

INTERMEDIATION AND RESALE IN NETWORKS

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ABSTRACT. We study intermediation in markets with a network structure. A good is sequentially resold via bilateral bargaining between linked intermediaries until it reaches one of several buyers. A decomposition of the network into layers captures intermediation power. Competitive forces allow intermediaries to demand the full surplus when they resell the good within a layer, while trades between layers involve hold-ups that entitle downstream parties to intermediation rents. A trader’s intermediation power does not depend directly on the number of intermediaries needed to access buyers, but on the competition among intermediation chains measured by the number of layers separating the trader from buyers. Only players serving as gateways to lower layers earn profits. Trade does not maximize welfare or minimize intermediation. The interplay between competition and hold-ups determines the extent of intermediation inefficiencies. The elimination of a middleman and the transfer of intermediation costs downstream increase seller profits. Horizontal and vertical integration have ambiguous effects on efficiency and seller, buyer, and consolidated intermediary profits.

1. INTRODUCTION

Intermediation plays an important role in many markets. Artwork and antiquities are sold to museums and individual collectors via long chains of dealers. Campbell (2013) reports that illicit trade in antiquities amounts to \$2.2 billion annually and documents cases in which antiquities were traded by a sequence of intermediaries “on an opportunity-to-opportunity basis,” with “money [being] exchanged at each interaction rather than collaborating together as a chain and partitioning profits after the final sale.” He identifies four intermediation roles that differ in specialization, costs, and risks: looters familiar with archaeological sites, middlemen with means to transport illicit artifacts across borders, art history experts who assess authenticity of artifacts and confer them legitimacy, and collectors. For example, the fourth century BC golden phial of Achyris [5, 6, 30] was looted from Sicily and acquired by Vincenzo Pappalardo, who exchanged it for goods worth \$20,000 with fellow Sicilian dealer Vincenzo Cammarata in 1980. Around 1988, Cammarata traded the phial with Zurich-based intermediary William Veres for artworks valued at \$90,000. In 1991, New York dealer Robert Haber purchased the phial from Veres for about \$1 million and resold it to collector

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Michael Steinhardt for \$1.2 million. Veres and Haber laundered the artifact in Switzerland by falsifying provenance documents. Veres made the largest profit in this intermediation chain due to the risks associated with transporting the phial across the Italian-Swiss border.¹

Illegal drugs are also smuggled through networks of intermediaries and account for a significant fraction of international trade.² For example, Colombia is the main supplier of cocaine. Farmers cultivate coca and sell its leaves to local dealers, who use chemicals to convert them into a paste. Coatsworth et al. (2015) report that in 1999 a dealer would pay a farmer \$200 for 200 kg of coca leaves, the amount necessary to produce 1 kg of paste. The dealer sells the paste for \$350 to an intermediary, who purifies it in a refinery. The intermediary sells 1 kg of cocaine for \$500 to a Colombian “exporter,” who then resells it to a courier for \$2,500. The courier delivers the cocaine to a wholesaler in New York (with the help of smugglers in Mexico or the Caribbean), who resells it to mid-level dealers for \$30,000. Low competition enables wholesalers to extract high profits from mid-level dealers.³

Intermediaries often facilitate bribery in public procurement, as detailed by the Organization for Economic Cooperation and Development (OECD) [22, 23]. Companies bidding for procurement projects engage intermediaries disguised as consultants, contractors, or suppliers to transfer money to corrupt officials. To conceal the graft, fake documentation misrepresents intermediaries’ services as legitimate business operations. Companies sometimes employ a “cascade of intermediaries” to “distance [themselves] from the crime and increase [their] chances of evading justice.” German engineering company Siemens designed many creative “one-shot” corruption schemes. The Securities and Exchange Commission found that Siemens used a “system of business consultants and intermediaries” to bribe officials around the world between 2001 and 2007 [27]. Siemens profited \$1.1 billion from corrupt activities involving \$1.4 billion in direct bribes to officials and \$391 million in bribes channeled through “payment intermediaries.” The OECD describes a high-voltage transmission line project in China for which Siemens funneled money through a fake consultant in Dubai, who transferred a fraction to a consultant in China, who then bribed Chinese officials.

Intermediation underlies legal economic activity as well. Financial institutions resell assets over the counter through networks of intermediaries [11, 16]. International trade is intermediated by exporters, importers, and distributors. In supply chains, firms sequentially transform and resell intermediate goods. Furthermore, an agent who navigates a network by acquiring access, goods, or services from nodes along the way faces strategic problems equivalent to intermediation. This class of applications encompasses truck drivers bargaining over bribes with authorities at checkpoints [24], infrastructure developers negotiating with landowners [34], and manufacturers employing contractors and suppliers.

¹His 1000% markup could further be explained by his expertise in certifying artifacts in Switzerland.

²The worldwide revenue of the illegal drug industry in 1995 was estimated at \$400 billion [32].

³In the 1990s, there were only 200 cocaine wholesalers in the US [33].

Intermediation networks involve competing paths between sellers and buyers with complex patterns of overlap and trader asymmetries. The number of middlemen, the cost of intermediation, and the value of final consumers vary across trading paths. Some market participants have access to more middlemen than others, who themselves enjoy a greater or smaller number of connections. Clearly, not all links are equally useful in generating intermediation rents. The bargaining power of each intermediary depends on both his distance to buyers and the nature of competition among his available trading routes. The network of connections among traders determines the path of trade and the profits that buyers, sellers, and intermediaries earn.

Given the prevalence of networks in markets where trade requires the participation of middlemen, it is important to develop models of intermediation in networks. Decentralized bargaining is at the heart of our opening examples. This paper puts forward a non-cooperative theory of sequential resale via bilateral bargaining. We study the following dynamic intermediation game. A seller is endowed with a single unit of an indivisible good, which is successively traded by pairs of linked intermediaries in a directed acyclic network until it reaches one of several buyers. Intermediaries have heterogeneous intermediation costs, and buyers have heterogeneous values. At every stage in the game, the current owner of the good selects a bargaining partner among his downstream neighbors in the network. The two traders negotiate according to a random proposer protocol: with probability p , the owner proposes a price and the partner decides whether to acquire the good at that price; roles are reversed with probability $1 - p$. In either event, if the offer is rejected, the owner keeps the good and gets a new chance to select a bargaining partner at the next stage. If the offer is accepted, then the owner incurs his intermediation cost, and the two traders exchange the good at the price agreed upon. If the new owner is an intermediary, he has the opportunity to resell the good to his downstream neighbors following the same protocol. Buyers consume the good upon purchase. Players have a common discount factor δ .

We address the following questions. How does an intermediary's position in the network affect his bargaining power and intermediation rents? Which players earn substantial profits? What trading paths are likely to emerge? How can upstream players exploit downstream competition? Is intermediation efficient? Does trade proceed along the shortest path? How do seller profits and total welfare respond to changes in network architecture such as the elimination of a middleman and the vertical or horizontal integration of intermediaries? What are the seller's preferences over the distribution of intermediation costs in the network?

Our analysis focuses on the limit of Markov perfect equilibria (MPEs) of the intermediation game as $\delta \rightarrow 1$. A player's resale value in an MPE is defined by his expected payoff in subgames where he owns the good. We prove that all MPEs generate identical limit resale

values as δ approaches 1. In the introduction, we use the shorthand “resale value” for these limit values; we follow the same convention for other (limit) equilibrium variables.

The main economic insight of this research is a decomposition of the network into layers of intermediation power that delineates the structure of equilibrium trading paths and determines resale values at every position in the network. Layer boundaries separate monopoly power from intermediation power. Competitive forces allow intermediaries to demand the full surplus when they resell the good within the same layer, while trades between layers involve hold-ups in which downstream parties extract intermediation rents. Hence, only intermediaries who serve as gateways to lower layers earn profits. Traders in the same layer have identical resale values, which decline exponentially as higher layers are reached. Thus, layers provide the appropriate metric for intermediation distance in the network: a trader’s intermediation power depends on the number of layers the good traverses between the trader and buyers. Since this metric is not directly related to the length of intermediation chains, trade does not always proceed along the shortest path from the seller to buyers. This finding refutes the standard intuition that sellers have incentives to minimize intermediation.

Another substantive contribution of the study is a systematic characterization of the inter-related structure of hold-ups and their effect on intermediation inefficiencies. The fact that hold-ups cause inefficiencies—even in markets with a single intermediary—is well known. Our analysis reveals how hold-ups arise endogenously in relation to competition in different parts of the network and how they shape trading paths and the division of gains from trade. This structure brings to light new types of inefficiency stemming from sellers’ incentives to pursue intermediation chains that exploit competition and avoid hold-ups. Such chains may lead to low value buyers or entail large intermediation costs.

The interplay between hold-ups, competition, and efficiency from our framework is novel to the literature. Gale and Kariv (2007) find that intermediation is efficient in a market where multiple units of a homogeneous good are resold between linked traders with heterogeneous values; their result is driven by the assumption that sellers have all the bargaining power. Blume et al. (2009) study a model where intermediaries simultaneously announce bid prices for sellers and ask prices for buyers and show that competitive forces lead to efficient trade.⁴ Similarly, in the general equilibrium model of trading in networks developed by Hatfield et al. (2013), the simultaneity of buying and selling decisions (along with the assumption of price-taking behavior) induces efficient competition. Choi et al. (2015) also prove the existence of efficient equilibria in a different network setting where intermediaries post prices and trade takes place along the least expensive path. In another recent paper, Siedlarek (2015) proposes a model in which players along a randomly selected intermediation chain

⁴Gale and Kariv (2009) confirm this conclusion in a laboratory experiment.

negotiate the division of gains from trade; the multilateral bargaining protocol assumes away hold-up problems and generates asymptotically efficient outcomes.

The distinguishing feature of our model is that it endogenizes competition among trading paths via strategic choices at every stage in the resale process. In a related paper, Gofman (2011) uses bilateral bargaining and second-price auctions to show that intermediation inefficiencies emerge when sellers cannot extract the full surplus in each trade. He also finds that welfare is not monotonic with respect to network density. Farboodi (2015) adopts the sharing rule embedded in Gofman’s bargaining solution to study endogenous intermediation in the financial sector.

Intermediation in networks constitutes an active area of research. More references can be found in the survey of Condorelli and Galeotti (2016). Rubinstein and Wolinsky (1987) pioneered the game theoretical analysis of intermediation.

Outline of Main Results. To gain intuition for the competitive forces induced by our bilateral bargaining protocol, we start with a simple version of the model in which there are no intermediaries. In this case, the seller bargains with a number of heterogeneous buyers. The MPE of the bargaining game without intermediaries is essentially unique and achieves asymptotic efficiency as $\delta \rightarrow 1$. The limit MPE outcome is determined as if the seller can choose between two scenarios: (1) a bilateral monopoly agreement whereby the seller bargains only with the highest value buyer and receives a share p of the proceeds; and (2) a second-price auction in which the seller exploits competition among buyers and demands the second highest value. In effect, the seller is able to take advantage of competition and ask for more than his bilateral monopoly share from the best buyer only if the threat of trading with the second best buyer is credible.

Now consider an MPE of the intermediation game. The strategic problem faced by the owner in any subgame reduces to a bargaining game in which his downstream neighbors act as surrogate “buyers” with valuations for the good induced by their resale values in the MPE. This situation resembles the game without intermediation, with one critical distinction: “buyers” may resell the good to one another, directly or through longer paths in the network. When a “buyer” acquires the good, other buyers may still enjoy positive continuation payoffs upon purchasing it in the subgame with the new owner. This possibility creates endogenous outside options for buyers. An important step in the analysis proves that buyers’ outside options are not “binding” in equilibrium: neighbors prefer buying the good directly from the current owner rather than acquiring it via an intermediation chain. This fact sets the foundation for a recursive formula establishing that the resale value of each trader is derived *as though* his downstream neighbors were final consumers in the bargaining game without intermediation with valuations given by their resale values. Building on the formula, we characterize the path of trade and the distribution of intermediation profits in equilibrium.

To examine the effect of an intermediary's position in the network on his resale value in more detail, we isolate network effects from other trader asymmetries by assuming that there are no intermediation costs and buyers have a common value v . We develop an algorithm that decomposes the network into a series of layers that capture intermediation power in such settings. The algorithm builds on the following observations. Any intermediary linked to two (or more) buyers uses his monopoly power to extract the full surplus of v when he resells the good. Moreover, any current owner linked to a pair of downstream players known to have resale value v also obtains a price of v due to competitive forces. We continue expanding the group of players with resale value v by adding traders who have two downstream links to existing group members; the final group is called layer 0. The recursive characterization of resale values implies that any remaining intermediary with a downstream neighbor in layer 0 has a resale value of pv . Competition between any pair of such intermediaries enables their upstream neighbors to charge a price of pv for the good. We iterate this argument to identify all players with resale value pv and refer to this set as layer 1. The composition of layer $\ell \geq 2$ derives analogously from layer $\ell - 1$. We prove that all players from layer ℓ have resale value $p^\ell v$. For positive intermediation costs and heterogeneous buyer values, resale values of layer ℓ players are bounded by an interval that collapses to a single point as intermediation costs and differences in buyer values become small.

The characterization of resale values by means of the network decomposition reveals that a trader's intermediation power does not depend explicitly on the number of intermediaries he needs to reach buyers, but on the competition among his intermediation chains as measured by the number of layer boundaries separating him from buyers. Moreover, only the first member of each layer trading the good in equilibrium earns profits. Thus, layers delimit monopoly power from intermediation power: full profit extraction is possible in agreements within layers, while intermediation rents are paid in transactions across layers.

Although our bargaining protocol generates an asymptotically efficient MPE allocation in the absence of intermediation, we show that the possibility of resale creates inefficiencies. One type of intermediation inefficiency stems from standard hold-up problems created by the bilateral nature of intermediation coupled with insufficient downstream competition. Severe hold-up problems can make trade infeasible even when some intermediation chains generate substantial surplus. Other types of inefficiency result from intermediaries' incentives to exploit downstream competition, which are not aligned with maximizing total gains from trade. We identify instances of allocation inefficiency, in which the good is not allocated to the highest value buyer even in the absence of intermediation costs, as well as cost inefficiency, where the good does not reach the buyer through the lowest cost path.

Finally, we provide comparative statics with respect to network architecture and cost distribution. Modifying the network by either adding a link or eliminating a middleman

cannot reduce seller profit; however, these network changes may create inefficiencies. We demonstrate that horizontal and vertical integration have ambiguous effects on welfare and the division of gains from trade among the seller, buyers, and intermediaries. Our last result establishes that any downward redistribution of costs in the network benefits the seller.

The rest of the paper is organized as follows. Section 2 introduces the intermediation game, and Section 3 analyzes the bargaining game without intermediation. In Section 4, we provide the recursive characterization of resale values, which is exploited in Section 5 to obtain the network decomposition into layers of intermediation power. Section 6 investigates the division of intermediation profits, and Section 7 examines the sources of intermediation inefficiencies. In Section 8, we present the comparative statics analysis. Section 9 discusses extensions, interpretations, and applications of the model and provides concluding remarks.

2. THE INTERMEDIATION GAME

A set of *players* $N = \{0, 1, \dots, n\}$ interacts in the market for a single unit of an indivisible *good*. Player 0, the *initial seller*, owns the good. Players $i = \overline{1, m}$ are *intermediaries* ($m < n$). We refer to each player $i = \overline{0, m}$ as a (potential) *seller*. Every seller $i = \overline{0, m}$ has an intermediation *cost* $c_i \geq 0$.⁵ Each player $j = \overline{m+1, n}$ is a *buyer* who has consumption *value* $v_j \geq 0$ for the good.

Players are linked by a *network* $G = (N, (N_i)_{i=\overline{0, m}}, (c_i)_{i=\overline{0, m}}, (v_j)_{j=\overline{m+1, n}})$. Formally, G is a *directed acyclic graph* with vertex set N . Seller $i = \overline{0, m}$ has *out-links* to “downstream” players in the set $N_i \subseteq \{i+1, \dots, n\}$; conversely, player $k \in N_i$ has an *in-link* from “upstream” seller i . Hence, a *link* is an ordered pair (i, k) with $k \in N_i$. For every link (i, k) , we say that k is a *downstream neighbor* of i , and i is an *upstream neighbor* of k . In the intermediation game to be introduced shortly, N_i represents the set of players to whom the current owner i can directly (re)sell the good. A *trading path* is a directed path connecting the initial seller to a buyer, i.e., a sequence of players $(i_0, i_1, \dots, i_{\bar{s}})$ with $i_0 = 0, m+1 \leq i_{\bar{s}} \leq n$, and $i_s \in N_{i_{s-1}}$ for all $s = \overline{1, \bar{s}}$. The *surplus* generated by the path (i_s) is given by $v_{i_{\bar{s}}} - \sum_{s=0}^{\bar{s}-1} c_{i_s}$. Without loss of generality, we assume that every player lies on a trading path and that buyers do not have out-links. For fixed $m < n$, a *linking structure* is any profile $(N_i)_{i=\overline{0, m}}$ that satisfies the properties above.

Figure 1 illustrates a network with nine sellers ($i = 0, 1, \dots, 8$) and two buyers ($j = 9, 10$). In all figures, buyers are represented by “terminal” nodes—those nodes without any out-links. Arrows indicate the possible direction of trade across each link. For instance, player 1 is an intermediary who can only purchase the good from player 0 and can then resell it to either intermediary 4 or 5 ($N_1 = \{4, 5\}$).

⁵For instance, intermediation costs in the market for antiquities account for excavation, theft, transportation, risk, restoration, false documentation, and appraisal. Cost components for cocaine trade include coca farming, equipment and chemicals required for producing cocaine, smuggling, bribes, and violence.

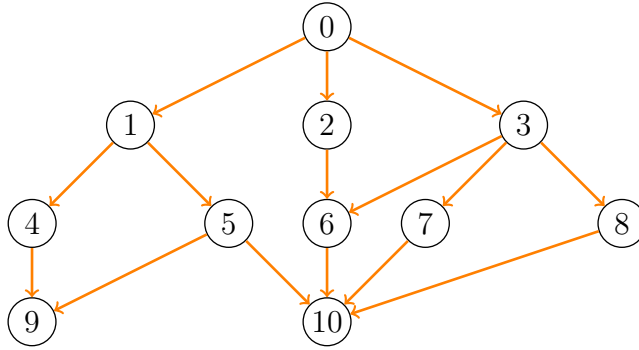


FIGURE 1. Network example

The good is sequentially resold via bilateral bargaining between linked players in the network G until it reaches one of the buyers. We consider the following dynamic non-cooperative *intermediation game*. At each date $t = 0, 1, \dots$ the history of play determines the current owner i_t . Player 0 is the owner at time 0, $i_0 = 0$. At date t , seller i_t selects a bargaining partner $k_t \in N_{i_t}$ among his downstream neighbors in the network G . With probability $p \in (0, 1)$, seller i_t proposes a price and partner k_t decides whether to purchase the good. Roles are reversed with probability $1 - p$. In either event, if the offer is rejected, the game proceeds to the next period with no change in ownership, $i_{t+1} = i_t$. If the offer is accepted, then i_t incurs the cost c_{i_t} (at date t), and i_t and k_t trade the good at the agreed price. If k_t is an intermediary, the game continues to date $t + 1$ with $i_{t+1} = k_t$. If k_t is a buyer, he consumes the good (at t) for a utility of v_{k_t} and the game ends. Players have a common discount factor $\delta \in (0, 1)$. Section 9 discusses extensions of the model and alternative interpretations of the game.

