country (Summers, 2005). If the Garin, Pries and Sims hypothesis is correct, we should observe jobless recoveries and acyclical ALP in countries that also had Great Moderations. In Section 7, I show that this is not something that we observe in the data. In fact, the coupling of jobless recoveries and acyclical ALP is only observed in the U.S., suggesting that we need a U.S. specific explanation for why these business cycle correlations changed in the mid-1980s.

The paper is organized as follows. In the next section, I document how the business cycle facts have changed. In Section 3, I present a simplified version of the model that conveys the main intuition. In Section 4, I build a general equilibrium model of establishment-level dynamics. In Section 5, I discuss my calibration and my steady state results. In Section 6, I add aggregate shocks and conduct the main policy experiment. In Section 7, I provide evidence that since the 1980s, labor market institutions have shifted to allow for more selective firing. Section 8 concludes.

2 Data Description and Empirical Evidence

The main source of data for the macro facts in this paper is the BLS labor productivity database. This database has quarterly information on output, total hours, employment and hours per worker from 1947 to the present. I follow the extant literature (Gali and van Rens, 2010) and use non-farm business as my main measure of output, however, the empirical results are robust to using GDP, all private business (including agriculture) and the non-financial sector instead. My preferred output series is constructed by excluding from GDP the following outputs: general government, nonprofit institutions, the farm sector, paid employees of private households, and the rental value of owner-occupied dwellings. Corresponding exclusions also are made to labor inputs. Hours and employment data for the major sector measures are drawn from the BLS Current Employment Statistics (CES) program, which provides monthly survey data on total employment and average weekly hours of production and nonsupervisory workers in nonagricultural establishments. Jobs rather than persons are counted. Weekly paid hours are adjusted to hours at work using data from the National Compensation Survey (NCS). I use total hours as my main measure of the labor input because it more closely corresponds to the labor measure in the model, since the
model does not include a distinction between the bodies and hours per worker. To make the evidence consistent with previous empirical work, I focus on the period 1960Q1-2011Q2. Since my focus is on labor market changes, I split the data into two sub-periods, pre-1984 (1960Q1-1983Q4) and post-1984 (1984Q1-2010Q2). The break date was chosen to be consistent with the existing literature (Gali and van Rens, 2010). None of the results are sensitive to this choice of break date.

I start by presenting evidence on the existence of jobless recoveries. Following the literature, I refer to jobless recoveries as a continued fall in aggregate labor input, accompanied by a continued increase in aggregate output (Aaronson et. al., 2004; Groshen and Potter, 2003; Bachmann, 2011). Figure 1 displays total hours and employment following recessions, where both series are shown in log-deviations. I do not HP-filter the data since this filter is not reliable near the endpoints making inference about the behavior of labor input during the Great Recession difficult.\(^5\) In each panel, all the series are normalized to zero at the NBER business cycle trough. The left panel of Figure 1, shows that relative to pre-1991 recessions (solid line), total hours fell as output recovered and did not recover to its pre-trough level for on average 7 quarters.

![Figure 1: The behavior of total hours and employment during business cycle recoveries](image)

The right panel of Figure 1 shows that the shift since pre-1991 recessions is even stronger for employment. This is because firms tend to increase hours per worker before they

\(^5\)Other than the most recent recession, the results are robust to HP-filtering the data.
hire new workers (Bachmann 2011). In Appendix A, I show that the recent jobless recoveries are not driven by trend declines in certain sectors or by cyclically sensitive sectors. For example, I show that jobless recoveries remain even after the manufacturing and construction sector have been removed from the aggregate employment series.

The cyclical dynamics of ALP also changed in the mid-1980s (Gali and Gambetti, 2009; Barnichon, 2010; Gali and van Rens, 2010). The left panel of Figure 2 shows the HP-filtered series of output and ALP. I HP-filter both series because end point issues are less of a concern here since the change occurred in the middle of the sample period. The shaded regions denote NBER recessions. Casual observation suggests that both ALP and output were strongly positively correlated before the mid-1980s, but the correlation is much weaker now. The right panel confirms this observation. It shows the eight year centered rolling correlation between output and ALP along with 95% confidence intervals. Before the 1980s, the correlation between output and ALP was high and stable at around 0.65, whereas the correlation is near zero or negative starting in the mid-1980s. It is also clear that the decline in the correlation of output and ALP was sharp and dramatic. Three features of the data are responsible for this quick decline: the eight year window, the fact that there were recessions approximately ten years apart, and the fact that ALP was strongly procyclical in one (the 1982 recession) and strongly countercyclical in the other (the 1992 recession).

Figure 2: The cyclical behavior of output and labor productivity

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6 I use a smoothing parameter of 1600.
Since the decline is stark, I performed multiple robustness checks to ensure that this trend break was not driven by my choice of data set. Research on U.S. labor productivity is complicated by the fact that a variety of measures are used, many of which differ along several dimensions. Fortunately, a recent data set constructed by Brugemann, Hagedorn and Manovskii (2010) is useful for addressing this question. They construct a consistent data set of both labor input and output series and then examine many different combinations of assumptions in order to see which ones matter and which ones do not. While the magnitude and level of the decline varies across different measures of inputs and outputs, Table 1a in the Appendix shows that the fact that there was a decline in the correlation of output and ALP during the 1980s is remarkably robust to the choice of data set. In particular, the decline is robust to using the aggregate employment measure from the household survey (CPS) as well as using GDP as the output measure. Finally, I show in Appendix A that the decline does not seem to be driven by a change in the behavior of entry and exit over the business cycle.

I also test formally whether the correlation between output and ALP changed in 1984. Table 1 displays the results: the change in correlations is strongly statistically significant.

Figure 3 shows that the behavior of output during recoveries has also changed since the mid-1980s. This figure is computed in the same manner as Figure 1. Relative to previous recoveries, output growth has been about 50% lower in the last three recoveries. Time zero denotes the trough of the business cycle and each series shows the cumulative percentage change since then. It is often argued that lower output growth during recoveries is the principal reason for jobless recoveries (Krugman, 2010; Orszag, 2011; Tasci, 2011). I address this question in Section 6.

<table>
<thead>
<tr>
<th></th>
<th>Pre-1984</th>
<th>Post-1984</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>HP</td>
<td>0.68***</td>
<td>0.09</td>
<td>-0.59***</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.11)</td>
<td>(0.13)</td>
</tr>
<tr>
<td>BP</td>
<td>0.69***</td>
<td>0.20**</td>
<td>-0.49***</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.09)</td>
<td>(0.14)</td>
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</tbody>
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SE in parenthesis. HP-filter with smoothing parameter of 1600.
Bandpass filter isolating periods [6-32]
Source: BLS Labor Productivity and Costs
3 Simple model

In this section, I introduce a simple, two-period model to illustrate how worker heterogeneity, learning, and selective firing interact to generate jobless recoveries and countercyclical productivity. I present the full, dynamic model in the next section. Workers in the model are matched to firms and productivity is match specific. The fact that human capital is match specific plays an important role in keeping the general model tractable as will become clear in the next section. Abowd, Kramarz and Margolis (1999) and Mortensen (2003) find evidence consistent with this assumption using evidence from matched employer-employee data.\footnote{More empirical support comes from Jacobson, Lalonde and Sullivan (1993). They show that employees who lost their jobs in plant closings experienced large and persistent earnings losses, which is consistent with a significant component of human capital being match specific.}

High and low productivity matches produce efficiency units $\theta_H$ and $\theta_L$, respectively, with $\theta_L \leq \theta_H$. When hiring, firms only observe the average match quality of a new hire, $p$, which is assumed to be constant in both periods.

The timing is as follows. In the first period, all firms are endowed with an initial TFP draw, $z_1$, and a certain number of employees, $L_1$, of whom a fraction $\alpha_1$ are high productivity matches. The subscript denotes the time period. I work with a continuum of workers so the firm always knows $\alpha_1$ precisely. At the end of the first period, firms learn the quality of its match with each of its workers. In the beginning of the second period, firms decide whether to hire new workers or fire existing ones.\footnote{This is a restriction, because a firm would be strictly better off if it could hire and fire within the same period. By firing all its known bad matches at the beginning of the period, a firm can always achieve a higher average match quality for a given workforce size. Firms are able to do this because they exchange certain bad matches with those that have a chance of being good.} I assume that employment adjustment is
costless. After making this restructuring decision, firms produce a homogenous good using a Cobb-Douglas production function where the only input is the total efficiency units of labor, \( \theta L_2 \). Here, \( \bar{\theta} \) is the average match quality of all a firm’s employees in efficiency units:

\[
\bar{\theta}(\alpha_2) = \alpha_2 \theta_H + (1 - \alpha_2) \theta_L
\]

Because productivity is match specific, the outside of option of each worker is the same at all firms. I therefore assume that wages are the same for all workers and are equal to one in both periods. Together, these assumptions imply that high productivity matches are strictly preferable because, per efficiency unit, high productivity matches are cheaper for the firm. Firms would like to be able to identify and fire their low productivity matches. Finally, I assume that \( \alpha_1 \geq p \). This assumption is (endogenously) satisfied in the fully dynamic model. In that model, a new firm, which has just hired by definition, has an average match quality equal to \( p \). Firing (weakly) increases match quality because of selection.

Consider the restructuring decision of a firm that starts the second period with \( L_1 \) workers, fraction \( \alpha_1 \) good matches, and TFP level \( z_1 \). The firm’s TFP draw in the second period is \( z_2 \). Given \( \alpha_1, L_1 \) and \( z_2 \), the firm’s optimal employment in the second period, \( L_2 \), solves:

\[
\max_{L_2} [\bar{\theta}(\alpha_2)L_2]^{\gamma} - L_2
\]

where \( \gamma \in (0, 1) \) and \( \alpha_2 \) is given by the following weighted average:

\[
\alpha_2 = \begin{cases} 
\alpha_1 \left( \frac{L_1}{L_2} \right) + p \left( \frac{L_2 - L_1}{L_2} \right) & \text{if hiring} \\
\alpha_1 \left( \frac{L_1}{L_2} \right) & \text{if firing fewer than } (1 - \alpha_1)L_1 \text{ employees} \\
1 & \text{if firing more than } (1 - \alpha_1)L_1 \text{ employees}
\end{cases}
\]

**Jobless Recoveries**

Let’s examine how average match quality at the firm affects the incentive to hire new workers. Suppose that productivity increases proportionately by \( \tau \) so that \( z_2 = (1 + \tau)z_1 \). Consider the homogenous worker model first (\( \theta = p\theta_H + (1 - p)\theta_L \)). This is a natural benchmark to consider since the homogeneous worker model exhibits the same employment dynamics as the heterogeneous worker model when there is either no learning or no selective firing.
In either case, average match quality for every firm is equal to $\bar{\theta}(p)$. The growth rate of employment is:

$$g_e = \left[ (1 + \tau)^{\frac{1}{1+\tau}} - 1 \right]$$  \hspace{1cm} (1)

A firm hires workers as long as $\tau > 0$, whenever TFP in the second period is greater than in the first period. Now consider the heterogeneous worker model with selective firing. In this case, the growth rate of employment is:

$$g_e = \left( \frac{\bar{\theta}(\alpha_1)}{\bar{\theta}(p)} \right) \left[ (1 + \tau)\frac{\bar{\theta}(p)}{\bar{\theta}(\alpha_1)} \right]^{\frac{1}{1+\tau}} - 1$$  \hspace{1cm} (2)

In the heterogeneous worker case (Equation 2), a firm only hires when $\tau$ exceeds a threshold, $\tau^*$:

$$\tau^* = \frac{\bar{\theta}(\alpha_1)}{\bar{\theta}(p)} - 1$$  \hspace{1cm} (3)

We learn two things from examining Equation (3). First, $\tau^* > 0$. Assume for a moment that $\tau \in (0, \tau^*)$. Notice that this implies that TFP in the second period is strictly greater than TFP in the first period. Equation (1) shows that a firm in the homogeneous worker model would want to hire if it had this aggregate TFP draw, whereas Equation (3) shows that a firm in the heterogeneous model would want to lay off workers if it received this same TFP draw in the second period. This means that relative to the homogeneous worker model, a firm needs a larger TFP shock in order to be willing to not lay off employees. The reason is because of firm learning and selective firing. At the end of the first period, the firm observes the exact quality of each worker, including the identity of all its worst matches. If $\tau \in (0, \tau^*)$, then the firm could increase its profits by shedding some of its low productivity ones. TFP must be high enough ($\tau > \tau^*$) to induce the firm to delay this restructuring process. The second point we learn from Equation (3) is that $\tau^*$ is increasing in $\alpha_1$. In words, firms with an initially better matched workforce (higher $\alpha_1$) need larger TFP to begin hiring than those with a more poorly matched workforce. The intuition is that new hires are on average less productive than incumbent workers, so that each additional hire dilutes average match quality, generating an additional margin of diminishing returns.

