

INTERMEDIATION IN NETWORKS

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ABSTRACT. We study intermediation in markets with an underlying network structure. A good is resold via successive bilateral bargaining between linked intermediaries until it reaches one of several buyers in the network. We characterize the resale values of all traders in the network. The seller's profit depends not only on the number of intermediaries involved in trade, but on the entire set of competing paths of access to buyers brokered by each intermediary. Local competition shapes the outcomes of intermediation. A decomposition of the network into layers of intermediation power describes the endogenous structure of local monopolies and trading paths. Local competition permits full profit extraction in transactions within the same layer, while hold-up problems impose intermediation rents on exchanges across layers. Layers delimit monopoly power from intermediation power. Only players who serve as gateways to lower layers earn significant profits. Resale values decline exponentially with each progressive layer. Trade does not maximize welfare or minimize intermediation. We provide comparative statics with respect to the network architecture and the distribution of intermediation costs. The elimination of a middleman and vertical integration increase the seller's profit, as does the transfer of costs downstream; horizontal integration has ambiguous effects.

1. INTRODUCTION

Intermediation plays an important role in many markets. In corrupt and bureaucratic institutions, bribes are shared through long chains of intermediaries in the hierarchical structure. Lobbyists gain access to powerful lawmakers by navigating the network of political connections and rewarding well-connected individuals for their influence and contacts. Illegal goods such as drugs and unregistered guns are also smuggled and dealt through networks of intermediaries. Financial institutions resell complex assets over the counter through networks of trusted intermediaries. Varying levels of intermediation are needed to bring agricultural goods from small farms to the tables of final consumers. Artwork and antiques are sold via the personal contacts of collectors and middlemen. Manufacturing in supply chains can also be regarded as a form of intermediation, whereby a number of firms sequentially transforms and resells a good until it becomes a finished product.

Date: December 22, 2014.

I thank Daron Acemoglu, Glenn Ellison, Drew Fudenberg, Ben Golub, Jerry Green, Matt Jackson, Fuhito Kojima, David Miller, Ben Olken, Michael Ostrovsky, and Jean Tirole for useful comments. I am also grateful to Gabriel Kreindler for careful proofreading.

In such markets there exist competing intermediation paths between sellers and buyers with a complex pattern of overlaps. The number of middlemen, the cost of intermediation, and the value of final consumers may vary across trading paths. Some market participants have access to more middlemen than others, who themselves enjoy a greater or smaller number of connections. Clearly, not all links are equally useful in generating intermediation rents. The bargaining power of each intermediary depends on both his distance to buyers and the nature of local competition among his available trading routes. The global network of connections among various intermediaries plays an essential role in determining the path of trade and the profits that buyers, sellers, and middlemen achieve.¹ Some fundamental questions arise: How does an intermediary's position in the network affect his intermediation rents? Which players earn substantial profits? What trading paths are likely to emerge? How can upstream players exploit downstream competition? Is intermediation efficient? Does trade proceed along the shortest path? How does the seller's profit respond to changes in the network architecture such as the elimination of a middleman and the vertical or horizontal integration of intermediaries?

Given the prevalence of networks in markets where trade requires the services of middlemen, it is important to develop non-cooperative models of intermediation in networks. Decentralized bilateral bargaining is at the heart of our opening examples. Without attempting to model closely any of those applications, this paper starts building the tools for a non-cooperative theory of dynamic bilateral bargaining among buyers, sellers, and intermediaries.² We study the following intermediation game. A seller is endowed with a single unit of an indivisible good, which can be resold via successive bilateral bargaining between linked intermediaries in a directed network until it reaches one of several buyers. Intermediaries have heterogeneous transaction costs and buyers have heterogeneous values. At every stage in the game, the current owner of the good selects a bargaining partner among his downstream neighbors in the network. The two traders negotiate the price of the good via a random proposer protocol. With probability p , the current seller makes an offer and the partner decides whether to acquire the good at the proposed price. Roles are reversed with probability $1 - p$. In either event, if the offer is rejected, the seller keeps possession of the good and gets a new chance to select a partner at the next stage. If the offer is accepted, then the seller incurs his cost and the two traders exchange the good at the agreed price. If the new owner is an intermediary, he is given the opportunity to resell the good to his downstream neighbors according to the same protocol. Buyers simply consume the good upon purchase. Players have a common discount factor. The main part of our analysis focuses

¹Chapter 11 of Easley and Kleinberg (2010) provides an extensive discussion on the importance of intermediation in networks.

²We defer the discussion of the related literature to Section 11. It is worth noting at this point that the only other existing models of intermediation in networks via bilateral bargaining, proposed by Gale and Kariv (2007) and Condorelli and Galeotti (2012), consider strategic interactions quite distinct from ours.

on limit Markov perfect equilibria (MPEs) of the intermediation game as players become patient.

To gain some intuition for the competitive forces induced by the chosen bargaining protocol, we start with the simple version of the model in which there are no intermediaries. In this special case, a seller bargains over the price of the good with a number of heterogeneous buyers. We prove that the bargaining game with no intermediaries has an essentially unique MPE, which achieves asymptotic efficiency as players become patient (Proposition 1). The limit MPE outcome is determined as though the seller can choose between two scenarios: (1) a bilateral monopoly settlement whereby the seller trades only with the highest value buyer and receives a share p of the proceeds; and (2) a second-price auction, in which the seller exploits the competition between top buyers and extracts the entire surplus generated by the second highest value buyer. In effect, the seller is able to take advantage of competition among buyers and demand more than the bilateral monopoly share from the first-best buyer only if the threat of trading with the second-best buyer is credible.

Consider next an MPE of the general intermediation game. We refer to a trader's equilibrium payoff conditional on having possession of the good as his resale value. The strategic situation faced by a current seller in the intermediation game reduces to a bargaining game with his downstream neighbors, who can be viewed as surrogate buyers with valuations for the good endogenously derived from their resale values in the MPE. This situation resembles the game with no intermediation, with one critical distinction: the current seller's trading partners may resell the good to one another, directly or through longer paths in the network. When the seller trades with a neighbor, other neighbors may still enjoy positive continuation payoffs upon purchasing the good subsequently in the subgame with the new owner. Such lateral intermediation rents act as endogenous outside options for the neighbors. A crucial step in the analysis establishes that these outside options are not binding in equilibrium: neighbors prefer buying the good directly from the current seller rather than acquiring it through longer intermediation chains. This finding leads to a recursive characterization of limit resale values. A key theoretical result proves that the resale value of any current seller is derived in the limit *as though* his downstream neighbors were final consumers in the game without intermediation with valuations given by their resale values (Theorem 1).

However, lateral intermediation rents influence the path of trade even when players are arbitrarily patient. We show by example that the current seller may trade with positive limit probability with partners who do not deliver the highest limit resale value (Remark 5). This finding conflicts with the intuition from the special case with no intermediation, where the seller trades with the highest value buyers almost surely in the limit. We also establish the existence of MPEs (Proposition 2).

To examine in more detail the effect of an intermediary's position in the network on his resale value, we isolate network effects from other asymmetries by assuming that there are no intermediation costs and buyers have a common value v . In such settings, any intermediary linked to two (or more) buyers wields his monopoly power to extract the full surplus of v in the limit. Moreover, any current seller linked to a pair of players known to have limit resale value v also exploits the local competition to obtain a limit price of v . The process of identifying players with a limit resale value of v continues until no remaining trader is linked to two others previously pinned down. We label the pool of players recognized in the course of this procedure as layer 0. The recursive characterization of resale values implies that any remaining intermediary linked to a single layer 0 player has a limit resale value of pv . We can then find additional traders who demand a limit price of pv due to competition among the latter intermediaries, and so on.

Building on the arguments above, we develop an algorithm that decomposes the network into a series of layers of intermediation power. Having defined layers 0 through $\ell - 1$, the construction of layer ℓ proceeds as follows. The first members of layer ℓ are those players who are directly connected to layer $\ell - 1$ and have not yet been assigned to a layer. Then the remaining traders with at least two links to established members of layer ℓ join the layer. Players linked to a pair of existing members of layer ℓ are also added to the layer, and so on. Our main result shows that a trader's intermediation power depends on the number of layer boundaries the good crosses to reach from the trader to a buyer. Specifically, all players from layer ℓ have a limit resale value of $p^\ell v$. Moreover, even for positive costs, the network decomposition leads to a uniform upper bound of $p^\ell v$ for the limit resale values of layer ℓ traders, as well as lower bounds that are close to $p^\ell v$ when costs are small (Theorem 2).

The characterization of resale values by means of the layer decomposition reveals that a seller's intermediation power is not only a function of the number of intermediaries needed to reach buyers, but also depends on the competition among intermediation chains. Indeed, layers measure the effective intermediation distance between players. In sparse networks with insufficient local competition, such as line networks and square lattices, the initial seller's profits decline exponentially with the distance to buyers. However, in denser networks with many alternate trading paths, such as triangular grids and small world networks, sellers arbitrarily far away from buyers capitalize on long chains of local monopolies and earn substantial profits.

Our network decomposition has implications for optimal network design in applications. The initial seller sets trade in motion only if his cost does not exceed the maximum resale value of his bargaining partners. Hence trade is possible only if the seller belongs to a sufficiently low layer. Therefore, in manufacturing, agriculture, and financial markets, where production and trade are beneficial, denser networks with short paths, strong downstream

competition, and extensive vertical integration are efficient. Such features allow the seller to extract an adequate share of the gains from trade to cover his costs. However, in markets where trade is not desirable, as in the case of bribery and illegal goods, sparser networks with long paths and a lack of downstream competition are preferable. In such networks, low profits may deter the seller from producing the (bad) good.

Although our bargaining protocol generates an asymptotically efficient MPE allocation in the absence of intermediation, we find that the possibility of resale may create inefficiencies (Proposition 4). We distinguish between two types of asymptotic inefficiency in the intermediation game. One type of inefficiency stems from hold-up problems created by the bilateral nature of intermediation coupled with insufficient local competition. A seller facing weak downstream competition is unable to charge a price close to the maximum resale value of his neighbors and has to surrender substantial intermediation rents to his chosen partner. The anticipation of such downstream rents reduces the gains from trade in upstream transactions and may make trade unprofitable even when positive surplus is available along some intermediation chains. The other type of intermediation inefficiency results from intermediaries' incentives to exploit local competition. Equilibrium trading paths capitalize on local monopolies, and this is not generally compatible with global surplus maximization. Relatedly, we find that in settings with homogeneous intermediation costs and a single buyer, trade does not always follow the shortest route from the seller to the buyer. This finding refutes the standard intuition that sellers have incentives to minimize intermediation.

