The Cost of Financial Frictions for Life Insurers

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Abstract

During the financial crisis, life insurers sold long-term policies at deep discounts relative to actuarial value. In January 2009, the average markup was −25 percent for 30-year term annuities as well as life annuities and −52 percent for universal life insurance. This extraordinary pricing behavior was a consequence of financial frictions and statutory reserve regulation that allowed life insurers to record far less than a dollar of reserve per dollar of future insurance liability. Using exogenous variation in required reserves across different types of policies, we identify the shadow cost of financial frictions for life insurers. The shadow cost was nearly $5 per dollar of excess reserve for the average insurance company in January 2009. (JEL G01, G22, G28)

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Traditional theories of insurance markets assume that insurance companies operate in an efficient capital market that allows them to supply policies at actuarially fair prices. Consequently, the market equilibrium is primarily determined by the demand side, either by life-cycle demand (Yaari, 1965) or informational frictions (Rothschild and Stiglitz, 1976). In contrast to these traditional theories, this paper shows that insurance companies are financial institutions whose pricing behavior can be profoundly affected by financial frictions and statutory reserve regulation.

Our key finding is that life insurers reduced the price of long-term policies in January 2009, when historically low interest rates implied that they should have instead raised prices. The average markup, relative to actuarial value (i.e., the present discounted value of future policy claims), was −25 percent for 30-year term annuities as well as life annuities at age 50. Similarly, the average markup was −52 percent for universal life insurance at age 30. These deep discounts are in sharp contrast to the 6 to 10 percent markup that life insurers earn in ordinary times (Mitchell, Poterba, Warshawsky, and Brown, 1999). In the cross section of insurance policies, the price reductions were larger for those policies with looser statutory reserve requirements. In the cross section of insurance companies, the price reductions were larger for those companies whose balance sheets were more adversely affected prior to January 2009.

This extraordinary pricing behavior was due to a remarkable coincidence of two circumstances. First, the financial crisis had an adverse impact on insurance companies’ balance sheets. Insurance companies had to quickly recapitalize to contain their leverage ratio and to avoid a rating downgrade. Second, statutory reserve regulation in the United States allowed life insurers to record far less than a dollar of reserve per dollar of future insurance liability in January 2009. Since rating agencies and state regulators assess insurance companies based on an accounting measure of liabilities, these companies ultimately care about accounting (rather than market) leverage. Insurance companies were able to lower their accounting leverage by selling policies at a price far below actuarial value, as long as that price was
above the reserve value.

We formalize our hypothesis in a dynamic model of insurance pricing that is otherwise standard, except for a leverage constraint that is familiar from macroeconomics and finance (e.g., Kiyotaki and Moore, 1997; Brunnermeier and Pedersen, 2009). The insurance company sets prices for various types of policies to maximize the present discounted value of profits, subject to a leverage constraint that the ratio of statutory reserves to assets cannot exceed a targeted value. When the leverage constraint binds, the insurance company optimally prices a policy below its actuarial value if its sale has a negative marginal impact on leverage. The Lagrange multiplier on the leverage constraint has a structural interpretation as the shadow cost of raising a dollar of excess reserve.

We test our hypothesis on panel data of nearly 35,000 observations on insurance prices from January 1989 through July 2011. Our data cover term annuities, life annuities, and universal life insurance for both males and females as well as various age groups. Relative to other industries, life insurance presents a unique opportunity to identify the shadow cost of financial frictions for two reasons. First, life insurers sell relatively simple products whose marginal cost can be accurately measured. Second, statutory reserve regulation specifies a constant discount rate for reserve valuation, regardless of the maturity of the policy. This mechanical rule generates exogenous variation in required reserves across policies of different maturities, which acts as relative shifts in the supply curve that are plausibly exogenous.

We find that the shadow cost of financial frictions is essentially zero for most of the sample, except around January 2001 and in January 2009. The shadow cost was nearly $5 per dollar of excess reserve for the average insurance company in January 2009. This cost varies from $1 to $13 per dollar of excess reserve for the cross section of insurance companies in our sample. We also find that those companies with the highest shadow costs were actively recapitalizing through two conventional channels. First, more constrained insurance companies received larger capital injections from their holding company, through the issuance of surplus notes or the reduction of stockholder dividends. Second, more constrained insurance companies
reduced their required risk-based capital by shifting to safer assets with lower risk charges, such as cash and short-term investments. Our findings suggest that conventional channels of recapitalization were insufficient at the height of the financial crisis, and insurance companies had to raise additional capital through a firesale of policies.

We rule out default risk as an alternative explanation for the deep discounts on long-term policies in January 2009. Since risk-based capital requirements assure that only a small share of life insurers' assets are risky, the implied recovery rate on their assets is too high for default risk to justify the magnitude of the discounts. We also find out-of-sample evidence against default risk based on the pricing of life annuities during the Great Depression. The absence of discounts during the Great Depression, when the corporate default spread was even higher than the heights reached during the recent financial crisis, is inconsistent with the hypothesis that default risk drives insurance prices. Only our explanation, based on financial frictions and statutory reserve regulation, is consistent with the evidence for both the Great Depression and the recent financial crisis.

Our finding that the supply curve for life insurers shifts down in response to a balance sheet shock, causing insurance prices to fall, contrasts with the evidence that the supply curve for property and casualty insurers shifts up, causing insurance prices to rise (Froot and O'Connell, 1999). Although these findings may seem contradictory at first, they are both consistent with our supply-driven theory of insurance pricing. The key difference between life insurers and property and casualty insurers is statutory reserve regulation. Life insurers were able relax their leverage constraint by selling new policies because their statutory reserve regulation allowed less than full reserve during the financial crisis. In contrast, property and casualty insurers must tighten their leverage constraint when selling new policies because their statutory reserve regulation always requires more than full reserve.

The remainder of the paper is organized as follows. Section 1 describes our data and documents key facts that motive our study of insurance prices. Section 2 reviews key features of statutory reserve regulation that are relevant for our analysis. In Section 3, we
develop a structural model of insurance pricing, which shows how financial frictions and statutory reserve regulation affect insurance prices. In Section 4, we estimate the shadow cost of financial frictions through the structural model of insurance pricing. In Section 5, we calibrate the structural model of insurance pricing to show that it explains the observed magnitudes of the price reductions and the shadow cost of financial frictions in January 2009. Using the calibrated model, we estimate that the welfare cost of the price reductions was an order of magnitude larger than the welfare cost that arises from markup pricing in ordinary times. Section 6 concludes with broader implications of our study for household finance and macroeconomics.

1. Annuity and Life Insurance Prices

1.1. Data Construction

1.1.1. Annuity Prices

Our annuity prices are from the Annuity Shopper (Stern, 1989), which is a semiannual (every January and July) publication of annuity price quotes from the leading life insurers. Following Mitchell, Poterba, Warshawsky, and Brown (1999), we focus on annuities that are single premium, immediate, and non-qualified. This means that the premium is paid upfront as a single lump sum, that the income payments start immediately after the premium payment, and that only the interest portion of the payments is taxable. Our data consist of three types of policies: term annuities, life annuities, and guaranteed annuities. For term annuities, we have quotes for 5- through 30-year maturities (every 5 years in between). For life and guaranteed annuities, we have quotes for males and females between ages 50 and 90 (every 5 years in between).

A term annuity is a policy with annual income payments for a fixed maturity of $M$ years. Because term annuities have a fixed income stream that is independent of survival, they are essentially bonds rather than insurance. An insurance company that issues a term annuity
must buy a portfolio of Treasury bonds to replicate its future cash flows. A portfolio of Aaa
corporate bonds, for example, does not perfectly replicate the cash flows because of default
risk. Therefore, the law of one price implies that the Treasury yield curve is the appropriate
cost of capital for the valuation of term annuities. Let $R_t(m)$ be the zero-coupon Treasury
yield at maturity $m$ and time $t$. We define the actuarial value of an $M$-year term annuity
per dollar of income as

$$V_t(M) = \sum_{m=1}^{M} \frac{1}{R_t(m)^m}. \tag{1}$$

We calculate the actuarial value for term annuities based on the zero-coupon yield curve for
off-the-run Treasury bonds (Gürkaynak, Sack, and Wright, 2007).

A life annuity is a policy with annual income payments until the death of the insured. Let $p_n$ be the one-year survival probability at age $n$, and let $N$ be the maximum attainable age according to the appropriate mortality table. We define the actuarial value of a life annuity at age $n$ per dollar income as

$$V_t(n) = \sum_{m=1}^{N-n} \left( \prod_{l=0}^{m-1} p_{n+l} \right) \frac{1}{R_t(m)^m}. \tag{2}$$

A guaranteed annuity is a variant of the life annuity whose income payments are guaran-
teed to continue for the first $M$ years, even if the insured dies during that period. We define the actuarial value of an $M$-year guaranteed annuity at age $n$ per dollar of income as

$$V_t(n, M) = \sum_{m=1}^{M} \frac{1}{R_t(m)^m} + \sum_{m=M+1}^{N-n} \left( \prod_{l=0}^{m-1} p_{n+l} \right) \frac{1}{R_t(m)^m}. \tag{3}$$

We calculate the actuarial value for life annuities based on the appropriate mortality
table from the American Society of Actuaries and the zero-coupon Treasury yield curve.
We use the 1983 Annuity Mortality Basic Table prior to December 2000, and the 2000 Annuity Mortality Basic Table since December 2000. These mortality tables are derived
from the actual mortality experience of insured pools, based on data provided by various insurance companies. Therefore, they account for adverse selection in annuity markets, that is, an insured pool of annuitants has higher life expectancy than the overall population. We smooth the transition between the two vintages of the mortality tables by geometrically averaging.

