Putting the Relationship First: Relational Contracts and Market Structure*

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Abstract

I examine relational contracts and investment in a market for intermediate goods using a tractable game of imperfect private monitoring. A downstream firm requires one of several products in each period from a market of suppliers. If output is not contractible, I show that the downstream firm relies on a small network of suppliers relative to first-best. These upstream firms choose to “put the relationship first.” By investing to produce many of the products that may be required by the downstream firm, they lock themselves into the relationship, thereby increasing effort provision. Using this framework, I consider why suppliers might resist socially efficient legal reform and discuss implications for employment and ex ante human capital investments.

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1 Introduction

Businesses in a relation-based system will expand at those margins where diminishing returns set in most slowly. This will mean preserving the closeness of the relation, even at the cost of undertaking a new activity that is not economically so close—not such a good complement in production or consumption. - Avinash Dixit (2007)

Repeat dealing, cultural homogeneity...and a lack of third-party enforcement...have been typical conditions. Under them transactions costs are low, but because specialization and division of labor is rudimentary, transformation costs are high. - Douglass North (1990)

Individuals and companies rely on informal relationships with one another to encourage cooperation when formal contracts are incomplete or unavailable. Participants in these relationships invest to strengthen their bonds, rather than simply cutting costs or maximizing productive efficiency. For example, suppliers might alter their production process or produce different goods in order to better meet a favored buyer’s needs, leading to markets that look structurally different from those with readily available formal contracts.

In this paper, I illustrate one way that relational contracts can affect investment decisions. A single downstream firm requires several inputs from a group of suppliers. Before the relationship begins, each supplier chooses a set of products to manufacture: highly specialized firms are very efficient at manufacturing a small number of products, while generalist suppliers can inefficiently produce many different goods. The game has imperfect private monitoring—each supplier is unable see the details of the downstream firm’s relationship with other suppliers—which prevents the upstream firms from jointly punishing a deviation by the downstream firm.

The main result of this paper links suppliers’ investments to the underlying contractual environment. When formal contracts are available, many upstream firms enter the market, and each specializes in a small set of products in order to minimize manufacturing costs. In contrast, generalist upstream firms have an advantage when enforceable contracts cannot be written. A generalist supplier can meet many of the downstream firm’s needs, increasing
the future value generated by that relationship. The supplier can then threaten to withhold production if the downstream firm does not adequately compensate it for output, which induces the downstream firm to pay bonuses and so encourages high effort. This leads to a tension between efficiency and adaptability in a relational contracting setting that is absent when formal contracts are available. I develop applications dealing with employment and legal reform that emphasize this connection between \textit{ex ante} investments and the underlying contracting environment.

As implied in the opening quotes by Dixit (2007) and North (1990), real markets are rife with interactions that are tailored to maximize the efficacy of relational contracts. In an attempt to mimic successful Japanese car companies, Chrysler revolutionized its production process in the early 1990s by developing close relationships with a small number of upstream firms.\footnote{See Dyer (1996) for an in-depth analysis of Chrysler’s transformation.} In the context of this model, Chrysler’s decision to use informal contracts naturally led to a reduction in the number of regular suppliers; I argue that the remaining suppliers exerted higher effort precisely \textit{because} they dramatically expanded the set of products they manufacture. Along this line, Liker and Choi (2004) point out that both Toyota and Honda ask their top-tier suppliers to “produce subsystems instead of components.” Similarly, Nistor (2012) finds that if a restaurant requires customized ingredients, it tends to source from fewer suppliers. In a survey, Guinipero (1990) reports that manufacturers implementing just-in-time techniques tend to both reduce the size of their supply networks and emphasize quality.

The fundamental trade-off in this model resembles Bernheim and Whinston’s (1990) analysis of multimarket contact. If duopolists compete in several different markets, they can threaten to revert to competition in \textit{every} market following a deviation from the collusive price. Bernheim and Whinston show that if the different markets exhibit certain kinds of heterogeneity, then the duopolists can exploit this heterogeneity in order to better sustain collusive outcomes. In my model, a \textit{generalist supplier} has a similar sort of “multimarket contact” with the downstream firm: it produces—and so can threaten to withhold—many different products. In my basic model, trade occurs in only one market in each round, which creates a very stark benefit for multimarket contact: parties who interact in multiple markets trade with one another more frequently. In Section 5.1, I consider a setting in which
generalist suppliers are sometimes optimal even if trade occurs in every market in every round.

While the seminal papers by Bull (1987) and Levin (2003) have instigated an extensive literature on relational contracts, relatively few papers consider the interaction between these informal arrangements and market structure (see Malcomson (2012) for a review of the large and growing relational contracting literature). Notable exceptions include Board (2011) and Calzolari and Spagnolo (2009), who argue that relational contracts tend to lead to small markets. In Board’s model, a downstream firm pledges future surplus to its suppliers in order to induce cooperation. Because the downstream firm must sacrifice some rent every time it contracts with a supplier, it chooses to contract with a strict subset of the available upstream firms. Similarly, Calzolari and Spagnolo argue that restricting entry in a procurement auction can increase bidders’ expected future surplus and thus induce higher effort. By considering a game that emphasizes the importance of ex ante investments, I generate insights that are complementary to both of these papers.

Many of the standard tools in game theory cannot be applied in environments with imperfect private monitoring; see Kandori (2002) for an overview. As a result, much of the theoretical literature focuses on either “Folk Theorem”-type results (investigating the set of equilibria among very patient players), or restricts attention to belief-free equilibria, which have a simplifying recursive structure. In contrast, I focus on a simple principal-agents game in which output produced by and bonuses paid to an agent are observed by the principal and that agent, but not by other agents. Hence, this model loosely resembles Ellison’s (1994) work on communal enforcement and Wolitzky’s (2011) analysis of public goods provision, albeit without any contagion-style punishments.

In Section 2, I discuss the timing of the model and introduce several important assumptions. Section 3 covers three different benchmark solutions that provide useful comparisons to the main results. I explore the central trade-off between specialization and adaptability in Section 4, along with a simple example that has a closed-form solution and is used in the extensions. Section 5 covers three applications: the first explores what happens if multiple products are required in each period, the second investigates why upstream firms might resist

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the introduction of formal contracts, and the third considers human capital investments by
employees. I conclude with discussion in Section 6.

2 Model

I propose a model in which a repeated game is preceded by *ex ante* investments. Section 2.1
describes the timing and monitoring structure of the game, and Section 2.2 gives definitions
and assumptions.

2.1 Timing

Consider an intermediate goods market with a single principal, who requires one of many
different products in each period. At the beginning of the game, agents decide whether to
enter the market or not. They also choose a *specialization* $\mathcal{P}_i \subseteq [0,1]$, which determines
the products that they can manufacture and their efficiency at producing each good. $\mathcal{P}_i$
captures the fundamental trade-off between efficiency and flexibility which lies at the heart
of the analysis: if the measure of $\mathcal{P}_i$ is large, then agent $i$ can make many different goods but
must pay a high fixed cost to produce. After entry decisions and specializations are observed
by everyone, the repeated game begins. In each period, a single good $\phi_t \in [0,1]$ is randomly
drawn as required by the principal. Any agent with $\phi_t \in \mathcal{P}_i$ is able to produce this good; the
principal pays a wage to each agent and asks one to produce. That agent pays a fixed cost
that depends on $\mathcal{P}_i$ and also chooses a private and costly effort $e_t$ that determines output $y_t$.
After observing output, the principal can choose whether to pay a discretionary bonus $\tau_{i,t}$
to agent $i$. Importantly, output $y_t$ is observable only by the principal and producing agent,
while wage $w_{i,t}$ and discretionary bonus $\tau_{i,t}$ are observable to only the principal and recipient
of these transfers.

Players share a common discount factor $\delta$ in the repeated game, and do not discount
between $t = 0$ and the repeated game $t = 1, 2, \ldots$. Formally, the game has the following
timing:

- *At the beginning of the game* $t = 0$:
1. A countably infinite number of agents simultaneously choose whether to enter or exit the market. Entry costs $F_E > 0$, and agents that do not enter have no additional actions. Let $\{1, \ldots, M\}$ be the set of agents in the market.

2. Each agent $i \in \{1, \ldots, M\}$ publicly chooses a measurable specialization $\mathcal{P}_i \subseteq [0, 1]$. Denote $\mu_i = \mu(\mathcal{P}_i)$ as the Lebesgue measure of this set.

3. The principal and agent $i$ simultaneously make transfer payments $\tau^A_{i,0} \geq 0$, $\tau^P_{i,0} \geq 0$, respectively, to one another. Let $\tau_{i,0} = \tau^A_{i,0} - \tau^P_{i,0}$ be the net transfer to agent $i$.\footnote{For all transfer payments, if $\tau_{i,0} \geq 0$, the convention is that $\tau^P_{i,0} = 0$, and similarly $\tau^A_{i,0} = 0$ if $\tau_{i,0} \leq 0$.}

4. In each round $t = 1, 2, \ldots$:

1. A required product $\phi_t \sim U[0, 1]$ is publicly observed.

2. The principal offers production to one agent $x_t \in \{\emptyset\} \cup \{1, 2, \ldots, M\}$. This offer is observed only by agent $x_t$.

3. For all $i \in \{1, \ldots, M\}$, the principal and agent $i$ simultaneously make wage payments $w^A_{i,t} \geq 0$, $w^P_{i,t} \geq 0$ to one another, with net wage to agent $i$ $w_{i,t} = w^A_{i,t} - w^P_{i,t}$. These payments are observed only by the principal and agent $i$.

4. Agent $x_t$ accepts or rejects production: $d_t \in \{0, 1\}$. This decision is observed only by the principal.

5. If $x_t$ accepts the contract, then he pays fixed cost $\gamma(\mu_i)$, where $\gamma : [0, 1] \to \mathbb{R}_+$, and privately chooses effort $e_t \in \{0, 1\}$ at cost $ce_t$.

6. Output $y_t \in Y \subseteq \mathbb{R}$ is realized, where $y_t \sim F(y|e_t)$ if both $\phi_t \in \mathcal{P}_i$ and $d_t = 1$, and $y_t = 0$ otherwise. $y_t$ is observed only by the principal and $x_t$.

7. For all $i \in \{1, \ldots, M\}$, the principal and agent $i$ simultaneously make bonus payments $\tau^A_{i,t} \geq 0$, $\tau^P_{i,t} \geq 0$, with net payment $\tau_{i,t} = \tau^A_{i,t} - \tau^P_{i,t}$. This transfer is observed only by $i$ and the principal.

8. Payoffs are realized: agent $i$ earns

$$u_{i,t} = (1 - \delta) \left( \tau_{i,t} + w_{i,t} - 1\{x_t = i\}d_t(\gamma(\mu_i) + ce_t) \right),$$
while the principal earns

\[ u_{0,t} = (1 - \delta) \left( 1 \{ x_t \neq \emptyset \} d_t y_t - \sum_{i=1}^{M} (\tau_{i,t} + w_{i,t}) \right). \]

Three of the assumptions in this model require special consideration. First, specialization determines the subset of goods \( \mathcal{P}_i \) that agent \( i \) can produce and his fixed cost of producing \( \gamma(\mu_i) \). This assumption is stark but cleanly captures the intuition that specialization increases efficiency at the cost of flexibility. Second, an agent’s specialization cannot be changed once it is chosen. This is a notion of lock-in: an agent tailors its production process to manufacture certain goods, and changing this role is prohibitively costly. Third, agents have no means of communicating with one another. While extreme, this assumption is a tractable way to model bilateral relationships between the principal and each agent.

### 2.2 Histories and Equilibrium

Because players observe different outcomes as the game progresses, I separately track each player’s private history over time.

**Definition 1** The set of baseline histories \( \mathcal{H}_B^T \) at time \( T \) is

\[
\mathcal{H}_B^T = \{ M, \{ \mathcal{P}_i \}_{i=1}^M, \{ \tau_{i,0}^A, \tau_{i,0}^P \}_{i=1}^M, \{ \phi_t, x_t, d_t, \{ w_{i,t}^A, w_{i,t}^P \}_{i=1}^M, e_t, y_t, \{ \tau_{i,t}^A, \tau_{i,t}^P \}_{i=1}^M \}_{t=1}^T \}
\]

The principal observes all actions except for effort \( e_t \), so the set of principal’s baseline histories at time \( T \) is \( \mathcal{H}_{B,i}^T = \{ h^T \setminus \{ e_t \}_{t=1}^T | h^T \in \mathcal{H}_B^T \} \). The set of agent \( i \)'s baseline histories at time \( T \) is

\[
\mathcal{H}_{B,i}^T = \left\{ M, \{ \mathcal{P}_i \}_{i=1}^M, \tau_{i,0}^A, \tau_{i,0}^P, \{ \phi_t, x_t, d_t I \{ x_t = i \}, w_{i,t}^A, w_{i,t}^P I \{ x_t = i \}, e_t, y_t, \tau_{i,t}^A, \tau_{i,t}^P I \{ x_t = i \} \}_{t=1}^T \right\}
\]

Let \( \mathcal{N} \) be the nodes of the stage game; then \( (h^T, n_{T+1}) \) indicates a history at node \( n_{T+1} \in \mathcal{N} \)

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4Why don’t specialized agents horizontally integrate in order to share information with one another? One possible answer is that “generalist agents” in this model are in fact horizontally integrated. Under this interpretation, \( \gamma(\mu) \) represents the production costs associated with a bloated organizational structure. There might also be legal or financial constraints that prevent horizontal integration, or behavioral restrictions that limit joint punishment by different divisions within a company.
of round $T + 1$. Let $\mathcal{I}(n_t)$ be agent $i$’s information set at stage-game node $n_t$, so that a private history is $(h_i^n, \mathcal{I}(n_{T + 1}))$.