We next investigate which intermediaries make significant profits. In settings with zero costs and homogeneous values, a player earns positive limit profits only following acquisitions in which he provides a gateway to a lower layer (Proposition 5). Thus layers delimit monopoly power from intermediation power. Full profit extraction is possible in agreements within the same layer, while intermediation rents are awarded in transactions across layers.

We also provide comparative statics with respect to the network architecture and the cost distribution. The addition of a new link to the network weakly increases the initial seller's limit profit, as does the elimination of a middleman (Proposition 6). In our context, eliminating a middleman entails a rewiring of the network whereby the middleman is removed and upstream neighbors for whom the middleman provides critical intermediation in the original network inherit his downstream links as well as his costs.

To discuss the effects of vertical and horizontal integration, we restrict attention to tier networks.³ We show that the vertical integration of two consecutive tiers weakly increases the limit profit of the initial seller (Proposition 7.1). The intuition is that the vertical integration of a pair of tiers amounts to the simultaneous elimination of all middlemen from the downstream tier. However, we find that the horizontal integration of a pair of same-tier

³Tier networks provide a natural framework for integration. Formal definitions can be found in Section 9. We emphasize that tiers do not correspond to (endogenous) layers of intermediation power.

intermediaries has ambiguous effects on the initial seller's welfare (Proposition 7.2). Horizontal integration exerts countervailing forces on upstream and downstream competition. The consolidated intermediary gains access to the joint pool of trading partners of the merging players, which may enhance downstream competition and generate a higher resale value. At the same time, horizontal integration eliminates competition between merged intermediaries and leaves upstream neighbors with fewer bargaining opportunities.

Finally, we provide comparative statics with respect to the distribution of costs in a fixed network. We establish that any downward redistribution of costs in the network can only benefit the initial seller (Theorem 3). This result has ramifications for optimal cost allocation in applications. In the case of bribery, lobbying, or illegal trade, an intermediary's transaction cost reflects the risk of getting caught red-handed. Crime can be prevented by shifting monitoring efforts and the severity of punishments towards the top of the hierarchy. In manufacturing, costs account for both production expenses and taxes. Our analysis suggests that costs should be transferred downstream for efficient production. Hence retail sales taxes are more efficient than value-added taxes, and subsidies are more effective in the initial stages of production.

The study of intermediation in networks constitutes an active area of research. Section 11 discusses some important contributions to this literature. The distinguishing feature of the present model is that it fully endogenizes competition among trading paths via strategic choices at each step. That, in turn, determines prices and intermediation rents, and finally delivers the layer structure of endogenous intermediation routes. The competition effects we identify in our framework are novel to the literature.

The rest of the paper is organized as follows. Section 2 introduces the intermediation game and discusses several interpretations and applications of the model. In Section 3, we analyze the version of the game without intermediation. Section 4 provides the recursive characterization of resale values in the intermediation game. The characterization is exploited in Section 5 to establish the relationship between intermediation power and the network decomposition into layers. The sources of intermediation inefficiencies are examined in Section 6. Section 7 investigates the division of intermediation profits. In Sections 8-10, we present the comparative statics results. Section 11 reviews the related literature, and Section 12 concludes. Proofs omitted in the body of the paper are available in the Appendix.

2. THE INTERMEDIATION GAME

A set of *players* $N = \{0, 1, \dots, n\}$ interacts in the market for a single unit of an *indivisible good*. The good is initially owned by player 0, the *initial seller*. Players $i = \overline{1, m}$ are *intermediaries* ($m < n$). We simply refer to each player $i = \overline{0, m}$ as a (potential) *seller*.

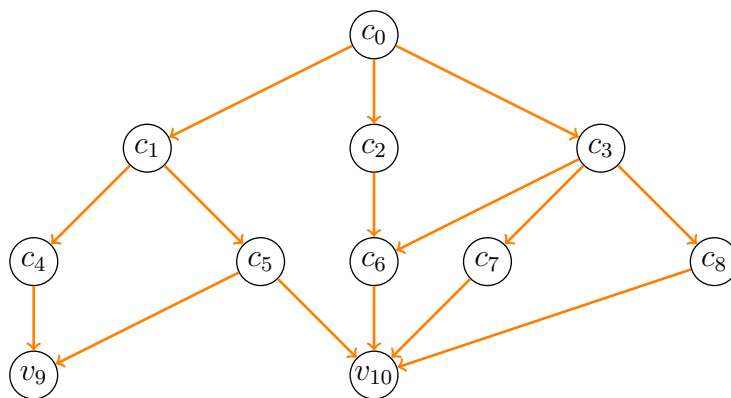


FIGURE 1. Network example

Every seller $i = \overline{0, m}$ has a transaction or production *cost* $c_i \geq 0$. Each player $j = \overline{m + 1, n}$ is a *buyer* with a consumption *value* $v_j \geq 0$ for the good.⁴

Players are linked by a *network* $G = (N, (N_i)_{i=\overline{0, m}}, (c_i)_{i=\overline{0, m}}, (v_j)_{j=\overline{m+1, n}})$. Formally, G is a *directed acyclic graph* with vertex set N . Each seller $i = \overline{0, m}$ has *out-links* to “downstream” players in the set $N_i \subset \{i+1, \dots, n\}$. Hence a *link* is an ordered pair (i, k) with $k \in N_i$. In the intermediation game to be introduced shortly, N_i represents the set of players to whom the current owner i can directly (re)sell the good. A *trading path* is a directed path connecting the initial seller to a buyer, i.e., a sequence of players $i_0, i_1, \dots, i_{\bar{s}}$ with $i_0 = 0, m+1 \leq i_{\bar{s}} \leq n$, and $i_s \in N_{i_{s-1}}$ for all $s = \overline{1, \bar{s}}$. Without loss of generality, we assume that every player lies on a trading path and that buyers do not have out-links. For fixed $m < n$, we refer to any profile $(N_i)_{i=\overline{0, m}}$ that satisfies the properties above as a *linking structure*.

Figure 1 illustrates a network with nine sellers and two buyers. The corresponding costs and values are displayed inside each node. Arrows indicate the possible direction of trade across each link. For instance, player 1 is an intermediary who can only purchase the good from player 0 and can then resell it to one of the intermediaries 4 and 5 ($N_1 = \{4, 5\}$).

The good is resold via successive bilateral bargaining between linked players in the network G until it reaches one of the buyers. We consider the following dynamic non-cooperative *intermediation game*. At each date $t = 0, 1, \dots$ the history of play determines the current owner i_t . Player 0 is the owner at time 0, $i_0 = 0$. At date t , player i_t selects a bargaining partner $k_t \in N_{i_t}$ among his downstream neighbors in the network G . With probability $p \in (0, 1)$, the seller i_t proposes a price and the partner k_t decides whether to purchase the good. Roles are reversed with probability $1 - p$. In either event, if the offer is rejected, the game proceeds to the next period with no change in ownership, $i_{t+1} = i_t$. If the offer is accepted, then i_t incurs the cost c_{i_t} (at date t), and i_t and k_t trade the good at the agreed price. If k_t is an intermediary, the game continues to date $t + 1$ with $i_{t+1} = k_t$. If k_t is a

⁴The implicit assumption that sellers cannot consume the good is easily relaxed.

buyer, he consumes the good (at time t) for a utility of v_{k_t} and the game ends. Players have a common discount factor $\delta \in (0, 1)$.

Note that all the elements of the game, including the network structure, are assumed to be common knowledge among the players. For simplicity, we assume that the game has perfect information.⁵ We focus on stationary Markov perfect equilibria. We refer to the latter simply as MPEs or equilibria, but intend to highlight stationarity explicitly in the statements of the main results. The natural notion of a Markov state in our setting is given by the identity of the current seller. An *MPE* is a subgame perfect (or sequential, in cases with imperfect information) equilibrium in which, after any history where player i owns the good at time t , player i 's (possibly random) choice of a partner k and the actions within the ensuing match (i, k) depend only on round t developments, recorded in the following sequence: the identity of the current seller i , his choice of a partner k , nature's selection of a proposer in the match (i, k) , and the offer extended by the proposer at t . In particular, strategies do not depend directly on the calendar time t .

Remark 1. The benchmark model assumes that one unit of time elapses between the moment an intermediary purchases the good and his first opportunity to resell it. All results can be adapted to an alternative model in which intermediaries may resell the good as soon as they acquire it and delays occur only following rejections.

Remark 2. The model can be extended to allow for link specific costs and consumption value for intermediaries. In the general version, seller i incurs a cost c_i^k upon trading with a neighbor $k \in N_i$ and may choose to consume the good for a utility v_i .

Remark 3. A more general specification of the model defines spot transaction costs c_i^k for all pairs of players $i < k$ and assigns a value $v(i_0, i_1, \dots, i_s)$ to each path (i_0, i_1, \dots, i_s) with $0 = i_0 < i_1 < \dots < i_s \leq n$. Note that the actual network architecture need not be explicitly modeled in this context. Indeed, the absence of a link from i to k may be captured by setting $c_i^k = \infty$ or assigning value 0 to all paths in which k immediately succeeds i . Upon purchasing the good along the path (i_0, i_1, \dots, i_s) , trader i_s has a choice between consuming the good for a value $v(i_0, i_1, \dots, i_s)$ (possibly 0) or reselling it to some player $k > i_s$. Markov states are then defined by histories of exchanges (i_0, i_1, \dots, i_s) .

We are interested in the limit equilibrium outcomes as players become patient ($\delta \rightarrow 1$). For this purpose, we generally refer to a *family of equilibria* as a collection that contains an MPE of the intermediation game for every $\delta \in (0, 1)$. Before proceeding to the equilibrium analysis, we define some welfare notions. Trade is said to be *asymptotically efficient* in a

⁵For our analysis, players do not need information about the entire history of past bargaining rounds as long as they observe the identity of the current owner.

family of equilibria if the sum of ex ante equilibrium payoffs of all players converges as $\delta \rightarrow 1$ to (the positive part of) the maximum total surplus achievable across all trading paths,

$$(2.1) \quad E := \max \left(0, \max_{\text{trading paths } i_0, i_1, \dots, i_{\bar{s}}} v_{i_{\bar{s}}} - \sum_{s=0}^{\bar{s}-1} c_{i_s} \right).$$

Trade is *asymptotically inefficient* if the limit inferior (\liminf) of the total sum of equilibrium payoffs as $\delta \rightarrow 1$ is smaller than E .

