Household Risk Management*

Adriano A. Rampini
Duke University

S. Viswanathan
Duke University

First draft: August 2009
This draft: November 2012

Preliminary and Incomplete

Abstract

Household risk management, that is, households' insurance against adverse shocks to income, assets, and financing needs, is limited and often completely absent, in particular for poor households. We explain this basic pattern in household insurance in an infinite horizon model in which households have access to complete markets subject to collateral constraints resulting in a trade-off between the financing needs for consumption and durable goods purchases and risk management concerns. We show that household risk management is monotone in household net worth and income, under quite general conditions, in economies with income risk, durable goods, and durable goods price risk. The limited participation in markets for claims which allow household risk management is a result of the financing risk management trade-off and should not be considered a puzzle.

JEL Classification: D86, D92, E22, E32, G31, G32.

Keywords: Household finance; Collateral; Risk management; Debt capacity; Financial constraints

*We thank João Cocco, Tomek Piskorski, Jeremy Stein, George Zanjani, and seminar participants at the 2012 American Economic Association Annual Meeting, Duke University, and the 2012 NBER-Oxford Saïd-CFS-EIEF Conference on Household Finance for helpful comments. Much of this paper was written while the first author was visiting the finance area at the Stanford Graduate School of Business and the economics department at Harvard University and their hospitality is gratefully acknowledged. Duke University, Fuqua School of Business, 100 Fuqua Drive, Durham, NC, 27708. Rampini: (919) 660-7797, rampini@duke.edu; Viswanathan: (919) 660-7784, viswanat@duke.edu.

1 Introduction

We argue that the absence of household risk management is due to the fact that households' financing needs exceed their hedging concerns. We provide a standard neoclassical model in which households' ability to promise to pay is limited by the need to collateralize such promises. Collateral constraints hence restrict both financing as well as risk management as both require households to issue promises to pay. Given this link, households limit their risk management and may completely abstain from hedging when financing needs are sufficiently strong. Thus, the absence of household risk management and the lack of markets that provide such insurance should not be considered a puzzle.

Households' primary financing needs are two: purchases of durable goods and the accumulation of human capital. First, households consume the services of durable goods, most importantly housing, and the purchase of such goods needs to be financed. Second, investment in education requires financing, and education and learning-by-doing imply an age-income profile which is upward sloping on average. The bulk of financing actually extended to households is for purchases of durable goods. Indeed, more than 90% of household liabilities are attributable to durable goods purchases, mainly real estate (around 80%) and vehicles (around 6%), and less than 4% of household liabilities are attributable to education purposes.[1] We study a model in which all household borrowing needs to be collateralized by households' stocks of durable goods. Since most household financing is comprised of such loans, our model is plausible empirically. While households are able to borrow for education only to a very limited extent, consistent with our model, education and learning-by-doing are nevertheless important as they result in age-income profiles that are upward sloping on average which means that households have an incentive to borrow against the future using other means, namely, by financing durable goods.

Shiller (1993) has argued that markets that allow households to manage their risks would significantly improve welfare and that the absence of such markets hence presents an important puzzle. For example, Shiller (2008) writes that "[t]he near absence of derivatives markets for real estate ... is a striking anomaly that cries out for explanation

[1] In the first quarter of 2009, data from the *Flow of Funds Accounts of the U.S.* suggests that home mortgages are 78% of household liabilities and consumer credit is about 19% and, according to the *Federal Reserve Statistical Release G.19*, 12% is non-revolving consumer credit (which includes automobile loans as well as non-revolving loans for mobile homes, boats, trailers, education, or vacations). Data from the *2007 Survey of Consumer Finances* on the purpose of debt suggests that in 2007, about 83% of household debt is due to the purchase or improvement of a primary residence or other residential property, about 6% is due to vehicle purchases, less than 4% is due to education, and about 6% is due to the purchase of goods or services which is not further broken out.

and for actions to change the situation." We provide a rationale why households may not use such markets even if they exist. And given this lack of demand from households, the absence of such markets may not be so puzzling after all. The explanation we provide is simple: households' primary concern is financing, that is, shifting funds from the future to today, not risk management, that is, not transferring funds across states in the future. Risk management would require households to make promises to pay in high income states in the future, but this would reduce households' ability to promise to pay in high income states to finance durable goods purchases today, because households' total promises are limited by collateral constraints. Our dynamic model of complete markets subject to collateral constraints allows an explicit analysis of the connection between financing and risk management, and shows that the cost of risk management may be too high.

Indeed, we show that household risk management is monotone in household net worth and income, under quite general conditions. We first show that optimal household risk management of risk averse households whose income follows a stationary Markov chain with a notion of positive persistence is monotone and incomplete, even in the long run, that is, under the stationary distribution of household net worth. We extend these results to an economy with durable goods that the households can borrow against, and show that the monotone risk management result generalizes. Finally, we consider durable goods price risk, in addition to income risk, and provide conditions for monotone risk management. Under some assumptions, households may partially hedge income risk but not hedge durable goods price risk at all.

Consistent with the view that financing needs may override risk management concerns, we discuss evidence on U.S. households which suggests that poor (and financially constrained) households are less well insured against many types of risks, such as health risks or flood risks, than richer (and less financially constrained) households. Furthermore, a similar positive relation between income and risk management has recently been documented for farmers in developing economies, and we summarize the pertinent evidence. In addition, there is evidence that firms' financial constraints affect corporate risk management. For example, Rampini, Sufi, and Viswanathan (2012) document a strong positive correlation between firms' net worth and the extent of corporate risk management.

In an environment similar to the household finance context analyzed in this paper, Rampini and Viswanathan (2010, 2013) argue that firms' financing needs may override their hedging concerns and thus severely constrained firms may abstain from risk management. This is in contrast to received theory, formalized by Froot, Scharfstein, and Stein (1993), which suggests that constrained firms should hedge. The extant results in the literature do not take into account firms' financing needs and this literature hence reaches a rather different conclusion. One important consequence of the absence of risk management by constrained households and firms is that such households and firms are then more susceptible to shocks.

Section 2 reviews the evidence on household and firm risk management. Section 3 analyzes household income risk management in an endowment economy with income risk only and derives the basic monotone household risk management result. Section 4 extends the model to an economy with durable goods and shows how the monotone risk management result generalizes and that financing needs for durable goods and education may override hedging concerns. Durable goods price risk management is analyzed in Section 5. Section 6 considers households' ability to rent durable goods and the interaction between the rent vs. buy decision and risk management.[2] Section 7 concludes. All proofs are in the Appendix.

2 Stylized Facts on Household Risk Management

In this section we briefly survey evidence of what we consider a stylized fact, namely that poor (and more financially constrained) households are less well insured than richer (and less financially constrained) households. Indeed, we think this is part of a much broader pattern applying to entrepreneurial households and firms as well, and we briefly discuss evidence on risk management by Indian farmers and U.S. corporations suggesting that financial constraints reduce risk management substantially.

Among U.S. households, health insurance coverage varies considerably with income and age according to data from the U.S. Census Bureau. Panel A of Table 1 reports the percentage of people without health insurance in the U.S., which varies from 25% of people with income less than $25,000 to 8% of people with income exceeding $75,000. Similarly, Panel B of Table 1 reports variation by age; for adults, the fraction without health insurance decreases from 28% and 26% for age group 18-24 and age group 25-34, respectively, to 14% for age group 45-64, and to less than 2% for age group 65 and up. Brown and Finkelstein (2007) report that participation in long-term care insurance by individuals aged 60 and over also varies substantially by wealth, increasing from about 3% for the bottom wealth quartile to about 20% for the top quartile in U.S. data, as reported

[2] The asset pricing implications of housing have recently been considered by Lustig and Van Nieuwerburgh (2005) and Piazzesi, Schneider, and Tuzel (2007) in economies with similar preferences over two goods, (nondurable) consumption and housing services. Both studies consider a frictionless rental market for housing unlike us, which reduces households' financing needs substantially. Lustig and Van Nieuwerburgh (2005) consider the role of solvency constraints similar to ours and Piazzesi, Schneider, and Tuzel (2007) study the frictionless benchmark.

in Panel C of Table 1. Browne and Hoyt (2000) find that flood insurance coverage, both in terms of the number of policies per capita and the amount of coverage per capita, is positively correlated with disposable personal income per capita using U.S. state level data. Clearly, the extent to which households are insured hence varies substantially with households' income. And, assuming that individuals in age group 18-24 and age group 25-34 are more financially constrained, households' insurance level also seems to vary with financial constraints. This evidence is consistent with the view that there is an important connection between household risk management and households' financial constraints. That said, there are certainly other reasons why insurance participation varies with income, such as crowding out of private insurance by public programs, as stressed, for example, by Brown and Finkelstein (2008).

Among farmers in rural India, Giné, Townsend, and Vickery (2008) find that participation in rainfall insurance programs increases in wealth and decreases with measures of borrowing constraints. Cole, Giné, Tobacman, Topalova, Townsend, and Vickery (forthcoming) provide evidence on the importance of credit constraints for the adoption of rainfall insurance using randomized field experiments in rural India. Farmers who are randomly surprised with a positive liquidity shock are much more likely to buy insurance. Moreover, the authors report that the most frequently stated reason for not purchasing insurance is "insufficient funds to buy insurance." Farmers might of course use other risk sharing mechanisms, including informal ones. To overcome the limitation of analyzing specific risk sharing mechanisms in isolation, Townsend (1994) studies data on Indian farmers' household consumption directly in a seminal paper. Townsend finds that, while the full insurance model provides a remarkably good benchmark, "[t]here is evidence that the landless are less well insured than their village neighbors in one of the three villages." That is, there is "a hint of a pattern by land class. Specifically, the landless and small farmers in Aurepalle and the small and medium farmers in Shirapur seem more vulnerable."

In the spirit of the Townsend (1994) critique, Blundell, Pistaferri, and Preston (2008) study the extent of insurance by analyzing the income and consumption distribution for U.S. households jointly. They find "some partial insurance of permanent shocks, especially for the college educated and those near retirement. [They] find full insurance of transitory shocks except among poor households." Overall, we conclude that there is a basic pattern in household insurance: richer households are better insured than poorer households.

For firms, Rampini, Sufi, and Viswanathan (2012) find a strong positive correlation between net worth and risk management both in the cross section and within firms over time in data on fuel price risk management by U.S. airlines. Moreover, they document a remarkable drop in fuel price hedging as airlines approach distress which reverses only slowly after distress. Relatedly, the corporate finance literature documents a strong size pattern in risk management, when measured by participation of firms in derivatives markets, among U.S. corporations overall. For example, Nance, Smith, and Smithson (1993) find that firms which do not hedge are smaller, and pay lower dividends, in survey data for large industrial firms. Similarly, Géczy, Minton, and Schrand (1997) find a strong positive relation between derivatives use and firm size among large U.S. firms.[3] The evidence on the relation between corporate risk management and other financial variables is more mixed (see, for example, Tufano (1996) as well as the aforementioned studies). Nevertheless, the basic pattern for corporate insurance seems to be the same as for households: better financed firms engage in more risk management and poorly financed firms engage at best in limited and typically in no risk management at all.