All elements of the game, including the network structure, are assumed to be common knowledge among players.⁶ For simplicity, we assume that the game has perfect information. We focus on *stationary Markov perfect equilibria*, which we call MPEs or equilibria for brevity. The natural notion of a Markov state in our setting is given by the identity of the current seller. An MPE is a subgame perfect equilibrium in which, after any history where i owns the good at time t , seller i 's (possibly random) choice of a partner $k \in N_i$ and the actions within

⁶Common knowledge of the network does not necessarily entail that all players know or can communicate with one other. Intermediaries may personally know their neighbors in the network and be aware of the overall network structure yet be unacquainted with neighbors of their neighbors. They may be informed about the existence of distant trading opportunities in the network but not about the exact identities or contacts of faraway traders. The assumption of an exogenous network with long intermediation chains is natural in contexts in which intermediaries perform specialized steps in the intermediation process that require local knowledge, technical expertise, or physical capital. The input of some intermediaries may be indispensable for realizing the gains from trade. Forging links with better positioned traders may be infeasible due to informational asymmetries, lack of trust, infrastructure, or institutional factors. For instance, the looter of the phial of Achyris may have been aware of the existence of a buyer in New York who was willing to pay \$1.2 million for the artifact but could not identify this buyer and lacked the means to smuggle, restore, launder, certify, export, and market the artifact.

the ensuing match (i, k) depend only on round t developments, recorded in the following sequence: the identity i of the current seller, his choice of a partner k , nature's selection of a proposer in the match (i, k) , and the offer extended by the proposer at t . In particular, strategies do not depend directly on the calendar time t . The economic intuitions we derive from the characterizations of MPE outcomes in the next two sections confer external validity to our equilibrium selection.

Our main results concern limit equilibrium outcomes as players become patient ($\delta \rightarrow 1$). Several examples presented in the Online Appendix in which we compute MPEs for every $\delta \in (0, 1)$ suggest that it is difficult to obtain analytical results for δ away from 1 in general networks.

Before proceeding to the equilibrium analysis, we define our welfare criterion. For this purpose, we consider *families of equilibria* that contain one MPE of the intermediation game for every $\delta \in (0, 1)$. We say that (trade in) a family of equilibria is *asymptotically efficient* if the sum of ex ante equilibrium payoffs of all players converges as $\delta \rightarrow 1$ to (the positive part of) the maximum surplus achieved by a trading path,

$$\max \left(0, \max_{\substack{\text{trading paths} \\ (i_0, i_1, \dots, i_{\bar{s}})}} v_{i_{\bar{s}}} - \sum_{s=0}^{\bar{s}-1} c_{i_s} \right).$$

Trade is *asymptotically inefficient* if the limit inferior (\liminf) of the total sum of equilibrium payoffs as $\delta \rightarrow 1$ is smaller than the expression above.

3. THE BARGAINING GAME WITHOUT INTERMEDIATION

To gain some intuition for the structure of MPEs, we start the analysis with the simple case in which there are no intermediaries, i.e., $m = 0$ in our model. In this game, the seller—player 0—bargains with the buyers—players $j = \overline{1, n}$ —following the protocol from the intermediation game. When the seller reaches an agreement with buyer j , the two parties exchange the good at the agreed price, the seller incurs his cost c_0 , and buyer j enjoys his consumption value v_j . The game ends when an exchange takes place. The next result provides a comprehensive characterization of MPEs.⁷

Proposition 1. *Suppose that $m = 0$, $v_1 \geq v_2 \geq \dots \geq v_n$ and $v_1 > c_0$. Then all MPEs are outcome equivalent.⁸ MPE expected payoffs converge as $\delta \rightarrow 1$ to*

- $\max(p(v_1 - c_0), v_2 - c_0)$ for the seller;

⁷Rubinstein and Wolinsky (1990; Section 5) introduced this bargaining game and offered an analysis for the case with identical buyers. A similar bargaining protocol appears in Abreu and Manea (2012). However, both studies focus on non-Markovian behavior.

⁸The *outcome* of a strategy profile is defined as the probability distribution it induces over agreements (including the agreement date, the identities of the buyer and the proposer, and the price) and the event that no trade ever takes place. Two strategies are *outcome equivalent* if they generate identical outcomes.

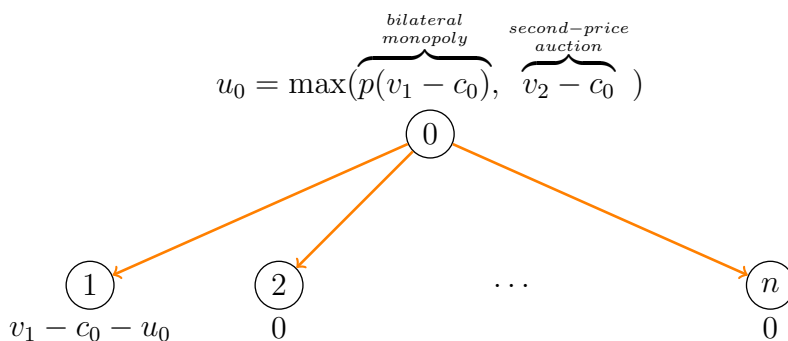


FIGURE 2. Limit equilibrium payoffs in the case without intermediation

- $\min((1 - p)(v_1 - c_0), v_1 - v_2)$ for buyer 1;
- 0 for all other buyers.

There exists $\underline{\delta} < 1$, such that in any MPE for $\delta > \underline{\delta}$,

- if $p(v_1 - c_0) \geq v_2 - c_0$, the seller trades exclusively with buyer 1;
- if $v_1 = v_2$, the seller trades with equal probability with all buyers j with $v_j = v_1$ and no others;
- if $v_2 - c_0 > p(v_1 - c_0)$ and $v_1 > v_2$, the seller trades with positive probability only with buyer 1 and all buyers j with $v_j = v_2$; the probability of trade with buyer 1 converges to 1 as $\delta \rightarrow 1$.

MPEs are asymptotically efficient.

Figure 2 illustrates the result. The intuition is that when there are positive gains from trade, the seller effectively chooses his favorite outcome between two scenarios in the limit $\delta \rightarrow 1$. In one scenario, the outcome corresponds to a *bilateral monopoly* agreement in which the seller bargains only with the (single) highest value buyer. It is well-known that in a two-player bargaining game with the protocol from the general model (formally, this is the case $m = 0, n = 1$), in which the seller has cost c_0 and the buyer has valuation v_1 , the seller and the buyer split the surplus $v_1 - c_0$ according to the ratio $p : (1 - p)$. The other scenario is equivalent to a *second-price auction*, in which the seller extracts the entire surplus $v_2 - c_0$ created by the second highest value buyer.

Thus, the seller is able to exploit the competition between buyers and extract more than the bilateral monopoly profits from player 1 only if the threat of dealing with player 2 is credible, i.e., $v_2 - c_0 > p(v_1 - c_0)$. In that case, the seller can extract the full surplus with player 2, since the “default” scenario in which he trades with player 1 leaves player 2 with zero payoff.⁹ Note that when $v_1 = v_2$, the seller bargains with equal probability with all buyers with value v_1 , so there is not a single default partner. If $v_1 > v_2$ and $v_2 - c_0 > p(v_1 - c_0)$, then for high δ a small probability of trade with buyer 2 is sufficient to drive buyer 1’s rents

⁹The role of outside options is familiar from the early work of Shaked and Sutton (1984).

down from the bilateral monopoly payoff of $(1-p)(v_1 - c_0)$ to the second-price auction payoff of $v_1 - v_2$. The threat of trading with buyer 2 is implemented with vanishing probability as $\delta \rightarrow 1$, and the good is allocated efficiently in the limit. An example in the Online Appendix illustrates MPE outcomes for discount factors δ away from 1.

The proof can be found in the Appendix.¹⁰ We show that the MPE is essentially unique: behavior is pinned down at all histories except those where the seller has just picked a bargaining partner who is not supposed to be selected (with positive probability) under the equilibrium strategies. Finally, note that when $v_1 \leq c_0$, the seller cannot create positive surplus with any of the buyers, and hence all players have zero payoffs in any MPE. Thus Proposition 1 has the corollary that when there are no intermediaries, all MPEs of the bargaining game are payoff equivalent and asymptotically efficient.¹¹

4. EQUILIBRIUM CHARACTERIZATION FOR THE INTERMEDIATION GAME

Consider now the general intermediation game. Fix an MPE σ for a given discount factor δ . By definition, σ induces equivalent behavior and outcomes in all subgames where player k possesses the good and has not yet selected a bargaining partner in the current period. We simply refer to all such circumstances as *subgame k* . In the equilibrium σ , every player h has the same expected payoff u_h^k in any subgame k . By convention, $u_h^k = 0$ whenever $k > h$ and $u_j^j = v_j/\delta$ for $j = \overline{m+1, n}$. The latter specification reflects the assumption that following an acquisition, buyers immediately consume the good, while intermediaries have the chance to resell it only one period later. This definition instates notational symmetry between buyers and sellers: whenever a player k acquires the good, every player h expects a *continuation payoff*—discounted at the date of k 's purchase—of δu_h^k .

Given the equilibrium σ , the strategic situation faced by a current seller i reduces to a bargaining game with “buyers” in N_i , in which each $k \in N_i$ has a (continuation) “value” δu_k^k . This *reduced game of seller i* is reminiscent of the bargaining game without intermediation analyzed in the previous section, with one important caveat. In the game with no intermediation, each buyer k has a continuation payoff of 0 when the seller trades with some other buyer h . By contrast, in the general intermediation model, player $k \in N_i$ may still enjoy positive continuation payoffs when another $h \in N_i$ acquires the good from seller i , if

¹⁰For brevity, some steps rely on more general arguments developed for the subsequent Theorem 1. Since the proof of the latter result does not invoke Proposition 1, there is no risk of circular reasoning.

¹¹However, when $v_2 - c_0 > p(v_1 - c_0)$ and $v_1 > v_2$, we can construct *non-Markovian* subgame perfect equilibria that are asymptotically inefficient. Indeed, if $v_2 - c_0 > p(v_1 - c_0)$, then for every δ , MPEs necessarily involve the seller mixing in his choice of a bargaining partner. We can construct subgame perfect equilibria that are not outcome equivalent with the MPE, even asymptotically as $\delta \rightarrow 1$, by simply modifying the seller's first period strategy to specify a deterministic choice among the partners selected with positive probability in the MPE. The proof of Proposition 1 shows that for every δ , the seller bargains with buyer 2 with positive probability in the MPE. Hence, for every δ , we can derive a subgame perfect equilibrium in which the seller trades with buyer 2 without delay. Such equilibria are asymptotically inefficient when $v_1 > v_2$.

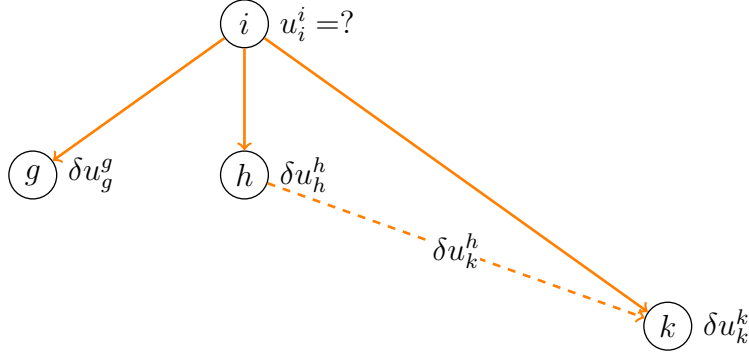


FIGURE 3. $N_i = \{g, h, k\}$. Player k may obtain positive continuation payoffs δu_k^h when intermediary h acquires the good from i .

k purchases it subsequently (directly from h or via a chain of trades). Hence, both i and k enjoy endogenous outside options: i can choose a different bargaining partner, while k may acquire the good from other players. As Figure 3 illustrates, the bargaining power u_k^i of player k in subgame i depends not only on u_i^i and the probability with which i selects k for bargaining (as it would in the absence of resale), but also on the probability of trade between i and h and on the possibly positive continuation payoff δu_k^h that k expects in subgame h .

In light of the discussion above, we refer to the payoff u_k^k as player k 's *resale value* and to u_k^h as player k 's *lateral intermediation rent* under (seller) h .¹² While lateral intermediation rents may be substantial, we find that they cannot be sufficiently high to induce a downstream neighbor of the current seller to wait for another neighbor to purchase the good with the expectation of acquiring it at a lower price later. The proof of the forthcoming Theorem 1 derives an upper bound on player k 's lateral intermediation rent under h in situations in which current seller i trades with intermediary h with positive probability in equilibrium. The bound relies on two observations:

- seller i 's incentives to choose h over k as a bargaining partner imply that the difference in resale values of k and h is not greater than the difference in subgame i expected payoffs of k and h , that is, $u_k^k - u_h^h \leq u_k^i - u_h^i$;
- k 's lateral intermediation rent under h , when positive, cannot exceed the difference in resale values of k and h , that is, $u_k^h \leq u_k^k - u_h^h$.

Under the conditions stated above, we find that $u_k^h \leq u_k^i - u_h^i$. In particular, $u_k^h \leq u_k^i$, which means that player k is better off at the beginning of subgame i rather than subgame h . In this sense, player k 's outside option is not “binding” in equilibrium.

¹²Note that player k receives positive lateral intermediation rents only if the initial seller is connected to k via directed paths of distinct lengths. Hence lateral intermediation rents do not feature in the analysis of networks in which all routes from the initial seller to any fixed player contain the same number of intermediaries. Two salient classes of linking structures satisfying this property are *arborescences* (networks in which there is a unique directed path from node 0 to any other node) and *tier networks* (where the set of nodes is partitioned into vertically ordered tiers and every trading path contains, in order, exactly one node from every tier).

Building on this intuition, Theorem 1 proves that lateral intermediation rents do not influence resale values in the limit as players become patient. Specifically, in any family of MPEs, the resale value of each seller i converges as $\delta \rightarrow 1$ to a limit r_i , which is a function only of the limit resale values $(r_k)_{k \in N_i}$ of i 's downstream neighbors. In the limit, seller i 's bargaining power in the reduced game is derived *as if* the players in N_i were buyers with valuations $(r_k)_{k \in N_i}$ in the game without intermediation.¹³

Theorem 1. *For any family of MPEs, resale values converge as $\delta \rightarrow 1$ to a vector $(r_i)_{i \in N}$, which is determined recursively as follows*

- $r_j = v_j$ for $j = m + 1, \dots, n$
- $r_i = \max(p(r_{N_i}^I - c_i), r_{N_i}^{II} - c_i, 0)$ for $i = m, m - 1, \dots, 0$, where $r_{N_i}^I$ and $r_{N_i}^{II}$ denote the first and the second highest order statistics of the vector $(r_k)_{k \in N_i}$, respectively.¹⁴

The proof appears in the Appendix. We provide the characterization of equilibrium payoffs and trading probabilities here. Fix a discount factor δ and a corresponding MPE σ with payoffs $(u_h^k)_{k, h \in N}$. Assume that the current seller i can generate positive gains by trading with one of his neighbors, i.e., $\delta \max_{k \in N_i} u_k^k > c_i$. Under this assumption, we find that if i selects $k \in N_i$ as a bargaining partner under σ , then the two players trade with probability 1. The equilibrium prices offered by i and k are $\delta u_k^k - \delta u_k^i$ and $\delta u_k^i + c_i$, respectively. If π_k denotes the probability that seller i selects neighbor k for bargaining in subgame i under σ , then we obtain the following equilibrium constraints for all $k \in N_i$:

$$(1) \quad u_k^i \geq p(\delta u_k^k - c_i - \delta u_k^i) + (1 - p)\delta u_k^i, \text{ with equality if } \pi_k > 0;$$

$$(2) \quad u_k^i = \pi_k (p\delta u_k^i + (1 - p)(\delta u_k^k - c_i - \delta u_k^i)) + \sum_{h \in N_i \setminus \{k\}} \pi_h \delta u_k^h.$$

For example, the right-hand side of equation (2) reflects the following equilibrium properties. Seller i trades with player k with probability π_k . At the time of purchase, the good is worth a discounted resale value of δu_k^k to k . If $\pi_k > 0$, seller i asks for a price of $\delta u_k^k - \delta u_k^i$ from k , while player k offers a price of $\delta u_k^i + c_i$ to i , with respective conditional probabilities p and $1 - p$. Furthermore, seller i trades with neighbor $h \neq k$ with probability π_h , in which event k enjoys a discounted lateral intermediation rent of δu_k^h .

The next result extends the characterization of buyer payoffs from Proposition 1 to every subgame of the intermediation game.

Proposition 2. *Suppose that $k \in N_i$ and $r_k = r_{N_i}^I \geq c_i$. Then for any family of MPEs, expected payoffs in subgame i converge as $\delta \rightarrow 1$ to*

¹³Whether the conclusion of Proposition 1 regarding outcome equivalence of MPEs generalizes to the intermediation game is an open question. However, this technical puzzle does not restrict the scope of our analysis since we focus on limit equilibrium outcomes as $\delta \rightarrow 1$, and Theorem 1 establishes that all MPEs generate identical limit resale values. In particular, the initial seller obtains the same limit profit in all MPEs.

¹⁴If $|N_i| = 1$, then $r_{N_i}^{II}$ can be defined to be any non-positive number.

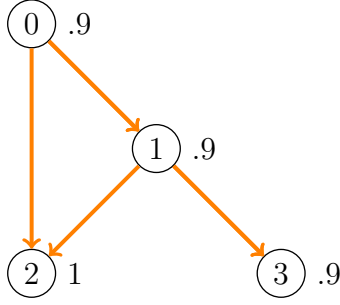


FIGURE 4. The seller trades with positive limit probability with the lowest resale value neighbor, intermediary 1.

- $\min((1-p)(r_k - c_i), r_k - r_{N_i}^{\text{II}})$ for player k ;
- 0 for any player $h \in N_i \setminus \{k\}$.

Theorem 1 and Proposition 2 effectively state that lateral intermediation rents do not affect the payoff formulae from Proposition 1. However, we discover that lateral intermediation rents play an unexpected role in determining the equilibrium path of trade and that the structure of agreements in the intermediation game differs substantially from the one described in Proposition 1. A natural extension of Proposition 1 would be that if $r_{N_i}^I > c_i$, then seller i trades only with neighbors $k \in N_i$ who have maximum limit resale values ($r_k = r_{N_i}^I$) almost surely as $\delta \rightarrow 1$. The next example demonstrates that this conjecture is false.

