14.662 Spring 2012, Lecture Note 5: Ricardian Models of Trade

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1 Outsourcing: Is it just another name for trade?

Outsourcing has become a leading political issue in many advanced countries. Should it be a leading economic issue as well? To answer this question, we need to define outsourcing. There is no agreed definition, but a non-controversial summary is that outsourcing is the process by which subcomponents of a production process (‘tasks’) are performed overseas, even while the final good is produced domestically. To see why this definition is not especially robust, note that many goods made in the U.S. use foreign components (intermediate inputs). So, one can simply say that outsourcing is trade in intermediate inputs. If outsourcing is really something different from that, it may be because it entails a finer international division of labor than typical trade in goods. Firms can potentially outsource very narrow production activities to other nations where these tasks are performed in close coordination with U.S. workers. Thus, one can view the production process as being geographically dispersed but otherwise tightly coupled.

There is a limited supply of useful empirical and theoretical work on outsourcing to date, though this is changing. The 2004 JEP article by Bagwati et al. is a helpful contribution in that clears up some of the confusion (‘muddles’) about what outsourcing is or is not, and attempts to answer the question of whether the outsourcing phenomenon requires a fundamentally new set of conceptual models to make it interpretable. The argument of the article is that outsourcing is simply international trade, and that it can be modeled using standard tools as a reduction in trading costs for some subset of factors. While the article is helpful, it is possible that the ‘nothing new under the sun’ view is overblown. The examples that the paper considers do not really capture the idea of a ‘finer’ international division of labor rather than simply a reduction in trading costs or opening to trade for some existing factor.

The 2008 AER paper by Gross and Rossi-Hansberg offers a model that is somewhat more closely attuned to the phenomenon of outsourcing as many lay people and some economists perceive it. In the model, workers of two types, $H$ and $L$, each perform a continuum of ‘tasks,’ some of which are more suitable for outsourcing than others. Their model explores what happens when the cost of outsourcing tasks declines, leading to an increase in the extent of task outsourcing among one or both skill groups, though not a complete elimination of tasks performed by either skill group. This model should not be viewed as the last word on its topic (closer to the first). There are some key assumptions—not all of them transparent—that make this model tick. One feature that makes it potentially controversial is that embeds a Ricardian (comparative advantage) framework inside of a standard Hecksher-Ohlin framework. This makes the model a bit complicated and fairly non-standard. Whether this approach will
prove to be a useful conceptual advance is not yet certain.

1.1 Model setup

Firms in the Home country produce two goods, X and Y, where X is relatively skill-intensive. There are two factors of production modeled (though others can be viewed as operating in the background), H labor, which performs H tasks, and L labor which performs L tasks. Production of each good requires a continuum of L and a continuum of H tasks. The measure of tasks of each type used in production of each good is normalized to 1. Each task within a continuum uses the same amount of the relevant type of labor if performed at home. That is, if L-tasks i and i' are undertaken at home in the course of producing good j, then firms use the same amount of domestic low-skilled labor to perform task i as they do to perform task i'. (This normalization is probably harmless since tasks could be made 'wider' or 'narrower' to guarantee that this condition holds.) If industries differ in their factor intensities, then it will be the case that they differ in the amount of L or H labor used to perform all tasks in the L or H continuum.

A key implicit assumption is that there is no substitution between tasks. Started more strongly, all tasks in a continuum are perfect complements since all must be performed for production to take place—so, each task must be performed at fixed intensity to produce a unit of output. This observation immediately implies that forces that reduce the cost of performing some subset of tasks within a continuum will, all else equal, raise demand for the other tasks in that continuum.

Specifically, in industry j, a firm needs $a_{fj}$ units of domestic factor f to perform a typical f task once. Since the measure of f tasks is normalized to one, this means that $a_{fj}$ is the total amount of domestic factor f needed to produce a unit of good j in the absence of offshoring. The fact that X is relatively skill intensive implies that:

$$\frac{a_{Hx}}{a_{Lx}} > \frac{a_{Hy}}{a_{Ly}}$$

Offshoring takes a very simple form. Firms can undertake tasks at home or abroad. It is assumed (reasonably) that tasks that are more ‘routine’ or ‘rules-based’ are easier (less costly) to offshore. GRH assume initially that only L tasks may be offshored, though the entire analysis carries through in parallel for H tasks. Order the L tasks in an industry by $i \in [0,1]$ so that the cost of offshoring is non-decreasing in i. One way to model this is that the unit labor requirements for tasks performed abroad are greater than or equal to unit labor requirements for tasks performed at home. In particular, if task i requires $a_{Lj}$ units of labor if performed domestically, it requires $a_{Lj} \beta t_j (i)$ units of foreign labor where $\beta$ is a
shift parameter and \( \beta t_j (i) \geq 1 \) for all \( i \) and \( j \) and \( t'(i) > 0 \) (making this inequality strict simplifies things considerably).

A key modeling choice is to ask which industry, \( X \) or \( Y \), finds offshoring less costly? GRH start from the logical baseline that the costs of offshoring are common for a given factor regardless of where it is employed. Thus both \( X \) and \( Y \) face identical outsourcing costs: \( t_X (i) = t_Y (i) = t (i) \). Substantively, this is equivalent to assuming that the unskilled tasks in \( X \) are no more skill-intensive than the unskilled tasks in \( Y \). This may not be a good assumption, but it is probably a good place to start. When you draw the Lerner diagrams implied by this paper (the paper does not provide diagrams), you will see how the combination of perfect complementarity between tasks performed by a skill group within an industry and equip-proportionate declines in the cost of offshoring tasks by a skill group between industries leads to a familiar theoretical case, though arrived at by different means.

It is further assumed that \( a_{Lj} \) and \( a_{Hj} \), the task 'intensities,' are endogenously chosen to minimize firms’ costs of production given the constraint that the chosen combination of intensities yields a unit of output.

Let \( w \) and \( w^* \) be the home and foreign wage, respectively, of \( L \) workers. Suppose that

\[
\begin{align*}
  w > \beta t (0) w^*,
\end{align*}
\]

so that it is profitable to outsource some \( L \) tasks.

Let \( I \) index the marginal task performed at home, so that:

\[
\begin{align*}
  w = \beta t (I) w^*.
\end{align*}
\]

If goods are produced competitively, price must equal input costs, so:

\[
\begin{align*}
  p_j \leq w a_{Lj} (\cdot) (1 - I) + w^* a_{Lj} (\cdot) \int_0^I \beta t (i) \, di + sa_{Hj} (\cdot) + ..., \text{ for } j = x, y.
\end{align*}
\]

Here, \( s \) is the wage of \( H \) labor and the notation \( a_{Lj} (\cdot) \) and \( a_{Hj} (\cdot) \) is meant to stress the dependence of the \( a' \)s on market conditions and technology so that these values are optimally chosen. This expression can be usefully rewritten as:

\[
\begin{align*}
  p_j \leq w a_{Lj} (\cdot) \Omega (I) + sa_{Hj} (\cdot) + ..., \text{ for } j = x, y,
\end{align*}
\]

where \( \Omega (I) = 1 - I + \int_0^I \frac{t (i) \, di}{t (I)} \).