3 Household Income Risk Management

In this section we consider household income risk management in an endowment economy. We show that optimal household income risk management is incomplete and monotone increasing in the households' net worth, that is, richer households are better insured. Moreover, we show that there is a sense in which "the poor cannot afford insurance." Finally, we characterize household risk management in the long run and in an economy in which households are eventually unconstrained.

3.1 Household Finance in an Endowment Economy

Consider household income risk management in an endowment economy. Time is discrete and the horizon is infinite. Households have preferences $E\left[\sum_{t=0}^{\infty}\beta^t u(c_t)\right]$ where we assume that $\beta \in (0,1)$ and $u(c)$ is strictly increasing, strictly concave, and satisfies $\lim_{c\to 0} u_c(c) = \infty$ and $\lim_{c\to\infty} u_c(c) = 0$. Households' income $y(s)$ follows a Markov chain on state space $s \in S$ with transition matrix $\Pi(s,s')$ describing the transition probability from state s to state s', and $\forall s, s_+,\ s_+ > s,\ y(s_+) > y(s) > 0$. We use the shorthand $y' \equiv y(s')$ for income in state s' next period wherever convenient and analogously for other variables.

[3] Approximately 41% of the firms with exposure to foreign currency risk in their data use currency derivatives and 59% use any type of derivative. Across firm size quartiles, currency derivative use increases from 17% for the smallest quartile to 75% for the largest quartile and the use of any derivatives increases from 33% to 90%; see their table 2.

Moreover, let $\underline{s} = \min\{s : s \in S\}$ and $\bar{s} = \max\{s : s \in S\}$ and analogously for $\underline{y}$ and $\bar{y}$ and let S also denote the cardinality of S in a slight abuse of notation.

Lenders are risk neutral and discount the future at rate $R^{-1} > \beta$, that is, are patient relative to the households, and have deep pockets and abundant collateral in all dates and states; lenders are thus willing to provide any state contingent claim at an expected return R.[4]

Enforcement is limited as follows: households can abscond with their income and cannot be excluded from markets for state contingent claims in the future. Extending the results in Rampini and Viswanathan (2010, 2013) to this environment, one can show that the optimal dynamic contract with limited enforcement can be implemented with complete markets in one-period ahead Arrow securities subject to short sale constraints (which are a special case of collateral constraints).

In some parts of the analysis, we consider Markov chains which exhibit the following notion of positive persistence:

Definition 1 *A Markov chain $\Pi(s, s')$ displays* ***first order stochastic dominance (FOSD)*** *if $\forall s, s_+, \hat{s}'$, $s_+ > s$, $\sum_{s' \leq \hat{s}'} \Pi(s_+, s') \leq \sum_{s' \leq \hat{s}'} \Pi(s, s')$.*

This definition requires that the distribution of states next period conditional on current state s_+ first order stochastically dominates the distribution conditional on current state s, for all $s_+ > s$. A Markov chain which is independent over time, that is, satisfies $\Pi(s, s') = \Pi(s')$, $\forall s \in S$, exhibits FOSD. For a symmetric two-state Markov chain, FOSD is equivalent to assuming that $\Pi(\bar{s}, \bar{s}) = \Pi(\underline{s}, \underline{s}) \equiv p \geq 1/2$, that is, that the autocorrelation ρ is positive, as $\rho = 2p - 1 \geq 0$. Arguably, such positive autocorrelation in household income is plausible empirically.

3.2 Household's Income Risk Management Problem

The household solves the following recursive problem by choosing (non-negative) consumption c and a portfolio of Arrow securities h' for each state s' (and associated net worth w') given the exogenous state s and the net worth w (cum current income),

$$v(w, s) \equiv \max_{c, h', w' \in \mathbb{R}_+ \times \mathbb{R}^{2S}} u(c) + \beta E[v(w', s')|s] \tag{1}$$

subject to the budget constraints for the current and next period, $\forall s' \in S$,

$$\begin{aligned} w &\geq c + E[R^{-1} h'|s], && (2) \\ y' + h' &\geq w', && (3) \end{aligned}$$

[4] We discuss the case in which $\beta = R^{-1}$ below.

and the short sale constraints, $\forall s' \in S$,

$$h' \geq 0. \tag{4}$$

Since the return function is concave, the constraint set convex, and the operator defined by the program in (1) to (4) satisfies Blackwell's sufficient conditions, there exists a unique value function v which solves the Bellman equation. The value function v is strictly increasing and strictly concave. Denoting the multipliers on the budget constraints (2) and (3) by μ and $\beta\Pi(s,s')\mu'$, respectively, and the multipliers on the short-sale constraints (4) by $\beta\Pi(s,s')\lambda'$, the first order conditions are

$$\mu = u_c(c), \tag{5}$$

$$\mu' = v_w(w', s'), \tag{6}$$

$$\mu = \beta R \mu' + \beta R \lambda'. \tag{7}$$

We have ignored the non-negativity constraint on consumption since it is not binding. The envelope condition is $v_w(w,s) = \mu$ (see Benveniste and Scheinkman (1979)).

3.3 Household Income Risk Management is Monotone

We now show that household risk management is monotone in household net worth. In particular, the set of states that the households hedge is increasing in net worth and richer households' net worth distribution next period dominates that of poorer households. Richer households moreover spend more on hedging.

Proposition 1 (Monotone household risk management) (i) *The set of states that the household hedges $S_h \equiv \{s' \in S : h(s') > 0\}$ is increasing in net worth w given the current state s, $\forall s \in S$.* **(ii)** *For $w_+ > w$ and denoting the net worth next period associated with w_+ (w) by w'_+ (w'), we have $w'_+ \geq w'$, $\forall s' \in S$, that is, w'_+ statewise dominates and hence FOSD w'; moreover, $h'_+ \geq h'$, $\forall s' \in S$, and $E[h'_+|s] \geq E[h'|s]$, $\forall s \in S$.*

Note that Proposition 1 does not impose any additional structure on the Markov process for income and hence does not determine which states are hedged. If we further assume that the Markov chain displays FOSD, then we can show that households hedge a lower interval of income realizations. Moreover, with this assumption household risk management is monotone in both net worth w and the current state s.

Proposition 2 *Assume that $\Pi(s, s')$ displays FOSD.* **(i)** *The marginal value of net worth $v_w(w, s)$ is decreasing in the state s.* **(ii) (Monotone household risk management with FOSD)** *The household hedges a lower interval of states, if at all, given net worth w and state s, that is, $S_h \equiv \{s' \in S : h(s') > 0\} = \{\underline{s}', \ldots, s'_h\}$, where the dependence of S_h and s'_h (and w_h below) on the current state w and s is suppressed for convenience; net worth next period w', hedging h', and the interval of states hedged S_h are all increasing in w and s, $\forall s, s' \in S$. If moreover $\Pi(s, s') = \pi(s')$, $\forall s, s' \in S$, then $w(s') = w_h$, $\forall s' \in S_h$, and w_h is increasing in w.* **(iii) (Monotone variance of net worth and consumption)** *If $\Pi(s, s') = \pi(s')$, $\forall s, s' \in S$, then the variance of net worth w' and consumption c' next period is decreasing in current net worth w.*

The key to the result is the fact that the marginal value of net worth $v_w(w, s)$ is decreasing not just in w, as before, but also in the state s. First order stochastic dominance means that if the household is in a higher state today, holding current net worth w constant, then the household's income next period is higher in a FOSD sense. This reduces the cost of hedging to a given level for each state tomorrow, as hedging decreases with the state, and hedging the same amount becomes less costly. The household partially consumes the resources that are thus freed up and partially uses them to buy additional Arrow securities, that is, purchase more insurance. Thus, parts (ii) and (iii) give a sense in which richer households are better insured.

Positive persistence in the income process hence means that a high income realization reduces the marginal value of net worth for two reasons: first, high current income raises current net worth, which lowers the marginal value of net worth due to concavity; and second, a high current income implies higher expected future income, further reducing the marginal value of net worth by the mechanism described above. In contrast, in a production economy with technology shocks, positive persistence has two effects which go in opposite directions: on the one hand, high current productivity implies high cash flow and thus raises current net worth, which lowers the marginal value of net worth due to the concavity of the value function; on the other hand, high current productivity increases the expected productivity which means firms would like to invest more, and this effect in turn raises the marginal value of net worth. Thus there are two competing effects when productivity shocks have positive persistence and if the second effect is sufficiently strong, firms hedge states with high productivity.[5]

The proof is of technical interest as we prove that the marginal value of net worth is (weakly) decreasing in s by showing that the Bellman operator maps functions which

[5] See Rampini and Viswanathan (2013) for an analysis of the effect of persistence on the extent of corporate risk management.

satisfy this property into functions which satisfy the property as well, and since the space of functions which satisfies the property is closed, the unique fixed point must satisfy the property, too.

Under the additional assumption of independent income shocks, the household ensures a minimum level of net worth next period, which is increasing in current net worth. Moreover, the variance of both net worth and consumption next period is decreasing in current net worth, that is, there is a strong sense in which richer households are better insured.[6]

3.4 Incomplete Household Risk Management

We have a more explicit characterization of optimal income risk management when the Markov chain displays FOSD:

Proposition 3 (Incomplete risk management) *Assume that $\Pi(s, s')$ displays FOSD.* **(i)** *At net worth $w = \underline{y}$ in state $\underline{s}$, the household does not hedge at all, i.e., $\lambda' > 0, \forall s' \in S$, and $S_h = \emptyset$.* **(ii)** *At net worth $w = \bar{y}$, the household does not hedge the highest state next period, that is, $\lambda(\bar{s}') > 0$ and $S_h \subsetneq S$, $\forall s \in S$.*

At net worth $\underline{y}$ (and in state $\underline{s}$) the household does not hedge at all, which can be interpreted as saying that "the poor can't afford insurance." Moreover, even at net worth $\bar{y}$, the household does not engage in complete risk management, and given monotonicity the household hence does not hedge the highest state for any level of wealth $w \leq \bar{y}$.

Figure 1 illustrates Propositions 2 and 3 for an economy with an independent, symmetric two state Markov chain. The top right panel illustrates the monotonicity of household risk management, with the middle right panel showing that the richer households actually spend a larger fraction of their budget on Arrow securities to hedge future income shocks.

3.5 Household Risk Management in the Long Run

How does household risk management behave in the long run, given that households can accumulate net worth? When the income process is independent, we show that the model

[6]If income is lower in a downturns and risk management consequently declines, then the cross sectional variation of consumption can be countercyclical, a property documented by Storesletten, Telmer, and Yaron (2004) that is of interest due to its asset pricing implications (see, for example, Mankiw (1986) and Constantinides and Duffie (1996)). Guvenen, Ozkan, and Song (2012) find that the left-skewness of idiosyncratic income shocks is countercyclical, rather than the variance itself, in earnings data from the U.S. Social Security Administration. Rampini (2004) provides a real business cycle model with entrepreneurs subject to moral hazard in which the cross sectional variation of the optimal incentive compatible allocation is similarly countercyclical.

induces a stationary distribution for household net worth. Under the unique stationary distribution, households never hedge completely. Notably, households abstain from risk management completely with positive probability under the stationary distribution. This means that even households whose current net worth is high, that are hit by a sufficiently long sequence of low income realizations, end up so constrained again that they no longer purchase any Arrow securities, that is, stop buying any insurance at all.

