Beyond Asset Ownership: Employment and Asset-less Firms in a Property-Rights Theory of the Firm (Job Market Paper)

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Abstract: Although most firms own alienable assets, many firms do not. This paper approaches the problem by embedding the Grossman-Hart-Moore (GHM) property rights model within a larger theoretical framework that can describe a richer spectrum of governance structures—including not only fully integrated firms and fully disintegrated market transactions, but also asset-less firms and exclusive dealing between firms. The framework operates by combining the GHM model with a model of *bargaining control rights*, yielding, in some cases, an allocation of ownership rights different from what the GHM model implies. When we interpret the model at the level of individuals, it can be efficient to prohibit employees from side-contracting with each other, and preventing other firms from side-contracting with one firm's employee could also improve efficiency. These results are consistent with what we observe in employment law. An important benefit of this approach is a clear interpretation of the employment relationship, i.e., an affiliation between the firm and its employees when there are multiple parties in the model. When we interpret the players at the business unit level, the model shows that dealing with a firm through an exclusive dealing contract could be more efficient than both dealing with a fully independent firm and producing through a fully integrated business unit, such as a division or subsidiary.

Keywords: Property Rights Theory; Asset-less Firm; Employment Relationship; Bargaining Control Rights; Theory of the Firm.

JEL Classification Numbers: D2; L2; Y4.

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1 Introduction

Although most firms own alienable assets, many firms do not. Professional-services firms such as law firms, accounting firms, consulting firms, design firms and many health care providers own few if any alienable assets. Instead, as Holmström and Roberts (1998) and others have observed, such firms rely on inalienable human assets that inhere in and move with the firm's employees. On the other hand, a key feature of the duly celebrated Grossman-Hart-Moore (GHM) theory of the firm (Grossman and Hart, 1986; Hart and Moore, 1990; Hart, 1995) is the role of alienable assets in explaining the boundaries of the firm. How then to explain asset-less firms?

This paper approaches the problem by embedding the GHM model within a larger theoretical framework that can describe a richer spectrum of governance structures—including not only fully integrated firms and fully disintegrated market transactions, but also asset-less firms and exclusive dealing between firms. This larger framework combines the GHM model of property rights with a model of *bargaining control rights*. Different from the residual rights of control over alienable assets that is endowed by asset ownership, bargaining control rights are *institutional restrictions* designed and controlled by the upper level of the economic organization imposed to limit the freedom to bargain of the lower-level parties. We find that the optimal governance structure often involves not only allocating property rights, as in GHM, but also restricting bargaining rights for some players. In some cases, we also find that the optimal allocation of property rights differs from what the GHM model implies.

When we interpret the model at the level of individuals as opposed to the level of business units, the paper shows that it can be efficient to prohibit employees within one firm from side-contracting with each other. Furthermore, the model shows that preventing other firms from side-contracting with one firm's employees could also improve efficiency. These results are consistent with what we observe in employment law. An important benefit of our approach is a clear interpretation of the employment relationship.

The GHM approach is close to silent on employment issues. For example, consider a model with three parties and two assets, and suppose that the GHM analysis prescribes non-integrated asset ownership as the optimal governance structure. Who, then, does the third party (the one without an asset) work for, if anyone? This paper enriches the GHM approach so as to answer this question.

The model assumes that all parties are free to bargain with all other parties, unless a party has endogenously restricted bargaining rights resulting from the *ex ante* institutional design. We interpret a party with unrestricted bargaining rights as the owner of a firm (although that firm might consist of only that party, in the case of self employment). If this party controls the bargaining rights of any other parties, then the controlling party is the boss and the controlled parties are subordinates, such as employees and divisions. That is, the owners of the firms, are free to bargain with their own subordinates. And firms are free to bargain with any other firms. However, the bosses of firms have bargaining control rights over their subordinates, who are endogenously restricted to bargaining with only their employer.

Many observations about the business firm fit the characteristics of bargaining control rights. When it comes to bargaining over decisions, the owner of the firm bargains for the firm *as a whole*.[1] She bargains, representing her employees, against other business firms and customers. And she also bargains against her own employees, representing the outside contractual relationships with other firms and customers. In this model, the boss can restrict its employees to bargain only with the firm itself.

To illustrate the model another way, the boss can block direct bargaining among the employees themselves as well as bargaining between her employee and any outside party in the transaction. For example, a grocer cannot deal with his favorite customer if he does not work for the supermarket anymore. And the customer of the supermarket cannot obtain services from her favorite grocer without shopping at the market he works for, which she might dislike. As another example, non-compete clauses in employment contracts are *ex ante* voluntarily engaged restriction over *ex post* bargaining freedom. Although non-compete clauses present issues regarding enforcement, they are still frequently observed in employment contracts between the firm and its critical employees. For example, Kaplan and Strömberg (2003) document that it is common—more than 70% of contracts in their sample—for venture capital firms to use non-complete clauses.

Why do we interpret those unrestricted parties as bosses and those restricted as employees? There are at least two factors that give the firm the advantage of bargaining control rights over employees, divisions and other internal entities. First, firms are legal persons in business contracts, whereas employees or divisions are not (Iacobucci and Triantis, 2007; Hansmann and Kraakman, 2000). With very few exceptions, all employees bargain with their employer over their employment contracts. In stark contrast, most employees do not participate directly in bargaining with other employees and with other outsiders. When they do, they bargain on behalf of their employer firm for the contract, not on behalf of themselves.

Second, it is a stylized fact that side contracts between employees within a firm or between

[1] We quote from Holmström (1999): "One possible explanation is that ownership strengthens the firm's bargaining power vis-a-vis outsiders. Suppliers and other outsiders will have to deal with the firm as a unit rather than as individual members... The general point though is that institutional affiliation, and not just asset allocation, can significantly influence the nature of bargaining."

an employee and other outsiders are rarely permitted in firms. Employees are forbidden, and rarely observed, to formally side-contract among themselves, such as to game the incentive systems of their employer. First, although employees are free to leave the firm, firms tend to implement the bargaining control rights by committing not to frequently renegotiate their employment contracts. Second, according to the employment laws, employees have a fiduciary duty to act in the best interest of their employer. So side-contracting among employees or between an employee and outside parties also tend to violate this legal restriction.

Bargaining control rights are not exclusive to the hierarchical structure within a firm. When we interpret the parties in the model at the level of business units, the parties whose bargaining rights are restricted are interpreted differently depending on their ownership of assets. If they do not own any asset, they are interpreted as internal business units within a firm, such as divisions or subsidiaries. If they own assets, then they are interpreted as firms under exclusive dealing contract with those firms who have bargaining control over them.

Similar to our modeling assumption, Segal and Whinston (2000) also consider bargaining control rights as designed instruments to govern transactions. Focusing their interpretation at the business unit level, Segal and Whinston (2000) characterize exclusive contracts as restricted bargaining rights between a seller-buyer relationship. The current model shares the common characteristic with their work in that we both emphasize the role of bargaining rights as a different instrument in the governance structure from asset ownership. But this paper departs from theirs in two aspects. First, we consider the effect of bargaining control rights in the presence of asset ownership, whereas they focus on studying bargaining control rights given fixed asset ownership structure. Second, we generalize their interpretion of bargaining control rights beyond the exclusive dealing contracts to associate with the boss-subordinate relationship, which consequently provides an interpretation of asset-less firms. In effect, one can also see the current paper as a generalization of Segal and Whinston (2000) that applies to the boundaries of the firm problem with asset allocation.

The paper proceeds as follows. Section 2 reviews some of the most related literature to highlight the paper's contributions. Section 3 describes the setup of the model as well as the rules of interpretation under the three-party case.[2] Section 4 provides an example to highlight the most important findings of the model. Section 5 provides an analysis of the three-party model and offers propositions that explain the observed patterns in the example. Section 6 presents the general setup of the model with any number of parties and any number of assets, offering some new insights that do not emerge from the three-party setup. Section 7 concludes.

[2]Because the key ingredient of the bargaining control rights is the ability of one party to bargain with a third party without going through the second one, the model operates with at least three parties.

2 Related Literature

Our model share the spirit of the subeconomy theory of the firm (Holmström and Milgrom, 1991; Holmström, 1999). In their works, the firm can use various incentive instruments for their employees to selectively isolate those employees from external incentives coming from other firms. In Holmström and Milgrom (1991), the principal can choose a set of *allowable tasks* for the agent. In Holmström (1999), the firm can "regulate trade within a firm" as a subeconomy in the sense that the principle is able to set rules over different activities of its employees, such as working from home. We do not study the problem from a contracting approach, nor do we emphasize the information or measurement problem in organizations. Instead, we analyze a structure that allows the firm to isolate outsiders and its employees from each other.

Rajan and Zingales (1998) is also a theory of the boundaries of the firm that does not rely on the ownership of assets and that sees the firm as a hierarchical structure. Assuming that the owner of the firm is fixed, Rajan and Zingales (1998) focus on the allocation of *ex ante* contractible *access* to the productive resource controlled by the owner. Those agents granted access become employees of the firm and those who do not have access are interpreted as outsiders. The present paper is different in several respects. First, I emphasize different characteristics of the firm. The model emphasizes the ability for the firm to bargain as a whole vis-à-vis different parties, not the right to grant or deny the access to the resources that are under the firm's control. Second, in their model, the identity of the party who controls the firm, as well as the ownership of the critical productive asset, are exogenous and fixed. By contrast, one of the major purposes of this model is precisely to answer these two questions: who should control the firm and who should own which assets? The answers to these two questions are the core endogenous results of the model. Third, their original model has only one focal firm, i.e., the firm except for the possible outside contractors. By contrast, the present model allows the number of firms involved in the transaction to be a fully endogenous choice; with a model of more than three parties, we can have multiple firms with subordinates. Although a simple extension of their model with multiple critical assets can also model an environment with multiple firms involved in the transaction, this feature is always exogenously fixed at the number of parties who control the critical assets. Fourth, We interpret the hierarchical structure differently. Their work interprets the party who gives out access as the boss, those who receive access as the subordinates, and those who do not receive access as the outsiders. This model interprets those who can freely bargain as the bosses, those who cannot freely bargain as the subordinates.

There have been studies of the GHM model with alternative bargaining solutions. Most importantly, de Meza and Lockwood (1998) consider alternating-offer bargaining in place of the Shapley value used in GHM.[3] The main purpose of their paper is to evaluate the robustness of the results in GHM when the model adopts a different bargaining solution. Our paper differs from their work in that we adopt a more general bargaining game which makes GHM a special case in our framework. And, more importantly, we use the generalized bargaining network to model an additional governance structure other than asset ownership. For this reason, our model is much closer to Segal and Whinston (2000) than to de Meza and Lockwood (1998).

de Fontenay and Gans (2005) and Kranton and Minehart (2000) are similar to this paper in that they both study vertical integration and networks. de Fontenay and Gans (2005) adopt the GHM framework to compare outcomes under upstream competition and monopoly. Both de Fontenay and Gans (2005) and the current paper study integrations and both involve endogenous incomplete bargaining networks. The main difference is that I focus on analyzing governance structures in one given complicated transaction that involves at least three parties with asset allocation. Whereas they study governance structures across multiple simple transactions without asset allocation. Most importantly, the network in our model represents status in the hierarchy, i.e. whether a party is free to bargain in the market as a firm or is restricted to bargain as a subordinate. However, in de Fontenay and Gans (2005), the network represents the various transaction flows across different upstream producers and downstream consumers.

Kranton and Minehart (2000) studies the tradeoff between a vertically integrated transaction versus a network of supplier relationships in an environment with specialization and individual demand shocks. Their network is different from mine in that it describes a supply structure involving, mostly, one buyer and multiple competing suppliers with uncertainty, whereas my network describes a chain of jointly producing parties without competition or uncertainty.

Our work is the first formal model that I am aware of in economic theory of the firm that provides tools to study asset-less firms and exclusive dealing contracts side-by-side with classical integrated and non-integrated firms. Other economic theories of the asset-less firms, such as Dow (1993), offer specialized models of this particular type of organization and do not consider integration between firms. Hansmann (1988) offers a framework to study a broad scope of various firm structures, but it does not consider asset ownership.

[3] The generalized Nash bargaining solution with equal bargaining power under the two-party case is a special case of the Shapley value.

3 A Model of Three Parties

In this section, we introduce the modeling framework with a three-party model. It illustrates all the key ingredients of the general model and delivers many (but not all) of the results.

3.1 Economic Environment

We consider a transaction involving three parties, $1, 2$ and 3. Three of them are needed to produce a final product or service. To govern their joint transaction, they agree on a *governance structure*, $g = (A, B)$, including the *asset ownership*, A, and the *bargaining control rights*, B, which we will specify later in this section.

Investment

Each party i makes *ex ante* non-contractible human-capital investment e_i with private cost $\Psi_i(e_i)$. The investments happen *ex ante* in the sense that the state of the world has not fully realized at the point of investment. They are non-contractible by the assumption that the investments are so complicated that they cannot be specified in a contract, nor can they be verified by any outside party, say the court.

We assume that the investment cost, $\Psi_i(e_i)$, is continuous, twice differentiable, increasing and convex in e_i.

To obtain the value of the final output, these three parties need access to a finite number of alienable assets, $\mathcal{M} = \{m_1, m_2, ...\}$. The assets are alienable in the sense that their ownership can be transferred between different parties.

Production

After the state of the world realizes, i.e. at the *ex post* stage, the three parties can make decisions over the assets they own and make use of the *ex ante* investments. These three parties can potentially engage in productions involving different coalitions among themselves. Specifically, any coalition $S \subseteq \{1, 2, 3\}$ can produce a value v_S. For instance, 1 and 2 might decide to produce together without 3, which will generate a value of v_{12}. For these three parties, there are seven production possibilities in total, including $v_{123}, v_{12}, v_{13}, v_{23}, v_1, v_2$ and v_3.

The value any coalition S can produce, $v_S(\mathbf{e}, A(S))$ is determined jointly by the vector of *ex ante* investments $\mathbf{e}$ *and* the asset ownership $A(S)$. Specifically, $A(S) \subseteq \mathcal{M}$ denotes the assets under control of coalition S. It is important to remark that the production function $v_S(\mathbf{e}, A(S))$ also depends on investment of parties who are not in S. This feature is called *cross-investment*, in the sense that one party's investment also benefit other parties' productions. For example, a firm's investment in R&D is likely to accumulate valuable experiences for the engineers and scientists. If these experiences are not entirely specific to the investor firm, then these investments increase the value of production for the engineers and scientists even if they do not work with the investor firm anymore.[4]

Following Hart and Moore (1990), we assume the following properties for the value functions $v_S(\mathbf{e}, A(S))$. *(i)* Given asset allocation A, $v_S(\mathbf{e}, A(S))$ is non-decreasing, continuous, twice differentiable and concave in e_i, for any $i \in \{1,2,3\}$. Moreover, an empty coalition produces nothing, $v_\varnothing(\mathbf{e}, A(\varnothing)) = 0$. *(ii)* Assets are complementary to the investments. That is $\frac{\partial v_S(\mathbf{e}, A'(S))}{\partial e_i} < \frac{\partial v_S(\mathbf{e}, A(S))}{\partial e_i}$ if $A'(S) \subset A(S)$ *(iii)* The investments are weak strategic complements, i.e. $\frac{\partial v_S^2(\mathbf{e}, A(S))}{\partial e_i \partial e_j} \geq 0$ for $i \neq j$. *(iv)* Other things equal, the value of production is superadditive. In other words, any two coalitions produce a smaller total value than they could if they were producing as a joint coalition. That is, given investment level $\mathbf{e}$, $v_{S'}(\mathbf{e}, A(S')) + v_{S \backslash S'}(\mathbf{e}, A(S \backslash S')) < v_S(\mathbf{e}, A(S))$ for any $S' \subset S$. To economize on notation, whenever the investment level $\mathbf{e}$ and asset ownership A is fixed, we write $v_S = v_S(\mathbf{e}, A)$.

As a result of the bargaining structure we adopt, the *ex post* renegotiation is always efficient.[5] Therefore under assumption *(iv)*, only the grand-coalition production v_{123} will be produced at the final stage. However, each party can use other production possibilities v_S as outside options to deviate a bigger share of the total payoff v_{123} toward herself during the bargaining.

Bargaining with Incomplete Networks

We apply the Myerson-Shapley value (Myerson, 1977), or Myerson value, to characterize the payoff for each party from the joint production. Myerson shows that this solution generalizes the Shapley value to bargaining on incomplete networks, in two senses: (i) the Myerson value equals the Shapley value when the bargaining network is complete; and (ii) the Myerson value is the unique solution satisfying axioms akin to those that produce the

[4] These following two examples are provided in Che and Hausch (1999): Nishiguchi (1994) p.138 reports that suppliers "send engineers to work with [automakers] in design and production. They play innovative roles in ... gathering information about [the automakers' long-term product strategies."

After Honda chose Donnelly Corporation as its sole supplier of mirrors for its U.S.-manufactured cars, "Honda sent engineers swarming over the two Donnelly plants, scrutinizing the operations for kinks in the flow. Honda hopes Donnelly will reduce costs about 2% a year, with the two companies splitting the savings" (Magnet, 1994).

[5] Grossman and Hart (1986) assumes Nash bargaining solution, which delivers efficient bargaining *ex post*. Here in this model, we adopt the Myerson value which allows for incomplete bargaining networks. But since the network is always assumed to be connected under the grand coalition, the *ex post* bargaining is till always efficient.

Shapley value.

In terms of rights to bargain, we require each party to be one of two types. A party is either *restricted to bargain*—she is restricted to bargain with one and only one other party. Or the party is *free to bargain*—she can bargain with any party who are free to bargain.[6]

The requirement that each party has to be restricted to bargain or free to bargain implies that the bargaining networks that we consider have to be *connected*.[7] We use $i : j$ to denote the bargaining *link* between any two parties i and j. A *bargaining network* is a set of bargaining links. For three parties, there are four possible connected bargaining networks (Table 1). There is one complete network, $B_c = \{1:2, 1:3, 2:3\}$, in which each party is free to bargain, so any coalition can freely form without restrictions. And there are three incomplete networks, $B_i = \{i : j, i : k\}$ for $i, j, k \in \{1, 2, 3\}$ and $i \neq j \neq k$. In these networks, party i is the only "connecting" party who can bargain with the other two parties. In this situation, we will sometimes refer to party i as the *nexus* of the network. We will also say *party i has bargaining control over party j* if j is restricted to bargain with i. As a result of the incomplete network, in B^i, j and k cannot bargain with each other without i. So j and k are not able to form coalition to produce v_{jk} together without the participation of party i.