2.1. Interpretations of the model. In the context of Remark 3, our framework is reincarnated as a *road network*. Olken and Barron (2009) considered an instance of a road network model in a study of bribes paid by truck drivers at checkpoints along two important transportation routes in Indonesia. Imagine a driver who has to transport some cargo across a network of roads, choosing a direction at every junction and negotiating the bribe amount with authorities at checkpoints along the way. In contrast to our benchmark model, the driver preserves ownership of his cargo and makes all the payments as he proceeds through the network. This interpretation suggests an alternative strategic model, in which player 0 carries the good through the network. Player i mans node i . Player 0 navigates the network by acquiring access from players at checkpoints along the way sequentially. Right after passing node i_s along the path (i_0, i_1, \dots, i_s) , player 0 can end the journey with a payoff of $v(i_0, i_1, \dots, i_s)$, as specified in Remark 3 (paths that do not lead to the desired final destination(s) generate zero value), or proceed to another node $i_{s+1} > i_s$.⁶

Remark 6 establishes a formal connection between MPEs in the two models. The intuition is that, as the good clears node i , the profits of downstream intermediaries do not depend on whether player i is entrusted with the good or player 0 preserves ownership. Either player bargains with the next trader along the path anticipating the same intermediation rents in downstream transactions.

Besides their literal interpretation, road networks open the door to other applications where a buyer advances through a network by bargaining for access to nodes along the path sequentially, with the goal of reaching certain positions in the network. For instance, in the case of corruption or lobbying an individual may seek access to influential decision makers by approaching lower ranking officials and rewarding them for connections to higher positions in the hierarchy. In this context, contrary to the benchmark model, trade is initiated by the buyer and proceeds upstream in the network. Then the results are transposed by recasting the buyer as the initial seller and reversing the direction of trade in the network.

⁶Since the two routes in the application of Olken and Barron (2009) are disconnected and there are no viable alternate roads, their theoretical analysis is naturally restricted to line networks. Our general framework accommodates multiple alternate routes that intersect at various hubs, so that the driver may be able to circumvent some checkpoints.

Ultimately, whether top-down or bottom-up bargaining is the right paradigm depends on the application. The top-down protocol is realistic in the context of institutionalized corruption, while the bottom-up one is appropriate for isolated instances of corruption. The top-down feature of the benchmark model is a metaphor for how prices are set at the stage where the head of the organization decides to condone corruption and accept bribes. At that initial stage, the highest ranking official negotiates his “price” for tolerating and facilitating the corrupt activity with immediate subordinates. Once prices are determined at the top level, the subordinates can “sell” the right to collect bribes to their inferiors, and so on.

In a different set of applications, the “road” is being built as a buyer makes headway through the network. Suppose that the buyer wishes to construct a highway, railway, or oil pipeline to reach from one location to a specific destination. Then the buyer advances towards the destination via sequential negotiations with each landowner along the evolving route. Similarly, the developer of a mall or residential community needs to acquire a series of contiguous properties. Manufacturing provides yet another application for road networks. For instance, in the garment, electronics and car industries, the main manufacturers out-source components and processes to contractors. A raw good or concept produced in-house is gradually assembled and converted into a finished product with contributions from several suppliers. The producer maintains ownership of the intermediate good throughout the process. In this application, trading paths represent competing contractors, different ordering and splitting of production steps, or entirely distinct production technologies.

3. THE CASE OF NO INTERMEDIATION

To gain some intuition into the structure of MPEs, we begin with the simple case in which there are no intermediaries, i.e., $m = 0$ in the benchmark model. In this game, the seller—player 0—bargains with the buyers—players $i = \overline{1, n}$ —following the protocol from the intermediation game. When the seller reaches an agreement with buyer i , the two parties exchange the good at the agreed price, the seller incurs his transaction cost c_0 , and buyer i enjoys his consumption value v_i . The game ends after an exchange takes place. Rubinstein and Wolinsky (1990; Section 5) introduced this bargaining game and offered an analysis for the case with identical buyers. A similar bargaining protocol appears in Abreu and Manea (2012). However, both studies focus on non-Markovian behavior. The next result provides the first comprehensive characterization of MPEs.

Proposition 1. *Suppose that $m = 0$, $v_1 \geq v_2 \geq \dots \geq v_n$ and $v_1 > c_0$. Then all stationary MPEs are outcome equivalent.⁷ MPE expected payoffs converge as $\delta \rightarrow 1$ to*

⁷The *outcome* of a strategy profile is defined as the probability distribution it induces over agreements (including the date of the transaction, the identities of the buyer and the proposer, and the price) and the event that no trade takes place in finite time. Two strategies are *outcome equivalent* if they generate the same outcome.

- $\max(p(v_1 - c_0), v_2 - c_0)$ for the seller;
- $\min((1 - p)(v_1 - c_0), v_1 - v_2)$ for buyer 1;
- 0 for all other buyers.

There exists $\underline{\delta} < 1$, such that in any MPE for $\delta > \underline{\delta}$,

- if $p(v_1 - c_0) \geq v_2 - c_0$, the seller trades exclusively with buyer 1;
- if $v_1 = v_2$, the seller trades with equal probability with all buyers j with $v_j = v_1$ and no others;
- if $v_2 - c_0 > p(v_1 - c_0)$ and $v_1 > v_2$, the seller trades with positive probability only with buyer 1 and all buyers j with $v_j = v_2$; as $\delta \rightarrow 1$, the probability of trade with buyer 1 converges to 1.

MPEs are asymptotically efficient.

The intuition for this result is that when there are positive gains from trade, the seller effectively chooses his favorite outcome between two scenarios in the limit $\delta \rightarrow 1$. In one scenario, the outcome corresponds to a *bilateral monopoly* agreement in which the seller bargains only with the (unique) highest value buyer. Indeed, it is well-known that in a two-player bargaining game with the same protocol as in the general model (formally, this is the case $m = 0, n = 1$), in which the seller has cost c_0 and the buyer has valuation v_1 , the seller and the buyer split the surplus $v_1 - c_0$ according to the ratio $p : (1 - p)$. The other scenario is equivalent to a *second-price auction*, in which the seller is able to extract the entire surplus $v_2 - c_0$ created by the second highest value buyer.

Thus the seller is able to exploit the competition between buyers and extract more than the bilateral monopoly profits from player 1 only if the threat of dealing with player 2 is credible, i.e., $v_2 - c_0 > p(v_1 - c_0)$. In that case, the seller can extract the full surplus from player 2, since the “default” scenario in which he trades with player 1 leaves player 2 with zero payoff.⁸ Note, however, that when $v_1 = v_2$, the seller bargains with equal probability with all buyers with value v_1 , so there is not a single default partner. If $v_1 > v_2$ and $v_2 - c_0 > p(v_1 - c_0)$, then for high δ a small probability of trade with buyer 2 is sufficient to drive buyer 1’s rents down from the bilateral monopoly payoff of $(1 - p)(v_1 - c_0)$ to the second-price auction payoff of $v_1 - v_2$. The threat of trading with buyer 2 is implemented with vanishing probability as $\delta \rightarrow 1$, and the good is allocated efficiently in the limit.

The proof can be found in the Appendix.⁹ We show that the MPE is essentially unique. Equilibrium behavior is pinned down at all histories except those where the seller has just picked a bargaining partner who is not supposed to be selected (with positive probability) under the equilibrium strategies. Finally, note that when $v_1 \leq c_0$ the seller cannot create

⁸The role of outside options is familiar from the early work of Shaked and Sutton (1984).

⁹For brevity, some steps rely on more general arguments developed for the subsequent Theorem 1. Since the proof of the latter result does not invoke Proposition 1, there is no risk of circular reasoning.

positive surplus with any of the buyers, and hence all players have zero payoffs in any MPE. Thus Proposition 1 has the corollary that when there are no intermediaries, all MPEs of the bargaining game are payoff equivalent and trade is asymptotically efficient.

If $v_2 - c_0 > p(v_1 - c_0)$ and $v_1 > v_2$, then we can construct non-Markovian subgame perfect equilibria that are asymptotically inefficient. Indeed, if $v_2 - c_0 > p(v_1 - c_0)$, then for every δ , any stationary MPE necessarily involves the seller mixing in his choice of a bargaining partner. We can thus construct subgame perfect equilibria that are not outcome equivalent with the MPEs, even asymptotically as $\delta \rightarrow 1$, by simply modifying the seller's first period strategy to specify a deterministic choice among the partners selected with positive probability in the MPEs. Incidentally, the proof of Proposition 1 shows that for every δ , the seller bargains with buyer 2 with positive probability in any MPE. Hence for every δ we can derive a subgame perfect equilibrium in which the seller trades with buyer 2 without delay. Such equilibria are asymptotically inefficient when $v_1 > v_2$.

4. EQUILIBRIUM CHARACTERIZATION FOR THE INTERMEDIATION GAME

Consider now the general intermediation game. Fix a stationary MPE σ for a given discount factor δ . All subgames in which k possesses the good and has not yet selected a bargaining partner in the current period exhibit equivalent behavior and outcomes under σ . We simply refer to such circumstances as *subgame k* . In the equilibrium σ , every player h has the same expected payoff u_h^k in any subgame k . By convention, $u_h^k = 0$ whenever $k > h$ and $u_j^j = v_j/\delta$ for $j = \overline{m+1, n}$. The latter specification reflects the assumption that, following an acquisition, buyers immediately consume the good, while intermediaries only have the chance to resell it one period later. This definition instates notational symmetry between buyers and sellers: whenever a player k acquires the good, every player h expects a *continuation payoff*—discounted at the date of k 's purchase—of δu_h^k .