Proposition 4 (Household risk management under the stationary distribution) *Suppose* $\Pi(s, s') = \pi(s'), \forall s, s' \in S$. **(i)** *There exists a unique stationary distribution of net worth.* **(ii)** *The support of the stationary distribution is* $[\underline{w}, \bar{w}]$ *where* $\underline{w} = \underline{y}$ *and* $\bar{w} = \bar{y}$. **(iii)** *Under the stationary distribution, household risk management is monotone, incomplete with probability 1, and completely absent with strictly positive probability.*

Figure 2 illustrates Propositions 4 for an independent two state Markov chain as in the example in Section 3.4 above. The top panel displays the unconditional stationary distribution whose support is between the low income realization ($\underline{y} = 0.5$ in the example) and the high income realization ($\bar{y} = 1.5$). The household never hedges the high state next period, which means the household's net worth conditional on a high realization is always $w(\bar{s}') = \bar{y}$ as the bottom panel shows. The household does hedge low realization on income, at least as long as net worth is sufficiently high, so starting from net worth $\bar{y}$ low income realization decrease the household's net worth gradually over time, as the middle panel illustrates; the probability mass decreases at a rate $\pi(\underline{s})$ in this range. Eventually, the household stops hedging, and subsequent realizations result in net worth $\underline{y}$ until a high income realization lifts the household's net worth again.

3.6 Risk Management when Households are Eventually Unconstrained

Consider the limit of the above economy where $\beta R = 1$, which means that households are eventually unconstrained. We show that the economy displays full insurance under the stationary distribution in the limit and that household net worth is nevertheless bounded in the limit. These results are related to results for the classic class of income fluctuations problems studied by Yaari (1976), Schechtman (1976), Bewley (1980), Aiyagari (1994), and others, in which households solve a consumption savings problem with non-contingent debt and borrowing constraints, that is, have access to incomplete markets only.[7] Our

[7] In a calibrated life-cycle model with incomplete markets, Fuster and Willen (2011) study the trade-off between insuring consumption across states and intertemporal smoothing quantitatively.

results are similar in that there is complete consumption insurance in the limit, but they are rather different in that net worth is bounded in the limit whereas it grows without bound in these related papers.

We emphasize that for net worth levels below $\bar{w}$, household risk management is incomplete and monotone in current net worth even when $\beta R = 1$, although such levels of net worth are transient. The main result of our paper hence obtains even in this case, albeit only in the transition.

When income is independent over time and $\beta R = 1$, we know from equation (7) and the envelope condition that $v_w(w) = v_w(w') + \lambda'$ and therefore $v_w(w)$ is non-increasing. Denoting the upper bound of net worth under the stationary distribution by $\bar{w}$, we hence have $v_w(\bar{w}) \geq v_w(w')$, but by strict concavity $v_w(\bar{w}) \leq v_w(w')$, and thus $\bar{w} = w'$, $\forall s' \in S$, that is, $\bar{w}$ is absorbing. Note that for $w < \bar{y}$, $\lambda(\bar{s}') > 0$ and $w(\bar{s}') = \bar{y}$. Moreover, suppose $\exists s' \in S$, such that $w' > \bar{y}$, then $v_w(\bar{y}) > v_w(w')$ and $\lambda' > 0$, that is, $w' = y' \leq \bar{y}$, a contradiction. Therefore, $w' = \bar{y}$, for all $s' \in S$. Thus, the stationary net worth distribution collapses to unit mass at $\bar{w} = \bar{y}$.

The full insurance result is general, that is, does not require independence of the income process:

Proposition 5 (Full insurance under the stationary distribution in the limit) *When $\beta R = 1$, the household engages in full insurance under the stationary distribution.*

In the case of a symmetric two state Markov chain for income, we can solve for the stationary distribution of net worth in closed form. Specifically, say $S = \{s_L, s_H\}$ with $s_L < s_H$, and $\Pi(s_H, s_H) = \Pi(s_L, s_L) \equiv p$. We use subscripts L and H where convenient. Using the fact that the stationary distribution of y is $(1/2, 1/2)$ and that $w_H = y_H$ since the household does not hedge the highest state next period given FOSD (see Proposition 3), $w_L = y_L + h_L$, $c_H = w_H - (1-p)R^{-1}h_L$, $c_L = w_L - pR^{-1}h_L$, and $c_H = c_L$, that is, full insurance, we obtain

$$h_L = \frac{R}{R-\rho}(y_H - y_L), \quad w_L - w_H = \frac{\rho}{R-\rho}(y_H - y_L), \quad c_H = c_L \equiv c = E[y] + \frac{1}{2}\frac{r}{R-\rho}(y_H - y_L)$$

where $\rho = 2p - 1 \geq 0$ and $r \equiv R - 1$. When income is independent over time, $p = 1/2$ and $\rho = 0$, we have $h_L = y_H - y_L$, $w_H = w_L = y_H$, and $c = E[y] + 1/2r/R(y_H - y_L)$.

Note that $w_L \geq w_H$ and that the difference $w_L - w_H$ is increasing in the persistence ρ. So net worth as we defined it is higher in the low state than in the high state. To see why this is, denote the present value of income (ex current income), that is, human capital, by PV_s, and note that

$$\begin{aligned} PV_H &= R^{-1}(p(y_H + PV_H) + (1-p)(y_L + PV_L)) \\ PV_L &= R^{-1}((1-p)(y_H + PV_H) + p(y_L + PV_L)) \end{aligned}$$

which implies that $PV_H - PV_L = w_L - w_H$ or $w_H + PV_H = w_L + PV_L$, that is, total wealth, (financial) net worth plus human capital, is constant across states. When the household has low current income, his (financial) net worth is high to compensate for the reduction in present value of future labor income. When income is independent over time, the present value of future labor income is constant across states and so is the household's (financial) net worth.

To sum up, when $\beta R = 1$, households are eventually unconstrained and fully insured, but their net worth remains finite, in contrast to the models with incomplete markets in which households accumulate infinite buffer stocks to smooth consumption in the limit.

3.7 Remark on Limit Behavior of Corporate Risk Management

There is an interesting related result for corporate risk management. Consider the limit as βR goes to 1 in the corresponding firm financing model (see Rampini and Viswanathan (2013)). In the limit, firms are eventually unconstrained and engage in complete risk management. This suggests that unconstrained firms should be thought of as being completely hedged. Instead, such firms are often interpreted as not hedging at all, because financial structure does not matter in a frictionless economy, the insight of the celebrated Modigliani and Miller (1958) theorem. It is of course true that at the limit, the corporate risk management policy is not uniquely determined, given the linearity of payoff function of a risk neutral firm, and the firm could hedge by retaining a sufficient buffer stock to cover any low income realizations in the future. But the unique limit is full risk management; in other words, the net worth minimal way of running a firm when $\beta R = 1$ involves complete risk management, not no risk management at all. This result may be empirically relevant as we are not aware of any evidence that corporate risk management declines when firms are very well capitalized.

4 Household Risk Management with Durable Goods

This section extends our model of household finance to include durable goods. The monotone household risk management results generalize to this environment to a large extent. Moreover, we show that for households with sufficiently low net worth financing needs override hedging concerns, and consider the additional financing needs for education purposes.

4.1 Household Finance with Durable Goods

Consider an extension of the economy of Section 3 with two goods, (non-durable) consumption c and durable goods k, which in practice comprises mainly housing. The environment, income process, and lenders are as before, but households have preferences $E[\sum_{t=0}^{\infty}\beta^t(u(c_t)+g(k_t)]$ where $g(k)$ is strictly increasing, strictly concave, and satisfies $\lim_{k\to 0} g_k(k)=+\infty$ and $\lim_{k\to\infty} g_k(k)=0$.

Durable goods depreciate at rate $\delta\in(0,1)$ and the price in terms of consumption goods is assumed to be constant and normalized to 1, $\forall s\in S$. Households can adjust their durable goods stock freely, but there is no rental market for durable goods and households have to purchase durable goods to consume their services. Durable goods are also used as collateral as we discuss below. We consider durable goods price risk in Section 5 and analyze the implications of households' ability to rent durables as well as purchase durables and borrow against them in Section 6.

Enforcement is limited as follows: households can abscond with their income and a fraction $1-\theta$ of durable goods, where $\theta\in(0,1)$, and cannot be excluded from markets for state contingent claims or durable goods. As before, one can show that the optimal dynamic contract with limited enforcement can be implemented with complete markets in one-period Arrow securities subject to collateral constraints that limit the household's promises b' in state s' next period as follows: $\theta k(1-\delta)\geq Rb'$, $\forall s'\in S$.

The only friction we add to the standard neoclassical environment is that claims need to be collateralized to enforce repayment. Moreover, we assume that there is no rental market for capital for now. Importantly, our environment is one with full information. Thus, households are able to trade contingent claims on all states of nature, which allows them to engage in risk management.

The simplest and equivalent formulation of the household's problem is to assume that the household levers durable assets fully, that is, borrows $\hat{b}'=R^{-1}\theta k(1-\delta)$, $\forall s'\in S$, and purchases Arrow securities in the amount $h'=\theta k(1-\delta)-Rb'$, $\forall s\in S$. Under this equivalent formulation, the collateral constraints on b' reduce to short sale constraints on h'. Moreover, since the household borrows as much as possible against durable assets, the household pays down $\wp\equiv 1-R^{-1}\theta(1-\delta)$ per unit of durable assets purchased only.

The household solves the following recursive problem by choosing (non-negative) consumption c, durable goods k, and a portfolio of Arrow securities h' for each state s' (and associated net worth w') given the exogenous state s and the net worth w (cum current income and durable goods net of borrowing),

$$v(w,s) \equiv \max_{c,k,h',w' \in \mathbb{R}^2_+ \times \mathbb{R}^{2S}} u(c) + \beta g(k) + \beta E[v(w',s')|s] \tag{8}$$

subject to the budget constraints for the current and next period, $\forall s' \in S$,

$$\begin{aligned} w &\geq c + \wp k + E[R^{-1}h'|s], & (9) \\ y' + (1-\theta)k(1-\delta) + h' &\geq w', & (10) \end{aligned}$$

and the short sale constraints (4), $\forall s' \in S$.

Note that the value function is written excluding the service flow of the current stock of durable goods. The return function $u(c)+\beta g(k)$ includes the service flow of durables purchased this period for use next period, which is deterministic given purchases of durables this period. This definition of the value function and net worth allows us to formulate the problem with one endogenous state variable, net worth w, only. Arguing analogously to before, there exists a unique value function which is strictly increasing and strictly concave. Note that there is no need to impose non-negativity constraints on consumption and durable goods as these are slack given our preference assumptions. Defining the multipliers as before, the first order conditions are (5) through (7) and

$$\wp\mu = \beta g_k(k) + E[\beta\mu'(1-\theta)(1-\delta)|s], \tag{11}$$

or written as an investment Euler equation for durable goods

$$1 = \beta \frac{1}{\mu} \frac{g_k(k)}{\wp} + E\left[\beta \frac{\mu'}{\mu} \frac{(1-\theta)(1-\delta)}{\wp} \middle| s \right]. \tag{12}$$

The first term on the right hand side is the (marginal) service flow of the durable goods purchased this period and consumed next period, that is, the (marginal) "dividend yield" of durables, and the second term on the right hand side is the return from the resale value of durables net of borrowing. Since durables are fully levered, $k(1-\delta) - R\hat{b}' = (1-\theta)k(1-\delta)$. The down payment requirement $\wp = 1 - R^{-1}\theta(1-\delta)$ is in the denominator as this is the amount of net worth the household has to invest per unit of durable assets.