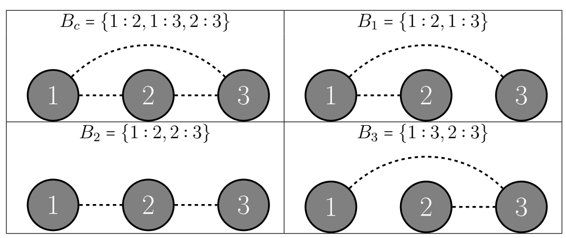


Table 1: Four Bargaining Graphs for Three Parties

To capture this feature, we define the following notation

$$v_S^B = \begin{cases} v_i + v_j & \text{if } S = \{i, j\} \text{ and } B = B^k \\ v_S & \text{otherwise} \end{cases} . \tag{1}$$

This definition is the key for us to model the incomplete bargaining network. By observation, only when the two parties $S = \{i, j\}$ meet *and* they cannot bargain directly in the network,

[6]In a model with more than three parties, we require that the free-to-bargain party needs to be able to bargain with at least two parties, and, moreover, all free-to-bargain parties are able to bargain with each and everyone of themselves. Within a three-party model, it is equivalent to the general definition to be able to bargain with the other two parties.

[7]See Section 6 for the proof in an N party model.

v_S^B alters v_S. It imposes that under network B_k, instead of producing v_{ij} through cooperation after bargaining, they can only produce separately and get $v_i + v_j$.[8]

Using this notation, the bargaining payoff of party i is defined by the Myerson value as

$$y_i = \phi_i(v^B) = \sum_{\substack{S \subseteq N = \{1,2,3\} \\ S \ni i}} p(S)\{v_S^B - v_{S \setminus \{i\}}^B\}, \tag{2}$$

where ϕ_i is the Shapley value operator; $p(S) = \frac{(|N|-|S|)!(|S|-1)!}{|N|!}$ and $|N|$, $|S|$ are the number of elements in the set N and S, respectively. Under complete network B_c, the bargaining payoff reduces to the original Shapley value payoff as is used in Hart and Moore (1990).

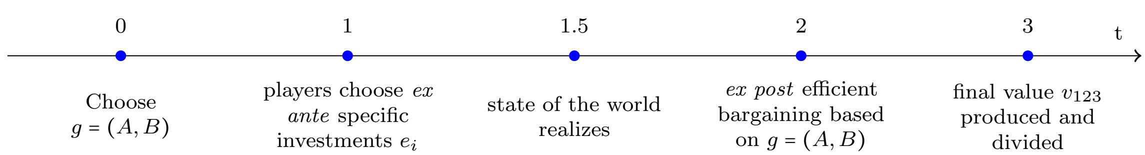
Governance Structure

The governance structure is a double $g = (A, B)$, where the asset ownership A describes who owns which assets. The bargaining network B characterizes who can bargain with whom. These two aspects jointly determine the bargaining payoffs of each party given investment level $\mathbf{e}$. Asset ownership A directly determines the value v_S produced by each coalition but has no effect on which production possibility will become available. Whereas bargaining network B has no effect on the value produced by each coalition, but determines whether some subcoalitional value, such as v_{23} can be produced.

Timing

We shall summarize the timing of the stage game described so far. The timing of this model is almost identical to that of the GHM model, with the only innovation that the governance structure is now enriched with a second dimension: bargaining networks.

At $t = 0$, information is symmetric, all parties agree on a governance structure $g = (A, B)$. At $t = 1$, parties make *ex ante* non-contractible relationship-specific investments. At $t = 1.5$, state of the world realizes. At $t = 2$, parties engage in *ex post* efficient bargaining based on the governance structure $g = (A, B)$. Finally at $t = 3$, the transaction is carried out and the final value is produced and divided by the parties according to the *ex post* bargaining result.



[8]The notation v^B is in fact a characteristic function game, which is formally introduced in Myerson (1977). The way we define it here is its special form applied to the three-party case under connected networks.

The only inefficiency in this model rises from the *ex ante* investment stage. Because parties maximize their individual returns from the bargaining instead of the joint return of the entire transaction, their investments are likely to be off the first-best level. The governance structure affects the efficiency of the transaction because the *ex ante* agreed governance structure determines the outcome of the *ex post* bargaining return of each individual, and thus it in turn governs each parties' investment decision *ex ante*. The most efficient governance structure, $g*$, is the one that generates the highest level of final product $v_{123}(\mathbf{e}, \{m, m_2\})$ net of the total private costs $\sum_i \Psi_i(e_i)$ with its associated *ex ante* investment level $\mathbf{e}^{g*}$.

An Example of Six Governance Structures

In the remaining part of this section, to eliminate redundant cases, we present the model in its simplest form by restricting our attention to a limited types of asset ownership and bargaining networks. These simplifications allow us to rule out many economically identical governance structures without losing any generality. However, it is still important to remark upfront that neither the modeling framework nor the propositions that follow in the analysis section hinge on these restrictions. We only put them in place to help build intuitions about the key features of the model.

In the simplest form of the model, we suppose that parties 2 and 3 are identical in production technologies and costs. In terms of asset ownership, A, we choose to follow the tradition of most applications of the GHM models to focus on the two cases that are most related to empirical works: the integrated asset ownership case, in which the assets are collectively owned *and* the non-integrated asset ownership case, in which the assets are separately owned. To evaluate these two cases, we assume that there are only two productive alienable assets, m and m_2. And we shall always assign ownership of m_2 to party 2 but choose between allocating ownership of m to either party 1 or party 2. We will then denote these two cases by $A = A_N$ for non-integrated asset ownership, i.e. if 1 owns m. And we denote $A = A_I$ for integrated asset ownership, i.e. if 2 owns m.[9]

Since the bargaining control rights are institutional restrictions on the ability to bargain, rather than technological difficulties that fundamentally block communication among parties, the three parties can always eventually reach agreements together. So in our model, we will only consider connected bargaining networks.[10] Because party 2 and party 3 are assumed to be identical in production technologies and costs, in terms of bargaining networks, B,

[9]Our assumptions reduce the space of A to 2 choices, so the choice of the correspondence A in a potentially large space reduces to the choice of a binary variable. Formally, in this case, $A \in \{A_N, A_I\}$, where $A_N(\{1\}) = \{m\}, A_N(\{2\}) = \{m_2\}$, and $A_I(\{1\}) = \{\varnothing\}, A_I(\{2\}) = \{m, m_2\}$.

[10]In fact, the network is connected as a result of the way we define the restricted-to-bargain parties and the free-to-bargain parties. See Section 6 for the specific statement.

we shall rule out B_3 and only consider three possible candidates for the optimal bargaining network: the original GHM complete bargaining network B_c *and* the incomplete bargaining networks B_1 and B_2, in which party 1 and party 2 are the nexus of the contracts, respectively.

The simplest model is thus a choice over 6 candidate governance structures, $g \in \{\{A_N, A_I\} \times \{B_c, B_1, B_2\}\}$. And they are presented graphically in Table 2. In these graphs, the dashed lines represents the bargaining links, which indicates the ability for any two parties to bargain with each other.

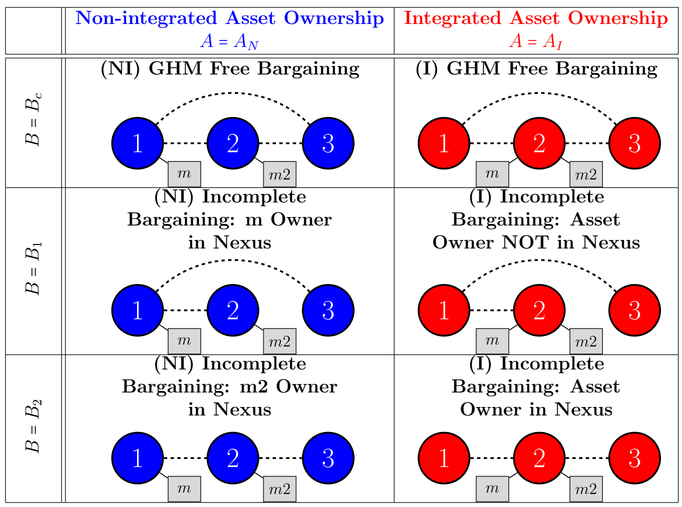


Table 2: Six Candidate Governance Structures in Bargaining Graphs

3.2 Interpreting Six Candidate Governance Structures

We spend this subsection discussing our interpretations of the two dimensional governance structures. The first part introduces our general interpretation of any party in a general environment. The second part interprets the six candidate governance structures introduced in the previous example.

General Interpretation Rules

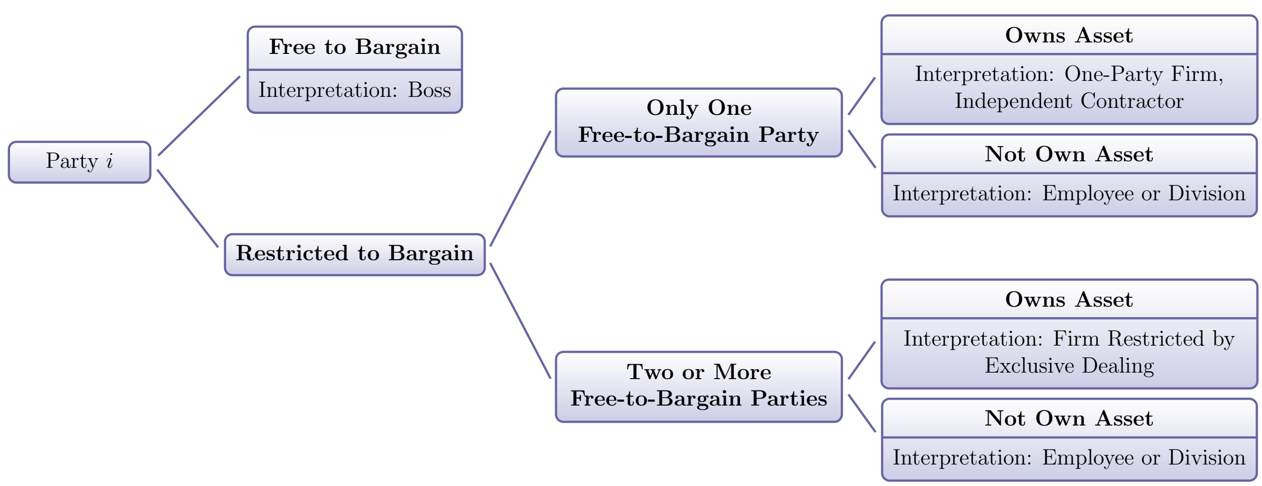
We interpret any party who is restricted to bargain as a *subordinate*, and the only party she can bargain with as *her boss*. Furthermore, we interpret someone who can bargain with everyone, except for possibly other bosses' subordinates, as a *boss*.

Since we can label any party in this model as either a boss or a subordinate, a natural interpretation of the business *firm* rises from the model without hinging on the ownership of assets. That is, a *firm* is consisted of a boss and her subordinates, if she has any.

Moreover, we will interpret the restricted-to-bargain parties who own assets as *boss of a one-party firms*. For example, in this three-party version of the model, when party 3 is restricted to bargaining with party 1, party 2 is automatically restricted in bargaining rights. This restriction, however, is due to a "degree of freedom" problem. We will thus interpret party 2 as a boss and a firm. Should we have a party 4 in the world as another boss, we would be able to identify whether party 2 is free to bargain, depending upon whether 2 can bargain with 4 without 1. If 2 is free-to-bargain, then she is interpreted as the boss of a one-party firm. Otherwise if 2 is restricted to bargaining with 1, then 2 is interpreted as a one-party firm that is restricted by exclusive dealing contract with party 1.

We are indeed aware of the fact that bargaining control rights exist beyond the hierarchical structure of the firm. The restriction in bargaining rights can be explicitly contracted on *ex ante*. As an example, exclusive dealing terms can be viewed as a specific form of bargaining control rights of one firm over another. Due to restriction of the simple structure in a three-party model, we cannot clearly identify the following two cases: (i) a firm who owns asset but can only bargain through another firm; versus (ii) an independent contractor who owns asset but cannot bargain with another firm's employee. However, as illustrated in the previous paragraph, the modeling framework we propose will be able to address this difference once we utilize a model with more than three parties. Section 6 provides an related example.

The following tree summarizes our rules of interpretation. In a three party model, we can have either three free-to-bargain parties, or having one free-to-bargain party and two restricted-to-bargain parties. The very lower branch of the tree is only relevant for the model with more than three parties.



Interpretation of the Example with Six Governance Structures

We interpret the six candidate governance structures, as is shown in Table 3. In each cell, we present the bargaining graphs in Table 2 on the top, and the interpretation graphs right below them. In the interpretation graphs, the vertical position represents our interpreted hierarchical structure. The parties outlined with think and black circles are bosses, and the parties outlined with thin circles are subordinates. We organize the rows in the table by the decreasing order of the number of firms involved in the transaction. We use color blue to denote all cases with non-integrated asset ownership, and color red to denote all cases with integrated asset ownership. We also use the darkness of the color to represent the number of firms involved in the transaction, the darkest being three firms each with no subordinate, the medium being two firms, and the lightest being one fully integrated firm. Because $g = (A_N, B_1)$ and $g = (A_N, B_2)$ both have two firms and both under non-integrated asset ownership, they share the same medium level blue color. So we apply an additional grid on the filling for $g = (A_N, B_1)$ to make the distinction.

Under the complete bargaining network B_c, every party has freedom to bargain with everyone else, so all three parties are interpreted as bosses, with or without assets. And thus the two GHM cases on the top row of Table 3 are interpreted as three firms dealing through contracts in the market.

The following two cases offer clearly identified employment relationship in the governance structure that we cannot always identify under classical GHM. Under network B_1, when asset ownership is non-integrated, 2 is interpreted as an independent firm because she has ownership over asset m_2. 3 is seen as the subordinate of 1 because he cannot bargain freely with 2. So for case $g = (A_N, B_1)$, we interpret it as the firm ran by boss 1 controlling asset m with subordinate 2 dealing with another firm 2 who controls asset m_2.

Similarly, $g = (A_N, B_2)$ is interpreted as a transaction involving two firms, each controlling one asset, dealing through the market. The only difference from the $g = (A_N, B_1)$ case is that party 3 is the subordinate of firm 2, instead of firm 1.

$g = (A_I, B_1)$ offers the interesting case of asset-less firm in a transaction. Party 1 is a boss with subordinate 3, dealing with another firm 2. Interestingly, the firm ran by 1 with subordinate 3 does not have control over any asset. All the assets needed for production is under control of firm 2. We interpret this case as a asset-less firm dealing through a business contract with another firm which is abundant with productive assets.

$g = (A_I, B_2)$ describes a classical firm in the sense that the owner of the firm is also the owner of the assets. In this case, party 2 owns all the assets but is also the boss of both 1

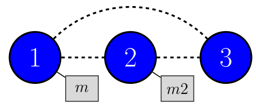
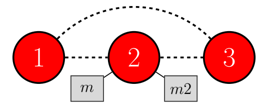
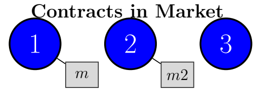
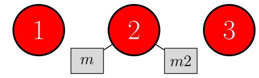
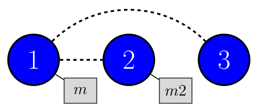
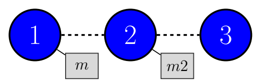
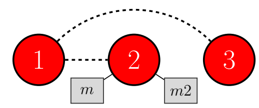
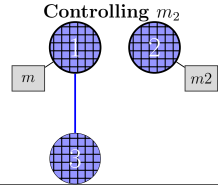
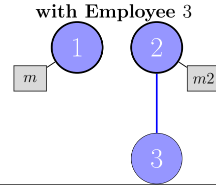
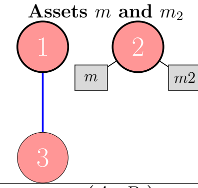
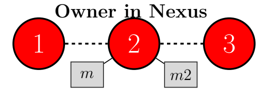
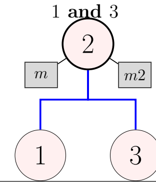
		Non-integrated Asset Ownership		Integrated Asset Ownership
Free Bargaining	Bargaining Network	$g = (A_N, B_c)$ **(NI) GHM Free Bargaining** 		$g = (A_I, B_c)$ **(I) GHM Free Bargaining** 
	Interpretation: Three Firms	**Three Firms with Non-integrated Asset Ownership Dealing through Contracts in Market** 		**Three Firms with Integrated Asset Ownership Dealing through Contracts in Market** 
Restricted Bargaining Rights	Bargaining Network	$g = (A_N, B_1)$ **(NI) Incomplete Bargaining: m Owner in Nexus** 	$g = (A_N, B_2)$ **(NI) Incomplete Bargaining: m2 Owner in Nexus** 	$g = (A_I, B_1)$ **(I) Incomplete Bargaining: Asset Owner NOT in Nexus** 
	Interpretation: Two Firms	**Firm 1 with Employee 3 Controlling Asset m Dealing with Firm 2 Controlling m_2** 	**Firm 1 Controlling Asset m Dealing with Firm 2 Controlling m_2 with Employee 3** 	**Asset-less Firm 1 with Employee 3 Dealing with Firm 2 Controlling Assets m and m_2** 
	Bargaining Network			$g = (A_I, B_2)$ **(I) Incomplete Bargaining: Asset Owner in Nexus** 
	Interpretation: One Firm			**Fulling Integrated Firm 2 Controlling Assets m and m_2, with Employees 1 and 3** 

Table 3: Interpreting Six Candidate Governance Structures

and 3. It can also be understood as a fully vertically integrated firm.

In the following sections, we will very often compare a governance structure with incomplete bargaining network, say g', with one that has a complete network, say g. In these comparisons, we will discuss it as if the governance structure changed from g to g'. To put it another way, in the thought experiments, we will pretend as if the party who has bargaining control under g' acquired the bargaining control rights over her subordinate. And we will refer to the boss in g' as *the integrating party*, and refer to the party who lost bargaining control and becomes a subordinate as *the integrated party*.

In the next section, we will use a parametrized example to demonstrate that these six governance structures can each be efficient under different situations. The four governance structures with incomplete bargaining networks can actually be more efficient than the classical GHM cases. And the optimal asset allocation structure can turn out to be different from what the GHM model implies.