These two observations explain why the model with worker heterogeneity and selective firing is able to generate jobless recoveries. After the recession, many firms have a well

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9 When firms cannot learn about match quality, firms fire at random; conversely, if firms cannot lay off selectively, firm knowledge about individual employees cannot be used in termination decisions. The only difference between the two models is a scale factor, $\bar{\theta}(p)$, which affects the level, but not the growth rate of employment.
matched workforce because they recently laid off their productive employees. Now they need much higher productivity/demand to begin hiring again relative to the homogenous worker model.

**Countercyclical Average Labor Productivity**

ALP is defined in the model as $Y/L$ where aggregate output and labor are defined as $Y = \int y_i$ and $L = \int L_i$ and $y_i = s_i^r L_i^r$ is gross output at firm $i$. Denote ALP initially and after selectively firing as $(Y/L)^i$ and $(Y/L)^f$ respectively. One can show that:

$$(Y/L)^f > Y/L$$

In other words, ALP is higher after selectively firing. If layoffs are concentrated early in recessions, then the model can generate countercyclical ALP. The intuition is that firms shed their least productive workers first, raising their average productivity and this composition effect dominates the decline in ALP due to the fall in TFP.

This simple model illustrates how worker heterogeneity, learning and selective firing interact to allow the model to qualitatively generate jobless recoveries and countercyclical ALP. In order to evaluate whether these insights survive the introduction of dynamics, general equilibrium and adjustment costs, we need to investigate a quantitative version of this model.

4 Model

4.1 Setup

Time is discrete and the horizon is infinite. The economy is inhabited by two kinds of entities: establishments and consumers. Establishments use labor to produce output. Consumers own establishments, supply labor, and consume. There are perfectly competitive output and labor markets. One homogenous good is produced and sold. The price of output is the numeraire and is normalized to 1. The only price we have to keep track of is the wage of the workers.

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10 In particular: $\frac{(Y/L)^f}{Y/L} = \left( \frac{\pi(\alpha_1)}{\pi(0)} \right)^{\gamma \frac{1-\sigma}{1+\sigma}} > 1$. Because of selection, $\frac{\pi(\alpha_1)}{\pi(0)} > 1$. Then, using the definition of the growth rate when firing workers, $g_f = \left( \frac{\pi(\alpha_1)}{\pi(0)} \right) \left[ \frac{(1-\sigma)\pi(0)}{\pi(\alpha_1)} \frac{1}{1+\gamma} - 1 \right]$ implies that $\frac{1-\sigma}{(1+g_f)^{1-\gamma}} > 1$. 

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Production

Workers have heterogeneous match qualities. There are two types of matches, with efficiency units $\theta_H$ and $\theta_L \leq \theta_H$. Match quality is specific to each establishment-worker pair. In other words, a worker who is a bad match at one establishment might be a good match at another establishment. The probability that a newly hired worker is a good match is equal to $p$. This assumption is justified if work is an experience good: the establishment only learns about the quality of the establishment-worker match through production as in Jovanovic (1979). Since $\theta$ is match specific, the composition over the potential match quality of the hiring pool is always constant. All matches are perfect substitutes.

Establishments have heterogeneous TFP. An establishment’s gross production can be written as $Y_i = s_i z f(\overline{\theta}(\alpha_i)L_i)$ where $\alpha_i$ is the proportion of good matches each establishment employs; $\overline{\theta}(\alpha_i) = \alpha_i \theta_H + (1 - \alpha_i) \theta_L$ is the total number of efficiency units at establishment $i$; $L_i$ denotes the labor input; $s_i$ and $z$ are the idiosyncratic and aggregate TFP levels, respectively; $f$ is a concave function. Establishments also face labor adjustment costs, $G(L', L)$, whose exact form will be discussed in detail during the calibration section. I work with a continuum of workers so the establishment always knows $\alpha$ precisely. For notational convenience, define $\Gamma$ as a shorthand for all the establishment specific state variables: $\Gamma = (\alpha, L, s)$.

Shocks

There are two types of TFP shocks in my model: aggregate and idiosyncratic. I assume that both follow an AR(1) in logs

$$
\log s'_i = a_s + \rho_s \log s_i + \epsilon'
$$

$$
\log z' = \rho_z \log z + \varepsilon'
$$

where both $\epsilon$ and $\varepsilon$ are mean zero Gaussian random variables.

Informational Structure

When establishments hire, they do not know initially what the match quality of any individual worker will be. However, when they decide to lay off workers, they can do so selectively. The exact quality of each matched worker is known with certainty after one period. This is a stark assumption but it is a useful shortcut for a more micro-founded learning process.\(^\text{11}\)

\(^{11}\)In general, models with bayesian learning with normal signals have the result that learning is quite fast (though not as fast as one quarter), so my assumption that complete learning only takes one period may not be that consequential. I am currently working on relaxing this assumption.
Wage outcomes

Competitive markets pin down a unique wage per unit of time worked.\textsuperscript{12} Since the quality of a match is specific to each establishment/worker pair, good and bad workers are paid the same wage regardless of their history. This arises because each worker’s outside option is determined by what she could get paid if she worked at another establishment (Felli and Harris, 1996). Because the labor market is competitive and human capital is establishment/worker specific, workers cannot appropriate any of the match specific surplus (Becker, 1964).

Entrants

Each period there is an unbounded mass of potential entrants, of which measure $J$ enter and measure $E$ actually decide to produce. Entrants decide to enter before observing their idiosyncratic TFP draw $s$. All establishments that enter pay an entry cost, $c_e$, whether they decide to produce or not.

Figure 4 shows the timing of events. First, the aggregate shock $z$ is revealed to

\textsuperscript{12}In the model, all workers earn the same wage regardless of their productivity, whereas there is significant wage dispersion in the data. In reality, all that needs to be true for firms to want to selectively fire is for wages to not increase 1-1 with productivity, so that firm profits are increasing in worker productivity. Is there evidence that profits are increasing in worker productivity? One indirect source of evidence is that the level of the unemployment rate is decreasing in the level of the workers education. Since wages are strongly correlated with educational attainment, this implies that firms prefer to employ high wage workers. Firms would only do this if these workers were more productive.
all establishments. Second, all incumbents receive their new idiosyncratic shock, \( s \). Third, each incumbent decides whether to exit or not. If it stays, it pays a flow fixed operating cost \( c_f \), decides whether to adjust employment or not, produces and pays workers. If the establishment exits, it avoids paying \( c_f \), but receives a zero payoff in all future periods. Fourth, entry occurs. If an establishment decides to enter, it pays the entry cost and receives its idiosyncratic TFP draw. Each entrant then decides whether to exit or not. If the entrant stays, it hires workers and produces and pays workers. Finally, the period ends.

### 4.2 Optimization

#### 4.2.1 Incumbents

The value of an incumbent establishment at the beginning of the period is described by the Bellman equation\(^{13}\)

\[
V(\alpha, L, s; z) = \max \{ -wG(0, L), \max_{L' \geq 0} \frac{s z (\theta(a')L')^{\gamma} - w L' - wc_f - wG(L', L) + \beta E_{s, z} [V(\alpha', L', s'; z')]} {\max_{\alpha' \geq 0}} \}
\]

The incumbent compares the value of exiting with the value of operating this period. If the establishment shuts down it has to pay the costs of getting rid of its entire workforce, \( wG(0, L) \), where \( g \) is the adjustment cost function. The policy function for this decision is denoted by \( X(\Gamma, z) \), where \( X(\Gamma, z) = \{0, 1\} \) captures the decision to continue/exit, respectively. Given the current state variables, the continuing establishment decides how much to adjust employment (if at all). The policy function for this decision is the net hiring decision, \( h(\Gamma, z) = L'(\Gamma, z) - L(1 - \delta) \) where \( \delta \) captures exogenous attrition of the labor force. Since all workers are indifferent about where they work, it is reasonable to assume that the level of attrition is the same for both high and low types. Finally, there is an indicator for whether the establishment adjusts its employment. If \( h(\Gamma, z) \neq 0 \) then \( \phi(\Gamma, z) = 1 \), otherwise, \( \phi(\Gamma, z) = 0 \).

Establishment hiring/firing decisions determine \( \alpha' \), tomorrows proportion of good matches. The hiring case is straightforward. After hiring \( h(\Gamma, z) > 0 \) new employees, who are well matched with probability \( p \), the proportion of matches that are of good quality is equal

\(^{13}\)This formulation of the incumbent’s problem does not allow for the possibility that the establishment can hire and fire in the same period. Relaxing this restriction did not change the quantitative results.
to:

\[
\alpha' = \left( \frac{L(1-\delta)}{L(1-\delta) + h(\Gamma, z)} \right) \alpha + \left( \frac{h(\Gamma, z)}{L(1-\delta) + h(\Gamma, z)} \right) p
\]

a weighted average of its current size of the workforce as well as on its average match quality. This equation depends on both the current measure of employees as well as the current proportion of good matches. Another important point is that hiring dilutes the average quality of an establishment's workforce. This is true because in equilibrium, \( \alpha \geq p \). Every entrant has \( \alpha = p \) when first hiring, and then \( \alpha \) (weakly) increases from there because of selective firing.

Selectivity combined with perfect substitutability (and the fact that all workers get paid the same wage) implies that when an establishment lays workers off it gets rid of the worst matches first. The transition rule for \( \alpha' \):

\[
\alpha' = \begin{cases} 
\frac{aL(1-\delta)}{L(1-\delta) - h(\Gamma, z)} & \text{if } -h(\Gamma, z) \in [0, (1-\alpha)L(1-\delta)] \\
1 & \text{if } -h(\Gamma, z) \in ((1-\alpha)L(1-\delta), L(1-\delta)]
\end{cases}
\]

The top case shows how \( \alpha' \) evolves if the establishment only fires bad matches. This is possible if the establishment wants to fire fewer than \((1-\alpha)L(1-\delta)\) workers. If the establishment desires to fire more than \((1-\alpha)L(1-\delta)\) workers, then the establishment has to fire some of its good matches as well and \( \alpha' = 1 \).