4.2 Monotonicity of Household Risk Management with Durable Goods

With durable goods, household risk management is monotone in net worth in the sense that the household's net worth w' next period is strictly increasing in current net worth. Unlike in the economy with income risk only in Section 3, we can no longer conclude that the household's purchases of Arrow securities necessarily increase in wealth, as the household also buys more durables which increases its net worth next period. With independence of the income process, the household hedges a lower set of income realizations and household risk management is incomplete under the stationary distribution.

Proposition 6 **(i) (Monotonicity)** *Consumption c, durable goods holdings k, and net worth next period w' are strictly increasing in net worth w, given state s, $\forall s, s' \in S$.* **(ii) (Incomplete risk management)** *Suppose $\Pi(s, s') = \pi(s')$, $\forall s' \in S$. Then the household hedges a lower interval of states, if at all, given net worth w, that is, $S_h \equiv \{s' \in S : h(s') > 0\} = \{\underline{s}', \ldots, s'_h\}$, possibly empty, where the dependence on w is again suppressed for convenience. For $w \leq \bar{w}$, the household never hedges the highest state next period, $h(\bar{s}') = 0$, where $\bar{w}$ is the highest wealth level attained under the stationary distribution.*

When the Markov process displays FOSD, we can again show that the marginal value of net worth $v_w(w, s)$ decreases in state s. Therefore, households hedge a lower set of income realizations and, among the states they hedge, hedge worse income realizations strictly more.

Proposition 7 *Assume that $\Pi(s, s')$ displays FOSD.* **(i)** *The marginal value of net worth $v_w(w, s)$ is decreasing in the state s.* **(ii) (Monotone household risk management with FOSD)** *The household hedges a lower interval of states, if at all, given net worth w and state s, that is, $S_h \equiv \{s' \in S : h(s') > 0\} = \{\underline{s}', \ldots, s'_h\}$, where the dependence of S_h and s'_h (and w_h below) on the current state w and s is suppressed for convenience; net worth next period w' is increasing in w, $\forall s, s' \in S$, and h' is strictly decreasing in s'. If moreover $\Pi(s, s') = \pi(s')$, $\forall s, s' \in S$, then $w(s') = w_h$, $\forall s' \in S_h$, and w_h is increasing in w.*

4.3 Financing Needs Override Risk Management Concerns

We now show that if a household's financing needs are sufficiently strong, then financing needs override hedging concerns. Noting that the budget constraint next period (3) binds in all states and that purchases of Arrow securities are limited by short-sale constraints (4), we know that net worth w' in state s' next period is bounded below, namely,

$$w' \geq y' + (1 - \theta)k(1 - \delta) > y'.$$

Households' limited ability to promise implies that their net worth w' next period in all states is bounded below. But this means that the household must be collateral constrained against all states s' next period if the household's current net worth w is sufficiently low, since the marginal value of net worth next period must be bounded above.

Proposition 8 (Financing needs override risk management concerns) *If a household's current net worth w is sufficiently low, the household is constrained against all states next period, and hence does not engage in risk management.*

Households' limited ability to credibly promise repayment means that households cannot pledge future income and households' net worth has to be at least future labor income. Moreover durable goods purchases require some down payment per unit of capital from the household and hence implicitly force households to shift additional net worth to the next period. Both these aspects imply that if current household net worth is relatively low, the household shifts resources to the present to the extent possible, that is, borrows as much as possible against durable goods.

4.4 Financing Education

Age-income profiles are upward sloping on average partly because of economic growth and partly presumably because of learning by doing, that is, skill accumulation with experience. These properties of the labor income process give households further incentives to borrow as much as they can against their durable goods, such as housing, and thus exhaust their debt capacity and abstain from risk management.

Suppose moreover that households are able to invest in education or human capital e. An amount of education e invested in the current period, which includes both foregone labor income and direct costs, results in income $A'f(e)$ in state s' next period, where f is strictly increasing and strictly concave, $\lim_{e\to 0} f_e(e) = +\infty$, and $\lim_{e\to\infty} f_e(e) = 0$, and the productivity of human capital $A' > 0$, for all $s' \in S$, is described by a Markov process also summarized by state s. Human capital depreciates at a rate $\delta_e \in (0, 1)$. Note that households in our model can borrow against neither future labor income nor human capital, as education capital is inalienable, and can only borrow against durable goods. The household's problem is to choose (non-negative) consumption c, durable goods k, education e, and a portfolio of Arrow securities h' (with associated net worth w') for each state s' given the exogenous state s and net worth w (cum current income, durable goods net of borrowing, and human capital) to maximize (8) subject to the budget constraints for the current and next period, $\forall s' \in S$,

$$
\begin{aligned}
w &\geq c + \wp k + e + E[R^{-1}h'|s], && (13)\\
A'f(e) + e(1-\delta_e) + (1-\theta)k(1-\delta) + h' &\geq w', && (14)
\end{aligned}
$$

and the short sale constraints (4), $\forall s' \in S$. Note that the household's problem is still well behaved, that is, the constraint set is convex.

Proposition 9 *In the problem with education, that is, investment in human capital, if a household's current net worth w is sufficiently low, the household is constrained against all states next period and hence does not engage in risk management.*

The household's Euler equation for education, that is, investment in human capital, can be written as

$$\begin{aligned} 1 &= E\left[\beta \frac{v_w(w', s')}{v_w(w, s)}(A' f_e(e) + (1-\delta_e)) \middle| s\right] \qquad (15) \\ &\geq \Pi(s, s')\beta \frac{v_w(w', s')}{v_w(w, s)}(A' f_e(e) + (1-\delta_e)), \quad \forall s, s' \in S. \end{aligned}$$

The budget constraint (13) implies that $w \geq e$ and hence as w goes to zero, so does e implying that $f_e(e)$ goes to $+\infty$. But then $\beta v_w(w', s')/v_w(w, s)$ must go to zero, $\forall s' \in S$, using the Euler equation for investment in education, and, using equation (7), $\beta\lambda'/\mu$ must go to R^{-1} implying that the multipliers on the short sale constraints $\lambda' > 0$, $\forall s' \in S$. The intuition is that if the household's net worth is sufficiently low, then the household's education decreases so much that the marginal rate of transformation on investment in human capital eventually exceeds the return on saving net worth for state s', for all states.

Investment in education is an additional reason why households are likely to have higher net worth later in life, giving them further incentives to finance as much of their durable goods purchases as they can, rather than using their limited ability to pledge to shift funds across states later on.

5 Durable Goods Price Risk Management

In this section we consider an economy with durable goods price risk. Suppose the price of durable goods $q(s)$ is stochastic, where the state s describes the joint evolution of income $y(s)$ and $q(s)$, and the economy is otherwise the same as in Section 4.[8] As in that section, we assume without loss of generality that the household levers durable assets fully, that is, borrows $\hat{b}' = R^{-1}\theta q' k(1-\delta)$, $\forall s' \in S$, and purchases Arrow securities in the amount h', $\forall s' \in S$. The collateral constraints again reduce to short sale constraints. Moreover, since the household borrows as much as possible against durable assets, the household pays down $\wp(s) \equiv q(s) - R^{-1}\theta E[q'|s](1-\delta)$ per unit of durable assets purchased

[8] In Section 4, the price of durable goods is constant and normalized to 1.

only. We assume that $q(s)$ and $\wp(s)$ are increasing in s, although some of our results obtain more generally.

The household's problem, formulated recursively, is to choose (non-negative) consumption c, durable goods k, and a portfolio of Arrow securities h' for each state s' (and associated net worth w') given the exogenous state s and the net worth w (cum current income and durable goods net of borrowing), to maximize (8) subject to the budget constraints for the current and next period, $\forall s' \in S$,

$$\begin{aligned} w &\geq c + \wp(s)k + E[R^{-1}h'|s], &(16)\\ y' + (1-\theta)q'k(1-\delta) + h' &\geq w', &(17)\end{aligned}$$

and the short sale constraints (4), $\forall s' \in S$.

Defining the multipliers as before, the first order conditions are (5) through (7) and

$$\wp(s)\mu = \beta g_k(k) + E[\beta\mu'(1-\theta)q'(1-\delta)|s]. \tag{18}$$

The durable goods price affects the down payment $\wp(s)$ in the current period and the resale value of durable goods next period. If the household cannot pledge the full resale value of durables, that is, if $\theta < 1$, then durable goods purchases force the household to implicitly save. Moreover, the household is then exposed to the price risk of durables in two ways: first, the resale value of durable goods affects the household's net worth next period, and second, the durable goods price affects the down payment which in turn affects the marginal value of net worth. If the household can pledge the full resale value of durables, that is, if $\theta = 1$, the second term on the right hand side of (18) is zero, and the first order condition simplifies to $\wp(s)\mu = \beta g_k(k)$. In this case, the durable goods price only affects the household's problem through the down payment. We are able to characterize the solution explicitly in the case of isoelastic preferences with coefficient of relative risk aversion $\gamma \leq 1$: household risk management is monotone. Specifically, we show that the economy is equivalent to an economy with income risk and preference shocks. With logarithmic preferences, households do not hedge the durable goods price risk at all, but may partially hedge income risk. With $\gamma < 1$, higher durable goods prices, and hence higher down payments, reduce the marginal value of net worth as the substitution effect dominates the income effect. In other words, lower house prices amount to investment opportunities and raise the marginal value of net worth.

Proposition 10 *Suppose $\theta = 1$ and preferences satisfy $u(c) = c^{1-\gamma}/(1-\gamma)$ and $g(k) = gk^{1-\gamma}/(1-\gamma)$ where $\gamma > 0$ and $g > 0$.* **(i)** *If $\gamma = 1$ (logarithmic preferences), then $v(w,s) = (1+\beta g)\hat{v}(w,s) + v_{\varphi}(s)$, where $\hat{v}(w,s)$ solves the income risk management* *problem (without durable goods) in equations (1) through (4) and $v_\varphi(s)$ is an exogenous function defined in the proof. Household risk management is monotone in the sense of Propositions 1 and 2 and the household does not hedge durable goods price risk at all.* **(ii)** *For $\gamma \neq 1$, the problem is equivalent to an income risk management problem in an economy with preference shocks where $\hat{u}(\hat{c}, s) = \phi(s)u(\hat{c})$ with $\hat{c}$ and $\phi(s)$ defined in the proof. Household risk management is monotone in the sense of Proposition 1. Moreover, if $\Pi(s, s')$ displays FOSD, $\wp(s)$ is increasing in s, and $\gamma < 1$, then the marginal value of net worth $v_w(w, s)$ is decreasing in s, the household hedges a lower set of states, and w', h', and S_h are all increasing in w and s, $\forall s, s' \in S$.*

More generally, when $\theta < 1$, a drop in the durable goods price lowers the household's net worth and hence raises the marginal utility of net worth, and, when $\gamma < 1$, the low durable goods price may further raise the marginal utility of net worth. Thus, households likely hedge low durable goods prices in this case. In contrast, when $\gamma > 1$, a drop in the durable goods price has two opposing effects, on the one hand lowering net worth and on the other hand raising the marginal utility of net worth due to the price effect. This additional effect reduces the household's hedging demand. Under plausible parameterizations, the direct effect on net worth arguably dominates nonetheless, but this quantitative question remains to be analyzed.[9]

6 Risk Management and the Buy vs. Rent Decision

In the analysis so far we have not considered households' ability to rent durable goods. If there were a frictionless rental market, ownership of a durable good and the use of its services would be separable. The need to collateralize claims might still limit risk sharing,[10] but tenure choice would not affect households' portfolio choice. Moreover, households' demand for housing services would not induce a substantial financing need in that case.