Consider the network from Figure 4 formed by the seller (player 0), a single intermediary (player 1), and two buyers (players 2 and 3). Assume that $c_0 = c_1 = 0, v_2 = 1, v_3 = 0.9$, and $p = 1/2$. Using Theorem 1, we immediately find that $r_0 = r_1 = 0.9$ (limit resale values are indicated next to the corresponding nodes, a convention we follow throughout). Let π_1 denote the probability with which the seller trades with player 1. In the Online Appendix, we show that for sufficiently high δ , the initial seller trades with both intermediary 1 and buyer 2 with positive probability in the MPE; the probability of trade with intermediary 1 converges to $(35 - \sqrt{649})/36 \approx 0.26$ as $\delta \rightarrow 1$. Therefore, the seller trades with intermediary 1 with positive limit probability as $\delta \rightarrow 1$ even though the seller is directly linked to buyer 2 and $r_1 = 0.9 < 1 = v_2 = r_2$. After acquiring the good, intermediary 1 resells it to the high value buyer 2 almost surely as $\delta \rightarrow 1$, so the positive limit probability of trade between the seller and his neighbor with the lowest resale value is not at odds with asymptotic efficiency in the context of this example.

The structure of equilibrium agreements changes drastically if we assume instead that $c_1 > 0$ in the example above. In this case, the equilibrium constraint (1) for $i = 0$ and $k = 2$ becomes $u_0^0 \geq p(1 - \delta u_2^0) + (1 - p)\delta u_0^0$, which leads to

$$\liminf_{\delta \rightarrow 1} (u_0^0 + u_2^0) \geq 1.$$

When $c_1 > 0$, this is possible only if the seller trades directly with buyer 2 with limit probability 1 as $\delta \rightarrow 1$ (any other trading path generates a surplus of at most $1 - c_1 < 1$). The limit outcome is again asymptotically efficient.

The discontinuity in the limit equilibrium path of trade as $\delta \rightarrow 1$ for $c_1 \rightarrow 0$ is puzzling but can be explained by the order of limits. It is remarkable that the discontinuity in trading probabilities does not affect the distribution of profits and the final allocation of the good in the example. The next result generalizes this conclusion. Whenever the highest limit resale value among the players in N_i is greater than c_i and is achieved by a single downstream neighbor k of seller i , player k eventually receives the good with limit probability 1 in subgame i ; moreover, k buys the good directly from seller i with limit probability 1 if i 's other downstream neighbors have positive intermediation costs. We view these findings as an expression of *local efficiency*. Section 7 shows that local efficiency does not imply global efficiency. The result also describes the structure of trading paths for instances in which there are ties in top resale values and characterizes the lateral intermediation rents relevant for equilibrium analysis.

Proposition 3. *The following statements hold for any family of MPEs when we take the limit $\delta \rightarrow 1$.*

- (1) *Suppose that $k \in N_i$ and $r_k = r_{N_i}^I > \max(r_{N_i}^{II}, c_i)$. Then player k acquires the good in subgame i with limit probability 1 either directly from i or via a chain of intermediaries; if $c_h > 0$ for all $h \in N_i \setminus \{k\}$,¹⁵ then k buys the good directly from i with limit probability 1. If seller i resells the good to intermediary $h \in N_i \setminus \{k\}$ with positive limit probability, then the lateral intermediation rent u_k^h of player k under h converges to $\min((1-p)(r_k - c_i), r_k - r_{N_i}^{II})$. The lateral intermediation rent u_h^k of every player $h \in N_i \setminus \{k\}$ under k is 0 for sufficiently high δ .*
- (2) *If $r_{N_i}^I = r_{N_i}^{II} > c_i$, then for sufficiently high δ , seller i offers the good in subgame i exclusively to neighbors $k \in N_i$ with $r_k = r_{N_i}^I$. For any pair (k, h) of such neighbors, the lateral intermediation rent u_h^k of h under k converges to 0.*

Suppose that $r_0 > 0$. We say that a trading path $(i_s)_{s=\overline{0}, \overline{s}}$ is *locally efficient* if $i_{s+1} = r_{N_{i_s}}^I$ for $s = \overline{0}, \overline{s} - 1$. If (i_s) is a locally efficient trading path such that there are no ties for the highest resale value of the downstream neighbors of any seller i_s (i.e., $r_{N_{i_s}}^I > r_{N_{i_s}}^{II}$ for $s = \overline{0}, \overline{s} - 1$), then (i_s) is the unique locally efficient path. In that case, the proof of Proposition 3 shows that for $s = \overline{0}, \overline{s} - 1$, player i_{s+1} acquires the good—either directly from i_s or via a chain of intermediaries—with limit probability 1 in equilibrium at a limit price of $r_{i_s} + c_{i_s}$ and enjoys a limit profit of $r_{i_{s+1}} - c_{i_s} - r_{i_s} = \min((1-p)(r_{i_{s+1}} - c_{i_s}), r_{i_{s+1}} - r_{N_{i_s}}^{II}) > 0$. Since these profits, along with player 0's profit r_0 , sum up to the surplus generated by the trading

¹⁵This condition can be weakened to require that any indirect trade between i and k involves higher intermediation costs than direct trade.

path $(i_s)_{s=\overline{0},\overline{s}}$, it must be that the ex ante equilibrium payoffs of all other players—including intermediaries who facilitate indirect trade between i_s and i_{s+1} —converge to 0 as $\delta \rightarrow 1$. Hence, only members of the locally efficient path $(i_s)_{s=\overline{0},\overline{s}}$ make *positive limit profits* in equilibrium. Furthermore, if all intermediation costs are nonzero, then trade takes place along the path (i_s) with limit probability 1 in equilibrium.

Fixing the linking structure and buyer values, consider the set of cost profiles $(c_i)_{i=\overline{0},\overline{m}}$ for which $r_0 > 0$. Clearly, this set has positive Lebesgue measure, and its subset for which there exist multiple locally efficient trading paths has measure 0, as does its subset in which some intermediaries have zero cost. Therefore, for generic cost profiles such that $r_0 > 0$, there exists a unique locally efficient path (i_s) , and trade proceeds along (i_s) with limit probability 1 in equilibrium at a sequence of limit prices $(r_{i_s} + c_{i_s})$.

Using the characterization of MPE payoffs and trade probabilities summarized by the system of constraints (1)-(2), the final result in this section establishes the existence of an equilibrium.

Proposition 4. *An MPE exists in the intermediation game.*

5. LAYERS OF INTERMEDIATION POWER

This section investigates how an intermediary's position in the network affects his resale value. In order to focus exclusively on network asymmetries, suppose momentarily that all sellers have zero costs and buyers have a common value v .¹⁶ Theorem 1 implies that competitive forces drive the (limit) resale price of any seller linked to two (or more) buyers up to v . Similarly, any trader linked to two downstream players with resale value v can exploit the competition between them to obtain a price of v . We use this property iteratively to identify players with resale value v until we reach a point where no remaining trader has two downstream neighbors known to have resale value v . Using Theorem 1 again, we can show that the resale value of each remaining trader who has a single downstream neighbor with resale value v is pv . We can then identify additional traders who charge a resale price of pv due to competition between downstream neighbors with resale value pv , and so on.

We are thus led to decompose the network into a sequence of *layers of intermediation power*, $(\mathcal{L}_\ell)_{\ell \geq 0}$, which characterizes every player's resale value. The recursive construction proceeds as follows. First all buyers are enlisted in layer 0. In general, for $\ell \geq 1$, having defined layers 0 through $\ell - 1$, the first players to join layer ℓ are those outside $\bigcup_{\ell' < \ell} \mathcal{L}_{\ell'}$ who have a downstream neighbor in $\mathcal{L}_{\ell-1}$. For every $\ell \geq 0$, given the initial membership of layer ℓ , new traders outside $\bigcup_{\ell' < \ell} \mathcal{L}_{\ell'}$ with at least two downstream links to established members of layer ℓ are added to the layer. We continue expanding layer ℓ until no remaining seller has

¹⁶In networks with identical buyer values, the zero-cost assumption implies welfare equality among trading paths of different lengths.

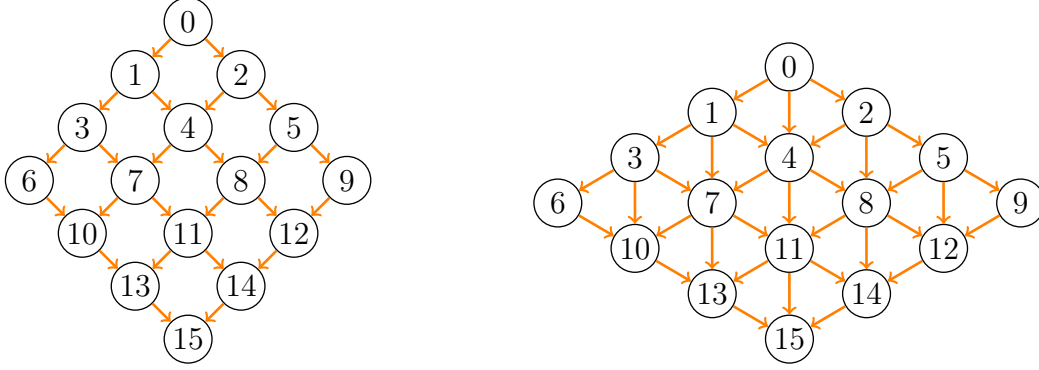


FIGURE 5. A square lattice and a triangular grid

two downstream neighbors among formerly recognized layer ℓ members. The players joining layer ℓ through this sequential procedure form the set \mathcal{L}_ℓ .¹⁷ The algorithm terminates when every player is assigned to a layer, $\bigcup_{\ell' \leq \ell} \mathcal{L}_{\ell'} = N$. For an illustration, in the network from Figure 1 the algorithm produces the layers $\mathcal{L}_0 = \{5, 9, 10\}$, $\mathcal{L}_1 = \{0, 1, 3, 4, 6, 7, 8\}$, $\mathcal{L}_2 = \{2\}$.

The definition of each layer is obviously independent of the order in which players join the layer. In fact, an equivalent description of the layer decomposition proceeds as follows. \mathcal{L}_0 is the largest set (with respect to inclusion) $M \subseteq N$, which contains all buyers, such that every seller in M has two (or more) out-links to other players in M . For $\ell \geq 1$, \mathcal{L}_ℓ is the largest set $M \subseteq N \setminus (\bigcup_{\ell' < \ell} \mathcal{L}_{\ell'})$ with the property that every node in M has out-links to either (exactly) one node in $\mathcal{L}_{\ell-1}$ or (at least) two nodes in M .

The arguments above suggest that if sellers have zero costs and buyers have value v , then players from layer ℓ have a resale value of $p^\ell v$. The next result proves a generalization of this claim for arbitrary profiles $(c_i)_{i=0, \overline{m}}$ and $(v_j)_{j=m+1, \overline{n}}$ by providing lower and upper bounds for the resale values of layer ℓ players that converge to the same point as intermediation costs and differences in buyer values vanish. Thus, the decomposition of the network into layers captures intermediation power.

Theorem 2. *Let \underline{v} and \bar{v} denote the minimum and the maximum buyer values, respectively. The limit resale value of any player $i \in \mathcal{L}_\ell$ satisfies*

$$r_i \in \left[p^\ell \underline{v} - \sum_{\ell'=0}^{\ell} p^{\ell-\ell'} \sum_{k \in \mathcal{L}_{\ell'}, i \leq k \leq m} c_k, p^\ell \bar{v} \right].$$

In particular, if intermediaries have zero costs and buyers have a common value v , then every layer ℓ player has a limit resale value of $p^\ell v$.

The layers of intermediation power have simple structures in some networks and more complex topologies in others. Consider the two networks with 15 sellers and one buyer from

¹⁷Note that layer ℓ has an analogous topology to layer 0 if we recast the initial members of layer ℓ as buyers.

Figure 5. In the square lattice from the left panel, we find the following layer decomposition:

$$\mathcal{L}_0 = \{15\}, \mathcal{L}_1 = \{11, 13, 14\}, \mathcal{L}_2 = \{4, 7, 8, 10, 12\}, \mathcal{L}_3 = \{0, 1, 2, 3, 5, 6, 9\}.$$

The initial seller, who is six links away from the buyer, belongs to layer 3. In the triangular grid on the right-hand side, the network decomposition is given by

$$\mathcal{L}_0 = \{15\}, \mathcal{L}_1 = \{0, 1, 2, 4, 7, 8, 11, 13, 14\}, \mathcal{L}_2 = \{3, 5, 10, 12\}, \mathcal{L}_3 = \{6, 9\}.$$

The initial seller, three links away from the buyer, is a member of layer 1.

Suppose now that there are no intermediation costs and there is a single buyer ($m = n - 1$) with valuation v . Let d denote the length \bar{s} of the shortest trading path $i_0 = 0, i_1, \dots, i_{\bar{s}} = n$ in the network. Since the initial seller belongs to a layer $\ell \leq d$, Theorem 2 implies that his limit profit is at least $p^\ell v$. This lower bound is achieved if G is a line network ($N_i = \{i + 1\}$ for $i = \overline{0, n - 1}$). In square lattices, the initial seller's limit payoff can be as high as $p^{d/2}v$. However, there exist networks with arbitrarily high d —e.g., scaled-up triangular grids—in which the initial seller belongs to layer 1 and makes a limit profit of pv . Therefore, a trader's intermediation power does not depend directly on the length of his intermediation chains but on the competition among them, which is reflected in the number of layers they intersect. In other words, layers measure the effective intermediation distance between traders.

The characterization of layers of intermediation power has practical implications. Suppose that the initial seller belongs to layer ℓ in our network decomposition. Then each neighbor $k \in N_0$ belongs to a layer $\ell' \geq \ell - 1$ and provides a resale value of at most $p^{\ell'} v \leq p^{\ell-1} v$. If $c_0 > p^{\ell-1} v$, then no transaction takes place for high δ . Hence trade is possible only if the initial seller belongs to a sufficiently low layer. Therefore, in financial markets and manufacturing, where trade is socially desirable, denser networks (downstream competition) with short paths (vertical integration) are optimal for promoting trade. Such networks enable the initial seller to obtain a significant share of the gains from trade in order to cover his costs. However, in markets where trade is not socially desirable, as in the case of illicit goods and bribery, sparser networks with long paths and many bottlenecks (checkpoints, bureaucracy) are preferable. In such networks, bargaining breaks down due to the large amount of anticipated downstream intermediation rents.

The construction of layers and the examples above suggest that players from layer ℓ are either directly linked to layer $\ell - 1$ or offer competing paths to layer $\ell - 1$. Every layer ℓ player is linked to at least two intermediaries added to layer ℓ earlier in the algorithm, who are each linked to two former members of the layer, and so on, until layer $\ell - 1$ players are eventually reached. Of course, there may be significant overlap among the paths traced in this fashion, but the possibility of branching out in at least two directions at every stage generates rich sets of paths connecting layer ℓ players to $\ell - 1$. In particular, the next result shows that every layer ℓ player not directly linked to layer $\ell - 1$ has two independent paths

of access to layer $\ell - 1$. In addition, any two players from layer ℓ can reach layer $\ell - 1$ via disjoint paths of layer ℓ intermediaries. In other words, every pair of intermediaries from layer ℓ can pass the good down to layer $\ell - 1$ without relying on each other or on any common layer ℓ intermediaries.

Proposition 5. *Every player from layer $\ell \geq 1$ has either a direct link to layer $\ell - 1$ or two non-overlapping paths of layer ℓ intermediaries connecting him to (possibly the same) layer $\ell - 1$ players. Moreover, any pair of distinct layer $\ell \geq 1$ players can reach some (possibly identical) layer $\ell - 1$ players via disjoint paths of layer ℓ intermediaries.*

6. POSITIVE PROFITS

We now study the structure of trading paths and the distribution of profits in the setting with no intermediation costs and homogeneous buyer values. The next result shows that players make positive limit profits only in transactions in which they constitute a *gateway* to a lower layer. Therefore, profits are shared along the equilibrium trading path among the first members of each layer. In this sense, layers delineate monopoly power from intermediation power. Local competition is exploited in exchanges within the same layer to extract the full amount of gains from trade, while intermediation rents are paid in agreements across layers.

Proposition 6. *Suppose that sellers have zero costs and buyers have a common value v . Then for sufficiently high δ , every layer ℓ player can acquire the good in equilibrium only from traders in layers ℓ and $\ell + 1$. Moreover, if player $k \in \mathcal{L}_\ell$ purchases the good from seller i with positive probability in subgame i for a sequence of δ 's converging to 1, then player k 's limit profit in subgame i is 0 if $i \in \mathcal{L}_\ell$ and $(1 - p)p^\ell v$ if $i \in \mathcal{L}_{\ell+1}$.*

Proposition 6 characterizes the limit profit of player k in subgame i . In order to evaluate player k 's ex ante equilibrium payoff, we need to compute the probability that i acquires the good and, more generally, the probability with which every trading path arises in the MPE. In Section 4, we argued that for generic profiles of intermediation costs, there exists a unique locally efficient path, along which trade takes place with limit probability 1 as $\delta \rightarrow 1$. However, the example analyzed in Section 4 demonstrates that when some intermediation costs are zero, there may be multiple locally efficient trading paths and all of them can emerge with positive limit probability in equilibrium. Determining the probability of each such path is intractable in general.

Surprisingly, we find that symmetries in resale values in the case with no intermediation costs and homogeneous buyer values do not translate into symmetries in equilibrium trading probabilities. Although traders from the same layer have identical limit resale values, it turns out that MPE trading paths may treat such traders asymmetrically, even in the limit as $\delta \rightarrow 1$. Indeed, as the next example illustrates, the seller may sell the good with unequal

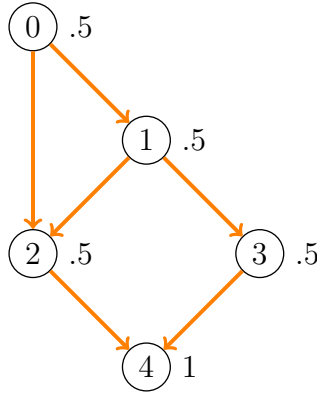


FIGURE 6. Asymmetric trading paths within layers

(positive) limit probabilities to a pair of downstream neighbors with identical limit resale values and lateral intermediation rents.

Consider the intermediation game induced by the network in Figure 6. Suppose that $p = 1/2$, all costs are zero, and $v_4 = 1$. Applying our layer decomposition algorithm, we find that all sellers belong to layer 1. Theorem 2 implies that the initial seller has a limit resale value of $1/2$, as do intermediaries 1 and 2. However, in the Online Appendix we argue that this game has a unique MPE, in which player 0 treats intermediaries 1 and 2 asymmetrically. For $\delta > 4/5$, player 0 chooses either intermediary for bargaining with positive probability in the MPE, but selects the better positioned player 2 more frequently. We prove that the seller trades with intermediary 1 with a probability that converges to $\pi_1^* := 1 - 1/\sqrt{2} \approx .29$ as $\delta \rightarrow 1$. The disparity in limit trading probabilities with players 1 and 2 is intriguing since all payoff asymmetries between the two players vanish in the limit: $\lim_{\delta \rightarrow 1} u_1^1 = \lim_{\delta \rightarrow 1} u_2^2 = 1/2$, $\lim_{\delta \rightarrow 1} u_2^1 = 0$, and $u_1^2 = 0$.