1.1.2. Life Insurance Prices

Our life insurance prices are from COMPULIFE Software, which is a computer-based quotation system for insurance brokers. We focus on guaranteed universal life policies, which are quoted for the leading life insurers since January 2005. These policies have constant guaranteed premiums and accumulate no cash value, so they are essentially “permanent” term life policies.¹ We pull quotes for the regular health category at the face amount of $250,000 in California. COMPULIFE recommended California for our study because it is the most populous state with a wide representation of insurance companies. We focus on males and females between ages 30 and 90 (every 10 years in between).

Universal life insurance is a policy that pays out a death benefit upon the death of the insured. The policy is in effect as long as the policyholder makes an annual premium payment while the insured is alive. We define the actuarial value of universal life insurance at age \( n \) per dollar of death benefit as

\[
V_{t}(n) = \left( 1 + \sum_{m=1}^{N-n-1} \prod_{l=0}^{m-1} \frac{p_{n+l}}{R_t(m)^m} \right)^{-1} \left( \sum_{m=1}^{N-n} \prod_{l=0}^{m-2} \frac{p_{n+l}(1 - p_{n+m-1})}{R_t(m)^m} \right).
\]  

(4)

This formula does not take into account the potential lapsation of policies, that is, the policyholder may drop coverage prior to the death of the insured. There is currently no agreed upon standard for lapsation pricing, partly because lapsations are difficult to model.

¹While COMPULIFE has quotes for various types of policies from annual renewable to 30-year term life policies, they are not useful for our purposes. A term life policy typically has a renewal option at the end of the guaranteed term. Because the premiums under the renewal option vary significantly across insurance companies, cross-sectional price comparisons are difficult and imprecise.
and predict. While some insurance companies price in low levels of lapsation, others take the conservative approach of assuming no lapsation in life insurance valuation.

We calculate the actuarial value for life insurance based on the appropriate mortality table from the American Society of Actuaries and the zero-coupon Treasury yield curve. We use the 2001 Valuation Basic Table prior to December 2008, and the 2008 Valuation Basic Table since December 2008. These mortality tables are derived from the actual mortality experience of insured pools, so they account for adverse selection in life insurance markets. We smooth the transition between the two vintages of the mortality tables by geometrically averaging.

1.1.3. Insurance Companies’ Balance Sheets

We obtain balance sheet data and A.M. Best ratings for insurance companies through the Best’s Insurance Reports CD-ROM for fiscal years 1992 through 2010. We merge annuity and life insurance prices to the A.M. Best data by company name. The insurance price observed in January and July of each calendar year is matched to the balance sheet data for the previous fiscal year (i.e., December of the previous calendar year).

1.2. Summary Statistics

We start with a broad overview of the industry that we study. Figure 1 reports the annual premiums collected for individual annuities and life insurance, summed across all insurance companies in the United States with an A.M. Best rating. In the early 1990’s, insurance companies collected nearly $100 billion in annual premiums for individual life insurance and about $50 billion for individual annuities. More recently, the annuity market expanded to $383 billion in 2008. The financial crisis had an adverse impact on annuity demand in 2009, which subsequently recovered in 2010.

Table 1 summarizes our data on annuity and life insurance prices. We have 988 observations on 10-year term annuities across 98 insurance companies, covering January 1989
through July 2011. The average markup, defined as the percent deviation of the quoted price from actuarial value, is 6.9 percent. Since term annuities have a fixed income stream that is independent of survival, we can rule out adverse selection as a source of this markup. Instead, the markup must be attributed to marketing and administrative costs as well as economic profits that may arise from imperfect competition. The fact that the average markup declines in the maturity of the term annuity is consistent with the presence of fixed costs. There is considerable cross-sectional variation in the pricing of 10-year term annuities across insurance companies, as captured by a standard deviation of 5.9 percent (Mitchell, Poterba, Warshawsky, and Brown, 1999).

We have 11,879 observations on life annuities across 106 insurance companies, covering January 1989 through July 2011. The average markup is 9.8 percent with a standard deviation of 8.2 percent. Our data on guaranteed annuities start in July 1989. For 10-year guaranteed annuities, the average markup is 5.5 percent with a standard deviation of 6.1 percent. For 20-year guaranteed annuities, the average markup is 4.2 percent with a standard deviation of 4.8 percent.

We have 3,989 observations on universal life insurance across 52 insurance companies, covering January 2005 through July 2011. The average markup is −4.2 percent with a standard deviation of 17.9 percent. The negative average markup does not mean that insurance companies systematically lose money on these policies. With a constant premium and a rising mortality rate, policyholders are essentially prepaying for coverage later in life. When a universal life policy is lapsed, the insurance company earns a windfall profit because the present value of the remaining premium payments is typically less than the present value of the future death benefit. Since there is currently no agreed upon standard for lapsation pricing, our calculation of actuarial value does not take lapsation into account. We are not especially concerned that the average markup might be slightly mismeasured because the focus of our study is the variation in markups over time and across policies of different maturities.
1.3. **Firesale of Insurance Policies**

Figure 2 reports the time series of the average markup on term annuities at various maturities, averaged across insurance companies and reported with a 95 percent confidence interval. The average markup varies between 0 and 10 percent, with the exception of a period of few months around January 2009. If insurance companies were to change annuity prices to perfectly offset interest-rate movements, the markup would be constant over time. Hence, the variation in average markup implies that insurance companies do not change annuity prices to perfectly offset interest-rate movements (Charupat, Kamstra, and Milevsky, 2012).

For 30-year term annuities, the average markup fell to an extraordinary $-24.8$ percent in January 2009. Much of this large negative markup arises from reductions in the price of 30-year term annuities from July 2007 to January 2009. For example, Allianz Life Insurance Company reduced the price of 30-year term annuities from $18.56 (per dollar of annual income) in July 2007 to $13.75 in January 2009, then raised it back up to $18.23 by July 2009. Such price reductions cannot be explained by interest-rate movements because relatively low Treasury yields implied a relatively high actuarial value for 30-year term annuities in January 2009. Appendix A shows that our finding is robust to making a conservative adjustment to the actuarial value to account for the special status of Treasury bonds in asset markets as they can serve as collateral.

In January 2009, there is a monotonic relation between the maturity of the term annuity and the magnitude of the average markup. The average markup was $-16.1$ percent for 20-year, $-7.7$ percent for 10-year, and $-3.0$ percent for 5-year term annuities. Excluding the extraordinary period around January 2009, average markup was negative for 20- and 30-year term annuities only twice before in our sample, in January 2001 and July 2002.

Figure 3 reports the time series of the average markup on life annuities at various ages. Our findings are similar to that for term annuities. For life annuities at age 50, the average markup fell to an extraordinary $-25.3$ percent in January 2009. There is a monotonic relation between age, which is negatively related to the effective maturity of the life annuity,
and the magnitude of the average markup. The average markup was \(-19.2\) percent at age 60, \(-11.1\) percent at age 70, and \(-3.3\) percent at age 80.

Figure 4 reports the time series of the average markup on universal life insurance at various ages. Our findings are similar to that for term and life annuities. For universal life insurance at age 30, the average markup fell to an extraordinary \(-52.2\) in January 2009. There is a monotonic relation between age and the magnitude of the average markup. The average markup was \(-47.0\) percent at age 40, \(-42.3\) percent at age 50, and \(-29.4\) percent at age 60.

1.4. Default Risk

Since insurance policies are ultimately backed by the state guaranty fund (e.g., up to $250k for annuities and $300k for life insurance in California), the only scenario in which a policyholder would not be fully repaid is if all insurance companies associated with the state guaranty fund were to systemically fail. During the financial crisis, the pricing of annuities and life insurance remained linear around the guaranteed amount, and the pricing was uniform across states with different guaranty provisions. The absence of kinks in pricing around the guaranteed amount rules out idiosyncratic default risk that affects only some insurance companies, but it does not rule out systematic default risk in which the state guaranty fund fails.

Even if we were to entertain an extreme scenario in which the state guaranty fund fails, the magnitude of the discounts in January 2009 are too large to be justified by default risk, given reasonable assumptions about the recovery rate. Since life insurers are subject to risk-based capital requirements, risky assets (e.g., non-investment-grade bonds, common and preferred stocks, non-performing mortgages, and real estate) only account for 16 percent of their assets (Ellul, Jotikasthira, and Lundblad, 2011). The remainder of their assets are in safe asset classes such as cash, Treasury bonds, and investment-grade bonds. Under an extreme assumption that risky assets lose their value entirely, a reasonable lower bound on
the recovery rate is 84 percent. To further justify this recovery rate, the asset deficiency in past cases of insolvency typically ranges from 5 to 10 percent and very rarely exceeds 15 percent (Gallanis, 2009).

Let \( d_t(l) \) be the risk-neutral default probability between year \( l-1 \) and \( l \) at time \( t \), and let \( \theta \in (0, 1) \) be the recovery rate conditional on default. Then the market value of an \( M \)-year term annuity per dollar of income is

\[
V_t(M) = \sum_{m=1}^{M} \frac{\theta + (1 - \theta) \prod_{l=1}^{m} (1 - d_t(l))}{R_t(m)^m}.
\] (5)

Given a recovery rate of 84 percent, an annual default probability of 6.7 percent at the 1- to 5-year horizon justifies a markup of −3.0 percent on 5-year term annuities. An annual default probability of 40.2 percent at the 6- to 10-year horizon justifies a markup of −7.7 percent on 10-year term annuities. This upward-sloping term structure of default probabilities is inconsistent with the notion that the financial crisis is a transitory shock that will dissipate in the long run. There are no default probabilities that can justify the discounts on term annuities with maturity greater than 15 years. This is because equation (5) implies that the discount cannot be greater than 16 percent (i.e., one minus the recovery rate), which is clearly violated for term annuities with maturity greater than 20 years.