In this section we consider a rental market which is not frictionless. Renting durable goods is possible, albeit costly, but relaxes collateral constraints as landlords or lessors can more easily repossess rented durables. A similar market for rented capital has been

[9]This result is reminiscent of the results in the consumption based asset pricing literature that show that investors' hedging demand in the presence of expected return variation depends in a similar way on the coefficient of relative risk aversion; investors hedge states with low expected returns when the coefficient of relative risk aversion exceeds 1 and otherwise hedge high expected returns (see, for example, Campbell (1996)).

[10]See, for example, Lustig and Van Nieuwerburgh (2005).

analyzed in a corporate finance context by Eisfeldt and Rampini (2009) and Rampini and Viswanathan (2013). Sufficiently constrained households choose to rent, which affects their risk management or portfolio choice. Because renting housing is costly, households will continue to have a strong incentive to own housing and hence face considerable financing needs for housing. We are able to characterize the interaction between risk management and home ownership since in our model markets are complete, although subject to collateral constraints. In contrast the literature typically studies the interaction of the risk of home ownership and portfolio choice under the assumption that markets are incomplete. Sinai and Souleles (2005) argue that both home ownership and renting are risky when households do not have access to complete markets. Our model may also provide a useful framework to study household interest rate risk management, which Campbell and Cocco (2003) model as the choice of mortgage type, specifically the choice between adjustable rate mortgages (ARMs) and fixed rate mortgages.

[TO BE COMPLETED.]

7 Conclusion

An explicit analysis of household risk management is provided in which households have access to complete markets subject to collateral constraints. We show the optimality of monotone household risk management, that is, risk management that increases in household net worth and income, under quite general conditions. Durable goods, most importantly housing, are used as collateral. In the absence of a frictionless rental market, households' demand for the services of consumer durables results in substantial financing needs. We show that if these financing needs are sufficiently strong, they override hedging concerns, which explains the almost complete absence of household risk management. In our view, proposals to introduce new markets providing household risk management tools are hence unlikely to be successful, as households may not use such markets even if they exist.

The fact that household risk management may require collateral in the form of margins has been recognized in the literature, but not explicitly analyzed. For example, Athanasoulis and Shiller (2000) write that "[m]argin requirements might deal with this [collection] problem, but only for people who have sufficient assets as margin. We will disregard these kinds of ... problems." Our work, in contrast, suggests that collateral constraints, together with households' other financing needs, are at the heart of the explanation why household risk management is limited.

Appendix

Proof of Proposition 1. Part (i): Suppose $\exists \hat{s}' \in S_h$ such that $\hat{s}' \notin S_{h+}$. Using (7), (6), the envelope condition, and strict concavity of the value function we have

$$\beta R v_w(w(\hat{s}'), \hat{s}') = v_w(w, s) > v_w(w_+, s) \geq \beta R v_w(w_+(\hat{s}'), \hat{s}'),$$

implying, again by strict concavity of the value function, that $w(\hat{s}') < w_+(\hat{s}')$. But $w(\hat{s}') = y(\hat{s}') + h(\hat{s}') > y(\hat{s}') = w_+(\hat{s}')$, a contradiction.

Part (ii): Note that $w'_+ \geq w$, $\forall s' \in S$, implies that $y' + h'_+ = w'_+ \geq w' = y' + h'$, that is, $h'_+ \geq h'$, $\forall s' \in S$, and hence $E[h'_+|s] \geq E[h'|s]$. To see that $w'_+ \geq w$, $\forall s' \in S$, suppose not, that is, suppose $\exists \hat{s}' \in S$, such that $w_+(\hat{s}') < w(\hat{s}')$, i.e., $h_+(\hat{s}') < h(\hat{s}')$. Proceeding as in part (i), since $h(\hat{s}') > 0$, $\beta R v_w(w(\hat{s}'), \hat{s}') = v_w(w, s) > v_w(w_+, s) \geq \beta R v_w(w_+(\hat{s}'), \hat{s}')$, implying that $w_+(\hat{s}') > w(\hat{s}')$, a contradiction. □

Proof of Proposition 2. Part (i) & (ii) Define the operator T as

$$Tv(w, s) \equiv \max_{c, h', w' \in \mathbb{R}_+ \times \mathbb{R}^{2S}} u(c) + \beta E[v(w', s')|s]$$

subject to equations (2) through (4). We show that if v has the property that $\forall s, s_+$, $s_+ > s$, $v_w(w, s_+) \leq v_w(w, s)$, then Tv inherits this property. Since the space of functions which have this property is closed, it follows that the fixed point has the property, too.

Suppose v has the property that $\forall s', s'_+$, $s'_+ > s'$, $v_w(w, s'_+) \leq v_w(w, s')$. For given w and s, suppose $\exists s'_+ > s'$ such that $h(s'_+) > h(s')$. Then $\beta R v_w(w(s'_+), s'_+) = v_w(w, s) \geq \beta R v_w(w(s'), s')$, implying, given the assumed property, that $w(s'_+) \leq w(s')$. But $w(s'_+) = y(s'_+) + h(s'_+) > y(s') + h(s') = w(s')$, a contradiction. Therefore, $h(s'_+) \leq h(s')$, $\forall s'_+ > s'$, that is, the household hedges lower income realizations (weakly) more. Hence, the household hedges a lower set of states, if at all.

Let $S = \{s_1, s_2, \ldots, s_S\}$. Denote the set of states that the household hedges by $S_h \equiv \{s' \in S : h(s') > 0\}$ and the highest state that the household hedges by $s_h = \max\{s' : s' \in S_h\}$. Take $s_+ > s$ and let S_{h+} and s_{h+} be associated with s_+ (and similarly for other variables). Suppose $\exists s_+ > s$, such that $Tv_w(w, s_+) > Tv_w(w, s)$ and, using the envelope condition, $c_+ < c$.

If $S_{h+} = \emptyset$, then equation (2) implies that $c_+ = w \geq c$, a contradiction. If $S_{h+} = \{s_1\}$, then

$$\beta R v_w(w_+(s'_1), s'_1) = Tv_w(w, s_+) > Tv_w(w, s) \geq \beta R v_w(w(s'_1), s'_1),$$

and therefore $w_+(s'_1) < w(s'_1)$ and $h_+(s'_1) < h(s'_1)$ since $y(s'_1) + h_+(s'_1) = w_+(s'_1) < w(s'_1) = y(s'_1) + h(s'_1)$. Using equation (2) and FOSD we have

$$w = c_+ + \Pi(s_+, s'_1) R^{-1} h_+(s'_1) < c + \Pi(s, s'_1) R^{-1} h(s'_1) \leq w,$$

a contradiction.

If $S_{h+} = \{s_1, s_2\}$, then proceeding as above we moreover have $w_+(s'_2) < w(s'_2)$ and $h_+(s'_2) < h(s'_2)$. Using equation (2), FOSD, specifically $\Pi(s, s'_1) \geq \Pi(s_+, s'_1)$ and $\Pi(s, s'_1) +$

$\Pi(s, s_2') \geq \Pi(s_+, s_1') + \Pi(s_+, s_2')$, and the fact that $h(s_1') \geq h(s_2')$, we have

$$
\begin{aligned}
w &= c_+ + \Pi(s_+, s_1')R^{-1}h_+(s_1') + \Pi(s_+, s_2')R^{-1}h_+(s_2') \\
&< c + \Pi(s_+, s_1')R^{-1}h(s_1') + \Pi(s_+, s_2')R^{-1}h(s_2') \\
&\leq c + [\Pi(s, s_1') + (\Pi(s_+, s_1') - \Pi(s, s_1'))]R^{-1}h(s_1') + [\Pi(s, s_1') + \Pi(s, s_2') - \Pi(s_+, s_1')]R^{-1}h(s_2') \\
&= c + \Pi(s, s_1')R^{-1}h(s_1') + \Pi(s, s_2')R^{-1}h(s_2') + (\Pi(s_+, s_1') - \Pi(s, s_1'))R^{-1}(h(s_1') - h(s_2')) \\
&\leq c + \Pi(s, s_1')R^{-1}h(s_1') + \Pi(s, s_2')R^{-1}h(s_2') \leq w,
\end{aligned}
$$

a contradiction.

If $S_{h+} = \{s_1, s_2, \ldots, s_{h+}\}$, then we have $w_+(s') < w(s')$ and $h_+(s') < h(s')$, $\forall s' \in S_{h+}$. Using equation (2), FOSD, and the fact that $h(s') \geq h(s_+')$, $\forall s_+' > s'$, we have

$$
\begin{aligned}
w &= c_+ + \Pi(s_+, s_1')R^{-1}h_+(s_1') + \cdots + \Pi(s_+, s_{h+}')R^{-1}h_+(s_{h+}') \\
&< c + \Pi(s_+, s_1')R^{-1}h(s_1') + \cdots + \Pi(s_+, s_{h+}')R^{-1}h(s_{h+}') \\
&= c + [\Pi(s, s_1') + (\Pi(s_+, s_1') - \Pi(s, s_1'))]R^{-1}h(s_1') + [\Pi(s, s_2') + (\Pi(s_+, s_2') - \Pi(s, s_2'))]R^{-1}h(s_2') \\
&\quad + \Pi(s_+, s_3')R^{-1}h(s_3') + \cdots + \Pi(s_+, s_{h+}')R^{-1}h(s_{h+}') \\
&\leq c + \Pi(s, s_1')R^{-1}h(s_1') + \Pi(s, s_2')R^{-1}h(s_2') \\
&\quad + [\Pi(s_+, s_1') + \Pi(s_+, s_2') - \Pi(s, s_1')) - \Pi(s, s_2')]R^{-1}h(s_2') \\
&\quad + \Pi(s_+, s_3')R^{-1}h(s_3') + \cdots + \Pi(s_+, s_{h+}')R^{-1}h(s_{h+}') \\
&\vdots \\
&\leq c + \Pi(s, s_1')R^{-1}h(s_1') + \cdots + \Pi(s, s_{h+-1}')R^{-1}h(s_{h+-1}') \\
&\quad + [\Pi(s_+, s_1') + \cdots + \Pi(s_+, s_{h+-1}') - \Pi(s, s_1') - \cdots - \Pi(s, s_{h+-1}')]R^{-1}h(s_{h+-1}') \\
&\quad + \Pi(s_+, s_{h+}')R^{-1}h(s_{h+}') \\
&\leq c + \Pi(s, s_1')R^{-1}h(s_1') + \cdots + \Pi(s, s_{h+}')R^{-1}h(s_{h+}') \\
&\quad + [\Pi(s_+, s_1') + \cdots + \Pi(s_+, s_{h+-1}') - \Pi(s, s_1')) - \cdots - \Pi(s, s_{h+-1}')]R^{-1}(h(s_{h+-1}') - h(s_{h+}')) \\
&\leq c + \Pi(s, s_1')R^{-1}h(s_1') + \cdots + \Pi(s, s_{h+}')R^{-1}h(s_{h+}') \leq w,
\end{aligned}
$$

a contradiction. Thus Tv inherits the property that $\forall s, s_+,\ s_+ > s,\ Tv_w(w, s_+) \leq Tv_w(w, s)$.