This expression is important because it expresses total labor costs for \( L \) tasks in relation to
the cost of performing all tasks domestically.

- In particular, \( w a_{Lj} (1 - I) \) is the actual domestic labor cost.
- The average foreign labor cost is \( w^* a_{Lj} \beta \int_0^I t(i) \, di \).
- Substituting \( w = \beta t(I) w^* \), the foreign labor cost is
  \[
  \frac{wa_{Lj} \left( \int_0^I t(i) \, di \right)}{t(I)}.
  \]
- Thus, total labor cost is:
  \[
  wa_{Lj} \left( 1 - I + \frac{\int_0^I t(i) \, di}{t(I)} \right).
  \]
- Logically, \( \Omega(I) < 1 \), which can be seen from the fact that \( t(i)/t(I) < 1 \) for \( i < I \) (thus the integral of \( t(i)/t(I) \) over \( [0, I] \) is less than \( I \)). Notice that \( \beta \) does not directly enter the expression for \( \Omega(\cdot) \), but it enters implicitly since \( I \) depends upon \( \beta \) (generally \( \partial I/\partial \beta < 0 \)).
- Market clearing in the market for \( L \) and \( H \) implies that
  \[
  a_{Lx} X + a_{Ly} Y = \frac{L}{1 - I},
  \]
  \[
  a_{Hx} X + a_{Hy} Y = H.
  \]

Thus, an increase in outsourcing \( I \) also has the effect of increasing domestic labor supply of \( L \) by the factor \( 1/(1 - I) \).

Assume that households have identical, homothetic preferences in all countries, and take the high skilled good \( X \) as the numeraire good, so \( p_x = 1 \), and \( p = p_y/p_x = p_y \). If the home country is small, \( p \) may be taken as parametric, otherwise not.

### 1.2 Comparative statics

Taking \( \Omega, p \) and \( I \) as exogenous for the moment, one can totally differentiate the equilibrium conditions to obtain an expression for the log change in the wage of low-skilled labor. In particular, let’s assume that there are only two factors and two sectors and both sectors are active (so we are in the cone of diversification). In this case, the four equilibrium equations are:
\begin{align*}
p &= w_{aL_y} (\cdot) \Omega (I) + s_{aH_y} (\cdot) \\
1 &= w_{aL_x} (\cdot) \Omega (I) + s_{aH_x} (\cdot) \\
L &= (1 - I) a_{L_x} X + (1 - I) a_{L_y} Y \\
H &= a_{H_x} X + a_{H_y} Y.
\end{align*}

The log change in \( w \) is equal to:
\[
\dot{w} = -\hat{\Omega} + \mu_1 \hat{p} + \mu_2 \frac{dI}{1-I},
\]  
(1)

where \( \mu_1 \) are the terms multiplying the price change and \( \mu_2 \) are the terms multiplying the labor supply term. This expression shows three separate channels by which outsourcing may impact the low-skilled wage.

### 1.3 Productivity effect

Consider the first term:
\[
\Omega (I) = 1 - I + \int_0^I \frac{t(i) \, di}{t(I)} \\
\frac{d\Omega (I)}{dI} = -1 + \frac{\partial t (I)^{-1}}{\partial I} \times \int_0^I t(i) \, di + t(I)^{-1} \times \frac{\partial}{\partial I} \int_0^I t(i) \, di \\
= -1 - \frac{t'(I)}{t(I)^2} \times \int_0^I t(i) \, di + 1 \\
= -\frac{\int_0^I t(i) \, di}{t(I)^2} \cdot t'(I) < 0
\]

It must be the case that \( \partial I/\partial \beta < 0 \), that is a fall in the costs of outsourcing raises the extent of outsourcing: \( \partial \Omega/\partial \beta > 0 \). But if outsourcing rises, \( \hat{\Omega} < 0 \), then the first term of (1) implies that the low-skilled wage rises. Why? It’s easiest to see by assuming that prices are exogenous, so the only moving part is \( \Omega \) and hence \( dp = 0 \). Rewrite the two price as equations
\begin{align*}
p &= w_{aL_y} (\Omega w/s) \Omega (I) + s_{aH_y} (\Omega w/s). \\
1 &= w_{aL_x} (\Omega w/s) \Omega (I) + s_{aH_x} (\Omega w/s).
\end{align*}

Here the parenthetical \((\Omega w/s)\) terms on the input coefficients are intended to stress the
dependence of the \( a' \)s on relative wages. If prices are parametric, then \( dp = 0 \). Also, we can use the envelope theorem to approximate \( \partial a (w/s) / \partial w \approx 0 \):

\[
dp = 0 = a_{Ly} (\Omega w/s) \Omega dw + a_{Lx} (\Omega w/s) wd\Omega
\]

\[
dw \cdot \Omega [a_{Ly} - a_{Lx}] + d\Omega \cdot w [a_{Ly} - a_{Lx}] = 0
\]

\[
dw \over w = -d\Omega \over \Omega
\]

\[
\partial \ln w = -\partial \ln \Omega
\]

\[
\dot{w} = -\dot{\Omega}.
\]

We can get a little more information by using the fact that

\[
w = w^\ast \beta t (I),
\]

\[
\dot{w} = \dot{\beta} + \dot{t} (I),
\]

\[
-\dot{\Omega} = \dot{\beta} + \dot{t} (I) = \frac{d\beta}{\beta} + \frac{t' (I)}{t (I)}.
\]

Rewriting:

\[
\frac{dw}{w} = \frac{dw^\ast \beta t (I)}{w^\ast \beta t (I)} = \frac{w^\ast t (I) d\beta + w^\ast \beta t' (I) dI}{w^\ast \beta t (I)} = \frac{d\beta}{\beta} + \frac{t' (I)}{t (I)} \frac{\partial I}{\partial \beta}
\]

Now, we substitute vigorously to get an expression for \( t' (I) / t (I) \):

\[
\frac{d\Omega}{\Omega} = -\frac{\int_0^1 \frac{t (I)}{t (I)^2} t' (I) \frac{dI}{d\beta}}{\Omega}
\]

\[
= \frac{w^\ast t (I) d\beta + w^\ast \beta t' (I) \frac{dI}{d\beta}}{w^\ast \beta t (I)}
\]

\[
= \frac{\partial I}{\partial \beta} + \frac{t' (I)}{t (I)} \frac{\partial I}{\partial \beta}
\]

\[
\frac{t' (I) \partial I}{I \partial \beta} = \frac{d\beta}{\beta} \left( \frac{\int_0^1 \frac{t (I)}{t (I)^2} - \Omega}{\Omega} \right)
\]

\[
= \frac{d\beta}{\beta} \left( \frac{\Omega}{\int_0^1 \frac{t (I)}{t (I)} - \Omega} \right).
\]
Now substitute for $dw/w$:

$$\frac{dw}{w} = t'(I) \frac{\partial I}{\partial \beta} + \frac{d\beta}{\beta} \Rightarrow \frac{dw}{w} = \frac{d\beta}{\beta} \left( \frac{\Omega}{\int_0^t t(i) \, di} + 1 \right)$$

$$\frac{dw}{w} = \frac{d\beta}{\beta} \left( \int_0^t t(i) \, di \right) \left[ \frac{1 - I + \int_0^t t(i) \, di}{t(I)} \right]$$

$$\frac{dw}{w} = -\frac{d\beta}{\beta} \left( \frac{\int_0^t t(i) \, di}{(1 - I) t(I)} \right)$$

$$\hat{w} = -\hat{\beta} \left( \frac{\int_0^t t(i) \, di}{(1 - I) t(I)} \right) = -\hat{\Omega}.$$ 