In Appendix B, we also find out-of-sample evidence against default risk based on the pricing of life annuities during the Great Depression. The absence of discounts during the Great Depression, when the corporate default spread was even higher than the heights reached during the recent financial crisis, is inconsistent with the hypothesis that default risk drives insurance prices. However, the absence of discounts is consistent with our explanation because the statutory reserve regulation that was in effect back then did not allow insurance companies to record liabilities at less than full reserve.
2. Statutory Reserve Regulation for Life Insurers

When an insurance company sells an annuity or life insurance policy, its assets increase by the purchase price of the policy. At the same time, the insurance company must record statutory reserves on the liability side of its balance sheet to cover future policy claims. In the United States, the amount of required reserves for each type of policy is governed by state law, but all states essentially follow recommended guidelines known as Standard Valuation Law (National Association of Insurance Commissioners, 2011, Appendix A-820). Standard Valuation Law establishes mortality tables and discount rates that are to be used for reserve valuation.

In this section, we review the reserve valuation rules for annuities and life insurance. Because these policies essentially have no exposure to market risk, finance theory implies that the market value of these policies is determined by the term structure of riskless interest rates. However, Standard Valuation Law requires that the reserve value of these policies be calculated using a mechanical discount rate that is a function of the Moody’s composite yield on seasoned corporate bonds. Insurance companies care about the reserve value of policies insofar as it is used by rating agencies and state regulators to determine the adequacy of statutory reserves.2 A rating agency may downgrade an insurance company whose asset value has fallen relative to its statutory reserves. In the extreme case, a state regulator may liquidate an insurance company whose assets are deficient relative to its statutory reserves.

2.1. Term Annuities

Let $y_t$ be the 12-month moving average of the Moody’s composite yield on seasoned corporate bonds, over the period ending on June 30 of the issuance year of the policy. Standard

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2In principle, rating agencies could calculate the market value of liabilities and base their ratings on market leverage. However, their current practice is to take reserve valuation at face value, so that ratings are ultimately based on accounting leverage (A.M. Best Company, 2011, p. 31).
Valuation Law specifies the following discount rate for reserve valuation of annuities:

\[
\hat{R}_t - 1 = 0.03 + 0.8(y_t - 0.03),
\]  

which is rounded to the nearest 25 basis point. This a constant discount rate that is to be applied to all expected future policy claims, regardless of maturity. The exogenous variation in required reserves that this mechanical rule generates, both over time and across policies of different maturities, allows us to identify the shadow cost of financial frictions for life insurers.

Figure 5 reports the time series of the discount rate for annuities, together with the 10-year zero-coupon Treasury yield. The discount rate for annuities has generally declined over the last 20 years as nominal interest rates have fallen. However, the discount rate for annuities has declined more slowly than the 10-year Treasury yield. This means that statutory reserve requirements for annuities have become looser over time because a high discount rate implies low reserve valuation.

The reserve value of an \( M \)-year term annuity per dollar of income is

\[
\hat{V}_t(M) = \sum_{m=1}^{M} \frac{1}{R_t^m}.
\]

Figure 6 reports the ratio of reserve to actuarial value for term annuities (i.e., \( \hat{V}_t(M)/V_t(M) \)) at maturities of 5 to 30 years. Whenever this ratio is equal to one, the insurance company records a dollar of reserve per dollar of future policy claims in present value. Whenever this ratio is greater than one, the reserve valuation is conservative in the sense that the insurance company records reserves that are greater than the present value of future policy claims. Conversely, whenever this ratio is less than one, the reserve valuation is aggressive in the sense that the insurance company records reserves that are less than the present value of future policy claims.

For the 30-year term annuity, the ratio reaches a peak of 1.20 in November 1994 and a
trough of 0.73 in January 2009. If the insurance company were to sell a 30-year term annuity at actuarial value in November 1994, its reserves would increase by $1.20 per dollar of policies sold. This implies a loss of $0.20 in capital surplus funds (i.e., total admitted assets minus total liabilities) per dollar of policies sold. In contrast, if the insurance company were to sell a 30-year term annuity at actuarial value in January 2009, its reserves would only increase by $0.73 per dollar of policies sold. This implies a gain of $0.27 in capital surplus funds per dollar of policies sold.

2.2. Life Annuities

The reserve valuation of life annuities requires mortality tables. The American Society of Actuaries produces two versions of mortality tables, which are called basic and loaded. The loaded tables, which are used for reserve valuation, are conservative versions of the basic tables that underestimate the mortality rates. The loaded tables ensure that insurance companies have adequate reserves, even if actual mortality rates turn out to be lower than those projected by the basic tables. For calculating the reserve value, we use the 1983 Annuity Mortality Table prior to December 2000, and the 2000 Annuity Mortality Table since December 2000.

Let \( \hat{p}_n \) be the one-year survival probability at age \( n \), and let \( N \) be the maximum attainable age according to the appropriate loaded mortality table. The reserve value of a life annuity at age \( n \) per dollar of income is

\[
\hat{V}_i(n) = \sum_{m=1}^{N-n} \frac{\prod_{l=0}^{m-1} \hat{p}_{n+l}}{R_i^m},
\]

where the discount rate is given by equation (6). Similarly, the reserve value of an \( M \)-year guaranteed annuity at age \( n \) per dollar of income is

\[
\hat{V}_i(n, M) = \sum_{m=1}^{M} \frac{1}{R_i^m} + \sum_{m=M+1}^{N-n} \frac{\prod_{l=0}^{m-1} \hat{p}_{n+l}}{R_i^m}.
\]
Figure 6 reports the ratio of reserve to actuarial value for life annuities, 10-year guaranteed annuities, and 20-year guaranteed annuities for males aged 50 to 80 (every 10 years in between). For these life annuities, the time-series variation in the ratio of reserve to actuarial value is quite similar to that for term annuities. In particular, the ratio reaches a peak in November 1994 and a trough in January 2009. Since the reserve valuation of term annuities depends only on the discount rates, the similarity with term annuities implies that discount rates, rather than mortality tables, have a predominant effect on the reserve valuation of life annuities.

2.3. Life Insurance

Let $y_t$ be the minimum of the 12-month and the 36-month moving average of the Moody’s composite yield on seasoned corporate bonds, over the period ending on June 30 of the year prior to issuance of the policy. Standard Valuation Law specifies the following discount rate for reserve valuation of life insurance:

$$\hat{R}_t(M) - 1 = 0.03 + w(M)(\min\{y_t, 0.09\} - 0.03) + 0.5w(M)(\max\{y_t, 0.09\} - 0.09),$$

which is rounded to the nearest 25 basis point. The weighting function for a policy with a guaranteed term of $M$ years is

$$w(M) = \begin{cases} 
0.50 & \text{if } M \leq 10 \\
0.45 & \text{if } 10 < M \leq 20 \\
0.35 & \text{if } M > 20
\end{cases}.$$ 

As with life annuities, the American Society of Actuaries produces basic and loaded mortality tables for life insurance. The loaded tables, which are used for reserve valuation, are conservative versions of the basic tables that overestimate the mortality rates. For calculating the reserve value, we use the 2001 Commissioners Standard Ordinary Mortality
Table. The reserve value of life insurance at age \( n \) per dollar of death benefit is

\[
\hat{V}_t(n) = \left(1 + \sum_{m=1}^{N-n} \frac{\prod_{l=0}^{m-1} \hat{P}_{n+l}}{\hat{R}_t(N - n)^m}\right)^{-1} \left(\sum_{m=1}^{N-n} \frac{\prod_{l=0}^{m-2} \hat{P}_{n+l}(1 - \hat{P}_{n+m-1})}{\hat{R}_t(N - n)^m}\right).
\] (12)

Figure 7 reports the ratio of reserve to actuarial value for universal life insurance for males aged 30 to 60 (every 10 years in between). Our earlier caveat regarding lapsation applies to this figure as well, so that we focus on the variation in the ratio of reserve to actuarial value over time and across policies of different maturities. In a period of few months around January 2009, the reserve value falls significantly relative to actuarial value. As shown in Figure 5, this is caused by the fact that the discount rate for life insurance stays constant during this period, while the 10-year zero-coupon Treasury yield falls significantly. If an insurance company were to sell universal life insurance to a 30-year old male in January 2009, its reserves would only increase by $0.87 per dollar of policies sold. This implies a gain of $0.13 in capital surplus funds per dollar of policies sold.

3. A Structural Model of Insurance Pricing

We now develop a model in which an insurance company sets prices for various types of policies to maximize the present discounted value of profits, subject to a leverage constraint that the ratio of statutory reserves to assets cannot exceed a targeted value. The model shows how financial frictions and statutory reserve regulation jointly determine insurance prices. We show that the model explains the magnitude of the price reductions in January 2009 through estimation in Section 4 and through calibration in Section 5.

3.1. An Insurance Company’s Maximization Problem

An insurance company sells \( I \) different types of annuity and life insurance policies, which we index as \( i = 1, \ldots, I \). These policies are differentiated not only by maturity, but also by sex and age of the insured. The insurance company faces a downward-sloping demand curve
$Q_{i,t}(P)$ for each policy $i$ in period $t$, where $Q'_{i,t}(P) < 0$. There are various micro-foundations that give rise to such a demand curve. For example, such a demand curve can be motivated as an industry equilibrium subject to search frictions (Hortaçsu and Syverson, 2004). We will take the demand curve as exogenously given because the precise micro-foundations are not essential for our purposes.