As a corollary of Proposition 1, w', h', and S_h are increasing in w given s, $\forall s' \in S$. To see that S_h is increasing in s given w, take $s_+ > s$ and suppose instead that $\exists \hat{s}'$ such that $h(\hat{s}') > 0$ but $h_+(\hat{s}') = 0$. Then $\beta R v_w(y(\hat{s}'), \hat{s}') \leq v_w(w, s_+) \leq v_w(w, s) = \beta R v_w(w(\hat{s}'), \hat{s}')$ which implies $w(\hat{s}') \leq y(\hat{s}')$, contradicting $w(\hat{s}') = y(\hat{s}') + h(\hat{s}') > y(\hat{s}')$. Thus, any state that the household hedges at s_+, the household hedges at $s < s_+$, that is, S_h is increasing in s. If the household hedges s' at s_+ but not at s, then clearly $w_+' > w'$ and $h_+' > h'$. If the household hedges s' at both s_+ and s, then $\beta R v_w(w_+', s') = v_w(w, s_+) \leq v_w(w, s) = \beta R v_w(w', s')$ and hence $w_+' > w'$ and $h_+' > h'$. Thus, w' and h' are increasing in s.

Part (iii): Take $w_+ > w$ and denote with a subscript $+$ the optimal policy associated with w_+. Let $\tilde{w}' \equiv w' - E[w']$, $\tilde{w}_h \equiv w_h - E[w']$, (and $\tilde{y}' = y' - E[w']$,) and analogously for $\tilde{w}_+'$, $\tilde{w}_{h+}$, and $\tilde{y}_+'$. We need to show that $var(\tilde{w}_+') \leq var(\tilde{w}')$. Note that $\tilde{w}' = \max\{\tilde{w}_h, \tilde{y}'\}$ and analogously for $\tilde{w}_+'$. If $\tilde{w}_{h+} = \tilde{w}_h$, then $\tilde{w}_+' = \tilde{w}'$ and the result is obvious. Assume instead that $\tilde{w}_{h+} > \tilde{w}_h$, w.l.o.g., and hence $E[w'_+] > E[w']$. Moreover, $\tilde{w}'_+ < \tilde{w}'$, $\forall s' \in S$ such that $\tilde{w}' > 0$ and $E[\tilde{w}'_+|\tilde{w}'_+ > 0] < E[\tilde{w}'|\tilde{w}' > 0]$. Let $\hat{\tilde{w}}'_+ \equiv \max\{\tilde{w}'_+, 0\}$, $\forall s' \in S$ such that $\tilde{w}' > 0$ and $\hat{\tilde{w}}'_+ \equiv \max\{\hat{\tilde{w}}_h, \tilde{y}'\}$ otherwise, where $\hat{\tilde{w}}_h$ such that $E[\hat{\tilde{w}}'_+] = 0$. Note that $\exists \hat{\tilde{w}}_h \in (\tilde{w}_h, \tilde{w}_{h+}]$ since $E[\tilde{w}'|\tilde{w}' > 0] > E[\tilde{w}'_+|\tilde{w}'_+ > 0] = E[\hat{\tilde{w}}'|\tilde{w}' > 0]$ and thus $E[\tilde{w}'_+|\tilde{w}'_+ \leq 0] = E[\hat{\tilde{w}}'_+|\tilde{w}' \leq 0] > E[\tilde{w}'|\tilde{w}' \leq 0]$. Since $|\hat{\tilde{w}}'_+| \leq |\tilde{w}'|$, $\forall s' \in S$, with strict inequality for some $s' \in S$, $var(\hat{\tilde{w}}'_+) < var(\tilde{w}')$. Moreover, $E[\hat{\tilde{w}}'_+ - \tilde{w}'_+] = 0$ and $\hat{\tilde{w}}'_+$ is a mean preserving spread of $\tilde{w}'_+$, that is, $var(\tilde{w}'_+) < var(\hat{\tilde{w}}'_+) < var(\tilde{w}')$. Moreover, consumption c is monotone and strictly increasing in net worth w. □

Proof of Proposition 3. Part (i): To be completed. **Part (ii):** At net worth $w = \bar{y}$, using (7) and the envelope condition, we have $v_w(\bar{y}, s) = \beta R v_w(w(\bar{s}'), \bar{s}') + \beta R \lambda(\bar{s}')$ which implies that $\lambda(\bar{s}') > 0$ since $w(\bar{s}') \geq \bar{y}$ and hence, by strict concavity of v and the fact that $v_w(w, s)$ is decreasing in s (see Part (i) of Proposition 2), $\beta R \lambda(\bar{s}') = v_w(\bar{y}, s) - \beta R v_w(w(\bar{s}'), \bar{s}') \geq (1 - \beta R) v_w(\bar{y}, s) > 0$. □

Proof of Proposition 4. Part (i): To be completed. **Part (ii):** Denoting the net worth at the upper bound of the stationary distribution by $\bar{w}$ and using (7) and the envelope condition, we have $v_w(\bar{w}) = \beta R v_w(\bar{w}) + \beta R \lambda(\bar{s}')$, implying that $\lambda(\bar{s}') > 0$ and hence $w(\bar{s}') = \bar{y}$. Suppose net worth w_t at time t is such that $w_t > \bar{y}$. Any path which reaches a state s_{t+n} against which the household is constrained in $t+n$ periods results in a household net worth $w(s_{t+n}) = y(s_{t+n}) \leq \bar{y}$ and indeed net worth is bounded above by $\bar{y}$ from then onwards. Consider a path along which the household is never constrained; since $v_w(w) = \beta R v_w(w')$ along such a path, $\exists n < \infty$, such that $v_w(w_{t+n}) = (\beta R)^{-n} v_w(w_t) > v_w(\bar{y})$ and hence again $w_{t+n} < \bar{y}$ at time $t + n$ and thereafter. Thus, net worth levels above $\bar{y}$ are transient. Since (3) holds with equality and using (4), $w' \geq y' \geq \underline{y}$; levels of net worth $w < \underline{y}$ are therefore transient.

Part (iii): Household risk management is monotone by Proposition 1 and incomplete with probability 1 since the stationary distribution of net worth is bounded above by $\bar{y}$ and risk management is monotone and incomplete at $w = \bar{y}$ by Part (ii) of Proposition 3. By Part (i) of that proposition, risk management is completely absent at $\underline{w} = \underline{y}$; by continuity, $\exists \varepsilon > 0$ such that for $w > \bar{w}$ with $|w - \bar{w}| < \varepsilon$, $v_w(w) > \beta R v_w(\underline{y})$, which means that the household does not hedge at all in this neighborhood. Clearly, $\bar{w}$ has positive probability under the stationary distribution since household income $\bar{y}$ has positive probability under the stationary distribution of income. If the household does not hedge $\underline{s}'$ at $\bar{w}$, then $\underline{w}$ has strictly positive probability. Consider instead a path along which the household continues to hedge the lowest income realization the following period, then $\exists n < \infty$ such that $v_w(w_{t+n}) = (\beta R)^{-n} v_w(\bar{w}) > v_w(\underline{y})$and hence $w_{t+n} < \underline{y}$, which is not possible. So the household must stop hedging the lowest state after a finite sequence of lowest income realizations, that is, the household does not hedge at all with positive probability under the stationary distribution. □

Proof of Proposition 5. From equation (7) and the envelope condition that $v_w(w, s) = v_w(w', s') + \lambda'$ and therefore $v_w(w, s)$ is non-increasing. Consider the marginal value of net worth at the upper bound of the stationary distribution for some state s, $v_w(\bar{w}(s), s)$; suppose there exists some state, say, w.l.o.g., next period, such that $v_w(\bar{w}(s), s) > v_w(w', s')$. But $v_w(w', s') \geq v_w(w'', s'')$, $\forall s'' \in S$, including $s'' = s$. But then, by concavity, $v_w(\bar{w}(s), s) \leq v_w(w'', s)$, a contradiction. Thus, $v_w(w, s) = v_w(w', s')$, $\forall (w, s), (w', s')$ in the support of the stationary distribution. □

Proof of Proposition 6. Part (i): Using the envelope condition and (5) we have $v_w(w, s) = u_c(c)$, and given the strict concavity of the value function, if $w_+ > w$, $v_w(w_+, s) < v_w(w, s)$ and hence $c_+ > c$, that is, c is increasing in w, given s.

To see that k is strictly increasing in w given s, take $w_+ > w$ and note that by strict concavity of v, $\mu_+ < \mu$. Suppose that $k_+ \leq k$, then $g_k(k) \leq g_k(k_+)$. Rewriting the Euler equation for durable goods purchases (12) we have

$$1 = \beta \frac{1}{\mu} \frac{g_k(k)}{\wp} + \sum_{s' \in S_h} \Pi(s, s') R^{-1} \frac{(1-\theta)(1-\delta)}{\wp} + \sum_{s' \in S \setminus S_h} \Pi(s, s') \beta \frac{\mu'}{\mu} \frac{(1-\theta)(1-\delta)}{\wp}.$$

Assume, without loss of generality, that $S_h = S_{h+}$. Since $g_k(k_+)/\mu_+ > g_k(k)/\mu$, it must be the case that $\exists \hat{s}' \in S \setminus S_h$ such that $\mu_+(\hat{s}')/\mu_+ < \mu(\hat{s}')/\mu$ and hence $\mu_+(\hat{s}') < \mu(\hat{s}')$, that is, $w_+(\hat{s}') > w(\hat{s}')$. But since $\hat{s}' \in S \setminus S_h$, $w_+(\hat{s}'_+) = y(\hat{s}') + (1-\theta)k_+(1-\delta) \leq y(\hat{s}') + (1-\theta)k(1-\delta) = w(\hat{s}')$, we have a contradiction.

To see that w' is strictly increasing in w given s, $\forall s' \in S$, assume again *w.l.o.g.* that $S_h = S_{h+}$. On S_h, $v_w(w, s) = \beta R v_w(w', s')$ and hence $w'_+ > w'$. On $S \setminus S_h$, $w'_+ = y' + (1-\theta)k_+(1-\delta) > y' + (1-\theta)k(1-\delta) = w'$.