Given the equilibrium σ , the strategic situation faced by a current seller i reduces to a bargaining game with “buyers” in N_i , in which each $k \in N_i$ has a (continuation) “value” δu_k^k . This *reduced game of seller i* is reminiscent of the bargaining game without intermediation analyzed in the previous section, with one important caveat. In the game with no intermediation, each buyer k has a continuation payoff of 0 when the seller trades with some other buyer h . By contrast, in the general intermediation model player $k \in N_i$ may still enjoy positive continuation payoffs when another $h \in N_i$ acquires the good from seller i , if k purchases it subsequently (directly from h or via a longer path of trades). Figure 2 illustrates this possibility. Both i and k enjoy endogenous outside options: i can choose a different bargaining partner, while k may acquire the good from other players. Hence the bargaining power u_k^i of player $k \in N_i$ in subgame i depends not only on u_i^i and the probability with which i selects k for bargaining (as it does in the game without intermediation), but also on

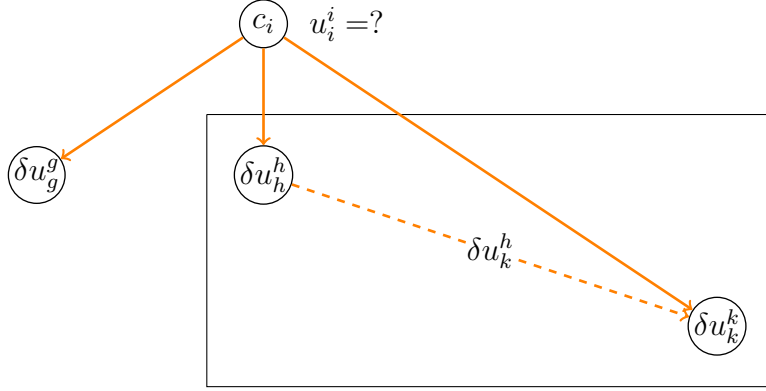


FIGURE 2. $N_i = \{g, h, k, \dots\}$. Player k may obtain positive continuation payoffs δu_k^h when intermediary h acquires the good from i , if k purchases it subsequently.

the probability of trade between i and any other player $h \in N_i$ and on the possibly positive continuation payoff δu_k^h that k expects in subgame h .

In light of the discussion above, we refer to the payoff u_k^k as player k 's *resale value* and to u_k^h as player k 's *lateral intermediation rent* under (seller) h .¹⁰ While lateral intermediation rents may be substantial, we find that they cannot be sufficiently high to induce a neighbor of the current seller to wait for another neighbor to purchase the good, with the expectation of acquiring it at a lower price later. The proof of the forthcoming Theorem 1 derives an upper bound on player k 's lateral intermediation rent under h in situations in which current seller i trades with intermediary h with positive probability in equilibrium. The bound relies on two observations:

- seller i 's incentives to choose h over k as a bargaining partner imply that the difference in resale values of k and h is not greater than the difference in subgame i expected payoffs of k and h , that is, $u_k^k - u_h^h \leq u_k^i - u_h^i$;
- k 's lateral intermediation rent under h , when positive, cannot exceed the difference in resale values of k and h , that is, $u_k^h \leq u_k^k - u_h^h$.

Under the conditions stated above, we find that $u_k^h \leq u_k^i - u_h^i$. In particular, $u_k^h \leq u_k^i$, which means that player k is better off at the beginning of subgame i rather than subgame h . In this sense, player k 's outside option is not binding in equilibrium.

Building on this intuition, Theorem 1 proves that lateral intermediation rents do not influence resale values in the limit as players become patient. Specifically, in any family of MPEs, the resale value of each seller i converges as $\delta \rightarrow 1$ to a limit r_i , which is a function

¹⁰Note that player k receives positive lateral intermediation rents only if the initial seller is connected to k via directed paths of distinct lengths. Hence lateral intermediation rents do not feature in the analysis for networks in which all routes from the initial seller to any fixed player contain the same number of intermediaries. A special class of such networks—tier networks—is defined in Section 9.

only of the limit resale values $(r_k)_{k \in N_i}$ of i 's neighbors. In the limit, seller i 's bargaining power in the reduced game is derived *as if* the players in N_i were buyers with valuations $(r_k)_{k \in N_i}$ in the game without intermediation. However, Remark 5 below shows that, due to lateral intermediation rents, the current seller does not necessarily trade with the highest resale value partner almost surely in the limit. In other words, the payoff formulae from Proposition 1 extend to the general intermediation model (see also Remark 4), but the structure of agreements does not.¹¹

Theorem 1. *For any family of stationary MPEs, resale values converge as $\delta \rightarrow 1$ to a vector $(r_i)_{i \in N}$, which is determined recursively as follows*

- $r_j = v_j$ for $j = m + 1, \dots, n$
- $r_i = \max(p(r_{N_i}^I - c_i), r_{N_i}^{II} - c_i, 0)$ for $i = m, m - 1, \dots, 0$, where $r_{N_i}^I$ and $r_{N_i}^{II}$ denote the first and the second highest order statistics of the collection $(r_k)_{k \in N_i}$, respectively.¹²

The proof appears in the Appendix. We sketch the main steps here. We proceed by backward induction on i , for $i = n, n - 1, \dots, 0$. To outline the inductive step for seller i , fix a discount factor δ and a corresponding MPE σ with payoffs $(u_h^k)_{k, h \in N}$. Assume that the current seller i can generate positive gains by trading with one of his neighbors, i.e., $\delta \max_{k \in N_i} u_k^k > c_i$. Under this assumption, we argue that if i selects $k \in N_i$ as a bargaining partner under σ , then the two players trade with probability 1. The equilibrium prices offered by i and k are $\delta u_k^k - \delta u_k^i$ and $\delta u_k^i + c_i$, respectively.

Let π_k denote the probability that seller i selects player k for bargaining in subgame i under σ . For all $k \in N_i$, we obtain the following equilibrium constraints,

$$(4.1) \quad u_k^i \geq p(\delta u_k^k - c_i - \delta u_k^i) + (1 - p)\delta u_k^i, \text{ with equality if } \pi_k > 0;$$

$$(4.2) \quad u_k^i = \pi_k (p\delta u_k^i + (1 - p)(\delta u_k^k - c_i - \delta u_k^i)) + \sum_{h \in N_i \setminus \{k\}} \pi_h \delta u_k^h.$$

For example, the right-hand side of (4.2) reflects the following equilibrium properties. Seller i trades with player k with probability π_k . At the time of purchase, the good is worth a discounted resale value of δu_k^k to k . If $\pi_k > 0$, seller i asks for a price of $\delta u_k^k - \delta u_k^i$ from k , while player k offers a price of $\delta u_k^i + c_i$ to i , with respective conditional probabilities p and $1 - p$. Furthermore, seller i trades with neighbor $h \neq k$ with probability π_h , in which event k enjoys a discounted lateral intermediation rent of δu_k^h .

¹¹Whether the conclusion of Proposition 1 regarding outcome equivalence of MPEs generalizes to the intermediation game is an open question. However, this technical puzzle does not restrict the scope of our analysis since we focus on limit equilibrium outcomes as players become patient and Theorem 1 establishes that all MPEs generate identical limit resale values. In particular, the initial seller obtains the same limit profit in all MPEs.

¹²If $|N_i| = 1$, then $r_{N_i}^{II}$ can be defined to be any non-positive number.

Consider now a family of MPEs for $\delta \in (0, 1)$ with payoffs $(u_h^k)_{k,h \in N}$ (the dependence of these variables on δ is suppressed for notational convenience). The main difficulties arise when $\sup\{\delta | \delta \max_{k \in N_i} u_k^k \leq c_i\} < 1$, so that bargaining does not break down for high δ . In this case, we have that $r_{N_i}^I \geq c_i$. It suffices to prove that $\liminf_{\delta \rightarrow 1} u_i^i \geq \max(p(r_{N_i}^I - c_i), r_{N_i}^H - c_i)$ and $\limsup_{\delta \rightarrow 1} u_i^i \leq \max(p(r_{N_i}^I - c_i), r_{N_i}^H - c_i)$.

The more delicate part of the argument shows that $\liminf_{\delta \rightarrow 1} u_i^i \geq p(r_{N_i}^I - c_i)$. Fix $k^1 \in \arg \max_{k \in N_i} u_k^k$ (k^1 is a function of δ). For high δ (such that $\delta u_{k^1}^{k^1} > c_i$), $u_{k^1}^i$ solves equation (4.2) with $k = k^1$. For all $h \in N_i$ with $\pi_h > 0$, the lateral intermediation rent of player k^1 under h satisfies $u_{k^1}^h \leq u_{k^1}^i$, as argued in the preamble of Theorem 1. Then (4.2) implies that

$$u_{k^1}^i \leq p\delta u_{k^1}^i + (1-p)(\delta u_{k^1}^{k^1} - c_i - \delta u_i^i).$$

Now substitute this bound on $u_{k^1}^i$ in the incentive constraint (4.1) for $k = k^1$. After algebraic manipulation, we obtain $u_i^i \geq p(\delta u_{k^1}^{k^1} - c_i)$. The induction hypothesis implies that $u_{k^1}^{k^1}$ converges as $\delta \rightarrow 1$ to $r_{N_i}^I$. Therefore, $\liminf_{\delta \rightarrow 1} u_i^i \geq p(r_{N_i}^I - c_i)$.

To show that $\liminf_{\delta \rightarrow 1} u_i^i \geq r_{N_i}^H - c_i$, note that the incentive constraint (4.1) leads to

$$\liminf_{\delta \rightarrow 1} (u_i^i + u_{k^1}^i) \geq \lim_{\delta \rightarrow 1} u_{k^1}^{k^1} - c_i = r_{N_i}^I - c_i.$$

A careful accounting of payoffs (Lemma 2 in the Appendix) in subgame i establishes that

$$\limsup_{\delta \rightarrow 1} \left(u_i^i + \sum_{k \in N_i} u_k^i \right) \leq r_{N_i}^I - c_i.$$

Then for any player $k^2 \in \arg \max_{k \in N_i \setminus \{k^1\}} u_k^k$, we have $\lim_{\delta \rightarrow 1} u_{k^2}^i = 0$. The incentive constraint (4.1) with $k = k^2$ leads to $u_i^i(1 - (1-p)\delta) + p\delta u_{k^2}^i \geq p(\delta u_{k^2}^{k^2} - c_i)$. Since $\lim_{\delta \rightarrow 1} u_{k^2}^{k^2} = r_{N_i}^H$ by the induction hypothesis, we obtain $\liminf_{\delta \rightarrow 1} u_i^i \geq r_{N_i}^H - c_i$.

We prove that $\limsup_{\delta \rightarrow 1} u_i^i \leq \max(p(r_{N_i}^I - c_i), r_{N_i}^H - c_i)$ by contradiction. If the claim is false, then there exists a sequence of discount factors converging to 1 along which (1) u_i^i converges to a limit greater than $\max(p(r_{N_i}^I - c_i), r_{N_i}^H - c_i)$, and (2) seller i either (i) trades exclusively with a (fixed) player k or (ii) trades with positive probability with two (fixed) players k and h such that $u_k^k \geq u_h^h$. In case (i), the payoff equations lead to $u_i^i = p(\delta u_k^k - c_i)$. Then the induction hypothesis implies that the limit of u_i^i along the sequence is $p(r_k - c_i) \leq p(r_{N_i}^I - c_i)$.