The insurance company incurs a fixed (marketing and administrative) cost $C_t$ in each period. Let $V_{i,t}$ be the actuarial value of policy $i$ in period $t$. The insurance company’s profit in each period is

$$\Pi_t = \sum_{i=1}^{I} (P_{i,t} - V_{i,t})Q_{i,t} - C_t.$$  

(13)

A simple way to interpret this profit function is that for each type of policy that the insurance company sells for $P_{i,t}$, it can buy a portfolio of Treasury bonds that replicate its expected future policy claims for $V_{i,t}$. For term annuities, this interpretation is exact since future policy claims are deterministic. For life annuities and life insurance, we assume that the insured pools are sufficiently large for the law of large numbers to apply.

We now describe how the sale of new policies affects the insurance company’s balance sheet. Let $A_{t-1}$ be its assets at the beginning of period $t$, and let $R_{A,t}$ be an exogenous rate of return on its assets in period $t$. Its assets at the end of period $t$, after the sale of new policies, is

$$A_t = R_{A,t}A_{t-1} + \sum_{i=1}^{I} P_{i,t}Q_{i,t} - C_t.$$  

(14)

As explained in Section 2, the insurance company must also record reserves on the liability side of its balance sheet. Let $L_{t-1}$ be its statutory reserves at the beginning of period $t$, and let $R_{L,t}$ be the return on its statutory reserves in period $t$. Let $\hat{V}_{i,t}$ be the reserve value of policy $i$ in period $t$. Its statutory reserves at the end of period $t$, after the sale of new policies,
policies, is

\[ L_t = R_{L,t}L_{t-1} + \sum_{i=1}^{I} \hat{V}_{i,t}Q_{i,t}. \]  

(15)

The insurance company chooses the price \( P_{i,t} \) for each type of policy to maximize firm value, or the present discounted value of its profits:

\[ J_t = \Pi_t + E_t[M_{t+1}J_{t+1}], \]  

where \( M_{t+1} \) is the stochastic discount factor. The insurance company faces a leverage constraint on the value of its statutory reserves relative to its assets:

\[ \frac{L_t}{A_t} \leq \phi, \]  

(17)

where \( \phi \leq 1 \) is the maximum leverage ratio. The underlying assumption is that exceeding the maximum leverage ratio leads to bad consequences, such as a rating downgrade or forced liquidation by state regulators.\(^3\) At fiscal year-end 2008, many highly rated insurance companies were concerned that the upward pressure on their leverage ratio would trigger a rating downgrade, which would have an adverse impact on their business.\(^4\)

To simplify notation, we define the insurance company’s excess reserves as

\[ K_t = \phi A_t - L_t. \]  

(18)

The leverage constraint can then be rewritten as

\[ K_t \geq 0. \]  

(19)

\(^3\)An alternative model, with similar implications to the leverage constraint, is that the insurance company faces a convex cost whenever the leverage ratio exceeds \( \phi \).

\(^4\)For example, A.M. Best Company (2009) reports that MetLife’s “financial leverage is at the high end of its threshold for the current rating level” at fiscal year-end 2008.
The law of motion for excess reserves is

\[ K_t = \phi R_{A,t} A_{t-1} - R_{L,t} L_{t-1} + \sum_{i=1}^{I} \left( \phi P_{i,t} - \hat{V}_{i,t} \right) Q_{i,t} - \phi C_t. \]  

(20)

### 3.2. Optimal Insurance Pricing

Let \( \lambda_t \geq 0 \) be the Lagrange multiplier on the leverage constraint (19). The Lagrangian for the insurance company’s maximization problem is

\[ \mathcal{L}_t = J_t + \lambda_t K_t. \]  

(21)

The first-order condition for the price of each type of policy is

\[ \frac{\partial \mathcal{L}_t}{\partial P_{i,t}} = \frac{\partial J_t}{\partial P_{i,t}} + \lambda_t \frac{\partial K_t}{\partial P_{i,t}} \]

\[ = \frac{\partial \Pi_t}{\partial P_{i,t}} + \lambda_t \frac{\partial K_t}{\partial P_{i,t}} \]

\[ = Q_{i,t} + (P_{i,t} - V_{i,t}) Q'_{i,t} + \lambda_t \left[ \phi Q_{i,t} + \left( \phi P_{i,t} - \hat{V}_{i,t} \right) Q'_{i,t} \right] = 0, \]  

(22)

where

\[ \lambda_t = \lambda_t + E_t \left[ M_{t+1} \frac{\partial J_{t+1}}{\partial K_t} \right]. \]  

(23)

Equation (22) implies that

\[ \lambda_t = - \frac{\partial \Pi_t}{\partial K_t}. \]  

(24)

That is, \( \lambda_t \) measures the marginal reduction in profits that the insurance company is willing to accept to raise its excess reserves by a dollar. Equation (23) implies that \( \lambda_t = 0 \) if the leverage constraint does not bind today (i.e., \( \lambda_t = 0 \)), and increasing excess reserves does
not relax future constraints (i.e., $E_t[M_{t+1} \partial J_{t+1}/\partial K_t] = 0$). Therefore, we refer to $\lambda_t$ as the shadow cost of financial frictions because it measures the importance of the leverage constraint, either today or at some future state.

Rearranging equation (22), the price of policy $i$ in period $t$ is

$$P_{i,t} = V_{i,t} \left(1 - \frac{1}{\epsilon_{i,t}}\right)^{-1} \left(1 + \lambda_t \frac{\hat{V}_{i,t}}{V_{i,t}}/1 + \lambda_t \phi\right),$$

(25)

where

$$\epsilon_{i,t} = -\frac{P_{i,t} Q'_{i,t}}{Q_{i,t}} > 1$$

(26)

is the elasticity of demand. If the shadow cost of financial frictions is zero (i.e., $\lambda_t = 0$), the price of policy $i$ in period $t$ is

$$P_{i,t} = V_{i,t} \left(1 - \frac{1}{\epsilon_{i,t}}\right)^{-1}.$$  

(27)

This is the standard Bertrand model of pricing, in which price is equal to marginal cost times a markup that is decreasing in the elasticity of demand.

If the shadow cost of financial frictions is positive (i.e., $\lambda_t > 0$), the price of policy $i$ in period $t$ satisfies the inequality

$$P_{i,t} \geq \overline{P}_{i,t} \text{ if } \frac{\hat{V}_{i,t}}{V_{i,t}} \geq \phi.$$  

(28)

That is, the price of the policy is higher than the Bertrand price if selling the policy tightens the leverage constraint on the margin. This is the case with property and casualty insurers (Froot and O’Connell, 1999), whose statutory reserve regulation requires that $\frac{\hat{V}_{i,t}}{V_{i,t}} > 1$ (American Academy of Actuaries, 2000). Conversely, the price of the policy is lower than the Bertrand price if selling the policy relaxes the leverage constraint on the margin. This
was the case with life insurers in January 2009.

When the leverage constraint binds, equation (25) and the leverage constraint (i.e., \( K_t = 0 \)) forms a system of \( I + 1 \) equations in \( I + 1 \) unknowns (i.e., \( P_{i,t} \) for each policy \( i = 1, \ldots, I \) and \( \lambda_t \)). Solving this system of equations for the shadow cost of financial frictions,

\[
\lambda_t = \frac{1}{\phi} \left( \sum_{i=1}^{I} \left( \phi V_{i,t}(1 - 1/\epsilon_{i,t})^{-1} - \hat{V}_{i,t} \right) Q_{i,t} + \phi R_{A,t} A_{t-1} - R_{L,t} L_{t-1} - \phi C_t \right) - \sum_{i=1}^{I} \hat{V}_{i,t}(\epsilon_{i,t} - 1)^{-1} Q_{i,t} - (\phi R_{A,t} A_{t-1} - R_{L,t} L_{t-1} - \phi C_t) \right). \tag{29}
\]

4. Estimating the Structural Model of Insurance Pricing

In this section, we estimate the shadow cost of financial frictions through the structural model of insurance pricing. Before doing so, we first present reduced-form evidence that is consistent with a key prediction of the model. Namely, the price reductions were larger for those companies that experienced more adverse balance sheet shocks just prior to January 2009, which are presumably the companies for which the leverage constraint was more costly.

4.1. Price Changes versus Balance Sheet Shocks

Figure 8 is an overview of how the balance sheet has evolved over time for the median insurance company in our sample. Assets grew by 3 to 14 percent annually from 1989 through 2010. The only exception to this growth is 2008 when assets shrank by 3 percent. The leverage ratio stays remarkably constant between 0.91 and 0.95 throughout this period, including 2008 when the leverage ratio was 0.93 for the median insurance company.

Figure 9 is a scatter plot of the percent change in annuity prices from July 2007 to January 2009 versus asset growth from fiscal year-end 2007 to 2008. The four panels represent term annuities, life annuities, and 10- and 20-year guaranteed annuities. The dots in each panel represent the insurance companies in our sample in January 2009. The linear regression line shows that there is a strong positive relation between annuity price changes and asset growth. That is, the price reductions were larger for those companies that experienced more
adverse balance sheet shocks just prior to January 2009.

Our joint interpretation of Figures 8 and 9 is that insurance companies were able to maintain a stable leverage ratio in 2008 and 2009 by taking advantage of statutory reserve regulation that allowed them to record far less than a dollar of reserve per dollar of future insurance liability. The incentive to reduce prices was stronger for those companies that experienced more adverse balance sheet shocks and, therefore, had a higher need to recapitalize.