4 A Parametrized Example

In this section, we use a specifically parametrized example to demonstrate that the incomplete bargaining networks with bargaining control rights can be more efficient than the complete bargaining networks in the classical GHM. Furthermore, we will observe a surprising result that after introducing the bargaining control rights as a part of the governance structure design, the optimal asset ownership can be different from what is predicted in the classical GHM model. In other words, the choice of optimal asset ownership A^* chosen as the jointly optimal governance structure $g^* = (A^*, B^*) \in \{A_N, A_I\} \times \{B_c, B_1, B_2\}$ can be different from the optimal asset ownership $g^{**} = A^{**} \in \{A_N, A_I\}$ given bargaining network B_c. Finally, in some situations, we will be able to see multiple rounds of asset ownership transfers of the same asset between the same dyad of parties as one's investment becomes more and more important relative to investments of other parties.

4.1 Model Setup

Bargaining Payoffs

At $t = 2$, after the state of the world is realized, the three parties engage in *ex post* efficient bargaining. Because of our assumption that production is superadditive, it is always jointly beneficial for the three parties to produce together and realize value v_{123}. The other subcoalitional bargaining possibilities, $v_S, \forall S \neq \{1, 2, 3\}$, serve as outside options that influence the division of the total value.[11]

Given asset allocation A and *ex ante* investments $\mathbf{e}$ fixed, by definition of Myerson value in equation (2), we can characterize the payoff for any party $i \in \{1, 2, 3\}$ under the three different networks B_c, B_i, B_j, where $i \neq j$. Let j, k denote the other two parties except for

[11] In non-cooperative bargaining terms, the subcoalitional values are off the equilibrium path, but they are used as threat points to influence the share of the final value each party can appropriate from the transaction.

party i, then the bargaining payoffs for i under the three bargaining networks are given by

$$Y_i^c(\mathbf{v}_S) = \frac{1}{3}v_{ijk} + \frac{1}{6}v_{ij} + \frac{1}{6}v_{ik} + \frac{1}{3}v_i - \frac{1}{3}v_{jk} - \frac{1}{6}v_j - \frac{1}{6}v_k; \tag{3}$$

$$Y_i^i(\mathbf{v}_S) = \frac{1}{3}v_{ijk} + \frac{1}{6}v_{ij} + \frac{1}{6}v_{ik} + \frac{1}{3}v_i - \frac{1}{2}v_j - \frac{1}{2}v_k; \tag{4}$$

$$Y_i^j(\mathbf{v}_S) = \frac{1}{3}v_{ijk} + \frac{1}{6}v_{ij} + \frac{1}{2}v_i - \frac{1}{3}v_{jk} - \frac{1}{6}v_j, \tag{5}$$

where Y_i^c, Y_i^i, Y_i^j denote party i's payoff under networks B_c, B_i and B_j respectively, and $\mathbf{v}_S$ stands for the vector of the production functions of all the possible coalitions $S \subseteq \{1,2,3\}$.[12]

Specific Parametrization of Production Functions

In this section, we rely on specific parametric assumptions over the production technology. We follow Whinston (2003)'s linear-quadratic setup to formulate the model. Each party i makes a one-dimensional *ex ante* non-contractible relationship-specific investment e_i.

We assume that the parties' investments have two potential benefits, it has a *self-investment* aspect and a *cross-investment* aspect. Self-investments means that the investments benefit the productions in which the investor participates. On the contrary, cross-investments means that investments benefit the productions that the investor is not a part of.[13] For example, if Apple invests in improving its iphone's compatibility with Google's map application, it is likely to not only benefit Apple, but also benefit Google by attracting more users who contributes data. And it may even benefit the downstream service carriers for bringing more customers and more revenue in data usage.

We assume these parties make investments at private costs with a quadratic form $\Psi_i(e_i) =$

[12]By the efficiency property of Myerson value and Shapley value, in a given network B, the sum of the payoffs to all parties equal to the final value that is produced, v_{ijk}. It can be readily checked that $\sum_{i\in\{1,2,3\}} Y_i^b = v_{123}$ for $b = c, i, j$.

[13]Cross investment is investments that not only benefit the investor, but also benefits others in the joint production. A similar concept is called *cooperative investment* in Che and Hausch (1999), which requires the investment to benefit the opponent more than it does for the investor herself.

$\frac{e_i^2}{2}$. The production functions for the seven possible coalitions are given as follows.

$$
\begin{aligned}
v_{123}(\mathbf{e}, A) &= \alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 \\
v_{12}(\mathbf{e}, A) &= m\big(k_s e_1 + k_s e_2 + \beta_{cross} k_c e_3\big) \\
v_{13}(\mathbf{e}, A) &= \big(\Omega_1 m + (1-\Omega_1)\big)\big(k_s e_1 + \beta_{cross} k_c e_2 + k_s e_3\big) \\
v_{23}(\mathbf{e}, A) &= \big(\Omega_1 + (1-\Omega_1)m\big)\big(\beta_{cross} k_c e_1 + k_s e_2 + k_s e_3\big) \\
v_1(\mathbf{e}, A) &= \big(\Omega_1 m + (1-\Omega_1)\big)\big(e_1 + \beta_{cross} e_2 + \beta_{cross} e_3\big) \\
v_2(\mathbf{e}, A) &= \big(\Omega_1 + (1-\Omega_1)m\big)\big(\beta_{cross} e_1 + e_2 + \beta_{cross} e_3\big) \\
v_3(\mathbf{e}, A) &= \beta_{cross} e_1 + \beta_{cross} e_2 + e_3
\end{aligned}
$$

In these equations, α_i is the marginal product of party i's investment in the final production. The higher α_i is, the more important is party i's investment. The multiplier m is the multiplicative effect of owning the alienable asset m. We assume the multiplier $m > 1$, so that the asset is always productive. If the asset is under control of party i, then the marginal product of all the productions that i participates in is multiplied by m.

k_s is the marginal product of self-investment in joint production of the investing party and any other party; whereas k_c is the marginal product of cross-investment in joint production of the other two parties. We assume $k_s, k_c > 2$ so the investments are more productive in bigger coalitions. β_{cross} is a binary variable controlling whether there is *cross investment*. If $\beta_{cross} = 0$, party i's investment does not have an effect on the productions that she does not participate in. Ω_1 is the binary variable indicating whether party 1 owns the asset m. $\Omega_1 = 1$ if $A = A_N$, and $\Omega_1 = 0$ if $A = A_I$.

Investment Choices Given $g = (A, B)$

At the *ex ante* stage, each party i chooses non-contractible investment e_i at private cost $\Psi_i(e_i)$ to maximize her own bargaining payoff Y_i. The network B_c, B_i or B_j determines which equations (3) to (5) is party i's bargaining payoff. The asset ownership A determines the values of productions by entering into the seven production functions v_S for $S \subseteq \{1, 2, 3\}$. And then affects the bargaining payoffs through $Y_i(v_S)$.

The equilibrium choice of e_i under governance structure g is thus characterized by

$$e_i^g = \arg\max_{e_i} \Big\{ Y_i^B\big(v_S(\mathbf{e}, A)\big) - \frac{e_i^2}{2} \Big\}.$$

The social surplus from the transaction under governance structure g is thus given by

$$\pi^g = Y_i^B\big(v_S(\mathbf{e}^g, A)\big) - \frac{(e_i^g)^2}{2}.$$

The most efficient governance structure is the one that generates the highest level of social surplus.

4.2 Horse Races Among Six Governance Structures

In the remaining part of this section, we compare the efficiency of the six governance structures in Table 3. We will show that, in this example, only when some party's investment has a cross-investment aspect, having bargaining control rights can be more efficient than using complete bargaining networks. Moreover, in some cases, after introducing the the incomplete bargaining network, the optimal asset ownership prediction can be different from the GHM result.

To demonstrate these findings, we discuss three different horse races. In Case I, every party's investment only has a self-investment aspect, we call it no-cross-investment case. Complete bargaining network is always more efficient. In Case II, we allow for the cross-investment aspect in production functions. Incomplete bargaining networks can be more efficient than complete bargaining networks, but the optimal asset allocation predictions remain the same as in GHM. In Case III, the optimal asset allocation predictions are different from the GHM predictions.

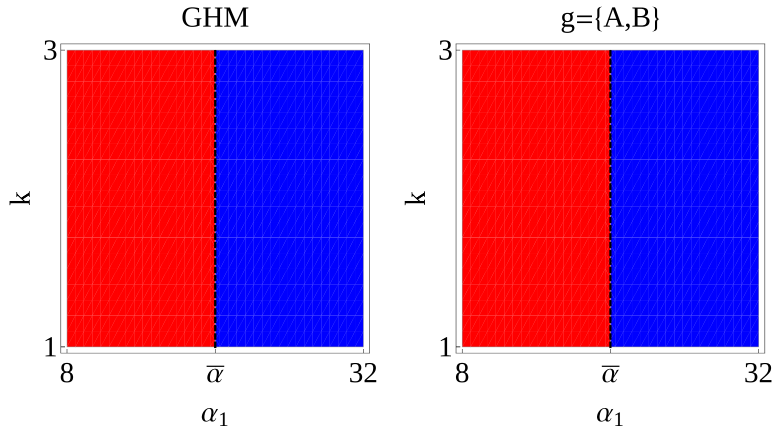
We choose to parametrize some variables and directly demonstrate the results with figures reporting the optimal governance structure under different parameter values. In what follows, in order to produce the figures, we fix $m = 2$, $\alpha_2 = \alpha_3 = \overline{\alpha} = 20$. We let $\beta_{cross}, \alpha_1, k_s$ and k_c vary as choice variables and report the optimal governance structures.[14]

Case I: No Cross-investment

We say there is *no cross-investment* if no party's investment has a marginal benefit in productions that she is not a part of. In the first case, we consider the situation where there is no cross-investment, i.e. $\beta_{cross} = 0$ in the production functions. The result is reported in Figure 1.

We use the same coloring and filling as in Table 3 to mark the governance structures. As the legend shows, we use color blue to mark all governance structures with non-integrated asset ownership, and use color red to mark all governance structures with integrated as-

[14] Ω_1 is not an exogenous choice variable, because it is determined endogenously by asset ownership A.



(a) (A_N, B_c) vs. (A_I, B_c) (b) All Six Gov. Structures

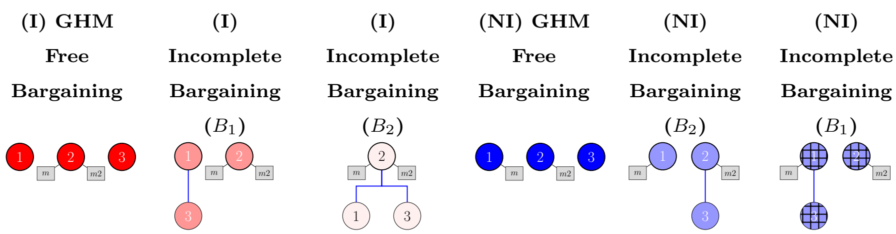


Figure 1: Optimal Governance Structures without Cross-investment

set ownership. We also use darkness of color to indicate the number of firms involved in the transaction. The darkest representing three firms, the medium representing two and the lightest representing one completely integrated firm. In the blue cases, (A_N, B_1) and (A_N, B_2) both have two firms in transaction so they share the same darkness. In this case, we use the grid filling to distinguish (A_N, B_1) from (A_N, B_2).

Figure 1a reports the optimal governance structures in the classical GHM world, where everyone has freedom to bargain. The choices of governance structure is between non-integrated asset ownership versus integrated asset ownership. Figure 1b reports the optimal governance structure when all six governance structures are in the horse race. Both graphs share identical horizontal and vertical axis. The horizontal axis, α_1, is the relative importance of party 1's investment. Party 1's investment is more important than 2 and 3 if α_1 is greater than $\overline{\alpha}$. The vertical axis, $k = k_c = k_s$, is set to be the value of the marginal benefit of investments in sub-coalitional productions, which, in this case, are assumed to be the same.

Figure 1a predicts that assets should be owned by the party who makes more important investments. When party 1's investment is less important than that of party 2, it is more efficient for party 2 to own asset m. But once 1's investment is more important than 2's investment, it is optimal to assign ownership of asset m to party 1. Figure 1b reports that, if there is no cross-investment, it is not efficient to have bargaining control rights. In other

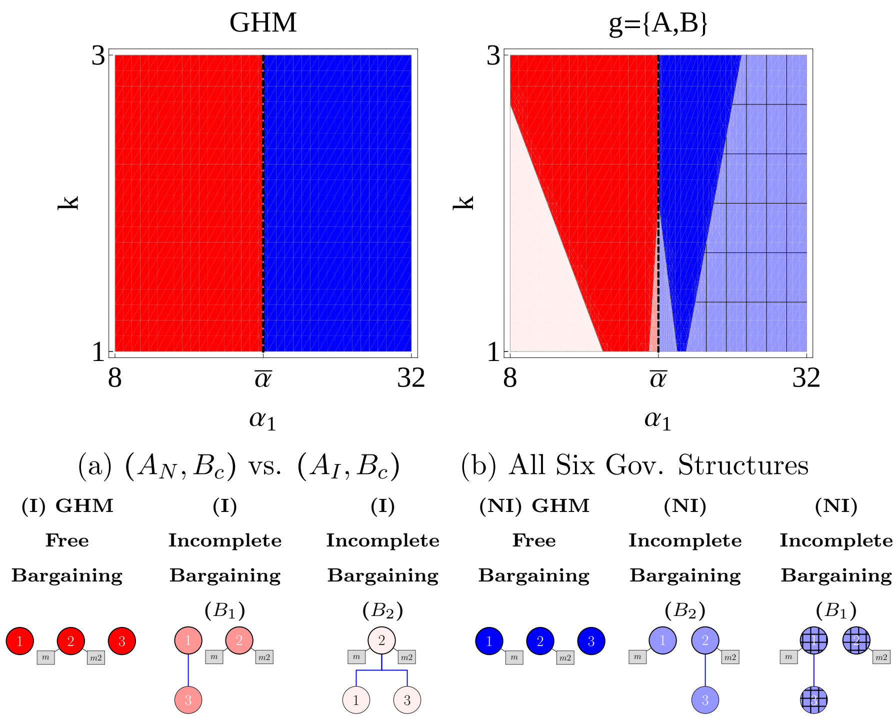


Figure 2: Optimal Governance Structures with Cross-investments

words, B_1 and B_2 are never more efficient than B_c. And the asset allocation predictions remain the same as that of GHM.[15]

Case II: with Cross-investment, $k_c = k_s$

In this case, we explore the alternative that there is cross-investment, i.e. $\beta_{cross} = 1$ in the production functions. The result is reported in Figure 2. The format of Figure 2 is identical to that of Figure 1.

The predictions under GHM is identical to the previous case—allocating asset to the party who makes the most important investment (Figure 2a). But the optimal governance structures are more complicated when we introduce bargaining control rights (Figure 2b).

Four observations emerge in this Figure. First, governance structures with bargaining control rights can be the most efficient sometimes. This shows that restricting bargaining rights can improve efficiency besides allocation of asset ownership.

Second, in this case, the model predicts identical optimal asset ownership as the classical GHM. That is, we see color red to the left of the vertical dashed line, which demarcates whether party 1 or party 2's investment is more important, and color blue to the right of

[15]This result hinges critically on the implicit assumption that $k_s > 1$, i.e. the investment is always self-investment superadditive at the margin. See Section 5 for details.

the line. So when 1's investment is less important than 2's, it is optimal for 2 to own the asset, and the opposite holds if the reverse is true.

Third, the boundary that determines which bargaining network is most efficient is not vertical or horizontal. This pattern reflects the interaction between the two instruments in governance structures.

The fourth observation is that we see a series of changes in the optimal governance structure. If we fix k and move from left to right, as party 1's investments becomes more important, it is efficient for her to own more assets, and have more bargaining rights. The optimal governance structure changes as party 1's investment becomes more and more important. When party 1's investment is very unimportant (left of Figure 2b), (A_I, B_2) wins. It is efficient to give party 2 all the asset ownership and the bargaining control over 1—2 integrating 1 to work as a subordinate. As 1 becomes more important, (A_I, B_c) is the most efficient. That is to give party 1 bargaining freedom and let her participate in the transaction as an independent contractor. As 1 becomes even more important but not more so than 2, it can be efficient to choose (A_I, B_1). That is to let 1 have bargaining control over 3 and deal with 2, who controls all the assets. This is the case in which party 1 runs an asset-less firm, such as a professional services firm, and deals with firm 2 that controls both productive assets, such as a manufacturing firm. As soon as party 1's investment becomes more important than 2's, (A_N, B_2) wins. The asset ownership shifts across the vertical line of $\overline{\alpha}$. But in order to balance 2's investment incentives, it is efficient to let 2 having bargaining control over party 3. When 1's investment gets more important, case (A_I, B_c) wins. It is efficient to give 1 and 3 their freedom to bargain with each other. And, finally, case (A_I, B_1) wins. Giving 1 both the bargaining control and the asset ownership is optimal when 1 is much more important than 2.

Case III: with Cross-investment, But $k_c \neq k_s$

Previously, we set the marginal benefit of investments on sub-coalitional productions, k_s and k_c to be the same. In this case, we make the distinction between the cross-investment aspect and self-investment aspect of the marginal benefits in sub-coalitional productions. We explore the optimal governance structure choice when $k_c \neq k_s$. As will be discussed extensively in the next section, other things the same, the greater is k_c, the greater the benefit is to have bargaining control rights. But the greater is k_s, the greater the cost is to use bargaining control rights. Whether incomplete bargaining network can be more efficient than the complete bargaining network is essentially a tradeoff between these two aspects. So we should expect to see the incomplete bargaining networks, B_1, B_2, being more likely to win if k_c is relatively large comparing to k_s, and B_c more likely to be efficient if the opposite

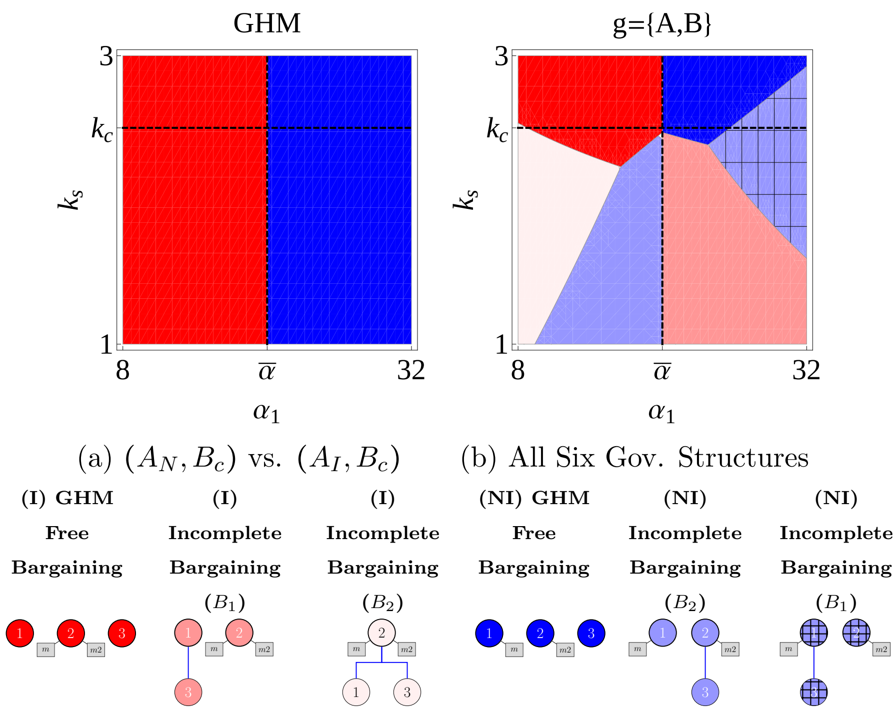


Figure 3: Optimal Governance Structures with Cross-investments: Fixed Cross-investment Superadditivity at the Margin (k_c)

holds. Figure 3 reports the result.