Part (ii): To see that the household hedges a lower interval of states, suppose that $s'_+ > s'$, and $h(s'_+) > 0$ but $h(s') = 0$; then

$$w(s'_+) = y(s'_+) + (1-\theta)k(1-\delta) + h(s'_+) > y(s') + (1-\theta)k(1-\delta) = w(s')$$

but $\beta R v_w(s'_+) = v_w(w) > \beta R v_w(w(s'))$ which implies $w(s') > w(s'_+)$, a contradiction.

Since $\bar{w}$ is the highest wealth level that is attained under the stationary distribution, we have at $\bar{w}$ that $v_w(\bar{w}) = \beta R v_w(\bar{w}') + \beta R \lambda(\bar{s}')$, so $\lambda(\bar{s}') > 0$. Now suppose $\exists w < \bar{w}$ such that the household hedges all states at w implying that $v_w(w) = \beta R v_w(w')$, $\forall s' \in S$, that is, net worth next period must be lower than net worth this period in all states. But then there must exists a $w_- < w$ such that $w_-(\bar{s}') > w(\bar{s}')$ (since otherwise $\bar{w}$ could not be attained), which implies that $w(\bar{s}') < w_-(\bar{s}') = y(\bar{s}') + (1-\theta)k_-(1-\delta) + h_-(\bar{s}')$, so $h_-(\bar{s}') > 0$. This in turn implies that $v_w(w_-) = \beta R v_w(w_-(\bar{s}'))$, that is, $v_w(w_-) < v_w(w))$, a contradiction. □

Proof of Proposition 7. Part (i): The proof proceeds analogously to the Proof of Part (i) of Proposition 2. We show that if the property that $v_w(w, s)$ is decreasing in s is satisfied by v next period, then Tv satisfies the property this period, and conclude that the fixed point satisfies the property as well. Moreover, as before, we observe that if the property is satisfied next period, then the household hedges a lower set of states and h' is decreasing in s'.

Now suppose that $\exists s_+ > s$, such that $Tv_w(w, s_+) > Tv_w(w, s)$, implying by the envelope condition that $c_+ < c$. From the budget constraint(2) we have

$$c_+ + \wp k_+ + \sum_{s' \in S} \Pi(s_+, s')h'_+ = w = c + \wp k + \sum_{s' \in S} \Pi(s, s')h',$$

and given FOSD and the fact that h' is decreasing in s' we have $\sum_{s' \in S} \Pi(s_+, s')h' \leq \sum_{s' \in S} \Pi(s, s')h'$, which implies that $x = \{c, k, h'\}$ is feasible at s_+ and hence $Tv(w, s_+) \geq Tv(w, s)$.

Suppose $k_+ \leq k$. There must exist an $\hat{s}'$ such that $w_+(\hat{s}') > w(\hat{s}')$, since otherwise $Tv(w, s_+) < Tv(w, s)$ as consumption of goods and durables and the net worth next period are all lower at s_+ than s. But then $h(\hat{s} > 0$ and therefore $\beta R \mu_+(\hat{s}') = \mu_+ > \mu = \beta R \mu(\hat{s}')$ implying $w_+(\hat{s}') < w(\hat{s}')$, a contradiction.

Now suppose $k_+ > k$. For $s' \in S_h \cap S_{h+}$, $\beta\mu'/\mu = R^{-1} = \beta\mu'_+/\mu_+$. For $s' \in S \setminus S_h \cap S \setminus S_{h+}$, $\beta\mu'/\mu > \beta\mu'_+/\mu_+$. For $s' \in S_h \cap S \setminus S_{h+}$, $\beta\mu'/\mu = R^{-1} \geq \beta\mu'_+/\mu_+$. Finally $S_h \cap S \setminus S_{h+} = \emptyset$, since for such s' we would have $\beta R \mu'_+ = \mu_+ > \mu \geq \beta R \mu'$, implying $w'_+ < w'$, whereas $w'_+ = y' + (1-\theta)k_+(1-\delta) + h'_+ > y' + (1-\theta)k(1-\delta) = w'$, a contradiction. Recalling that $R^{-1} \geq \beta\mu'/\mu$ and that the right hand side is decreasing in s', the Euler equation for durables (12) implies

$$\begin{aligned}
1 &= \beta \frac{1}{\mu_+} \frac{g_k(k_+)}{\wp} + \left[\sum_{s' \in S_{h+}} \Pi(s_+, s')R^{-1} + \sum_{s' \in s \setminus S_{h+}} \Pi(s_+, s')\beta \frac{\mu'_+}{\mu_+} \right] \frac{(1-\theta)(1-\delta)}{\wp} \\
&< \beta \frac{1}{\mu} \frac{g_k(k)}{\wp} + \left[\sum_{s' \in S_h} \Pi(s_+, s')R^{-1} + \sum_{s' \in s \setminus S_h} \Pi(s_+, s')\beta \frac{\mu'}{\mu} \right] \frac{(1-\theta)(1-\delta)}{\wp} \\
&\leq \beta \frac{1}{\mu} \frac{g_k(k)}{\wp} + \left[\sum_{s' \in S_h} \Pi(s, s')R^{-1} + \sum_{s' \in s \setminus S_h} \Pi(s, s')\beta \frac{\mu'}{\mu} \right] \frac{(1-\theta)(1-\delta)}{\wp} = 1,
\end{aligned}$$

a contradiction.

Part (ii): Arguing analogously to Part (i) of Proposition 2, since the property in Part (i) above is satisfied, the household hedges a lower set of states and w' and h' is decreasing in s' since for two states $s'_+ > s'$ which are hedged we have $v_w(w'_+, s'_+) = v_w(w', s') \geq v_w(w', s'_+)$, that is, $w' > w'_+$. If $\Pi(s, s') = \pi(s')$, $\forall s, s'$, then for any two states $s'_+ > s'$ that are hedged we have $v_w(w'_+) = v_w(w')$, that is, $w' = w'_+ \equiv w_h$. □

Proof of Proposition 8. To be completed. □

Proof of Proposition 9. To be completed. □

Proof of Proposition 10. Part (i): When $\theta = 1$, the investment Euler equation for durable goods (18) simplifies to

$$\wp(s)\mu = \beta g_k(k), \tag{19}$$

which in the case of logarithmic utility further simplifies to $k = (\beta g/\wp(s))c$. Define the total expenditure on consumption and durable goods as $\hat{c} = c + \wp(s)k = (1 + \beta g)c$. Substituting for c and k in the return function we have

$$\hat{u}(\hat{c}, s) = u(c) + \beta g(k) = (1 + \beta g)u(\hat{c}) + \varphi(s),$$

where $\varphi(s) = -(1+\beta g)\log(1+\beta g)+\beta g\log(\beta g)-\beta g\log(\wp(s))$. The problem with durable goods can then be written as an income risk management problem with preference shocks

$$v(w, s) = \max_{\hat{c},h',w'\in\mathbb{R}_+\times\mathbb{R}^{2S}} \hat{u}(\hat{c}, s) + \beta E[v(w', s')|s] \tag{20}$$

subject to

$$w \geq \hat{c} + E[R^{-1}h'|s], \tag{21}$$

(3), and (4).

Let $\hat{v}(w, s)$ solve the following income risk management problem without preference shocks

$$\hat{v}(w, s) = \max_{\hat{c},h',w'\in\mathbb{R}_+\times\mathbb{R}^{2S}} u(\hat{c}) + \beta E[\hat{v}(w', s')|s]$$

subject to (21), (3), and (4). This is in fact the problem considered in Section 4. Noting that the preference shock component of utility $\varphi(s)$ is separable and defining $v_\varphi(s)$ recursively as

$$v_\varphi(s) \equiv \varphi(s) + \beta E[v_\varphi(s')|s],$$

we have $v(w, s) = (1 + \beta g)\hat{v}(w, s) + v_\varphi(s)$ as can be verified by substituting into equation (20).

Part (ii): With isoelastic preferences, (19) simplifies to $k = (\beta g/\wp(s))^{1/\gamma}c$. Define the total expenditure on consumption and durable goods as $\hat{c} = c + \wp(s)k = (1 + \wp(s)(\beta g/\wp(s))^{1/\gamma})c$. Substituting for c and k in the return function we have

$$\hat{u}(\hat{c}, s) = u(c) + \beta g(k) = \phi(s)u(\hat{c}),$$

where $\phi(s) = (1+(\beta g)^{1/\gamma}\wp(s)^{(\gamma-1)/\gamma})^\gamma$. The proof of Proposition 1 applies without change.

Suppose $\Pi(s, s')$ satisfies FOSD and $\wp(s)$ is increasing in s. To prove that $v_w(w, s)$ is decreasing in s when $\gamma < 1$, first observe that $\phi(s)$ is decreasing in s in that case (whereas it is increasing in s if $\gamma > 1$). We can now proceed as in the proof of the first part of Part (ii) of Proposition 2, that is, we assume that the property is satisfied by $v(\cdot)$ next period and then show that it has to be satisfied by $Tv(\cdot)$ in the current period as well. As before, note that if the property is satisfied next period, the household hedges a lower set of states and h' decreases in s'. Suppose the opposite, that is, suppose $\exists s_+ > s$, such that $Tv_w(w, s_+) > Tv_w(w, s)$, implying by the envelope condition that $\phi(s_+)u(\hat{c}_+) = \mu_+ > \mu = \phi(s)u(\hat{c})$ and therefore $u(\hat{c}_+) > \phi(s)/\phi(s_+)u(\hat{c}) \geq u(\hat{c})$, which further implies that $\hat{c}_+ < \hat{c}$. Since h' is decreasing in s', $E[R^{-1}h'|s] \leq E[R^{-1}h'|s_+]$ and $\{\hat{c}, h', w'\}$ is feasible at s_+. Since $\hat{c}_+ < \hat{c}$, $\exists \hat{s}'$ such that $w_+(\hat{s}') > w(\hat{s}')$ since otherwise $\{\hat{c}_+, h'_+, w'_+\}$ would achieve lower utility than switching to $\{\hat{c}, h', w'\}$, contradicting optimality. But then $y(\hat{s}') + h_+(\hat{s}') = w_+(\hat{s}') > w(\hat{s}') = y(\hat{s}') + h(\hat{s}')$ and $h_+(\hat{s}') > h(\hat{s}') \geq 0$, so $\beta R\mu_+(w_+(\hat{s}'), \hat{s}') = \mu_+ > \mu \geq \beta R(w(\hat{s}'), \hat{s}')$, implying $w_+(\hat{s}') < w(\hat{s}')$, a contradiction. Therefore $v_w(w, s)$ is decreasing in s, and the rest of the proposition obtains from the proof of Part (ii) of Proposition 2 without change. $\square$

References

Aiyagari, S. Rao, "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics*, 109 (1994), 659-684.

Athanasoulis, Stefano G., and Robert J. Shiller, "The Significance of the Market Portfolio," *Review of Financial Studies*, 13 (2000), 301-329.

Benveniste, Lawrence M., and José A. Scheinkman, "On the Differentiability of the Value Function in Dynamic Models of Economics," *Econometrica*, 47 (1979), 727-732.