4.2. Empirical Specification

Let $i$ index the type of policy, $j$ index the insurance company, and $t$ index time. Based on pricing equation (25), we model the markup as a nonlinear regression model:

$$
\log \left( \frac{P_{i,j,t}}{V_{i,t}} \right) = - \log \left( 1 - \frac{1}{\epsilon_{i,j,t}} \right) + \log \left( \frac{1 + \lambda_{i,j,t} \hat{V}_{i,t}/V_{i,t}}{1 + \lambda_{j,t} L_{j,t}/A_{j,t}} \right) + e_{i,j,t},$$

(30)

where $e_{i,j,t}$ is an error term with conditional mean zero.

We model the elasticity of demand as

$$\epsilon_{i,j,t} = 1 + \exp\{-\beta' y_{i,j,t}\},$$

(31)

where $y_{i,j,t}$ is a vector of policy and insurance company characteristics. In our baseline specification, the policy characteristics are sex and age. The insurance company characteristics are the A.M. Best rating, the leverage ratio, asset growth, and log assets. We also include a full set of time dummies to control for any variation in the elasticity of demand over the business cycle. We interact each of these variables, including the time dummies, with dummy variables that allow their impact on the elasticity of demand to differ across term annuities, life annuities, and life insurance.

In theory, the shadow cost of financial frictions depends only on insurance company characteristics that appear in equation (29). However, most of these characteristics do not
have obvious counterparts in the data except for $\phi$, which is equal to the leverage ratio when the constraint binds (i.e., $\phi = L_t/A_t$). Therefore, we model the shadow cost of financial frictions as

$$
\overline{\lambda}_{j,t} = \exp\{-\gamma'z_{j,t}\},
$$

where $z_{j,t}$ is a vector of insurance company characteristics. In our baseline specification, the insurance company characteristics are the leverage ratio and asset growth. Our use of asset growth is motivated by the reduced-form evidence in Figure 8. We also include a full set of time dummies and their interaction with insurance company characteristics to allow for the fact that the leverage constraint may only bind at certain times.

4.3. Identifying Assumptions

If the elasticity of demand is correctly specified, the regression model (30) is identified by the fact that the markup has a nonnegative conditional mean in the absence of financial frictions (i.e., $-\log(1 - 1/\epsilon_{i,j,t}) > 0$). Therefore, a negative markup must be explained by a positive shadow cost of financial frictions whenever the ratio of reserve to actuarial value is less than the leverage ratio (i.e., $\hat{V}_{i,t}/V_{i,t} < L_{j,t}/A_{j,t}$).

Even if the elasticity of demand is potentially misspecified, the shadow cost of financial frictions is identified by exogenous variation in the ratio of reserve to actuarial value across different types of policies. To illustrate this point, we approximate the regression model (30) through first-order Taylor approximation as

$$
\log\left(\frac{P_{i,j,t}}{V_{i,t}}\right) \approx \alpha_{j,t} + \frac{1}{1/\overline{\lambda}_{j,t} + L_{j,t}/A_{j,t}} \left(\frac{\hat{V}_{i,t}}{V_{i,t}} - \frac{L_{j,t}}{A_{j,t}}\right) + v_{i,j,t},
$$

where

$$
v_{i,j,t} = -\alpha_{j,t} - \log\left(1 - \frac{1}{\epsilon_{i,j,t}}\right) + e_{i,j,t}
$$
is an error term with conditional mean zero. For a given insurance company $j$ at a given time $t$, the regression coefficient $\lambda_{j,t}$ is identified as long as $\hat{V}_{i,t}/V_{i,t}$ is orthogonal to $v_{i,j,t}$.

More intuitively, Standard Valuation Law generates relative shifts in the supply curve across different types of policies that an insurance company sells, which we exploit to identify the shadow cost of financial frictions.

### 4.4. Estimating the Shadow Cost of Financial Frictions

Since the data for most types of annuities are not available prior to July 1998, we estimate the structural model on the sub-sample from July 1998 through July 2011. Table 2 reports our estimates for the elasticity of demand in the regression model (30). Instead of reporting the raw coefficients (i.e., $\beta$), we report the average marginal effect of the explanatory variables on the markup. The average markup on policies sold by A or A− rated insurance companies is 3.13 percentage points higher than that for policies sold by A++ or A+ rated companies. The leverage ratio and asset growth have a relatively small economic impact on the markup through the elasticity of demand. Every 1 percentage point increase in the leverage ratio is associated with a 6 basis point increase in the markup. Every 1 percentage point increase in asset growth is associated with a 4 basis point increase in the markup.

Figure 10 reports the time series of the shadow cost of financial frictions for the average insurance company (i.e., at the conditional mean of the leverage ratio and asset growth). The leverage constraint is not costly for most of the sample period. There is evidence that the leverage constraint was costly around January 2001 with a point estimate of $0.79$ per dollar of excess reserve. The leverage constraint was clearly costly in January 2009 with a point estimate of $4.58$ per dollar of excess reserve. That is, the average insurance company was willing to accept a marginal reduction of $4.58$ in profits to raise its excess reserves by a dollar. The 95 percent confidence interval ranges from $2.78$ to $6.39$ per dollar of excess reserve.

In Table 3, we report the shadow cost of financial frictions for the cross section of insurance
companies in our sample that sold annuities in January 2009. The table shows that there is considerable heterogeneity in the shadow cost of financial frictions. The shadow cost of financial frictions is positively related to the leverage ratio and negatively related to asset growth. In January 2009, MetLife was the most constrained insurance company with a shadow cost of $13.38 per dollar of excess reserve. Metlife had a relatively high leverage ratio of 0.97 at fiscal year-end 2008 and suffered a balance sheet loss of 10 percent from fiscal year-end 2007 to 2008. American General was the least constrained insurance company with a shadow cost of $1.41 per dollar of excess reserve, which is explained by the bailout of its holding company as we discuss below.

4.5. Conventional Channels of Recapitalization

Since insurance companies cannot issue public debt or equity, they essentially have three channels of raising capital surplus funds (i.e., accounting equity). The first, which we emphasize in this paper, is the sale of new policies at a price above reserve value, which generates accounting profits. The second is a direct capital injection from its holding company (that can issue public debt or equity), through the issuance of surplus notes or the reduction of stockholder dividends. The third is the reduction of required risk-based capital by shifting to safer assets with lower risk charges, such as cash and short-term investments (A.M. Best Company, 2004). We now provide evidence that these three channels were complementary during the financial crisis.

For the same set of insurance companies as Table 3, the left panel of Figure 11 reports the inflow of capital surplus funds for fiscal years 2008 and 2009, as a percentage of capital surplus funds at fiscal year-end 2007. The linear regression line shows that there is a strong positive relation between the inflow of capital surplus funds and the shadow cost of financial frictions in January 2009. In particular, MetLife had both the highest inflow of capital surplus funds (224 percent) and the highest shadow cost ($13.38 per dollar of excess reserve). American General is an outlier in Figure 11 with a relatively high inflow of capital surplus funds
(158 percent), despite having the lowest shadow cost ($1.41 per dollar of excess reserves). This is explained by the bailout of its holding company, American International Group, in September 2008.

For the same set of insurance companies as Table 3, the right panel of Figure 11 reports the change in cash and short-term investments in fiscal years 2008 and 2009, as a percentage of capital surplus funds at fiscal year-end 2007. The linear regression line shows that there is a strong positive relation between the change in cash and short-term investments and the shadow cost of financial frictions in January 2009. In particular, MetLife had both the highest increase in cash and short-term investments (142 percent) and the highest shadow cost ($13.38 per dollar of excess reserve).

The picture that emerges from Figure 11 is that those companies that were financially constrained received capital injections from their holding company (Berry-Stötzle, Nini, and Wende, 2011) and reduced their required risk-based capital. However, these conventional channels of recapitalization were insufficient at the height of the financial crisis, and insurance companies had to raise additional capital through a firesale of policies.

5. Calibrating the Structural Model of Insurance Pricing

In this section, we calibrate the structural model of insurance pricing and solve explicitly for the insurance company’s policy and value functions. Compared to the estimation in the last section, the advantage of this approach is that we gain additional insight into how optimal insurance pricing is related to firm value and the shadow cost of financial frictions. The disadvantage is that we must make additional parametric assumptions regarding asset returns, demand, and fixed costs. The two approaches provide complementary evidence that the model explains the magnitude of the price reductions in January 2009.
5.1. Additional Parametric Assumptions

Our goal is to calibrate and solve the simplest version of the model in Section 3 that captures the essence of our empirical findings. Therefore, we start with the version in which the insurance company sells only one type of policy. We assume that the return on assets and statutory reserves are constant and equal to the riskless interest rate (i.e., \( R_{A,t} = R_{L,t} = R \)). The stochastic discount factor is constant and equal to the inverse of the riskless interest rate (i.e., \( M_t = 1/R \)). We assume that both the reserve and the actuarial value of the policy are constant and denote them as \( \hat{V} \) and \( V \), respectively. Under these assumptions, the law of motion for excess reserves (20) simplifies to

\[
K_t = RK_{t-1} + \left( \phi P_t - \hat{V} \right) Q_t - \phi C_t.
\]  

(35)

We parameterize demand as

\[
Q_t = X_t P_t^{-\epsilon},
\]

(36)

where \( \epsilon > 1 \) is a constant elasticity of demand. The demand shock follows a geometric random walk:

\[
\Delta X_t = \frac{X_t}{X_{t-1}} = \exp \left\{ ut - \frac{\sigma^2}{2} \right\},
\]

(37)

where \( u_t \sim N(0, \sigma^2) \). Finally, we parameterize the fixed cost as

\[
C_t = cX_t^{\omega}X_{t-1}^{1-\omega}V^{1-\epsilon}.
\]

(38)

The parameter \( c \in [0,1) \) determines the size of the fixed cost, and \( \omega \geq 1 \) determines its sensitivity to demand shocks. The presence of fixed costs creates operating leverage, which causes the leverage constraint to bind for sufficiently adverse demand shocks.
We calibrate the parameters of the model, summarized in Table 4, to explain the pricing of 30-year term annuities in January 2009. As reported in Figure 7, the ratio of reserve to actuarial value for 30-year term annuities was 0.73 in January 2009. A riskless interest rate of 0.5 percent is based on the 1-year nominal Treasury yield in January 2009. An elasticity of demand of 11 generates a realistic markup of 10 percent when the leverage constraint does not bind. A standard deviation of 30 percent for demand shocks is based on the standard deviation of the growth rate for annual premiums on individual annuities in Figure 1. We calibrate the size of the fixed cost to match the typical general expense ratio (excluding commissions) of 2 percent for individual annuities. As explained in Appendix C, we calibrate the sensitivity of the fixed cost to demand shocks so that the leverage constraint binds for sufficiently adverse demand shocks. The maximum leverage ratio is 0.97 to match the highest leverage ratio for the cross section of insurance companies in Table 3.