Finally, the establishment also always has the option of not adjusting. In this case the problem is trivial with \( h(\Gamma, z) = 0 \) and \( \alpha' = a \).

4.2.2 Establishment entry

Entrants compare the expected value of entering

\[
V^e(z) = \int V(0, 0, s; z) d\varphi(s)
\]

to the cost of entering, \( w_c \). If they do not enter, they earn a zero payoff forever. If they enter, their problem is identical to the production decision of an incumbent establishment which faces TFP draws \((s, z)\) and currently employs no workers. Because the entry decision is made before the idiosyncratic TFP level \( s \) is drawn, some entrants choose to exit immediately. Free entry implies that \( V^e(z) \leq w_c \) with equality when entry occurs.
4.3 Households

A representative consumer maximizes expected utility

\[ U = E \left[ \sum_{t=0}^{\infty} \beta^t \left( C_t - \frac{A\varepsilon}{1+\varepsilon} L_t^{1+\varepsilon} \right) \right] \]

choosing \( C_t \), consumption of the numeraire good and \( L_t \), total hours. \( \beta \in (0,1) \) is the discount factor, \( A > 0 \) measures the disutility of labor supply and \( \varepsilon \) is the Frisch elasticity of labor supply. The consumption good is perishable and there are no financial markets, so the consumer cannot save or borrow. The budget constraint each period is

\[ C_t = w_t L_t + \Pi_t \]

where \( \Pi_t \) is the sum of all profits made in the economy and \( w_t \) is the wage rate, both expressed in terms of the consumption good numeraire.

The FOC are given by:

\[ \rho_t = u_C(C_t, L_t) = 1 \]
\[ w_t = -u_N(C_t, L_t) = \frac{AL_t^{1/\varepsilon}}{\rho_t} \]

where \( \rho_t \) is the pricing kernel. Notice that the pricing kernel is pinned down by my assumption that the consumer has linear utility in consumption. This assumption is not innocuous but greatly simplifies the computational burden of this problem. This simplification enables me to apply the consumer discount factor \( \beta \) to the incumbent establishment’s maximization problem. Since the focus of this paper is on the labor margin, it is more important to incorporate a realistic labor supply elasticity.

4.4 Equilibrium

The distribution of establishments over \((\alpha, L, s)\) can be summarized using the probability measure \( \mu \). This distribution evolves from the current period to the next according to the mapping \( \mu' = T(\Gamma, z, \mu, E) \), which establishments take as given. For a given \( \mu_0 \), a recursive competitive equilibrium consists of (i) value functions \( V(\Gamma, z) \) and \( V^e(z) \), (ii) policy functions \( X(\Gamma, z) \) and \( h(\Gamma, z) \) (iii) bounded sequences of non-negative wages \( \{w_t\}_{t=0}^{\infty} \), incumbent measures \( \{\mu_t\}_{t=0}^{\infty} \), and entrant measures \( \{E_t\}_{t=0}^{\infty} \) such that
1. Establishment optimality: $V(\Gamma, z), h(\Gamma, z),$ and $X(\Gamma, z)$ solve the incumbent’s problem.

2. Household optimality: Given $\{w_t\}_{t=0}^{\infty}, L^* = \left(\frac{w_t}{\mathcal{A}_t}\right)^\varepsilon \forall t \geq 0$

3. Labor market clearing\(^1\):

$$L^*(w_t) = \int (L_t(1 - \delta) + h_t(\Gamma_t, z_t) + G(L_t, L_{t-1}) + c_f) d\mu_t +$$

$$\int (L_t(1 - \delta) + h_t(\Gamma_t, z_t) + G(L_t, L_{t-1}) + c_f) d\varphi(s_t) + c_e J_t$$

4. Measure of actual entrants: $\forall t \geq 0$,

$$E_t = J_t \int_{B_e} [1 - X_t(\Gamma_t, z_t)] d\varphi(s_t)$$

where $B_e = \{(\Gamma, z) \text{ s.t: } V^c(z) \geq c_e \text{ and } X(\Gamma, z) = 0 \}$

5. Model consistent dynamics, $T(\mu_t, J_{t+1})$

$$\mu_{t+1} = \int \int_B [1 - X_{t+1}(\Gamma_{t+1}, z_{t+1})] dH(s_{t+1} | s_t) d\mu_t + E_{t+1}$$

with $B = \{(\Gamma, z) \text{ s.t: } V(\Gamma, z) > -w(G(L, 0) \text{ and } X(\Gamma, z) = 0 \}$

5 Calibration and Steady State Results

5.1 Calibration

I calibrate my model at quarterly frequency, so that its steady state equilibrium, where $z = 1$ forever, matches establishment level facts for the U.S. non-farm sector. Then I add aggregate shocks, $z$, to evaluate the model’s business cycle performance. The details of the computation of the steady-state model are described in Appendix C. I use four data sources to calibrate and estimate my model. The first three sources are collected by the BLS. The Job Openings and Labor Turnover Survey (JOLTS) produces monthly information on job openings, hires, and separations for a representative sample of establishments. The Business Employment Dynamics (BED) program creates quarterly series on gross job gains and gross job losses derived from the universe of unemployment insurance records.

\(^1\)This is a slight abuse of notation. Here $G(L_t, L_{t-1})$ denotes the labor adjustment costs excluding the disruption costs, which are denominated in units of out (see the calibration section), rather than total employment adjustment costs.
<table>
<thead>
<tr>
<th>Empirical moments</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual real interest rate</td>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>Employment rate</td>
<td>$A$</td>
<td>1.66</td>
</tr>
<tr>
<td>Frisch elasticity</td>
<td>$\varepsilon$</td>
<td>1.0</td>
</tr>
<tr>
<td>Labor share</td>
<td>$\gamma$</td>
<td>0.70</td>
</tr>
<tr>
<td>Quit rate*</td>
<td>$\delta$</td>
<td>0.057</td>
</tr>
<tr>
<td>Reallocation rate**</td>
<td>$(p_s, \sigma_s)$</td>
<td>(0.95, 0.19)</td>
</tr>
<tr>
<td>Exit rate**, average size of establishments***</td>
<td>$(c_f, a_s)$</td>
<td>(3.13, 0.04)</td>
</tr>
<tr>
<td>Steady state wage level***</td>
<td>$c_e$</td>
<td>5.50</td>
</tr>
<tr>
<td>Average size of entrants**</td>
<td>$b$</td>
<td>1.0</td>
</tr>
<tr>
<td>Hazard rate of worker separation by tenure***</td>
<td>$(\theta_H/\theta_L, p)$</td>
<td>(1.2, 0.6)</td>
</tr>
</tbody>
</table>

*Source: BLS Job Openings and Labor Turnover Survey  
**Source: BLS Business Employment Dynamics  
***Source: BLS Current Population Survey

<table>
<thead>
<tr>
<th>Table 2: Baseline Parameters</th>
</tr>
</thead>
</table>
The Current Population Survey (CPS) is a monthly survey of households that provides comprehensive information on household’s labor market status and earnings. The last source of information is the Longitudinal Business Database (LBD) collected by the U.S. Census. The LBD covers all business establishments in the U.S. private non-farm economy that file payroll taxes with the IRS (Haltiwanger, Jarmin and Miranda, 2010). As such, it covers the universe of establishments in the U.S. nonfarm business sector with at least one paid employee. The LBD includes information on establishment size, age and industry.

I set $\beta = 0.99$ which corresponds to a 4% annual real interest rate. The parameter $\gamma$ corresponds to the labor share, so I set it equal to 0.7. The parameters $A$ and $\varepsilon$ in the utility function are chosen so that the employment rate is 60% ($L = 0.6$) and so that the average Frisch elasticity of labor supply is equal to 1. I set the exogenous attrition rate of workers equal to 0.057, which is the average quarterly quit rate from the JOLTS data set.

As is standard in the literature on employer learning, I identify the worker heterogeneity parameters using the hazard rate of worker separation by duration. I normalize $\theta_L = 1$ and choose $\theta_H$ and $p$ to match the estimated hazard rate following procedure detailed in Pries (2003). I use cross-sectional data from the CPS to construct the empirical hazard rate. I then smooth this hazard and pick the values of $\theta_H$ and $p$ that provide the best fit to this estimated hazard rate. Matching the hazard rate is informative for identification because holding $\theta_H/\theta_L$ constant, as $p \to 1$ the initial hazard rate becomes less steeply decreasing since newly hired workers are always good matches and there is not reason to get rid of them the next period. The relative productivity between good and bad matches is also informative about the hazard rate: as $\theta_H/\theta_L$ increases, the incentive to restructure increases, and average
Moments in the data

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average size of continuing establishments</td>
<td>16.1</td>
<td>15.4</td>
</tr>
<tr>
<td>Average size of entering establishments</td>
<td>5.99</td>
<td>6.16</td>
</tr>
<tr>
<td>Establishment exit rate</td>
<td>5.18</td>
<td>5.81</td>
</tr>
<tr>
<td>Establishment entry rate</td>
<td>5.40</td>
<td>5.81</td>
</tr>
<tr>
<td>Reallocation rate</td>
<td>14.68</td>
<td>15.89</td>
</tr>
</tbody>
</table>

Source: BED. Average of quarterly data, 1992q3-2011q2.

Table 3: Calibration targets

tenure per worker declines.

For the idiosyncratic TFP process, I choose $\rho = 0.95$ and $\sigma = 0.19$ which correspond to a persistence of 0.81 and a standard deviation of 0.38 at an annual frequency. This level of persistence is higher than estimates from Cooper and Haltiwanger (2006) but consistent with other studies (Mukoyama and Lee, 2011). Conditional on the shock process, I estimate the adjustment cost parameters using the SMM procedure of Cooper and Haltiwanger (2006). I use moments of the distribution of employment changes that are representative of the whole U.S. economy. I discuss this procedure in more detail below.

The average employment size and the exit rate of establishments are used to determine the flow fixed cost, $c_f$, and the mean level of the idiosyncratic shock, $a_s$. There is a unique choice of $c_f$ and $a_s$ that matches these two numbers. This follows from the fact that the threshold productivity level, $s^*$, below which establishments choose to exit is increasing in $c_f$ and $a_s$ and that, given $s^*$, the average size is increasing in $a_s$ and independent of $c_f$. I normalize the wage rate, $w$, in the benchmark to 1. This pins down the entry cost, $c_e$, in the model. I pin down the pdf of the shocks entrants receive, $\varphi$, using the average size of new entrants. The specific functional form I assume is $\varphi(s) = be^{-bs}$. This distributional choice delivers the result that new establishments are on average smaller than incumbent establishments, a fact that is present in the data. The parameter $b$ is chosen to match the initial employment distribution of entrants. So in essence, I give myself one degree of freedom to match the average size of new entrants. Table 3 shows that the model gets close to matching the calibration targets.