In the MPE limit, the good is traded along each of the paths $(0, 1, 2, 4)$ and $(0, 1, 3, 4)$ with probability $\pi_1^*/2$ and along the shorter path $(0, 2, 4)$ with probability $1 - \pi_1^*$. The example can be easily adapted so that the value of π_1^* affects the distribution of limit profits in the network. Suppose, for instance, that the buyer is replaced by two unit value buyers, one linked to intermediary 2 and the other to 3. Then the limit MPE payoffs of the two buyers are $1/2 - \pi_1^*/4$ and $\pi_1^*/4$, respectively.

We close this section with the observation that intermediaries who are not essential for trade can make substantial profits. Suppose that all sellers have zero costs and the buyer has unit value in the network depicted in Figure 7. Note that intermediary 1 is not essential for trade, as the initial seller can access buyer 4 via intermediaries 2 and 3 in this network. Goyal and Vega-Redondo (2007) posit that inessential intermediaries like player 1 should make zero profits. Siedlarek (2015) finds support for this hypothesis in his multilateral bargaining model. However, the hypothesis is not borne out by our model. Indeed, limit resale values

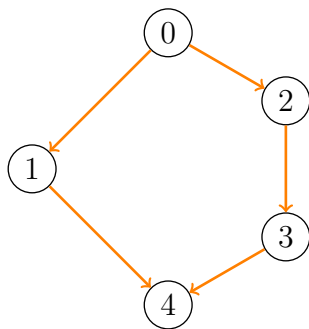


FIGURE 7. Intermediary 1 is not essential for trade, but makes positive profit.

in the network are immediately computed from Theorem 1: $r_1 = r_3 = p, r_0 = r_2 = p^2$. By Proposition 2, intermediary 1 earns a positive limit profit of $(1 - p)p$.¹⁸

7. INTERMEDIATION INEFFICIENCIES

In contrast to the bargaining model with no intermediaries analyzed in Section 3, intermediation may create trade inefficiencies. There are two distinct sources of asymptotic inefficiency, which constitute opposite sides of the same coin. One source, already well understood [16], resides in *hold-up* problems induced by the bilateral nature of intermediation combined with weak downstream competition. Consider a subgame in which a current seller i obtains positive net profit by trading with the highest resale value neighbor, but cannot capture the entire surplus available in the transaction, that is, $r_i = \max(p(r_{N_i}^I - c_i), r_{N_i}^{II} - c_i, 0) < r_{N_i}^I - c_i$. Then $r_{N_i}^I > \max(r_{N_i}^{II}, c_i)$ and the player with the highest resale value secures positive rents of $\min((1 - p)(r_{N_i}^I - c_i), r_{N_i}^I - r_{N_i}^{II})$ (Proposition 2). The rent amount is independent of the history of transactions; in particular, the payment i made to procure the good is sunk. Such rents are anticipated by upstream traders and diminish the gains they share. In some cases, the dissipation of surplus is so extreme that trade becomes unprofitable even though some intermediation chains generate positive surplus. We refer to this possibility as *no-trade inefficiency*.

To see the simplest manifestation of no-trade inefficiency, consider the intermediation network from Figure 8 that connects the initial seller to a single intermediary, who provides access to one buyer. Suppose that $v_2 > c_0 > 0$ and $c_1 = 0$. In the MPE, upon purchasing the good, the intermediary expects a payoff of pv_2 in the next period from reselling it to the buyer. Trade between the seller and the intermediary is then possible only if the seller's cost does not exceed the intermediary's continuation payoff, $c_0 \leq \delta pv_2$. Hence for $c_0 \in [pv_2, v_2)$, bargaining breaks down in the MPE for every δ , and traders fail to realize the positive gains

¹⁸More generally, in networks with a single buyer and no intermediation costs, which consist of two non-overlapping paths of unequal length connecting the seller to the buyer, no individual intermediary is essential for trade, yet intermediaries along the shorter path obtain positive intermediation rents in the limit.



FIGURE 8. No-trade inefficiency

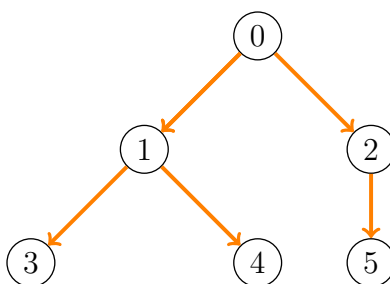


FIGURE 9. Allocation inefficiency

$v_2 - c_0$. The MPE is asymptotically inefficient in this case. As discussed above, the source of asymptotic inefficiency is that the buyer holds up the intermediary for a profit of $(1 - p)v_2$. Then, at the initial stage, the seller and the intermediary bargain over a reduced limit surplus of $pv_2 - c_0$, rather than the total amount of $v_2 - c_0$.¹⁹

Another source of inefficiency lies in sellers' incentives to exploit *local competition*, which are not aligned with global welfare maximization. This behavior has two negative welfare consequences. First, it may lead to *allocation inefficiency*—the good is not allocated to the highest value buyer in networks with no intermediation costs. Second, it can create *cost inefficiency*—the good is not traded along the intermediation chain with the lowest total cost in networks with a single buyer.

For an example of allocation inefficiency, consider the network from Figure 9, where $c_0 = c_1 = c_2 = 0$ and $v_3 = v_4 \in (pv_5, v_5)$. By Theorem 1, $r_1 = v_3 > pv_5 = r_2$. Proposition 3 then implies that the seller trades in the MPE with intermediary 1 with limit probability 1 as $\delta \rightarrow 1$ at a limit price $r_0 = pv_5$ derived from the second-price auction. Hence the good is allocated almost surely in the limit to one of the low value buyers 3 and 4 instead of the high value buyer 5.²⁰

¹⁹Blanchard and Kremer (1997) discuss similar hold-up problems in line networks.

²⁰Gofman (2011) provides an earlier study of allocation inefficiency in a setting with sequential auctions.

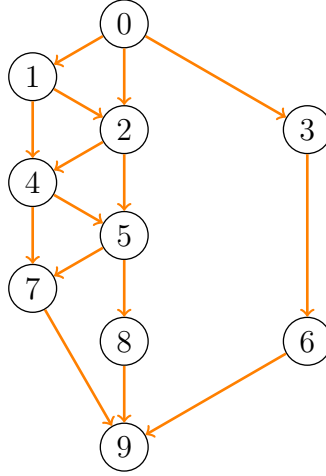


FIGURE 10. Cost inefficiency and long intermediation chains

To illustrate cost inefficiency, we analyze the network from Figure 10, in which the buyer value v_9 is normalized to 1 and sellers are assumed to have a common cost

$$\kappa \in \left[0, \min \left(\frac{p(1-p)}{4-p^2}, \frac{p(1-p)}{5-3p-p^2} \right) \right).$$

In this network, the layer decomposition algorithm leads to

$$\mathcal{L}_0 = \{9\}, \mathcal{L}_1 = \{0, 1, 2, 4, 5, 6, 7, 8\}, \mathcal{L}_2 = \{3\}.$$

Resale values are easily computed from Theorem 1. In particular, we find that $r_0 = p - (5 + p)\kappa$, $r_1 = p - (4 + p)\kappa$, $r_2 = p - (3 + p)\kappa$, and $r_3 = p^2 - p(1 + p)\kappa$. For the range of κ considered, we have $r_2 \geq r_1 > r_3$ and $r_0 = r_1 - \kappa > p(r_2 - \kappa) > 0$. Hence, the initial seller obtains the second-price auction profits in a bargaining game in which his neighbors act as buyers with values (r_1, r_2, r_3) .

Since $r_0 = r_1 - \kappa > r_3 - \kappa$, the initial seller does not trade with intermediary 3 for high δ . Equilibrium trade proceeds via intermediary 1 or 2 in order to exploit local competition within layer 1. The trading path involves at least three intermediaries from the set $\{1, 2, 4, 5, 7, 8\}$, so the realized gains from trade do not exceed $1 - 4\kappa$. However, the intermediation chain formed by traders 3 and 6 achieves a total welfare of $1 - 3\kappa$. Thus, the MPE is asymptotically inefficient for $\kappa > 0$.

It is interesting to note that even in settings where the network constitutes the sole source of asymmetry among intermediaries, trade does not proceed along the shortest route from seller to buyer.²¹ In the example above, the shortest trading path involves intermediaries 3 and 6. However, at least three intermediaries from layer 1 are employed to transfer the good

²¹In a result of a similar flavor but different substance, Glode and Opp (2016) show that longer intermediation chains may be more efficient in line networks with incomplete information about the value of the good if every intermediary possesses more information than his upstream neighbor.

to the buyer in equilibrium for high δ . Intermediary 3 from layer 2 never gains possession of the good, even though he brokers the shortest intermediation chain, because the seller prefers to deal with intermediaries 1 and 2 from layer 1.

We conclude this section by noting that inefficiencies disappear if sellers have all the bargaining power. Indeed, for p close to 1, the current seller is able to extract most of the surplus even in a bilateral monopoly scenario. Hence the hold-up friction vanishes and competition becomes redundant as $p \rightarrow 1$. The recursive characterization from Theorem 1 implies that player i 's limit resale value converges as $p \rightarrow 1$ to the maximum surplus generated by intermediation chains originating at i . Then there are no efficiency losses as p approaches 1. The corresponding limit resale values constitute the maximum competitive equilibrium prices in the framework with bilateral contracts of Hatfield et al. (2013).

8. COMPARATIVE STATICS

This section provides comparative statics with respect to the network architecture and the distribution of intermediation costs.

8.1. Adding Links and Eliminating Middlemen. We first investigate how the seller's profit responds to the addition of a link or the elimination of a middleman. Fix a network $G = (N, (N_i)_{i=\overline{0,m}}, (c_i)_{i=\overline{0,m}}, (v_j)_{j=\overline{m+1,n}})$. For seller i and player $k > i$ with $k \notin N_i$, *adding the link* (i, k) to G results in a new network \tilde{G} which differs from G only in that player i 's downstream neighborhood \tilde{N}_i in \tilde{G} incorporates k ($\tilde{N}_i = N_i \cup \{k\}$). Similarly, if $k \in N_i$, *removing the link* (i, k) from G generates a network where i 's downstream neighborhood excludes k .

Imagine now a scenario in which trader k could eliminate middleman $i \in N_k$ and gain direct access to i 's pool of trading partners N_i . Intuitively, such a rewiring of the network should benefit trader k , since k avoids paying intermediation rents to i . Consistent accounting of intermediation costs requires that player k bear i 's costs if i provides essential connections. We say that trader k (*directly*) *relies* on intermediary $i \in N_k$ if the removal of the link (k, i) from the network changes k 's limit resale value (which implies that k and i trade with positive probability in subgame k in any MPE for sufficiently high δ). For $i = \overline{1,m}$, the *elimination of middleman* i from G is the procedure that generates a network $\tilde{G} = (\tilde{N}, (\tilde{N}_i), (\tilde{c}_i), (v_j))$, which excludes node i ($\tilde{N} = N \setminus \{i\}$), such that traders k who rely on i in G inherit i 's links and costs ($\tilde{c}_k = c_k + c_i$, $\tilde{N}_k = N_k \cup N_i \setminus \{i\}$) and traders k who do not simply lose existing links to i ($\tilde{N}_k = N_k \setminus \{i\}$); all other elements of \tilde{G} are specified as in G .²²

²²In the benchmark model, a seller's cost is independent of his trading partner. Given this assumption, if seller k has to cover the cost of an eliminated middleman i , the cost increase is artificially reflected in k 's trades with all his downstream neighbors. It is then reasonable to require that k incur i 's cost only if k relies on i . A more straightforward conceptualization of eliminating middlemen is available in the extension of the model with link specific costs (see Section 9.1 for details). We can also prove a version of the result for the

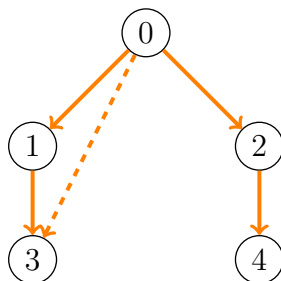


FIGURE 11. $v_3 \in (pv_4, v_4)$. The addition of the link $(0, 3)$ decreases total welfare.

Proposition 7. *Both the addition of a new link and the elimination of a middleman weakly increase the initial seller's limit profit.*

While beneficial for the initial seller, the addition of a link and the elimination of a middleman may be detrimental to efficiency.²³ For an example in which a new link has negative welfare consequences, consider the network formed by the solid links from Figure 11, where all costs are 0 and buyer values satisfy $v_3 \in (pv_4, v_4)$. In this network, the high value buyer 4 acquires the good in equilibrium with limit probability 1. If we add the dashed link from the seller to buyer 3, then the seller switches to trading with buyer 3, whose value v_3 is greater than intermediary 2's resale value pv_4 . Thus, the addition of the link $(0, 3)$ decreases total welfare from v_4 to v_3 . As we discuss in the next section, the elimination of middleman 3 in the network from Figure 14 (which is equivalent to the vertical integration of intermediaries 1 and 3 in that example) also affects welfare negatively.

8.2. Horizontal and Vertical Integration. Here we examine the effects of horizontal and vertical integration on the division of gains from trade and intermediation efficiency. For simplicity, we define horizontal and vertical integration in our model assuming that all intermediation costs are zero.²⁴ Fix a network G with this property. The *horizontal integration* (or *merger*) of a pair of intermediaries $i < j$ not linked in G generates a network that excludes node j , in which node i inherits all of j 's in- and out-links from G (the definition applies only when links are directed from lower to higher index nodes in the resulting network). The *vertical integration* of a pair of intermediaries $i < j$ linked in G leads to a network that excludes node j and all of its in-links, where node i inherits all of j 's out-links from G . For either definition, we ignore duplication in node i 's (expanded) collection of links and refer to player i as the *consolidated* or *merged* intermediary in the new network. We use the term *consolidated intermediary profits* when we compare the limit

case in which a middleman is bypassed by a single upstream neighbor, but is not removed from the network and retains his other in- and out-links.

²³Gofman (2011) develops a general analysis of welfare effects of network density in his model.

²⁴Horizontal and vertical integration have been studied in various settings with multiple goods, demand externalities, and market shocks but small numbers of players and limited intermediation [4, 18, 20, 21].

(if it exists) of the sum of equilibrium payoffs of i and j in the network prior to the merger with the limit equilibrium payoff of i in the post-merger network. *Buyer* and *total welfare* are measured by the limit of the sum of equilibrium payoffs of all buyers and all players in the network, respectively. *Seller profit* stands for the initial seller's limit equilibrium payoff r_0 .

Horizontal integration has opposite effects on downstream and upstream competition. On the one hand, the horizontal integration of intermediaries i and j may lead to more downstream competition and a higher resale value for the consolidated player i , as his pool of trading partners expands. On the other hand, horizontal integration leaves upstream neighbors with fewer (re)selling options and eliminates the competition between i and j , which may reduce upstream resale values. The enhanced bargaining position of intermediary i may provide incentives to redirect trade towards i , while the deteriorating position of i 's and j 's upstream neighbors may have the opposite effect. Thus, horizontal integration may change the path of trade, the distribution of profits in the network, and the final allocation of the good. We show that these changes have ambiguous effects on seller and merged intermediary profits as well as buyer and total welfare.

We identify similar countervailing forces that underlie vertical integration. In a vertical merger along the link (i, j) , the upstream neighbors of j lose their links to j , which may lower their resale values. At the same time, if i does not have directed trading paths to any such neighbor, then the resale values of i 's downstream neighbors different from j remain unchanged, and the addition of links from i to j 's downstream neighbors can only increase i 's resale value. The enhanced competitiveness of intermediary i and diminished competitiveness of j 's former upstream neighbors may either increase the resale values of i 's upstream neighbors and provide incentives to reroute trade towards i or decrease their resale values and move trade away from i . This affects the distribution of profits and the ultimate allocation of the good in the network. We find that vertical mergers have indeterminate effects on the four equilibrium variables of interest.

Proposition 8. *Horizontal integration has ambiguous effects on seller and consolidated intermediary profits and buyer and total welfare. The same statement is true for vertical integration.*

To demonstrate these claims, we first examine the effects of the horizontal integration of intermediaries 1 and 2 in the network depicted on the left-hand side of Figure 12. The network resulting from the merger is illustrated on the right-hand side of the figure, a convention we follow for subsequent examples. If $pv_6 < v_4 = v_5 < v_6$, then Theorem 1 implies that the initial seller's limit profit increases from pv_4 prior to the merger to pv_6 afterwards. The merger enables the consolidated intermediary 1 to exploit the competition between buyers 4 and 5. This boosts intermediary 1's resale value from pv_4 to v_4 , and player 0 captures

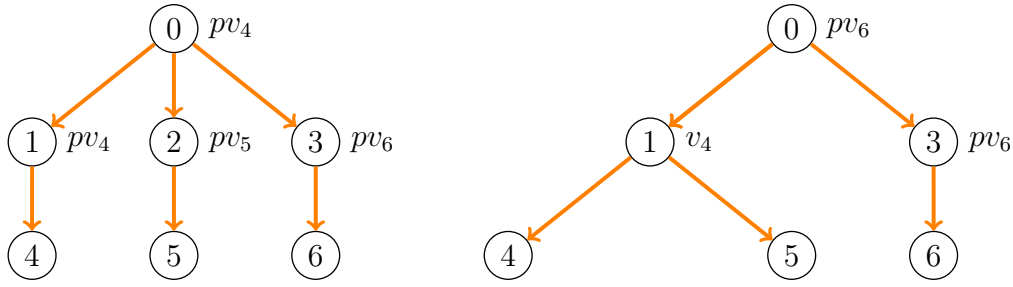


FIGURE 12. $pv_6 < v_4 = v_5 < v_6$. The horizontal integration of intermediaries 1 and 2 increases seller profit from pv_4 to pv_6 and consolidated intermediary profits from 0 to $v_4 - pv_6$, while it decreases buyer welfare from $(1 - p)v_6$ to 0 and total welfare from v_6 to v_4 .