5.2. Optimal Insurance Price and Firm Value

Figure 12 reports the optimal insurance price, firm value, and the shadow cost of financial frictions as functions of initial excess reserves. The figure is shown for a $−3.70$ standard deviation demand shock, which is sufficiently adverse for the leverage constraint to bind even with positive initial excess reserves. The leverage constraint does not bind when initial excess reserves are greater than 44 percent of firm value. In this region of the state space, the insurance company sells its policies at a markup of 10 percent, and its firm value is $100$. In the region of the state space where the leverage constraint binds, both the optimal insurance price and firm value are decreasing in initial excess reserves.

When initial excess reserves are 4 percent of firm value, the insurance company sells its policies at a markup of $−12$ percent, and its firm value is $58$. Put differently, the insurance price falls by 22 percent, and the firm value falls by 42 percent relative to when the leverage constraint does not bind. The shadow cost is $4.71$ per dollar of excess reserve. When we decompose this shadow cost through equation (23), the impact of future constraints (i.e.,
$E_t[M_{t+1}\partial J_{t+1}/\partial K_t]$ only accounts for $0.01$ of the shadow cost. These magnitudes in the calibrated model are consistent with our empirical findings. Namely, 30-year term annuities sold at a markup of $-25$ percent, and the shadow cost was $4.58$ per dollar of excess reserve for the average insurance company in January 2009.

5.3. Welfare Cost of Deviations from Actuarially Fair Pricing

Since Treasury bonds and bond mutual funds are close substitutes for 30-year term annuities, we can interpret deviations from actuarially fair pricing (i.e., zero markup) as a deadweight loss in consumer financial markets. Following the standard approach in microeconomics, we can quantify this deadweight loss by computing the area between the demand curve and the marginal cost curve. In the context of our calibrated model, the welfare cost of deviations from actuarially fair pricing, as a share of sales under actuarially fair pricing, is

$$\int_{V}^{P_t} \frac{(Q_t(P) - Q_t(P_t))dP}{VQ_t(V)} = \frac{1}{1 - \epsilon} \left[ \left( \frac{P_t}{V} \right)^{1-\epsilon} - 1 \right] - \left( \frac{P_t}{V} - 1 \right) \left( \frac{P_t}{V} \right)^{-\epsilon}.$$  \hspace{1cm} (39)

Figure 13 reports the welfare cost of deviations from actuarially fair pricing. When the markup is positive, deadweight loss arises from the fact that the consumer surplus lost exceeds the profits gained by the insurance company, represented by the green shaded area in the left panel. When the markup is 10 percent, the welfare cost is equivalent to 3 percent of sales under actuarially fair pricing, represented by the green dot in the right panel. When the markup is negative, deadweight loss arises from the fact that the profits lost by the insurance company exceeds the consumer surplus gained, represented by the red shaded area in the left panel. When the markup is $-12$ percent, the welfare cost is equivalent to 25 percent of sales under actuarially fair pricing, represented by the red dot in the right panel. Our calibrated model suggests that the welfare cost of statutory reserve regulation, which effectively subsidized insurance policies in January 2009, was an order of magnitude larger than the welfare cost that arises from markup pricing in ordinary times.
Our finding suggests a simple modification to statutory reserve regulation that would stabilize prices and improve social welfare. Suppose the required reserve were equal to the targeted leverage ratio times the actuarial value (i.e., $\hat{V}_{i,t} = \phi V_{i,t}$). Then pricing equation (25) implies that the insurance price would always be the Bertrand price (27), even when the leverage constraint binds. Although this simple reserve rule may not be socially optimal in a fully specified model, we think it is a good starting point for discussions of optimal regulation.

6. Conclusion

This paper shows that financial frictions and statutory reserve regulation have a large and measurable impact on insurance prices and social welfare. More broadly, we show that frictions on the supply side have a large and measurable impact on consumer financial markets. The previous literature on household finance has mostly focused on frictions on the demand side of these markets, such as household borrowing constraints, asymmetric information, moral hazard, and near rationality. While these frictions on the demand side are undoubtedly important, we feel that financial and regulatory frictions on the supply side are also important for our understanding of market equilibrium and social welfare.

Another broader implication of our study is that we provide micro evidence for a class of macro models based on financial frictions, which is a leading explanation for the Great Recession (see Gertler and Kiyotaki, 2010; Brunnermeier, Eisenbach, and Sannikov, 2012, for recent surveys of the literature). We feel that this literature would benefit from additional micro evidence on the cost of these frictions for other types of financial institutions, such as commercial banks and health insurance companies. The empirical approach in this paper may be extended to estimate the shadow cost of financial frictions for other types of financial institutions.
References


Table 1: Summary Statistics for Annuity and Life Insurance Prices

The markup is defined as the percent deviation of the quoted price from actuarial value. The actuarial value is based on the appropriate basic mortality table from the American Society of Actuaries and the zero-coupon Treasury yield curve. The sample is semiannual from January 1989 through July 2011.

<table>
<thead>
<tr>
<th>Type of policy</th>
<th>Sample starts in</th>
<th>Observations</th>
<th>Insurance companies</th>
<th>Mean</th>
<th>Median</th>
<th>Standard deviation</th>
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</thead>
<tbody>
<tr>
<td>Term annuities:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5-year</td>
<td>January 1993</td>
<td>732</td>
<td>83</td>
<td>6.7</td>
<td>6.5</td>
<td>8.4</td>
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<tr>
<td>10-year</td>
<td>January 1989</td>
<td>988</td>
<td>98</td>
<td>6.9</td>
<td>7.0</td>
<td>5.9</td>
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<tr>
<td>15-year</td>
<td>July 1998</td>
<td>418</td>
<td>62</td>
<td>4.3</td>
<td>4.8</td>
<td>5.6</td>
</tr>
<tr>
<td>20-year</td>
<td>July 1998</td>
<td>414</td>
<td>62</td>
<td>3.8</td>
<td>4.4</td>
<td>6.6</td>
</tr>
<tr>
<td>25-year</td>
<td>July 1998</td>
<td>339</td>
<td>53</td>
<td>3.4</td>
<td>3.7</td>
<td>7.5</td>
</tr>
<tr>
<td>30-year</td>
<td>July 1998</td>
<td>325</td>
<td>50</td>
<td>2.9</td>
<td>2.8</td>
<td>8.8</td>
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<tr>
<td>Life annuities:</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>Life only</td>
<td>January 1989</td>
<td>11,879</td>
<td>106</td>
<td>9.8</td>
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<td>10-year guaranteed</td>
<td>July 1998</td>
<td>7,885</td>
<td>66</td>
<td>5.5</td>
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<td>66</td>
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<td>Universal life insurance</td>
<td>January 2005</td>
<td>3,989</td>
<td>52</td>
<td>-4.2</td>
<td>-5.5</td>
<td>17.9</td>
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</table>
Table 2: Estimated Model of Insurance Pricing

This table reports the average marginal effect of the explanatory variables on the markup through the elasticity of demand in percentage points. The model for the elasticity of demand also includes time dummies and its interaction effects for life annuities and life insurance, which are omitted in this table for brevity. The omitted categories for the dummy variables are term annuities, A++ or A+ rated, male, and age 50. The t-statistics, reported in parentheses, are based on robust standard errors clustered by insurance company, type of policy, sex, and age. The sample is semiannual from July 1998 through July 2011.