5.2 Estimating adjustment costs

Following Cooper and Haltiwanger (2006) and Cooper, Gang and Yan (2010), I allow the adjustment cost function to be different depending on whether the plant is increasing or

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15Ideally, I would calibrate the TFP process and adjustment costs jointly. I do not because there exists no establishment level information on output for all sectors. Most papers in the literature calibrate the TFP process under the assumption that there are no adjustment costs in the model (Hopenhayn and Rogerson, 1993; Mukoyama and Lee, 2011). This procedure is problematic because it is difficult to distinguish between the dynamics of the driving process and adjustment costs.
reducing employment. I assume that adjustment costs are on gross flows.\textsuperscript{16} I start with the following cost function

\[
G^+(\alpha', L', \alpha, L, s) = \kappa^+(L' - L(1 - \delta)) + \nu^+ \left( \frac{(L' - L(1 - \delta))}{L(1 - \delta)} \right)^2 L(1 - \delta) + (1 - \lambda^+)R(\alpha', L', \alpha, L, s) \tag{8}
\]

if an establishment hires,

\[
G^-(\alpha', L', \alpha, L, s) = \kappa^-(L' - L(1 - \delta)) + \nu^- \left( \frac{(L' - L(1 - \delta))}{L(1 - \delta)} \right)^2 L(1 - \delta) + (1 - \lambda^-)R(\alpha', L', \alpha, L, s) \tag{9}
\]

if it lays workers off. If \( L' = L(1 - \delta) \) then \( G(\alpha', L', \alpha, L, s) = 0 \).

There are three forms of adjustment costs. First, I allow for linear adjustment costs, parameterized by \( \kappa \) to capture, for example, severance payments to workers. The second is a quadratic adjustment cost, parameterized by \( \nu \). This cost captures the idea that large changes in hiring and firing rates are increasingly costly. Finally, I allow for disruption style adjustment costs parameterized by \( \lambda \). This parameter can be interpreted as the opportunity cost of adjustment: the plant loses a fraction \((1 - \lambda)\) of its revenues \((R(\alpha', L', \alpha, L, s))\) when it adjusts its labor force. One interpretation of this cost is that it captures the disruption to the production process when a plant hires or fires workers. For example, shedding workers may require shutting down an assembly line for some period of time. While the former interpretation is natural for manufacturing, for the service sector it is more natural to think of this cost as capturing the time cost of adjusting employment. This interpretation is reasonable if managerial time or effort is an important input into the production process.

I estimate the adjustment cost parameters by SMM. This approach revolves around finding the vector of structural parameters to minimize the weighted difference between simulated and actual data moments. I follow Cooper, Gang and Yan (2010) in my choice of moments to target, which are moments of the employment distribution derived from the micro data underlying the LBD.\textsuperscript{17} I use information for the period 1985-1999 since my focus is on recent business cycles.\textsuperscript{18} I do not try to estimate all six parameters at the same time, instead I focus on a subset of the adjustment cost parameters. In particular, I assume that the costs of hiring and firing are the same \((G^+(\alpha', L', \alpha, L, s) = G^-(\alpha', L', \alpha, L, s))\).

\textsuperscript{16}For expositional simplicity, I do not show the adjustment cost function for the case when the establishment can hire and fire in the same period. In this case, the establishment has to pay the adjustment costs when hiring and when firing.

\textsuperscript{17}I graciously thank Giuseppe Moscarini for providing these moments from a validation of results of the U.S. Census Bureau’s Synthetic LBD program: http://www.vrdc.cornell.edu/news/data/lbd-synthetic-data/

\textsuperscript{18}1999 is the more recent year available in the synthetic LBD.
The seven moments I match are derived from the distribution of unweighted employment changes for continuing establishments. The first moment is the inaction rate, the fraction of establishments that do not change employment in a given quarter. The other six moments are the fraction of establishments that have job creation and destruction rates less than 10% (JC10/JD10), between 10% and 20% (JC1020/JD1020), and greater than 30% (JC30/JD30), respectively. While there is no clear 1-1 mapping between all the parameters, it is clear how to identify some of them. First, roughly speaking, the quadratic adjustment is identified off of the small employment changes (JC10/JD10). Second, the large amount of inaction observed even at an annual frequency suggests that non-convex or fixed adjustment costs are important. The simulated moments are obtained by solving the dynamic programming problem in equation (4) for a given value of the parameters. The resulting decision rules are used to simulate a panel data set. The simulated moments are calculated from that data set.

Table 4 displays the results. The first row shows the raw empirical moments from the LBD that the procedure is trying to match. Consistent with the evidence in Davis, Haltiwanger and Schuh (1996), there is an incredible amount of heterogeneity in establishment level growth rates. On average over thirty percent of establishments have employment growth rates that exceed 30%, while another thirty percent of establishments are not changing their size at all.\textsuperscript{19} The next two rows display the results for each specification. The columns display how well each model matches a particular moment. Overall, the model matches the distribution of employment changes well although the model predicts too little inaction and too much job destruction.\textsuperscript{20} This is not surprising since the introduction of worker heterogeneity and learning creates strong incentives for layoffs. The estimated parameter values are: \((\kappa, \lambda, \nu) = (1.6, 0.975, 0.22)\) These parameter values imply that average adjustment costs conditional on adjusting are about 2.8% of annual revenues or about 8

\textsuperscript{19}Since I only have annual data on job not worker flows, there is a possibility that the inaction rate is significantly overestimated. Establishments could be hiring/firing all the time and just by coincidence have the same employment level each year in the month when the data were collected. Since 1991, the BED produces a quarterly series on the fraction of establishments that do not change employment in a given quarter. On average, this fraction is greater than 50% suggesting that inaction is not an artifact of time aggregation.

\textsuperscript{20}In future work, I plan to allow the adjustments to be different if firms are hiring or firing.
Table 5: Size and employment distribution

<table>
<thead>
<tr>
<th>Estabs Size</th>
<th>Estabs share</th>
<th>Employment share</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>1-19 emps</td>
<td>0.866</td>
<td>0.807</td>
</tr>
<tr>
<td>20-99 emps</td>
<td>0.111</td>
<td>0.154</td>
</tr>
<tr>
<td>100-499 emps</td>
<td>0.020</td>
<td>0.036</td>
</tr>
<tr>
<td>500-999 emps</td>
<td>0.002</td>
<td>0.001</td>
</tr>
<tr>
<td>1000+ emps</td>
<td>0.001</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Source: County Business Patterns. Average of annual, 1984-2008

working days. This estimate is slightly larger though roughly consistent with other estimates in the literature.\(^{21}\) Furthermore, the estimated magnitude of the disruption cost is in the middle of previous estimates.\(^{22}\) The reason estimated adjustment costs are somewhat bigger than other estimates in the literature is attributable to my assumption that learning about match quality takes place completely after one quarter. If learning was slower, then the option value for each new hire would be larger so establishments would wait longer before shedding bad matches. In the absence of slow learning, I need larger adjustment costs to prevent establishments from adjusting employment too frequently.

5.3 Steady-State Results

Tables 5 evaluates the distributional performance of the model. I do not force the model to match either the size or employment distribution of establishments, so this is a good test of fit. Table 5 show the size distribution of establishments in both the data and the model. The moments of the data are computing using data from the County Business Patterns, an annual series of the number of establishments, employment and payroll information collected by the U.S. Census Bureau. Overall the fit is good. There are slightly too many mid-sized establishments in my model but the model is quite close to matching the establishment size distribution. The last two columns show the employment share by establishment size class in both the data and the model. Once again, the model over predicts the amount of employment in the middle size classes, but otherwise the model fits the data well.

\(^{21}\)Bloom (2009) finds that average employment adjustment costs are aproximately 2.1% of annual revenues; Bachmann (2011) finds that average adjustment costs are approximately 2.05% of annual revenues.

\(^{22}\)Cooper, Haltiwanger and Willis (2004) estimate \(\lambda = 0.983\) for the U.S. manufacturing sector using data from 1972-1980. Cooper, Gong and Yang (2010) find a mean estimate of \(\lambda = 0.95\) for private establishments in China. Estimates of the magnitude of disruption costs for investment are much larger (close to \(\lambda = 0.70\)). Cooper and Haltiwanger (2006).
6 Results with Aggregate Shocks

6.1 Computation

As a result of free entry, labor demand becomes perfectly elastic—there is one wage level that is consistent with the labor market equilibrium, regardless of the level of employment. Given the assumption of linear utility for consumption, the wage depends only on aggregate TFP and is not affected by the labor demand of incumbents.\footnote{Given preference assumptions, there is an exact 1-1 mapping between the equilibrium amount of labor supply (and hence labor demand) \( L_t = \left( \frac{w_t}{A_t} \right)^{\alpha} \).} Aggregate shocks affect the wage only through the value of entry. Using this property, I can perform the optimization by establishments and determine the wage without considering the labor-market equilibrium. After the wage is determined, the labor-market equilibrium determines the equilibrium quantities, in particular the mass of actual entrants, \( E \).

Free entry is a very convenient assumption computationally as there is no need for a Krusell-Smith approximation. However, free entry has some significant downsides. First, it implies that the entry rate is too volatile relative to the data. Second, it makes total labor demand a deterministic function of the aggregate shock. Thus, it is uninteresting to examine the business cycle behavior of total employment because it follows the dynamics of the exogenous driving process. Instead, I compute all the aggregate business cycle statistics on the labor demand and output of incumbents. While this is a limitation of my modeling approach, it is consistent with shortcomings in how aggregate employment is measured in the Current Employment Statistics, the main source of national employment information. The BLS only adds new businesses to this survey twice a year, noting that "there is an unavoidable lag between the birth of a new firm and its appearance on the sampling frame and availability for selection."\footnote{http://www.bls.gov/ces/cesfaq.htm#revisions16} I am currently working on relaxing the free entry assumption. In particular, I am solving the model with a fixed number entrants which is much more computationally demanding problem.\footnote{With a fixed number of entrants, the model solution is no longer block recursive so that I must use the trick of Krusell-Smith (1998) to approximate the cross-sectional distribution of establishment's employment and match quality. The problem is similar in structure to the problem described in Appendix E of Mukoyama and Lee (2011), except that my problem has one additional idiosyncratic and aggregate state variable making it significantly more computationally demanding.} This computational approach is discussed in detail in Appendix D. Since the approach is computationally demanding, I only have preliminary results for this procedure. The main results survive eliminating the assumption of unbounded entrants, however, in general all the main results are somewhat attenuated. The reason the results are weaker is that in this version of the model the wage is much less procyclical. This makes
adjustment costs less procyclical since two components of the adjustment cost function are
denominated in units of labor. This weakens the restructuring mechanism in the model
since less restructuring is concentrated in recessions leading to weaker results. Nonetheless,
the model still explains about 30-40% of the decline in the procyclicality of ALP that we
observe in the data and generates substantial jobless recoveries.

To analyze business cycles, I add aggregate shocks to the model. Since output growth
coming out of the last 3 recessions has been much lower than before, I calibrate the aggregate
productivity shock to match the cyclical properties of HP-filtered output for the post-1984
period. I find that $\rho_z = 0.88$ and $\sigma_z = 0.006$ provide a good fit: the persistence and standard
deviation of HP-filtered non-farm business output for the period 1984q1-2011q2 are 0.89 and
0.015, respectively.

I perform two experiments with three models: the standard Hopenhayn model with-
out and with labor adjustment costs, and my model with heterogeneous workers, learning
and the ability to selectively fire. The goal of the experiments is to evaluate whether each
model can replicate the macro facts presented in Section 2.