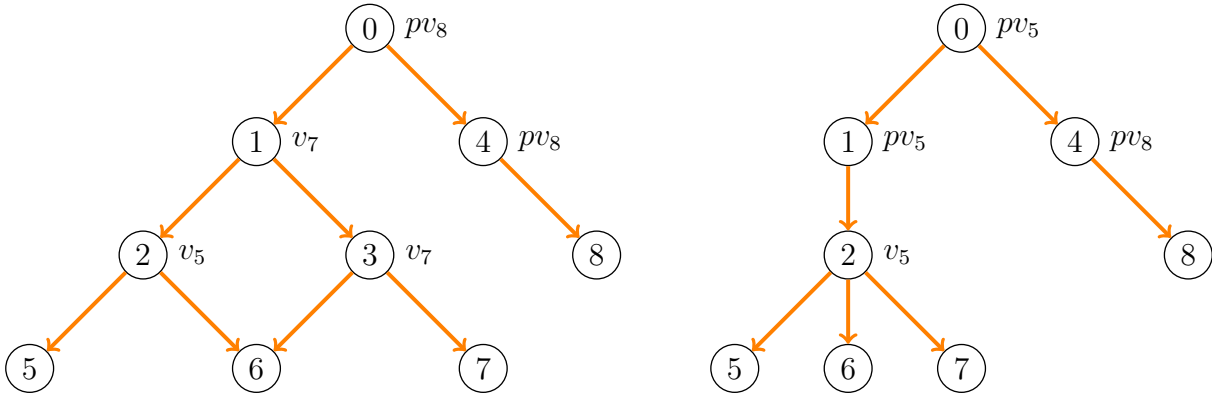


FIGURE 13. $pv_8 < v_7 < v_5 = v_6 < v_8$. The horizontal integration of intermediaries 2 and 3 decreases seller profit from pv_8 to pv_5 and consolidated intermediary profits from $v_5 - v_7$ to 0, while it increases buyer welfare from 0 to $(1 - p)v_8$ and total welfare from v_5 to v_8 .

part of the gain. The extra profit is transferred to player 0 indirectly: Proposition 3 implies that player 0 trades with intermediary 3 in the original network almost surely as $\delta \rightarrow 1$, but switches to intermediary 1 after the merger. Joint profits for the integrated intermediaries grow from 0 to $v_4 - pv_6$. In response to the merger, the allocation of the good changes from buyer 6, who enjoys a limit equilibrium payoff of $(1 - p)v_6$ in the original network, to buyers 4 and 5, who compete away all buyer surplus in the new network. Moreover, total welfare decreases from v_6 to v_4 .

In the next example, horizontal integration has exactly opposite effects on the four equilibrium variables. Consider the horizontal integration of intermediaries 2 and 3 in the network from Figure 13. If $pv_8 < v_7 < v_5 = v_6 < v_8$, then sellers 0, 1, 2, and 3 take advantage of downstream competition to extract the corresponding second-price auction profits (conditional on

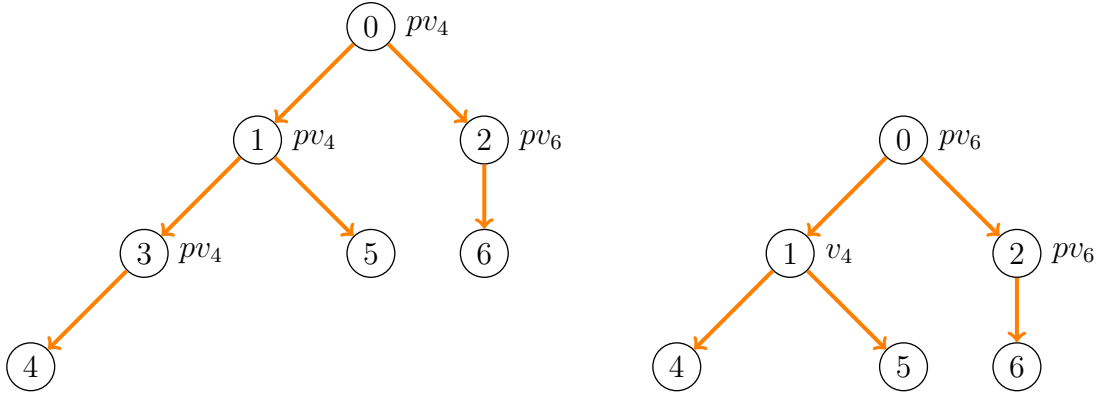


FIGURE 14. $pv_6 < v_4 = v_5 < v_6$. The vertical integration of intermediaries 1 and 3 increases seller profit from pv_4 to pv_6 and consolidated intermediary profits from 0 to $v_4 - pv_6$, while decreasing buyer welfare from $(1 - p)v_6$ to 0 and total welfare from v_6 to v_4 .

having the good), while intermediary 4 is held up by buyer 8 in a bilateral monopoly. Using Theorem 1, we derive the resale values indicated next to each node in the figure. By Proposition 3, the seller trades with intermediary 1, who then resells the good to intermediary 2, who in turn resells it to either buyer 5 or 6 (with limit probability 1). In the original network, player 1 attains second-price auction profits by exploiting the competition between intermediaries 2 and 3, but after the pair merges, player 1 has to settle for the bilateral monopoly profits with the consolidated intermediary; consequently, his resale value declines from v_7 to pv_5 . The seller then prefers to trade with intermediary 4 in the new network and suffers a drop in profits from pv_8 to pv_5 ; combined payoffs for intermediaries 2 and 3 thus fall from $v_5 - v_7$ to 0. The allocation of the good changes accordingly from buyers 5 and 6 to buyer 8. In the original network, competition between buyers 5 and 6 drives their payoffs down to 0, while in the new network buyer 8 is able to extract his bilateral monopoly share $(1 - p)v_8$ when bargaining with intermediary 4. Hence, the merger enhances buyer welfare. Total welfare increases as well, from v_5 to v_8 , reflecting the efficient post-merger allocation of the good.

Consider next the consequences of the vertical integration of intermediaries 1 and 3 in the network of Figure 14. If $pv_6 < v_4 = v_5 < v_6$, then applying Theorem 1 for the prior and post merger networks leads to the resale values shown next to the corresponding nodes. Proposition 3 implies that the seller trades with intermediary 2 with limit probability 1 in the equilibrium for the original network and realizes a second-price auction profit matching intermediary 1's resale value of pv_4 . Intermediary 2 then resells the good to buyer 6 at the bilateral monopoly price of pv_6 . Following the merger, intermediary 1 exploits the direct competition between buyers 4 and 5, which increases his resale value to v_4 . The seller then

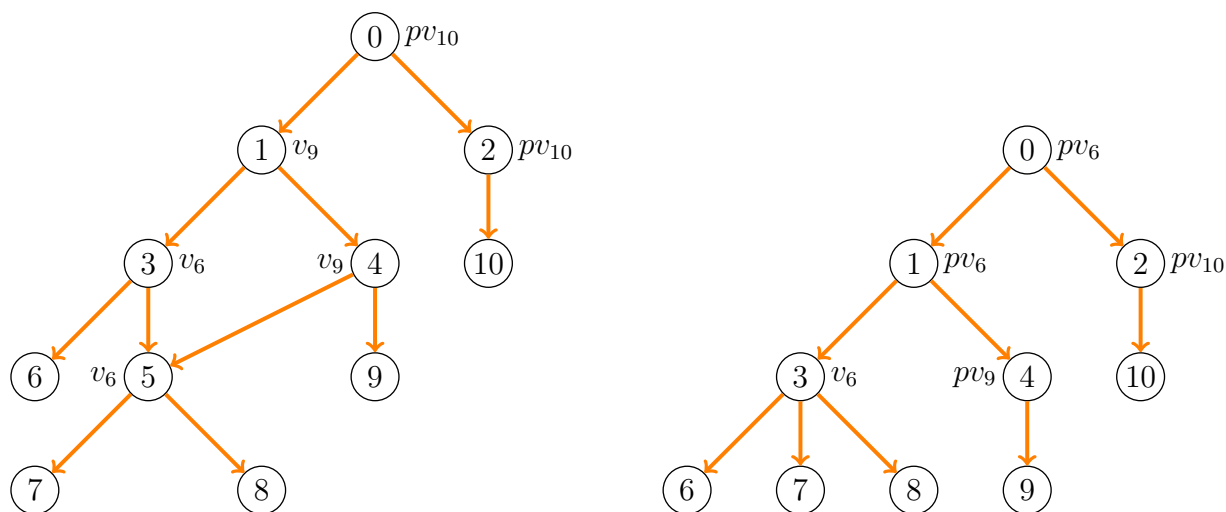


FIGURE 15. $pv_{10} < v_9 < v_6 = v_7 = v_8 < v_{10}$. The vertical integration of intermediaries 3 and 5 decreases seller profit from pv_{10} to pv_6 and consolidated intermediary profits from $v_6 - v_9$ to 0, while increasing buyer welfare from 0 to $(1 - p)v_{10}$ and total welfare from v_6 to v_{10} .

prefers to trade with intermediary 1 rather than 2 and sees profits surge from pv_6 to pv_4 . Combined profits of intermediaries 1 and 3 grow from 0 to $v_4 - pv_6$. The ensuing reallocation of the good from buyer 6 to either buyer 4 or 5 is inefficient—total welfare falls from v_6 to v_4 . Buyer welfare drops from $(1 - p)v_6$ to 0 because in the original network buyer 6 holds up intermediary 2, while in the new network buyers 4 and 5 compete to acquire the good from the consolidated intermediary.

We now construct an intermediation network in which following a vertical merger, the four equilibrium variables move in different directions from the previous example. Consider the vertical integration of intermediaries 3 and 5 in the network from Figure 15. If buyer values satisfy $pv_{10} < v_9 < v_6 = v_7 = v_8 < v_{10}$, then Theorem 1 implies the resale values indicated in the figure. By Proposition 3, in the original network, the seller trades with intermediary 1, who then resells the good to intermediary 3 at a price of v_9 reflecting the outside option provided by intermediary 4. The good is eventually acquired by one of the buyers 6, 7, and 8 at a price of v_6 derived from the second-price auction. Intermediary 3 realizes a profit of $v_6 - v_9$, while 5 obtains no profit. Since intermediary 3 captures all gains from trade available in subgame 3 in the original network, his vertical merger with 5 does not affect his resale value. However, the loss of the link with the integrated player 5 reduces intermediary 4's resale value from v_9 to pv_9 post merger. In consequence, intermediary 1's resale value declines from v_9 in the original network to pv_6 in the new network. The decreased competitiveness of intermediary 1 induces the seller to shift trade from intermediary 1 to 2. After acquiring

the good, intermediary 2 resells it to buyer 10 at price pv_{10} . The seller's profit drops from pv_{10} to pv_6 , while combined profits of intermediaries 3 and 5 fall from $v_6 - v_9$ to 0. Buyer welfare increases from 0 to $(1 - p)v_{10}$, and total welfare increases from v_6 to v_{10} .

The conclusion of the example from Figure 14 admits the following generalization for arborescences (defined in footnote 12). The proof of this result is relegated to the Online Appendix.

Proposition 9. *Neither seller profits nor consolidated intermediary profits can fall after the vertical integration of two nodes in an arborescence in which all intermediation costs are 0.*

8.3. Comparative Statics for Costs. In the previous sections we discussed what network architectures are more profitable for the seller. Here we fix the network structure and provide comparative statics with respect to the distribution of costs in the network. We use the following partial order to compare cost profiles: $\tilde{c} = (\tilde{c}_i)_{i=0,\overline{m}}$ *upstream dominates* $c = (c_i)_{i=0,\overline{m}}$ if for every directed path connecting the seller to an intermediary, $0 = i_0, i_1, \dots, i_{\bar{s}} \leq m$ ($i_s \in N_{i_{s-1}}, s = \overline{1, \bar{s}}$), including the degenerate case with $\bar{s} = 0$, we have

$$\sum_{s=0}^{\bar{s}} \tilde{c}_{i_s} \geq \sum_{s=0}^{\bar{s}} c_{i_s}.$$

Intuitively, \tilde{c} upstream dominates c if costs are relatively more concentrated at the “top” under \tilde{c} . The next result shows that the downward redistribution of costs in the network entailed by the shift from \tilde{c} to c benefits the initial seller. The idea is that if the bilateral monopoly outcome emerges in a transaction between seller i and downstream neighbor k , then k “shares” a fraction $1 - p$ of i 's cost (Proposition 2). Hence, higher downstream costs reduce intermediation rents in downstream exchanges, leaving more gains from trade for upstream players.

Theorem 3. *Let c and \tilde{c} be two distinct cost profiles in otherwise identical networks. If \tilde{c} upstream dominates c , then the initial seller's limit profit is at least as high under c as under \tilde{c} .*

Theorem 3 has ramifications for optimal cost distribution in the applications discussed in the introduction. In the case of illegal trade and bribery, intermediation costs include the risk of getting caught engaging in illicit activity and the potential penalties. Then crime can be discouraged by ratcheting up the severity of punishments at the top of intermediation chains. If monitoring expenses need to be budgeted across the network, the optimal targets for investigations and audits are antiquities dealers operating near archaeological sites, areas suspected of harboring cocaine refineries,²⁵ and companies with large stakes in overseas

²⁵Given the large number of substitutable antiquities looters and coca farmers, law enforcement at the “root” may be ineffective.

procurement contracts. In the case of manufacturing, costs may capture both production expenditures and taxes. In order to implement efficient production, Theorem 3 suggests that costs have to be pushed downstream to the extent possible. Thus retail sales taxes are more efficient than value-added taxes. Similarly, subsidies are more effective at earlier stages of production than at later ones. The result also sheds light on the optimal allocation of manufacturing processes in supply chains where each step of production is performed by a tier of specialized firms. If every step involves essential processes that require specialization as well as certain generic tasks (bells and whistles) that can be performed by downstream firms, then the generic features should be added as far along as possible in production. Section 9.2 explores additional implications of Theorem 3.

9. DISCUSSION AND EXTENSIONS

This section discusses extensions and alternative interpretations of the model. It also comments on the relationship between our results and double marginalization and provides concluding remarks.

9.1. Extensions of the Model. The benchmark model assumes that one unit of time elapses between the moment an intermediary purchases the good and his first opportunity to resell it. The results carry over to an alternative model in which intermediaries may resell the good as soon as they acquire it, and delays occur only following rejections.

The model can also be extended to allow for link specific costs and consumption for intermediaries. In the general version, seller i incurs a cost c_i^k upon trading with a neighbor $k \in N_i$ and may choose to consume the good for a utility $v_i \geq 0$. The recursive formula for resale values in this setting becomes

$$r_i = \max(p(r_k - c_i^k)_{k \in N_i}^I, (r_k - c_i^k)_{k \in N_i}^{II}, v_i) \text{ for } i = m, m-1, \dots, 0,$$

where $(r_k - c_i^k)_{k \in N_i}^I$ and $(r_k - c_i^k)_{k \in N_i}^{II}$ denote the first and the second highest order statistics of the vector $(r_k - c_i^k)_{k \in N_i}$, respectively. To extend the proofs of Theorem 1 and the supporting Lemma 2, we need to assume that direct connections are not more costly than indirect ones. Formally, for any link (i, k) and all directed paths $i = i_0, i_1, \dots, i_{\bar{s}} = k$ ($i_s \in N_{i_{s-1}}$ for all $s = \overline{1, \bar{s}}$) connecting i to k , it must be that $c_i^k \leq \sum_{s=0}^{\bar{s}-1} c_{i_s}^{i_{s+1}}$. Note that the condition above is satisfied whenever c_i^k depends exclusively on i —as in our benchmark model—or on k —as in the road network version of the model we discuss in Section 9.2 below. Moreover, the condition is vacuously satisfied in the class of networks described in footnote 12.

In the setting with link specific costs, we can prove a version of Proposition 7 that sidesteps the endogenous notion of relying on intermediaries in the definition of eliminating a middleman. The alternative definition of eliminating middleman i entails the removal of node i

from the network, accompanied by rewiring the network such that all of i 's upstream neighbors inherit his downstream links (if $i \in N_k$ then the new downstream neighborhood of k is $\tilde{N}_k = N_k \cup N_i \setminus \{i\}$) and (re)defining the costs of links (k, h) with $i \in N_k$ and $h \in N_i$ to be $\tilde{c}_k^h = \min(c_k^h, c_k^i + c_i^h)$ (with the convention that $c_k^h = \infty$ if $h \notin N_k$). Furthermore, the notion of cost dominance and the comparative statics for cost redistributions (Theorem 3) admit straightforward generalizations to the version of the model with link specific costs.

9.2. Alternative Interpretation of the Model: Navigation in Networks. Our framework can alternatively serve as a model of navigation in networks. This interpretation is related to Olken and Barron's (2009) study of bribes paid by truck drivers at checkpoints along two important transportation routes in Indonesia. Imagine a driver who has to transport some cargo across a network of roads, choosing a direction at every junction and negotiating the bribe with authorities at checkpoints along the way. In contrast to our benchmark model, the driver preserves ownership of his cargo and makes all the payments as he proceeds through the network. This interpretation suggests an alternative strategic model, in which player 0 "carries" the good through the network. Player i mans node i . There is a set of terminal nodes $j = \overline{m+1, n}$, and the good is worth v_j at node j . Player 0 navigates the network by acquiring access from players at checkpoints along the way sequentially. After clearing node i , player 0 can proceed to any node $k \in N_i$. Upon reaching a terminal node j , player 0 realizes the value v_j .²⁶

The characterization of MPEs in the intermediation game substantiates the road network interpretation above. Indeed, the system of equilibrium conditions (1)-(2) also captures the MPE constraints in the road network model, where u_i^i and u_k^i represent the expected payoffs of players 0 and $k \in N_i$, respectively, conditional on player 0 having just cleared checkpoint i . The cost of intermediary i has to be replaced by the cost c_k of checkpoint k . Both models are naturally embedded in the common framework with link specific costs (or the less general setting with no intermediation costs). The equivalence holds because in the benchmark model seller i effectively internalizes the downstream intermediation rents forfeited by player 0 as he advances beyond node i in the road network. Indeed, as the good clears node i , the profits of downstream intermediaries do not depend on whether player i is entrusted with the good or player 0 preserves ownership. Either player bargains with the next trader along the path anticipating the same intermediation rents and final values in downstream transactions.

Besides their literal interpretation, road networks open the door to other applications in which the buyer "builds" the "road" as he makes headway through the network. In order

²⁶Since the two routes in the application of Olken and Barron (2009) are disconnected and there are no viable alternate roads, their theoretical analysis is naturally restricted to line networks. Our general framework accommodates multiple routes that intersect at various hubs, so that the driver may circumvent some checkpoints.

to construct a highway, railway, or oil pipeline on private land, the government conducts sequential negotiations with landowners along the route. Similarly, the developer of a mall or residential community needs to acquire a series of contiguous properties. If the developer can bargain with only one landowner at a time, Theorem 3 implies that the developer should approach owners with smaller opportunity costs for their land earlier in the bargaining process.²⁷ This special case of the result was independently proved by Xiao (2015).

Manufacturing provides another application for navigation in networks. In the garment, electronics and car industries, the main manufacturers outsource components, processes, and services to contractors. A raw good or concept produced in-house is gradually assembled and converted into a finished product with contributions from several contractors. The producer maintains ownership of the intermediate good throughout the process. In this application, trading paths represent competing contractors, different ordering and splitting of production steps, or entirely distinct production technologies. Theorem 3 implies that if there is flexibility in the order of production steps, then the manufacturer prefers dealing with the costliest contractors in the last stages.