So, the bottom line here is

$$\hat{w} = -\hat{\beta} \left( \frac{\int_0^t t(i) \, di}{(1 - I) t(I)} \right) = -\hat{\Omega}.$$

*Since $\hat{\beta} < 0$ implies that outsourcing costs fall, we see that the wage effect is strictly positive.*

This is easiest to see in a Lerner diagram. The reduction in the cost of performing $L$ tasks on the interval 0 through $I$ is a like a factor augmenting technical change that raises the productivity of $L$ labor in both sectors. However, the extent of savings is larger in the $Y$ sector, meaning that the cost of producing $Y$ falls by more. Thus, both sectors become more $H$ intensive, and the relative wage ratio must shift favorably towards $L$ so that the relative cost of producing a bundle of $X$ and $Y$ is unchanged (otherwise, they cannot both be produced). $L$ labor will be freed from both sectors, and this can be accommodate by an expansion of the $L$ intensive sector.

### 1.4 Terms of trade and labor supply effects

If prices were parametric, this would be the end of the discussion. But if the home country is large, or if the fall in $\beta$ is not unilateral (so multiple countries experienced a fall in outsourcing costs at once), then there would also be an output expansion effect, leading to an adverse terms of trade effect. The price of $Y$ would fall as relative supply increased, and this would reduce the unskilled wage through the standard Stolper-Samuelson channel. This is the $\mu_1\hat{p}$ term in the above equation. These two terms (productivity gain, terms of trade) would be present in any standard two-by-two trade environment where a factor augmenting technical change differentially raised the productivity of the $L$ factor.
The peculiar nature of outsourcing creates a third effect. In addition to the employment reallocation that results from the rise in \( w \) (due to increased \( L \) productivity), there is a mechanical effect coming from direct labor displacement (seen in \( dI/(1-I) \)). This is not present in a standard factor-augmentation case. It is as if outsourcing is a 'machine' that increases the productivity of a subset \( 1-I \) workers while directly displacing the other \( I \) workers. Clearly, the term \( \mu_2dI/(1-I) \) must also be negative.

Thus, a reduction in \( \beta \) that raises \( I \) can have a net negative or positive effect on the \( L \) wage. This effect will be positive in the case where a small open economy experiences a unilateral reduction in its outsourcing costs. Obviously, such a case is highly stylized, and it is exactly this type of conceptual exercise that is critiqued by Krugman in 2000 (though Grossman and Rossi-Hansberg are sensitive to this criticism and acknowledge that this case is not realistic).

Meanwhile, for \( H \) labor, the log wage change equation can be written:

\[
\hat{s} = -\mu_3\hat{p} + \mu_4 \frac{dI}{1-I}.
\]

There is no direct factor-augmentation effect here \((\hat{\Omega})\). The price effect will have opposite sign for \( H \) than \( L \) labor in the two-by-two case. The labor supply effect will generally be positive for the \( H \) wage due to standard q-complementarity effects.

### 1.5 More realistic cases

The paper discusses numerous extensions that expand from this narrow special case.

- So far, the paper assumes that the extent of outsourcing is identical in \( X \) and \( Y \). It should be apparent that, holding \( p \) constant, if outsourcing possibilities increase differentially in \( Y \), this is better for the wages of \( L \) and if they increase differentially in \( X \), this is better for the wages of \( H \). Draw the Lerner diagram and you will see why.

- It should also be clear that once we leave the small country case, an expansion in Outsourcing will have an adverse relative price effect on \( p \), which (all else equal) will lower \( w \). This works to the benefit of \( S \) and against \( w \). It’s possible, however, for both \( H \) and \( L \) to gain if the price effect does not swamp the productivity effect.

- The labor supply effect is implicitly included in \( \hat{p} \) in most cases since domestic factor supplies do not impact relative wages within the cone of diversification except inasmuch as they change world relative output. Thus, the labor supply effect becomes relevant
when $L$ is additionally used in a non-traded activity. In this case, the labor-supply effect of an increase in outsourcing works to the benefit of $H$ and against $L$.

- All results hold in mirror image for a case in which offshoring of high skill tasks becomes feasible or increases. One interesting addendum here is that it is reasonable to assume (if we are thinking about industrial economies) that Home is relatively skill intensive relative to the rest of the world. Thus, even an uniform rise in outsourcing of $H$ and $L$ tasks differentially increases output of the $X$ good (since this is the good in which Home is specialized). This means that potentially both $H$ and $L$ can gain from increased productivity, but the terms of trade effect will augment the benefits for $L$ and reduce the benefits for $H$.

1.6 Conclusions from Grossman/Rossi-Hansberg

Whether or not you find this paper persuasive, you should certainly understand the conceptual mechanism. Among the paper’s virtues is that the conceptual model is able to nest a large number of interesting cases in a relatively parsimonious form. The trick here is the paper’s integration of Ricardian and Hecksher-Ohlin frameworks, which allows considerable flexibility, though at the cost of some transparency.

A central, under-examined, assumption of the paper is that all $L$ tasks within an industry are complementary, and similarly for $H$ tasks. At some level, this assumption seems right: senior programmers at IBM still need to architect the software that WiPro coders develop in Bangalore; physically present customer service representatives still need to sell mobile phones at the local Verizon store, even if the tech support is done from the Philippines. Thus, so long as some tasks need to be performed at Home, there is a fundamental complementarity between domestic and offshored tasks.

However, one may legitimately ask if the tasks ’left behind’ tend to favor a particular ’skill group’ within the distribution of $H$ or $L$ workers. One might loosely suspect that the low-skill tasks within the set of $H$ tasks and the high-skill tasks within the set of $L$ tasks are offshored (i.e., low-level programmers and moderately-skilled customer support positions are outsourced; high-level programmers and low-skill in-person sales agents are retained). This observation would be in the spirit of the ALM task framework, which suggests that it is ’routine’ tasks that are increasingly subject to automation and offshoring (with routine tasks typically performed by the least educated among the highly educated and the most educated among the low educated). If so, it may not be innocuous to assume, as in GRH, that workers within a skill group $H$ and $L$ are homogenous. In particular, the $L$ workers displaced by offshoring may be harmed if relegated to performing the remaining $L$ tasks (if
these are particularly low-skill tasks), and the $H$ workers displaced may be harmed if they are relegated to performing the remaining $H$ tasks (if these are particularly high-skill tasks). Incorporating these observations would require a model with a larger number of skill types, or one in which there are only two skill types but that members of each type have heterogeneous abilities in various tasks. Such a model would be more complicated to write. But one should bear in mind Einstein’s razor (a contrast to Occam’s razor), “Everything should be made as simple as possible, but not simpler.”