We highlight three observations in this case. First, the incomplete bargaining networks tend to be efficient when k_c is relatively large comparing to k_s. When $k_s < k_c$, the benefit of having bargaining control rights tends to overweight its cost. The two GHM governance structures are dominated towards the bottom part of Figure 3b. As k_s gets closer to the magnitude of k_c and goes above, the structures using bargaining control rights start to lose to GHM.

Second, this model predicts that, once we introduce bargaining control rights, the optimal asset ownership can be different from what is predicted in GHM. In Figure 3b, the optimal governance structures are not all red to the left of the vertical dashed line and blue to the right. This indicates that it can be efficient for party 1 to control the asset even though her investment is not as important as 2's. The intuition for this case is the following. When the benefit of using bargaining control is relatively large comparing to its cost, having bargaining control can more effectively motivate investment. In this case, bargaining control rights become a more effective instrument than asset ownership. The party who makes relatively more important investment should have the bargaining control rights. So as party 1's investment gets important but not more so than 2, it is efficient to have her run an independent firm with asset (case (A_N, B_2)), rather than making her control a firm with the subordinate (case (A_I, B_1)). This pattern is in stark contrast with what is predicted in the previous case where $k_c = k_s$. In fact, in the lower part of Figure 3b, when 1's investment is less important than 2's, 2 always has bargaining control rights over 1. And it is always efficient for 1 to hold bargaining control rights over 2 once 1's investment becomes more important.

Third, fixing k_s and moving from the left to the right, as α_1 increases, there are multiple rounds of transfers of asset ownership. When α_1 is very small, the asset m is controlled by party 2. As α_1 gets greater and approaches $\overline{\alpha} = \alpha_2 = \alpha_3$, it is efficient for 1 to control the asset. We see another round of transfer of asset ownership once α_1 becomes greater than $\overline{\alpha}$. When α_1 crosses the vertical dashed line of $\overline{\alpha}$, the asset ownership changes back to party 2, then changes back again to party 1 as α_1 gets very large relative to $\overline{\alpha}$.

We briefly summarize the findings in this section. Among the observations, two of them stand out being most interesting. First, the model shows that with cross-investment, introducing bargaining control rights as instruments in the governance structure can further improve the efficiency of transactions in addition to using allocation of asset ownership. Second, the model can predict different optimal asset ownership as GHM does.

5 Analysis of the Model of Three Parties

After observing some of the interesting features in the previous section, we devote this section to more rigorous analysis of the incomplete bargaining networks. The different propositions provide the general intuitions behind the patterns we observe previously in the example. Furthermore, we offer further discussions of the propositions regarding their interpretations in terms of integration of the firm. All proofs of the propositions are omitted and included in the Appendix A.

We analyze the model backwards. First, we analyze the bargaining payoffs at the *ex post* stage under different governance structures. Then we move on to study how these bargaining payoffs affect the three parties' *ex ante* investment incentives. From the associated investment incentives, we are able to draw some conclusions regarding the choice of the optimal governance structure.

5.1 *ex post* Bargaining Payoffs

Having characterized the bargaining payoffs for the three-party case under different governance structures in equations (3) through (5), we start by analyzing observations that follow from them.

By subtracting the three equations from each other, we have

$$Y_i^i - Y_i^c = \frac{1}{3}\big(v_{jk} - v_j - v_k\big); \tag{6}$$

$$Y_i^j - Y_i^c = -\frac{1}{6}\big(v_{ik} - v_i - v_k\big). \tag{7}$$

By the assumption that the production is superadditive, $v_{ij} > v_i + v_j, \forall i, j = 1, 2, 3$, we have the following result.

Remark 1. Given fixed *ex ante* investment levels and fixed asset allocation, bargaining control rights provide extra bargaining payoff. Specifically, $Y_i^i > Y_i^c > Y_i^j$.[16]

i obtains a higher bargaining payoff under B_i because, comparing to B_c, she is no longer jointly threatened by k and j together. B_i prevents j and k from bargaining with each other to form a contract without i. Intuitively, the result follows because an employee is unable to reach a side-contract with an outside firm or with another employee at the same firm. Thus they are unable to jointly make a credible threat against the employer firm for a more favorable term in their respective contracts. As a consequence, j and k's bargaining payoffs are lower comparing to those under B_c.

In all the incomplete bargaining networks, the control over other parties' ability to bargain diverts a greater share of final value from those who lost the bargaining rights to the party who obtains bargaining control.

By observation from equations (3) through (5), the following proposition becomes obvious.

Proposition 1. *Comparing to all other cases in which party j is free to bargain, if some party i has bargaining control rights over party j, then we have (**i. Insulation Effect**) the outside option v_{jk} between j and the party other than i is insulated from every parties' bargaining payoff. Specifically, for any $k \neq i$, $\frac{\partial Y_l^b}{\partial v_{jk}} \neq 0, \forall l = 1, 2, 3$ for $b \neq i$. But $\frac{\partial Y_l^i}{\partial v_{jk}} = 0, \forall l = 1, 2, 3$. (**ii. Condensation Effect**) the individual outside options v_j and v_k weigh more in every parties' bargaining payoff. Specifically, for any $k \neq i$, $|\frac{\partial Y_l^i}{\partial v_j}| > |\frac{\partial Y_l^c}{\partial v_j}|$ and $|\frac{\partial Y_l^i}{\partial v_k}| > |\frac{\partial Y_l^c}{\partial v_k}|, \forall l = 1, 2, 3$.*

The intuition behind the insulation effect is that if party j can only bargain directly with party i, no one other than i is able to form an agreement with j without going through i. Consequently, v_{jk} is no longer a credible threat for either j or k against i. As a result,

[16]In terms of the timing of the model, this result confirms that the bargaining control over other party is sub-game perfect. That is, once a party obtains the bargaining control (become the nexus) from the agreed governance structure, she will *not* give up the control right in the *ex post* bargaining stage to let the other two parties freely bargain with each other.

j and k will have no incentive to invest *ex ante* in v_{jk}. The benefit of this effect is that if party i's investment has an cross-investment aspect that also benefits v_{jk}, she will have greater incentive to invest. Because she need not be concerned about increasing v_{jk} that will turn into a potential threat against her own payoff. More specific discussions regarding the influence of this property will continue in our analysis about the *ex ante* stage investments.

Following our interpretation of the bargaining control rights as a hierarchical structure, the proposition says that integration of party j by party i fundamentally changes the payoff structure of every party. This effect has a very broad influence across all parties involved in the transaction. It does not only influence the integrating firm i and the integrated firm j, but also every other firm k that deals with both of them in the transaction.[17]

The insulation effect describes the benefit of bargaining control rights. By removing some potential outside options from all the parties involved in the transaction, it can help align the interests of all the parties with the social interest, v_{123}.

Unsurprisingly, the bargaining control rights comes with a cost as well. The condensation effect highlights the cost side of limited bargaining rights. Although restricting some parties' ability to bargain with each other removes the sub-coalitional outside option, it does not remove parties' incentives in quasi-rent expropriation by pursuing outside options. Equations (6) and (7) highlights that restriction in bargaining rights only shifts parties interests from pursuing a joint sub-coalitional outside option to pursuing individual outside options. [18] The efficiency of using bargaining control rights depends on the tradeoff between lighter weights spread on more outside options and heavier weights condensed on less smaller-scale outside options.

Following our interpretation, Proposition 1 describes that as a result of integration, by which we mean obtaining control over another party's bargaining rights, the incentives of all the parties involved in the transaction become more *focused*. On one hand, they are more focused in the sense that they care about less types of outside options (the insulation effect). One the other hand, they are more focused because they put heavier weights on

[17]In a three-party model, one might argue that in B_i, j and k simultaneously lose their bargaining rights to party i. So it seems too strong to make the point that the insulation effect also affects those parties who are not integrated. However, we show that the insulation effect indeed generalizes to a model with any number of parties. Following the integration of any party, all outside options that involves joint production with this party are insulated from all parties' payoffs. Specifically, in any network B that j can only bargain with i, $\frac{\partial Y_l}{\partial v_S} = 0$, for all parties l and *all* coalitions S such that $S \not\ni i$ and $S \ni j$. For the specific statement and proof, see Appendix C Proposition C.1.

[18]However, it offers an efficiency improving opportunity if putting more concerns over the individual outside option, in place of the joint sub-coalitional outside options, improves the productive investment incentives or reduces the wasteful investment incentives. Hold-up can be a friend. Removing outside options may be harmful, see for example Gibbons (2005). Also, some investments may be harmful, then reducing these investment incentives can improve efficiency, see Holmström and Milgrom (1991).

some smaller-scale outside options (the condensation effect).

This theory predicts that integration of one other firm fundamentally changes outside options for all transaction-related parties. Integration immunes the integrating firm from joint hold-up threats that involves the integrated party. And integration removes all other, integrated or not-yet-integrated, parties' incentives to invest toward these sub-coalitional outside options. However, as its downside, it creates more narrow minded parties who puts a heavier weight on their own outside opportunities.

Bargaining Payoffs under Different Asset Ownership

Previously we have only discussed the bargaining payoffs given a fixed asset ownership structure. In this part of the section, we show that asset ownership can have interacting effects with bargaining control rights.

In this model, the asset ownership affects the *ex post* bargaining payoffs through the assets' roles in the production functions, $v_S(\mathbf{e}, A(S))$. We can obtain the bargaining payoff for party i under governance structure $g = (A_a, B_b)$ for $a = N, I$ and $b = c, i, j$ as

$$Y_i^{a,b} = Y_i^b|_{A=A_a}, \tag{8}$$

where Y_i^b is given in equations (3) through (5).

Let us define the following operation $\Delta_{N-I}\big(v_S(\mathbf{e})\big) = v_S(\mathbf{e}, A_N(S)) - v_S(\mathbf{e}, A_I(S))$ as the difference in the production value v_S under the two asset ownership structures for coalition S. In a similar form as equations (6) and (7), we have

$$Y_i^{N,i} - Y_i^{I,i} = Y_i^{N,c} - Y_i^{I,c} + \frac{1}{3}\Delta_{N-I}\big(v_{jk} - v_j - v_k\big); \tag{9}$$

$$Y_i^{N,j} - Y_i^{I,j} = Y_i^{N,c} - Y_i^{I,c} - \frac{1}{6}\Delta_{N-I}\big(v_{ik} - v_i - v_k\big). \tag{10}$$

The following result follows immediately from these two equations.

Proposition 2. *The change of asset ownership can have different effects on payoffs under different bargaining networks. Specifically, there is difference in payoffs across different networks if the asset ownership changes the superadditivity in sub-coalitional cooperation, i.e.* $\Delta_{N-I}\big(v_{jk} - v_j - v_k\big) \neq 0$.

Proposition 2 offers the interaction between the two dimensions of the seemingly independent governance structures. It says that the effect of the asset ownership can vary across different allocations of bargaining control rights.

With our interpretation, Proposition 2 predicts that the transfer of ownership over the same asset between the same pair of parties can cause different changes in payoff distribution. The amount of payoff each party can gain or lose from the transfer can depend on the level of integration in the transaction. Suppose there are two cases, in the first, i and j are both free to bargain and controls no other party; whereas in the second case, i has bargaining control over some other party k. Then the *ex post* rent distribution can differ in these two cases following a transfer of the same asset from i to j.[19]

To summarize our analysis up to now, bargaining control rights diverts a greater bargaining payoff from those parties who become restricted to bargain toward those who have control. This shift removes all the outside options of joint productions that involve the integrated parties. It shifts the parties' interests to focus more heavily on outside options involving less parties. The asset ownership and the allocation of bargaining control rights can interact with each other. The *ex post* benefit or loss from obtaining the ownership of the same asset from the same party may differ depending on the bargaining control rights. The answer regarding whether restricting bargaining rights can improve efficiency, however, depends on the specific nature of investments. The following subsection studies these implications in further detail.

5.2 *ex ante* Investment Incentives

In the *ex ante* stage, each party i chooses her non-contractible relationship-specific investment level e_i at private cost $\Psi_i(e_i)$ to maximize her future bargaining payoff given the agreed upon governance structure. In this section, we analyze different investment incentives under different governance structures. And consequently, we are able to draw some implications from the model regarding the efficiency of the respective structures.

First-best Benchmark

Before specifying the *ex ante* investment problem under any specific governance structure, we will analyze the first-best investment level as a benchmark.

The first-best level of investment e_i^{FB} is the choice of e_i that maximizes the final value

[19] With more than three parties, we can possibly identify a firm under exclusive dealing restrictions in the model. A generalization of Proposition 2 then implies that the payoff changes following a transfer of the same asset between a firm restricted by exclusive dealing and another firm can be different should the restricted party were an independent firm.

of production $v_{123}(\mathbf{e}, A)$ given the costs $\Psi_i(e_i)$ for all parties. It is characterized by

$$\frac{\partial v_{123}(\mathbf{e}, \{m, m_2\})}{\partial e_i} = \Psi_i'(e_i). \tag{11}$$

Investments Given Fixed Asset Ownership

We first characterize the *ex ante* investment levels, e_i^{A,B_c}, e_i^{A,B_1} and e_i^{A,B_2} under the three different bargaining networks given fixed asset ownership A.[20]

Party i obtains her associated payoff Y_i under the particular bargaining network. Under B_c, party i will obtain Y_i^c *ex post*, so e_i^{A,B_c} is characterized by

$$\frac{\partial Y_i^c(\mathbf{v}_S)}{\partial e_i} = \Psi_i'(e_i). \tag{12}$$

where $Y_i^c(v_S)$ is given in equation (3), and each v_S in vector $\mathbf{v}_S$ is a function of both investment level $\mathbf{e}$ and asset allocation rule A.

Similarly, e_i^{A,B_i} and e_i^{A,B_j} are characterized by $\frac{\partial Y_i^i(\mathbf{v}_S(\mathbf{e},A))}{\partial e_i} = \Psi_i'(e_i)$ and $\frac{\partial Y_i^j(\mathbf{v}_S(\mathbf{e},A))}{\partial e_i} = \Psi_i'(e_i)$, respectively. But we can utilize equations (6) and (7) to rewrite them as

$$\frac{\partial Y_i^c(\mathbf{v}_S)}{\partial e_i} + \frac{1}{3}\frac{\partial(v_{jk} - v_j - v_k)}{\partial e_i} = \Psi_i'(e_i). \tag{13}$$

$$\frac{\partial Y_i^c(\mathbf{v}_S)}{\partial e_i} - \frac{1}{6}\frac{\partial(v_{ik} - v_i - v_k)}{\partial e_i} = \Psi_i'(e_i). \tag{14}$$

Assumption 1. *We assume that the marginal product of each party i's investment e_i is strictly smaller in the sub-coalitional productions comparing to that in the production of the grand coalition, i.e.* $\frac{\partial v_S}{\partial e_i} < \frac{\partial v_N}{\partial e_i}, \forall S \subset N$.[21]

Proposition 3. *Under assumption 1, there is always under-investment in any bargaining network B_c, B_i and B_j. That is $e_i^{A,B} < e_i^{FB}$ for any $i \in \{1,2,3\}$ and any $B \in \{B_c, B_i, B_j\}$.*

Proposition 4. *If any governance structure g induces a higher investment vector $\boldsymbol{e}^g$ than the alternative g' does, then g is more efficient than g'. That is $v_{123}(\boldsymbol{e}^g, \{m, m_2\}) - \sum_i \Psi_i(e_i^g) \geq v_{123}(\boldsymbol{e}^{g'}, \{m, m_2\}) - \sum_i \Psi_i(e_i^{g'})$ if $\boldsymbol{e}^g \geq \boldsymbol{e}^{g'}$.*[22]

[20] The efficiency implications regarding the optimal asset ownership given the free bargaining network B_c is very well studied in the seminal work of Hart and Moore (1990).

[21] Assumption 1 is in place so we can anchor the relative relationship between the first-best and second-best investment levels. We do not think the assumption is substantive as long as the sign of the inequality is consistently positive or negative. The sign can be understood as the direction we choose to interpret the nature of the investment.

[22] Proposition 3 and Proposition 4 together are the counterparts of Proposition 1 in Hart and Moore (1990).

Having laid the ground for evaluating the relative efficiencies of different governance structures, we move on to compare the complete bargaining network B_c with the incomplete bargaining networks B_i.

At this point, it is convenient for what follows to introduce some definitions.

Definition. We say *there is cross investment for* e_i if for any $S \not\ni i$, $\frac{\partial v_S}{\partial e_i} > 0$.[23]

Definition. We say the investment e_i *is cross-investment superadditive at the margin (CSM) with respect to coalition* S if for coalition $S \not\ni i$ and $S' \subset S$, $\frac{\partial v_S}{\partial e_i} > \frac{\partial v_{S'}}{\partial e_i} + \frac{\partial v_{S \setminus S'}}{\partial e_i}$ We say the investment e_i *is cross-investment superadditive at the margin* if e_i is cross-investment superadditive at the margin with respect to all coalitions..

One sufficient condition for investments to satisfy CSM is if the nature of the investment is (i) *non-specific to the investor* ($\frac{\partial v_{S_{-i}}}{\partial e_i} > 0$ for some $S_{-i} \not\ni i$), such as investment in capabilities, knowledge, process or routine that benefits other parties, but (ii) generates more marginal benefits when other parties jointly participate with their resources ($\frac{\partial v_{jk}}{\partial e_i} > \frac{\partial v_j}{\partial e_i} + \frac{\partial v_k}{\partial e_i}$). One such example is investment in workers' skills to operate a information system that are not specific to the investor but specific to, say, the supplier company of the investor. For another instance, investment in a complicated early-stage R&D project that requires joint work of designing specialists and marketing specialists.