In case (ii), (4.1) implies that $u_i^i(1 - (1-p)\delta) \leq p(\delta u_h^h - c_i)$. Note that $u_k^k \geq u_h^h$, along with the induction hypothesis, leads to $r_k = \lim_{\delta \rightarrow 1} u_k^k \geq \lim_{\delta \rightarrow 1} u_h^h = r_h$. Hence the limit of u_i^i along the sequence does not exceed $r_h - c_i \leq r_{N_i}^H - c_i$. In either case, we reach a contradiction.

Remark 4. The proof shows that the characterization of buyer payoffs from Proposition 1 also extends to the intermediation game. The intermediation rent u_k^i extracted by player

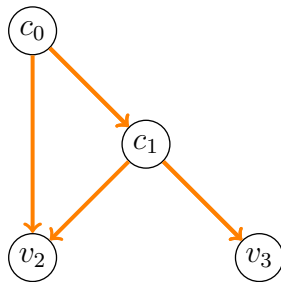


FIGURE 3. The initial seller trades with positive limit probability with the second highest resale value neighbor, intermediary 1.

$k \in N_i$ in subgame i converges as $\delta \rightarrow 1$ to $\min((1-p)(r_{N_i}^I - c_i), r_{N_i}^I - r_{N_i}^H)$ if $r_k = r_{N_i}^I \geq c_i$ and zero otherwise.

Remark 5. While lateral intermediation rents do not directly influence limit resale values, they may play an important role in determining the path of trade. Suppose that $r_{N_i}^I > c_i$, which means that seller i can generate positive gains from trade for high δ . A natural extension of Proposition 1 would be that seller i trades only with neighbors $k \in N_i$ who have maximum limit resale values, i.e., $r_k = r_{N_i}^I$, almost surely as $\delta \rightarrow 1$. However, this conjecture is incorrect. Consider the two-seller and two-buyer network from Figure 3 with $c_0 = c_1 = 0, v_2 = 1, v_3 = 0.9$. Assume that $p = 1/2$. We immediately find the limit resale values using Theorem 1: $r_0 = 0.9, r_1 = 0.9, r_2 = 1, r_3 = 0.9$. Computations available upon request (similar to those for the example solved in Section 7) show that the initial seller trades with intermediary 1 with limit probability $(35 - \sqrt{649})/36 \approx 0.26$ as $\delta \rightarrow 1$, even though his limit resale value is smaller than buyer 2's value. This finding is not at odds with asymptotic efficiency in the context of this example, since intermediary 1 trades with the higher value buyer 2 almost surely as $\delta \rightarrow 1$.

Remark 6. The characterization of MPEs in the intermediation game substantiate the road network interpretation from Subsection 2.1. Indeed, (4.1)-(4.2) also capture the MPE constraints in the road network model, where u_i^i and u_k^i represent the expected payoffs of players 0 and $k > i$, respectively, conditional on player 0 having just cleared checkpoint i . The cost of intermediary i has to be replaced by the cost c_k of checkpoint k . Both models are naturally embedded in the common framework with link specific costs from Remark 2 (or the less general setting with no intermediation costs). The equivalence holds because in the benchmark model seller i internalizes the downstream intermediation rents forfeited by player 0 as he advances beyond node i in the road network.

Remark 7. The recursive formula for resale values in the setting with link specific costs from Remark 2 becomes

$$r_i = \max(p(r_k - c_i^k)_{k \in N_i}^I, (r_k - c_i^k)_{k \in N_i}^H, 0) \text{ for } i = m, m-1, \dots, 0,$$

where $(r_k - c_i^k)_{k \in N_i}^I$ and $(r_k - c_i^k)_{k \in N_i}^{II}$ denote the first and the second highest order statistics of the collection $(r_k - c_i^k)_{k \in N_i}$, respectively. To extend the proofs of Theorem 1 and the supporting Lemma 2 to this more general setting, we need to assume that direct connections are not more costly than indirect ones. Formally, for any link (i, k) and all directed paths $i = i_0, i_1, \dots, i_{\bar{s}} = k$ ($i_s \in N_{i_{s-1}}$ for all $s = \overline{1, \bar{s}}$) connecting i to k , it must be that

$$c_i^k \leq \sum_{s=0}^{\bar{s}-1} c_{i_s}^{i_{s+1}}.$$

Note that the condition above is satisfied whenever c_i^k depends exclusively on i —as in our benchmark model—or on k —as in the road network version.

4.1. Relationship to double marginalization. Consider a line network ($N_i = \{i + 1\}$ for $i = \overline{0, n-1}$) with a single buyer (player n) and no intermediation costs ($c_i = 0$ for $i = \overline{0, n-1}$). The formula for limit resale values from Theorem 1 implies that $r_i = p^{n-i} v_n$ for $i = \overline{0, n-1}$. This conclusion is reminiscent of *double marginalization* (Spengler 1950). However, there does not seem to be a deep theoretical connection between intermediation in networks and multiple marginalization in a chain of monopolies. The two models capture distinct strategic situations. In the classic double marginalization paradigm, upstream monopolists set prices for downstream firms, which implicitly determine the amount of trade in each transaction. Downstream firms have no bargaining power. By contrast, in the line network application of our intermediation game, downstream neighbors derive bargaining power from the opportunity of making offers to upstream sellers, which arises with probability $1 - p$ at every date. Every link represents a bilateral monopoly. Moreover, since our environment presumes that a single unit of the indivisible good is available, there is no discretion over the quantity traded in each agreement. It is worth emphasizing here that our focus is on understanding how competition among trading paths shapes the outcomes of intermediation. Since competition is entirely absent in line networks, any relationship to double marginalization is peripheral to our main contribution.

4.2. Equilibrium existence. Using the properties of MPEs uncovered by the analysis above, we establish the existence of an equilibrium.

Proposition 2. *A stationary MPE exists in the intermediation game.*

5. LAYERS OF INTERMEDIATION POWER

This section investigates how an intermediary's position in the network affects his resale value. In order to focus exclusively on network asymmetries, suppose momentarily that sellers have zero costs and buyers have a common value v (or that there is a single buyer, i.e., $m = n - 1$). Since no resale value can exceed v , Theorem 1 implies that any seller linked

to two (or more) buyers has a limit resale value of v . Then any seller linked to at least two of the latter players also secures a payoff of v , acting as a monopolist for players with limit continuation payoffs of v . More generally, any trader linked to two players with a resale value of v also obtains a limit price of v . We continue to identify players with a resale value v in this fashion until we reach a stage where no remaining seller is linked to two others known to have resale value v .

Using Theorem 1 again, we can show that each remaining trader linked to one value v player has resale value pv . We can then identify additional traders who command a resale price of pv due to competition between multiple neighbors with resale value pv , and so on.

We are thus led to decompose the network into a sequence of *layers of intermediation power*, $(\mathcal{L}_\ell)_{\ell \geq 0}$, which characterizes every player's resale value. The recursive construction proceeds as follows. First all buyers are enlisted in layer 0. In general, for $\ell \geq 1$, having defined layers 0 through $\ell - 1$, the first players to join layer ℓ are those outside $\bigcup_{\ell' < \ell} \mathcal{L}_{\ell'}$ who are linked to a node in $\mathcal{L}_{\ell-1}$. For every $\ell \geq 0$, given the initial membership of layer ℓ , new traders outside $\bigcup_{\ell' < \ell} \mathcal{L}_{\ell'}$ with at least two links to established members of layer ℓ are added to the layer. We continue expanding layer ℓ until no remaining seller is linked to two of its formerly recognized members. All players joining layer ℓ through this sequential procedure constitute \mathcal{L}_ℓ .¹³ The algorithm terminates when every player is uniquely assigned to a layer, $\bigcup_{\ell \leq \ell} \mathcal{L}_\ell = N$. For an illustration, in the network example from Figure 1 the algorithm produces the layers $\mathcal{L}_0 = \{5, 9, 10\}$, $\mathcal{L}_1 = \{0, 1, 3, 4, 6, 7, 8\}$, $\mathcal{L}_2 = \{2\}$.

The definition of each layer is obviously independent of the order in which players join the layer. In fact, an equivalent description of the layer decomposition proceeds as follows. \mathcal{L}_0 is the largest set (with respect to inclusion) $M \subset N$, which contains all buyers, such that every seller in M has two (or more) out-links to other players in M . For $\ell \geq 1$, \mathcal{L}_ℓ is the largest set $M \subset N \setminus (\bigcup_{\ell' < \ell} \mathcal{L}_{\ell'})$ with the property that every node in M has out-links to either (exactly) one node in $\mathcal{L}_{\ell-1}$ or (at least) two nodes in M .

The informal arguments above suggest that if sellers have zero costs and buyers have the same value v , then all players from layer ℓ have a resale value of $p^\ell v$. The next result proves a generalization of this claim for arbitrary cost structures. It provides lower and upper bounds anchored at $p^\ell v$ for the resale values of layer ℓ players. The precision of the bounds for player $i \in \mathcal{L}_\ell$ depends on the sum of costs of sellers $k \geq i$ from layers $\ell' \leq \ell$ weighted by $p^{\ell-\ell'}$. In particular, the bounds become tight as intermediation costs vanish. Thus the decomposition of the network into layers captures intermediation power.

¹³Note that layer ℓ has an analogous topology to layer 0, if we recast the initial members of layer ℓ as buyers.

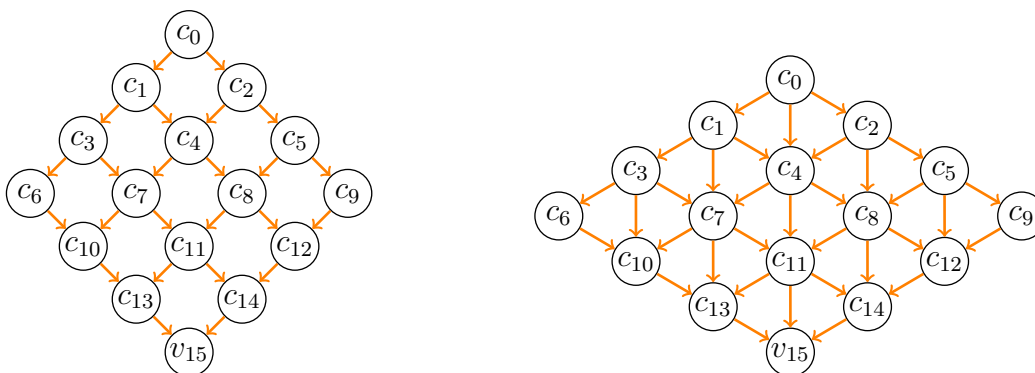


FIGURE 4. A square lattice and a triangular grid

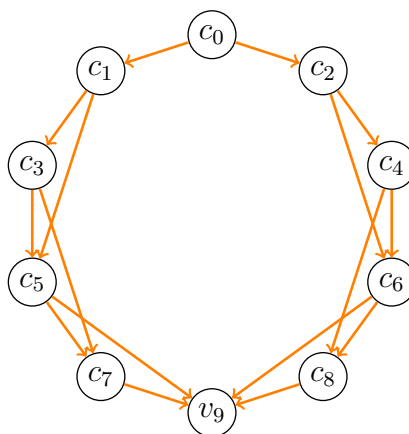


FIGURE 5. An enhanced circle network

Theorem 2. *Suppose that all buyers have the same value v . Then the limit resale value of any player $i \in \mathcal{L}_\ell$ satisfies*

$$r_i \in \left[p^\ell v - \sum_{\ell'=0}^{\ell} p^{\ell-\ell'} \sum_{k \in \mathcal{L}_{\ell'}, i \leq k \leq m} c_k, p^\ell v \right].$$

In particular, if all costs are zero, then every layer ℓ player has a limit resale value of $p^\ell v$.