An interesting exercise is to consider the effect of outsourcing in the Acemoglu-Autor Handbook model, where outsourcing is a technical change or price change that causes a subset of tasks previously assigned to a domestic skill group ($L$, $M$, or $H$) to be supplied by foreign labor. It might be useful to simplify down to two skills groups in the AA model and, for consistency with GRH, assume that the tasks that are outsourced are exclusively from the $L$ subset of the task continuum (that is, they lie below $I_l$ at its pre-outsourcing value). How would outsourcing of these tasks affect the relative wage of $H$ versus $L$? How would it affect the real wage of $H$? How about the real wage of $L$ (harder)? Why (and in what ways) do the implications of the AA model differ from those of GRH for this case? Which assumption(s) account for these differences? Which set of predictions strikes you as more plausible and why?

## 2 Ricardian models of trade

Four empirical regularities are not readily understood in the standard HO model:

1. Trade between countries diminishes with distance

2. Prices vary across locations, with greater price differences between countries that are further apart

3. Factor rewards do not appear to be equalized across countries (that is, FPE does not hold)

4. Countries’ relative productivities vary across industries (inconsistent with the HO model in which variation in endowments rather than technology determines patterns of country specialization)

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1The actual quotation from Einstein’s 1933 Herbert Spencer Lecture is, “It can scarcely be denied that the supreme goal of all theory is to make the irreducible basic elements as simple and as few as possible without having to surrender the adequate representation of a single datum of experience.”
Distinct from H-O models that are based on differences in factor endowments with common technologies, the canonical conceptual model of international trade starts with Ricardo, who articulated the principle of comparative advantage: countries specialize in the activities in which they are relatively more productive. This productivity differences stem from differences in technology or skills (labor productivity). The Ricardian idea seems to many much more intuitively appealing than the H-O model, yet this idea had failed to have much traction in contemporary economic thinking until recently. Why not, you ask? The draft *Journal of Economic Perspectives* paper by Eaton and Kortum, “Putting Ricardo to Work,” answers this question eloquently. A one word summary of their answer: tractability.

In its basic form, the Ricardian model produces knife-edge predictions—a series of special cases rather than general results. This made the entire apparatus unattractive. The EK paper explains how economists have dealt with this problem historically and then lays out the solution that they and others have recently developed (their 2002 *Econometrica* paper is the classic reference). The solution, as we’ll see, is not so much an “aha—that solves it!” as a workaround: a way to make the model tractable by choosing functional forms carefully and applying probability theory to get a continuum of results rather than a discrete number of special cases. Whether you find this approach appealing or merely a “trick” is somewhat a matter of personal taste. Inarguably, their innovations have cleared a logjam in applying the Ricardian model in both the theoretical and empirical realm. This has resulted in a flowering of applied trade research that is arguably more relevant and appealing than the brand of trade economics that preceded it.

This section of the lecture note lays out the apparatus. You can get the same take from Eaton-Kortum’s lovely paper. (I partly summarize the insights here for my own benefit. Note for your academic futures: if you are unsure whether you fully understand a topic, write a set of lecture notes on it. When you’re done, you’ll either understand the topic thoroughly or you’ll be thoroughly clear that you don’t get it.)

### 2.1 The basic Ricardian two-by-two model

Ricardo imagined two countries making two goods each. Let’s take the case of Brazil and Costa Rica trading sugar and coffee. Assume that their labor requirements to make 100 kilos of each are:

<table>
<thead>
<tr>
<th></th>
<th>Coffee</th>
<th>Sugar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>100</td>
<td>75</td>
</tr>
<tr>
<td>Costa Rica</td>
<td>120</td>
<td>150</td>
</tr>
</tbody>
</table>
In this example, Brazil has an absolute advantage in both activities—that is, its unit labor requirements are lower in both sectors. However, it has a comparative advantage in Sugar since its unit labor requirements in Sugar relative to Coffee are 0.75 versus 1.25 in Costa Rica. Assume that the world relative price of coffee and sugar is 1. Clearly, Brazil will export sugar and Costa Rica will export Coffee. Thus, Brazil can get 100 kilos of Coffee with only 75 units of labor instead of 100 if produced domestically, and Costa Rica can get 100 kilos of sugar for only 120 units of labor rather than 150 units if produced domestically.

Even this simple example is obviously incomplete however. Three types of outcomes are possible in this setting: (i) Brazil only produces sugar and Costa Rica only produces coffee; (ii) Brazil makes only sugar and Costa Rica produces both goods; (iii) Costa Rica makes only coffee and Brazil produces both goods. With incomplete specialization, relative prices of goods also have to meet market clearing conditions within a country—that is, if Brazil makes both goods, then the marginal product of Brazilian labor must be equated between coffee and sugar production. Once those prices are pinned down, we have to check whether consumer demands are consistent with market clearing. If not, we’ve got to check alternative cases. So, even in this simple example, the model is clumsy. One might speculate that it’s not going to get prettier when we add more goods and more countries.

2.2 The chain of comparative advantage

Rather than assuming that the price of sugar and coffee are equated, let’s normalize Brazil’s wage to 1 and then determine the wage in Costa Rica, $w$, that is consistent with equilibrium. Assuming that trade occurs, it must be the case that the world price of coffee and sugar will be equated in both countries in purchasing power terms. Let’s write these as $p(c)$ and $p(s)$. Competition assures that costs are minimized, so it must be the case that

$$p(c) = \min \{120w, 100\}, \quad p(s) = \min \{150w, 75\}.$$  

For a country to be able to trade, it must also be a producer. Thus, each country must be the minimum cost producer of at least one good. Applying this logic to the numbers above, it’s evident that $w$ must be at least 0.5 but no greater than 0.80. (Since Costa Rica has an absolute disadvantage in both goods, it must also be the case that $w < 1$, but this is automatically satisfied by the prior inequalities.)

Let’s write unit labor requirements as $a_B(c), a_B(s)$ and $a_{CR}(c), a_{CR}(s)$ for Brazil and Costa Rica respectively. Since Brazil has a comparative advantage in sugar, we can write

$$a_B(c) / a_B(s) > a_{CR}(c) / a_{CR}(s).$$
One can readily extend this logic to three-plus goods to form a so-called “chain of comparative advantage”—that is, a series of inequalities that express the comparative advantage in one country relative to the other. An equilibrium is the $w$ that breaks the chain such that one set of goods is produced in Brazil, another in Costa Rica. At most one good can be produced in common; this occurs if at wage $w$, Brazil and Costa Rica have identical costs for producing the marginal good $j$. Note that this set of inequalities is not by itself sufficient to pin down the equilibrium. One needs further assumptions on demands and endowments to close the model.