Definition. We say the investment e_i *is self-investment superadditive at the margin (SSM) with respect to coalition* S if for coalition $S \ni i$ and $S' \subset S$, $\frac{\partial v_S}{\partial e_i} > \frac{\partial v_{S'}}{\partial e_i} + \frac{\partial v_{S \setminus S'}}{\partial e_i}$. We say the investment e_i *is self-investment superadditive at the margin* if e_i is self-investment superadditive at the margin with respect to all coalitions.

One sufficient condition for investments to satisfy SSM is if the nature of the investment is specific to the investor ($\frac{\partial v_j}{\partial e_i} = 0$), such as investment in assets that's currently under control, but complementary to other parties' existing resources ($\frac{\partial v_{ij}}{\partial e_i} > \frac{\partial v_i}{\partial e_i}$). For example, investment in firm-specific human capital.

Some investment can be both SSM and CSM. For example, investment in knowledge ($\frac{\partial v_{ij}}{\partial e_i} > 0$) that is specific to the particular transaction ($\frac{\partial v_j}{\partial e_i} = 0$), but not specific to the investor ($\frac{\partial v_{jk}}{\partial e_i} > 0$).

Moving on to the analysis, equations (13) and (14) provides two interesting observations regarding the effect of bargaining control rights on the investment incentives.

First, comparing to the complete bargaining network case, obtaining bargaining control over another party only increases the marginal benefit of this party's investment if and only if her investment is *CSM.* This is shown by the second term in equation (13), $\frac{\partial (v_{jk} - v_j - v_k)}{\partial e_i}$.

[23] This definition of cross investment is also introduced in Whinston (2003).

Second, comparing to the complete bargaining network case, losing bargaining rights to some other party j reduces the marginal benefit of this party's investment if and only if her investment is *SSM*, which is shown by the second term in equation (14), $\frac{\partial(v_{ik}-v_i-v_k)}{\partial e_i}$.

Although the first-order effects of bargaining control rights is clear, the net effect on the equilibrium investment levels are ambiguous in general conditions due to second-order interactions in parties' investments. The following Remark summarizes these "asymmetric" first-order effects under a special environment.

Definition. We say the investments of any two parties i and j are *technologically independent* if their investments has no effect on each other's marginal product, i.e. $\frac{\partial v_S^2}{\partial e_i \partial e_j} = 0, \forall S$.

Remark 2. If all parties' investments are technologically independent, then comparing to the baseline of complete bargaining network, suppose party i obtains bargaining control rights over party j, e_i increases after the fact if and only if it is CSM with respect to coalition $\{jk\}$; e_j and e_k decreases after the fact if and only if they are SSM with respect to coalition $\{jk\}$.

The following remark is a counterpart of the previous one presented in a comparative-static manner.

Remark 3. Comparing to the baseline of complete bargaining network, suppose party i obtains bargaining control over party j, (i) if only i makes investment, then the change is more efficient if and only if e_i is CSM with respect to coalition $\{jk\}$; (ii) if only j (or k) makes investment, then the change is less efficient if and only if e_j (or e_k) is SSM with respect to coalition $\{jk\}$.

Remark 3 provides the basis for a thought experiment under the general environment where every party makes investments. The efficiency of having bargaining control rights depends on whether the increased investment incentives by alleviating investor's concern in cross-investment can overweight the reduced investment incentives due to restricted outside options.[24]

Indeed, remark 3 is the counterpart of the result in Hart and Moore (1990) regarding the optimal governance structure if only one party makes investment. GHM predicts that if only one party makes investment, she should own all the assets as long as her investments are complementary with the assets. Our model predicts that the only investor should obtain bargaining control rights over others if and only if her investment supports other parties' cooperation without her.

[24]Reducing self-interested investments need not be efficiency reducing, we have this result because there is always under investment. This is not the case, if the investment is purely rent-seeking without being productive. But the predictions for the latter situation can be easily induced from our results with minimal differeces in the signs. This case can be readily studied by a straightforward extension of the current framework with a multi-tasking agent model.

The following proposition outlines the tradeoff in an extreme case *without* assuming technological independence in investments.

Proposition 5. *If there is no CSM, and every parties' investments are SSM with respect to all coalitions $S \subseteq \{1,2,3\}$, then it is never efficient to have bargaining control rights, i.e. B_c is always more efficient.*

Corollary 1. *If there is no cross investment, then under Assumption 1, it is never efficient to have bargaining control rights, i.e. B_c is always more efficient.*

We interpret Proposition 5 and Corollary 1 in the backward order.

Indeed, Corollary 1 is a very strong result based on a simple, although not necessarily weak, assumption. The environment without cross-investment corresponds to a situation where the effects of every party's investment is well-contained in the productions that she is a part of. Loosely speaking, this property describes a world without externality. If we follow our interpretation that bargaining control rights is a hierarchy in the firm, we can read Corollary 1 as saying that if there is no externality, there should not be vertically integrated firms in the transaction. In this situation, market transaction, B_c, is the most efficient governance structure. In other words, by stating that a hierarchical structure is inefficient without externality, Corollary 1 implies that the firm is an institution that helps reducing certain externalities among those parties involved in a transaction.

Proposition 5 describes the specific type of externality on which integration has effect. Should the investment be CSM, integration would help motivate investment of the integrating party by protecting her from joint hold-up threats. But if her investment is not CSM, then replacing the joint hold-up threats with individual hold-up threats actually lowers her investment incentives. Proposition 5 says that integration into a hierarchical structure is never efficient if protecting the owner from larger-scope joint threats worsens her overall hold-up concerns, even though Proposition 1 shows the integrating party obtains a higher level of payoff.

As a comparison to Proposition 5, we provide the following result, which is an opposite result that describes an extreme condition in which it is always efficient to use bargaining control rights.

Proposition 6. *If all parties' investments are only SSM with respect to coalitions that include party i, and suppose party i's investment is weakly CSM with respect to other coalitions, then it is always optimal for i to have bargaining control rights over others.*[25]

[25]By "weakly CSM", we refer to a condition $\frac{\partial v_S}{\partial e_i} \geq \frac{\partial v_{S'}}{\partial e_i} + \frac{\partial v_{S\setminus S'}}{\partial e_i}$, which need not necessarily hold in its strict form.

To interpret, loosely speaking, Proposition 6 says that if every parties' investments are only "complementary" to one party, then this party should be the boss of everyone. In other words, all parties should be integrated into the same firm that is controlled by this party who is complementary to every one's investments.

We can relate the main results in this section to the classical Coasean tradeoff between the cost to use the market and the cost to use fiat. In this model, the cost of using the market is exposing the integrating party to potential joint hold-up by others. Integration can help protect investment incentives by reducing the externality from her investments and replacing it with several individual level hold-up threats. Integration would help in this case only if the investment is productive to other parties' productions and helpful for other parties' cooperation. But it comes with the cost of lowering the investment incentives for the integrated party due to a worse agency problem. Moreover, our model highlights that integration also worsens the agency problem for all other parties involved in the transaction.

Most interestingly, although the benefit of integration is rooted in externality, the cost of integration is not. All these parties' investment incentives tend to be lower because they are restricted to work with their boss, which in turn restricts their outside options.

Investments under Different Asset Ownership

From observations of equations (9) and (10), we find that the effect of a given asset ownership change over the marginal benefit of the *ex ante* investments can vary depending on the allocation of bargaining rights.

The following remark compares the "likelihood" of a asset being owned by one party rather than another in a fixed dyad under different bargaining networks. Taking derivatives of equations (9) and (10) with respect to the *ex ante* investments yields the following remark.

Remark 4. Under different bargaining networks, a given transfer of asset ownership between two parties can have different first-order effects on parties' marginal benefit of investments. Specifically, compare the transfer of the asset m from j to i under network B_i and B_c. Suppose all other things equal. (i) If losing m decreases (increases) the level of SSM for party j and k, then the transfer is associated with less (more) of a drop in e_j and e_k under B_i than under B_c. (ii) If gaining m increases (decreases) the level of CSM for party i, then the transfer is associated with more (less) of an increase in e_i under B_i than under B_c.

Roughly speaking, Remark 4 states the conditions which increase the likelihood that bargaining control rights and ownership of assets are allocated to the same party. In other words, given it is efficient for a party to have bargaining control rights, it might be more likely for her to have asset ownership in the optimal governance structure.

For example, if the ownership of an asset plays an important role in the cooperation between j and k (decreases the level of SSM for party j and k), then after party i obtains bargaining control over one of j or k (under B_i), this asset is more likely to be owned by party i, instead of one of j or k. In this case, bargaining control rights and ownership of assets are likely to be jointly owned.

The logic of Remark 4 provides the intuition behind the pattern in Section 4 Case III. In fact, in Section 4 Case III, bargaining control rights and asset ownership are likely to be owned together. Because as k_s increases, the SSM decreases when someone loses the asset m.[26] This is why, in the lower part of the figure (b), the two boundary lines demarcating the shift of assets between 2 and 1 (lines separating blue from red, except for the middle line) tilt toward the center. In the south-west part, given the it is optimal for party 2 to have bargaining control rights, fix the importance of party 1's investment, α_1, as k_s increases, it is more likely for 2 to own the asset. Similarly, in the south-east part, given that 1 has bargaining control is optimal, fix α_1, it is more likely for party 1 to own the asset as k_s increases.

6 A Model of n Parties

This section provides the generalized setup of the framework and analysis with any finite number of parties and any finite number of assets. The n-party model provides some new insights regarding the effect of bargaining control rights on the bargaining payoffs for different parties under a richer environment. Most importantly, we will show when a firm obtains bargaining control rights of another firm, there is no effect on the bargaining payoff of the existing subordinates of the firm. Consequently, this type of integration has no effect on the existing subordinates' marginal benefit of investment. For the proofs of these propositions, see Appendix B.

The main propositions under the 3-party model generalize to the n-party case without additional assumptions, but since the results are basically restating the previous propositions, we leave the statements and the proofs in Appendix C.

[26] In the example, the SSM for party j and k when they own m is $k_{self} - 2$, the SSM when i owns m is $mk_{self} - m - 1$. Subtracting one term from another, the change in SSM before and after losing the asset to be $(m-1)(1-k_{self})$, which is decreasing in k_{self}. The change in SSM for party 1 and 3 before and after losing the asset to 2 is exactly the same.

6.1 Setup of the Model

Let there be a finite set of risk neutral players $N = \{1, 2, ..., n\}$ and a finite set of alienable assets $M = \{m_1, m_2, \ldots, m_N\}$. There is a contractual network B connecting all parties in N. For given coalition $S \subseteq N$, we denote the asset allocation rule by $A(S) \to M$, which assigns each asset to a certain party. Each party makes *ex ante* noncontractible, relationship-specific investments $\mathbf{e} = \{e_1, e_2, \ldots, e_n\}$ at private cost $\Psi_i(e_i)$. The production function, or characteristic function of the coalitional form game, is a function of the coalition S, the asset allocation rule by A and the *ex ante* noncontractible investments x of different players, formally we write $v_S(\mathbf{e}, A) \in \mathbb{R}$.

The governance structure in the general model is a two-dimensional object $g = (A, B)$, including the asset allocation rule A, and the bargaining network structure B.

The timing of the stage game and the assumptions on Ψ_i and v_S are exactly the same as in Section 3.

Bargaining Networks

We follow terminologies and notations in Myerson (1977). A *link* is an unordered set $\{i : j\}$ for $i, j \in N$. And a *network (graph)* on the players N is a set of links, such as $B = \{1:2, 2:3, 3:4, 1:4\}$ for $1, 2, 3, 4 \in N$. Let B_c be the complete network that contains all links between any two parties in N, i.e. $B_c = \{i : j | i, j \in N, i \neq j\}$.

Definition. A party, i, is *restricted to bargain under a network* B if she is connected with one and only one party, j, under network B, i.e. $i : j \in B$, and $i : k \notin B, \forall k \neq j$. And we denote the set of all the restricted-to-bargain parties under network B by R_B.

Definition. We say a party, i, is *free to bargain under a network* B if she can bargain with at least two parties, and she can bargain with any other party who is not restricted to bargain. Specifically, we define the set of parties who are free to bargain under bargaining network B as $F_B = \{i | i : j, i : k \in B \text{ for some } j \neq k, \text{ and } i : j \in B \text{ for any } j \in N \backslash R_B\}$.

In this model, we are interested in two types of parties. One that behaves like a firm, who acts as nexus of contracts and is able to form employment contracts with its employees, as well as forming business contracts directly with any other firms. The other type of party, however, behaves like subordinates in the firm, such as employees, divisions or subsidiaries. They are usually disciplined by the contract with their employers or headquarters. The subordinates are incapable to bargain and form contracts directly with outside suppliers, downstream customers or even other employees while still working for their employer. Their role in the transaction is governed by a vertical relationship closely related with their firms. But they do not directly involve in contracts with outside parties or with each other.

We require the parties to be either restricted to bargain, or be free to bargain. The restricted-to-bargain parties should be able to bargain with only one party. And this party should be free-to-bargain, since she represents the subordinates' boss. We define the set of bargaining networks as $\mathcal{B} = \{B | i \in R_B \text{ or } i \in F_B, \forall i; j \in F_B \text{ for } i \in R_B, i : j \in B\}$.

By definition, R_B and F_B are mutually exclusive. Thus our definition of $\mathcal{B}$ immediately implies that for any bargaining network $B \in \mathcal{B}$, the two sets R_B and F_B form a partition of N. Furthermore, under this definition, all networks in $\mathcal{B}$ are necessarily connected.

Definition. A network B is *connected* if for any $i, j \in N$, there exists a path $\{i : k_1, k_1 : k_2, k_2 : k_3, \ldots, k_p : j\} \subseteq B$ linking i and j in B for some $k_1, \ldots, k_p \in N$.

Lemma 1. *Any network $B \in \mathcal{B}$ is connected.*

Thus for any network B, we can uniquely define a function $f_B(i) : R_B \to F_B$ for all $i \in R_B$ to identify the free-to-bargain party that is uniquely linked with the restricted-to-bargain party i. The definition of R_B requires, $i \in R_B$, must be associated with one and only one free-to-bargain party, j.

Definition. We say j has *bargaining control over i* under network B if $i \in R_B$ and $f_B(i) = j$.

Lemma 2. *Given the players N, a free-to-bargain set $F \subset N$ and a mapping $f : N \backslash F \to F$ uniquely defines a network $B \in \mathcal{B}$.*[27]

By Lemma 2, we can refer to a bargaining network $B \in \mathcal{B}$ as $B(F_B, f_B(\cdot))$, where the restricted to bargain set of parties under network B is $R_B = N \backslash F_B$, whose unique link to the rest of the network is identified by $f_B(\cdot)$.

Because F_B and f_B uniquely define the bargaining network, for any incomplete network, i.e. $B \in \mathcal{B}$ such that $R_B \neq \varnothing$, and the complete network, i.e. B_c such that $R(B_c) = \varnothing$, it is obvious that we can convert B to B_c in finite steps by moving one party from R_B to F_B at a time. And we can also convert from B_c to B by moving parties from F to R and setting f correspondingly. Therefore, any two networks $B_1 \neq B_2 \in \mathcal{B}$ can be converted to each other.[28] The basic step of the change between two different networks B_1 and B_2 is to move one party from F to R or from R to F, and to set the corresponding function f.

[27]Notice, however, the same network $B \in \mathcal{B}$ can possibly be written as different $(F, f(\cdot))$ pairs.

[28]We can convert B_1 to B_c and convert B_c to B_2. Each step only involves moving one party between F and R, and set f.

Interpretation: Definition of the Firm

When we jointly allocate the bargaining network B and the asset allocation A, we can clearly define the boundaries of the firm from the governance structure $g = (A, B)$.

Definition. Any free-to-bargain party $i \in F_B$ is the boss of a *firm* FM_i, independent of whether i owns any assets.

Definition. Any restricted-to-bargain party $j \in R_B$ who does not own asset is a *subordinate of the firm* controlled by $f(j) \in F_B$. In other words, $f(j) \in F_B$ is the boss of $j \in R_B$.

Definition. Any restricted-to-bargain party $j \in R_B$ who owns asset is a *firm restricted by exclusive dealing terms* controlled by $f(j) \in F_B$.

Denote the set of firms by $\{FM_1, FM_2, \ldots, FM_n\}$. The following lemma shows that there is no party who belongs to two firms, and there is no party who is left out of any firm either.

Lemma 3. *$\{FM_1, FM_2, \ldots, FM_n\}$ partitions N.*

An Example of Five Parties with Subsidiary

In Table 4, we provide an example with five parties. Unlike the three party case, in this example, we can clearly identify the firm under exclusive dealing terms, who is restricted to bargain but owns asset. The first row involves four firms in the transaction, while the second row involves only three firms.

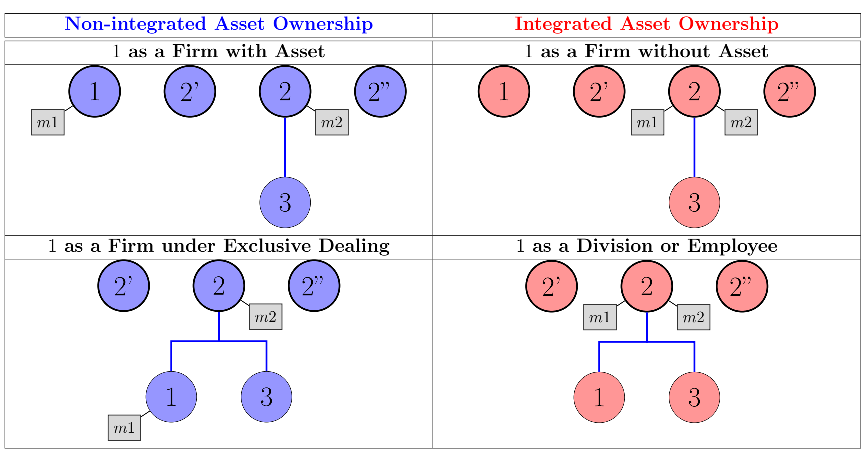


Table 4: Independent Firm, Subsidiary and Division

Partitions of a Coalition by Network and Bargaining under the Incomplete Network

At this point, we make a detour to formally introduce the Myerson value definition under a general N party environment. After the definition, we will be able to characterize the bargaining payoffs of each party under any bargaining network.