The layers of intermediation power have simple structures in some networks and more complex topologies in others. Consider the two networks with 15 sellers and one buyer from Figure 4. In the square lattice from the left panel, layers are formed by symmetric corners around the buyer. The initial seller, who is six links away from the buyer, belongs to layer 3. In the triangular grid on the right-hand side, layers form vertical strips around the buyer. The initial seller, three links away from the buyer, is a member of layer 1. Figure 5 depicts an enhanced circle network, exhibiting small world properties, in which all sellers belong to layer 1.

Suppose that there are no intermediation costs and there is a single buyer ($m = n - 1$) with valuation v . Let d denote the length \bar{s} of the shortest trading path $i_0 = 0, i_1, \dots, i_{\bar{s}} = n$ in the network. Since the initial seller belongs to a layer $\ell \leq d$, Theorem 2 implies that his limit profit is at least $p^d v$. The latter bound is achieved if G is a line network ($N_i = \{i + 1\}$ for $i = \overline{0, n - 1}$). In square lattices, the initial seller's limit payoff can be as high as $p^{d/2} v$. However, there exist networks with arbitrarily high d —e.g., scaled-up triangular grids and enhanced circles—in which the initial seller belongs to layer 1 and makes a limit profit of $p v$.

The examples discussed above demonstrate that a seller's intermediation power is not only a function of the number of intermediaries needed to reach a buyer, but also depends on the local monopolies each intermediary enjoys in the network. Indeed, a trader's intermediation power is determined by the number of layer boundaries the good needs to cross to get from the trader to a buyer. In other words, layers measure the effective intermediation distance between traders.

The characterization of layers of intermediation power has practical implications for applications. Suppose that the initial seller belongs to layer ℓ in our network decomposition. Then each neighbor $k \in N_0$ belongs to a layer $\ell' \geq \ell - 1$ and provides a resale value of at most $p^{\ell'} v \leq p^{\ell-1} v$. If $c_0 > p^{\ell-1} v$, then no transaction takes place for high δ . Hence trade is possible only if the initial seller belongs to a sufficiently low layer.

Therefore, in markets for socially beneficial assets, such as manufacturing, agriculture and finance, denser networks with short paths (downstream competition, vertical integration) are optimal for encouraging trade. Such networks enable the initial seller to obtain a significant share of the gains from trade in order to cover his costs. However, in markets where trade is not socially desirable, as in the case of bribery and illegal goods, sparser networks with long paths (few subordinates, bureaucracy) are preferable. In such networks, bargaining breaks down due to the large amount of anticipated downstream intermediation rents. The analysis provides guidance for designing network architectures (e.g., hierarchical structures in public institutions) and setting costs (e.g., transaction fees, legal punishments) that implement the desired social outcomes.

5.1. Competing trading paths. The construction of layers and the examples above suggest that players from layer ℓ are either directly linked to layer $\ell - 1$ or offer competing paths to layer $\ell - 1$. Every layer ℓ player is linked to at least two intermediaries added to layer ℓ earlier in the algorithm, who are each linked to two former members of the layer, and so on, until layer $\ell - 1$ players are eventually reached. Of course, there may be significant overlap among the paths traced in this fashion, but the possibility of branching out in at least two directions at every stage generates rich sets of paths connecting layer ℓ players to $\ell - 1$. In particular, the next result shows that every layer ℓ player not directly linked to layer $\ell - 1$ has two independent paths of access to layer $\ell - 1$. In addition, any two players from layer ℓ

can reach layer $\ell - 1$ via disjoint paths of layer ℓ intermediaries. In other words, every pair of intermediaries from layer ℓ can pass the good down to layer $\ell - 1$ without relying on each other or on any common layer ℓ intermediaries.

Proposition 3. *Every player from layer $\ell \geq 1$ has either a direct link to layer $\ell - 1$ or two non-overlapping paths of layer ℓ intermediaries connecting him to (possibly the same) layer $\ell - 1$ players. Moreover, any pair of distinct layer $\ell \geq 1$ players can reach some (possibly identical) layer $\ell - 1$ players via disjoint paths of layer ℓ intermediaries.*

It is possible to prove the following version of the result for layer 0. Every seller from layer 0 is connected to two different buyers via non-overlapping paths of layer 0 intermediaries. Furthermore, any pair of sellers from layer 0 can reach distinct buyers using disjoint paths of layer 0 intermediaries.

6. INTERMEDIATION INEFFICIENCIES

In contrast to the bargaining model with no intermediaries analyzed in Section 3, intermediation may create trade inefficiencies. There are two distinct sources of asymptotic inefficiency. One source resides in *hold-up* problems induced by the bilateral nature of intermediation combined with weak downstream competition. Consider a subgame in which a current seller i creates positive net profit by trading with the highest resale value neighbor, but cannot capture the entire surplus available in the transaction, that is, $r_i = \max(p(r_{N_i}^I - c_i), r_{N_i}^H - c_i, 0) < r_{N_i}^I - c_i$. Then $r_{N_i}^I > \max(r_{N_i}^H, c_i)$ and the (unique) player with the highest resale value secures positive rents of $\min((1-p)(r_{N_i}^I - c_i), r_{N_i}^I - r_{N_i}^H)$ (Remark 4). The rent amount is independent of the history of transactions; in particular, the payment i made to procure the good is sunk. Such rents are anticipated by upstream traders and diminish the gains they share.¹⁴ In some cases, the dissipation of surplus is so extreme that trade becomes unprofitable even though some intermediation chains generate positive total surplus.

Figure 6 illustrates a simple intermediation network that connects the initial seller to a single intermediary, who provides access to one buyer. Suppose that $v_2 > c_0 > 0$ and $c_1 = 0$. In the MPE, upon purchasing the good, the intermediary expects a payoff of pv_2 in the next period from reselling it to the buyer. Trade between the initial seller and the intermediary is then possible only if the seller's cost does not exceed the intermediary's continuation payoff, $c_0 \leq \delta pv_2$. Hence for $c_0 \in [pv_2, v_2)$, bargaining breaks down and traders fail to realize the positive gains $v_2 - c_0$. The MPE is asymptotically inefficient in this case. As discussed above, the source of asymptotic inefficiency is that the buyer holds up the intermediary for

¹⁴By contrast, Gofman (2011) and Farboodi (2014) assume that surplus is divided along intermediation chains according to an exogenous rule, which grants a fixed proportion of the gains from trade to downstream players.



FIGURE 6. Hold-up inefficiencies

a profit of $(1 - p)v_2$. Then at the initial stage the seller and the intermediary bargain over a reduced limit surplus of $pv_2 - c_0$, rather than the total amount of $v_2 - c_0$.¹⁵ The conclusion of this example can be immediately extended to show that in any setting with at least one intermediary ($m \geq 1$) there exist configurations of intermediation costs and buyer values such that trade is asymptotically inefficient.

Proposition 4. *For any linking structure $(N_i)_{i=\overline{0,m}}$ with $m \geq 1$ intermediaries there exist cost and value configurations $((c_i)_{i=\overline{0,m}}, (v_j)_{j=\overline{m+1,n}})$ such that any family of MPEs induced by the network $(N, (N_i)_{i=\overline{0,m}}, (c_i)_{i=\overline{0,m}}, (v_j)_{j=\overline{m+1,n}})$ is asymptotically inefficient.*

For a sketch of the proof, note that for any linking structure $(N_i)_{i=\overline{0,m}}$ with $m \geq 1$ there must be a trading path $i_0 = 0, i_1 = 1, \dots, i_{\bar{s}}$ with $\bar{s} \geq 2$. If we set $c_0 \in (p^{\bar{s}-1}, 1)$, $c_{i_s} = 0$ for $s = \overline{1, \bar{s}-1}$, $v_{i_{\bar{s}}} = 1$, the costs of all remaining intermediaries to 1, and the values of all buyers different from $i_{\bar{s}}$ to 0, then no trade takes place in equilibrium even though the path $(i_s)_{s=\overline{0, \bar{s}}}$ generates positive surplus.

Another source of inefficiency lies in sellers' incentives to exploit *local competition*, which are not aligned with global welfare maximization. Consider the network from Figure 7, in which the buyer value v_9 is normalized to 1 and sellers are assumed to have a common cost

$$c \in \left[0, \min \left(\frac{p(1-p)}{4-p^2}, \frac{p(1-p)}{5-3p-p^2} \right) \right).$$

In this network, the layer decomposition algorithm leads to

$$\mathcal{L}_0 = \{9\}, \mathcal{L}_1 = \{0, 1, 2, 4, 5, 6, 7, 8\}, \mathcal{L}_2 = \{3\}.$$

Resale values are easily computed from Theorem 1. In particular, we find that $r_0 = p - (5 + p)c$, $r_1 = p - (4 + p)c$, $r_2 = p - (3 + p)c$, and $r_3 = p^2 - p(1 + p)c$. For the range of c considered, we have $r_2 \geq r_1 > r_3$ and $r_0 = r_1 - c > p(r_2 - c) > 0$. Hence the initial seller

¹⁵Blanchard and Kremer (1997) and Wong and Wright (2011) discuss similar hold-up problems in line networks.

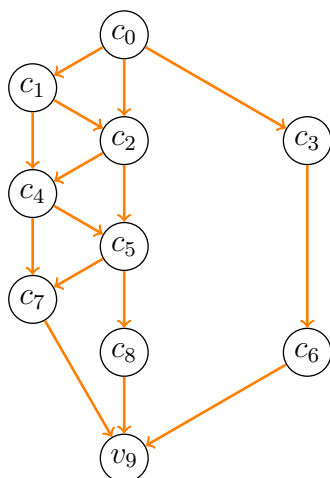


FIGURE 7. Trade does not minimize intermediation or maximize welfare.

obtains the second-price auction profits in a game in which his neighbors act as buyers with the collection of values (r_1, r_2, r_3) .