Figure 1 in the EK JEP paper provides a nice illustration of wage determination ($w^*$) in the reference country (here Costa Rica, in their paper, England) as a function of labor supply. This figure makes a crucial point, which is that the demand curve with a finite number of goods will be decidedly non-smooth. In regions of the demand curve where the cost of the marginal good $j$ is the same in the two comparison countries, demand for labor is perfectly elastic in each country since $j$ can be produced in either country at the same cost. The conceptual experiment in Figure 1 of their paper corresponds to a case where England’s (Costa Rica’s) share of world labor supply expands such that it begins to take over production of additional goods. In the real where the two countries (England/Portugal, Costa Rica/Brazil) are producing the same good, demand for labor is perfectly elastic (no wage effects). When Costa Rica takes over production of the marginal good entirely—so the two countries produce no goods in common, we are in a realm where demand is again elastic; the more of the original $j$ good that Costa Rica produces, the more its price falls. If labor supply expands further, Costa Rica will eventually become competitive in the next good, $j'$. We are then in another flat spot where Brazil and Costa Rica are producing the (new) marginal good and the demand that each country faces is again perfectly elastic at the market price. It’s easy to see why this demand structure will create problems for simple comparative static exercises.

### 2.3 A continuum of goods

The problem with the setup above is that demand for labor in each country is decidedly non-smooth as we transition between regions in which there is a marginal good $j$ and no marginal good. A famous AER paper by Dornbusch, Fischer and Samuelson in 1977 (DFS) developed a nice workaround for this clunky structure. They assumed a continuum of goods $j$ arrayed on the unit interval $j \in [0, 1]$ where the ratio $A(j) = a_B(j) / a_{CR}(j)$ is non-increasing in $j$ and, more restrictively, $A(j)$ is smooth and strictly decreasing. Thus, Brazil’s comparative advantage is rising in the index $j$. These assumptions guarantee that the chain of comparative advantage has no flat spots; there’s always a marginal good that is equally costly to produce.
in both countries. In particular, continuing to use the normalization that the wage in Brazil is equal to one and the wage in Costa Rica is equal to \( w \), the marginal good \( \bar{j} \) satisfies

\[
a_B(\bar{j}) = w \cdot a_{CR}(\bar{j}) \Rightarrow a_B(\bar{j}) / a_{CR}(\bar{j}) = w \Rightarrow A(\bar{j}) = w.
\]

Thus, Costa Rica produces goods \( j \leq \bar{j} \) and Brazil produces goods \( j \geq \bar{j} \). A fall in \( w \) increases the range of goods that Costa Rica produces and contracts the range of goods that Brazil produces. Notice by the way that the Acemoglu-Autor (2010 Handbook chapter uses the same modeling tool. If you were to label the goods in DFS as tasks and the countries as skill groups, you’d have the foundation of the Acemoglu-Autor model (though what AA do with the model is distinct from what DFS use it for). Figure 2 of EK 2012 provides a nice illustration of the operation of the DFS model.

### 2.4 Trade costs

A second innovation of DFS is to introduce trade costs. These are missing from standard H-O models but arguably of first order importance in the real world. How do we know this is true? First, most countries consume a disproportionate share of their own output, which is consistent with either significant trade costs or strong home-bias in tastes. Second, distant countries trade less with one another and remote countries trade less with everyone. These facts suggest that trade costs have a large impact on equilibrium outcomes. Trade costs also immediately introduce a greater semblance of reality into the frictionless H-O setting. If trade between two countries is costly, then competition is imperfect. The same good can have different prices in different markets, and moreover, the low-cost producer of a good for one country may not be the low-cost producer for another country if trade costs between these two different country pairs differ. Thus, trade costs are not just a nuisance, they’re a crucial toe hold on reality.

DFS model trade costs as a kind of “iceberg” transportation cost: part of the cargo decays (melts) in transit, and in general, the amount of decay will be proportional to transit time or distance. Specifically, delivering one unit of a good from country \( i \) to \( k \) will require shipping \( d_{ik} > 1 \) units of the good. Note that \( d_{ik} \) will differ among country pairs. In general, it is assumed that \( d \) will not differ among goods within a country pair, but this can be relaxed. It’s also typical to invoke the “triangle inequality,” \( d_{ik} \times d_{km} \geq d_{im} \), which says that the cost of shipping a good from country \( i \) to \( k \) and then from \( k \) to \( m \) is weakly greater than the cost of shipping it from \( i \) to \( m \). This is a no-arbitrage condition.
2.5 Adding more countries

So far so good. But now it’s about to get messy again. With many goods and many countries, the chain of comparative advantage is not likely to retain its useful ‘smoothness’ unless we impose strong restrictions on the shape of comparative advantage. (That’s precisely what Acemoglu-Autor do for skill groups \( L, M \) and \( H \). Arguably, it’s justifiable to do this for skill groups, where productivity rankings might plausibly be monotone. For countries, this is harder to rationalize).

A classic paper by Jones (1961) shows why this doesn’t work. The following table from Jones gives unit labor requirements for three goods in three countries:

<table>
<thead>
<tr>
<th></th>
<th>U.S.</th>
<th>Britain</th>
<th>Europe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corn</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Linen</td>
<td>5</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>Wool</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

Each country will produce one good, so the question is which? Normalize the U.S. wage at 1 and let \( w_B \) and \( w_E \) equal the wage rate in Britain and Europe respectively.

Here are two possible assignments

<table>
<thead>
<tr>
<th></th>
<th>Option 1</th>
<th>Option 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corn</td>
<td>Britain</td>
<td>U.S.</td>
</tr>
<tr>
<td>Linen</td>
<td>U.S.</td>
<td>Europe</td>
</tr>
<tr>
<td>Wool</td>
<td>Europe</td>
<td>Britain</td>
</tr>
</tbody>
</table>

You can check all of the inequalities for any two countries and two goods to confirm that both assignments satisfy comparative advantage. That is, for any pair of goods that two countries are producing, we can confirm that they would not want to switch places (so, in Option 1, we compare US and Britain in Linen and Corn, US and Europe in Linen and Wool, and Britain and Europe in Corn and Wool).

\[
\begin{align*}
\frac{a_{US}(C)}{a_B(C)} > \frac{a_{US}(L)}{a_B(L)} & \iff \frac{10}{10} > \frac{5}{7} \\
\frac{a_{US}(W)}{a_E(W)} > \frac{a_{US}(L)}{a_E(L)} & \iff \frac{4}{2} > \frac{5}{3} \\
\frac{a_B(W)}{a_E(W)} > \frac{a_B(C)}{a_E(C)} & \iff \frac{3}{2} > \frac{10}{10}
\end{align*}
\]
For Option 2, we compare U.S. and Europe in Corn and Linen, U.S. and Britain in Corn and Wool, and Europe and Britain in Linen versus Wool:

\[
\begin{align*}
\frac{a_{US}(L)}{a_E(L)} &> \frac{a_{US}(C)}{a_E(C)} \iff \frac{5}{3} > \frac{10}{10} \\
\frac{a_{US}(W)}{a_B(W)} &> \frac{a_{US}(C)}{a_B(C)} \iff \frac{4}{3} > \frac{10}{10} \\
\frac{a_B(L)}{a_E(L)} &> \frac{a_B(W)}{a_E(W)} \iff \frac{7}{3} > \frac{3}{2}
\end{align*}
\]

Notice that both assignments satisfy Ricardo’s inequality.