Strategies are denoted $\sigma_i$ for agent $i \in \{1, ..\}$ and $\sigma_0$ for the principal, with profile $\sigma = \{\sigma_0, ....\}$. A relational contract is a Perfect Bayesian Equilibrium (PBE) of the repeated game.\footnote{A Perfect Bayesian Equilibrium consists of a strategy profile $\sigma$ and belief system $\rho = \{\rho_i\}_{i=0}^\infty$ over true histories for each player such that (1) given beliefs $\rho_i(h_i^n)$, $\sigma_i$ maximizes player $i$’s continuation surplus, and (2) $\rho_i$ updates according to Bayes Rule whenever it is well-defined. When Bayes Rule is not well-defined, $\rho_i$ assigns weight only to histories that are consistent with agent $i$’s information but is otherwise unconstrained.} A relational contract is stationary if on the equilibrium path, actions in period $t$ depend only on variables observe in period $t$, and is optimal if it maximizes total \textit{ex ante} expected surplus. Because monitoring is imperfect and private, I cannot use standard recursive techniques in this analysis and so rely on other methods.

The cost function $\gamma(\mu)$ is constrained so that a meaningful trade-off between specialization and flexibility exists.

**Assumption 1** $\gamma(\mu)$ is differentiable with $\gamma', \gamma'' > 0$ and $\gamma'(0) = 0$, and $\mu(E[y|e = 1] - c - \gamma(\mu))$ is strictly increasing in $\mu$.

Assumption 1 implies that if agent $i$ is allocated production of every good in $\mathcal{P}_i$ and works hard, then the total surplus produced by $i$ is increasing in $\mu_i$. In addition to driving the tension between adaptability and productive efficiency, this assumption ensures that it is optimal for specializations $\{\mathcal{P}_i\}_{i=1}^M$ to cover the entire interval if the agents are expected to work hard.

Finally, I constrain $F(y|e)$ so that it is efficient for an agent to accept production if and only if he works hard.

**Assumption 2** 1. F first-order stochastically increases in effort: 

$$F(y|e = 1) >_{\text{FOSD}} F(y|e = 0).$$

2. $e = 1$ is strictly efficient: $E[y|e = 1] - c - \gamma(\mu) > 0 \geq E[y|e = 0] - \gamma(\mu), \forall \mu \in [0, 1]$.

It will turn out in this analysis that the critical determinant of the strength of agent $i$’s relationship with the principal can be measured by the \textit{total surplus produced by agent}
\( i \), which is determined by whether (1) \( i \) is allocated production of \( \phi_t \in \mathcal{P}_i \), (2) \( i \) accepts production if it is offered, and (3) \( i \) works hard if he accepts production.

**Definition 2** The total per-period surplus produced by agent \( i \) in round \( t \) is

\[
\pi_{i,t}^{TOT} = (1 - \delta) \mathbb{I}\{x_t = i\} d_t (y_t - c e_t - \gamma(\mu_i)).
\]

The principal’s per-period surplus from agent \( i \) is

\[
\pi_{0,t}^i = (1 - \delta) \mathbb{I}\{x_t = i\} d_t y_t - w_{i,t} - \tau_{i,t}.
\]

**Given strategy profile \( \sigma \) and history \((h^{t-1}, n_t)\), the continuation surplus for agent \( i \) is**

\[
U_i(h^{t-1}, n_t; \sigma) = E_{\sigma} \left[ \sum_{t' = t+1} \delta^{t' - t - 1} u_{i, t' | h^{t-1}, n_t} \right]
\]

**and the continuation surplus for the principal from agent \( i \) is**

\[
U_{0,i}(h^{t-1}, n_t; \sigma) = E_{\sigma} \left[ \sum_{t' = t+1} \delta^{t' - t - 1} \pi_{0,t}^i | h^{t-1}, n_t \right],
\]

with the principal’s total continuation payoff equal to \( U_0 = \sum_{i=1}^M U_{0,i} \).

When unambiguous, I suppress the notation \((h^{t-1}, n_t; \sigma)\) in \( U_i, U_{0,i}, \) and \( U_0 \). By construction, \( \pi_{i,t}^{TOT} = \pi_{0,t}^i + u_{i,t} \) and the total surplus produced in a period is \( \sum_{i=1}^M \pi_{i,t}^{TOT} \). Therefore, \( \pi_{i,t}^{TOT} \) captures the **contribution of each agent to total surplus** in a period. Intuitively, this total surplus is at stake in each relationship in the sense that it is lost as a punishment if either the principal or agent \( i \) reneges on a promised payment. If agent \( i \) works hard when he is allocated production, then giving production to agent \( i \) more frequently increases \( \pi_{i,t}^{TOT} \) and thus increases the size of this punishment. Because \( \mu(\mathcal{P}_i) \) determines how frequently agent \( i \) is able to produce, the size of \( \mu(\mathcal{P}_i) \) determines the maximum value of \( \pi_{i,t}^{TOT} \), which in turn determines the set of credible bonuses that can be supported in a relational contract. This fundamental tension drives my main result.
Following Levin (2003) and much of the subsequent relational contracting literature, I focus on the optimal equilibrium. In this setting, any PBE is payoff equivalent to a PBE in which agents do not condition on their past efforts, so I consider only relational contracts that are independent of past effort decisions.

3 Benchmarks - First Best, One-Shot, and Public Monitoring

Three different benchmarks are relevant in this model. The first characterizes the first-best by supposing that output $y_t$ is contractible, the second considers the one-shot game, and the final considers optimal relational contracting if monitoring were public.

3.1 Optimal Entry and Specializations with Formal Contracts

Suppose that output $y_t$, entry, and specializations $\mathcal{P}_i$ are contractible at the beginning of the game. Then the principal can efficiently induce high effort from every agent, since all parties are risk-neutral and have deep pockets. Because each agent exerts high effort regardless of her specialization $\mathcal{P}_i$, these specializations are chosen to balance the fixed costs $\gamma(\mu_i)$ against the cost of entering the market $F_E$. Therefore, it is straightforward to calculate the efficient number of entrants and their specializations.

**Proposition 1** Let Assumptions 1 and 2 hold. Every optimal equilibrium entails $M^{FB}$ entrants and takes the following form:

1. For any agents $i, k$ in the market, $\mu(\mathcal{P}_i) = \mu(\mathcal{P}_k) = \mu$, where $\mu = \frac{1}{M^{FB}}$.

2. $\forall t$, effort is $e_t = 1$.

The first best number of firms $M^{FB}$ decreases in $F_E$ and is independent of $\delta$.

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6The proof of this fact may be found in Andrews and Barron (2012), Appendix B.
A principal who has access to formal incentive contracts can always motivate her workers, so the number of firms in the market is determined by setting marginal production costs $\frac{1}{M} \gamma'(\frac{1}{M})$ equal to the cost of entry $F_E$ (subject to integer constraints). In Section 4, I will show that $M^{FB}$ is large and specializations $\mu = \frac{1}{M^{FB}}$ is specialized relative to the optimal relational contract when players are impatient. Intuitively, when formal contracts are available, market size is limited only by the cost of entry and each firm specializes in a narrow band of products.

The distributions $F(y|e = 1)$ and $F(y|e = 0)$ are statistically distinguishable, so some contract exists that induces high effort. I record this result as a corollary.

**Corollary 1** There exists a bounded set of transfers $\tau(y)$ that induce the agent to exert high effort, and such that $\infty > \sup_y \tau(y) - \inf_y \tau(y)$. For any transfers $\tau(y)$ that induces high effort, $\sup_y \tau(y) - \inf_y \tau(y) > 0$.

**Proof:** One such contract with bounded transfers is exhibited in the proof of Proposition 1. Note that the agent’s IC constraint is

$$E[\tau(y)|e = 1] - E[\tau(0)|e = 0] \geq c$$

and so $\sup_y \tau(y) - \inf_y \tau(y) \geq c > 0$ as desired. ■

Rather than explicitly calculating the optimal incentive contract in what follows, I will instead use Corollary 1 to argue that an agent’s payoff must vary with $y$ in order to motivate him to work hard. In a relational contract, the principal must prefer to pay this bonus rather than renege and face a punishment by the betrayed agent, and the role of $P_i$ is to ensure that the principal is indeed willing to do so.

### 3.2 Equilibrium in the One-Shot Game

A second important benchmark is the one-shot version of the repeated game. The principal cannot credibly promise to reward the agent for high output, so no surplus is generated in
equilibrium. Because entry is costly and low effort is inefficient, no agents enter the market ($M_{SPOT} = 0$).

**Proposition 2** The unique payoff in the spot game is 0, with $M_{SPOT} = 0$.

**Proof:** Using backwards induction, $\tau_i(y) = 0$, $\forall i, y \in \{0, y_H\}$. Therefore, $e^* = 0$ in equilibrium, and so total surplus generated by any firm in the market is no larger than 0 because $E[y|e = 0] - \gamma(\mu) \leq 0$. But then no firm chooses to enter the market, since they must incur the cost $F_E > 0$ to do so. ■

Proposition 2 demonstrates that repeated interaction between the principal and each agent is required to induce the agents to work hard. While all players earn their min-max payoff in this one-shot equilibrium, Section 4 will prove that punishments following a deviation that is not publicly observed do not typically mix-max the deviator.

### 3.3 What Happens if Monitoring is Public?

It is instructive to consider the optimal equilibrium if $x_t, y_t, \{w_{i,t}\}$, and $\{\tau_{i,t}\}$ were publicly observed so that the game was one of imperfect public monitoring. This benchmark will highlight role of private monitoring in the baseline model.

Many of the tools developed in the foundational relational contracting paper by Levin (2003) can be adapted to this setting. As a first step, I show that stationary contracts are optimal in this setting.\(^7\)

**Lemma 1** There exists an optimal relational contract that is stationary in the public monitoring game.

**Proof:** See Appendix A.

The proof of Lemma 1 constructs a stationary optimal relational contract and is similar to Levin (2003), Theorem 2. In this equilibrium, the principal earns all of the surplus produced

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\(^7\)Note that utility is not transferrable between agents in this setting, and so the model differs from Levin (2003). Indeed, Andrews and Barron (2012) show in a related model that if the set of relational contracts were restricted so that the agents earned a positive fraction of the surplus they produced, then a tension similar to what is explored in this paper continues to be relevant.
in each period, and is punished by reversion to the static equilibrium following any deviation. As a result, whenever the agents are willing to work hard in any equilibrium, the principal is willing to pay them an incentive scheme that motivates them to work hard in each period of a stationary equilibrium.

The next result demonstrates that the efficient contract either has $M = 0$, or $M \geq M^{FB}$.

**Proposition 3** In an optimal relational contract, $\mu(P_i \cap P_k) = 0$, $\forall i, k \in \{1, ..., M\}$. $\mu(P_i) = \frac{1}{M}$, and either $M^{Pub} = 0$ or $M^{Pub} \geq M^{FB}$ firms enter the market.

**Proof:** See Appendix A.

With public monitoring and transfers between agents, at least $M^{FB}$ firms enter the market. As in the first best, additional entry leads to lower production costs $\gamma(\frac{1}{M})$. This reduction in costs has two benefits: it both directly increases surplus and indirectly allows the principal to credibly promises larger bonuses in equilibrium, since she would lose her entire continuation surplus—which does not include the cost of entry $F_E$—were she to renege on this promise. The optimal relational contract weighs both of these benefits against the cost $F_E$ of additional entry to determine the efficient number of entrants, whereas the first-best weighs only the first of these benefits against $F_E$. Therefore, $M^{Pub} \geq M^{FB}$.

## 4 Putting the Relationship First

In this section, I consider the optimal relational contract in the baseline game. The optimal equilibrium is substantially different than the first-best benchmark from Section 3.1: a relatively small number of agents enter the market, each of whom is able to produce a broad range of products. I prove the general result in Section 4.1, while Section 4.2 considers a specific example with a complete closed-form solution.

### 4.1 The Role of Flexibility in Relational Contracts

This section proves that if formal contracts are unavailable, the optimal market structure involves a small number of agents that are not very specialized. Because agents cannot
communicate with one another, they are unable to coordinate and jointly punish deviations by the principal. As a result and unlike the case with public monitoring, the future surplus generated by each agent determines the largest bonus that can be paid to that agent. *Ex ante* investments that focus on flexibility (i.e., a large \( \mu_i \)) increase the total surplus generated by agent \( i \), at the cost of reducing the maximum feasible total surplus.