Since $r_0 = r_1 - c > r_3 - c$, Lemma 1 from the Appendix implies that the initial seller does not trade with intermediary 3 for high δ . Equilibrium trade proceeds via intermediary 1 or 2 in order to exploit local competition within layer 1. The trading path involves at least three intermediaries from the set $\{1, 2, 4, 5, 7, 8\}$. Any such path generates a limit surplus of at most $1 - 4c$. However, the path from the seller to the buyer intermediated by players 3 and 6 achieves a total welfare of $1 - 3c$. Thus the MPE is asymptotically inefficient if c is positive.

It is interesting to note that even in settings where the network constitutes the sole source of asymmetry among intermediaries, trade does not proceed along the shortest route from seller to buyer. In the example above, the shortest trading path involves intermediaries 3 and 6. However, at least three intermediaries from layer 1 are employed to transfer the good to the buyer in equilibrium for high δ . Intermediary 3 from layer 2 never gains possession of the good, even though he lies on the shortest trading path, because the initial seller prefers to deal with intermediaries 1 and 2 from layer 1.

Remark 8. Note that for p close to 1 the current seller is able to extract most of the surplus even in a bilateral monopoly scenario. Hence both the hold-up friction and local competition effects vanish as $p \rightarrow 1$. Using the recursive characterization from Theorem 1, one can prove by induction that trader i 's limit resale value converges as $p \rightarrow 1$ to the (positive part of the) maximum surplus generated by all paths connecting i to a buyer. In particular, the initial seller's limit price converges to E , so there are no efficiency losses as p approaches 1. The corresponding limit resale values constitute the maximum competitive equilibrium prices defined in the general framework with bilateral contracts of Hatfield et al. (2013). In

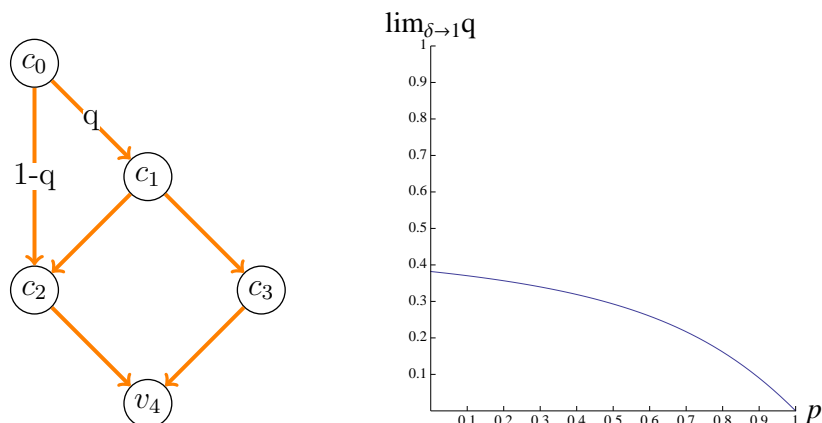


FIGURE 8. Asymmetric trading paths within layers

contrast to the findings of this section, competitive equilibria in the latter model are always efficient. We elaborate on the modeling assumptions that underlie the divergent predictions in the literature review (Section 11).

7. POSITIVE PROFITS

It is important to determine which intermediaries make significant profits. This question is challenging because the path of trade is not necessarily deterministic in equilibrium. We reinstate the zero costs and homogeneous values assumptions of Section 5. Although traders from the same layer have identical limit resale values, it turns out that MPE trading paths may treat such traders asymmetrically, even in the limit as players become patient. Indeed, as the next example illustrates, two trading partners with vanishing differences in resale values and lateral intermediation rents may acquire the good from the seller with unequal positive limit probabilities.

Consider the intermediation game induced by the network in the left panel of Figure 8. Suppose all costs are zero and $v_4 = 1$. Applying our layer decomposition algorithm, we find that all sellers belong to layer 1. Then Theorem 2 implies that the initial seller, as well as intermediaries 1 and 2, has a limit resale value of p . However, we argue next that equilibrium trade treats intermediaries 1 and 2 asymmetrically. Player 0 chooses both intermediaries for bargaining with positive limit probabilities, but selects the better positioned player 2 more often in the unique MPE.

We solve the intermediation game backward. Subgames 2 and 3 are standard two-player bargaining games. We can easily compute the payoffs, $u_2^2 = u_3^3 = p$.

We next roll back to subgame 1. Since there are no intermediation chains between players 2 and 3, this subgame is strategically equivalent to the bargaining game without intermediaries in which player 1 may sell the good to either “buyer” 2 or 3, who have a common (discounted

resale) “value” δp . This game has a unique MPE by Proposition 1. By symmetry, player 1 trades with equal probability with intermediaries 2 and 3. Then payoffs in subgame 1 solve the following system of equations

$$\begin{aligned} u_1^1 &= p(\delta u_2^2 - \delta u_2^1) + (1-p)\delta u_1^1 \\ u_2^1 &= \frac{1}{2}(p\delta u_2^1 + (1-p)(\delta u_2^2 - \delta u_1^1)) \\ u_3^1 &= u_2^1. \end{aligned}$$

Substituting in $u_2^2 = p$, we immediately find

$$\begin{aligned} u_1^1 &= \frac{\delta(2-\delta)p^2}{2(1-\delta) + \delta p} \\ u_2^1 &= \frac{\delta(1-\delta)p(1-p)}{2(1-\delta) + \delta p}. \end{aligned}$$

Consider now the bargaining problem faced by the initial seller. An agreement with intermediary 1 generates a continuation payoff of δu_2^1 for player 2; this payoff is positive for $\delta \in (0, 1)$, as intermediary 1 sells the good with probability 1/2 to player 2 in subgame 1. However, in the event of an agreement between the initial seller and intermediary 2, player 1’s continuation payoff is 0 since he cannot purchase the good subsequently. Let q denote the probability that the initial seller selects intermediary 1 for bargaining in an MPE. Then the analysis of Section 4 leads to the following payoff equations

$$\begin{aligned} u_0^0 &= p(q(\delta u_1^1 - \delta u_1^0) + (1-q)(\delta u_2^2 - \delta u_2^0)) + (1-p)\delta u_0^0 \\ u_1^0 &= q(p\delta u_1^0 + (1-p)(\delta u_1^1 - \delta u_0^0)) \\ u_2^0 &= q\delta u_2^1 + (1-q)(p\delta u_2^0 + (1-p)(\delta u_2^2 - \delta u_0^0)), \end{aligned}$$

where u_1^1, u_2^1 , and u_2^2 have been computed previously.

For sufficiently high δ , it is impossible that $q \in \{0, 1\}$. For instance, $q = 1$ implies that $u_0^0 = p\delta u_1^1, u_1^0 = (1-p)\delta u_1^1$, and $u_2^0 = \delta u_2^1$. In particular, the MPE payoffs of players 1 and 2 converge to $p(1-p)$ and 0, respectively, as $\delta \rightarrow 1$. Then for high δ , we have $\delta u_2^2 - \delta u_2^0 > \delta u_1^1 - \delta u_1^0$, so the initial seller prefers to bargain with intermediary 2 instead of 1. A similar contradiction obtains assuming that $q = 0$ for high δ .

Hence we need $q \in (0, 1)$ for high δ . Then the seller’s indifference between the two intermediaries requires that $\delta u_1^1 - \delta u_1^0 = \delta u_2^2 - \delta u_2^0$. Appending this constraint to the system of equations displayed above, we find that for δ sufficiently close to 1 there is a unique solution $q \in (0, 1)$, which satisfies

$$\lim_{\delta \rightarrow 1} q =: q^* = \frac{3}{2} - p - \sqrt{(1-p)^2 + \frac{1}{4}} \in \left(0, \frac{3-\sqrt{5}}{2} \approx .38\right).$$

The graph in the right panel of Figure 8 traces the relationship between the limit q^* and the parameter p .

The good is traded along each of the paths 0, 1, 2, 4 and 0, 1, 3, 4 with probability $q/2$ and along the shorter path 0, 2, 4 with probably $1 - q$. The example can be easily adapted so that the value of q affects the distribution of profits in the network. Suppose, for instance, that the buyer is replaced by two unit value buyers, one connected to intermediary 2 and the other to 3. Then the limit MPE payoffs of the two buyers are $(1 - p)(1 - q^*/2)$ and $(1 - p)q^*/2$, respectively.

The intricate derivation of q in the example above suggests that in general networks it is difficult to compute the random path of trade. However, the next result shows how profits are shared along any trading path that emerges with positive probability in equilibrium. More specifically, it identifies the exchanges in which the new owner secures positive intermediation rents.

Proposition 5. *Suppose that sellers have zero costs and buyers have a common value v . Then for sufficiently high δ , every layer ℓ player can acquire the good in equilibrium only from traders in layers ℓ and $\ell + 1$. If player $k \in \mathcal{L}_\ell$ purchases the good from seller i with positive probability in subgame i for a sequence of δ 's converging to 1, then player k 's limit profit conditional on being selected by i as a trading partner is $p^\ell(1 - p)v$ if $i \in \mathcal{L}_{\ell+1}$ and zero if $i \in \mathcal{L}_\ell$.*

This result establishes that players make positive limit profits only in transactions in which they constitute a *gateway* to a lower layer. Hence layers delineate monopoly power from intermediation power. Local competition is exploited in exchanges within the same layer to extract the full amount of gains from trade, while intermediation rents need to be surrendered in agreements across layers.

We close this section with the observation that intermediaries who are not essential for trade can make substantial profits. Suppose that all sellers have a common cost $c < pv_4/(1 + p)$ in the network depicted in Figure 9. Note that intermediary 1 is not essential for trade, as the initial seller can access buyer 4 via intermediaries 2 and 3 in this network. Goyal and Vega-Redondo (2007) posit that inessential intermediaries like player 1 should make zero profits. However, this assumption is not borne out by our model. Indeed, limit resale values in the network are immediately computed from Theorem 1: $r_1 = r_3 = p(v_4 - c)$, $r_0 = r_2 = p(pv_4 - (1 + p)c)$. By Remark 4, intermediary 1 turns a positive limit profit of $(1 - p)(r_1 - c) = (1 - p)(pv_4 - (1 + p)c)$.¹⁶

¹⁶More generally, in networks with a single buyer and no intermediation costs, which consist of two non-overlapping paths of unequal length connecting the seller to the buyer, no intermediary is essential for trade, yet those along the shorter path obtain positive intermediation rents in the limit.