Which assignment is superior? Footnote 10 of Eaton-Kortum shows that efficiency demands the assignment that minimizes the product of labor requirements. For Option 1, that’s \(a_B(C) \times a_U(L) \times a_E(W) = 10 \times 5 \times 2 = 100\). For Option 2, that’s \(a_B(W) \times a_U(C) \times a_E(L) = 3 \times 10 \times 3 = 90\). So, the second assignment is cost-minimizing.

Clearly, this is iterative approach is messy: 1) test all feasible assignments to determine which satisfy comparative advantage; 2) calculate labor requirements; 3) choose the cost-minimizing feasible assignment.

Here’s where the Eaton-Kortum 2002 *Econometrica* paper comes in. Their paper takes the next logical step—akin to DFS 1977—in turning a messy discreet problem into a tractable, continuous problem.

### 3 A probabilistic approach

One solution to the many-goods, many-countries problem would be to assume a continuum of countries, as DFS did when assuming a continuum of goods. But this is a bit silly. There are arguably an arbitrarily large number of goods that can be traded, but there is only a finite, countable set of countries. In place of a continuum, EK assume a probability distribution and use then use probability theory (and the law of large numbers) to get predictions. Here’s how they do it:

Continue to assume a continuum of goods \(j \in [0, 1]\) as in DFS and add an integer number of countries \(i = 1, 2, ..., I\). We’ll continue to work with only Labor as an input for now. Write the unit labor requirements for good \(j\) in country \(i\) as \(a_i(j)\).

Here’s where the cleverness starts. Think of these \(a\)'s as random draws from particular probability distributions. Using continuous distributions makes the problem smooth again. In addition, we don’t have to keep track of individual values of the many \(a\)'s as we work through the problem. We can simply use cumulative distribution functions to determine
the probability that a country is the low cost producer of a specific good. The choice of functional forms is quite important here. We need a distributional family where conditional distributions inherit the property of marginal distributions, which moves us towards the exponential family (and not any exponential will due). The family that EK use is the Fréchet distribution. Why this distribution? If inventions (productivity realizations) are drawn from a Pareto distribution and only the most efficient draws are retained, then the order statistics from these draws will be characterized by the type II extreme value (Fréchet) distribution. Specifically, their assumption is that

\[ \Pr \left[ a_i (j) < x \right] = 1 - e^{-(A_i x) \theta}. \]

Here, \( A_i \) is country \( i \)'s absolute advantage in all goods: a higher \( A_i \) means country \( i \) will in expectation have lower draws of labor requirements in all goods \( j \). The parameter \( \theta > 1 \) is inversely related to the variance of labor requirements. A larger \( \theta \) means that a country’s labor requirements are typically closer to its country-specific mean; a smaller \( \theta \) means that these requirements have greater dispersion.

A convenient simplification in this model is to impose the assumption that \( \theta \) is common across countries. In that case, \( \theta \) also describes the type of competition that producers will face. If \( \theta \) is large, so there’s very little variance in productivity draws, what will matter is a country’s absolute advantage as well as its trade costs vis-a-vis other countries. Competition in this setting is close to perfectly competitive. Small perturbations in a country’s cost vis-a-vis its competitors could cause it to gain or lose the entire market for a specific good \( j \). Conversely, when \( \theta \) is small, productivity draws will be very disperse. Even countries with low absolute advantage may be the low-cost producer of certain goods. Moreover, since the dispersion of countries’ draws for a given good will be substantial, small perturbations in costs may have little effect on the demand that a country faces for its output of good \( j \). That is, if a country has strong comparative advantage in producing some good, then a modest increase in costs might not have a first order effect on its world market share for that good.

### 3.1 The price distribution

As before, let \( w_i \) equal the labor cost in country \( i \) (no normalization is needed, since it’s only relative \( w's \) that matter) and continue to assume iceberg transport costs such that \( d_{i\nu} > 1 \) and \( d_{ii} = 1 \). Thus, the cost of producing good \( j \) in country \( i \) and delivering it country \( n \) is

\[ c_{ni} (j) = a_i (j) w_i d_{ni}. \]
The price that country \( n \) pays for \( j \) is of course the minimum of all prices available to it. That is,

\[
c_n(j) = \min \{ c_{ni}(j) \}.
\]

Note that this does not mean that each country pays the minimum world price for each good. With non-zero trade costs that differ among country pairs, the lowest available price for a good \( j \) will vary across countries.

The distribution of the cost of good \( j \) produced in country \( i \) and offered in country \( n \) is given by

\[
\text{Pr}[c_{ni}(j) < c] = 1 - e^{-(cA_i/w_id_{ni})^\theta}.
\]

Note this is only the price that \( n \) faces for good \( j \) from country \( i \). The cumulative distribution of prices for good \( j \) that country \( n \) faces across all supplier countries is

\[
\text{Pr}[p_n(j) < p] = 1 - \prod_i \text{Pr}[c_{ni}(j) > p] = 1 - e^{-(\bar{A}_np)^\theta},
\]

where \( \bar{A}_n = \left[ \sum_{i=1}^{I} (A_i/w_id_{ni})^\theta \right]^{\frac{1}{\theta}} \).

This term \( \bar{A}_n \) is a country specific purchase price parameter. In EK 2002, they explain its meaning as follows:

The price parameter \( \bar{A}_n \) is critical to what follows. It summarizes how (i) states of technology around the world, (ii) input costs around the world, and (iii) geographic barriers govern prices in each country \( n \). International trade enlarges each country’s effective state of technology with technology available from other countries, discounted by input costs and geographic barriers. At one extreme, in a zero-gravity world with no geographic barriers (\( d_{ni} = 1 \) for all \( n \) and \( i \) ), \( \bar{A}_n \) is the same everywhere and the law of one price holds for each good. At the other extreme of autarky, with prohibitive geographic barriers (\( d_{ni} \to \infty \) for \( n \neq i \) ), \( \bar{A}_n \) reduces to \( A_n/w_n \), country \( n \) ’s own state of technology down-weighted by its input cost.

Finally, we want to add preferences. Without any loss of relevance, we’ll consider the simplest case: preferences are symmetric Cobb-Douglas, with equal shares of income spent on all goods. In this case, the ideal price index is simply the geometric mean of the price distribution. So we can write

\[
p_n = \frac{\gamma}{\bar{A}_n},
\]
where \( \gamma \) is a constant.\(^2\) **Lower** values of \( p_n \) mean higher purchasing power. Since \( \bar{A}_n \) enters inversely, a higher value of \( \bar{A}_n \) corresponds to **higher** purchasing power. Factors that raise \( \bar{A}_n \) (lowering \( p_n \)) are: higher own-productivity (\( A_i \)); lower bilateral trade costs (\( d_{ni} \)); and lower input costs (\( w_i \)). Thus, the model nicely gives rise to differences in PPP as a function of the model’s primitives.