### 6.2 Experiment 1: Cyclicality of ALP

The first experiment tests whether each of the models with aggregate shocks can
match the decline in the procyclicality of labor productivity that was observed in the data.
I simulate each model economy 500 times for 110 periods after a burn-in period. I choose
110 periods because that is the same number of observations I have in the data. I HP-filter
both output and ALP series and compute the correlation between the two variables. I report
the level of this correlation for the data and all models in the second column of Table 6. The
first three rows show the level of the correlation in the data for the pre and post 1984 period.
As documented earlier, there was a large decline in the correlation between these periods.

The fourth and fifth rows show that the standard Hopenhayn model with and without
adjustment costs generates a correlation that is far too positive relative to the data. This is
perhaps surprising because the standard model has a potential mechanism to generate coun-
tercyclical ALP through the cleansing of low productivity establishments during recessions.
However, as others have found, selection on the extensive margin is quantitatively small
(Berger, 2011; Mukoyama and Lee, 2011). The intuition behind this result is simple. There
are two main effects which determine the strength of the cleansing margin. In response to a
negative aggregate shock, ceteris paribus, the threshold level of productivity below which all establishments exit rises because establishments who were indifferent between exiting and continuing at the previous level of the aggregate shock now want to exit. But wages also respond to the aggregate shock, reducing the wage bill and the fixed costs of operations and making exit more attractive. It turns out that in most versions of the model, wages decreases just enough so that the same establishments that were previously marginal are also (virtually) indifferent between exiting and continuing at the level of aggregate TFP. In other words, these models behave very similarly to a competitive RBC model where ALP is proportional to a strongly procyclical real wage.26

The sixth row shows the results for my model with selective firing. The third column shows the change in the correlation from the pre-1984 period to the post-1984 period. Relative to the heterogeneous worker version of the Hopenhayn model without selective firing (equivalent to the Hopenhayn model with adjustment costs), the model with heterogeneous workers reduces the correlation between output and ALP to 0.661, or 55% of the drop in the correlation that we observe in data.27 Given my assumptions, this comparison model is isomorphic to the Hopenhayn model with labor adjustment costs, since if establishments fire at random then \( \alpha = p \) for all establishments. This comparison highlights the role selective firing plays in generating the quantitative results.

My model is not able to match the level of the correlation for the post-1984 period (0.67 vs 0.09). This failure is not surprising given that the model can only generate a rise in ALP during recessions through a composition effect. Since the aggregate driving force is a TFP shock, ALP, can only rise in recessions if endogenous match quality increases enough or labor demand declines enough. Furthermore, in my model there is only one type of aggregate

26 The details of this argument are fully fleshed out in Berger (2011), available upon request.
27 The correlation drops 0.593, so we have \((0.990-0.661)/0.593 = 0.554\)
shock, whereas in the data there are many. I can always lower the level of the correlation by adding a preference shock in my model.\textsuperscript{28} The seventh and eighth rows show the results after I introduce a preference shock perfectly correlated with aggregate TFP. I pick the variance of the preference shock to match the observed correlation of output and ALP in the pre-1984 period. The two shock model provides a much closer match to the pre-1984 data and also generates a substantial though smaller decline (45\%) in the procyclicality of ALP. The reason the decline is smaller is that adding the preference shock makes the real wage less procyclical. Since two of the components of the adjustment cost specification are denominated in units of labor, a less cyclical real wage reduces the incentive for firms to restructure during recessions, which attenuates the results.

6.3 Experiment 2: Jobless Recoveries

Next, I investigate each model’s ability to generate jobless recoveries. I simulate a sequence for aggregate TFP that generates a long boom and a shallow recession. This is meant to capture in a simple way the recessions we observed in 1991 and 2001. I compute turning points in the data and investigate how long it takes labor input to return to trend in each model. To be consistent with Figure 1, I do not HP-filter the model generated data, but look at the percent change since the trough of the business cycle. The dotted black line in Figure 5 shows the results from the standard Hopenhayn (1992) model denoted "NH" for no heterogeneity. This model has dynamics similar to a frictionless model: as soon as output starts growing (which occurs at quarter 0), aggregate labor input starts to increase.

The dashed red line shows the results after adding adjustment costs (the model is labeled "NH + AC"). This is a similar model to the one considered by Bachmann (2011), which can generate significant jobless recoveries. The intuition is that after a long boom and a shallow recession, establishments have relatively too many employees. Rather than pay the adjustment cost and shed workers, establishments would rather reduce their workforce via attrition. As a result, employment falls for a few quarters but then starts to pick up by quarter 3 when establishments begin hiring again. Finally, the long-dashed green line shows the results for the heterogeneous worker model ("WH"). Here we see that the quantitative impact of adding worker heterogeneity and selective firing is to increase the average duration

\textsuperscript{28}I introduce a preference shock in the model, by changing the period utility function to \( \eta_t C_t - \frac{1}{1 + \delta} L_t^{1 + \gamma} \). When the preference shock \( \eta_t \) is high, consumers demand more consumption today, which increases output, but since the shock does not make establishments more productive, it tends to lower the marginal productivity of labor due to decreasing returns to scale of the production function.
of a jobless recovery by about a quarter and a half relative to the model with adjustment costs. The intuition for this increase is that after restructuring during recessions, many establishments employ almost exclusively good matches, which deters firms’ hiring, since hiring dilutes match quality and lowers the marginal product of labor.

Next I examine whether the models can generate jobless recoveries after the Great Recession. Figure 6 shows that heterogeneous worker model is able to, whereas the model with adjustment costs only cannot. To construct this figure, I pick the sequence of aggregate shocks to exactly match the growth rate of GDP over the period 1960-2011. Then, I feed this sequence of shocks into the model and compute what the recovery in total hours is in the model in the same manner as in Figure 1. The solid blue line in Figure 6 is the data for the 2009-2011 recovery. The dashed red line is the model with adjustment costs and homogenous workers. There is essentially no jobless recovery in this model. The reason this model does not generate a jobless recovery in the Great Recession is that this recession was deep. The decline in output was large enough so that most establishments were willing to pay the adjustment cost and lay workers off, over coming the labor hoarding motive in the model. Once output started to recover in 2009, establishments started hiring immediately, leading to a quick employment recovery. The long-dashed green line shows the results for the heterogeneous worker model with selective. This model is able to generate jobless recovery of about 3 quarters.
6.4 Growthless Recoveries or Productivity Gains?

It is possible and indeed has been argued that the main reason the employment recovery in 2009 has been jobless is that the output recovery was anemic relative to previous recoveries: if output growth during the recovery had been as robust as it was in the pre-1991 period then there would not have been a jobless recovery (Krugman, 2010; Orszag, 2011; Tasci, 2011). In order to evaluate this statement quantitatively, I perform the following computational experiment. I simulate the model using the same path of output in the data until 2009q2. After that period, I feed in the sequence of shocks needed to produce the output recovery that we observed after the 1982 recession. These results are shown in Figure 7. The green long-dashed and the solid blue line lines show the recovery in total hours in the heterogeneous worker model and the data, respectively. The purple dotted line shows the counterfactual recovery in total hours. Consistent with the conventional wisdom, total hours recover faster with larger shocks. Nonetheless, even with a more robust recovery, it still takes total hours 2 quarters to return to trend suggesting that the recovery still would have been somewhat jobless.

6.5 Discussion

What have we learned? So far I have shown that the model with worker heterogeneity and selective firing (WH) can match the post-1984 facts: it replicates both acyclical ALP (Table 6, row 8) and generates substantial jobless recoveries (The long-dashed green line
in Figures 5 and 6). However, as the purple dotted line in Figure 7 shows, the model under predicts the speed and magnitude of the employment recovery in the pre-1984 period. Conversely, the model with worker heterogeneity without selective firing does a good job at replicating the pre-1984 facts (Table 6, row 5; The dashed red line in Figure 7), but it does a poor job post-1984. In short, each model describes one time period well but neither can explain both. These two models are connected by selective firing, which suggests that if establishments acquired the ability to selectively fire starting in the mid-1980s then this model would be able to replicate employment and productivity dynamics for the entire post-war period.29

This intuition is captured in Figure 8. The red dashed line displays the performance of the worker heterogeneity model without selective firing (WH - SF) model during the recovery from the 1982 recession. To create this figure, I feed in the sequence of shocks needed to produce the output recovery that we observed after the 1982 recession.30 For ease of comparison, the dot-dashed brown line shows the employment recovery in the data after the 1982 recession. The WH - SF model does a good job of replicating the observed employment recovery in the pre-1984 period. I then allow establishments in the WF - SF model able to selectively fire. This is the WH model. The long dashed green line and the blue line are the employment recovery in the model and data, respectively, after the Great

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29 For the sake of clarity, the ability to selectively fire is a binary decision in my model. One can easily imagine only allowing a fraction of establishments the ability to do so.

30 One weakness of this empirical approach is that I calibrated adjustment costs using post-1984 data. I am currently working on calibration that only uses pre-1984 information.
Recession. The WH model with selectively firing is able to match the observed jobless recovery quite closely. In other words, the introduction of selectively layoffs allows the model with worker heterogeneity to replicate employment and productivity dynamics for the entire post-war business period.

Figure 8: Employment recoveries after the 1982 and 2009 recessions in the data and the model

How prevalent is selective firing? There is significant anecdotal evidence that it is practiced by many firms. Jack Welch, perhaps the most influential CEO of the last 20 years, introduced a forced ranking system at General Electric in the early 1980s. Every year, each employee was ranked, those in the top 10% were given bonuses and those in the bottom 10% were fired (Welch, 2001). Evidence from firm level data sets that include objective measures of worker quality confirm these anecdotes. Kahn and Lange (2010) show that receiving a lower performance score predicts employee exit. This is consistent with selective firing, however, Kahn and Lange are not able to distinguish between quits and layoffs in their data set, so it could be that firing is random, but quits are selective. Capelli and Conyon (2011) circumvent this problem by using more detailed data from a large Fortune 500 company. They show that receiving a lower score on a performance evaluation is strongly associated with a higher probability of being laid off.

More generally, there is a burgeoning empirical literature documenting that firms respond to adverse trade shocks by terminating low performing establishments and product lines (Bernard and Jensen, 2007; Bernard, Redding and Schott, 2010; Arkolakis and
Muendler, 2011). While the empirical context in these papers is exporters, it is likely that product switching and establishment shut downs are a more general feature of firm dynamics and one that leads to significant labor market reallocation. The employees who are laid off in response to these negative shocks are not low quality per se, rather they just happen to produce a product for which there is little demand. Thus, a more holistic interpretation of what constitutes restructuring suggests that selectively "pruning" is quite likely a pervasive feature of reality.

7 What Happened in the 1980’s?

Figure 2 shows that ALP went from being strongly procyclical to mildly countercyclical in only 10 years during the 1980s. The dramatic nature of this decline suggests that possible explanations for the change should also be relatively sharp. The first obvious candidate is the Great Moderation. As is well known, there was a large decline in the volatility of U.S. output in 1984, almost exactly when the correlation between output and ALP changed. Furthermore, Garin, Pries and Sims (2011) argue that the Great Moderation led to a change in the composition of shocks whereby reallocation shocks became more important. They present a model that can explain both the decline in the cyclicity of ALP and jobless recoveries. If their hypothesis is correct, we should observe a sharp decline in the correlation of output and labor productivity soon after the Great Moderation begins. Figure 9 shows that the Great Moderation occurred in many G-7 nations. The dashed red lines show the eleven year centered rolling correlation between output and output per hour and the solid blue lines show the eleven year rolling standard deviation of GDP. The top left panel, shows the results for the United States. Consistent with the evidence presented in Garin, Pries and Sims (2011), this correlation declines at the same time as the Great Moderation began in the U.S.