Together, the next lemmas provide the basic ingredients for the main result. First, I argue that a principal that deviates in her relationship with agent \( i \) is punished by no worse than the *bilateral breakdown* of trade with \( i \). To ensure that each agent is sufficiently valuable, the equilibrium measure of specialization \( \mu(P_i) \) must be bounded from below by some \( \mu^*(\delta) \geq \frac{1}{1-M_{FB}} \). Moreover, if specializations do not overlap (so that \( \mu(P_i \cap P_k) = 0 \)), then the efficient relational contract can be replicated by a stationary relational contract. Proposition 4 combines these arguments to demonstrate the main result: when relational contracts are used and players are impatient, the number of agents in the market is smaller and each agent specializes in a broader range of tasks than in the first-best equilibrium.

Other agents do not observe when the principal betrays one of their number, so the punishment in each principal-agent dyad depends on the surplus generated by that relationship. More precisely, if an equilibrium transfer is not made to (or by) agent \( i \), then the size of the punishment following this deviation is bounded above by the value of the relationship to the player paying the bonus. Thus, the contribution of each agent to total surplus is the key variable that determines what is at stake in each relationship and thus the incentive pay \( \tau_{i,t} \) that can be offered to each agent.

**Lemma 2** Let Assumption 2 hold, and suppose \( \sigma^* \) is an equilibrium. Fix a history \((h^{t-1}, n_t)\), where \( n_t \) is a node immediately following output \( y_t \). Then

\[
(1 - \delta)E_{\sigma} [\tau_i, t| h_i^{t-1}, I_i(n_t)] \leq \delta E_{\sigma^*} [U_i(h^t)| h_i^{t-1}, I_i(n_t)]
\]

\[
-(1 - \delta)E_{\sigma} [\tau_i, t| h_i^{t-1}, I_i(n_t)] \leq \delta E_{\sigma^*} [U_i(h^t)| h_i^{t-1}, I_i(n_t)]
\]

**Proof:** See Appendix A.

The proof of Lemma 2 argues that the player responsible for paying \( \tau_{i,t} \) always has the option of refusing to pay and suffering a breakdown in the relationship. Because agent \( i \)
interacts only with the principal, a breakdown in this realtionship would hold $i$ at his outside option 0. On the other hand, the principal can continue to allocate business to the other agents following a breakdown with agent $i$. Therefore, she can “cut her losses” following a breakdown and so $(1 - \delta)\tau_{i,t}$ is bounded by the value of *that agent* to the principal, $U_{i0}$. The bound (1) on the principal’s punishment is not necessarily tight, since she may be able to do strictly better by surreptitiously altering her allocation rule following a deviation. For instance, if the principal could award product $\phi_t$ to either firm $i$ or firm $j$ in round $t$, then she may be able to reward the product to $j$ following a breakdown with $i$ without triggering any further punishment.

For Lemma 2 to hold, agents must be unable to communicate; otherwise, they could coordinate to jointly punish a deviation by the principal.\textsuperscript{8} Hence, there is an incentive to aggregate production in a small group of firms in order to limit reneging temptation. This impulse towards aggregation leads to a deviation from the first best market structure, since a specialized firm produces at a low cost but is unable exact a revenge sufficiently large to deter the principal from deviating.

The next lemma demonstrates that it is in fact easier to induce high effort from an agent with a large $P_i$, particularly when that agent is the sole producer of every product in $P_i$.

**Lemma 3** Fix the number of entrants $M$ and specializations $\{P_i\}$. A necessary condition for there to exist a PBE in which agent $i$ chooses $e_t = 1$ on the equilibrium path is that there exists an incentive scheme $\tau(y)$ that induces $e_t = 1$ and

$$\sup_y \tau_i(y) - \inf_y \tau_i(y) \leq \frac{\delta}{1 - \delta} \mu_i \left(E[y|e = 1] - c - \gamma(\mu_i)\right)$$

(2)

**Proof:** See Appendix A.

Agent $i$ can be motivate to work hard using both contemporaneous transfers $\tau_{i,t}$ and continuation payoffs $U_i$. From Lemma 2, the total variation in these incentives must satisfy

$$0 \leq (1 - \delta)\tau_{i,t} + \delta U_i \leq \delta(U_{i0} + U_i).$$

The sum $U_i + U_{i0}$ is bounded above by the right-hand side of (2), which is the surplus produced by agent $i$ if he is awarded every $\phi \in P_i$ in each product

\textsuperscript{8}Alternatively, I could assume that agents can communicate but are behaviorally restricted to “bilateral breakdown” following a deviation by the principal. Such a behavioral restriction is non-trivial to define because agents’ beliefs about how the principal allocates business might change when they observe a deviation.
period and always works hard. If agent $i$ were unwilling to choose $e_t = 1$ if she were the sole producer of every $\phi_t \in P_i$ in each round, then she would also be unwilling to choose $e_t = 1$ in any relational contract. Intuitively, allocating production to agent $i$ whenever possible maximizes the size of the punishment following a deviation in two ways. First, if $i$ produces every $\phi \in P_i$ then she can threaten to withhold production of these goods, which creates a powerful incentive to maintain the relationship. Moreover, all other agents also expect agent $i$ to produce every $\phi \in P_i$, so the principal cannot deviate from this allocation rule without being punished by every agent. Thus, if the principal were to renege on agent $i$, she would be unable to reallocate production without triggering a punishment.

The next corollary shows that the incentive condition (2) is also sufficient if the agents have specializations that do not overlap.

**Corollary 2** If $\mu(P_i \cap P_j) = 0 \ \forall i, j \in \{1, ..., M\}$, then there exists a stationary optimal relational contract. In this equilibrium, agent $i$ picks $e_t = 1 \ \forall t$ on the equilibrium path iff $\exists$ transfers $\tau_i(y)$ that induce high effort and satisfy (2).

**Proof:** See Appendix A.

Consider an equilibrium such that $P_i \cap P_j = \emptyset$ for every agent. In round $t$, the principal optimally allocates production of $\phi_t \in [0, 1]$ to the sole agent who can produce, so the maximum feasible continuation surplus in each relationship does not depend on the principal’s allocation rule. Hence, a stationary relational contract is optimal for similar reasons to Levin (2003), and in particular there exists an optimal stationary equilibrium in which the principal earns all of the surplus in each period. In such a stationary contract, the left-hand side of (2) captures the temptation to renege on a promised bonus payment, whereas the right-hand side is the amount of surplus lost by the principal if she does not pay the equilibrium bonus $\tau_i$.

So long as $P_i \cap P_j = \emptyset$, (2) implies that the specialization $P_i$ is the sole determinant of whether agent $i$ works hard in the optimal relational contract. In general, equilibrium behavior in this game can be quite complicated, but much of this complexity stems from how the principal allocates business over time. If this allocation rule is independent of
the history—which is natural when specializations don’t overlap—then simple contracts are optimal.

The incentive constraint (2) suggests an important definition: the minimum specialization required to induce high effort in equilibrium, $\mu^*(\delta)$.

**Corollary 3** There exists a continuous function $\mu^*(\delta)$ that is defined on $\delta \in [\hat{\delta}, 1)$ for some $\hat{\delta} < 1$ such that (1) $\mu^*(\delta)$ is decreasing in $\delta$, (2) $\mu^*(\hat{\delta}) = 1$, and (3) $\lim_{\delta \to 1} \mu^*(\delta) = 0$, and $\exists$ a PBE in which agent $i$ chooses $e_t = 1$ at some history only if $\mu_i \geq \mu^*(\delta)$. Moreover, if $\mu_i \geq \mu^*(\delta)$, $\forall i$, and $\mu(\mathcal{P}_i \cap \mathcal{P}_k) = 0$, $\forall i, k$, then all agents choose $e_t = 1$, $\forall t$ along the equilibrium path.

**Proof:** See Appendix A.

The critical threshold $\mu^*(\delta)$ is the smallest interval of specialization such that inequality (2) holds for some incentive scheme that induces high effort. In an optimal equilibrium, every agent that enters the market must satisfy $\mu_i \geq \mu^*(\delta)$; otherwise, that firm would never exert high effort, and so should instead stay out of the market to save the entry cost $F_E$. Moreover, if $\mathcal{P}_i \cap \mathcal{P}_k = \emptyset$, $\forall i, k$, then $\mu_i \geq \mu^*(\delta)$ is sufficient to induce high effort from $i$.

Corollary 3 illustrates the central intuition of the model: if $\delta$ is far from 1, $\mu^*(\delta) > \frac{1}{M\sup}$, so firms must choose a broader specialization than in the first-best. In other words, each firm “puts the relationship first:” rather than specializing to minimize manufacturing costs, as they would if formal contracts were available, firms in the market instead inefficiently produce a broad array of different products. Broad specializations—$\mu(\mathcal{P}_i) > \frac{1}{M\sup}$—lead to lower total surplus given $e = 1$ but also increase *dyad-specific surplus* $U_i^0 + U_i$, so that better relational incentive contracts can be implemented in equilibrium.

Proposition 4 puts the preceding steps together to show that a market reliant on relational contracts typically has fewer entrants and broader specializations than a market in which formal contracts are available. If participants are more impatient, then the number of agents in the market is smaller and each entrant is responsible for a broader set of products.

**Proposition 4** Suppose Assumptions 1 and 2 hold. Then:
1. For every \( M \leq M^{FB} \), there exists \( \bar{\delta}(M) \) such that for all \( \delta \geq \bar{\delta}(M) \), at least \( M \) firms will enter the market in any optimal equilibrium.

2. For every \( M \leq M^{FB} \), there exists an open interval \( \Delta(M) \) that satisfies \( \sup \Delta(M) \leq \inf \Delta(M + 1) \), such that for every optimal equilibrium with \( \delta \in \Delta(M) \):

   (a) \( M \) firms are optimal;

   (b) \( \mu(\mathcal{P}_i \cap \mathcal{P}_k) = 0 \), \( \mu(\mathcal{P}_i) = \frac{1}{M} \), and the efficient equilibrium can be replicated by a stationary equilibrium.

**Proof:** See Appendix A.

The proof of Proposition 4 relies on the fact that \( \mu(\mathcal{P}_i) \geq \mu^*(\delta) \forall i \) in the efficient equilibrium. So long as \( \mu^*(\delta)M \leq 1 \), there is no reason for specializations to overlap; because \( \gamma(\mu) \) is increasing and strictly convex, it is instead optimal for each firm to specialize in a disjoint subset of the same size, \( \mu(\mathcal{P}_i) = \frac{1}{M} \). In this situation, \( \frac{1}{M} \geq \mu^*(\delta) \) and so the agents exert high effort in the optimal equilibrium and together produce the entire interval of goods. For \( M < M^{FB} \), surplus from this equilibrium is strictly increasing in \( M \) conditional on high effort, so any equilibrium with \( M < M^{FB} \) firms and \( (M + 1)\mu^*(\delta) \leq 1 \) is dominated by an equilibrium with \( M + 1 \) firms with disjoint specializations of measure \( \frac{1}{M+1} \). This bounds the number of entrants \( M \) from below. Moreover, if \( \mu^*(\delta)M = 1 \), then it is uniquely optimal for exactly \( M \) firms to enter and choose disjoint specializations with \( \mu(\mathcal{P}_i) = \mu^*(\delta) \). This equilibrium both induces high effort and minimizes entry and fixed costs. Because \( F_E > 0 \), it remains optimal for \( M \) firms with disjoint specializations to enter the market on an open interval \( \Delta(M) \) about the \( \delta \) for which \( \mu^*(\delta)M = 1 \).

Together, Proposition 1 and Proposition 4 illustrate the central trade-off between flexibility and efficiency. When formal contracts are available, a large number of agents enter the market and each specializes in a relatively small subset of products to minimize the fixed cost \( \gamma(\mu) \). In contrast, when high effort can only be induced through a relational contract and players are impatient, the optimal equilibrium often involves a small number of relatively inefficient firms, each of whom is responsible for producing a wide variety of products and
generating substantial surplus. Because the set of products is fixed and each agent specializes in a large subset of that set, fewer agents to needed to satisfy the principal’s needs.

On the intervals $\Delta(M)$, $\mu(P_i \cap P_k) = 0$ and there is no overlap between specializations. In other words, each product is “single-sourced:” agents never compete to produce the same set of products. Single-sourcing is not uncommon within supplier networks, particularly when the firms rely on relational contracting—for instance, the promise to “carry out business with...suppliers without switching to others” is enshrined in Toyota’s 1939 Purchasing Rules.\(^9\) In this model, multi-sourcing—in which several agents can produce the same inputs—increases the principal’s outside option and thus her reneging temptation, which makes it more difficult to sustain high effort within a relationship. Now, suppose that multi-sourcing did occur, perhaps for some unmodeled reason. Even in this case, the tension between adaptability and efficiency presented in Proposition 4 would be unlikely to disappear. Indeed, multi-sourcing makes it harder to induce high effort; because flexibility is a way to increase lock-in and effort, flexible \textit{ex ante} investments are one way to mitigate the deleterious effects of multi-sourcing on relationships. In short, the trade-off between efficiency and flexibility seems likely to hold, even if multi-sourcing were optimal for other (unmodeled) reasons.