9.3. Relationship to double marginalization. Consider a line network ($N_i = \{i + 1\}$ for $i = \overline{0, n - 1}$) with a single buyer (player n) and no intermediation costs ($c_i = 0$ for $i = \overline{0, n - 1}$). The formula for limit resale values from Theorem 1 implies that $r_i = p^{n-i}v_n$ for $i = \overline{0, n - 1}$. This conclusion is reminiscent of *double marginalization* [31]. However, there does not seem to be a deep theoretical connection between intermediation in networks and multiple marginalization in a chain of monopolies. The two models capture distinct strategic situations. In the classic double marginalization paradigm, upstream monopolists set prices for downstream firms, which implicitly determine the amount of trade in each transaction. Downstream firms have no bargaining power. By contrast, in the line network application of our intermediation game, downstream neighbors derive bargaining power from the opportunity to make offers to upstream sellers, which arises with probability $1 - p$ at every date. Every link represents a bilateral monopoly. Moreover, since our environment presumes that a single unit of the indivisible good is available, there is no discretion over the quantity traded in each agreement. Our research focuses on understanding how competition among trading paths shapes the outcomes of intermediation. Since competition is absent in line networks, the relationship to double marginalization is peripheral to our main contribution.

9.4. Concluding Remarks. This paper investigates how competing paths of intermediation, which naturally call for a network formulation, determine the terms of trade between buyers, sellers, and middlemen. We discover a network decomposition into layers of intermediation power that provides insights into the equilibrium interplay between hold-ups,

²⁷Any ordering of costs in a line network upstream dominates the ordering in which costs are increasing.

competition, and efficiency. We demonstrate how layers determine resale values, intermediation profits, and the structure of trading paths. In a first attempt at building a dynamic model of resale in networks via non-cooperative bargaining, we make a number of simplifying assumptions, among which we enumerate: (1) the network structure, intermediation costs, and buyer values are exogenous²⁸ and commonly known;²⁹ (2) the intermediation network is directed and acyclic;³⁰ (3) traders are patient; (4) a single good is traded in the network. In future work, it would be useful to relax these assumptions.

APPENDIX A. PROOFS

Proof of Proposition 1. Consider an MPE of the bargaining game with no intermediation and discount factor δ . Let u_k denote the expected payoff of player $k \in N$ and π_j be the probability that the seller selects buyer $j = \overline{1, n}$ for bargaining (following a history along which the good has not yet been traded) in this equilibrium. Since $v_1 > c_0$, the general arguments from the proof of Theorem 1 establish that the seller reaches an agreement with conditional probability 1 with any buyer j such that $\pi_j > 0$. Furthermore, for all j with $\pi_j > 0$, we obtain the following payoff equations,

$$\begin{aligned} u_0 &= p(v_j - c_0 - \delta u_j) + (1 - p)\delta u_0 \\ u_j &= \pi_j (p\delta u_j + (1 - p)(v_j - c_0 - \delta u_0)). \end{aligned}$$

Expressing variables in terms of u_0 (≥ 0), we get

$$(3) \quad u_j = \frac{v_j - c_0}{\delta} - \frac{1 - \delta + \delta p}{\delta p} u_0$$

$$(4) \quad \pi_j = \frac{1 - \delta + \delta p}{\delta p} - \frac{(1 - \delta)(1 - p)(v_j - c_0)}{\delta p(v_j - c_0 - u_0)}.$$

The (equilibrium) condition $u_j \geq 0$ implies that $u_0 \leq p(v_j - c_0)/(1 - \delta + \delta p)$ if $\pi_j > 0$. It follows immediately that $\pi_j = 0$ whenever $u_0 \geq p(v_j - c_0)/(1 - \delta + \delta p)$ (if $u_0 = p(v_j - c_0)/(1 - \delta + \delta p)$ then the right-hand side of (4) becomes 0).

Moreover, $u_0 < p(v_j - c_0)/(1 - \delta + \delta p)$ implies that $\pi_j > 0$. Indeed, if $\pi_j = 0$ then $u_j = 0$. The seller has the option to choose buyer j as his bargaining partner in the first

²⁸The characterization of equilibrium payoffs for any given network developed here can serve as a building block for the study of strategic network formation. In most applications—like in our model—the seller would prefer to form a direct link with the highest value buyer. However, as discussed in the introduction and in footnote 6, trade between sellers and buyers may be impossible without the participation of intermediaries. In order to study incentives for link formation, one has to model explicitly the roles of intermediaries (e.g., providing access, information, logistics, technology, capital, or inputs) and the costs involved in replicating the underlying intermediation services. The specification of linking costs, trade restrictions, and substitutability of intermediation steps needs to be tailored to the desired application.

²⁹More broadly, research has made little progress in the game theoretic analysis of incomplete information about network structure. The static model of Galeotti et al. (2010) is one notable exception.

³⁰Condorelli et al. (2015) approach the issue of incomplete information regarding traders' valuations in an undirected intermediation network.

period, which leads to the incentive constraint $u_0 \geq p(v_j - c_0) + (1 - p)\delta u_0$, or equivalently $u_0 \geq p(v_j - c_0)/(1 - \delta + \delta p)$.

We established that selection probabilities in any MPE are described by the following functions of u_0

$$(5) \quad \tilde{\pi}_j(u_0) = \begin{cases} \frac{1-\delta+\delta p}{\delta p} - \frac{(1-\delta)(1-p)(v_j-c_0)}{\delta p(v_j-c_0-u_0)} & \text{if } u_0 < (v_j - c_0) \frac{p}{1-\delta+\delta p} \\ 0 & \text{if } u_0 \geq (v_j - c_0) \frac{p}{1-\delta+\delta p} \end{cases}.$$

One can easily check that each function $\tilde{\pi}_j$ is strictly decreasing over the interval $[0, p(v_j - c_0)/(1 - \delta + \delta p)]$ and continuous over $[0, p(v_1 - c_0)/(1 - \delta + \delta p)]$. Hence the expression $\sum_{j=1}^n \tilde{\pi}_j(u_0)$ is strictly decreasing and varies continuously for $u_0 \in [0, p(v_1 - c_0)/(1 - \delta + \delta p)]$. Note that

$$\sum_{j=1}^n \tilde{\pi}_j(0) = \frac{|\{j|v_j > c_0\}|}{\delta} > 1 \text{ and } \sum_{j=1}^n \tilde{\pi}_j\left(\frac{p(v_1 - c_0)}{1 - \delta + \delta p}\right) = 0.$$

Therefore, there exists a unique u_0 for which $\sum_{j=1}^n \tilde{\pi}_j(u_0) = 1$. This value pins down all other variables describing the equilibrium. Hence all MPEs are outcome equivalent. However, we cannot pin down equilibrium actions in subgames in which the seller selects a buyer j such that $\pi_j = 0$ and $v_j - c_0 \leq \delta u_0$. Behavior in these off the equilibrium path subgames does not affect expected payoffs.

The limit MPE payoffs as $\delta \rightarrow 1$ for this special instance of the intermediation game are derived in the proof of Theorem 1. We next establish properties of MPEs for high δ . For every $\delta \in (0, 1)$, formula (5) implies that $\pi_1 \geq \pi_2 \geq \dots \geq \pi_n$.

Define $\Delta = \{\delta \in (0, 1) | \pi_1 = 1\}$. Assume first that $\sup \Delta = 1$. For $\delta \in \Delta$, the payoffs satisfy

$$\begin{aligned} u_0 &= p(v_1 - c_0 - \delta u_1) + (1 - p)\delta u_0 \\ u_1 &= p\delta u_1 + (1 - p)(v_1 - c_0 - \delta u_0) \\ u_j &= 0, j = \overline{2, n}. \end{aligned}$$

We immediately obtain that $u_0 = p(v_1 - c_0)$ and $u_1 = (1 - p)(v_1 - c_0)$. The seller's incentives then imply that $v_1 - c_0 - \delta(1 - p)(v_1 - c_0) \geq v_2 - c_0$ for all $\delta \in \Delta$. Since $\sup \Delta = 1$, it must be that $p(v_1 - c_0) \geq v_2 - c_0$.

Conversely, if $p(v_1 - c_0) \geq v_2 - c_0$ then $v_1 - c_0 - \delta(1 - p)(v_1 - c_0) \geq v_2 - c_0$ for all $\delta \in (0, 1)$, and one can construct an MPE in which $\pi_1 = 1$ for every $\delta \in (0, 1)$. By the first part of the proof, this constitutes the unique MPE for each δ . The arguments above establish that $\sup \Delta = 1$ if and only if $\pi_1 = 1$ for every $\delta \in (0, 1)$ if and only if $p(v_1 - c_0) \geq v_2 - c_0$.

Suppose now that $\sup \Delta < 1$, which is equivalent to $v_2 - c_0 > p(v_1 - c_0)$. Consider a buyer j with $v_j < v_2$. Since $\lim_{\delta \rightarrow 1} u_2 = 0$, we have $v_2 - c_0 - \delta u_2 > v_j - c_0 - \delta u_j$ for sufficiently high δ . Hence the seller prefers trading with buyer 2 rather than j , so $\pi_j = 0$ for high δ .

Therefore, if $v_1 = v_2$ then the seller bargains only with the highest value buyers when players are sufficiently patient. Formula (5) implies that all such buyers are selected as partners with equal probability.

If $v_1 > v_2$ then trade with any buyer other than player 1 generates a surplus of at most $v_2 - c_0 < v_1 - c_0$. Since $\lim_{\delta \rightarrow 1} u_0 + u_1 = v_1 - c_0$, it must be that $\lim_{\delta \rightarrow 1} \pi_1 = 1$. Since $\sup \Delta < 1$, we have $\pi_2 > 0$ for sufficiently high δ . By formula (5), $\pi_j = \pi_2$ whenever $v_j = v_2$. For high δ , we argued that $\pi_j = 0$ if $v_j < v_2$. This completes the characterization of MPE outcomes for all parameter ranges. \square

We next present two lemmata that will be used in the proof of Theorem 1.

Lemma 1. *Suppose that conditional on being the current owner, seller i selects player $k \in N_i$ for bargaining with positive probability in the MPE. Then in every subgame in which i has just chosen k as a bargaining partner, i expects a payoff of u_i^i . Moreover, resale values satisfy $u_i^i \leq \max(\delta u_k^k - c_i, 0) \leq u_k^k$.*

Proof. The first part follows from standard equilibrium mixing conditions. Suppose that conditional on being the current owner, seller i selects player $k \in N_i$ for bargaining with positive probability in the MPE. Then player i expects a payoff of u_i^i from bargaining with k . However, in the MPE player k never accepts a price above δu_k^k or offers a price above $\delta u_i^i + c_i$ to i . Moreover, player i 's continuation payoff in case of disagreement is δu_i^i . Hence $u_i^i \leq \max(\delta u_k^k - c_i, \delta u_i^i)$. Since $\delta \in (0, 1)$, it follows that $u_i^i \leq \max(\delta u_k^k - c_i, 0) \leq u_k^k$. \square

Lemma 2. *For any seller i and any subset of players $M \not\ni i$,*

$$u_i^i + \sum_{k \in M} u_k^i \leq \max(u_i^i, \delta \max_{k \in M} u_k^k - c_i).$$

For intuition, note that the left-hand side of the inequality above represents the total expected profits accruing to players from $M \cup \{i\}$ in subgame i . Consider a history of subgame i in which the good is traded within $M \cup \{i\}$ for a number of periods, until it reaches some intermediary $k \in M$, who resells it to a player outside M . Assume that the good does not return to M thereafter. The net contribution of transfers between pairs of players in $M \cup \{i\}$ to the sum of expected payoffs of $M \cup \{i\}$ in this scenario is zero. The only other contributions of the history to the sum are the costs of sellers from $M \cup \{i\}$ along the trading path and the price received by k . Seller k expects a price of $u_k^k + c_k$ and incurs the cost c_k at the time of the sale, which takes place at least one period into subgame i . Thus the conditional expected contribution of the history to the sum, discounted at the beginning of subgame i , does not exceed $\delta u_k^k - c_i$. The general argument is more involved, as it has to account for intermediation chains that exit and re-enter $M \cup \{i\}$ several times. The proof exploits the monotonicity of expected prices along equilibrium trading paths, which is a corollary of Lemma 1.

Proof. We prove the following claim by backward induction on i , for $i = n, n-1, \dots, 0$. For all $M \subseteq N \setminus \{i\}$,³¹

$$u_i^i + \sum_{k \in M} u_k^i \leq \max(u_i^i, \delta \max_{k \in M} u_k^k - c_i).$$

For buyers $i = \overline{m+1, n}$, the claim clearly holds (for arbitrary specifications of c_i) as $u_k^i = 0$ for all $k \neq i$. Assuming the claim is true for $n, \dots, i+1$, we prove it for i . Fix $M \subseteq N \setminus \{i\}$. Let $M' = \{h \in N \setminus \{i\} | u_h^h \leq \max_{k \in M} u_k^k\}$. We set out to show that

$$u_i^i + \sum_{k \in M'} u_k^i \leq \max(u_i^i, \delta \max_{k \in M} u_k^k - c_i).$$

The sum $u_i^i + \sum_{k \in M'} u_k^i$ represents the total payoffs of the players in $M' \cup \{i\}$ in subgame i . It may be evaluated as an expectation over contributions from several events unfolding in the first round of the subgame:

- (1) player i trades with some player $h \in M'$;
- (2) player i trades with some player $h \notin M'$;
- (3) player i does not reach an agreement with his selected bargaining partner.

In the first event, players i and h exchange the good for a payment. Besides the change in ownership, the net contribution of this transaction to the total payoffs of i and h is the cost c_i . The conditional continuation payoff of each player $k \in M'$ is given by δu_k^h . The sum of continuation payoffs of the players in $M' \cup \{i\}$ conditional on the transaction between i and h is thus $\delta \sum_{k \in M'} u_k^h - c_i$. By the induction hypothesis applied for player h ($h \in N_i \Rightarrow h > i$) and the set $M' \setminus \{h\}$, the expression above does not exceed $\delta \max(u_h^h, \delta \max_{k \in M' \setminus \{h\}} u_k^k - c_h) - c_i \leq \delta \max_{k \in M'} u_k^k - c_i = \delta \max_{k \in M} u_k^k - c_i$.

In the second event, player i trades with some $h \notin M'$. By definition, $u_h^h > \max_{k \in M'} u_k^k$. Since Lemma 1 implies that resale values are non-decreasing along every path on which trade takes place with positive probability, it must be that once h acquires the good, no player in M' ever purchases the good at a later stage. Thus conditional on i selling the good to h , the expected payoffs of all players in M' are 0. By Lemma 1, player i 's conditional expected payoff (when event (2) has positive probability) is u_i^i . Hence event (2) contributes to the sum of payoffs of the players in $M' \cup \{i\}$ with the amount u_i^i .

The third event contributes to the expectation with a term $\delta(u_i^i + \sum_{k \in M'} u_k^i)$.

Since $u_i^i + \sum_{k \in M'} u_k^i$ is evaluated as an expectation of contributions from events of one of the three types analyzed above, it must be that

$$u_i^i + \sum_{k \in M'} u_k^i \leq \max \left(u_i^i, \delta \max_{k \in M} u_k^k - c_i, \delta \left(u_i^i + \sum_{k \in M'} u_k^i \right) \right).$$

³¹By convention, let $c_i = 0$ for every buyer $i = \overline{m+1, n}$.

Then $u_i^i + \sum_{k \in M'} u_k^i \geq 0$, $u_i^i \geq 0$ and $\delta \in (0, 1)$ imply that

$$u_i^i + \sum_{k \in M'} u_k^i \leq \max(u_i^i, \delta \max_{k \in M} u_k^k - c_i).$$

As $M \subseteq M'$ and $u_k^i \geq 0$ for all $k \in M'$, the latter inequality leads to

$$u_i^i + \sum_{k \in M} u_k^i \leq \max(u_i^i, \delta \max_{k \in M} u_k^k - c_i),$$

which concludes the proof of the inductive step. \square

Proof of Theorem 1. We prove by backward induction on i , for $i = n, n-1, \dots, 0$, that u_i^i converges as δ goes to 1 to a limit r_i , which satisfies $r_i = v_i$ for $i = \overline{m+1, n}$ and $r_i = \max(p(r_{N_i}^I - c_i), r_{N_i}^H - c_i, 0)$ for $i = \overline{0, m}$. The base cases $i = n, \dots, m+1$ (corresponding to buyers) are trivially verified. Assuming that the induction hypothesis holds for players $n, \dots, i+1$, we seek to prove it for seller i ($\leq m$).

Let $\Delta = \{\delta \in (0, 1) \mid \delta \max_{k \in N_i} u_k^k \leq c_i\}$. If $\sup \Delta = 1$, then the induction hypothesis implies that $\max_{k \in N_i} \delta u_k^k$ converges to $r_{N_i}^I \leq c_i$ as $\delta \rightarrow 1$. Hence the maximum gains that i can create by trading with any of his neighbors vanish as $\delta \rightarrow 1$. It follows that u_i^i converges to 0 as $\delta \rightarrow 1$, so $\lim_{\delta \rightarrow 1} u_i^i = 0 = \max(p(r_{N_i}^I - c_i), r_{N_i}^H - c_i, 0)$.

For the rest of the proof, assume that $\sup \Delta < 1$ and restrict attention to $\delta > \sup \Delta$. Then $r_{N_i}^I \geq c_i$ and $\delta \max_{k \in N_i} u_k^k > c_i$. Fix $k^1 \in \arg \max_{k \in N_i} u_k^k$. By Lemma 1,

$$(6) \quad u_i^i \leq \max(\delta u_{k^1}^{k^1} - c_i, 0) = \delta u_{k^1}^{k^1} - c_i.$$

Then Lemma 2 implies that

$$(7) \quad u_i^i + \sum_{k \in N_i} u_k^i \leq \delta u_{k^1}^{k^1} - c_i.$$

If $\delta u_{k^1}^{k^1} - c_i \leq \delta u_{k^1}^i$ then the inequality above leads to

$$u_i^i + \sum_{k \in N_i} u_k^i \leq \delta u_{k^1}^{k^1} - c_i \leq \delta u_{k^1}^i,$$

which is possible only if $u_k^i = 0$ for all $k \in N_i \cup \{i\}$. However, if $u_{k^1}^i = 0$ then player i can select k^1 for bargaining and offer him an acceptable price arbitrarily close to $\delta u_{k^1}^{k^1}$. This deviation leads to a profit approaching $\delta u_{k^1}^{k^1} - c_i > 0$ in the event that i is chosen as the proposer, contradicting $u_i^i = 0$.

Hence $\delta u_{k^1}^{k^1} - c_i > \delta u_{k^1}^i$. Since player k^1 accepts any price z with $\delta u_{k^1}^{k^1} - z > \delta u_{k^1}^i$, player i can secure a positive profit by bargaining with k^1 and, when selected to make an offer to k^1 , proposing a price arbitrarily close to $\delta u_{k^1}^{k^1} - \delta u_{k^1}^i > c_i$. Thus $u_i^i > 0$.