<table>
<thead>
<tr>
<th>Explanatory variable</th>
<th>Average marginal effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rating: A to A-</td>
<td>3.13 (17.69)</td>
</tr>
<tr>
<td>Rating: B++ to B-</td>
<td>9.16 (13.28)</td>
</tr>
<tr>
<td>Leverage ratio</td>
<td>6.13 (23.55)</td>
</tr>
<tr>
<td>Asset growth</td>
<td>3.91 (15.08)</td>
</tr>
<tr>
<td>Log assets</td>
<td>2.31 (40.53)</td>
</tr>
<tr>
<td>Interaction effects for life annuities:</td>
<td></td>
</tr>
<tr>
<td>Rating: A to A-</td>
<td>-2.26 (-17.94)</td>
</tr>
<tr>
<td>Rating: B++ to B-</td>
<td>-8.77 (-11.02)</td>
</tr>
<tr>
<td>Leverage ratio</td>
<td>16.88 (26.27)</td>
</tr>
<tr>
<td>Asset growth</td>
<td>-5.58 (-19.58)</td>
</tr>
<tr>
<td>Log assets</td>
<td>-1.89 (-44.69)</td>
</tr>
<tr>
<td>Female</td>
<td>0.27 (10.18)</td>
</tr>
<tr>
<td>Age 55</td>
<td>0.25 (1.74)</td>
</tr>
<tr>
<td>Age 60</td>
<td>0.60 (3.90)</td>
</tr>
<tr>
<td>Age 65</td>
<td>0.83 (11.94)</td>
</tr>
<tr>
<td>Age 70</td>
<td>1.14 (10.25)</td>
</tr>
<tr>
<td>Age 75</td>
<td>1.45 (2.99)</td>
</tr>
<tr>
<td>Age 80</td>
<td>1.80 (10.60)</td>
</tr>
<tr>
<td>Age 85</td>
<td>2.36 (10.41)</td>
</tr>
<tr>
<td>Age 90</td>
<td>3.28 (6.58)</td>
</tr>
<tr>
<td>Interaction effects for life insurance:</td>
<td></td>
</tr>
<tr>
<td>Rating: A to A-</td>
<td>-23.21 (-5.12)</td>
</tr>
<tr>
<td>Leverage ratio</td>
<td>21.78 (3.02)</td>
</tr>
<tr>
<td>Asset growth</td>
<td>-30.05 (-5.27)</td>
</tr>
<tr>
<td>Log assets</td>
<td>-13.21 (-7.36)</td>
</tr>
<tr>
<td>Female</td>
<td>0.18 (1.03)</td>
</tr>
<tr>
<td>Age 30</td>
<td>2.38 (0.21)</td>
</tr>
<tr>
<td>Age 40</td>
<td>0.62 (0.03)</td>
</tr>
<tr>
<td>Age 60</td>
<td>0.18 (0.00)</td>
</tr>
<tr>
<td>Age 70</td>
<td>0.64 (0.31)</td>
</tr>
<tr>
<td>Age 80</td>
<td>0.65 (0.27)</td>
</tr>
<tr>
<td>Age 90</td>
<td>24.12 (4.74)</td>
</tr>
<tr>
<td>$R^2$ (percent)</td>
<td>48.51</td>
</tr>
<tr>
<td>Observations</td>
<td>29,570</td>
</tr>
</tbody>
</table>
Table 3: Shadow Cost of Financial Frictions in January 2009

This table reports the shadow cost of financial frictions for the cross section of insurance companies in our sample that sold annuities in January 2009, implied by the estimated model of insurance pricing. The leverage ratio is at fiscal year-end 2008. The growth in total admitted assets is from fiscal year-end 2007 to 2008.

<table>
<thead>
<tr>
<th>Insurance company</th>
<th>A.M. Best rating</th>
<th>Leverage growth</th>
<th>Asset growth</th>
<th>Shadow cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>MetLife Investors USA Insurance Company</td>
<td>A+</td>
<td>0.97</td>
<td>-0.10</td>
<td>13.38</td>
</tr>
<tr>
<td>Allianz Life Insurance Company of North America</td>
<td>A</td>
<td>0.97</td>
<td>-0.03</td>
<td>10.47</td>
</tr>
<tr>
<td>Lincoln Benefit Life Company</td>
<td>A+</td>
<td>0.87</td>
<td>-0.45</td>
<td>8.76</td>
</tr>
<tr>
<td>OM Financial Life Insurance Company</td>
<td>A-</td>
<td>0.95</td>
<td>-0.04</td>
<td>8.31</td>
</tr>
<tr>
<td>Aviva Life and Annuity Company</td>
<td>A</td>
<td>0.95</td>
<td>0.12</td>
<td>4.44</td>
</tr>
<tr>
<td>Presidential Life Insurance Company</td>
<td>B+</td>
<td>0.91</td>
<td>-0.06</td>
<td>4.33</td>
</tr>
<tr>
<td>EquiTrust Life Insurance Company</td>
<td>B+</td>
<td>0.95</td>
<td>0.13</td>
<td>4.12</td>
</tr>
<tr>
<td>Integrity Life Insurance Company</td>
<td>A+</td>
<td>0.92</td>
<td>0.03</td>
<td>3.85</td>
</tr>
<tr>
<td>United of Omaha Life Insurance Company</td>
<td>A+</td>
<td>0.91</td>
<td>-0.03</td>
<td>3.65</td>
</tr>
<tr>
<td>Genworth Life Insurance Company</td>
<td>A</td>
<td>0.90</td>
<td>0.00</td>
<td>3.13</td>
</tr>
<tr>
<td>North American Company for Life and Health Insurance</td>
<td>A+</td>
<td>0.94</td>
<td>0.24</td>
<td>2.44</td>
</tr>
<tr>
<td>American National Insurance Company</td>
<td>A</td>
<td>0.87</td>
<td>-0.02</td>
<td>1.84</td>
</tr>
<tr>
<td>American General Life Insurance Company</td>
<td>A</td>
<td>0.87</td>
<td>0.05</td>
<td>1.41</td>
</tr>
</tbody>
</table>
Table 4: Parameters in the Calibrated Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Riskless interest rate</td>
<td>$R - 1$</td>
<td>0.5%</td>
</tr>
<tr>
<td>Ratio of reserve to actuarial value</td>
<td>$\tilde{V}/V$</td>
<td>0.73</td>
</tr>
<tr>
<td>Elasticity of demand</td>
<td>$\epsilon$</td>
<td>11</td>
</tr>
<tr>
<td>Standard deviation of demand shocks</td>
<td>$\sigma$</td>
<td>30%</td>
</tr>
<tr>
<td>Size of the fixed cost</td>
<td>$c$</td>
<td>2%</td>
</tr>
<tr>
<td>Sensitivity of the fixed cost to demand shocks</td>
<td>$\omega$</td>
<td>2.91</td>
</tr>
<tr>
<td>Maximum leverage ratio</td>
<td>$\phi$</td>
<td>0.97</td>
</tr>
</tbody>
</table>
Figure 1: Annual Premiums for Individual Annuities and Life Insurance

This figure reports the total annual premiums collected for individual annuities and life insurance, summed across all insurance companies in the *Best’s Insurance Reports*. The sample is from fiscal year 1992 through 2010.
The markup is defined as the percent deviation of the quoted price from actuarial value. The actuarial value is based on the zero-coupon Treasury yield curve. The average markup is estimated from a regression of markups onto dummy variables for A.M. Best rating and time. The conditional mean for policies sold by A++ and A+ rated companies is reported. The confidence interval is based on robust standard errors clustered by insurance company. The sample is semianual from January 1989 through July 2011.
Figure 3: Average Markup on Life Annuities

The markup is defined as the percent deviation of the quoted price from actuarial value. The actuarial value is based on the appropriate basic mortality table from the American Society of Actuaries and the zero-coupon Treasury yield curve. The average markup is estimated from a regression of markups onto dummy variables for A.M. Best rating, sex, and time. The conditional mean for policies sold to males by A++ and A+ rated companies is reported. The confidence interval is based on robust standard errors clustered by insurance company, sex, and age. The sample is semiannual from January 1989 through July 2011.
The markup is defined as the percent deviation of the quoted price from actuarial value. The actuarial value is based on the appropriate basic mortality table from the American Society of Actuaries and the zero-coupon Treasury yield curve. The average markup is estimated from a regression of markups onto dummy variables for A.M. Best rating, sex, and time. The conditional mean for policies sold to males by A++ and A+ rated companies is reported. The confidence interval is based on robust standard errors clustered by insurance company, sex, and age. The sample is semiannual from January 2004 through July 2011.
Figure 5: Discount Rates for Annuities and Life Insurance
This figure reports the discount rates used for statutory reserve valuation of annuities and life insurance (with guaranteed term greater than 20 years), together with the 10-year zero-coupon Treasury yield. The sample is monthly from January 1989 through July 2011.
Figure 6: Reserve to Actuarial Value for Annuities
The reserve value is based on the appropriate loaded mortality table from the American Society of Actuaries and the discount rate specified by Standard Valuation Law. The actuarial value is based on the appropriate basic mortality table from the American Society of Actuaries and the zero-coupon Treasury yield curve. The sample is monthly from January 1989 through July 2011.
Figure 7: Reserve to Actuarial Value for Universal Life Insurance
The reserve value is based on the appropriate loaded mortality table from the American Society of Actuaries and the discount rate specified by Standard Valuation Law. The actuarial value is based on the appropriate basic mortality table from the American Society of Actuaries and the zero-coupon Treasury yield curve. The sample is monthly from January 2005 through July 2011.
Figure 8: Asset Growth and the Leverage Ratio for Life Insurers
This figure reports the growth rate of total admitted assets and the leverage ratio for the median insurance company in our sample. The leverage ratio is the ratio of total liabilities to total admitted assets. The sample is from fiscal year 1989 through 2010.
Figure 9: Price Change versus Asset Growth in January 2009
The percent change in annuity prices is from July 2007 to January 2009. The growth in total admitted assets is from fiscal year-end 2007 to 2008. For term annuities, the average price change is estimated from a regression of the price change onto dummy variables for insurance company and maturity. The conditional mean for 30-year term policies is reported. For life annuities, the average price change is estimated from a regression of the price change onto dummy variables for insurance company, sex, and age. The conditional mean for policies sold to males aged 50 is reported. The linear regression line weights the observations by annual premiums collected for individual annuities in fiscal year 2008.
Figure 10: Shadow Cost of Financial Frictions
This figure reports the shadow cost of financial frictions for the average insurance company, implied by the estimated model of insurance pricing. The confidence interval is based on robust standard errors clustered by insurance company, type of policy, sex, and age. The sample is semiannual from July 1998 through July 2011.
Figure 11: Conventional Channels of Recapitalization in 2008–2009
The inflow of capital surplus funds and the change in cash and short-term investments for fiscal years 2008 and 2009 are reported as a percentage of capital surplus funds at fiscal year-end 2007. The inflow of capital surplus funds includes the issuance of surplus notes and the reduction stockholder dividends. The shadow cost of financial frictions in January 2009 is for the same set of insurance companies as Table 3.
This figure reports the optimal insurance price (i.e., $P_t/V - 1$), firm value (i.e., $J_t$), and the shadow cost of financial frictions (i.e., $\lambda_t$) as functions of initial excess reserves (i.e., $K_{t-1}$) when the realized demand shock is $-3.70$ standard deviations. Initial excess reserves are normalized by the firm value corresponding to the highest value of initial excess reserves. Firm value is normalized to $100$ at the highest value of initial excess reserves. Table 4 reports the parameters of the calibrated model.
Figure 13: Welfare Cost of Deviations from Actuarially Fair Pricing