### 3.2 Equilibrium

There’s a whole lot more one can do with this model. One thing we haven’t done so far is close it. In a model where all income is labor income, this is straightforward. Let \( L_i \) be the labor endowment of country \( i \). Then country \( i \)’s total income is

\[
 w_i L_i = \sum_{n=1}^{I} \pi_{ni} (w_n L_n + D_n) \tag{2}
\]

where \( D_n \) is the trade deficit in county \( n \) (meaning that it spends in excess of its labor income) and \( \pi_{ni} \) is the share \( n \)'s consumption purchased from \( i \). We derive an expression for \( \pi \) immediately below. Equation (2) is actually a system of \( I \) linear equations. It will generally have to be solved numerically.

### 3.3 Putting the model to work: Trade shares and 'gravity'

We can now derive a bunch of useful comparative trade for trade flows.

- The probability \( \pi_{ni} \) that country \( i \) is the lowest cost supplier of any specific good \( j \) to country \( n \) is

\[
 \pi_{ni} = \Pr [c_{ni}(j) = p_n(j)] = \left( \frac{A_i/w_id_{ni}}{\bar{A}_n} \right)^{\theta}.
\]

We don’t subscript this probability by \( j \) because the probability does not differ across goods. Notice that \( \theta \) plays the role of an elasticity. Imports of goods from \( i \) to country \( n \) decrease with elasticity \( \theta \) in response to a rise in \( w_i \) or \( d_{ni} \). Notice that higher world productivity (net of trade costs) measured by \( \bar{A}_n \) lowers the probability that country \( i \) is the low cost producer of \( j \) for country \( n \).

- With a continuum of goods, \( \pi_{ni} \) is also the share of all goods (on the continuum) consumed in \( n \) that are supplied by \( i \).

\(^2\gamma = e^{-\epsilon/\theta} \) and \( \epsilon \) is Euler’s constant.
• Since $\pi_{ni}$ is the fraction of goods that country $n$ buys from $i$, and since the expected price of goods does not vary by source (conditional on purchase), $\pi_{ni}$ is also the fraction of $n$’s expenditure spent on goods from $i$. Let $X$ equal expenditure:

$$\pi_{ni} = \frac{X_{ni}}{X_n} = \left(\frac{A_i/w_id_{ni}}{\bar{A}_n}d_{ni}\right)^\theta$$  

(3)

• Some further manipulation provides a useful expression. Let $Q_i$ equal total sales by exporter $i$. Country $i$’s total sales to all countries $m$ is:

$$Q_i = \sum_{m=1}^N X_{mi} = \left(\frac{A_i}{w_i}\right)^\theta \sum_{m=1}^N \frac{d_{mi}^{-\theta}X_m}{\bar{A}_m^\theta}.$$  

(4)

• We can use (4) to solve for $(A/w_i)^\theta = (X_{ni}/X_n) \times (\bar{A}_n/d_{ni})^\theta$ and substituting into (3) gives an expression for $X_{ni}$, which is the consumption of goods in $n$ produced in $i$ (AKA exports from $i$ to $n$):

$$X_{ni} = \frac{X_n \cdot Q_i \left(\bar{A}_n d_{ni}\right)^{-\theta}}{\sum_{m=1}^N \left(\bar{A}_m d_{mi}\right)^{-\theta} X_m} = \frac{X_n \cdot Q_i \left(\gamma d_{ni} p_n\right)^{-\theta}}{\sum_{m=1}^N \left(\frac{\gamma d_{mi} p_m}{p_n}\right)^{-\theta} X_m} = \frac{X_n \cdot Q_i \left(\frac{d_{ni}}{p_n}\right)^{-\theta}}{\sum_{m=1}^N \left(\frac{d_{mi}}{p_m}\right)^{-\theta} X_m}$$  

(5)

This central equation says that imports to $n$ from $i$ are:

– Increasing in $i$’s total exports

– Declining in bilateral trade costs $d_{ni}$

– Rising in the importer’s total purchases $X_n$

– In all cases, the geographic barrier $d_{ni}$ between $i$ and the importer is deflated by the importer’s price level $p_n$. The lower are goods prices in the destination market, the more that geographic barriers between $n$ and $i$ reduce trade. That is, import costs matter more when the destination market is more competitive.

– The denominator of this expression, $\left(\frac{d_{mi}}{p_m}\right)^{-\theta} X_m$, is the size of each destination market $m$ as perceived by $i$. Higher $X_m$ means that $i$ has a larger market into
which to sell. Higher bilateral trade costs $d_{ni}$ and a lower price level $p_{m}$ in market $m$ reduces $i$’s sales into $m$. Thus, county $n$’s share of $i$’s imports is $n$’s share of $i$’s effective world market—that is, the size of the world market perceived by $i$, after accounting for transports costs to $i$ and prices and market size in all other countries.

- Historically, the gravity model for trade among bilateral country pairs has been estimated something like this:

\[
\ln (X_{ni}) = \beta_0 + \beta_1 \ln (M_n) + \beta_2 \ln (M_i) + \beta_3 \ln (d_{ni}) + e_{ni},
\]

where $M_n, M_i$ are the economic ‘masses’ of country’s $n$ and $i$, $d_{ni}$ is the trade cost (usually distance), and the expected signs of $\beta_1$ and $\beta_2$ are positive and the expected sign of $\beta_3$ is negative. Equation (5) also suggests this relationship, if we substitute $X_n$ and $Q_i$ for $M_n$ and $M_i$.

\[
\ln X_{ni} = \ln X_n + \ln Q_i - \theta \ln d_{ni} + \theta \ln p_n - \kappa,
\]

where $\kappa = \ln \left[ \sum_{m=1}^{N} \left( \frac{d_{mi}}{p_m} \right)^{-\theta} X_m \right]$.

- Equation (5) also implies a relationship between trade flows and price differences:

\[
\frac{X_{ni}/X_n}{X_{ii}/X_i} = \left( \frac{p_i d_{ni}}{p_n} \right)^{-\theta}.
\]

The left-hand side of this expression is country $i$’s share in $n$’s output relative to $i$’s consumption of its own output. This share is falling in the bilateral trade cost and rising as overall prices in market $n$ rise relative to prices in market $i$. Higher values of $\theta$ magnify these effects; when competition is more intensive (price heterogeneity smaller), trade flows are more responsive to given trade costs and price differentials.

### 3.4 Gains from Trade

One very nice application of the framework is to derive the gains from trade. Real income in country $i$ is

\[
\frac{w_i}{p_i} = \gamma^{-1} A_i \pi_i^{-1/\theta}.
\]

This expression says that a country’s income is increasing in its absolute advantage $A_i$ (you can substitute the term “number of ideas” for $A_i$ if you like) and its declining in its home
share in consumption (because it’s gaining less from trade). Taking logs to simplify we get

\[
\ln \left( \frac{w_i}{p_i} \right) = -\ln \gamma + \ln A_i - \frac{1}{\theta} \ln \pi_{ii}.
\]

Note that if a country doesn’t trade at all, \( \pi_{ii} = 1 \) and hence the final term in this expression is zero, thus the real wage is determined entirely by domestic productivity \( A_i \). Let’s say that \( \theta \approx 4 \). That is, the elasticity of trade with respect to price of traded goods is equal to 4. So, if country \( i \)'s import share went from zero to 0.25, so \( \pi_{ii} \) declined from 1 to 0.75, then its welfare would rise by \( -\frac{1}{4} \ln 0.75 \approx 0.072 \), i.e., 7.2 log points or about 7.5 percent. This insight (with a lot more depth) is the basis of a recent, influential \( AER \) paper by Arkilakos, Costinot and Rodriguez-Claire.