Definition. Suppose for any coalition $S \subseteq N$, the network $B \subset B_c$ contains the a path linking i and j and stays within S, such as $\{i : k_1, k_1 : k_2, k_2 : k_3, \ldots, k_n : j\} \subseteq B$ for $i, j, k_1, \ldots, k_n \in S$, then we say i and j are *connected in S under B*.

By connectedness, the network B uniquely partitions the coalition S into groups of connected players. We denote the partition $S/B = \{\{i | i \text{ and } j \text{ are connected in } S \text{ under } B\} | j \in S\}$. For example, if $N = \{1, 2, 3\}$ and $B_1 = \{1 : 2, 1 : 3\}$, then $N/B_1 = \{\{1, 2, 3\}\}$ because everyone is connected in N, but $\{2, 3\}/B_1 = \{\{2\}, \{3\}\}$ because 2 and 3 are not connected without player 1. But instead, for $\tilde{B}_c = \{1 : 2, 1 : 3, 2 : 3\}$, $\{2, 3\}/\tilde{B}_c = \{\{2, 3\}\}$ because without 1, 2 and 3 can still maintain a coalition under network B_c.

We define the following operation v^B as

$$v_S^B = \sum_{T \subseteq S/B} v_T. \tag{15}$$

The Myerson value then defines the coalitional bargaining return as in Equation (2).[29] When the network is complete, i.e. $B = B_c$, the Myerson value corresponds with the Shapley value. Therefore in this model, the organization with the complete bargaining network is exactly the same as Hart and Moore (1990).

6.2 Generalized Results

Our analysis shows that all the key insights obtained in the 3-party model generalize to the n-party model. Two new observations present themselves in the model with more than three-parties. In Proposition 8, we learn that when the firm integrates an existing free-to-bargain party, the payoff for this firm's employees remain the same. Corollary 2 thus states that the marginal benefits of investments of these existing subordinates of the firm is also unaffected by the integrations or dis-integrations of this firm in terms of bargaining control.

[29]The Myerson-Shapley value is the unique bargaining rule if the allocation rule Y is fair, i.e. $Y_i(B) - Y_i(B \backslash \{i : j\}) = Y_j(B) - Y_j(B \backslash \{i : j\}), \forall B \in \mathcal{B}, \forall i : j \in B$ (Myerson, 1977). The fairness of property requires a notion of *equal bargaining power* among all parties. In other words, when a contract is established (broken), the benefit or loss (loss or benefit) is equally shared by the two parties involved in the relationship. Note that this assumption does not necessarily require a positive gain from the bargaining relation.

Bargaining Payoffs

We denote the bargaining return for party i under network B as Y_i^B. Furthermore, for coalition $S \subseteq N$ and network $B \in \mathcal{B}$, we denote the set of parties that includes all the free-to-bargain parties in S and their associated subordinates in S by $T_B(S)$. Specifically, $T_B(S) = \{i | i \in F_B \cap S \text{ or } f(i) \in S \text{ for } i \in S\}$. Notice that, from S, $T_B(S)$ filters out all the restricted-to-bargain parties who are disconnected with others in S, i.e. $S \backslash T_B(S) = \{i | f(i) \notin S \text{ for } i \in S\}$.

We also introduce another notation $R_B^i(S) = \{k | f_B(k) = i \text{ and } k \in S\}$ as the set of parties that are under bargaining control of party i in coalition S under network B.

The following Proposition characterizes the bargaining payoff for any party i under any bargaining network $B \in \mathcal{B}$ with production function v_S.

Proposition 7. *Each party's bargaining payoff under network $B \in \mathcal{B}$ is given by*

$$Y_i^B(v_S) = \begin{cases} \sum_{S \ni i} p(S) \left[v_{T_B(S)} - v_{T_B(S) \backslash \{i\} \backslash R_B^i(S)} - \sum_{k \in R_B^i(S)} v_k \right] & \text{if } i \in F_B \\ \sum_{\substack{S \ni i \\ S \not\ni f_B(i)}} p(S) v_i + \sum_{\substack{S \ni i \\ S \ni f_B(i)}} p(S) \left[v_{T_B(S)} - v_{T_B(S) \backslash \{i\}} \right] & \text{if } i \in R_B \end{cases} \tag{16}$$

Change in the Bargaining Payoffs Following a Change in the Bargaining Network

In order to simplify the statement of the following proposition, we introduce the following assumption, we will be explicitly called for whenever it is needed for the result.

Assumption 2. *The production function v_S is convex with respect to the size of the coalition. That is, fix $\boldsymbol{e}$ and A, for any party i, and any coalitions $S' \subset S$ such that $i \in S'$, $v_S - v_{S \backslash \{i\}} > v_{S'} - v_{S' \backslash \{i\}}$.*

Assumption 2 states that the marginal contribution of a given member increases in the size of the group that she is cooperating with.

The following proposition generalizes Proposition 1 to consider the payoff changes when some party i loses bargaining control to party j.

Proposition 8. *For any bargaining network $B \in \mathcal{B}$ that has a party i who is free-to-bargain but controls no other party, i.e. $i \in F_B$ and $R_B^i(N) = \varnothing$. Let there be another network $\tilde{B}$ that is identical to B except that party i is restricted to bargain with party $\tilde{i}$, i.e. $\tilde{B} = B \backslash \cup_{k \neq \tilde{i}} \{i : k\}$.*

Then we have

$$
\begin{aligned}
&Y_{\tilde{i}}^{\tilde{B}}(v_S) - Y_{\tilde{i}}^{B}(v_S) \geq 0 && \text{for } \tilde{i} = f_{\tilde{B}}(i)\\
&Y_{i}^{\tilde{B}}(v_S) - Y_{i}^{B}(v_S) \leq 0 && \text{for } i\\
&Y_{\tilde{j}}^{\tilde{B}}(v_S) - Y_{\tilde{j}}^{B}(v_S) \leq 0 && \text{for any } \tilde{j} \in F_B \text{ and } \tilde{j} \neq f_{\tilde{B}}(i) \text{ if Assumption 2 holds}\\
&Y_{j}^{\tilde{B}}(v_S) - Y_{j}^{B}(v_S) \leq 0 && \text{for any } j \in R_B \text{ and } f_B(j) \neq f_{\tilde{B}}(i) \text{ if Assumption 2 holds}\\
&Y_{i'}^{\tilde{B}}(v_S) - Y_{i'}^{B}(v_S) = 0 && \text{for any } i' \in R_B \text{ and } f_B(i') = f_{\tilde{B}}(i)
\end{aligned}
$$

Corollary 2. $|\frac{\partial Y_{i'}^{\tilde{B}}}{\partial e_{i'}}| = |\frac{\partial Y_{i'}^{B}}{\partial e_{i'}}|$ *for any party i' such that $f_B(i') = \tilde{i}$.*

Proposition 8 generalizes Proposition 1.[30] The proposition describes the changes in the bargaining returns associated with every party in the network when one party obtains bargaining control rights over another party. Since any network in $\mathcal{B}$ can be constructed from another one by finite number of moves which shifts one party between the restricted-to-bargain set R and the free-to-bargain set F, Proposition 8 can help us predict the changes in bargaining returns when the bargaining network changes.

For example, suppose under network B_1, party k is under bargaining control of party i. We further suppose that network B_2 has the identical structure as B_1 except that, in B_2, k is under bargaining control of party j. Given *ex ante* investment level fixed, Proposition 8 can help us understand the absolute payoff changes as a consequence of such a change in the bargaining network from, say, B_1 to B_2. We can interpret this change as one firm integrating another firm's division.

We can decompose the change from B_1 to B_2 into two steps. Suppose there is a third bargaining network B_3 which is identical to B_1 and B_2, except that party k is free to bargain. Then the change from B_1 to B_2 can be broken down to a two-step change from B_1 to B_3, then from B_3 to B_2. Proposition 8 offers payoff changes for each party in the network in each of these two steps.

From B_1 to B_3, k obtains freedom to bargain. The payoff of his boss under B_1, party i, decreases. The payoffs of all other restricted-to-bargain parties under party i remain the same. And the payoff of all other parties, including k, increases. From B_3 to B_2, party j obtains bargaining control over party k. Party j's payoff increases. The payoffs of all other restricted-to-bargain parties under party j remain the same. Party i, along with all other parties, including k, obtains lower payoffs. As a net result, party i's payoff decreases, so does all restricted-to-bargain parties under i except for k. Party j's payoff increase, so does

[30]It confirms that the once some party obtains bargaining control over another party, it is at her best interest to enforce the restriction in bargaining *ex post*. In other words, bargaining control rights is sub-game perfect.

all restricted-to-bargain parties under j except for k. Party k and all other parties' payoff changes are ambiguous.

In terms of its interpretation, Proposition 8 says that when a subordinate, either an employee or a division, of firm i is integrated by firm j, firm i's ex post bargaining payoff decreases, including that for both its boss and subordinates. On the contrary, firm j's *ex post* bargaining payoff increases for both its boss and subordinates. The effect in payoff for the recently integrated party and all other firms involved in the transaction remain ambiguous.

Following our interpretation, Corollary 2 says that, in terms of bargaining control rights, any integration or dis-integration for a firm of another free-to-bargain party does not affect its existing subordinates' first-order incentives. This result is very strong and robust, and it resembles the idea similar to Holmström (1999) that the firm is a subeconomy like an island that insulates the outside market from its inside incentive systems. For instance, in Table 4, party 3's bargaining return and investment incentives remain unchanged before and after the integration of party 1 by party 2 in the two respective columns.[31] The model thus implies that establishing control over another firm through exclusive dealing terms or integrating existing independent contractors does not affect the investment incentives for existing subordinates of the integrating firm.

As a comparison to Corollary 2, it requires a much stronger condition for a change in the asset ownership to have a similar "neutral" impact on the existing subordinates. Suppose instead that party j, who has bargaining control over k, integrated an asset from any other party i. In this scenario, we have the following proposition.

Proposition 9. *Suppose party i' is under bargaining control of party i, then compare two almost identical governance structures, g_i and g_j, that are otherwise the same, except that asset m is owned by i in $g_i = (A_i, B)$ but owned by j in $g_j = (A_j, B)$. Then the bargaining payoffs, thus the first-order investment incentives, for party i' under g_i and g_j are identical if and only if $\left(v^{A_i}_{T_B(S)} - v^{A_i}_{T_B(S)\setminus\{i'\}}\right) - \left(v^{A_j}_{T_B(S)} - v^{A_j}_{T_B(S)\setminus\{i'\}}\right) = 0$ for all $S \ni i', S \ni i$, $S \not\ni j$, where v^A_S is short for $v_S(\boldsymbol{e}, A)$ given $\boldsymbol{e}$ fixed.*

Roughly speaking, in order for the existing subordinates' payoff remain constant following an acquisition of an asset by his boss, the subordinates' contribution to all productions with his boss should remain the same, with or without the asset. Broadly speaking, the statement is true if the subordinates' participation is not complementary to the asset. This is a much stronger condition comparing to Proposition 8 and Corollary 2, which holds true without

[31] As a caveat, Corollary 2 is *not* arguing that after the integration, the existing subordinates' *investment levels* remain constant, although their investment incentives do. Their investment levels may change because the second-order effects from other parties' investment levels will likely influence the subordinates' equilibrium choice of investment, although their own objective payoffs remain the same.

any additional assumptions for the existing subordinate, i', of the integrating party.

7 Concluding Remarks

Main Results

This paper studies the endogenous institutional restriction that limits the ability of firms' subordinates to bargain freely with other firms in the transaction. In this particular model, we embed this idea in the framework of the property-rights theory of the firm to evaluate whether introducing such restrictions in bargaining rights can improve efficiency in addition to using allocation of property rights over assets. Our main finding is that, when there is cross-investment, restricting some parties' ability to bargain with others in the transaction can improve efficiency in addition to using asset ownership. Furthermore, the predicted optimal asset allocation can differ from the result prescribed in classical property-rights model without restriction in bargaining rights.

Other results from the model include: (i) Restricting bargaining rights insulates some of the outside options from *all* parties' objectives, but replaces them with smaller-scale outside options. (ii) Bargaining control rights and asset ownership can interact with each other. (iii) Under mild assumptions, cross-investment is a necessary condition for the efficiency of restricting any party's bargaining rights. (iv) In the presence of cross-investment, it tends to be optimal to allocate bargaining control rights to the party who makes important non-contractible investments. (v) When one party obtains or loses bargaining control rights of some party, it does not affect the investment incentives for those parties who are already under bargaining control of the first party.

Interpretation and Discussion

We interpret this modeling framework to match many observed governance structures in the real world. We claim that the bargaining control rights resembles the vertical hierarchical structure in a business firm. This interpretation and our model together offer a theory of the boundary of the firm without relying on the asset ownership. This feature allows us to expand the scope of the traditional theories of the firm to understand asset-less firms, employment relationships and subsidiaries. The model suggests that all these different forms of governance structures can be rationalized within the same framework. The answer regarding the optimal choice of governance structure depends on the specific characteristics of the industry and technology.

The efficiency of the rich set of governance structures under different scenarios helps us rationalize the real life counterparts of these structures. Traditionally, in theory, some of these structures are considered as outliers or special cases, such as asset-less firms and subsidiaries. Furthermore, our model also justifies the efficiency of non-compete contracts as a voluntarily engaged restriction in *ex post* bargaining.

Using our interpretation of the model, this paper predicts that (i) Asset-less firms are efficient governance structures adapted to different economic environments. (ii) Integration insulates the firm's subordinates from contractual externalities in the market, and it also attenuates the externalities for outside firms and their subordinates; but it worsens the agency problems. (iii) Under some conditions, the firm that has bargaining control rights tends to own all the productive alienable assets. (iv) Under mild conditions, cross-investment is a necessary condition for the firm to be a more efficient governance structure than the market. (v) In the presence of cross-investment, the party who makes important non-contractible investments should control the firm. (vi) Establishing control over another firm through exclusive dealing or integrating existing independent contractors does not affect the investment incentives for existing subordinates of the integrating firm.

It is worth noting that the insulation effect and condensation effect from the model together resembles the spirit of the subeconomy view of the firm (Holmström, 1999). The model suggests that the firm is an institution that reduces externalities and trade it off with motivation problems. This is the case in this model because the firm isolates its subordinates from outside options involving external parties in the transaction. On one hand, this isolation can possibly better align the incentives of the subordinates and the external parties with those of the boss of the firm to protect the investment incentives of the boss. But, as its cost, the isolation dulls the motivation of all other parties.

Robustness of the Results

It may occur to some readers that firms may not have the full control over subordinates' bargaining power to totally block bargaining between the outsiders and subordinates. For instance, different states in the U.S. treats non-compete clauses very differently in court Garmaise (2011). However, given the reasons we have discussed in the introduction, it is likely that the real-world firms have significant control over their subordinates' bargaining rights. Thus the reality seems to lie somewhere between the two extremes.

The qualitative implications of our analysis holds true even if the firm has imperfect bargaining control rights. To see this point, consider a straightforward extension of our model. The firms are assigned with an exogenous value describing the intensity of bargaining control rights, which may be determined by the local institutions, such as enforcement of non-compete clauses. Let the intensity, q be a value between 0 and 1. Then the bargaining payoff for each party is modeled by a linear combination of the payoff under complete network and the payoff under the corresponding incomplete network, such as $(1-q)Y_i^c + qY_i^i$. In this model, all the qualitative implications would be identical to our current model.

Future Directions

The current modeling framework has the potential to be extended to study the difference among independent firms, subsidiaries and divisions. In non-wholly owned subsidiaries, each parent firm may not have residual rights of control over the assets that are legally owned the subsidiary. Classical GHM model does not have enough details in governance structures to distinguish an independent firm from a subsidiary that owns assets. However, our model sketches one aspect that differentiates the subsidiaries from independent firms—bargaining control rights. The non-wholly owned subsidiary can be modeled as a party who owns assets but under bargaining control of its parent firm. In this aspect, this paper provides an elemental model that can potentially contribute to a more sophisticated model to study the differences among independent firms, subsidiaries and divisions.

Although left unmodeled, our results provide a hint that incentives provided within the firm can never, and should not, resemble those at the market. This idea echos previous models such as Baker et al. (2002), but holds by a different logic in this paper. In our model, even with the same asset ownership allocation profile, every party has very different objectives regarding the outside options when some parties are restricted to bargain comparing to the alternative case in which every party is free to bargain. Therefore, the same incentive contract between independent firms would perform differently if it were used within a firm.

To maintain the generality of our modeling framework, we chose not to impose much specific institutional or technological details in this paper. As a consequence, this paper does not speak as closely as it potentially could to the span of interesting governance structures, such as the asset-less firm. In future works, this modeling framework can potentially offer a workhorse model to analyze these problems.

As a restriction, this paper starts with the assumption that firms are able to control the bargaining rights of its subordinates without going into the microeconomic details regarding how the employment contract, or ownership of the firm translates into the control of bargaining rights. We suspect that one important channel that links the two ends lies in specialization through job assignments. Microfounding any possible channel that links the ownership of the firm to the bargaining control rights may provide more insights about the theory of the firm.

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A Omitted Proofs for Propositions in Section 3

Proposition 3. *Under assumption 1, there is always under-investment in any bargaining network B_c, B_i and B_j. That is $e_i^{A,B} < e_i^{FB}$ for any $i \in \{1,2,3\}$ and any $B \in \{B_c, B_i, B_j\}$.*

Proof. For any coalition S such that $S \subset \{1,2,3\}$ and $S \ni i$, assumption 1 guarantees that $\frac{\partial v_{123}(\mathbf{e},\{m,m_2\})}{\partial e_i} > \frac{\partial v_S(\mathbf{e},\{m,m_2\})}{\partial e_i}$. Furthermore, by the assumption that assets are complementary to investments, under any asset ownership A, $\frac{\partial v_S(\mathbf{e},\{m,m_2\})}{\partial e_i} \geq \frac{\partial v_S(\mathbf{e},A)}{\partial e_i}$ because $A(S) \subseteq \{m, m_2\}$. So $\frac{\partial v_{123}(\mathbf{e},\{m,m_2\})}{\partial e_i} > \frac{\partial v_S(\mathbf{e},A)}{\partial e_i}$.