Outside of the intervals $\Delta(M)$ in Proposition 4, specializations might overlap $P_i \cap P_j \neq \emptyset$. Andrews and Barron (2012) explore the non-stationary allocation rules that are optimal in such a setting.

\subsection*{4.2 A Simple Framework for Applications}

This subsection presents an example for which a simple closed-form optimal relational contract exists. While this example is a special case, it starkly illustrates the broader trade-off between flexibility and specialization.

Consider the following binary output: if $e_t = 0$ or $\phi_t \notin P_i$, then $y_t = 0$; if $e_t = 1$ and $\phi_t \in P_i$, then $y_t = y_H > 0$ with probability $p$ and otherwise $y_t = 0$. Suppose the cost function

\(^9\)As referenced in Sako (2004).
\[ \gamma(\mu) = \begin{cases} 0, & \mu \leq \frac{1}{M} \\ \gamma, & \mu > \frac{1}{M} \end{cases} \]  

(3)

where \( M \geq 2 \) is an integer and \( \gamma \) satisfies \( y_H p - c - \gamma > \frac{1}{M}(y_H p - c) > F_E \). In this market, each agent faces a very simple choice: they either specialize in a subset \( \frac{1}{M} \) of the market—what I’ll call a “specialist”—or they choose to become a “generalist,” able to inefficiently produce whatever product is required. For the purposes of this analysis, assume \( MF_E < \gamma \), so that it is optimal to have \( M \) specialists enter the market if output is contractible.

**Proposition 5** Define \( \bar{\delta} \) by \( c_p = \frac{\bar{\delta}}{1} - \frac{1}{M}(y_H p - c) \) and \( \underline{\delta} \) by \( c_p = \frac{\underline{\delta}}{1-M}(y_H p - c - \gamma) \). In this example with the assumptions given above, any optimal equilibrium satisfies:

1. If \( \delta \geq \bar{\delta} \), then \( M \) firms enter the market and specialize in subsets \( P_i \subseteq [0,1] \) with \( \mu(P_i) = \frac{1}{M} \) and \( S_i \cap S_j = \emptyset \). Firm \( i \) produces every \( j \in P_i \).

2. If \( \delta \in [\bar{\delta}, \underline{\delta}) \), then a single firm enters the market, specializes in \( P_1 = [0,1] \), and manufactures every \( j \in [0,1] \).

3. If \( \delta < \bar{\delta} \), then no firms enter the market.

**Proof:**

First, consider optimal entry and specialization supposing that \( e_t = 1 \) in every period. Because \( MF_E < \gamma \), the maximum surplus if there are \( S \leq M \) specialists and \( G \leq 1 \) generalists is

\[ \frac{S}{M}(y_H p - c) + G \frac{M - S}{M}(y_H p - c - \gamma) - (S + G)F_E. \]

The derivative of this expression with respect to \( S \) is \( \frac{1}{M}(y_H p - c) - \frac{G}{M}(y_H p - c - \gamma) - F_E \). If \( G = 1 \), then this derivative is positive if \( \gamma > MF_E \), and if \( G = 0 \), it is strictly positive if \( \frac{1}{M}(y_H p - c) > F_E \). These inequalities hold by assumption, so \( S = M \) and \( G = 0 \) is optimal if each specialist chooses \( e_t = 1 \), \( \forall t \).

Now, consider the optimal relational contract, and define \( \tau_H \) and \( \tau_0 \) to be a bonus scheme
with minimal $|\tau_H - \tau_0|$ such that

$$p\tau_H + (1 - p)\tau_0 - c \geq \tau_0.$$  

It is clear that one such bonus scheme is $\tau_H = \frac{c}{p}$, $\tau_0 = 0$. If $\delta \geq \bar{\delta}$, then by Corollary 2 first-best effort can be induced in a relational contract with $M$ entrants and $\mathcal{P}_i \cap \mathcal{P}_k = \emptyset$, and so first-best can be attained. If instead $\delta < \bar{\delta}$, Corollary 3 implies that in order for agent $i$ to choose $e = 1$ in equilibrium, it must be that $\mu_i > \frac{1}{M}$. Because all $\mu_i > \frac{1}{M}$ have the same fixed cost $\gamma$ of production and $y_Hp - c - \gamma > F_E$, it is optimal for a single agent to enter with $\mathcal{P}_1 = [0, 1]$. By Corollary 2 and using the bonus scheme $\tau_0, \tau_H$, that agent chooses $e_t = 1 \forall t$ so long as

$$\frac{c}{p} \leq \frac{\delta}{1 - \delta} (y_Hp - c - \gamma)$$

By assumption, $y_Hp - c - \gamma > \frac{1}{M}(y_Hp - c)$, so there exists a range $\delta \in (\bar{\delta}, \bar{\delta})$ in which a single agent enters the market and chooses $\mathcal{P}_1 = [0, 1]$. ■

This example provides a very sharp result. For sufficiently patient firms, the first best market structure can be achieved: a large number of agents enter the market, and each specializes in a small subset of products. As $\delta$ decreases, however, the market abruptly collapses to a single agent.\textsuperscript{10} This lone remaining agent instead prioritizes his relationship with the principal in order to preserve its own incentive to exert high effort. Because the first-best market structure involves $M$ entrants specializing in $\mu(\mathcal{P}_i) = \frac{1}{M}$, Proposition 5 reiterates that relational contracts tend to involve fewer agents and more flexible investments than formal contracts.

5 Applications

The availability and quality of formal contracts differ dramatically between countries, industries, and jobs. In this section, I extend the basic model to explore several implications for

\textsuperscript{10} Notice that this discontinuity is the result of the discontinuous cost function $\gamma$ and is different than Proposition 4, which proves that for continuous $\gamma$ any $M \leq M^{PB}$ is optimal for an open set of discount factors.
production and investment. Extension 5.1 shows that the basic trade-off between flexibility and specialization may persist even if every product in required in every round. Extension 5.2 illustrates that entrenched firms might agitate against socially beneficial legal change because they fear that it would render their investment in flexibility obsolete. Extension 5.3 considers the employment relationship and suggests important differences between the skill sets and assigned tasks of employees and independent contractors.

5.1 Extension 1 - Multiple Required Products in Each Round

In the baseline model, I assume that the principal requires a single product in each round, which implies that agents with broader specializations interact more frequently with the principal. The purpose of this extension is to demonstrate that the trade-off between flexibility and specialization may be relevant even when every product is required in each round. This result follows because the principal can tailor the relational contract to provide stronger incentives to a generalist agent without increasing the maximal reneging temptation.

To make this point, I consider a very simple model that departs from the baseline model in two key ways. First, the products required by the principal are drawn from a finite set \( \{1, \ldots, M\} \), so that agents specialize in a subset \( \mathcal{P}_i \subseteq \{1, \ldots, M\} \). This assumption ensures that there is aggregate uncertainty in the market conditional on effort, which is required for the result to hold. Second, all products are required in every period, so the principal chooses one agent \( x_{\phi,t} \) to produce each good \( \phi \in \{1, \ldots, M\} \) in each round \( t \). An agent \( i \) assigned the set of products \( \chi_{i,t} \subseteq \{1, \ldots, M\} \) exerts effort \( e_{\phi,t} \) on each \( \phi \in \chi_{i,t} \) at cost \( c \sum_{\phi \in \chi_{i,t}} e_{\phi,t} \). The principal earns surplus \( \sum_{\phi=1}^{M} y_{\phi,t} y_{\phi,t} \in \{0, y_H\} \), where

\[
Prob\{y_{\phi,t} = y_H | e_{\phi}\} = \begin{cases} 
    p & e_{\phi} = 1 \\
    0 & e_{\phi} = 0
\end{cases}
\]

if \( \phi \in \mathcal{P}_{x_{\phi,t}} \), and \( y_{\phi,t} = 0 \) if \( \phi \notin \mathcal{P}_{x_{\phi,t}} \). As in Section 4.2, I assume the specialization function \( \gamma(\cdot) \) takes the simple form that \( \gamma(\mathcal{P}_i) = 0 \) if \( \mathcal{P}_i \) is a singleton and otherwise \( \gamma(\mathcal{P}_i) = \gamma \). I also assume that \( F_E = 0 \).

Proposition 6 illustrates that a single generalist agent can sometimes be optimal in this
Proposition 6  Consider the model in this section, and suppose that \( p < \frac{M-1}{M} \) and \( \frac{M-1}{M}(y_H p - c) > \gamma \). In an optimal equilibrium with the minimal number of agents in the market, there exists a \( \gamma^* \) such that if \( \gamma \leq \gamma^* \), there exist cutoffs \( \tilde{\delta}_S > \bar{\delta}_S \) such that:

1. If \( \delta > \tilde{\delta}_S \), then \( M \) firms enter the market and specialize in a single product;
2. If \( \delta \in (\bar{\delta}_S, \tilde{\delta}_S) \), then a single firm enters the market, specializing in \( M \) products;
3. Otherwise, no firms enter the market.

Proof:  See Appendix A.

The extent of cooperation in a relational contract depends on the shape of the bonus scheme \( \tau \): as in Lemma 3, agent \( i \) works hard only if there exists some bonus scheme \( \tau \) defined on the possible outcomes of products in \( \chi_{i,t} \) that induces high effort and satisfies

\[
\sup \tau - \inf \tau \leq \frac{\delta}{1 - \delta} (U_i^i + U_{i0}^0)
\]

Suppose agent \( i \) manufactures every product \( \{1, \ldots, M\} \); then he must only be motivated to work hard on each \( \phi \in \{1, \ldots, M\} \) based on the expected realizations of \( (y_1, \ldots, y_M) \). Because \( M \) is finite, agent \( i \) faces aggregate uncertainty about the vector of realized outputs. If it is unlikely that \( y_j = y_H \) for every product \( j \), the optimal contract can offer a relatively low payment for this outcome while still providing incentives for effort. The reneging temptation depends only on the largest and smallest bonuses paid, so it increases less rapidly than the continuation value produced by a single agent as the number of products made that agent increases. Hence, a generalist agent can be induced to work hard when there is aggregate uncertainty, even if a specialist agent would be unwilling to do so.

5.2 Extension 2 - Resistance to Contractual Innovation

Formal contracts unambiguously increase total surplus in this setting, since they lead to smaller specializations and higher effort. Suppose now that the agents have an unanticipated opportunity to make formal contracts available after they choose their specializations,
perhaps by supporting legal reform or codifying the production process. Although the principal would benefit from having access to formal contracts, the existing agents have already chosen to be generalists and so might resist any reform that leads to the entry of specialist competitors.\footnote{Unlike Baker, Gibbons, and Murphy (1994), the introduction of formal contracts in this extension has an unambiguously positive effect on social welfare.}

To make this argument formal, I make several restrictive assumptions. Suppose that after the agents have entered the market and chosen their specializations, they have an unanticipated opportunity to make $y_t$ contractible for the principal. For instance, the agents might be able to codify their knowledge of the production process, develop an internal auditing scheme that could be appropriated by the principal, or push for legal reform to eliminate corruption in the courts. Importantly, once this contracting technology is generated, it is a public good in that the principal can write formal contracts with any agent.

In order for the agents to be motivated to resist legal reform, they must have some stake in the relationship in the sense that each expects to earn positive profits whenever he produces. Therefore, I restrict attention to relational contracts in which each agent earns a fraction $\alpha \in (0, 1)$ of the total expected surplus if he is called upon to produce, and earns 0 otherwise. To simplify the argument, I assume that $F_E = 0$ and consider the optimal relational contract with the minimal number of entrants.

Formally, consider the following additions to the timing at the end of $t = 0$:

1. Legal reform becomes available: unless the current agents pay $F_R > 0$, output $y_t$, entry, and specializations become contractible. Assume that no players anticipate this stage before it occurs.

2. If legal reform occurs, the principal writes a long-term formal contract with existing and potential new entrants, who then choose whether or not to enter at cost $F_E = 0$.

3. If legal reform does not occur, players continue according to an optimal relational contract.\footnote{In particular, this assumptions implies that agents are not punished for resisting legal reform. One justification for this assumption is that by successfully resisting legal reform, and agent might ensure that the principal was never even aware of the opportunity for that reform.}


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These modifications to the game are ad hoc—they are meant to represent a market that is ripe for legal reform after operating for a long time. To make this point in the starkest way, assume the specialization cost $\gamma(\mu)$ is as in Section 4.2.

**Proposition 7** Suppose that

$$
\gamma(\mu) = \begin{cases} 
0, & \mu \leq \frac{1}{M} \\
\gamma, & \mu > \frac{1}{M}
\end{cases}
$$

Assume $\delta \in [\bar{\delta}, \bar{\delta})$, where $\bar{\delta}$ and $\bar{\delta}$ are defined in Proposition 5, and consider only relational contracts of the following form: there exists an $\alpha > 0$ such that $\forall t, h^{t-1}$, agent $i$’s payoff is at least $\alpha E[\pi_{i,t}^{TOT} | h^{t-1}_i, \phi_t, 1\{x_t = i\}] \geq 0$.\(^{13}\) In the optimal relational contract in this class, legal reform does not occur if $F_R \leq \alpha(y_{HP} - c - \gamma)$.