This figure reports the welfare cost of deviations from actuarially fair pricing, as a percentage of sales under actuarially fair pricing. The consumer surplus lost, net of profits gained by the insurance company, is represented by the green shaded area in the left panel and the green dot in the right panel. The profits lost by the insurance company, net of consumer surplus gained, is represented by the red shaded area in the left panel and the red dot in the right panel. Table 4 reports the parameters of the calibrated model.
Appendix

A. Special Status of Treasury Bonds

The low Treasury yields in January 2009 may reflect to some extent the special status of Treasury bonds as they can serve as collateral. To understand the magnitude of this effect, we calculate the actuarial value based on the Aaa corporate yield curve, instead of the zero-coupon Treasury yield curve. Unfortunately, the yield spread between Aaa corporate and Treasury bonds not only measures the special status of Treasury bonds, but also the difference in credit risk. The latter may be significant during the financial crisis. Krishnamurthy and Vissing-Jørgensen (2012) estimate that 64 percent of the yield spread between Aaa corporate and Treasury bonds can be attributed to the special status of Treasury bonds, and the remainder to credit risk. As a conservative upper bound, we assume that the entire yield spread is due to the special status of Treasury bonds in the following calculations.

Table A1 shows that the Aaa corporate yield curve reduces the implied discounts on term annuities in January 2009. However, this comes at the cost of generating puzzlingly high markups in July 2007. The average markup on 30-year term annuities under the Aaa corporate yield curve is 10.6 percent in July 2007 and −8.2 percent in January 2009. This 18.8 percent reduction in the average markup during the financial crisis is substantial, but smaller than the 29.6 percent reduction implied by the Treasury yield curve. We conclude that life insurers sold long-term policies at deep discounts in January 2009, even under this conservative upper bound that accounts for the special status of Treasury bonds.

B. Life Annuities during the Great Depression

Following Warshawsky (1988), our prices on life annuities from 1929 through 1938 are from annual editions of The Handy Guide (The Spectator Company, 1929). We focus on quotes for males between ages 50 and 80 (every 10 years in between). We match the quoted price for each year of The Handy Guide to the actuarial value in January of that year. We
calculate the actuarial value based on the annuitant mortality table from M’Clintock (1899) and the zero-coupon Treasury yield curve. We derive the zero-coupon yield curve based on the constant-maturity yield curve in Cecchetti (1988).

Figure B1 reports the time series of the average markup on life annuities at various ages, averaged across insurance companies and reported with a 95 percent confidence interval. The key finding is that the markup remained positive throughout the Great Depression. In particular, the average markup for life annuities at age 50 was 28 percent in 1932, when the corporate default spread was even higher than the heights reached during the recent financial crisis.

Prior to the adoption of Standard Valuation Law in the mid-1940’s, individual states had their own standards for reserve valuation. However, many states used the annuitant mortality table from M’Clintock (1899) and a constant discount rate for reserve valuation (e.g., 3.5 percent in California). Figure B2 reports the ratio of reserve to actuarial value for life annuities for males aged 50 to 80 (every 10 years in between) at the discount rate of 3.5 percent. The ratio of reserve to actuarial value remained close to or above one throughout the Great Depression. This implies that insurance companies could not lower their leverage ratio by selling life annuities at a price below actuarial value, which is consistent with the absence of discounts in Figure B1.

C. Solving the Model by Dynamic Programming

Because demand follows a geometric random walk, we must scale both the value function and excess reserves by market size to make the model stationary. We rewrite the value function as

\[ j_t = \frac{J_t + C_t}{X_t V^{1-\epsilon}} = j(k_t) = \left( \frac{P_t}{V} - 1 \right) \left( \frac{P_t}{V} \right)^{-\epsilon} + \frac{1}{R} E_t \left[ \Delta X_{t+1} \left( j_{t+1} - \frac{c}{\Delta X_{t+1}} \right) \right]. \]  

(C1)
We rewrite the law of motion for excess reserves as

\[ k_{t+1} = \frac{RK_t - \phi C_{t+1}}{X_{t+1}V^{1-\epsilon}} = \frac{R}{\Delta X_{t+1}} \left[ k_t + \left( \phi \frac{P_t}{\hat{V}} - \frac{\hat{V}}{V} \right) \left( \frac{P_t}{V} \right)^{-\epsilon} \right] - \frac{\phi c}{\Delta X_{t+1}^\omega}. \]  

(C2)

We rewrite the leverage constraint as

\[ k_t + \left( \phi \frac{P_t}{\hat{V}} - \frac{\hat{V}}{V} \right) \left( \frac{P_t}{V} \right)^{-\epsilon} \geq 0. \]  

(C3)

The insurance company chooses \( P_t \) to maximize firm value (C1) subject to the law of motion for excess reserves (C2) and the leverage constraint (C3).

We discretize the state space into 50 grid points, which we denote as \( \{k_s\}_{s=1}^S \). We also discretize the demand shock into 7 grid points by Gauss-Hermite quadrature, which we denote as \( \{\Delta X_n\}_{n=1}^N \). Equation (25) implies that \( P_t \geq (\hat{V}/\phi)(1 - 1/\epsilon)^{-1} \) when \( \hat{V}/V < \phi \). Therefore, the leverage constraint (C3) can be satisfied as long as initial excess reserves, prior to the sale of new policies, satisfies

\[ k_t \geq k_1 = -\frac{\phi}{\epsilon} \left( \frac{\hat{V}}{V} \right)^{1-\epsilon} \left( 1 - \frac{1}{\epsilon} \right)^{-1}. \]  

(C4)

Equations (C2) and (C3) imply that \( k_{t+1} \geq k_1 \) if \( -\phi c/\Delta X_1^\omega \geq k_1 \). Therefore, we set the sensitivity of the fixed cost to demand shocks to

\[ \omega = \frac{\log(-\phi c/k_1)}{\log(\Delta X_1)}. \]  

(C5)

This assures that the lower bound of the state space (i.e., \( k_1 \)) can be realized with the worst possible demand shock (i.e., \( \Delta X_1 \)).

Starting with an initial guess for the policy function, \( P_1(k_s) = V(1 - 1/\epsilon)^{-1} \), we solve the model by value iteration.

1. Iterate on equation (C1) to compute the value function \( j_i(k_s) \) corresponding to the
current policy function $P_i(k_s)$.

2. For each point $k_s$ on the grid, find $P_{i+1}(k_s)$ that maximizes equation (C1) with $j_{t+1} = j_i(k_s)$.

3. If $\max_{k_s} |P_{i+1}(k_s) - P_i(k_s)|$ is less than the convergence criteria, stop. Otherwise, return to step 1.
Table A1: Average Markup on Term Annuities under the Aaa Corporate Yield Curve
The average markup on term annuities is calculated under the zero-coupon Treasury yield curve and the Aaa corporate yield curve. The Aaa corporate yield curve is estimated based on the yield spread between Moody’s seasoned Aaa corporate bonds and the 20-year constant-maturity Treasury bond as well as the yield spread between 3-month Aa nonfinancial commercial paper and the 3-month Treasury bill.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Treasury yield curve</th>
<th>Aaa corporate yield curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 years</td>
<td>7.4</td>
<td>-3.0</td>
</tr>
<tr>
<td>10 years</td>
<td>7.2</td>
<td>-7.7</td>
</tr>
<tr>
<td>15 years</td>
<td>6.5</td>
<td>-11.9</td>
</tr>
<tr>
<td>20 years</td>
<td>5.6</td>
<td>-16.1</td>
</tr>
<tr>
<td>25 years</td>
<td>5.6</td>
<td>-20.2</td>
</tr>
<tr>
<td>30 years</td>
<td>4.8</td>
<td>-24.8</td>
</tr>
</tbody>
</table>
The markup is defined as the percent deviation of the quoted price from actuarial value. The actuarial value is based on the annuitant mortality table from M’Clintock (1899) and the zero-coupon Treasury yield curve. The average markup is estimated from a regression of markups onto dummy variables for time. The confidence interval is based on robust standard errors clustered by insurance company and age. The sample is annual from January 1929 through January 1938.
Figure B2: Reserve to Actuarial Value for Life Annuities: 1929–1938
The reserve value is based on the annuitant mortality table from M’Clintock (1899) and a constant discount rate of 3.5 percent. The actuarial value is based on the annuitant mortality table from M’Clintock (1899) and the zero-coupon Treasury yield curve. The sample is monthly from January 1929 through January 1938.