Then for bargaining payoffs under B_c, by equation (3), we have $\frac{\partial Y_i^c}{\partial e_i} < \partial\Big[\frac{1}{3}v_{123} + \frac{1}{6}v_{ij} + \frac{1}{6}v_{ik} + \frac{1}{3}v_i\Big]/\partial e_i < \frac{\partial v_{123}(\mathbf{e},\{m,m_2\})}{\partial e_i}$, the first inequality holds because of the assumption that any production v_S is increasing in investments e_i. Therefore by equations (11) and (12), $e_i^{A,B_c} < e_i^{FB} \forall i = 1,2,3$. Similar reasoning also applies to e_i^{A,B_1} and e_i^{A,B_2}. □

Proposition 4. *If any governance structure g induces a higher investment vector $\boldsymbol{e}^g$ than the alternative g' does, then g is more efficient than g'. That is $v_{123}(\boldsymbol{e}^g, \{m,m_2\}) - \sum_i \Psi_i(e_i^g) \geq v_{123}(\boldsymbol{e}^{g'}, \{m,m_2\}) - \sum_i \Psi_i(e_i^{g'})$ if $\boldsymbol{e}^g \geq \boldsymbol{e}^{g'}$.*

Proof. By Proposition 3 and the assumption that v_{123} is non-decreasing in investments, an increase in investment vector $\mathbf{e}$ increases the social surplus. The result then follows for $\mathbf{e}^g > \mathbf{e}^{g'}$. □

Proposition 5. *If there is no cross-investment superadditivity at the margin, it is never efficient to have bargaining control rights. B_c is always more efficient.*

Proof. Suppose i obtains bargaining control, i.e. the new governance structure is under network B_i. Then by equation (14) and self-investment superadditivity at the margin, $e_j^{A,B_i} < e_j^{A,B_c}$ for any party $j \neq i$ who does not gain bargaining control.

But by equation (13), if there is no cross-investment superadditivity at the margin, $e_i^{A,B_i} \leq e_i^{A,B_c}$ for the party who obtains bargaining control rights. Further by complementarity assumption $\frac{\partial v_S^2}{\partial e_i \partial e_j} > 0$, B_c induces at least as high investments as in B_i even for the party i who gains bargaining control.

Thus by Proposition 4, B_c is always more efficient than any incomplete network B_i. □

Proposition 6. *If all parties' investments are only SSM with respect to coalitions that include party i, and suppose party i's investment is weakly CSM with respect to other coalitions, then it is always optimal for i to have bargaining control rights over others.*

Proof. The result follows from Remark 2. If all parties' investments are only SSM with respect to coalitions that include party i, then comparing to B_c, under network B_i, except for party i, no party's marginal benefit of investment is lower.

And because party i's investment is weakly CSM with respect to other coalitions, party i's marginal benefit of investment is at least as high under B_i than under B_c. Therefore B_i necessarily induces a higher investment vector than B_c. So the result follows by Proposition 4. □

B Omitted Proofs for Propositions in Section 6

Lemma 1. *Any network $B \in \mathcal{B}$ is connected.*

Proof. We will prove that any two parties $i, j \in N$ are connected under any given network $B \in \mathcal{B}$.

If $i, j \in F_B$, i and j are connected by definition of F_B. If $i \in R_B$ and $j \in F_B$, then $i : \tilde{i} \in B$ for some $\tilde{i} \in F_B$ by definition of R_B. But we also have $\tilde{i} : j \in B$ because $\tilde{i}, j \in F_B$. Thus i and j are connected.

Otherwise if $i, j \in R_B$, then $i : \tilde{i} \in B$ and $j : \tilde{j} \in B$ for some $\tilde{i}, \tilde{j} \in F_B$. But it must be either $\tilde{i} = \tilde{j}$ or $\tilde{i} : \tilde{j} \in B$. So i and j are connected. □

Lemma 2. *Given the players N, a free-to-bargain set $F \subset N$ and a mapping $f : N \backslash F \to F$ uniquely defines a network $B \in \mathcal{B}$.*

Proof. Suppose F and $f(\cdot)$ define both B and $\tilde{B} \in \mathcal{B}$ s.t. $B \neq \tilde{B}$. Then for $F = F_B = F_{\tilde{B}}$ and $f = f_B = f_{\tilde{B}}$, there must exist a link $i : \tilde{i}$ in one of the networks B or $\tilde{B}$, but not in the other, for some $i \in N$. With out loss of generality, we suppose $i : \tilde{i} \in B$ but $i : \tilde{i} \neq \tilde{B}$.

Since $i : \tilde{i} \neq \tilde{B}$, either i or $\tilde{i}$ is not in F by the definition of F. Without loss of generality, let $i \in F$ and $\tilde{i} \in N \backslash F = R$. Since $i : \tilde{i} \neq \tilde{B}$, $i \neq f(\tilde{i})$.

But since F and $f(\cdot)$ also defines B, for $\tilde{i} \in N \backslash F, i : \tilde{i} \in B$ implies that $i = f(\tilde{i})$ by definition of R. Thus it must be that $f_B \neq f_{\tilde{B}}$, therefore we reach a contradiction. □

Lemma 3. *$\{FM_1, FM_2, \ldots, FM_n\}$ partitions N.*

Proof. First, we show that any party $i \in N$ is in a firm.

Sets F_B and R_B partitions N by definition. Suppose party $i \in F$. Then i is a firm. Suppose, instead, $i \in R$, then i is the subordinate for firm $f_B(i) \in F$. The above categorization exhausts N, thus all parties in N is in a firm.

Next, we show that no $i \in N$ belongs to two firms.

Suppose $i \in FM_1$ and $i \in FM_2$ for $FM_1 \neq FM_2$. By definition of $\mathcal{B}$ and the definition of firms, i cannot be a subordinate for both firms. And i cannot be a subordinate for one firm and the boss the other because R_B and F_B partition N. Moreover, by definition of the boss, a party cannot be the boss for two firms. This concludes the proof. □

Proposition 7. *Each party's bargaining payoff under network $B \in \mathcal{B}$ is given by*

$$Y_i^B(v_S) = \begin{cases} \sum_{S \ni i} p(S) \big[v_{T_B(S)} - v_{T_B(S) \backslash \{i\} \backslash R_B^i(S)} - \sum_{k \in R_B^i(S)} v_k \big] & \text{if } i \in F_B \\ \sum_{\substack{S \ni i \\ S \not\ni f_B(i)}} p(S) v_i + \sum_{\substack{S \ni i \\ S \ni f_B(i)}} p(S) \big[v_{T_B(S)} - v_{T_B(S) \backslash \{i\}} \big] & \text{if } i \in R_B \end{cases}$$

Before proving the Propositions, it is convenient to prove some lemmas first.

Lemma B.1. *$T_B(S)$ is the only element in S/B that contains more than one party.*

Proof. Suppose there exists $T'_B(S) \cap T_B(S) = \varnothing$ such that $i, j \in T'_B(S)$ for some $i \neq j$.

Because $T'_B(S) \in S/B$, by definition, i and j are connected in S under B. Thus there must be a link $\{i : k_1, k_1 : k_2, ..., k_n : j\} \subseteq B$ for $i, j, k_1, ..., k_n \in S$. By definition of $\mathcal{B}$, it cannot be the case that none of them is in F while being connected to each other. But suppose any one of them is in F_B, $T'_B(S) \cap T_B(S) \neq \varnothing$, we reach a contradiction. □

Lemma B.2. *For all $S \subseteq N$, we have*

$$v_S^B = v_{T_B(S)} + \sum_{\substack{k \in S \\ k \notin T_B(S)}} v_k. \tag{B.1}$$

Proof. By Lemma B.1, all $k \notin T_B(S)$ are singleton components containing only one party. For any $S \subseteq N$, S/B contains only one connected non-singleton component $T_B(S)$ and a group of other unconnected singleton components. Then the result follows by the definition of v_S^B. □

Lemma B.3. *For all $S \subseteq N$, we have*

$$v_{S\backslash\{i\}}^B = \begin{cases} v_S^B - v_i & \text{if } i \notin T_B(S) \\ v_{T_B(S)\backslash\{i\}\backslash R_B^i(S)} + \sum_{\substack{k \in S\backslash\{i\} \\ f_B(k) \notin S}} v_k + \sum_{k \in R_B^i(S)} v_k & \text{if } i \in T_B(S) \end{cases} \tag{B.2}$$

Proof. Lemma B.2 helps unpack v_S^B into the form of v_S. We can also apply Lemma B.2 again to unpack $v_{S\backslash\{i\}}^B$.

By Lemma B.2,

$$v_{S\backslash\{i\}}^B = v_{T_B(S\backslash\{i\})} + \sum_{\substack{k \in S\backslash\{i\} \\ k \notin T_B(S\backslash\{i\})}} v_k.$$

Furthermore, by definition of $T_B(S)$,

$$T_B(S\backslash\{i\}) = \begin{cases} T_B(S) & \text{if } i \notin T_B(S), \\ T_B(S)\backslash\{i\}\backslash R_B^i(S) & \text{if } i \in T_B(S). \end{cases}$$

Therefore, if $i \notin T_B(S)$,

$$\begin{aligned} v_{S\backslash\{i\}}^B &= v_{T_B(S)} + \sum_{\substack{k \in S\backslash\{i\} \\ k \notin T_B(S)}} v_k \\ &= v_{T_B(S)} + \sum_{\substack{k \in S \\ k \notin T_B(S)}} v_k - v_i \\ &= v_S^B - v_i. \end{aligned}$$

Otherwise if $i \in T_B(S)$,

$$\begin{aligned} v_{S\backslash\{i\}}^B &= v_{T_B(S)\backslash\{i\}\backslash R_B^i(S)} + \sum_{\substack{k \in S\backslash\{i\} \\ k \notin T_B(S)\backslash\{i\}\backslash R_B^i(S)}} v_k \\ &= v_{T_B(S)\backslash\{i\}\backslash R_B^i(S)} + \sum_{\substack{k \in S\backslash\{i\} \\ f_B(k) \notin S}} v_k + \sum_{\substack{k \in S\backslash\{i\} \\ f_B(k) = i}} v_k \\ &= v_{T_B(S)\backslash\{i\}\backslash R_B^i(S)} + \sum_{\substack{k \in S\backslash\{i\} \\ f_B(k) \notin S}} v_k + \sum_{k \in R_B^i(S)} v_k. \end{aligned}$$

□

Proof of Proposition 7

Proof. By definition of the Myerson value, to specify the bargaining payoff $Y_i^B(v_S)$, we need to specify the term $v_S^B - v_{S\setminus\{i\}}^B$.

Subtract Equation (B.2) from Equation (B.1), we have

$$v_S^B - v_{S\setminus\{i\}}^B = \begin{cases} v_i & \text{if } i \notin T_B(S), \\ v_{T_B(S)} - v_{T_B(S)\setminus\{i\}\setminus R_B^i(S)} - \sum_{k\in R_B^i(S)} v_k & \text{if } i \in T_B(S). \end{cases} \tag{B.3}$$

Plug Equation (B.3) into the definition of Myerson value, we have

$$Y_i^B(v_S) = \sum_{\substack{S\ni i \\ T_B(S)\not\ni i}} p(S)v_i + \sum_{\substack{S\ni i \\ T_B(S)\ni i}} p(S)\Big\{v_{T_B(S)} - v_{T_B(S)\setminus\{i\}\setminus R_B^i(S)} - \sum_{k\in R_B^i(S)} v_k\Big\} \tag{B.4}$$

When i is free to bargain, the first term in Equation (B.4) drops out, which yields the payoff for any free-to-bargain party. When i is restricted to bargain with a given party, the first term in Equation (B.4) remains. And the second term in Equation (B.4) reduces to $v_{T_B(S)} - v_{T_B(S)\setminus\{i\}}$ because $R_B^i(S) = \varnothing$ when i is restricted to bargain. Therefore we have

$$Y_i^B(v_S) = \begin{cases} \sum_{S\ni i} p(S)\big[v_{T_B(S)} - v_{T_B(S)\setminus\{i\}\setminus R_B^i(S)} - \sum_{k\in R_B^i(S)} v_k\big] & \text{if } i \in F_B \\ \sum_{\substack{S\ni i \\ S\not\ni f_B(i)}} p(S)v_i + \sum_{\substack{S\ni i \\ S\ni f_B(i)}} p(S)\big[v_{T_B(S)} - v_{T_B(S)\setminus\{i\}}\big] & \text{if } i \in R_B \end{cases}$$

□

Proposition 8. *For any bargaining network $B \in \mathcal{B}$ that has a party i who is free-to-bargain but controls no other party, i.e. $i \in F_B$ and $R_B^i(N) = \varnothing$. Let there be another network $\tilde{B}$ that is identical to B except that party i is restricted to bargain with party $\tilde{i}$, i.e. $\tilde{B} = B\setminus \cup_{k\neq\tilde{i}} \{i:k\}$. Then we have*

$$\begin{aligned} &Y_{\tilde{i}}^{\tilde{B}}(v_S) - Y_{\tilde{i}}^B(v_S) \geq 0 && \textit{for } \tilde{i} = f_{\tilde{B}}(i) \\ &Y_i^{\tilde{B}}(v_S) - Y_i^B(v_S) \leq 0 && \textit{for } i \\ &Y_{\tilde{j}}^{\tilde{B}}(v_S) - Y_{\tilde{j}}^B(v_S) \leq 0 && \textit{for any } \tilde{j} \in F_B \textit{ and } \tilde{j} \neq f_{\tilde{B}}(i) \\ &Y_j^{\tilde{B}}(v_S) - Y_j^B(v_S) \leq 0 && \textit{for any } j \in R_B \textit{ and } f_B(j) \neq f_{\tilde{B}}(i) \\ &Y_{i'}^{\tilde{B}}(v_S) - Y_{i'}^B(v_S) = 0 && \textit{for any } i' \in R_B \textit{ and } f_B(i') = f_{\tilde{B}}(i) \end{aligned}$$

Lemma B.4. *Given B and $\tilde{B}$ defined in Proposition 8, for any $S \subseteq N$ and $\tilde{i} = f_{\tilde{B}}(i)$,*

$$T_{\tilde{B}}(S) = \begin{cases} T_B(S) & \textit{if } i \notin S \textit{ or if } \tilde{i} \in S \\ T_B(S)\setminus\{i\} & \textit{if } i \in S \textit{ but } \tilde{i} \notin S \end{cases}$$

Proof. First of all, for any $j \neq i$, $j \in F_{\tilde{B}}$ if and only if $j \in F_B$, and $j \in R_{\tilde{B}}$ if and only if $j \in R_B$ with $f_{\tilde{B}}(j) = f_B(j)$. So $j \in T_B(S)$ if and only if $j \in T_{\tilde{B}}(S)$ for any $j \neq i$ and any $S \subseteq N$.

Thus if $i \notin S$, then $\forall j \in S, j \in T_B(S)$ if and only if $j \in T_{\tilde{B}}(S)$. So $T_{\tilde{B}}(S) = T_B(S)$ if $i \notin S$.

If $i \in S$ and $\tilde{i} \in S$, then $i \in T_{\tilde{B}}(S)$ if $i \in S$, and $i \notin T_{\tilde{B}}(S)$ if $i \notin S$. Under network B, we also have $i \in T_B(S)$ if and only if $i \in S$ because $i \in F_B$. Thus $T_{\tilde{B}}(S) = T_B(S)$ if $i \in S$ and $\tilde{i} \in S$.

Otherwise if $i \in S$ and $\tilde{i} \notin S$, then $i \in T_B(S)$ because $i \in F_B$, but $i \notin T_{\tilde{B}}(S)$ since $\tilde{i} \notin S$. Yet as is shown, for all other $j \neq i$, $j \in T_B(S)$ if and only if $j \in T_{\tilde{B}}(S)$. So $T_{\tilde{B}}(S) = T_B(S)\setminus\{i\}$. □

Proof of Proposition 8

Proof. We will use Lemma B.4 repeatedly in the following calculations to help us simplify the expressions.

For party i, who becomes restricted to bargain under party $\tilde{i}$, we have, by Proposition 7,

$$\begin{aligned}
Y_i^{\tilde{B}}(v_S) - Y_i^B(v_S) &= \sum_{\substack{S \ni i \\ S \not\ni \tilde{i}}} p(S) v_i + \sum_{\substack{S \ni i \\ S \ni \tilde{i}}} p(S) \big[v_{T_{\tilde{B}}(S)} - v_{T_{\tilde{B}}(S) \setminus \{i\}} \big] \\
&\quad - \sum_{S \ni i} p(S) \big[v_{T_B(S)} - v_{T_B(S) \setminus \{i\} \setminus R_B^i(S)} - \sum_{k \in R_B^i(S)} v_k \big] \\
&= \sum_{\substack{S \ni i \\ S \not\ni \tilde{i}}} p(S) v_i + \sum_{\substack{S \ni i \\ S \ni \tilde{i}}} p(S) \big[v_{T_B(S)} - v_{T_B(S) \setminus \{i\}} \big] - \sum_{S \ni i} p(S) \big[v_{T_B(S)} - v_{T_B(S) \setminus \{i\}} \big] \\
&= - \sum_{\substack{S \ni i \\ S \not\ni \tilde{i}}} p(S) \big[v_{T_B(S)} - v_{T_B(S) \setminus \{i\}} - v_i \big]. \qquad \text{(B.5)}
\end{aligned}$$

The second step is by Lemma B.4. So $Y_i^{\tilde{B}}(v_S) - Y_i^B(v_S) \leq 0$ by the assumption that production functions v_S are superadditive.

For party $\tilde{i}$, who obtains bargaining control over party i, by definition,

$$R_{\tilde{B}}^{\tilde{i}}(S) = \begin{cases} R_B^{\tilde{i}}(S) \cup \{i\} & \text{if } S \ni i \\ R_B^{\tilde{i}}(S) & \text{if } S \not\ni i \end{cases}.$$

Therefore, we have, again by Proposition 7,

$$\begin{aligned}
Y_{\tilde{i}}^{\tilde{B}}(v_S) - Y_{\tilde{i}}^B(v_S) &= \sum_{S \ni \tilde{i}} p(S) \big[v_{T_{\tilde{B}}(S)} - v_{T_{\tilde{B}}(S) \setminus \{\tilde{i}\} \setminus R_{\tilde{B}}^{\tilde{i}}(S)} - \sum_{k \in R_{\tilde{B}}^{\tilde{i}}(S)} v_k \big] \\
&\quad - \sum_{S \ni \tilde{i}} p(S) \big[v_{T_B(S)} - v_{T_B(S) \setminus \{\tilde{i}\} \setminus R_B^{\tilde{i}}(S)} - \sum_{k \in R_B^{\tilde{i}}(S)} v_k \big] \\
&= \sum_{\substack{S \ni \tilde{i} \\ S \not\ni i}} p(S) \big[v_{T_B(S)} - v_{T_B(S) \setminus \{\tilde{i}\} \setminus R_B^{\tilde{i}}(S)} - \sum_{k \in R_B^{\tilde{i}}(S)} v_k \big] \\
&\quad + \sum_{\substack{S \ni \tilde{i} \\ S \ni i}} p(S) \big[v_{T_B(S)} - v_{T_B(S) \setminus \{\tilde{i}\} \setminus R_B^{\tilde{i}}(S) \setminus \{i\}} - \sum_{k \in R_B^{\tilde{i}}(S)} v_k - v_i \big] \\
&\quad - \sum_{S \ni \tilde{i}} p(S) \big[v_{T_B(S)} - v_{T_B(S) \setminus \{\tilde{i}\} \setminus R_B^{\tilde{i}}(S)} - \sum_{k \in R_B^{\tilde{i}}(S)} v_k \big] \\
&= \sum_{\substack{S \ni \tilde{i} \\ S \ni i}} p(S) \big[v_{T_B(S) \setminus \{\tilde{i}\} \setminus R_B^{\tilde{i}}(S)} - v_{T_B(S) \setminus \{\tilde{i}\} \setminus R_B^{\tilde{i}}(S) \setminus \{i\}} - v_i \big]. \qquad \text{(B.6)}
\end{aligned}$$

Again, by the assumption that production functions v_S are superadditive, $Y_{\tilde{i}}^{\tilde{B}}(v_S) - Y_{\tilde{i}}^B(v_S) \geq 0$.