**Proof:** As in Proposition 5, one agent enters the market at the beginning of the game and specializes in $P_1 = [0, 1]$. Consider the continuation equilibrium if legal reform occurs. Because $(y_{HP} - c - \gamma) < (y_{HP} - c)$ and $F_E = 0$, the principal will write a long-term contract inducing high effort in each period with $M$ new entrants following legal reform. Those $M$ agents enter the market and specialize in disjoint subsets of measure $\mu_i = \frac{1}{M}$. Call agent 1 the original entrant in the market, while agents $2, ..., M + 1$ are the $M$ new entrants following legal reform.

Following legal reform, agent 1 is never allocated production and the principal has no incentive to pay him, so agent 1 earns 0. If legal reform does not occur, the continuation relational contract induces high effort from agent 1, who earns $\alpha(y_{HP} - c - \gamma)$ by the assumption that each agent earns at least $\alpha E[\pi_{i,t}^{TOT} | h^{t-1}_i, \phi_t, 1\{x_t = i\}]$. Therefore, agent 1 is willing to pay any $F_R \leq \alpha(y_{HP} - c - \gamma)$ in order to prevent legal reform. ■

This example highlights a tension inherent in developing markets that might prevent or delay the introduction of better contracts. The agents in a relational market prize flexibility, which is only valuable if relational contracts are required—if legal reform succeeds, these

\(^{13}\)That is, if $x_t = i$, then $i$ earns a percent $\alpha$ of the total surplus in the period, and if $x_t \neq i$, then $i$ earns 0.
agents are displaced by a larger group of highly specialized agents. Importantly, the principal cannot credibly commit to share the efficiency gains from legal reform. Any such promise would have to satisfy the principal’s reneging constraint to be credible, and if formal contracts are available, then the principal’s payoff following reneging is too attractive to support these promises.

5.3 Extension 3 - The Employment Relationship

In the context of the employment relationship, Proposition 4 suggests that employees and independent contractors (such as consultants) may make systematically different investments in human capital. To make this point precise, I modify the example from Section 4.2 so that the “principal” actually represents two different employers. At the beginning of the game, agents choose whether to enter at cost $F_E > 0$ and, if they do enter, choose specializations $P_i$. The fixed cost of a specialization $P_i$ is given by (3), and specializing in a product implies that the agent can make that product for either employer.

In each period, two products $\phi_1^t, \phi_2^t \sim U[0, 1]$ are independently drawn as required for that period, and for each product $\phi_i^t$, the principal asks one agent $x_i^t$ to produce. Critically, each agent can only manufacture a single product in each period, so $x_i^1 \neq x_i^2$ unless $x_i^1 = x_i^2 = \emptyset$. For product $k$, agent $x_i^k$ accepts or rejects production, chooses effort $e_i^k \in \{0, 1\}$ at cost $c e_i^k$, and produces binary output $y_i^k \in \{0, y_H\}$, where the probability of $y_H > 0$ is $p$ if $e_i^k = 1$ and $\phi^k \in P_{x_i^k}$, and 0 otherwise. The monitoring structure for all variables is just like the baseline game, and transfers between the principal and each agent $w_{i,t}, \tau_{i,t}$ are the same as that model.

One interpretation of this set-up is that the principal actually represents two different employers, each of whom has a single task $\phi_i^k$ that must be accomplished in each period. This interpretation is unusual because these two (presumably independent) employers act as a single player in the game. By modeling them as a single player, I ensure that the employers will always act in their joint best interest in order to highlight an interaction between employment and human capital investments. If I instead modeled the two employers as two separate players, then the optimal equilibrium might entail contagion-style punishments in which one employer serves to disseminate information to the agents when the other employer
reneges on a bonus. In order to completely rule out such contagion punishments (which may or may not be realistic in a given setting), I model the employers as a single player.

Agent $i$ is a contractor if $\Pr\{x^1_t = i\}, \Pr\{x^2_t = i\} \in (0,1) \forall t$ on the equilibrium path, so that $i$ produces either $\phi^1_t$ or $\phi^2_t$ with positive probability in each period. An agent is an employee if he exclusively produces one of $\phi^1_t$ or $\phi^2_t$, so that $\exists k \in \{1,2\}$ with $\Pr\{x^k_t = i\} = 0 \forall t$.

Formalizing this logic, Proposition 8 proves that for some parameters, employees—who are generalists and produce only one of the products $k \in \{1,2\}$—are (non-uniquely) optimal if output is non-contractible, while contractors—who work for both firms and specialize in a small subset of products—are optimal if output is contractible.

**Proposition 8** Consider the example presented in this section. Let

$$\frac{\delta}{1 - \delta}(y_{hp} - c - \gamma) \geq \frac{c}{\delta} > \frac{\delta}{1 - \delta}(1 - (1 - \frac{1}{M})^2)(y_{hp} - c)$$

and suppose that $\max \left\{ \bar{M}\bar{F}_E, y_{hp} - c - M^2\bar{F}_E \right\} < \gamma < \frac{M-1}{M}(y_{hp} - c)$. Then:

1. If output is contractible, $M$ workers enter the market in any equilibrium. Each specializes in $\frac{1}{M}$ of the interval $[0,1]$ and manufactures products for both principals.

2. If output is not contractible, then $2$ workers enter in any equilibrium. Each specializes in the interval $[0,1]$.

Moreover, if output is not contractible, there exists an optimal equilibrium in which each worker works exclusively for a single principal.

**Proof:** See Appendix A.

If agent $i$ is a specialist with $\mu(\mathcal{P}_i) = \frac{1}{M}$, then the principal can use that agent to produce either $\phi^1_t$ or $\phi^2_t$ when one of those products happens to lie in $\mathcal{P}_i$. if $M$ is large, then the probability of both $\phi^1_t \in \mathcal{P}_i$ and $\phi^2_t \in \mathcal{P}_i$ is very small, and so the principal only needs

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14For instance, the following chain of events might occur: if principal $A$ reneges on a bonus for agent $i$, then agent $i$ reneges on principal $B$. When principal $B$ observes this, she reneges on every other employee, who renege on employer $A$ in turn.
a single agent to specialize in the region \( \mathcal{P}_i \). In contrast, if \( i \) is a generalist with \( \mu(\mathcal{P}_i) = 1 \), then they will always be able and needed to produce one of the products, so the principal might assign \( i \) to be an employee for a single task \( k \in \{1, 2\} \). Generalists tend to be optimal when formal contracts are unavailable by a logic similar to the previous sections.

Proposition 8 can be interpreted to suggest a difference between employees and contractors. Contractors (1) produce contractible output, (2) are likely to be highly specialized and very good at their chosen tasks, and (3) work for multiple firms. In contrast, employees (1) produce non-contractible output, (2) are flexible, though perhaps not very efficient at any given task, and (3) are locked into a bilateral relationship with a single firm. In this model the decision whether to hire long-term or short-term workers is driven by the human capital investments required to sustain high effort. Employees and contractors are distinguished both by the differences in the contractibility of their jobs, and by their different \textit{ex ante} specializations. Put another way, the contractibility of output drives the \textit{ex ante} human capital acquired by a worker, and consequently whether that worker operates as a contractor or a employee. If formal labor contracts are not available, employees—who are locked into a bilateral relationship and can flexibly produce whatever product is required of them—might be favored over contractors.

6 Conclusion

In closing, I informally discuss a few more applications in the context of this model. First, markets sometimes interact with cultural norms to inhibit trade. For instance, Dixit (2011) points out that in institution-poor environments, firms from developing countries tend to have an advantage over multinationals that are based in developed nations. His explanation is that firms from developing countries understand how to navigate inefficient or corrupt institutions—they know whom to influence to get things done. I present a complementary story: a firm from a developed country might also be \textit{too specialized} to work well in a relational setting. Suppose that a “downstream” domestic firm (a retailer, perhaps) would like to contract with a multinational “upstream” firm. Such a multinational might be a very cost-effective producer of a few inputs, but it lacks the flexibility to be a close partner if
formal contracts are not available. As a result, foreign firms face multiple barriers to entry in developing markets: not only do they need to learn how to work around inefficient or corrupt formal institutions, they must also alter their production process to prize adaptability over efficiency.

Second, relational contracts sometimes build upon existing social ties—for instance, a CEO might hire a close relative to supply a service, rather than choosing the most cost-effective producer. In a relational contract, such behavior might be eminently justified, since ostracism and social sanctions are powerful forces that imply the parties can punish one another very harshly if one of them deviates from their agreement, which in turn prevents reneging. In Section 5.2, close social ties can also be used to induce an otherwise unwilling agent to accept legal reform rather than resist it. Unlike informal contracts, familial relationships exist independently of business ties. Therefore, social ties provide a “stick” that helps the principal commit to rewarding its suppliers for acquiescing to changes, even when those changes weaken their market relationship. While an outsider might observe inefficient production methods and attribute them to nepotism, closely intertwined business and social networks could also serve a valuable purpose by inducing higher effort and mitigating resistance to efficiency-enhancing institutional changes.

Finally, this model presents a simplistic view of firm specialization and capital acquisition, and it would be worthwhile to expand on the notion of *ex ante* investments and precisely illustrate how they interact with relational contracts. For example, suppliers often choose whether to invest in general or relationship-specific capital. In a static setting, relationship-specific investments can be appropriated ex post; in a repeated game, however, these investments increase lock-in and hence effort provision in a relationship (a point made by Klein and Leffler (1981)). Discussing the hold-up problem in this context requires an assumption about how surplus is split in a relational contract, and hence requires a theory of bargaining in repeated games.
References


A Omitted Proofs

A.1 Proof of Proposition 1:

This proof proceeds in four steps. A long-term contract can be written before entry and utility is transferrable, so the optimal contract will set \( M, \{ \mathcal{P}_i \}, \) and \( x_t, e_t, \forall t \) to maximize total surplus.

1. Optimal one-shot contract. Consider the following formal contract \( \tau(y) \): let \( \bar{y} \) be such that \( F(\bar{y}|e=0) - F(\bar{y}|e=1) \in (0, 1) \), which must exist because \( F(\cdot|e=0) >_{FOSD} F(\cdot|e=1) \). Then \( \tau(y) = \frac{c}{F(\bar{y}|e=0) - F(\bar{y}|e=1)} \) if \( y \geq \bar{y} \), and otherwise \( \tau(y) = 0 \). Under this contract, an agent chooses \( e = 1 \) since

\[
\frac{c}{F(\bar{y}|e=0) - F(\bar{y}|e=1)} (1 - F(\bar{y}|e=1)) - c \geq \frac{c}{F(\bar{y}|e=0) - F(\bar{y}|e=1)} (1 - F(\bar{y}|e=0)).
\]

It is an optimal static equilibrium for the principal of offer this formal contract, and so \( e_t = 1 \) in any round of the repeated game for which \( \phi_t \in \mathcal{P}_i \).

2. If agents choose \( e_t = 1, \forall t \), then specializations do not overlap. Suppose that \( \mu(\mathcal{P}_i \cap \mathcal{P}_k) > 0 \), and consider the alternative with \( e_t = 1, \forall t, \hat{\mathcal{P}}_j = \mathcal{P}_j \forall j \neq k \), and \( \hat{\mathcal{P}}_k = \mathcal{P}_k \setminus \mathcal{P}_i \). By claim 1, each agent chooses \( e_t = 1 \) in the static equilibrium with these alternative specializations. If \( \hat{\mathcal{P}}_k = \emptyset \), then \( k \) need not enter the market, which improves total surplus by \( F_E > 0 \); If \( \hat{\mathcal{P}}_k \neq \emptyset \), then firm \( k \)'s cost is \( \gamma(\hat{\mu}_k) < \gamma(\mu_k) \), also increasing total surplus.

3. \( \mu_i = \mu_j, \forall i,j \). Suppose \( M \) firms enter the market and choose \( \{ \mathcal{P}_i \}_{i=1}^M \). By claim 1, \( e_t = 1 \) in the efficient equilibrium. By claim 2, there is no overlap in specializations and so the problem is

\[
\min_{\{\mu_i\}} \sum_{i=1}^M \mu_i \gamma(\mu_i)
\]

subject to

\[
\sum_{i=1}^M \mu_i \leq 1
\]

\( \gamma(\cdot) \) is convex and increasing, so \( \mu \gamma(\mu) \) is convex. Hence, the solutions to this problem is \( \mu_i = \mu \forall i \) and some \( \mu \geq 0 \).
(4) Optimal entry. The measure $\mu$ is chosen to solve

$$\max_{\mu} M\mu(E[y|e = 1] - c - \gamma(\mu)) \quad (\text{subject to } M\mu \leq 1)$$

But $M\mu < 1$ cannot be a solution, since $\mu(E[y|e = 1] - c - \gamma(\mu))$ is strictly increasing in $\mu$. Hence, the optimal equilibrium sets $\mu = \frac{1}{M}$ and $P_i \cap P_j = \emptyset$, $\forall i, j$.