Bewley, Truman, "The Permanent Income Hypothesis: A Theoretical Formulation," *Journal of Economic Theory*, 16 (1977), 252-292.

Blundell, Richard, Luigi Pistaferri, and Ian Preston, "Consumption Inequality and Partial Insurance," *American Economic Review* 98 (2008), 1887-1921.

Brown, Jeffrey R., and Amy Finkelstein, "Why is the Market for Long-term Care Insurance so Small?" *Journal of Public Economics*, 91 (2007), 1967-1991.

Brown, Jeffrey R., and Amy Finkelstein, "The Interaction of Public and Private Insurance: Medicaid and the Long-term Care Insurance Market," *American Economic Review*, 98 (2008), 1083-1102.

Browne, Mark J., and Robert E. Hoyt, "The Demand for Flood Insurance: Empirical Evidence," *Journal of Risk and Uncertainty*, 20 (2000), 291-306.

Campbell, John Y., "Understanding Risk and Return," *Journal of Political Economy*, 104 (1996), 298-345.

Campbell, John Y., and João F. Cocco, "Household Risk Management and Optimal Mortgage Choice," *Quarterly Journal of Economics*, 118 (2003), 1449-1494.

Cole, Shawn, Xavier Giné, Jeremy Tobacman, Petia Topalova, Robert Townsend, and James Vickery, "Barriers to Household Risk Management: Evidence from India," *American Economic Journal: Applied Economics*, forthcoming.

Constantinides, George M., and Darrell Duffie, "Asset Pricing with Heterogeneous Consumers," *Journal of Political Economy*, 104 (1996), 219-240.

Eisfeldt, Andrea L., and Adriano A. Rampini, "Leasing, Ability to Repossess, and Debt Capacity," *Review of Financial Studies*, 22 (2009), 1621-1657.

Froot, Kenneth A., David S. Scharfstein, and Jeremy C. Stein, "Risk Management: Coordinating Corporate Investment and Financing Policies," *Journal of Finance*, 48 (1993), 1629-1658.

Fuster, Andreas, and Paul S. Willen, "Insuring Consumption Using Income-Linked Assets," *Review of Finance*, 15 (2011), 835-873.

Géczy, Christopher, Bernadette A. Minton, and Catherine Schrand, "Why Firms Use Currency Derivatives," *Journal of Finance*, 52 (1997), 1323-1354.

Giné, Xavier, Robert Townsend, and James Vickery, "Patterns of Rainfall Insurance Participation in Rural India," *World Bank Economic Review*, 22 (2008), 539-566.

Guvenen, Fatih, Serdar Ozkan, and Jae Song, "The Nature of Countercyclical Income Risk," University of Minnesota Working Paper, 2012.

Lustig, Hanno N., and Stijn G. Van Nieuwerburgh, 2005, "Housing Collateral, Consumption Insurance, and Risk Premia: An Empirical Perspective," *Journal of Finance*, 60 (2005),1167-1219.

Mankiw, N. Gregory, "The Equity Premium and the Concentration of Aggregate Shocks," *Journal of Financial Economics*, 17 (1986), 211-219.

Modigliani, Franco, and Merton H. Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment," *American Economic Review* 48 (1958), 261-297.

Nance, Deana R., Clifford W. Smith, Jr., and Charles W. Smithson, "On the Determinants of Corporate Hedging," *Journal of Finance*, 48 (1993), 267-284.

Piazzesi, Monika, Martin Schneider, and Selale Tuzel, "Housing, Consumption and Asset Pricing," *Journal of Financial Economics*, 83 (2007), 531-569.

Rampini, Adriano A., "Entrepreneurial Activity, Risk, and the Business Cycle, *Journal of Monetary Economics*, 51 (2004), 555-573.

Rampini, Adriano A., Amir Sufi, and S. Viswanathan, "Dynamic Risk Management," Duke University Working Paper, 2012.

Rampini, Adriano A., and S. Viswanathan, "Collateral, Risk Management, and the Distribution of Debt Capacity," *Journal of Finance*, 65 (2010), 2293-2322.

Rampini, Adriano A., and S. Viswanathan, "Collateral and Capital Structure," *Journal of Financial Economics* (2013), forthcoming.

Schechtman, Jack, "An Income Fluctuation Problem," *Journal of Economic Theory*, 12 (1976), 218-241.

Shiller, Robert J., *Macro Markets: Creating Institutions for Managing Society's Largest Economic Risks* (New York, NY: Oxford University Press, 1993).

Shiller, Robert J., "Derivatives Markets for Home Prices," NBER Working Paper 13962, 2008.

Sinai, Todd, and Nicholas S. Souleles, "Owner-occupied Housing as a Hedge Against Rent Risk," *Quarterly Journal of Economics*, 120 (2005), 763-789.

Storesletten, Kjetil, Chris I. Telmer, and Amir Yaron, "Cyclical Dynamics in Idiosyncratic Labor-Market Risk," *Journal of Political Economy*, 112 (2004), 695-717.

Townsend, Robert M., "Risk and Insurance in Village India," *Econometrica*, 65 (1994), 539-592.

Tufano, Peter, "Who Manages Risk? An Empirical Examination of Risk Management Practices in the Gold Mining Industry," *Journal of Finance*, 51 (1996), 1097-1137.

Yaari, Menahem E., "A Law of Large Numbers in the Theory of Consumer's Choice under Uncertainty," *Journal of Economic Theory*, 12 (1976), 202-217.

Table 1: Evidence on Households' Insurance Coverage Across Wealth and Age

This table reports the data on insurance coverage across households with different wealth and age from various sources. Panels A and B present data on people without health insurance coverage by income and age, respectively, from Table 6 of the U.S. Census Bureau's Report on *Income, Poverty, and Health Insurance Coverage in the United States: 2007.* Panel C presents data on private long-term care insurance ownership rates among individuals aged 60 and over from the *2000 Health and Retirement Survey* as reported by Brown and Finkelstein (2007), Table 1.

Panel A: People without Health Insurance Coverage by Income (in Thousands)					
	Total	$\leq$ \$25	\$25-\$49	\$50-\$74	$\geq$ \$75
Percentage uninsured	15.3	24.5	21.1	14.5	7.8

Panel B: People without Health Insurance Coverage by Age (in Years)							
	Total	≤ 18	18-24	25-34	35-44	45-64	≥ 65
Percentage uninsured	15.3	11.0	28.1	25.7	18.3	14.0	1.9

Panel C: Private Long-term Care Insurance Coverage Rates by Wealth Quartile					
	Total	Bottom	Third	Second	Top
Coverage rate (%)	10.5	2.8	6.0	11.3	19.6

Figure 1: Monotone Household Risk Management

This figure displays household income risk management when household income follows an independent two state Markov process. The solid (dashed) lines plot the policies for the low (high) state next period. Top left: value function $v(w)$; top right: hedging h'; middle left: net worth next period w' and 45 degree line (dotted); middle right: consumption as percent of current net worth c/w; bottom left: marginal value of net worth next period $v_w(w')$ and marginal value of current net worth $v_w(w)$ (dash-dotted); and bottom right: (scaled) multiplier on the short sale constraint $\beta\lambda'/\mu$. The parameter values are: $\beta = 0.80$, $R = 1.05$, $\Pi(\underline{s}, \underline{s}) = \Pi(\bar{s}, \bar{s}) = 0.50$, $y(\underline{s}) = 0.50$, $y(\bar{s}) = 1.50$, and preferences $u(c) = c^{1-\gamma}/(1-\gamma)$ with $\gamma = 2$.

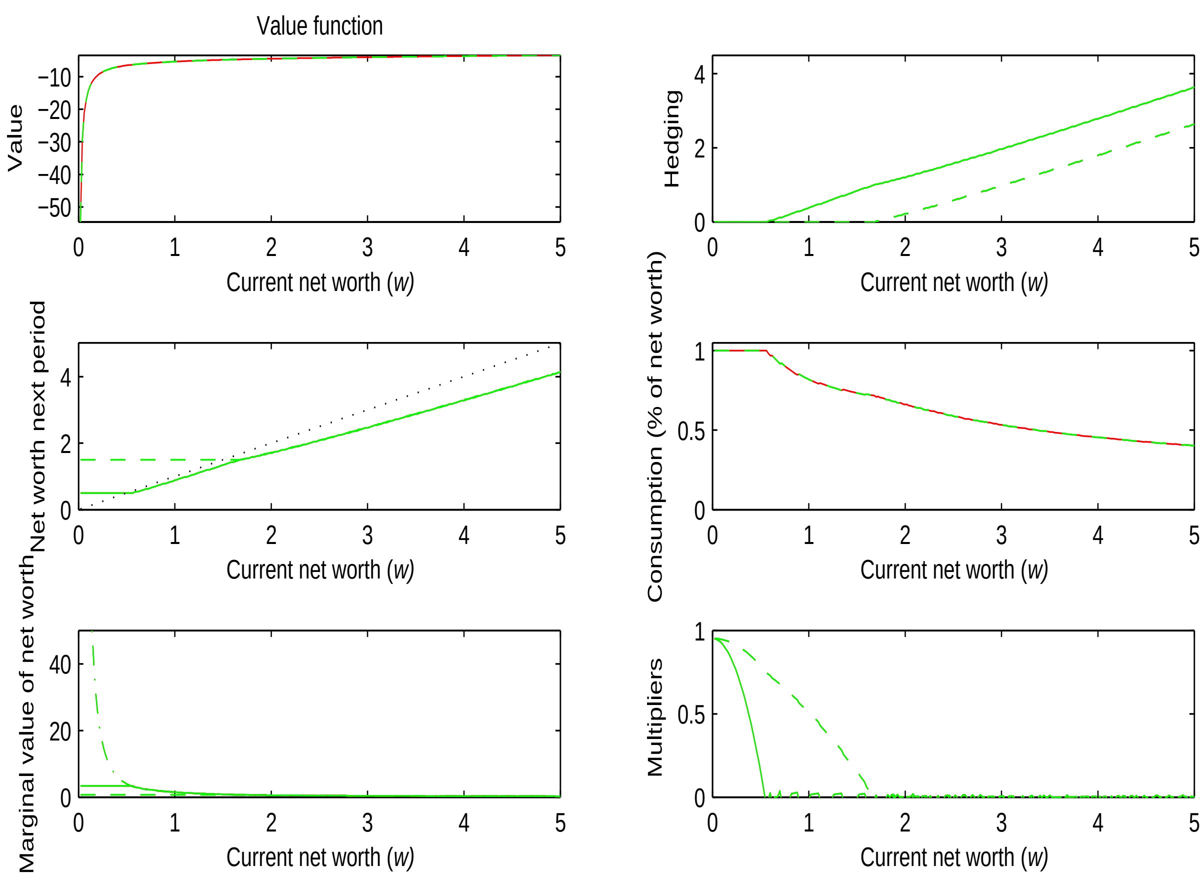

Figure 2: Stationary Distribution of Household Net Worth

This figure displays the stationary distribution of net worth from Proposition 4 for a household income risk management economy when household income follows an independent two state Markov process as in Figure 1 (see the caption of that figure for parameter values). Top: unconditional distribution of net worth; middle: distribution conditional on the low state; bottom: distribution conditional on the high state.