Assume that the current seller i selects $k \in N_i$ for bargaining with positive probability. By Lemma 1, player i expects a payoff of u_i^i conditional on choosing k . Note that player k would never offer i a price higher than $\delta u_k^k + c_i$, and i 's continuation payoff in case of

disagreement is δu_i^i . Since $u_i^i > 0$, it must be that k is willing to accept a price offer z from i with $z - c_i > u_i^i$. Then $\delta u_k^k - z \geq \delta u_k^i$, which leads to $\delta u_k^k - \delta u_k^i \geq z > u_i^i + c_i$. It follows that $\delta(u_i^i + u_k^i) < \delta u_k^k - c_i$. Standard arguments then imply that conditional on i selecting k as a bargaining partner, the two players trade with probability 1. When selected as the proposer, either player offers a price that makes the opponent indifferent between accepting and rejecting the offer. The equilibrium prices offered by i and k are $\delta u_k^k - \delta u_k^i$ and $\delta u_i^i + c_i$, respectively. Let π_k denote the probability that seller i selects player $k \in N_i$ for bargaining in subgame i . The arguments above lead to the equilibrium conditions (1) and (2) displayed following the statement of Theorem 1.

For all $h \in N_i$ with $\pi_h > 0$, seller i 's incentive constraints (1) lead to $\delta u_h^h - c_i - \delta u_h^i \geq \delta u_{k^1}^{k^1} - c_i - \delta u_{k^1}^i$, or equivalently,

$$(8) \quad u_{k^1}^{k^1} - u_h^h \leq u_{k^1}^i - u_h^i.$$

By Lemma 2, we have $u_h^h + u_{k^1}^h \leq \max(u_h^h, \delta u_{k^1}^{k^1} - c_h)$, which along with $\delta \in (0, 1)$ and $c_h \geq 0$, implies that $u_{k^1}^h \leq \max(0, u_{k^1}^{k^1} - u_h^h) = u_{k^1}^{k^1} - u_h^h$. Then (8) leads to

$$(9) \quad u_{k^1}^h \leq u_{k^1}^i - u_h^i \leq u_{k^1}^i.$$

Combining (9) with (2) for $k = k^1$, we obtain

$$u_{k^1}^i \leq \pi_{k^1} \left(p \delta u_{k^1}^i + (1-p)(\delta u_{k^1}^{k^1} - c_i - \delta u_i^i) \right) + \sum_{h \in N_i \setminus \{k^1\}} \pi_h \delta u_{k^1}^i.$$

Since $u_{k^1}^i \geq 0$ and $\delta u_{k^1}^{k^1} - c_i - \delta u_i^i \geq 0$ (6), it follows that

$$u_{k^1}^i \leq \frac{\pi_{k^1}}{1 - \delta \sum_{h \in N_i \setminus \{k^1\}} \pi_h} \left(p \delta u_{k^1}^i + (1-p)(\delta u_{k^1}^{k^1} - c_i - \delta u_i^i) \right) \leq p \delta u_{k^1}^i + (1-p)(\delta u_{k^1}^{k^1} - c_i - \delta u_i^i),$$

which leads to

$$(10) \quad u_{k^1}^i \leq \frac{(1-p)(\delta u_{k^1}^{k^1} - c_i - \delta u_i^i)}{1 - p\delta}.$$

By the incentive constraint (1) for $k = k^1$,

$$(11) \quad u_i^i \geq \frac{p(\delta u_{k^1}^{k^1} - c_i - \delta u_{k^1}^i)}{1 - (1-p)\delta}.$$

Substituting the bound on $u_{k^1}^i$ from (10) into (11) and collecting the u_i^i terms, we find that $u_i^i \geq p(\delta u_{k^1}^{k^1} - c_i)$. By the induction hypothesis, $u_{k^1}^{k^1}$ converges to $r_{N_i}^I$ as $\delta \rightarrow 1$. Therefore, $\liminf_{\delta \rightarrow 1} u_i^i \geq p(r_{N_i}^I - c_i)$.

For $k = k^1$, (1) can be rewritten as $u_i^i(1 - (1-p)\delta) + p\delta u_{k^1}^i \geq p(\delta u_{k^1}^{k^1} - c_i)$, which implies that (recall that k^1 is a function of δ)

$$(12) \quad \liminf_{\delta \rightarrow 1} (u_i^i + u_{k^1}^i) \geq \lim_{\delta \rightarrow 1} u_{k^1}^{k^1} - c_i = r_{N_i}^I - c_i.$$

Since (7) holds for every $\delta \in \Delta$, we obtain

$$(13) \quad \limsup_{\delta \rightarrow 1} \left(u_i^i + \sum_{k \in N_i} u_k^i \right) \leq r_{N_i}^I - c_i.$$

Consider a player $k^2 \in \arg \max_{k \in N_i \setminus \{k^1\}} u_k^k$ (k^2 is also a function of δ). The last two displayed equations imply that $\lim_{\delta \rightarrow 1} u_{k^2}^i = 0$. The incentive constraint (1) for $k = k^2$ leads to $u_i^i(1 - (1-p)\delta) + p\delta u_{k^2}^i \geq p(\delta u_{k^2}^k - c_i)$. Since $\lim_{\delta \rightarrow 1} u_{k^2}^i = 0$ and, by the induction hypothesis, $\lim_{\delta \rightarrow 1} u_{k^2}^k = r_{N_i}^I$, we obtain $\liminf_{\delta \rightarrow 1} u_i^i \geq r_{N_i}^I - c_i$.

Thus far, we established that

$$(14) \quad \liminf_{\delta \rightarrow 1} u_i^i \geq \max(p(r_{N_i}^I - c_i), r_{N_i}^I - c_i).$$

We next prove by contradiction that

$$(15) \quad \limsup_{\delta \rightarrow 1} u_i^i \leq \max(p(r_{N_i}^I - c_i), r_{N_i}^I - c_i).$$

Suppose that (15) does not hold. Then there exists a sequence \mathcal{S} of δ approaching 1,³² along which u_i^i converges to a limit greater than $\max(p(r_{N_i}^I - c_i), r_{N_i}^I - c_i)$. \mathcal{S} can be chosen to satisfy in addition one of the following properties:

- (i) there exists $k \in N_i$ such that for all $\delta \in \mathcal{S}$, current seller i selects k for bargaining with conditional probability 1 in the MPE;
- (ii) there exist $k \neq h \in N_i$ such that for all $\delta \in \mathcal{S}$, $u_k^k \geq u_h^h$ and current seller i bargains with positive conditional probability with both k and h .

In case (i), the analysis above establishes that for all $\delta \in \mathcal{S}$,

$$\begin{aligned} u_i^i &= p(\delta u_k^k - c_i - \delta u_k^i) + (1-p)\delta u_i^i \\ u_k^k &= p\delta u_k^i + (1-p)(\delta u_k^k - c_i - \delta u_k^i). \end{aligned}$$

The system of equations is immediately solved to obtain $u_i^i = p(\delta u_k^k - c_i)$. Then the induction hypothesis implies that $\lim_{\delta \in \mathcal{S}} u_i^i = p(r_k - c_i) \leq p(r_{N_i}^I - c_i)$.

In case (ii), it must be that $u_i^i = p(\delta u_h^h - c_i - \delta u_h^i) + (1-p)\delta u_i^i$, which implies that

$$(16) \quad u_i^i(1 - (1-p)\delta) \leq p(\delta u_h^h - c_i).$$

Note that $u_k^k \geq u_h^h$, along with the induction hypothesis, leads to $r_k = \lim_{\delta \rightarrow 1} u_k^k \geq \lim_{\delta \rightarrow 1} u_h^h = r_h$. In particular, $r_h \leq r_{N_i}^I$. Taking the limit for δ along \mathcal{S} in (16), we immediately find that $\lim_{\delta \in \mathcal{S}} u_i^i \leq r_{N_i}^I - c_i$.

In either case, we obtained a contradiction with the assumption that $\lim_{\delta \in \mathcal{S}} u_i^i > \max(p(r_{N_i}^I - c_i), r_{N_i}^I - c_i)$. Therefore, (15) holds, implying along with (14) that u_i^i converges as $\delta \rightarrow 1$ to

³²To simplify notation, we write $\delta \in \mathcal{S}$ to represent the fact that δ appears in the sequence \mathcal{S} and use the shorthand $\lim_{\delta \rightarrow 1} f(\delta)$ for the limit of a function $f : (0, 1) \rightarrow \mathbb{R}$ along the sequence \mathcal{S} .

$r_i = \max(p(r_{N_i}^I - c_i), r_{N_i}^H - c_i) = \max(p(r_{N_i}^I - c_i), r_{N_i}^H - c_i, 0)$ (recall that $r_{N_i}^I \geq c_i$ if $\sup \Delta < 1$). This completes the proof of the inductive step. \square

Proof of Proposition 2. Suppose that $k \in N_i$ and $r_k = r_{N_i}^I \geq c_i$. The result is obvious for $r_k = c_i$. If $r_k > c_i$, then the analysis supporting (12) and (13) in the proof of Theorem 1 implies that

$$\limsup_{\delta \rightarrow 1} (u_i^i + u_k^i) \leq \limsup_{\delta \rightarrow 1} \left(u_i^i + \sum_{h \in N_i} u_h^i \right) \leq r_k - c_i \leq \liminf_{\delta \rightarrow 1} (u_i^i + u_k^i).$$

Since $\lim_{\delta \rightarrow 1} u_i^i = r_i$, it must be that u_k^i converges to $r_k - c_i - r_i = \min((1-p)(r_k - c_i), r_k - r_{N_i}^H)$ and u_h^i converges to 0 for $h \in N_i \setminus \{k\}$ as $\delta \rightarrow 1$. \square

Proof of Proposition 3. Suppose that $k \in N_i$ and $r_k = r_{N_i}^I > \max(r_{N_i}^H, c_i)$. Then, by Proposition 2,

$$(17) \quad \lim_{\delta \rightarrow 1} u_k^i = r_k - c_i - r_i = \min((1-p)(r_k - c_i), r_k - r_{N_i}^H) > 0.$$

Consider a family of MPEs for a sequence of δ 's converging to 1 in which trade takes place in subgame i along the path $i_0 = i, i_1, \dots, i_{\bar{s}} = k$ (so k eventually receives the good) with positive probability. The proof of Theorem 1 establishes that trading prices on this path converge to $(r_{i_s} + c_{i_s})_{s=0, \overline{s-1}}$ as $\delta \rightarrow 1$. By Lemma 1, equilibrium payoffs satisfy $u_{i_s}^{i_s} \leq u_{i_{s+1}}^{i_{s+1}} - c_{i_s}$ for δ along the sequence, which in the limit $\delta \rightarrow 1$ implies that $r_{i_{s+1}} \geq r_{i_s} + c_{i_s}$. Stringing together these inequalities, we find that the limit price $r_{i_{\bar{s}-1}} + c_{i_{\bar{s}-1}}$ at which k acquires the good from $i_{\bar{s}-1}$ is greater than or equal to $r_i + c_i + \sum_{s=1}^{\bar{s}-1} c_{i_s}$. Since k obtains a limit payoff of r_k in subgame k , player k 's limit profit when trade takes place along the path $(i_s)_{s=0, \overline{s}}$ does not exceed $r_k - c_i - r_i - \sum_{s=1}^{\bar{s}-1} c_{i_s}$.

If for some family of MPEs, the probability with which k acquires the good in subgame i (directly from i or via a chain of intermediaries) does not converge to 1 as $\delta \rightarrow 1$, then there exist $z < 1$ and a sequence of δ 's converging to 1 such that for δ in the sequence: (1) player k receives the good with probability less than z in subgame i , and (2) the probability of every sequence of agreements in subgame i converges. Since player k 's limit profit in subgame i for any trading path that delivers the good to him in equilibrium cannot exceed $r_k - c_i - r_i$, it must be that

$$\lim_{\delta \rightarrow 1} u_k^i \leq z(r_k - c_i - r_i) < r_k - c_i - r_i,$$

where the last inequality relies on $r_k - c_i - r_i > 0$ and $z < 1$. This contradicts (17).

If $c_h > 0$ for all $h \in N_i \setminus \{k\}$ and the probability that i resells the good directly to k in subgame i does not converge to 1 as $\delta \rightarrow 1$ in some family of MPEs, we reach a contradiction similarly. In that case, there exist $z > 0$ and a sequence of δ 's converging to 1 for which: (1) trade takes place in subgame i with probability greater than z along a (fixed) path $i_0 = i, i_1 = h, \dots, i_{\bar{s}} = k$ with $h \in N_i \setminus \{k\}$; and (2) the probability of each sequence of

agreements in subgame i converges. Player k 's limit profit for δ along the sequence when trade takes place on the path (i_s) does not exceed $r_k - c_i - r_i - \sum_{s=1}^{\bar{s}-1} c_{i_s} \leq r_k - c_i - r_i - c_h$. Since k 's limit profit is at most $r_k - c_i - r_i$ for any other trading path in subgame i , we obtain the contradiction that $\lim_{\delta \rightarrow 1} u_k^i \leq (1-z)(r_k - c_i - r_i) + z(r_k - c_i - r_i - c_h) < r_k - c_i - r_i$.

To prove the claims about lateral intermediation rents, consider a family of MPEs for $\delta \in (0, 1)$. The analysis leading to (9) in the proof of Theorem 1 implies that for sufficiently high δ , equilibrium payoffs satisfy $u_k^h \leq u_k^i$ for all $h \in N_i \setminus \{k\}$ with $\pi_h > 0$. Fix $h \in N_i \setminus \{k\}$ such that $\lim_{\delta \rightarrow 1} \pi_h > 0$. The payoff equation (2) implies that for high δ ,

$$u_k^i \leq \pi_k (p\delta u_k^i + (1-p)(\delta u_k^k - c_i - \delta u_i^i)) + \pi_h \delta u_k^h + (1 - \pi_k - \pi_h) \delta u_k^i.$$

Reorganizing this inequality and taking limits, we obtain $\lim_{\delta \rightarrow 1} \pi_h \liminf_{\delta \rightarrow 1} (u_k^h - u_k^i) \geq 0$. Then $\lim_{\delta \rightarrow 1} \pi_h > 0$ leads to $\liminf_{\delta \rightarrow 1} (u_k^h - u_k^i) \geq 0$. Since $u_k^h - u_k^i \leq 0$ for high δ , this is possible only if $u_k^h - u_k^i$ converges to 0 as $\delta \rightarrow 1$. Therefore, $\lim_{\delta \rightarrow 1} u_k^h = \lim_{\delta \rightarrow 1} u_k^i$.

Since $r_k = r_{N_i}^I > r_{N_i}^H$, there exists $\underline{\delta} < 1$ such that the MPE payoffs for $\delta > \underline{\delta}$ satisfy $u_k^k > u_h^h$ for every $h \in N_i \setminus \{k\}$. Then Lemma 1 implies that for $\delta > \underline{\delta}$, no player $h \in N_i \setminus \{k\}$ obtains the good in subgame k , so $u_h^k = 0$.

The proof for the second part of the result uses similar (but simpler) arguments and is omitted. \square

Proof of Proposition 4. We recursively construct a profile $(u_{k'}^k)_{k, k' \in N}$ that describes the payoffs in each subgame in an MPE of the intermediation game. For $k = \overline{m+1, n}$ we simply set $u_k^k = v_k/\delta$ and $u_{k'}^k = 0$ for all $k' \neq k$ as discussed in Section 4. For $i \leq m$, having defined the variables $u_{k'}^k$ for all $k > i$ and $k' \in N$, we derive the payoffs $(u_{k'}^i)_{k' \in N}$ as follows.

If $\delta \max_{k \in N_i} u_k^k \leq c_i$, then let $u_k^i = 0$ for all k . MPEs for subgame i in which the current seller i never trades the good generating the desired payoffs off the equilibrium path are easily constructed.

Assume next that $\delta \max_{k \in N_i} u_k^k > c_i$. The proof of Theorem 1 shows that the payoffs $(u_k^i)_{k \in N_i \cup \{i\}}$ and the selection probabilities $(\pi_k)_{k \in N_i}$ describing MPE outcomes in the first round of subgame i solve the system of equations

$$(18) \quad u_i^i = \sum_{k \in N_i} \pi_k (p(\delta u_k^k - c_i - \delta u_i^i) + (1-p)\delta u_i^i)$$

$$(19) \quad u_k^i = \pi_k (p\delta u_k^i + (1-p)(\delta u_k^k - c_i - \delta u_i^i)) + \sum_{h \in N_i \setminus \{k\}} \pi_h \delta u_k^h, \forall k \in N_i,$$

where the variables $(u_k^h)_{k, h \in N_i}$ have been previously specified.

Consider an arbitrary vector $\pi = (\pi_k)_{k \in N_i}$ describing a probability distribution over N_i . Let f^π denote the function that takes any $u^i := (u_k^i)_{k \in N_i \cup \{i\}}$ (slight abuse of notation) to $\mathbb{R}^{N_i \cup \{i\}}$ with component $k \in N_i \cup \{i\}$ defined by the right-hand side of the equation for u_k^i in the system (18)-(19). Then u^i solves the system for the given π if and only if it is a fixed

point of f^π . One can easily check that f^π is a contraction of modulus δ with respect to the sup norm on $\mathbb{R}^{N_i \cup \{i\}}$. By the contraction mapping theorem, f^π has a unique fixed point. This means that the system of linear equations (18)-(19) with unknowns u^i is non-singular and can be solved using Cramer's rule. The components of the unique solution, which we denote by $\tilde{u}(\pi)$, are given by ratios of determinants that vary continuously in π .

For any $u^i \in \mathbb{R}^{N_i \cup \{i\}}$, let $\tilde{\pi}(u^i)$ denote the set of probability mass functions over N_i that are consistent with optimization by seller i , given the variables u^i and the previously determined resale values $(u_k^k)_{k \in N_i}$. That is, $\tilde{\pi}(u^i)$ contains all π such that $\pi_k > 0$ only if $k \in \arg \max_{h \in N_i} u_h^h - u_h^i$. Clearly, $\tilde{\pi}$ has a closed graph and is convex valued.

By Kakutani's theorem, the correspondence $\pi \mapsto \tilde{\pi}(\tilde{u}(\pi))$ has a fixed point π^* . Then $(u_k^i)_{k \in N_i \cup \{i\}} = \tilde{u}(\pi^*)$ and $(\pi_k)_{k \in N_i} = \pi^*$ satisfy the equilibrium constraints (1)-(2), as well as the system of equations (18)-(19). Also define $u_k^i = \sum_{h \in N_i} \pi_h u_k^h$ for $k \notin N_i \cup \{i\}$. One can easily specify strategies that constitute an MPE for subgame i and yield the desired payoffs provided that $u_k^i \geq 0$ for all $k \in N_i \cup \{i\}$. The rest of the proof demonstrates that the constructed values are indeed non-negative.