While Figure 9 is far from conclusive, it shows that there is less evidence for the predicted relationship in other G-7 countries. In the U.K., Japan and Canada, the correlation between output and ALP actually increases after the Great Moderation starts, which is the opposite prediction of their theory. The evidence from Italy and France is more supportive.

31 For example, the USPS is shutting down 3600 post offices around the U.S., 3000 of which had less than $27,500 in annual sales (Courson and Liberto, 2011).
32 Figure 8 excludes Germany because the reunification of East and West Germany in 1989 leads to large trend breaks in the output and hours series in the middle of the sample.
33 I use a centered, eleven year window rather than an eight year window (as I used in figure 2) because this figure is made with annual data. The results are robust to reasonable changes in window size.
of their hypothesis. However, even for these two countries, the correlation between output and ALP increases before rather than after the Great Moderation, suggesting perhaps, that the causality runs in the opposite direction. All in all, the evidence in Figure 9 suggests that we should be looking for a U.S. specific explanation.

Figure 9: Evidence on the timing of the Great Moderation and the change in the correlation between output and ALP from the G-7 countries

Another candidate explanation is the decline in union power. This story lines up well with the observed timing of the decline in the procyclicality of ALP. Farber and Western (2002) document a sharp decline in the number of union certification elections in the early 80s (shown in Figure 10). They interpret this as evidence for an “unfavourable political climate which raises the costs of unionization”, induced by Reagan’s policies and in particular his handling of the air-traffic controllers’ strike in 1981. Union contracts often include legal restrictions which limit the ability of a firm to adjust its workforce, in particular rules governing layoffs. Abraham and Medoff (1984) find that 92% of union firms have written rules to deal with permanent layoffs while only 24% of nonunion firms have such written layoff policies, and that 58% of nonunion firms have a practice of sometimes laying off a more senior worker if a junior worker is believed to be worth more on net, as compared

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34Gali and van Rens (2010) also suggest that the decline in union power can help explain the decline in the procyclicality of ALP. Less union power means lower adjustment costs which leads to decreased labor hoarding. However, this explanation does not explain why the recoveries have become jobless. Closely related to the union power hypothesis is the "disposable worker hypothesis" of Gordon (2010). He argues that there was an increase in managerial power and a decline in labor power that contributed to both the rise in inequality and the increasingly cyclically sensitive labor market that we have observed. A closely related explanation is that there was a dramatic shift in corporate philosophy towards maximizing shareholder value, especially since 1990 (Sinai 2010).
to 17% of union employers. Further suggestive evidence comes from Dunne, Klimek and Schmitz (2010). They show that there were dramatic changes in the structure of union contracts in U.S. cement industry in the mid-1980s which gave establishments much more scope to fire workers based on performance rather than just tenure, and as a result ALP and TFP in the industry dramatically increased.

![Graph showing number of NLRB certification elections from 1960 to 1998](image)

Figure 10: Number of union elections certified by the National Labor Relations Board from 1960-1998

While the specific industry studies and Figure 10 are suggestive, in order to address the question more directly, I examine cross-state variation in union coverage rates, the percentage of the workforce covered by collective bargaining agreements. I test whether states which had larger percentage declines in coverage rates had larger declines in the correlation of output and labor productivity. I use annual output and employment data for each state from the BEA’s Regional Economic Accounts and state level information on union coverage rates from the CPS.\(^{35}\) Since I lack a measure of total hours by state, my measure of labor productivity is output per worker. My estimating equation is:

\[
(\rho_{i,\text{post}95} - \rho_{i,\text{pre}85}) = \alpha + \beta \log(UC_{i,\text{post}95} / UC_{i,\text{pre}85}) + \epsilon
\]

Here, \(\rho_{i,\text{post}95}\) and \(\rho_{i,\text{pre}85}\) denote the correlation between output and output per worker for state \(i\) in the post-1995 and pre-1985 periods, respectively. These dates were chosen to ensure that the correlation was computed using the same number of observations in each

\(^{35}\)The unionization data are available at unionstats.com. Ideally, I would use cross-state variation in the number of NLRB certification elections as my measure of union power since Figure 10 shows that the number of elections declined sharply in the 1980s, whereas the decline in union coverage rates was more smooth. Due to data limitations I was unable to pursue this strategy, however, I am currently working on collecting this data.
Table 7: Regression results: Union power and the decline in corr(Y,Y/E)

<table>
<thead>
<tr>
<th></th>
<th>Union coverage rates</th>
<th></th>
<th>Union membership rates</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Change in corr(Y,Y/E)</td>
<td>% change in corr(Y,Y/E)</td>
<td>Change in corr(Y,Y/E)</td>
<td>% change in corr(Y,Y/E)</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.083</td>
<td>0.182</td>
<td>0.050</td>
<td>0.138</td>
</tr>
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<td></td>
<td>(0.192)</td>
<td>(0.291)</td>
<td>(0.186)</td>
<td>(0.281)</td>
</tr>
<tr>
<td>(\beta)</td>
<td>0.565*</td>
<td>0.878**</td>
<td>0.455*</td>
<td>0.723*</td>
</tr>
<tr>
<td></td>
<td>(0.292)</td>
<td>(0.432)</td>
<td>(0.253)</td>
<td>(0.377)</td>
</tr>
<tr>
<td>Observations</td>
<td>51</td>
<td>51</td>
<td>51</td>
<td>51</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.079</td>
<td>0.084</td>
<td>0.069</td>
<td>0.076</td>
</tr>
</tbody>
</table>

Robust SE in parentheses. All observations weighted by mean employment pre-1985

Source: BEA’s Regional Economic Accounts and Current Population Statistics

time period, but the results are robust to reasonably different choices. The correlations were computed using logged and HP-filtered data with a smoothing parameter of 6.25. \(\bar{UC}_{s,post95}\) and \(\bar{UC}_{s,pre82}\) are the mean union coverage rates for state \(i\) in the pre and post periods. The coefficient \(\beta\) is the object of interest. It captures the association between the change in the level of the correlation between output and output per worker and the percentage change in the union coverage rates from the pre-1985 to the post-1995 eras. The union power hypothesis predicts that \(\beta > 0\): states which had larger percentage declines in unionization rates should have larger declines in the level of the correlation between output and labor productivity.

The results are shown in Table 7 and graphically in Figure 11. I weight each observation by the average employment per state in the pre-1985 period. The observations are weighted because I want to capture the average effect of the decline in unionization and I do not want the results to be unduly influenced by small state outliers.\(^{36}\) The first column presents the results where the change in correlation is in levels. Consistent with the union power hypothesis, the estimate \(\hat{\beta}\) is significantly positive. A 50% decline in the union coverage rate (close to the mean decline for this time period) is associated with a 0.32 decline in the level of the correlation between output and output per worker. The second column presents the results when the dependent variable \(corr(Y,Y/E)\) is in percentage changes rather than levels. Once again, the estimate \(\hat{\beta}\) is positive and significant, which is supportive of the union power hypothesis. As an additional robustness check, the third and fourth columns of Table 7 show the results when using union membership rather than union coverage as the measure of union power. The qualitative results are unchanged.

As an additional robustness test of the union power hypothesis, I split my sample

\(^{36}\)The results are robust to weighted the observations by union coverage and membership levels in the pre-1982 period.
Right-to-work states | Non right-to-work states
--- | ---
Change in corr(Y,Y/E) | % change in corr(Y,Y/E) | Change in corr(Y,Y/E) | % change in corr(Y,Y/E)
$\hat{\alpha}$ | -0.218 | -0.026 | 0.231 | 0.341
(0.392) | (0.648) | (0.209) | (0.335)
$\hat{\beta}$ | 0.095 | 0.528 | 0.902** | 1.257**
(0.468) | (0.812) | (0.338) | (0.553)
Observations | 21 | 21 | 29 | 29
R-squared | 0.001 | 0.019 | 0.127 | 0.100

Robust SE in parentheses. All observations weighted by mean employment pre-1985. Dependent variable is union coverage rates. Source: BEA's Regional Economic Accounts and Current Population Statistics

Table 8: Regression results: Union power and the decline in corr(Y,Y/E)

and estimate Equation (10) in right-to-work and non right-to-work states separately. It is common in the empirical literature on union power to characterize right-to-work states as "probusiness" and non right-to-work states as "anti-business" (Holmes, 1998), though this decomposition is obviously a coarse measure at best. If the union power hypothesis is true, one would expect to observe a statistically stronger relationship between the decline in union coverage rates and the change in the cyclicality of ALP in non right-to-work states because these are the states where unions were historically more powerful. The results are shown in Table 8 and are consistent with the union power hypothesis. There is no significant statistical relationship between the change in union coverage rates and the change in the level of the correlation between output and ALP in right-to-work states, but a strong relationship in the non right-to-work states.
Next I examine the relationship between the change in union coverage rates and the change in the cyclicality of ALP across U.S. industries. I use annual output and employment data for each industry from the BEA’s Annual Industry Accounts and industry level information on union coverage rates from the CPS. I estimate Equation (10) where the only difference is that $i$ now refers to industry $i$. The results are shown in Figure 12. Consistent with my cross-state evidence and the union power hypothesis, I find a positive relationship between the change in union coverage rates and the change in the cyclicality of ALP across U.S. industries. Despite the fact that this relationship is significant at the 10% level for some specifications, I view this cross-industry evidence as merely suggestive due to the lack of observations.\footnote{I am only able to obtain a long enough output series for 25 two-digit NAICS industries. Changes in the definitions of these industries from the early 1980s to the mid-1990s (relevant for constructing the union coverage data) further restricts the sample to 16 industries.} Nonetheless, all the evidence together suggests that the union power hypothesis has some merit.

The union power hypothesis is consistent with an increase in the frequency of selective firing in the 1980s. This makes the union power hypothesis easy to interpret through the lens of my quantitative model. As discussed in the previous section (see Figure 8), turning selective firing on/off allows the model with worker heterogeneity to match employment and productivity dynamics over the entire post-war period.\footnote{My WH model predicts that tenure and productivity are positively correlated because workers that survive are good matches. So when switching from random to selective firing there should be a sharp decline in the cyclicality of ALP (firms have a lot of bad matches), but then over time this effect is diminished somewhat since incumbents do not exit frequently and employ mostly good workers. Nonetheless, there is upper bound to this effect even with perfect match quality because establishments exit and because good matches are lost via exogenous attrition.} Could it be that the decline in

![Figure 12: Union power and the decline in corr(Y,Y/E): Evidence from U.S. Industries](image-url)
union power led only to a fall in adjustment costs and not an increased ability to selectively fire? This is the hypothesis advanced by Gali and van Rens (2010). The behavior of the black dotted line in Figure 5 shows the problem with this hypothesis: it is inconsistent with jobless recoveries. Thus in my model, declining adjustment costs are not enough. In order to match the post-1984 facts, it must be the case that firms are more able to selectively fire.