For any other free-to-bargain party $\tilde{j} \in F_B$ who does not gain bargaining control over party i,

$\tilde{j} \neq f_{\tilde{B}}(i)$, we have $R^{\tilde{j}}_{\tilde{B}}(S) = R^{\tilde{j}}_{B}(S), \forall S$. So we have

$$\begin{aligned}
Y^{\tilde{B}}_{\tilde{j}}(v_S) - Y^{B}_{\tilde{j}}(v_S) &= \sum_{S \ni \tilde{j}} p(S)\big[v_{T_{\tilde{B}}(S)} - v_{T_{\tilde{B}}(S)\setminus\{\tilde{j}\}\setminus R^{\tilde{j}}_{\tilde{B}}(S)} - \sum_{k \in R^{\tilde{j}}_{\tilde{B}}(S)} v_k\big] \\
&\quad - \sum_{S \ni \tilde{j}} p(S)\big[v_{T_B(S)} - v_{T_B(S)\setminus\{\tilde{j}\}\setminus R^{\tilde{j}}_{B}(S)} - \sum_{k \in R^{\tilde{j}}_{B}(S)} v_k\big] \\
&= \sum_{\substack{S \ni \tilde{j} \\ S \ni i, S \not\ni \tilde{i}}} p(S)\big[v_{T_{\tilde{B}}(S)} - v_{T_{\tilde{B}}(S)\setminus\{\tilde{j}\}\setminus R^{\tilde{j}}_{\tilde{B}}(S)}\big] - \sum_{\substack{S \ni \tilde{j} \\ S \ni i, S \not\ni \tilde{i}}} p(S)\big[v_{T_B(S)} - v_{T_B(S)\setminus\{\tilde{j}\}\setminus R^{\tilde{j}}_{B}(S)}\big] \\
&= \sum_{\substack{S \ni \tilde{j} \\ S \ni i, S \not\ni \tilde{i}}} p(S)\big[v_{T_B(S)\setminus\{i\}} - v_{T_B(S)\setminus\{i\}\setminus\{\tilde{j}\}\setminus R^{\tilde{j}}_{\tilde{B}}(S)}\big] - \sum_{\substack{S \ni \tilde{j} \\ S \ni i, S \not\ni \tilde{i}}} p(S)\big[v_{T_B(S)} - v_{T_B(S)\setminus\{\tilde{j}\}\setminus R^{\tilde{j}}_{B}(S)}\big] \\
&= -\sum_{\substack{S \ni \tilde{j} \\ S \ni i, S \not\ni \tilde{i}}} p(S)\big[(v_{T_B(S)} - v_{T_B(S)\setminus\{i\}}) - (v_{T_B(S)\setminus\{\tilde{j}\}\setminus R^{\tilde{j}}_{B}(S)} - v_{T_B(S)\setminus\{\tilde{j}\}\setminus R^{\tilde{j}}_{\tilde{B}}(S)\setminus\{i\}})\big]
\end{aligned} \tag{B.7}$$

By Assumption 2, the production function is convex in participation. Party i's marginal contribution is greater in a larger coalition. Thus $Y^{\tilde{B}}_{\tilde{j}}(v_S) - Y^{B}_{\tilde{j}}(v_S) \leq 0$.

For any restricted-to-bargain party j under any party other than $\tilde{i}$, i.e. $f_B(j) = f_{\tilde{B}}(j) = \tilde{j} \neq f_{\tilde{B}}(i) = \tilde{i}$, we have

$$\begin{aligned}
Y^{\tilde{B}}_{j}(v_S) - Y^{B}_{j}(v_S) &= \sum_{\substack{S \ni j \\ S \not\ni \tilde{j}}} p(S) v_j + \sum_{\substack{S \ni j \\ S \ni \tilde{j}}} p(S)\big[v_{T_{\tilde{B}}(S)} - v_{T_{\tilde{B}}(S)\setminus\{j\}}\big] \\
&\quad - \sum_{\substack{S \ni j \\ S \not\ni \tilde{j}}} p(S) v_j - \sum_{\substack{S \ni j \\ S \ni \tilde{j}}} p(S)\big[v_{T_B(S)} - v_{T_B(S)\setminus\{j\}}\big] \\
&= \sum_{\substack{S \ni j, S \ni \tilde{j} \\ S \ni i, S \not\ni \tilde{i}}} p(S)\big[v_{T_{\tilde{B}}(S)} - v_{T_{\tilde{B}}(S)\setminus\{j\}}\big] - \sum_{\substack{S \ni j, S \ni \tilde{j} \\ S \ni i, S \not\ni \tilde{i}}} p(S)\big[v_{T_B(S)} - v_{T_B(S)\setminus\{j\}}\big] \\
&= \sum_{\substack{S \ni j, S \ni \tilde{j} \\ S \ni i, S \not\ni \tilde{i}}} p(S)\big[v_{T_B(S)\setminus\{i\}} - v_{T_B(S)\setminus\{j\}\setminus\{i\}}\big] - \sum_{\substack{S \ni j, S \ni \tilde{j} \\ S \ni i, S \not\ni \tilde{i}}} p(S)\big[v_{T_B(S)} - v_{T_B(S)\setminus\{j\}}\big] \\
&= -\sum_{\substack{S \ni j, S \ni \tilde{j} \\ S \ni i, S \not\ni \tilde{i}}} p(S)\big[(v_{T_B(S)} - v_{T_B(S)\setminus\{i\}}) - (v_{T_B(S)\setminus\{j\}} - v_{T_B(S)\setminus\{j\}\setminus\{i\}})\big]
\end{aligned} \tag{B.8}$$

Again, by Assumption 2, $Y^{\tilde{B}}_{j}(v_S) - Y^{B}_{j}(v_S) \leq 0$.

For any restricted-to-bargain party i' under party $\tilde{i}$, i.e. $f_B(i') = f_{\tilde{B}}(i') = f_{\tilde{B}}(i) = \tilde{i}$, we have

$$\begin{aligned}
Y_{i'}^{\tilde{B}}(v_S) - Y_{i'}^{B}(v_S) &= \sum_{\substack{S \ni i' \\ S \not\ni \tilde{i}}} p(S) v_{i'} + \sum_{\substack{S \ni i' \\ S \ni \tilde{i}}} p(S)\big[v_{T_{\tilde{B}}(S)} - v_{T_{\tilde{B}}(S)\setminus\{i'\}}\big] \\
&\quad - \sum_{\substack{S \ni i' \\ S \not\ni \tilde{i}}} p(S) v_{i'} - \sum_{\substack{S \ni i' \\ S \ni \tilde{i}}} p(S)\big[v_{T_B(S)} - v_{T_B(S)\setminus\{i'\}}\big] \\
&= \sum_{\substack{S \ni i' \\ S \ni \tilde{i}}} p(S)\big[v_{T_{\tilde{B}}(S)} - v_{T_{\tilde{B}}(S)\setminus\{i'\}}\big] - \sum_{\substack{S \ni i' \\ S \ni \tilde{i}}} p(S)\big[v_{T_B(S)} - v_{T_B(S)\setminus\{i'\}}\big] \\
&= 0 \qquad \text{(B.9)}
\end{aligned}$$

Therefore, for any party i' who is already under bargaining control of party $\tilde{i}$, when $\tilde{i}$ obtains bargaining control over some other party i, i''s bargaining payoff does not change. □

Proposition 9. *Suppose party i' is under bargaining control of party i, then compare two almost identical governance structures, g_i and g_j, that are otherwise the same, except that asset m is owned by i in $g_i = (A_i, B)$ but owned by j in $g_j = (A_j, B)$. Then the bargaining payoffs, thus the first-order investment incentives, for party i' under g_i and g_j are identical if and only if $\big(v^{A_i}_{T_B(S)} - v^{A_i}_{T_B(S)\setminus\{i'\}}\big) - \big(v^{A_j}_{T_B(S)} - v^{A_j}_{T_B(S)\setminus\{i'\}}\big) = 0$ for all $S \ni i', S \ni i$, $S \not\ni j$, where v_S^A is short for $v_S(\boldsymbol{e}, A)$ given $\boldsymbol{e}$ fixed.*

Proof. Given network B such that i' is under bargaining control of party $\tilde{i}$, and party $\tilde{j}$ is free-to-bargain.

Let's denote the production functions as v_S^i and v_S^j for asset allocations A_i and A_j, respectively. Then we have for party i''s payoff following an asset transfer from $\tilde{j}$ to $\tilde{i}$ as

$$\begin{aligned}
Y_{i'}^{B}(v_S^i) - Y_{i'}^{B}(v_S^j) &= \sum_{\substack{S \ni i' \\ S \ni \tilde{i}}} p(S) v_{i'}^i + \sum_{\substack{S \ni i' \\ S \ni \tilde{i}}} \big[v^i_{T_B(S)} - v^i_{T_B(S)\setminus\{i'\}}\big] - \sum_{\substack{S \ni i' \\ S \ni \tilde{i}}} p(S) v_{i'}^j - \sum_{\substack{S \ni i' \\ S \ni \tilde{i}}} \big[v^j_{T_B(S)} - v^j_{T_B(S)\setminus\{i'\}}\big] \\
&= \sum_{\substack{S \ni i' \\ S \ni \tilde{i}}} \Big[\big(v^{A_i}_{T_B(S)} - v^{A_i}_{T_B(S)\setminus\{i'\}}\big) - \big(v^{A_j}_{T_B(S)} - v^{A_j}_{T_B(S)\setminus\{i'\}}\big)\Big] \\
&= \sum_{\substack{S \ni i' \\ S \ni \tilde{i}, S \not\ni \tilde{j}}} \Big[\big(v^{A_i}_{T_B(S)} - v^{A_i}_{T_B(S)\setminus\{i'\}}\big) - \big(v^{A_j}_{T_B(S)} - v^{A_j}_{T_B(S)\setminus\{i'\}}\big)\Big]
\end{aligned}$$

The last step follows because if both $\tilde{i}$ and $\tilde{j}$ are in S, then $v_S^i = v_S^j$. □

C Omitted Statements and Proofs for the General n-Party Model

Proposition C.1. ***(Insulation Effect)*** *Let S_j be any non-singleton coalition that include j but not $\tilde{j}$. Under any network $\tilde{B}$ such that $j \in R_{\tilde{B}}$, $f_{\tilde{B}}(j) = \tilde{j}$, we have $\frac{\partial Y_i^{\tilde{B}}}{\partial v_{S_j}} = 0$ for any party $i \in N$. Otherwise under any network B such that $j \in F_B$, we have $\frac{\partial Y_i^{B}}{\partial v_{S_j}} \neq 0$ for any party $i \in N$.*

Proof. By definition of $T_{\tilde{B}}(S)$, if $j \in R_{\tilde{B}}$, we have $j \notin T_{\tilde{B}}(S)$ for any non-singleton set $S \not\ni \tilde{j}$. In other words, $T_{\tilde{B}}(S)$ cannot be a non-singleton set that includes j. So there is no coalition S that has a corresponding $S_j = T_{\tilde{B}}(S)$ that is non-singular, contains j but not $\tilde{j}$. By Proposition 7, the bargaining payoff for any party i does not include the term v_{S_j}. Thus $\frac{\partial Y_i^{\tilde{B}}}{\partial v_{S_j}} = 0$ for any $i \in N$.

Instead, if $j \in F_B$, we always have $j \in T_B(S_j)$ as long as $S_j \ni j$. Therefore for any $S \ni j$, we have $S_j = T_B(S)$ that is non-singleton, including j, and not including some other party $\tilde{j}$. Again, by Proposition 7, $v_{S_j} = v_{T_B(S)}$ shows up in the payoff function for party i. Furthermore, whenever $i \in S_j$, the weight on $v_{S_j} = v_{T_B(S)}$ is always positive, and otherwise, the weight is negative. Thus $\frac{\partial Y_i^B}{\partial v_{S_j}} \neq 0$.

□

Proposition C.2. ***(Condensation Effect)*** *$|\frac{\partial Y_i^{\tilde{B}}}{\partial v_j}| > |\frac{\partial Y_i^B}{\partial v_j}|$ for any party i such that $f_B(i) \neq \tilde{j}$. Moreover, let S_{-j} be any coalition such that $S \not\ni j, S \not\ni \tilde{j}$. Then we have $|\frac{\partial Y_i^{\tilde{B}}}{\partial v_{S_{-j}}}| > |\frac{\partial Y_i^B}{\partial v_{S_{-j}}}|$ for any party i such that $f_B(i) \neq \tilde{j}$.*

Proof. The result follows directly taking derivatives from equations (B.5) to (B.9) with respect to v_i and v_S for $S \ni i, S \not\ni \tilde{i}$. □

Proposition C.3. *The shift of asset ownership can have different effects on payoffs under different bargaining networks.*

Proof. Using operation Δ_{N-I}, we can apply the same operations to equations (B.5) to (B.9), then a similar result to Proposition 2 follows. □

Proposition C.4. *Under any governance structure $g = (A, B)$, there is always under-investment. That is $e_i^{A,B} < e_i^{FB}$ for any $i \in N$.*

Proof. The first-best level of investment is characterized by $\frac{\partial v_N}{\partial e_i} = \Psi_i'(e_i)$. And the second-best investments are characterized by $\frac{\partial Y_i^g(\mathbf{e},A)}{\partial e_i} = \Psi_i'(e_i)$.

By definition of Myerson value

$$\begin{aligned} Y_i^B &= \sum_{S \ni i} p(S)\{v_S^B - v_{S\setminus\{i\}}^B\} \\ &< \sum_{S \ni i} p(S) v_S^B \\ &< \sum_{S \ni i} p(S) v_S \\ &< \sum_{S \ni i} p(S) v_N, \end{aligned}$$

where the last inequality holds by Assumption 1.

Thus $\frac{\partial Y_i^g(\mathbf{e},A)}{\partial e_i} < \frac{\partial v_N}{\partial e_i}$, which implies that the second-best investment is strictly less than the first-best level. □

Proposition C.5. *If there is no CSM, and every parties' investments are SSM with respect to all coalitions $S \subseteq N$, then it is never efficient to have bargaining control rights.*

Proof. Suppose in network B_K, there are K parties who are restricted to bargain. We can compare the network B_K with a similar network, B_{K-1}, that is otherwise identical, but with only $K-1$ parties restricted to bargain. Without loss of generality, label this party as i, then the payoff comparisons between these two networks for any party k, $Y_k^{B_K}(v_S) - Y_k^{B_{K-1}}(v_S)$, are given by equations (B.5) to (B.9), depending on the bargaining rights of each party.

If there is no cross-investment superadditivity at the margin, it can be readily verified from equations (B.5) and (B.6) that $\frac{\partial Y_k^{B_K}(v_S)}{e_k} - \frac{\partial Y_k^{B_{K-1}}(v_S)}{e_k} < 0$ for party $k = i$ and party $k = \tilde{i}$.

We can rewrite equation (B.7) as

$$\begin{aligned} Y_{\tilde{j}}^{\tilde{B}}(v_S) - Y_{\tilde{j}}^{B}(v_S) &= - \sum_{\substack{S \ni \tilde{j} \\ S \ni i, S \not\ni \tilde{i}}} p(S) \big[(v_{T_B(S)} - v_{T_B(S) \setminus \{i\}}) - (v_{T_B(S) \setminus \{\tilde{j}\} \setminus R_B^{\tilde{j}}(S)} - v_{T_B(S) \setminus \{\tilde{j}\} \setminus R_{\tilde{B}}^{\tilde{j}}(S) \setminus \{i\}}) \big] \\ &= - \sum_{\substack{S \ni \tilde{j} \\ S \ni i, S \not\ni \tilde{i}}} p(S) \big[(v_{T_B(S)} - v_{T_B(S) \setminus \{i\}} - v_i) - (v_{T_B(S) \setminus \{\tilde{j}\} \setminus R_B^{\tilde{j}}(S)} - v_{T_B(S) \setminus \{\tilde{j}\} \setminus R_{\tilde{B}}^{\tilde{j}}(S) \setminus \{i\}} - v_i) \big]. \end{aligned}$$

By self-investment superadditivity at the margin, the partial derivative of the first term in bracket with respect to $e_{\tilde{j}}$ is positive. And since there is no cross-investment superadditivity at the margin, the the partial derivative of the second term in bracket with respect to $e_{\tilde{j}}$ is negative. So overall, $\frac{\partial Y_k^{B_K}(v_S)}{e_k} - \frac{\partial Y_k^{B_{K-1}}(v_S)}{e_k} < 0$ for all free-to-bargain parties $\tilde{j} \neq \tilde{i}$. Same logic applies to equation (B.8) and so the result also follows for all k such that $f_{B_{K-1}} \neq \tilde{i}$.

By equation (B.9), $\frac{\partial Y_k^{B_K}(v_S)}{e_k} - \frac{\partial Y_k^{B_{K-1}}(v_S)}{e_k} = 0$ for all k such that $f_{B_{K-1}} = \tilde{i}$.

Therefore, we have $e_i^{B_K} \leq e_i^{B_{K-1}}$. Thus given asset allocation A, bargaining network B_K is strictly less efficient than B_{K-1}.

We can then repeat the same logic and iterate all the way through $K = 1$ and compare it with the complete bargaining network B_c. As a consequence, B_c is strictly more efficient than any incomplete bargaining network. □

Corollary C.1. *If there is no cross-investment, then under Assumption 1, it is never efficient to have bargaining control rights.*

Proof. If there is no cross-investment, there cannot be cross-investment superadditivity at the margin. Then the result follows from Proposition C.5. □