We prove that $u_k^i > 0$ by contradiction. Suppose that $u_k^i \leq 0$. Fix $k^1 \in \arg \max_{k \in N_i} u_k^k$. Since $\pi \in \tilde{\pi}(u^i)$, it must be that $u_h^h - u_h^i \geq u_{k^1}^{k^1} - u_{k^1}^i$ whenever $\pi_h > 0$. By construction, $(u_k^h)_{k \in N}$ constitute MPE payoffs for subgame h . Retracing the steps that establish (9) in Theorem 1, we find that $u_{k^1}^h \leq u_{k^1}^i$ if $\pi_h > 0$. Then (19) implies that

$$(20) \quad u_{k^1}^i \leq \pi_{k^1} \left(p \delta u_{k^1}^i + (1-p)(\delta u_{k^1}^{k^1} - c_i - \delta u_{k^1}^i) \right) + \sum_{h \in N_i \setminus \{k^1\}} \pi_h \delta u_{k^1}^i.$$

Setting $k = k^1$ in (1), we obtain

$$(21) \quad u_{k^1}^i \geq p(\delta u_{k^1}^{k^1} - c_i - \delta u_{k^1}^i) + (1-p)\delta u_{k^1}^i.$$

Under the assumptions $u_{k^1}^i \leq 0$ and $\delta u_{k^1}^{k^1} > c_i$, we have

$$p \delta u_{k^1}^i + (1-p)(\delta u_{k^1}^{k^1} - c_i - \delta u_{k^1}^i) = \delta u_{k^1}^{k^1} - c_i - \left(p(\delta u_{k^1}^{k^1} - c_i - \delta u_{k^1}^i) + (1-p)\delta u_{k^1}^i \right) > 0.$$

Then (20) implies that

$$(22) \quad u_{k^1}^i \leq \frac{\pi_{k^1}}{1 - \delta \sum_{h \in N_i \setminus \{k^1\}} \pi_h} \left(p \delta u_{k^1}^i + (1-p)(\delta u_{k^1}^{k^1} - c_i - \delta u_{k^1}^i) \right) \leq p \delta u_{k^1}^i + (1-p)(\delta u_{k^1}^{k^1} - c_i - \delta u_{k^1}^i).$$

As in the proof of Theorem 1, inequalities (21) and (22) imply that $u_{k^1}^i \geq p(\delta u_{k^1}^{k^1} - c_i) > 0$, a contradiction.

We showed that $u_k^i > 0$. If $\pi_k > 0$, then $u_k^i = p(\delta u_k^k - c_i - \delta u_k^i) + (1-p)\delta u_k^i$, which implies that $\delta u_k^k - c_i - \delta u_k^i > \delta u_k^i$. It follows that $\delta u_k^k - c_i - \delta u_k^i > \delta u_k^i$ whenever $\pi_k > 0$. Then (19) leads to $u_k^i \geq \pi_k \delta u_k^i + \sum_{h \in N_i \setminus \{k\}} \pi_h \delta u_k^h$ for all $k \in N_i$. Therefore, $u_k^i \geq \delta \sum_{h \in N_i \setminus \{k\}} \pi_h u_k^h / (1 - \delta \pi_k) \geq 0$ for all $k \in N_i$. \square

Proof of Theorem 2. We prove that

$$(23) \quad i \in \mathcal{L}_\ell \implies r_i \in \left[p^\ell \underline{v} - \sum_{\ell'=0}^{\ell} p^{\ell-\ell'} \sum_{k \in \mathcal{L}_{\ell'}, i \leq k \leq m} c_k, p^\ell \bar{v} \right]$$

by backward induction on i , for $i = n, n-1, \dots, 0$. The base cases $i = n, \dots, m+1$ (corresponding to buyers) are trivially verified. Assuming that the induction hypothesis holds for players $n, \dots, i+1$, we seek to prove it for seller i ($\leq m$). Suppose that $i \in \mathcal{L}_\ell$.

We first show that $r_i \leq p^\ell \bar{v}$. By Theorem 1, $r_i = \max(p(r_h - c_i), r_{h'} - c_i, 0)$ for some $h, h' \in N_i$ with $r_h = r_{N_i}^I \geq r_{N_i}^{II} = r_{h'}$.³³ Since $i \in \mathcal{L}_\ell$, the layer structure entails that $h, h' \in \bigcup_{\ell' \geq \ell-1} \mathcal{L}_{\ell'}$. Then the induction hypothesis for player h implies that $r_h \leq p^{\ell-1} \bar{v}$.

Similarly, if $h' \notin \mathcal{L}_{\ell-1}$, then the induction hypothesis leads to $r_{h'} \leq p^\ell \bar{v}$. Assume instead that $h' \in \mathcal{L}_{\ell-1}$. If $h \in \mathcal{L}_{\ell-1}$ as well, then the construction of layer $\ell-1$ implies that $i \in \mathcal{L}_{\ell-1}$, a contradiction. Hence $h \in \bigcup_{\ell' \geq \ell} \mathcal{L}_{\ell'}$. Then the induction hypothesis for player h leads to $r_h \leq r_h \leq p^\ell \bar{v}$. In either case, $r_{h'} \leq p^\ell \bar{v}$. Since $r_h \leq p^{\ell-1} \bar{v}$, $r_{h'} \leq p^\ell \bar{v}$ and $c_i, \bar{v} \geq 0$, we obtain that $r_i = \max(p(r_h - c_i), r_{h'} - c_i, 0) \leq p^\ell \bar{v}$.

We next show that $r_i \geq p^\ell \underline{v} - \sum_{\ell'=0}^{\ell} p^{\ell-\ell'} \sum_{k \in \mathcal{L}_{\ell'}, i \leq k \leq m} c_k$. Since $i \in \mathcal{L}_\ell$, it must be that i has out-links to either (exactly) one layer $\ell-1$ player or (at least) two layer ℓ players. In the first case, let $\{h\} = N_i \cap \mathcal{L}_{\ell-1}$. By Theorem 1, $r_i \geq p(r_h - c_i)$. The induction hypothesis implies that $r_h \geq p^{\ell-1} \underline{v} - \sum_{\ell'=0}^{\ell-1} p^{\ell-\ell'-1} \sum_{k \in \mathcal{L}_{\ell'}, h \leq k \leq m} c_k$. Therefore,

$$r_i \geq p^\ell \underline{v} - p c_i - \sum_{\ell'=0}^{\ell-1} p^{\ell-\ell'} \sum_{k \in \mathcal{L}_{\ell'}, h \leq k \leq m} c_k \geq p^\ell \underline{v} - \sum_{\ell'=0}^{\ell} p^{\ell-\ell'} \sum_{k \in \mathcal{L}_{\ell'}, i \leq k \leq m} c_k.$$

In the second case, N_i contains at least two players from layer ℓ . Hence there exists $h \in N_i \cap \mathcal{L}_\ell$ with $r_h \leq r_{N_i}^{II}$. By Theorem 1, $r_i \geq r_h - c_i$. The induction hypothesis implies that $r_h \geq p^\ell \underline{v} - \sum_{\ell'=0}^{\ell} p^{\ell-\ell'} \sum_{k \in \mathcal{L}_{\ell'}, h \leq k \leq m} c_k$. It follows that

$$r_i \geq p^\ell \underline{v} - c_i - \sum_{\ell'=0}^{\ell} p^{\ell-\ell'} \sum_{k \in \mathcal{L}_{\ell'}, h \leq k \leq m} c_k \geq p^\ell \underline{v} - \sum_{\ell'=0}^{\ell} p^{\ell-\ell'} \sum_{k \in \mathcal{L}_{\ell'}, i \leq k \leq m} c_k.$$

This completes the proof of the inductive step. The last claim of the theorem is an immediate consequence of (23). \square

Proof of Proposition 5. Note that the first part of the theorem follows immediately from the second. Indeed, every player from layer ℓ not directly connected to layer $\ell-1$ has two downstream neighbors in layer ℓ . If the second part of the result is true, the two neighbors provide non-overlapping paths of layer ℓ intermediaries to layer $\ell-1$.

To establish the second part, fix $\ell \geq 1$ and assume that $\mathcal{L}_\ell = \{i_1, i_2, \dots, i_{\bar{s}}\}$ with $i_1 < i_2 < \dots < i_{\bar{s}}$. Two directed paths in G (possibly degenerate, consisting of a single node) are

³³The argument only becomes simpler if $|N_i|=1$ and no such h' exists.

called *independent* if they connect disjoint sets of layer ℓ players and end with nodes that are directly linked to layer $\ell - 1$. We say that players i_s and $i_{s'}$ *have independent paths* if there exist independent paths that originate from nodes i_s and $i_{s'}$, respectively.

We prove by backward induction on s , for $s = \bar{s}, \bar{s} - 1, \dots, 1$, that i_s and $i_{s'}$ have independent paths for all $s' > s$. For the induction base case $s = \bar{s}$, the claim is vacuously true.

Assuming that the induction hypothesis is true for all higher indices, we set out to prove it for s . Fix $s' > s$. If i_s is directly linked to a layer $\ell - 1$ node, then the degenerate path consisting of node i_s alone forms an independent pair with any layer ℓ path connecting node $i_{s'}$ to a player linked directly to layer $\ell - 1$. Player i_s does not belong to any such path because $i_s < i_{s'}$.

By the construction of \mathcal{L}_ℓ , if i_s is not directly linked to $\mathcal{L}_{\ell-1}$, then i_s must have at least two neighbors in \mathcal{L}_ℓ . Hence i_s is linked to some player $i_{s''} > i_s$ different from $i_{s'}$. The induction hypothesis applied for step $\min(s', s'') > s$ implies the existence of two independent paths originating from $i_{s'}$ and $i_{s''}$, respectively. If we append the link $(i_s, i_{s''})$ to the path originating from $i_{s''}$, we obtain a new path, starting with i_s , which forms an independent pair with the path from $i_{s'}$. The constructed paths are indeed disjoint. Player i_s does not belong to the path of $i_{s'}$ because $i_s < i_{s'} \leq i$ for all nodes i along the latter path. This completes the proof of the inductive step. \square

Proof of Proposition 6. Suppose that current seller $i \in \mathcal{L}_{\ell'}$ sells the good with positive conditional probability to trader $k \in \mathcal{L}_\ell$ for a sequence \mathcal{S} of δ 's converging to 1. By Theorem 2, the limit resale values of players i and k are $p^{\ell'}v$ and $p^\ell v$, respectively. Lemma 1 implies that resale values satisfy $u_i^i \leq u_k^k$ for $\delta \in \mathcal{S}$. Taking the limit $\delta \rightarrow 1$ in \mathcal{S} , we obtain $p^{\ell'}v = r_i \leq r_k = p^\ell v$, which means that $\ell' \geq \ell$. As $k \in N_i \cap \mathcal{L}_\ell$, it must be that $\ell' \leq \ell + 1$ and thus $\ell' \in \{\ell, \ell + 1\}$. This shows that for sufficiently high δ , player $k \in \mathcal{L}_\ell$ acquires the good in equilibrium only from sellers in layer ℓ or $\ell + 1$. If $\ell' = \ell$, then Theorem 2 and the construction of layers lead to $|N_i \cap \mathcal{L}_\ell| \geq 2$ and $r_k = r_{N_i}^I = r_{N_i}^{II} = p^\ell v$, which along with Proposition 2 imply that player k 's limit profit in subgame i is 0. If instead $\ell' = \ell + 1$, then k is i 's only downstream neighbor in $\mathcal{L}_\ell \cup \mathcal{L}_{\ell-1} \cup \dots \cup \mathcal{L}_0$, so $r_k = r_{N_i}^I = p^\ell v$ and $r_{N_i}^{II} \leq p^{\ell+1}v$, and Proposition 2 implies that player k 's subgame i limit profit is $(1 - p)p^\ell v$. \square

Proof of Proposition 7. By Theorem 1, the addition of a link (i, k) to a network weakly increases the (limit) resale value of player i and does not affect the resale value of any player $h > i$. Then a simple inductive argument invoking Theorem 1 proves that the resale values of all players $h \leq i$, including the initial seller, weakly increase when the link (i, k) is added.

We next prove that the elimination of a middleman weakly increases the initial seller's limit profit. Let $\tilde{G} = (\tilde{N}, (\tilde{N}_i), (\tilde{c}_i), (v_j))$ be the network obtained by eliminating middleman

i from network $G = (N, (N_i)_{i=\overline{0,m}}, (c_i)_{i=\overline{0,m}}, (v_j)_{j=\overline{m+1,n}})$. Let $(r_k)_{k \in N}$ and $(\tilde{r}_k)_{k \in N \setminus \{i\}}$ denote the vectors of resale values in G and \tilde{G} , respectively.

Fix k such that $i \in N_k$. To elucidate the implications of k relying on i in G , note that the removal of the link (k, i) from G does not affect the resale value of any players with labels greater than k ; in particular, it does not change the resale values of players in N_k . Then, by Theorem 1, player k relies on i if and only if

$$(24) \quad (r_k =) \max(p(r_{N_k}^I - c_k), r_{N_k}^{II} - c_k, 0) > \max(p(r_{N_k \setminus \{i\}}^I - c_k), r_{N_k \setminus \{i\}}^{II} - c_k, 0),$$

and does not if and only if inequality is replaced by equality above.

We now show that if k relies on i then $r_k \leq r_i - c_k$. Indeed, if k relies on i then (24) implies that either $r_k = p(r_{N_k}^I - c_k) > p(r_{N_k \setminus \{i\}}^I - c_k)$ or $r_k = r_{N_k}^{II} - c_k > r_{N_k \setminus \{i\}}^{II} - c_k$. In the first case, we have $r_{N_k}^I > r_{N_k \setminus \{i\}}^I$, which means that $r_i = r_{N_k}^I$. Hence $r_k = p(r_i - c_k) \geq 0$, giving rise to $r_k \leq r_i - c_k$. In the second case, we obtain that $r_{N_k}^{II} > r_{N_k \setminus \{i\}}^{II}$, which is possible only if $r_i \geq r_{N_k}^{II}$. Then $r_k = r_{N_k}^{II} - c_k \leq r_i - c_k$. In both cases, we established that $r_k \leq r_i - c_k$.

We prove by reverse induction on k , for $k = n, n-1, \dots, i+1, i-1, \dots, 0$, that $r_k \leq \tilde{r}_k$. For the base cases $k = n, n-1, \dots, i+1$, it is obvious that $r_k = \tilde{r}_k$. Assuming that the induction hypothesis holds for all players different from i with labels greater than k , we aim to prove it for player $k < i$. Consider two cases:

- (i) $i \in N_k$ and k relies on i ;
- (ii) $i \notin N_k$ or $i \in N_k$ but k does not rely on i .

In case (i), since k relies on i , we have $r_k \leq r_i - c_k$. Then Theorem 1 leads to

$$\begin{aligned} r_k \leq r_i - c_k &= \max(p(r_{N_i}^I - c_i) - c_k, r_{N_i}^{II} - c_i - c_k, -c_k) \\ &\leq \max(p(r_{N_i}^I - c_k - c_i), r_{N_i}^{II} - c_k - c_i, 0) = \max(p(\tilde{r}_{N_i}^I - \tilde{c}_k), \tilde{r}_{N_i}^{II} - \tilde{c}_k, 0) \\ &\leq \max(p(\tilde{r}_{N_k}^I - \tilde{c}_k), \tilde{r}_{N_k}^{II} - \tilde{c}_k, 0) = \tilde{r}_k. \end{aligned}$$

The sequence of equalities and inequalities above uses the following conditions: $c_k \geq 0, r_h = \tilde{r}_h, \forall h \in N_i \subseteq \tilde{N}_k$ and $\tilde{c}_k = c_k + c_i$.

In case (ii), either $i \notin N_k$ or $i \in N_k$ and k does not rely on i implies that $r_k = \max(p(r_{N_k \setminus \{i\}}^I - c_k), r_{N_k \setminus \{i\}}^{II} - c_k, 0)$. By the induction hypothesis, $r_h \leq \tilde{r}_h$ for all $h \in N_k \setminus \{i\}$. Theorem 1 then leads to

$$r_k = \max(p(r_{N_k \setminus \{i\}}^I - c_k), r_{N_k \setminus \{i\}}^{II} - c_k, 0) \leq \max(p(\tilde{r}_{N_k \setminus \{i\}}^I - c_k), \tilde{r}_{N_k \setminus \{i\}}^{II} - c_k, 0) = \tilde{r}_k.$$

This completes the proof of the induction step. Step $k = 0$ yields the desired inequality, $r_0 \leq \tilde{r}_0$. \square

Proof of Theorem 3. It is useful to generalize the concept of cost domination as follows. For $\kappa \geq 0$, a cost pattern \tilde{c} κ -dominates another c at some node $i \in N$ if for every path of sellers

originating at i , $i = i_0, i_1, \dots, i_{\bar{s}} \leq m$ ($i_s \in N_{i_{s-1}}, s = \overline{1, \bar{s}}$), including the case $\bar{s} = 0$,

$$(25) \quad \kappa + \sum_{s=0}^{\bar{s}} \tilde{c}_{i_s} \geq \sum_{s=0}^{\bar{s}} c_{i_s}.$$

The condition above is vacuously satisfied if i is a buyer since there are no seller paths originating at i in that case.

We prove by backward induction on i , for $i = n, n-1, \dots, 0$, that for any c and \tilde{c} such that \tilde{c} κ -dominates c at node i for $\kappa \geq 0$, we have $r_i + \kappa \geq \tilde{r}_i$, where r_i and \tilde{r}_i denote player i 's limit resale values in the intermediation game under cost structures c and \tilde{c} , respectively. The result to prove is a special case of the claim above, for $i = 0$ and $\kappa = 0$.

For buyers i , we have $r_i = \tilde{r}_i = v_i$, which along with $\kappa \geq 0$ proves the base cases $i = n, \dots, m+1$. Assuming that the induction hypothesis holds for players $n, \dots, i+1$, we seek to prove it for seller i ($\leq m$). Suppose that \tilde{c} κ -dominates c at i , and let r and \tilde{r} denote the vectors of resale values under c and \tilde{c} , respectively.

Fix $k \in N_i$. Considering all seller paths starting with the link (i, k) in (25), we can check that \tilde{c} ($\kappa + \tilde{c}_i - c_i$)-dominates c at k . The domination inequality (25) for the degenerate path consisting of the single node i becomes $\kappa + \tilde{c}_i - c_i \geq 0$. The induction hypothesis applied to $k > i$ implies that $r_k + \kappa + \tilde{c}_i - c_i \geq \tilde{r}_k$, or equivalently, $r_k - c_i + \kappa \geq \tilde{r}_k - \tilde{c}_i$. Since the latter inequality holds for all $k \in N_i$, it must be that $r_{N_i}^I - c_i + \kappa \geq \tilde{r}_{N_i}^I - \tilde{c}_i$ and $r_{N_i}^{II} - c_i + \kappa \geq \tilde{r}_{N_i}^{II} - \tilde{c}_i$. Then Theorem 1 leads to

$$\begin{aligned} \tilde{r}_i &= \max(p(\tilde{r}_{N_i}^I - \tilde{c}_i), \tilde{r}_{N_i}^{II} - \tilde{c}_i, 0) \leq \max(p(r_{N_i}^I - c_i + \kappa), r_{N_i}^{II} - c_i + \kappa, 0) \\ &\leq \max(p(r_{N_i}^I - c_i), r_{N_i}^{II} - c_i, 0) + \kappa = r_i + \kappa, \end{aligned}$$

where the last inequality relies on $\kappa \geq 0$. This completes the proof of the inductive step. \square

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