Finally, my model provides an alternative mechanism that can explain why restructuring became more prevalent in the 1980s - employers become more able to learn about worker quality. For my restructuring margin to operate, firms must both be able to learn about worker quality and be able to selectively fire. I have chosen, for empirical reasons, to focus on the case where the speed and scope of employer learning are held constant while the ability of firms to selectively fire is increased, however, I think it is quite reasonable to believe that firms have also become more able to measure worker quality. Anecdotally at least, we know that firms have invested enormous resources in information technology and that these investments have allowed firms to become more able to measure in real time the sales of their product lines, divisions, establishments, working groups and in some case individual workers. While we lack comprehensive evidence that firms use this information when they are deciding which products to discontinue or workers to lay off, I think it is reasonable to think that they do. At the very least, providing comprehensive evidence of whether firms use this technology in making termination decisions is an interesting topic for future research.

8 Conclusion

Since the mid-1980s, average labor productivity has become markedly less cyclical at the same time as employment recoveries have become markedly more jobless. In this paper, I argue that these two facts are related and investigate whether the last three employment recoveries have been jobless because firms emerged from recessions more productive and efficient. I develop a model that can math these two facts. The model captures the idea that firms grow "fat" during booms but then aggressively restructure their workforce during recessions. In the model, firms employ unproductive workers because learning about match quality takes time and because adjustment is costly. In recessions, firms shed unproductive workers causing ALP to increase endogenously. Firms enter the recovery "lean and mean",

\footnote{The fact that more workers are employed in the service sector, where is it on average more difficult to identify the quality of the worker, is a countervailing effect.}
with a greater ability to meet expanding demand without hiring additional workers.

I calibrate and compute a quantitative competitive industry model with endogenous entry and exit, worker heterogeneity, labor adjustment costs and aggregate shocks. I find that the model can generate a 55% of the decline in the procyclicality of ALP (relative to the model without selective firing) and can generate jobless recoveries that average four months in duration. In contrast to previous work, I find that the model generates a substantial jobless recovery during the Great Recession. I then investigate whether the employment recovery after the Great Recession would have been jobless if the recovery in output had been as strong as it was after the 1982 recession. I find that even if we had observed a much more robust recovery in output, there would still have been a two quarter long jobless recovery.

References


The robustness of the macro facts

A.1 Decline in the cyclicalality of ALP

I show that the decline in the procyclicality of ALP is robust to using different output and employment measures, occurs most strongly in the U.S. and is not driven by a change in entry and exit dynamics. Following Brugemann, Hagedorn and Manovskii (2010), I use the following shorthand to refer to the possible permutations of data source, labor input, sector and filter.

1. Data source: I use "cps" and "lpc" to refer to data from the Current Populations Statistics (BLS household survey) and Labor Productivity and Costs (BLS Establishment survey), respectively

2. Sector: I use "nfb" for nonfarm business and "all" for total economy

The results are shown in Table 9. The first three columns show the correlation between output and output/hour for the pre-1984, post-1984 as well as the change in the correlations. In every case, the change in the correlations between the sub-periods is statistically significant and quite large. The fifth column examines whether we can formally detect a trend break in the correlations without specifying the break point ex-ante. To do this I implement the procedure described by Andrews and Ploberger (1994). For the LPC measures of labor input, one can reject the null of no structural break at the 1% significance level. The results for CPS measures of labor input are less clean, however, one can reject the null of no structural break at the 10% level for the nonfarm-business productivity measure.

The Conference Board collects annual data on GDP and total hours across countries. I use this data to analyze whether the decline in the procyclicality in ALP during the mid-1980s was unique to the U.S. among G7 countries. The results are shown in the first three columns of Table 10. The first and second columns show the pre and post 1984 levels of the correlation between HP-filtered output and ALP. I used a smooth parameter of 6.25 though the results are not sensitive to this choice. They show that while the decline in procyclicality of ALP occurred in 6 of the 7 G-7 nations, the decline was only statistically significant in the U.S. As a robustness check, I also allow for the two time periods to vary by country. In particular, I choose the end of the first period to be the year in which the Great Moderation started in that country using the country specific dates from Summers (2005). The results are shown in the last three columns of Table 10. The main difference between
Correlation with Output Trend break statistics

<table>
<thead>
<tr>
<th></th>
<th>Pre-84</th>
<th>Post-84</th>
<th>Change</th>
<th>Obs</th>
<th>Andrews/Ploberger (1994)</th>
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<tbody>
<tr>
<td>LPC (nfb)</td>
<td>0.64***</td>
<td>-0.02</td>
<td>-0.67***</td>
<td>254</td>
<td>-7.48***</td>
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<td></td>
<td>(0.06)</td>
<td>(0.12)</td>
<td>(0.13)</td>
<td>(1987Q2)</td>
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<tr>
<td>CPS (nfb)</td>
<td>0.96***</td>
<td>0.25**</td>
<td>-0.70***</td>
<td>138</td>
<td>-4.41*</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.10)</td>
<td>(0.14)</td>
<td>(1987Q3)</td>
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<tr>
<td>LPC (all)</td>
<td>0.51***</td>
<td>0.05</td>
<td>-0.46***</td>
<td>246</td>
<td>-6.38***</td>
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<td></td>
<td>(0.06)</td>
<td>(0.14)</td>
<td>(0.15)</td>
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<tr>
<td>CPS (all)</td>
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<td>-3.33</td>
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<td></td>
<td>(0.06)</td>
<td>(0.13)</td>
<td>(0.14)</td>
<td>(1980Q2)</td>
<td></td>
</tr>
</tbody>
</table>

Standard Errors in parenthesis. NFB is "non-farm business" and "all" is the entire US economy.

See Brugemann, Hagedorn and Manovskii (2010) for details about the data construction.

Table 9: Robustness checks: correlation between output and ALP

<table>
<thead>
<tr>
<th></th>
<th>Pre-84</th>
<th>Post-84</th>
<th>Change</th>
<th>Pre-GM</th>
<th>Post-GM</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>0.46***</td>
<td>-0.27</td>
<td>-0.73**</td>
<td>0.46***</td>
<td>-0.27</td>
<td>-0.73**</td>
</tr>
<tr>
<td></td>
<td>(0.14)</td>
<td>(0.26)</td>
<td>(0.29)</td>
<td>(0.14)</td>
<td>(0.26)</td>
<td>(0.14)</td>
</tr>
<tr>
<td>Japan</td>
<td>0.83***</td>
<td>0.69***</td>
<td>-0.14</td>
<td>0.78***</td>
<td>0.72***</td>
<td>-0.06</td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
<td>(0.15)</td>
<td>(0.20)</td>
<td>(0.14)</td>
<td>(0.12)</td>
<td>(0.12)</td>
</tr>
<tr>
<td>UK</td>
<td>0.47**</td>
<td>0.37**</td>
<td>-0.10</td>
<td>0.53***</td>
<td>0.35*</td>
<td>-0.18</td>
</tr>
<tr>
<td></td>
<td>(0.18)</td>
<td>(0.19)</td>
<td>(0.26)</td>
<td>(0.16)</td>
<td>(0.20)</td>
<td>(0.23)</td>
</tr>
<tr>
<td>France</td>
<td>0.65***</td>
<td>0.35**</td>
<td>-0.30</td>
<td>0.85***</td>
<td>0.30*</td>
<td>-0.55**</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
<td>(0.17)</td>
<td>(0.24)</td>
<td>(0.15)</td>
<td>(0.16)</td>
<td>(0.22)</td>
</tr>
<tr>
<td>Italy</td>
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<td>0.68***</td>
<td>-0.16</td>
<td>0.74***</td>
<td>0.71***</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
<td>(0.18)</td>
<td>(0.22)</td>
<td>(0.14)</td>
<td>(0.17)</td>
<td>(0.25)</td>
</tr>
<tr>
<td>Canada</td>
<td>0.37**</td>
<td>0.47***</td>
<td>0.10</td>
<td>0.37**</td>
<td>0.47**</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
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<td>(0.14)</td>
<td>(0.21)</td>
<td>(0.15)</td>
<td>(0.20)</td>
<td>(0.19)</td>
</tr>
</tbody>
</table>

Data are from the Conference Board. The data spans 1960-2009.
Data exclude Germany because reunification leads to trend breaks in the data in 1989.
Great Moderation (GM) start date for each country taken from Summers (2005).

Table 10: Correlation of output and ALP (G7 Nations)

The baseline results is that now the decline in the correlation is significant for France as well as the U.S. All the other results are consistent across the two specifications. Overall then, Table 10 provides suggestive evidence that we should explore U.S. specific explanations - as opposed to global explanations - when seeking to explain what changed in the mid-1980s.

Was the change in the cyclicality of ALP due to a change in the nature of entry and exit since the mid-1980s? I use data from the Business Dynamics Statistics (BDS) for the period 1977-2009 to address this question. The BDS can be downloaded here: http://www.ces.census.gov/index.php/bds
rate (dashed blue) over the business cycle. The shaded regions denote NBER recessions from peak to trough. Other than a slight trend decline in both rates, there do not seem to be any dramatic differences in the pre and post 1985 period. The right panel shows how the fraction of total job creation due to establishment entry (solid red) and the fraction of total job destruction due to establishment exit (dashed blue) varies over the business cycle. The figure shows that the fraction of job creation contributed by entrants rose and the fraction of job destruction contributed by exiters fell in each of the last four recessions. Once again, there is no evidence that the behavior of entrants and exiters changed in the 1980s. This suggests that the decline in the procyclicality of ALP was not due to a change in the behavior of entry and exit over the business cycle in the mid-1980s.

![Figure 13: Left panel: Establishment entry and exit rates over the business cycle; Right panel: Share of job creation and destruction by entrants and exiters over the business cycle. (Both in percentages)](image)

### A.2 Jobless recoveries

It is often argued that the jobless recoveries are merely an artifact the secular decline in manufacturing employment. To show that this is not the case, I recreate the standard jobless recovery figures excluding manufacturing (RHS of Figure 14) and excluding manufacturing and construction employment (LHS of Figure 14). The series come from the Current Establishment Survey. All series are at a quarterly frequency and seasonally adjusted. In total, Figure 14 shows that removing manufacturing (and construction) attenuates jobless recoveries but it does not remove them altogether, especially for Great Recession.
B Computing the model: steady state

In computing the steady-state model, the basic logic is that the wage is pinned down by establishment entry, the ergodic distribution of establishments $\mu^*$ is determined by iterating on the transition rule $T(\mu, J)$ when initial entry has measure 1. The algorithm is described in detail below. I refer to the values of the three establishment specific state variables as $\Gamma = (s, \alpha, L)$ as a shorthand. The algorithm is:

1. Guess an initial value for the wage, $w$.

2. Given this wage, solve the incumbent’s problem shown in Equation (4) by value function iteration. Calculate the optimal decision rules for employment, exit and adjustment ($h(\Gamma, z)$, $X(\Gamma, z)$ and $\phi(\Gamma, z)$).

3. Compare the value of entering versus the entry cost shown in Equation (5). By free entry, they must be equal. If the value of entering exceeds the cost of entering, choose a higher value for the wage since this will decrease the value of enter and increase the cost of entering (since the cost of entering is measured in units of labor). Otherwise, choose a lower value for the wage. I found that a simple bisection algorithm works well.