The efficient number of entrants $M$ solves

$$\max_{M} (E[y|e = 1] - c - \gamma(\frac{1}{M})) - F_E M$$

with first-order condition $\gamma'(\frac{1}{M}) = M^2 F_E$.

The left- and right-hand sides of this expression are strictly decreasing and increasing in $M$, respectively, so some (non-integer) $M^*$ equates the two sides. Total surplus is continuous in $M$ and $M^{FB}$ must be a natural number, so $M^{FB}$ is either the floor or the ceiling of $M^*$.

A.2 Proof of Lemma 1:

A deviation in any variable other than $e_t$ in round $t$ is publicly observed and hence can be punished by breakdown in the market, which leaves every player at the min-max payoff 0. Let $\sigma$ be an optimal equilibrium. If $e_t = 0$ at every history of $\sigma$, then it can be trivially replicated by a stationary contract. Otherwise, let $(h^{t-1}, n_t)$ be a history on the equilibrium path immediately following $e_t = 1$ and output $y_t$. Let $\tilde{\tau}(y_t) = E_\sigma[\tau_{x_t}|h^{t-1}, n_t]$ be the expected transfer to the producing agent in this history, and $U_i(y_t) = E_\sigma[U_i|h^{t-1}, n_t]$ $\forall i \in \{0, ..., M\}$ the expected continuation surplus (where $i = 0$ is the principal).

For $e_t = 1$ in equilibrium,

$$(1 - \delta)\tilde{\tau}(y_t) + \delta U_i(y_t) - c \geq (1 - \delta)\tilde{\tau}(y_t) + \delta U_i(y_t). \quad (4)$$
For transfer $\tilde{\tau}(y_t)$ to be supported in equilibrium,

\[
(1 - \delta)\tilde{\tau}(y_t) \leq \delta U_0(y_t) \quad \forall y_t
\]

\[
-(1 - \delta)\tilde{\tau}(y_t) \leq \delta U_{x_t}(y_t)
\]

since otherwise at least one player would prefer to deviate, not pay a bonus, and earn min-max payoff 0 in the rest of the game.

Define $\tau(y_t) = \tilde{\tau}(y_t) + \frac{\delta}{1 - \delta} U_{x_t}(y_t)$, and note that $\tau(y_t) \geq 0$ by (5). Consider the following stationary equilibrium: At $t = 0$, $M$ firms enter and specialize as in $\sigma$, and $w_{i,0} = F_E$. For all $t > 0$ on the equilibrium path, $x_t$ is the agent with the smallest specialization $\mu_{x_t}$, such that $\phi_t \in P_{x_t}$, $w_{x_t,t} = c + \gamma(\mu_{x_t}) - E[\tau(y_t)]$, and $w_{i,t} = 0 \ \forall i \neq x_t$. Agent $x_t$ accepts ($d_t = 1$) and chooses $e_t = 1$. Following output $y_t$, $\tau_{x_t} = \tau(y_t) \geq 0$ and $\tau_i = 0 \ \forall i \neq x_t$. Any deviation is immediately punished by reversion to the static equilibrium.

At $t = 0$, the agent is indifferent between entering or not. The principal is willing to pay $w_{i,0} = F_E$ because $MF_E$ is smaller than the principal’s total continuation surplus in an optimal equilibrium. For all $t > 0$, each agent earns 0. For each $\phi_t \in [0,1]$, let $\mu^m(\phi_t)$ be the smallest specialization $\mu(P_i)$ such that $\phi_t \in P_i$. Then the principal earns

\[
U_0 = \int_0^1 1\{\exists i \in \{1, ..., M\} \text{ s.t. } \phi_t \in P_i\} E[y_t - c - \gamma(\mu^m(\phi_t))|e_t = 1]d\phi_t
\]

from every on-path history in this relational contract.

The principal or agent $x_t$ is willing to pay $w_{x_t,t}$ because doing so earns player at least 0, while failing to do so earns them 0. For all $t$, agent $x_t$ is willing to work hard because (4) holds for $\tau(y_t)$. The principal is willing to pay $\tau(y_t) \geq 0$ because

\[
(1 - \delta)\tau(y_t) = (1 - \delta)\tilde{\tau}(y_t) + \delta U_{x_t}(y_t) \leq \delta U_0(y_t) + U_{x_t}(y_t) \leq \delta U_0
\]

where the first inequality follows from (5) and the second inequality is because $U_0$ is the maximum expected surplus attainable given entry $M$ and specializations $\{P_i\}$. Thus, we have found a stationary equilibrium that produces at least as much total surplus as the posited original equilibrium. ■
A.3 Proof of Proposition 3:

Let \( V(M, \{P_i\}) \) be the expected total surplus in the optimal stationary equilibrium with \( M \) firms and specializations \( \{P_i\} \). Byemma 1, there exists a stationary optimal continuation equilibrium in which the principal earns total surplus:

\[
V(M, \{P_i\}) = \begin{cases} 
-M F_E, & \text{if (6) does not hold} \\
\int_0^1 \{ i \text{ s.t. } \phi_i \in P_i \} E[y_t - c - \gamma(\mu^m(\phi_i))|c_t = 1]d\phi_t - M F_E, & \text{otherwise} 
\end{cases}
\]

Suppose that at the beginning of the game, the total-surplus maximizing number of agents \( M^{Pub} \) enter and choose optimal specializations \( \{P_i\}_{i=1}^M \). The principal pays \( F_E \) to each entrant and play continues according to the optimal stationary equilibrium, while any deviation is punished by a breakdown of the market. Since \( V(M,\{P_i\}) \geq 0 \), the principal would rather pay \( M F_E \) than suffer relational breakdown, so firms are willing to enter because they earn 0 and hence this is an equilibrium. Therefore, it suffices to find the \( M \) and \( \{P_i\} \) that maximize \( V \).

Suppose that \( \mu(P_i \cap P_k) \neq 0 \), and consider the alternative equilibrium in which agent \( k \) does not specialize in \( P_i \cap P_k \). If \( P_k \setminus P_i \subseteq \bigcup_{i \neq k} P_i \), then agent \( k \) can exit the market without affecting total surplus; otherwise, the fixed cost \( \gamma(\mu_k) \) of agent \( k \) strictly decreases. In either case, \( V(M,\{P_i\}) \) strictly increases, so it must be that \( \mu(P_i \cap P_k) = 0 \) in the optimal equilibrium.

If \( \#(M,\{P_i\}) \) such that \( V(M,\{P_i\}) > 0 \), then \( M = 0 \) is optimal in equilibrium. Otherwise, \( \mu_i = \frac{1}{M} \) because \( \gamma(\mu) \) is convex, as in Proposition 1, so the optimal equilibrium maximizes total surplus subject to satisfying (6). Ignoring this constraint, \( M^{FB} \) maximizes surplus by definition. The constraint (6) is relaxed as \( M \) increases, so the optimal number of entrants is either \( M = 0 \) or \( M \geq M^{FB} \).

A.4 Proof of Lemma 2:

Towards contradiction. Suppose that

\[-(1 - \delta)E_\sigma [\tau_{i,t}|h_{i}^{t-1}, I_{i}(n_t)] > \delta E_{\sigma^*} [U_i(h^t)|h_{i}^{t-1}, I_{i}(n_t)].\]
Consider the following deviation for agent \( i \): do not pay this transfer, and in future rounds pay no transfers, never accept production, and never exert effort. This deviation yields payoff 0, so is profitable given agent \( i \)'s information set \((h_{i}^{t-1}, I_{i}(n_{t}))\).

Suppose now that \( (1 - \delta)E_{\sigma} [\tau_{i,t}|h_{i}^{t-1}, I_{i}(n_{t})] \geq \delta E_{\sigma^{*}} [U_{0}^{i}(h^{t})|h_{i}^{t-1}, I_{i}(n_{t})] \). Then it must be that \( \tau_{i,t} > U_{0}^{i}(h^{t}) \) for at some history in this information set. Consider the following deviation for the principal: do not pay \( \tau_{i,t} \) in round \( t \). In the continuation game, replicate equilibrium play following history \((h_{t-1}^{t}, n_{t})\), with the exception that \( w_{i,t} = \tau_{i,t} = 0 \) in every future period whenever \( x_{t} = i \) is specified by the equilibrium, instead play \( x_{t} = \emptyset \). Because \( w_{i,t}, \tau_{i,t}, \) and \( x_{t} = \emptyset \) are not observed by any agents \( j \neq i \), all other agents do not detect a deviation. The principal earns no less than 0 from agent \( i \) in each future period, so her continuation surplus is bounded below by

\[
\delta E \left[ \sum_{k \neq i} U_{0}^{i}(h^{t})|h^{t-1}, n_{t} \right] > -(1 - \delta)E_{\sigma} [\tau_{i}|h^{t-1}, n_{t}] + \delta E \left[ \sum_{i=1}^{N} U_{0}^{i}(h^{t})|h^{t-1}, n_{t} \right]
\]

and hence this deviation is profitable. ■

A.5 Proof of Lemma 3:

Suppose there exists an equilibrium \( \sigma \) and an on-path history \((h_{i}^{t-1}, n_{t})\) immediately preceding effort \( e_{t} \) such that \( x_{t} = i \) and \( e_{t} = 1 \). Then

\[
E_{\sigma} \left[ (1 - \delta)\tau_{x_{t},t} + \delta U_{x_{t}}(h^{t}) \mid h_{x_{t}}^{t-1}, I_{x_{t}}(n_{t}), e_{t} = 1 \right] - c \geq E \left[ (1 - \delta)\tau_{x_{t},t} + \delta U_{x_{t}}(h^{t}) \mid h_{x_{t}}^{t-1}, I_{x_{t}}(n_{t}), e_{t} = 0 \right]
\]

(7)

for \( e_{t} = 1 \) to be incentive compatible. Define \( \tilde{\tau}(y_{t}) = E_{\sigma^{*}} [\tau_{x_{t},t} + \frac{\delta}{1 - \delta} U_{x_{t}}|h_{x_{t}}^{t-1}, I_{x_{t}}(n_{t}), y_{t}] \).

By Lemma 2, the following inequalities must hold for \( \tau_{i} \) to be incentive compatible \( \forall y_{t} \):

\[
(1 - \delta)E_{\sigma} [\tilde{\tau}|h_{i}^{t-1}, I_{i}(n_{t}, y_{t})] \leq \delta E_{\sigma^{*}} [U_{0}^{i}(h^{t}) + U_{0}(h^{t})|h_{i}^{t-1}, I_{i}(n_{t}, y_{t})]
\]
\[-(1 - \delta)E_{\sigma} [\tilde{\tau}|h_{i}^{t-1}, \mathcal{I}(n_{t}, y_{t})] \leq 0\]

By definition,
\[U_{0}^{i}(h_{i}^{t-1}, n_{t}) + U_{i}(h_{i}^{t-1}, n_{t}) = E_{\sigma} [\sum_{t'=t+1}^{\infty} \delta^{t'-t-1}(u_{i} + \pi_{0}^{i})|h_{i}^{t-1}, n_{t}] = \mu_{i}(E[y_{t}|e = 1] - c - \gamma(\mu_{i}))\]

because \(\mu_{i}(E[y_{t}|e = 1] - c - \gamma(\mu_{i}))\) is the maximum surplus produced by agent \(i\) given \(\mathcal{P}_{i}\).

\(\tilde{\tau}(y)\) satisfies (7) and so is an incentive scheme that induces high effort, and moreover
\[(1 - \delta) \left( \sup_{y} \tilde{\tau}(y) - \inf_{y} \tilde{\tau}(y) \right) \leq \mu_{i}(E[y - \gamma(\mu_{i})|e = 1]) .\]

Hence, whenever agent \(i\) chooses \(e_{t} = 1\) in equilibrium, there exists an incentive scheme \(\tilde{\tau}(y)\) that satisfies (7) and (2). \(\blacksquare\)

**A.6 Proof of Corollary 2:**

Suppose that \(\mu(\mathcal{P}_{i} \cap \mathcal{P}_{k}) = 0\) for every \(i, k \in \{1, ..., M\}\). By Lemma 3, agent \(i\) exerts high effort in equilibrium only if (2) holds; I construct a stationary equilibrium in which this condition is also sufficient.

Fix some \(\tau(y)\) that minimizes \(\sup_{y} \tau(y) - \inf_{y} \tau(y)\) among all incentive schemes such that (7) holds, and assume without loss that \(\inf_{y} \tau_{i}(y) = 0\). Consider the following strategy profile: the principal allocates \(\phi_{t}\) to \(i\) such that \(\phi_{t} \in \mathcal{P}_{i}\) (if multiple agents are available—which occurs with probability 0—then the principal chooses the lowest-numbered agent that has the smallest \(\mu_{i}\) among those for which (2) holds). If (2) holds for \(x_{t}\), then \(w_{x_{t},t} = c + \gamma(\mu_{i}) - E[\tau_{i}(y)|e = 1]\), \(d_{t} = e_{t} = 1\), and \(\tau_{x_{t},t} = \tau(y_{t}) \geq 0\) following output \(y_{t}\). If (2) does not hold, then the principal pays \(w_{x_{t},t} = \tau_{x_{t},t} = 0\) and the agent rejects production. In either case, \(w_{i,t} = \tau_{i,t} = 0\) \(\forall i \neq x_{t}\). After a commonly observed deviation, the entire market breaks down. After a deviation observed by agent \(i\) and the principal, agent \(i\) thereafter rejects production, \(w_{i,t} = \tau_{i,t} = 0\) in each period, and \(x_{t} = \emptyset\) whenever \(i\) is the only agent able to produce. The principal plays the on-path actions for all agents \(j \neq i\).