### 3.5 What is this model for?

Other than a lot algebra, what have we gotten out of this model? From my perspective, here are some of the key ‘deliverables’ we’ve obtained:

1. A succinct, closed analysis of the operation of comparative advantage in a full GE setting. This is not a small accomplishment. Of course, it requires strong distributional assumptions. But it’s likely that trade economists will make progress in relaxing them.

2. A plausible rationalization of the four key trade facts set out above: trade between countries diminishes with distance; prices vary across locations, with greater price differences between countries that are further apart; factor rewards do not appear to be equalized across countries; and countries’ relative productivities vary across industries.

3. Perhaps most empirically relevant: This model relates trade flows to their potential effects on destination markets. In the standard HO model, it is only prices and not flows that are informative about the potential impact of trade on a given market. In the EK model, trade flows (and their changes over time) are directly informative about changes in comparative advantage and transport costs. These changes both induce trade flows and affect the final demand for the output of each competing country; hence, they impact welfare in competing economies.

In summary, this paper offers the rudiments of an empirical toolkit for analyzing how trade affects trading economies. I would say that this toolkit was largely lacking in the HO model. (Though see Krugman 2000 for a justification of the use of factor contents, which was a labor tool applied to trade questions, which trade economists such as Leamer (2000) had previously condemned as irrelevant.) As one partial example of this toolkit, the Autor, Dorn,
Hanson (2011) paper uses the EK model to consider how an exogenous rise in productivity in one country (China) would affect the demand for goods produced in each competing local economy, which ADH operationalize as ‘Commuting Zones.’ (The derivation of the ADH approach is given below. It is a very simple extension)

The EK (2002) paper is now a decade since publication, and there has been much subsequent progress. Some of the key papers are:


And these are just the theory papers. Important empirical papers in this literature include:


While the literature is progressing rapidly, it is a literature that for the most part lacks a labor economist’s sensibility: tight identification, a focus on labor market consequences (not just prices and goods), and attention to institutions and frictions that may drive a wedge between the lovely theory and the complexities of reality. Indeed, the labor market is notably absent from much (but not all) of contemporary trade literature, including Eaton-Kortum 2002. This lacuna creates an opportunity for contemporary labor scholars. Trade and labor have been too long been two countries divided by a common language. That is changing.
4 Using the Eaton-Kortum (2002) model to motivate a labor market approach

A simple but potentially useful application of EK to labor markets is found in Autor-Dorn-Hanson 2011. Their reduced-form EK application is motivated as follows:

- Let the demand for labor in industry \( j \) by region \( i \) be given by \( L_{ij} = L^d(w_{ij}, Q_{ij}) \), where \( w_{ij} \) is unit production costs and \( Q_{ij} \) is output.

- For region \( i \), sales to destination market \( n \) in industry \( j \) are a function of its technological capability (\( A_{ij} \)), unit production costs (\( w_{ij} \)), and bilateral trade costs (\( \tau_{nij} \)), as well as expenditure in destination market \( n \) for goods of industry \( j \) (\( X_{nj} \)).

- Technological capability, \( T_{ij} \), is a parameter that determines the position of the distribution of firm productivities in an industry and region. Using the EK model, region \( i \)'s sales in industry \( j \) to destination market \( n \) can be written as

\[
X_{nij} = \frac{A_{ij}(w_{ij}\tau_{nij})^{-\theta}}{\bar{A}_{nj}}X_{nj},
\]

where \( \theta \) is a parameter describing the dispersion in productivity among firms and \( \bar{A}_{nj} = \sum_h A_{nj}(w_{nj}\tau_{nhj})^{-\theta} \) describes the “toughness” of competition in destination market \( n \) in industry \( j \), reflecting production and trade costs in the locations that supply products to market \( n \).

- Region \( i \) will capture a larger share of market \( n \)'s purchases in industry \( j \) when it has high productivity, low production costs, and low trade costs relative to other suppliers. Define \( A_{ij} = A_{ij}w_{ij}^{-\theta} \) to be the cost-adjusted productivity of region \( i \) in industry \( j \). Then, summing over destination markets for region \( i \), its total output in industry \( j \) is

\[
Q_{ij} = A_{ij} \sum_n X_{nj}\tau_{nij}^{-\theta}/\bar{A}_{nj}.
\]

- China will be among the countries with which each U.S. region competes in serving destination markets. When China’s productivity expands or its foreign trade costs fall, it increases the value of \( \bar{A}_{nj} \) in each destination market, diverting product demand away from U.S. regions that also serve these markets.

- To show this formally, consider the change in \( Q_{ij} \) that would result were China to experience exogenous productivity growth (i.e., an increase in \( T_{cj} \), where \( c \) indexes
China) or a reduction in trade costs, do, say, to China’s accession to the WTO. The direct effect of changes in China’s productivity and trade costs on $Q_{ij}$ is

$$
\dot{Q}_{ij} = -\sum_n X_{nij} X_{ncj} Q_{ij} (\dot{A}_{cj} - \theta \dot{\tau}_{ncj})
$$

(7)

where $\dot{x} \equiv d\ln x$, $X_{nij}/Q_{ij}$ is the share of exports to destination market $n$ in region $i$’s output in industry $j$, and $X_{ncj}/X_{nj}$ is the share of imports from China in spending by destination market $n$ in industry $j$.

• Equation (7) implies that the fall in region $i$’s output in industry $j$ is larger the higher is cost-adjusted productivity growth in China and the larger is the reduction in trade costs facing China, where the impact of these shocks is larger the more dependent region $i$ is on market $n$ and the more important China is as a source of supply to market $n$. In applying equation 7, we will focus on competition that CZs face from China in the U.S. market, thus limiting the summation above to $n = u$, that is, to outputs produced and consumed in the United States.

• In general equilibrium, changes in China’s productivity and trade costs may also cause wages and other factor prices to change in the countries with which China competes. These changes in factor prices, in turn, may cause changes in aggregate spending by countries, as the effects of shocks to China reverberate through the global economy. Equation (7) thus shows only the direct effect of shocks to Chinese productivity and trade costs on the demand for output in region $i$, ignoring the indirect effects of these changes on factor prices and spending in region $i$ and in other regions and countries.

The useful feature of this setup is that it provides a link between observed changes in quantities of goods imported from China and changes in the demand for the output of a local economy (e.g., a Commuting Zone). Thus, unlike the H-O models we considered in the earlier lectures, this model provides an empirical toehold for quantities of goods traded to local demand for labor. This linkage is almost non-existent in the H-O framework.