4. Repeat steps 1-3 until the free entry condition is satisfied. Call $w^*$ the wage that satisfies this condition.

5. Solve for the ergodic distribution $\mu$ of establishments using Equation (7) and the property, common to models with heterogeneous firms and endogenous entry and exit that
the transition rule $T(\mu_t, J_t)$ is homogeneous of degree 1 in both $\mu$ and $J$. Thus, I set initial entry to have measure 1 ($J = 1$), guess an initial value for $\mu_0$ and plug the decision rules from step 2 into Equation (7). Notice, that given an initial guess for $\mu_0$, the transition rule outputs another value of the ergodic distribution $\mu_1$. I repeat this procedure until $|\mu_t - \mu_{t-1}| < \epsilon$. At this point the ergodic distribution has converged. I refer to this object as $\mu^* = T(\mu^*, 1, w^*)$.

6. The labor market clearing condition allows use to pin down the equilibrium measure of establishment entry. Using $\mu^*$ and the optimal decision rules $h(\Gamma, z)$ and $X(\Gamma, z)$, compute aggregate labor demand using Equation (6). Adjust the amount of entry if labor demand does not equal labor supply at wage $w^*$. Specifically, if labor demand is greater than labor supply, decrease the amount of establishment entry; otherwise increase it. A simple bisection algorithm works well to quickly find the equilibrium $J^*$.

I solve the problem in Step (2) using value function iteration on a grid. Knotek and Terry (2008) argue that discretizing fixed adjustment cost models has robustness advantages versus collocation or other interpolation methods. Nevertheless, earlier versions of my model were solved using linear interpolation and the results were unchanged. The random variables are discretized using the method of Tauchen (1986). In the benchmark analysis, 120 grid points were used for the employment grid, 24 grid points were used for the idiosyncratic productivity grid, 18 grid points were used for the $\alpha$ grid and 5 grid points were used for the aggregate productivity grid. Results were unchanged when more grid points were added.

C Computing the model with aggregate shocks

The model with aggregate can be solved using a without using a Krusell-Smith approximation for the cross-sectional distribution. The reason for this is free entry. The model is set up so that anyone can pay the entry costs and start producing. As a result of free entry, aggregate labor demand is perfectly elastic—there is one wage level that is consistent with the labor market equilibrium, regardless of the level of employment. This model turns out to be easy to analyze, even with aggregate productivity shocks, because the equilibrium wage only depends upon the aggregate productivity level.

Assume there are $N$ aggregate states $\{z_1, \ldots, z_N\}$ ordered from their smallest to highest values. Denote the menu of wages associated with the aggregate shocks as $\{w(z_1), w(z_2), \ldots, w(z_N)\}$. Given a guess for wages, we can solve the incumbent’s problem shown in Equation (4)
gives the value of being an incumbent at every state. Note that this value already includes the value of exit. Next, we check if the free entry condition holds:

\[ w(z) c_e = \int V(0, 0, s; z) d\varphi(s) \]  

(11)

Here, \( w(z) c_e \) denotes the current cost of entering. A prospective entrant compares this cost to the value of entering before observing his idiosyncratic productivity draw \( s \). Conditional on entering, the entrant has the value function \( V(0, 0, s; z) \), that is, the entrant has no initial workforce (the two zeros) and has shocks \( (s, z) \). This value depends on the entire menu of wage rates (through the continuation value), the current wage rate and the Markov transition matrix for the aggregate shock. For example, it is much more desirable to enter when the aggregate state is high if this state is highly persistent. An equilibrium is reached when the free entry condition is satisfied at every aggregate states. In short, the algorithm is (starting in iteration \( i \)):

1. Guess an initial menu of wages \( w^i = \{ w^i(z_1), ..., w^i(z_N) \} \)

2. Solve the incumbent’s problem (Equation 4) using this menu of wages, \( w^i \). Keep the associated value function, \( V^i \)

3. Plug the associate value functions and wages \( (V^i \text{ and } w^i) \) into the free entry condition (Equation 11).

4. If Equation (11) is satisfied at every aggregate state then stop. Otherwise, we need to update the wage guess. We update the wage guess in the following way: starting with \( w^i(z_1) \), find the value of this wage \( w^*(z_1) \) that satisfies the free entry condition for aggregate state \( z_1 \) holding constant the wages for every other aggregate state at the level that was guessed. That is, fix \( \{ w^i(z_2), ..., w^i(z_N) \} \) and then vary \( w(z_1) \) until the free entry condition is satisfied for state \( z_1 \). Notice that this requires the incumbent’s problem for each new guess of \( w(z_1) \). Once \( w^*(z_1) \) is found, update our guess for this wage guess by setting \( w^{i+1}(z_1) = w^*(z_1) \). Then proceed to state 2 and repeat the procedure; holding the wages of the other aggregate states constant at the values \( \{ w^{i+1}(z_1), ..., w^i(z_3), ..., w^i(z_N) \} \), find the value of \( w(z_2) \) such that the free entry condition is satisfied for state \( z_2 \). Repeat this procedure for every aggregate state. This updates our menu of wage guesses from iteration \( i \) to iteration \( i + 1 \). Then return to step 1.
Computing the model with aggregate shocks and a fixed number of potential entrants

I now solve the model for a fixed number of entrants $J$. This model is substantially more difficult to analyze as compared to the model in section 4 (I refer to this model as the basic model). This is because the wage $w_t$ is now simultaneously determined with total labor demanded, $L_t$ in labor market equilibrium. We cannot separate the determination of $w_t$ from the determination of equilibrium quantities as in section 4. This, in turn, implies that the plants which are making dynamic decisions have to predict what will happen to the equilibrium prices and quantities in the future. Because the equilibrium quantities in the future depend on the distribution of plants in the current period, the entire distribution of the plants enters into the set of relevant information for the plant’s decision problem. I utilize a variant of Krusell and Smith’s (1998) method to solve for the general equilibrium of this model.

The model primitives are almost identical to the basic model. The only difference is the entry process. Instead of free entry with an unbounded number of potential entrants, $J$, there is a fixed number of potential entrants $J$. Every period, these potential entrants decide whether to enter by paying $w_{ce}$. All other timings and aspects are the same. In particular, the consumer’s optimization problem is identical to the basic model and labor supply is given by

$$L^s_t = \left( \frac{w_t}{A} \right)^\varepsilon$$

(12)

The establishment side of the model is different as a consequence of the modification in the entry process. In particular, because the wage is now determined by the labor market equilibrium, in order to predict the future wage each establishment has to predict where the labor demand curve lies. Because the future labor demand depends on the current distribution of employment, match quality and idiosyncratic productivity shocks, the establishment has to incorporate a lot more information when making decisions in the current period.

The value of an incumbent establishment at the beginning of the period is described by the Bellman equation

$$V(s_{i,t}, \alpha_{i,t-1}, L_{i,t-1}, z_t, \Omega_{t-1}) = \max \left\{ V^E(s_{i,t}, \alpha_{i,t-1}, L_{i,t-1}, z_t, \Omega_{t-1}), V^C(s_{i,t}, \alpha_{i,t-1}, L_{i,t-1}, z_t, \Omega_{t-1}) \right\}$$

(13)
where $\Omega_{t-1}$ denotes the information set of the establishment at the end of period $t-1$. It includes the distribution of establishments over productivity, average match quality and employment. As before, $V^E(s_{i,t}, \alpha_{i,t-1}, L_{i,t-1}; z_t, \Omega_t)$ captures the exit decision. If an incumbent decides to continue producing, it solves:

$$V^C(s_{i,t}, \alpha_{i,t-1}, L_{i,t-1}; z_t; \Omega_{t-1}) = \max_{L_t \geq 0} s_{i,t} z_t (\bar{\theta}(\alpha_{i,t}) L_{i,t})^\gamma - w(N_t) L_{i,t} - w(L_t) c_f - w(N_t) G(L_{i,t}, L_{i,t-1})$$

$$+ \beta E_{z_t} [V(s_{i,t+1}, \alpha_{i,t}, L_{i,t+1}; z_{t+1}; \Omega_t)]$$

Note that I denote the wage as as $w(N_t)$ because once we know aggregate labor $L_t$, the wage is known through equation (12). $\Omega_t$ evolves according to $\Omega_t = \Phi(z_t, \Omega_{t-1})$. Total employment $L_t$ is the sum of employment $L_{i,t}$ across all establishments and is thus part of $\Omega_t$. The manner is which labor is chosen and thus the way average match quality at the establishments evolves is the same as in the basic model.

In equilibrium, the combination $(w_t, L_t)$ has to clear the labor market. Total labor supply is given by equation (12) and total labor demand is given by

$$L^D_t = L^I_t + E_t L^E_t$$

where $L^I_t$ and $L^E_t$ are the total amount of labor demand from incumbents and entrants respectively. The equilibrium $(w_t, L_t)$ are given by these two equations. Once $(w_t, L_t)$ is given, the values of the other variables can be determined in the same manner as in the basic model.

The computation of this model is potentially much more complex than the basic model, since the optimization (potentially) involves many more state variables. To overcome this difficulty, we follow Krusell and Smith (1998) in using limited information instead of the entire state variables to perform optimization, and check whether the “forecast” using this limited information is accurate by simulation. In particular, what is necessary for optimization is to forecast the value of $L_t$ and to forecast average match quality in the economy $A_t$. $A_t$ is the average of $\alpha_t$ across establishments.

Following Mukoyama and Lee (2011), I guess a log-linear prediction rule for $L_t$ and $A_t$:

$$\log N_t = a_0 + a_1 \log(L_{t-1}) + a_2 \log(z_t) + a_3 \log(A_{t-1}) + a_4 \log(L_{t-1})x \log(z_t) + a_5 \log(A_{t-1})x \log(z_t) +$$

$$+ a_6 \log(A_{t-1})x \log(L_{t-1}) + a_7 I(z_t \neq z_{t-1})$$
\[
\log A_t = b_0 + b_1 \log(L_{t-1}) + b_2 \log(z_t) + b_3 \log(A_t) + b_4 \log(L_{t-1})x \log(z_t) + b_5 \log(A_t)x \log(z_t) \\
+ b_6 \log(A_t)x \log(L_{t-1}) + b_7 I(z_t \neq z_{t-1})
\]

where \(I()\) is an indicator function which takes the value 1 if the statement in the parenthesis is true and 0 if it is false. By adopting this formulation, we are reducing the aggregate state variable from \((z_t, \Omega_{t-1})\) to \((z_t, z_{t-1}, L_{t-1}, A_{t-1})\). This is a computationally demanding problem but (potentially) feasible. As is standard there are two loops: and outer simulation loop and an inner optimization loop.

The computational algorithm is as follows:

1. Guess an initial value of total labor and average quality \((L_0, A_0)\) and initial coefficients \((a_0, a_1, a_2, a_3, a_4, a_5, a_7, b_0, b_1, b_2, b_3, b_4, b_5, b_6, b_7)\)

2. Optimization step. Given the guess for the aggregate transition rule, solve the incumbent’s problem (equation 13). The aggregate transition rule is used to forecast total labor and hence wages. Iterate until the value function converges. Calculate all decision rules.

3. Simulation step: Simulate the economy. The basic idea is to create a series of \((z, L, A)\) to update this series. Every period compute the labor demand of incumbents and entrants. The compute total labor demanded. Let \(L^D_t = L^I_t + E_t L^E_t\) use this to compute the wage \(w(L) = A(L^s)^{1/\varepsilon}\). Compute \(A_t\) using the measure of incumbents and entrants.

4. From simulation, we have the time series data of \((z, L, A)\) which we can use the to update the forecast rule. Check if the initial guess on the coefficients is correct. If not, adjust these coefficients and go back to step 1. Repeat until the coefficients in the decision rule change by less than 1%.