I claim that this strategy profile is an equilibrium. Let \(\mathcal{U}\) be the set of agents for which
(2) holds. In each period on the equilibrium path, every agent earns 0 and so the principal earns the total expected surplus for that period. The principal has no profitable deviation from the allocation rule on the equilibrium path, since either there is only one available agent or the principal is allocating production to the agent that maximizes expected surplus in that period. If (2) does not hold, then play is a mutual best response both on and off the equilibrium path. If (2) holds, then the agent is indifferent between accepting and rejecting production and willing to choose \( e = 1 \) by construction of \( \tau_i(y) \). Whoever is responsible for paying \( w_{x,t} \) is willing to do so because paying \( w_{x,t} \) yields higher continuation surplus by construction. If the principal does not pay \( \tau(y) \), then she no longer earns any surplus from agent \( i \). Because \( \mu(\mathcal{P}_i \cap \mathcal{P}_k) = 0 \) for every \( i, k \in \{1, \ldots, M\} \), the total loss from this deviation is
\[
\delta \mu_i (E[y|e = 1] - c - \gamma(\mu_i))
\]
and the total gain from the deviation is \( \tau_i(y) \). This is not a profitable deviation because (2) holds.

Off the equilibrium path, strategies among those that have observed the deviation are a mutual best response by construction. Pay among those who have not observed are likewise a mutual best-response. Therefore, this strategy profile is an equilibrium that induces high effort from every agent \( i \) whose \( \mathcal{P}_i \) satisfies (2). It is optimal because (2) is a necessary condition for high effort. \( \square \)

A.7 Proof of Corollary 3:

From Lemma 3, agent \( i \) is only willing to choose \( e = 1 \) in an equilibrium if (2) holds. The same set of transfer schemes induce high effort regardless of \( \mu_i \) or \( \delta \). Define \( \mu^*(\delta) \) to solve the following
\[
\inf_{\{\tau\} \in \{\tau\}} \left\{ \sup_y \tau(y) - \inf_y \tau(y) \right\} = \frac{\delta}{1 - \delta} \mu^*(\delta) (E[y|e = 1] - c - \gamma(\mu^*(\delta)))
\]
whenever this is well-defined. Because \( \sup \tau_i - \inf \tau_i \geq c > 0 \), \( \mu^*(\delta) > 0 \) defines the minimum measure of specialization required to induce an agent to choose \( e = 1 \). Because \( \gamma(\mu) \) is
continuous and \( \mu(E[y|e = 1] - c - \gamma(\mu)) \) is increasing in \( \mu \) for \( \mu \in [0,1] \), \( \mu^*(\delta) \) exists, is continuous, and satisfies \( \lim_{\delta \rightarrow 1} \mu^*(\delta) = 0 \) and \( \lim_{\delta \rightarrow \hat{\delta}} \mu^*(\delta) = 1 \) for some \( \hat{\delta} < 1 \).

If \( \mu(S_i \cap S_k) = 0 \), then a stationary contract is optimal and this condition is sufficient to induce high effort by Corollary 2, so \( e = 1 \) at every history in the optimal equilibrium iff \( \mu_i \geq \mu^*(\delta), \forall i \).

**A.8 Proof of Proposition 4:**

To prove this result, I introduce an equilibrium with the desired properties and show that it is optimal.

First, fix the number of entrants \( M \leq M^{FB} \), and consider the continuation game. Suppose that \( \mu^*(\delta)M \leq 1 \). I first claim that the optimal equilibrium with \( M \) firms satisfies \( \mu(\mathcal{P}_i \cap \mathcal{P}_k) = 0 \) and \( \mu(\mathcal{P}_i) = \mu_M, \forall i, k \) and some \( \mu_M \in [0,1] \). Suppose not, and assume that the optimal equilibrium generates total surplus \( v^* \).

Let \( \tilde{v} \) is the surplus generated under the same allocation rule if \( e_t = 1 \) in each period. Then \( v^* \leq \tilde{v} \), where this inequality hold strictly if \( v^* \) ever has \( e_t = 0 \) on the equilibrium path.

\[
\tilde{v} = E_\sigma \left[ \sum_{t=0}^{\infty} \delta^t (1 - \delta) \sum_{i=1}^M \int_{\mathcal{P}_i} 1\{x_t = i\} \{E[y|e = 1] - c - \gamma(\mu(\mathcal{P}_i))\} \right].
\]

In turn, \( \tilde{v} \) is dominated by the strategy profile in which (1) \( e_t = 1 \) in each period, (2) specializations do not overlap, and (3) the principal allocates to the unique producer in each period. Define \( \{\mathcal{P}_i^{NO}\} \) as the set of disjoint specializations created by removing \( \mathcal{P}_i \cap \mathcal{P}_k \) from one of \( i, k \)'s specializations. Let \( v_{NO} \) be the resulting surplus, so \( v_{NO} \geq \tilde{v} \) with strict inequality if \( \mu(\mathcal{P}_i \cap \mathcal{P}_k) > 0 \) for some \( i \neq k \).

\[
v_{NO} = E_\sigma \left[ \sum_{t=0}^{\infty} (1 - \delta)\delta^t \left\{ \mu(\mathcal{P}_1^{NO} \cup \ldots \cup \mathcal{P}_M^{NO}) \{E[y|e = 1] - c\} - \sum_{i=1}^M \mu(\mathcal{P}_i^{NO}) \gamma(\mu(\mathcal{P}_i^{NO})) \right\} \right].
\]

Because \( \mu \gamma(\mu) \) is strictly convex, \( v_{NO} \) is dominated by the surplus generated if all agents choose \( e_t = 1 \) and have equally-sized, non-overlapping specializations \( \mu^{ES} = \frac{1}{M} \mu(\mathcal{P}_1^{NO} \cup \ldots \cup \mathcal{P}_M^{NO}) \).
This alternative strategy profile generates total surplus \( v_{ES} \), where \( v_{ES} \geq v_{NO} \) and this inequality holds strictly if specializations \( \{ P_i^{NO} \} \) are not of equal size.

\[
v_{ES} = M \mu^{ES} \left( E[y|e = 1] - c - \gamma(\mu^{ES}) \right).
\]

Finally, recall that \( \mu(E[y|e = 1] - c - \gamma(\mu)) \) is increasing in \( \mu \). Therefore, \( v_{ES} \) is dominated by the surplus generated if each agent picks \( e = 1 \), has an equally-sized specialization, and specializations collectively cover \([0,1]\). Define

\[
v_{OPT} \equiv \sum_{t=0}^{\infty} \delta^t \left( E[y|e = 1] - c - \gamma(\frac{1}{M}) \right)
\]

as the surplus in this alternative.

By Lemma 3 and Corollary 2, because \( \frac{1}{M} \geq \mu^*(\delta) \), \( v_{OPT} \) can be generated in an equilibrium of the repeated game, provided that (1) \( \mu_i = \frac{1}{M} \) with \( P_i \cap P_k = \emptyset \), and (2) \( M \leq M^{FB} \) firms enter the market. It remains to show that such an equilibrium exists.

Consider the following strategies:

1. \( M \) agents enter the market, labelled \( \{1,\ldots,M\} \).

2. agent \( i \in \{1,\ldots,M\} \) specializes in the interval \( P_i = \left[ \frac{i-1}{M}, \frac{i}{M} \right] \).

3. \( i \in \{1,\ldots,M\} \), \( w_{i,0} = \max \{0, F_E - U_i\} \), where \( U_i \) is the surplus earned by agent \( i \) in the continuation game.

4. Play continues as in the optimal stationary contract with agents \( \{1,\ldots,M\} \).

5. If either fewer or more than \( M \) firms enter, or specializations differ from those specified in step 2, then continuation play specifies \( w_i = \tau_i = 0 \) and \( e = 0, \forall i \), at every history.

Because both specializations and the set of entrants are public knowledge, the punishment strategy specified in step 5 is feasible. Hence, the number of entrants will not exceed \( M \), and entrants will specialize in the specified interval. If agent \( i \) expects to be paid \( w_{i,0} = \max \{0, F_E - U_i\} \), then she weakly prefers to enter the market, since the surplus from entering is \( w_i + U_i - F_E \geq 0 \). Moreover, the principal is willing to pay \( w_i \): if she does not, her
loss is $\frac{1}{M} (E[y|e = 1] - c - \gamma(\frac{1}{M})) - U_i$, which is larger than $w_i$ because $F_E \leq \frac{1}{M} (E[y|e = 1] - c - \gamma(\frac{1}{M}))$ for $M \leq M^{FB}$.

If $\mu^*(\delta) \leq \frac{1}{M}$, the total surplus generated by the efficient equilibrium is

$$E[y|e = 1] - c - \gamma(\frac{1}{M}) - M F_E$$

This expression is increasing in $M$ for $M < M^{FB}$ by definition of $M^{FB}$. Thus, the number of entrants in the optimal equilibrium is bounded below by $M(\delta) = \min\left\{ M^{FB}, \text{floor}\left\{ \frac{1}{\mu^*(\delta)} \right\} \right\}$.

To prove statements 2 and 3 of the Proposition, suppose that $M\mu^*(\delta) = 1$. For $M \leq M^{FB}$, such a $\delta$ exists because $\mu^*(\delta)$ varies continuously on $[0,1]$. By the argument above, at least $M$ firms enter the market. If exactly $M$ firms enter the market and specialize in disjoint subsets of length $\mu^*(\delta)$, then total surplus is

$$\sum_{t=0}^{\infty} \delta^t \{y_{HP} - c - \gamma(\frac{1}{M})\} - M F_E$$

For $M < M^{FB}$, (8) is increasing in $M$. $\mu^*(\delta) = \frac{1}{M}$, so $\gamma(\mu_i) \geq \frac{1}{M}$ by Corollary 3 and hence (8) is the largest surplus that can be generated in an equilibrium with discount rate $\delta$.

Therefore, every efficient equilibrium has $M$ firms enter the market and specialize in subsets $\frac{1}{M}$ of the unit interval. Hence, when $M\mu^*(\delta) = 1$, exactly $M$ firms enter.

Now, consider a small increase in $\delta$. At least $M$ agents will enter the market by statement 1, so suppose that $M + K$ agents enter. Then total surplus is no more than

$$\sum_{t=0}^{\infty} (1 - \delta)\delta^t \{y_{HP} - c - \gamma(\mu^*(\delta))\} - (M + K) F_E$$

But $\mu^*$ and $\gamma$ are both continuous, and $F_E > 0$. Hence, there exists some open set $\Delta(M)$ such that for $\delta \in \delta(M)$, (8) is strictly larger than (9) and so the optimal equilibrium entails $M$ agents. ■
A.9 Proof of Proposition 6:

I first claim that first-best can be attained if and only if

\[ \frac{c}{p} \leq \frac{\delta}{1-\delta} (y_{HP} - c). \]

Suppose this inequality holds, and consider the following strategy profile:

1. \( M \) agents enter the market, each specializing in a single product with \( P_i \cap P_k = \emptyset \forall i, k \). The principal pays each entrant \( w_{i,0} = F_E \).

2. In each period, the principal allocates production to the unique \( i \) with \( \phi_t \in P_i \).

3. The principal offers wage \( w_{i,t} = 0 \); the agent accepts and chooses \( e_t = 1 \).

4. If \( y_t = 0 \), \( \tau_{x_t,t} = 0 \); if \( y_t = y_H \), \( \tau_{x_t,t} = \frac{c}{p} \). \( \tau_{i,t} = 0 \) for all \( i \neq x_t \).

5. If the principal deviates on \( x_t \), then every agent rejects and chooses \( e_t = 0 \) in each future round, and \( w_{i,t} = \tau_{i,t} = 0 \ \forall i \). If either principal or agent deviates on \( w_{i,t} \) or \( \tau_{i,t} \), then whenever \( x_t = i \), the agent rejects and chooses \( e_t = 0 \) in every future round, and \( w_{i,t} = \tau_{i,t} = 0 \) for that agent, but otherwise play continues as on the equilibrium path.

The principal earns \( M(y_{HP} - c) \) in continuation surplus following any on-path history, and the agent earns 0. The principal loses \( (y_{HP} - c) \) following a deviation in \( w_{i,t} \) or \( \tau_{i,t} \), and earns 0 following a deviation in \( x_t \). Each agent is indifferent between entering the market or not, and the principal is willing to pay \( w_{i,0} = F_E \) because \( y_{HP} - c - F_E > 0 \). The principal and agent are trivially willing to pay the wage, and the principal is willing to pay \( \tau_{x_t,t} \) so long as

\[ \frac{c}{p} \leq \frac{\delta}{1-\delta} (y_{HP} - c) \]

which holds by assumption. The agent is willing to work hard because \( \frac{c}{p} - c = 0 \). Players mutually best-respond off the equilibrium path, so this describes an equilibrium that attains first-best.
Suppose instead that \( \frac{c}{p} > \frac{\delta}{1-\delta} (y_HP - c) \), and fix a strategy profile \( \sigma \) that attains first-best. By an argument similar to Lemmas 2 and 3, it must be that

\[
(1 - \delta) \left( \sup_y E_\sigma [\tau_i h_{i-1} | I_i(n_t, y_t)] - \inf_y E_\sigma [\tau_i h_{i-1} | I_i(n_t, y_t)] \right) \leq \delta (y_HP - c).
\]

Plugging in the optimal contract, \( \frac{c}{p} \leq \frac{\delta}{1-\delta} (y_HP - c) \), which contradicts the assumption.

Next, I claim that there exists a set of \( \delta \) for which \( \frac{c}{p} > \frac{\delta}{1-\delta} (y_HP - c) \) but a single agent with \( \mathcal{P}_1 \) can be motivated to work hard. By the argument above, any firm specializing in one product will never exert high effort under this condition. Therefore, every firm entering must specialize in at least two products, and so the maximum surplus in the market is \( M(y_HP - c - \gamma) \). Using an argument similar to Lemmas 2 and 3, a stationary relational contract can be constructed similarly to above, but with a single entrant who specializes in \( \mathcal{P}_1 = \{1, ..., M\} \) and \( w_{1,t} = \gamma \) in each period.

Define an incentive scheme as \( \{\tau^M, \tau^{M-1}, ..., \tau^0\} \), where \( \tau^k \) is the bonus payment if \( y_H \) is observed for \( k \) products. This incentive scheme can be implemented in the stationary relational contract if and only if

\[
\sup_k \tau^k - \inf_k \tau^k \leq \frac{\delta}{1-\delta} M(y_HP - c - \gamma)
\]

I claim that there exists a threshold \( \gamma^* \) such that if \( \gamma \leq \gamma^* \), high effort can be supported with a single firm but not with \( M \) firms. It suffices to show that for some \( \delta \) such that \( \frac{c}{p} > \frac{\delta}{1-\delta} (y_HP - c) \), there exists a bonus scheme \( \{\tau^i\} \) that induces an agent to work hard on all \( M \) products and satisfies \( \sup_i \tau^i - \inf_i \tau^i \leq \frac{\delta}{1-\delta} M(y_HP - c - \gamma) \).

To begin, consider the incentive scheme \( \tilde{\tau}^k = k \frac{c}{p} \). Define \( \Psi_K \) as the event that \( e_{k,t} = 1 \) for \( K \) of the \( M \) products. Then under the incentive scheme \( \{\tilde{\tau}^k\} \), I first argue that

\[
\frac{c}{p} E[\#\{y = y_H\} | \Psi_K] - Kc
\]

is weakly increasing in \( K \). Fix every outcome except \( y_k \), and denote this vector of \( M - 1 \) outputs \( y_{-k} \). For any fixed \( y_{-k} \), the value of choosing \( e_k = 1 \) is \( \frac{c}{p} p - c \geq 0 \). Therefore, regardless of the outcome of the other tasks, the agent always weakly prefers to pick \( e_k = 1 \)
under the specified contract, so the agent’s payoff is weakly increasing in $K$.

Now, consider the IC constraints for an arbitrary incentive scheme $\{\tau^k\}$. The agent is willing to choose $e_{k,t} = 1$, $\forall k, t$, if

$$E_k[\tau^k | \Psi_M] - Mc \geq E_k[\tau^k | \Psi_K] - Kc, \forall K \in \{0, ..., M\}$$

Consider the incentive scheme $\tau^k = \frac{c}{p^k}$, $\forall k < M - 1$, $\tau^{M-1} > \frac{c}{p} (M - 1)$, and $\tau^M < \frac{c}{p} M$, where $\tau^{M-1}$ and $\tau^M$ are chosen so that

$$E_k[\tau^k | \Psi_M] \geq \frac{c}{p} E[\#\{y = y_H\} | \Psi_M]$$

Such an incentive scheme exists so long as $p < 1$. I claim that this alternative contract continues to satisfy the IC constraints. Since $\tau^k = \tilde{\tau}^k$ for all $k < M - 1$ and $\tau^{M-1} > \tilde{\tau}^{M-1}$, $E_k[\tau^k | \Psi_K] - Kc$ is increasing for all $K \leq M - 1$. Therefore, it suffices to show that

$$E_k[\tau^k | \Psi_M] - Mc \geq E_k[\tau^k | \Psi_{M-1}] - (M - 1)c$$

or

$$\sum_{k=0}^{M} \left( \begin{array}{c} M \\ k \end{array} \right) p^k (1 - p)^{M-k} \tau^k \geq \sum_{k=0}^{M-1} \left( \begin{array}{c} M - 1 \\ k \end{array} \right) p^k (1 - p)^{M-1-k} \tau^k + c$$

strictly slackens when $\tau^{M-1}$ increases, since this slack can then be used to decrease $\tau^M$.

The coefficient on $\tau^{M-1}$ is $\frac{M!}{(M-1)!} p^{M-1}(1-p)$ on the left-hand side, and $\frac{(M-1)!}{(M-1)!} p^{M-1} = p^{M-1}$ on the right-hand side. Thus, increasing $\tau^{M-1}$ strictly relaxes the IC constraint if $p < \frac{M-1}{M}$.

Under this parameter restriction, $\sup_k \tau^k < M \frac{p}{p}$ when a single agent enters the market and manufactures every product. Hence, for $\gamma > 0$ sufficiently small, there exists an open interval of $\delta$ for which $M \frac{p}{p} > \frac{\delta}{1-\delta} (y_H p - c)$ but

$$\sup_i \tau^i - \inf_i \tau^i \leq \frac{\delta}{1-\delta} (y_H p - c - \gamma)$$

On this interval, a single generalist firm enters the market in the optimal equilibrium. ■
A.10 Proof of Proposition 8:

First, suppose that output is not contractible. I claim that an agent with specialization $\mu_i \leq \frac{1}{M}$ will never choose $e_t = 1$ in a relational contract. As in the baseline model, following any deviation in $\tau_{i,t}$ with agent $i$, the principal can always allocate business as if no deviation has occurred, but set $\tau_{i,t} = w_{i,t} = 0$ and $x_{k}^{t} = \emptyset$ whenever he would have chosen $x_{i}^{t} = i$ on the equilibrium path. Similarly, agent $i$ can always set $\tau_{i,t} = w_{i,t} = 0$ and $d_t = 0$. As a result, in equilibrium it must be that

$$(1 - \delta)\tau_{i,t} \leq \delta E [U^{i}_{0}(h^{t})|h^{t-1}, n_{t}]$$

$- (1 - \delta)\tau_{i,t} \geq \delta E [U^{i}(h^{t})|h^{t-1}, I_{i}(n_{t})]$$

for any $(h^{t-1}, n_{t})$ immediately following the realization of output $y_{t}$. Worker $i$ can produce only once per round and only if $\phi_{k}^{t} \in P_{i}$ for some $k \in \{1, 2\}$. Therefore, agent $i$ is able to product at least one of the products in a period with probability $(1 - (1 - \mu_{i})^{2})$. By an analogous argument to Lemma 3, in any PBE in which agent $i$ chooses $e_t = 1$, it must be that

$$\frac{c}{p} \leq \frac{\delta}{1 - \delta} (1 - (1 - \mu_{i})^{2})(y_{H}p - c).$$

By assumption, this inequality does not hold for $\mu_{i} \leq \frac{1}{M}$ because the right-hand side of (10) is increasing in $\mu_{i}$.

Hence, in any optimal equilibrium every entrant specializes in $\mu_{i} > \frac{1}{M}$. Then the maximum feasible surplus is $2(y_{H}p - c - \gamma) - 2\bar{F}_{E}$, which is attained in the following stationary equilibrium:

1. Two workers enter and specialize in $\mu(P_{1}) = [0, 1]$, $\mu(P_{2}) = [0, 1]$. $w_{i,0} = \bar{F}_{E}$

2. $\phi_{1}^{t}$ is always assigned to $i = 1$, $\phi_{2}^{t}$ is always assigned to $i = 2$.

3. $w_{i,t} = \gamma$ for both agents. Agents accept and choose $e = 1$.

4. If $y_{t} = y_{H}$, $\tau_{i} = \frac{\xi}{p}$; otherwise, $\tau_{i} = 0$.

The agents are willing to follow the equilibrium entry and effort decisions by construction.
The principal is willing to pay \( w_{i,t} \) for reasons similar to those argued in Proposition 6, and willing to pay \( \tau_i \) so long as
\[
\frac{c}{p} \leq \frac{\delta}{1 - \delta}(yHp - c - \gamma)
\]
which holds by assumption. Note that each agent is an employee in this relational contract.

Suppose instead that output is contractible, so that every agent chooses \( e_t = 1 \) in every period. Note that it is weakly optimal if \( \forall \) agent \( i, \mu_i \in \{\frac{1}{M}, 1\} \). Call \( i \) a “specialist” if \( \mu_i = \frac{1}{M} \) and a “generalist” if \( \mu_i = 1 \). Because only two products are required in equilibrium, there will be at most two generalists in the market. Fix the number of generalists in the market and consider the number of specialists. Suppose it is optimal for at least one specialist to enter the market, which is implied by the condition \( \gamma > M\tilde{F}_E \). Then it is optimal for at least \( M \) specialists to enter the market, because these specialists can choose disjoint specializations and each generate the same additional surplus as the first specialist. If \( M \) specialists enter the market, then it is not optimal to have two generalists in the market.

More than \( 2M \) specialists will never enter the market in the optimal equilibrium. Fixing the number of generalists \( G \in \{0, 1, 2\} \), suppose that \( 2M \geq K > M \) specialists enter the market. Because only two products are required in each period, it is never optimal for more than two specialists to produce the same product. The total measure of production is \( 2 \geq \frac{K}{M} > 1 \). Let \( \mu^1 \) be the measure of products that have exactly one producer and \( \mu^2 \) the measure of products that have two producers. Then surplus can be written
\[
(1 - \mu^1 - \mu^2)G(yHp - c - \gamma) + \mu^1(yHp - c + 1\{G \neq 0\}(yHp - c - \gamma)) + \mu^2 2(yHp - c)
\]
subject to the constraint that measures of specialization are between 0 and 1 and sum to \( \frac{K}{M} \):
\[
\mu^1 + 2\mu^2 = \frac{K}{M}, \quad \mu^k \in [0, 1], k \in \{1, 2\}.
\]

Plugging in summing-up constraint yields
\[
(1 - \mu^1 - \mu^2)G(yHp - c - \gamma) + \mu^1(yHp - c + 1\{G \neq 0\}(yHp - c - \gamma)) + \left(\frac{K}{M} - \mu^1\right) (yHp - c)
\]
or
\[(1 - \frac{1}{2} \mu^1 - \frac{1}{2} \frac{K}{M})G(y_Hp - c - \gamma) + \mu^1 1\{G \neq 0\}(y_Hp - c - \gamma) + \frac{K}{M}(y_Hp - c).\]

Notice that total surplus increases linearly in $K$, so if $K > M$ specialists are optimal then it is optimal to have $K = 2M$. We have already shown 2 generalists are dominated by $M$ specialists and one generalist, so we need only consider whether $M$ specialists, $M$ specialists and 1 generalist, or $2M$ specialists are optimal.

If $2M$ specialists enter the market, then total surplus is $2(y_Hp - c) - 2M \tilde{F}_E$. If $M$ specialists enter the market, then the same specialist is required for both products with probability $\frac{1}{M^2}$. Hence, total surplus is
\[2(y_Hp - c) - \frac{1}{M^2}(y_Hp - c) - M \tilde{F}_E\]

Finally, if $M$ specialists and a generalist enter the market, then total surplus is
\[2(y_Hp - c) - \frac{1}{M^2}\gamma - (M + 1) \tilde{F}_E\]

So long as $M \tilde{F}_E > \frac{1}{M^2}(y_Hp - c)$, $M$ specialists strictly dominates $2M$ specialists. So long as $\frac{1}{M^2}(y_Hp - c - \gamma) < \tilde{F}_E$, $M$ specialists strictly dominates $M$ specialists and a generalist. Rearranging these conditions, we have that $M$ specialists are optimal so long as
\[M^2 \tilde{F}_E > \max\{\frac{1}{M}(y_Hp - c), (y_Hp - c - \gamma)\}\]

By assumption, we know that $\frac{1}{M}(y_Hp - c) < (y_Hp - c - \gamma)$, so we require only that $M^2 \tilde{F}_E > y_Hp - c - \gamma$ which holds by assumption. Therefore, under the given conditions, it is optimal for $M$ specialists to enter the market when formal contracts are available. Each specialist is allocated either $x^1_i$ or $x^2_i$ with positive probability in each period, proving the claim. ■