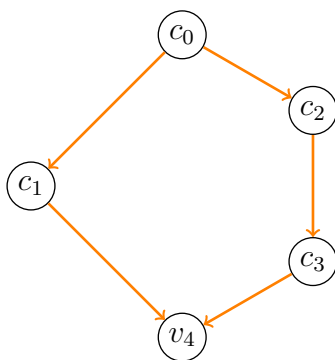


FIGURE 9. Intermediary 1 is not essential for trade, but makes positive limit profit.

8. PRELIMINARY COMPARATIVE STATICS

We seek comparative statics for changes in the initial seller’s profit in response to the addition of a link or the elimination of a middleman. Fix a network $G = (N, (N_i)_{i=\overline{0,m}}, (c_i)_{i=\overline{0,m}}, (v_j)_{j=\overline{m+1,n}})$. For $0 \leq i \leq m, i < k \leq n$ with $k \notin N_i$, adding the link (i, k) to G results in a new network \tilde{G} which differs from G only in that player i ’s downstream neighborhood \tilde{N}_i in \tilde{G} incorporates k ($\tilde{N}_i = N_i \cup \{k\}$). Similarly, if $k \in N_i$, removing the link (i, k) from G results in a network where i ’s downstream neighborhood excludes k .

Imagine now a scenario where a trader k could eliminate a middleman $i \in N_k$ and gain direct access to i ’s pool of trading partners N_i . Intuitively, such a rewiring of the network should benefit trader k , since he avoids paying intermediation rents to i .¹⁷ However, for consistency with the production technology, we acknowledge that player k might also have to bear i ’s intermediation costs, if i provides essential connections.

Formally, we say that trader k (*directly*) *relies* on intermediary $i \in N_k$ if the removal of the link (k, i) from the network changes the limit resale value of k . The latter condition implies that k and i trade with positive probability in subgame k in any MPE for sufficiently high δ . For $i = \overline{1, m}$, the *elimination of middleman i* from G is the procedure that generates a network $\tilde{G} = (\tilde{N}, (\tilde{N}_i), (\tilde{c}_i), (v_j))$, which excludes node i ($\tilde{N} = N \setminus \{i\}$), such that traders k who rely on i in G inherit the cost and links of i ($\tilde{c}_k = c_k + c_i, \tilde{N}_k = N_k \cup N_i \setminus \{i\}$) and traders k who do not simply lose existing links to i ($\tilde{N}_k = N_k \setminus \{i\}$); all other elements of \tilde{G} are specified as in G .

Proposition 6. *Both*

- (1) *the addition of a new link to the network*
- (2) *the elimination of a middleman*

weakly increase the initial seller’s limit profit.

¹⁷This intuition has a flavor similar to the logic suggesting that vertical integration eliminates the losses caused by double marginalization.

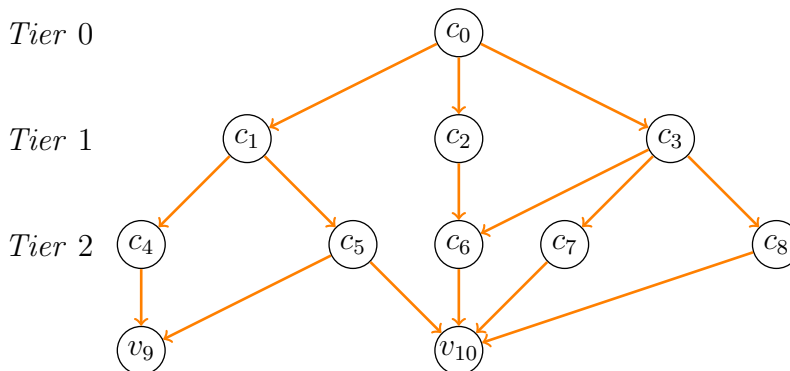


FIGURE 10. A tier network ($c_1 = c_2 = c_3$; $c_4 = \dots = c_8$)

Remark 9. In the benchmark model each seller's cost is independent of his choice of a trading partner. If seller k has to cover the cost of an eliminated middleman i , the cost increase must be reflected in k 's transactions with all downstream neighbors. It is then reasonable to assume, as we have, that k incurs i 's cost only if i provides key access in the network for k . Alternatively, we can sidestep the endogenous notion of relying on a middleman and prove a version of Proposition 6 in the extension of the model with link specific costs discussed in Remark 2. In that context, we simply redefine the intermediation cost across the link (k, h) in the network obtained by eliminating middleman i to be $\min(c_k^h, c_k^i + c_i^h)$ whenever $i \in N_k$ and $h \in N_i$ (with the convention that $c_k^h = \infty$ if $h \notin N_k$).

Remark 10. Proposition 6 assumes that the elimination of a middleman entails his complete removal from the network, along with the rewiring of links and the appropriate redefinition of costs. It is possible to establish a version of the result in which a middleman is bypassed by a single upstream neighbor, but retains his other in- and out-links.

9. VERTICAL AND HORIZONTAL INTEGRATION

To define vertical and horizontal integration in our model, we need to impose more structure on the intermediation network. Specifically, we focus on tier networks. In a *tier network* all trading paths have the same length, and all intermediaries at a given distance from the initial seller have identical costs. In such networks, directed paths with identical endpoints have the same length. Players at distance τ from the initial seller form *tier* τ . Figure 10 provides an illustration. We refer to the common cost of these players as the *cost of tier* τ . In the context of industrial organization, each tier corresponds to a specific step in the production process.

Fix a tier network G . The *vertical integration* of tiers τ and $\tau + 1$ of sellers in G generates a tier network \tilde{G} , which satisfies the following conditions:¹⁸

- \tilde{G} excludes the nodes from tier $\tau + 1$ in G

¹⁸We preserve the labels from G when referring to nodes in \tilde{G} .

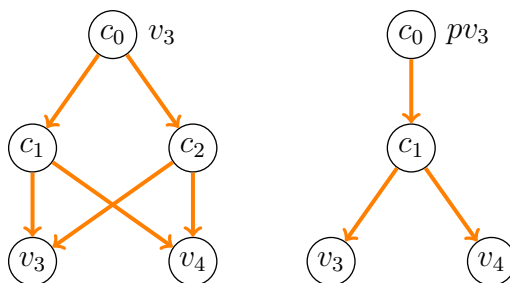


FIGURE 11. Upstream effects of horizontal integration

- each tier τ player gets directly linked in \tilde{G} to all tier $\tau + 2$ traders from G with whom he is connected in G via tier $\tau + 1$ intermediaries
- the cost of tier τ in \tilde{G} is the sum of the costs of tiers τ and $\tau + 1$ in G
- all other elements of \tilde{G} (links, costs, buyer values) are specified as in G .

The *horizontal integration* of two same-tier intermediaries i and i' in G leads to a tier network \tilde{G} with the following properties:

- \tilde{G} excludes node i'
- player i inherits all in- and out-links of i' (ignoring duplication)
- in all other respects, \tilde{G} retains the structure and parameters of G .

Proposition 7. *Suppose that G is a tier network.*

- (1) *The vertical integration of any two consecutive tiers of sellers in G weakly increases the limit profit of the initial seller.*
- (2) *The horizontal integration of a pair of same-tier intermediaries in G has an ambiguous effect on the initial seller's welfare.*

The vertical integration of tiers τ and $\tau + 1$ can be understood as the simultaneous elimination of *all* middlemen (in the sense of Section 8) from tier $\tau + 1$. Thereby the intuition for part (1) of the result is similar to Proposition 6. The formal proof can be found in the Appendix.

For part (2), note that horizontal integration has opposite effects on upstream and downstream competition. On the one hand, the horizontal integration of two intermediaries may lead to more downstream competition and a higher resale value for the consolidated player, as his pool of trading partners expands. On the other hand, horizontal integration leaves upstream neighbors with fewer trading options and eliminates the competition between merged intermediaries. The next examples show that either effect can drive the net change in upstream profits.

Consider the tier networks from the left-hand sides of Figures 11 and 12. In either network, we assume that all costs are zero and analyze the effects of the horizontal integration of

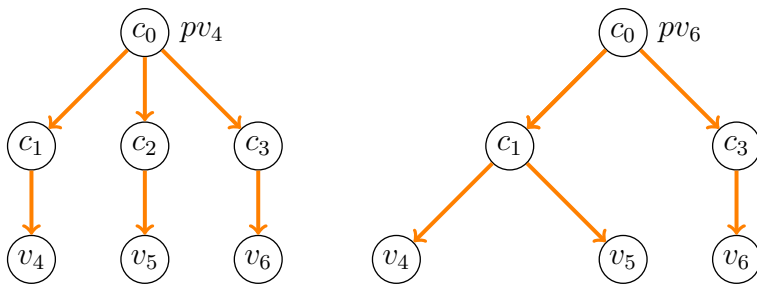


FIGURE 12. Downstream effects of horizontal integration

intermediaries 1 and 2. The networks resulting from integration are represented on the right-hand side of the respective figures. In the network from Figure 11, if $v_3 = v_4 > 0$, then the upstream effect of horizontal integration dominates. Indeed, using Theorem 1 we find that player 0's limit profit declines from v_3 in the initial network to pv_3 in the integrated one. Initially, player 0 could attain the second-price auction profits by exploiting the competition between the two intermediaries, but after their merger he has to settle for the bilateral monopoly profits with the single remaining intermediary. The merger of players 1 and 2 has no downstream advantages, as the consolidated intermediary deals with the same pool of buyers and obtains the same resale value as traders 1 and 2 in the original network.

By contrast, in the network from Figure 12 the downstream effect of horizontal integration is dominant for a range of buyer values. If $pv_6 < v_4 = v_5 < v_6$, then Theorem 1 implies that the initial seller's limit profit increases from pv_4 prior to the integration of intermediaries 1 and 2 to pv_6 afterwards. The merger of the two intermediaries allows the consolidated player 1 to exploit the competition between buyers 4 and 5. This boosts the resale value of intermediary 1, and player 0 is able to capture part of the gain. Note that the gain is transferred to player 0 indirectly, since in the initial network he trades with intermediary 3 almost surely as $\delta \rightarrow 1$, but in the integrated network he switches to intermediary 1.

10. COMPARATIVE STATICS FOR COSTS

In the previous sections we discussed what network architectures are socially desirable. Here we fix the network structure and provide comparative statics with respect to the distribution of costs in the network. We use the following partial order to compare cost patterns. A cost profile $\tilde{c} = (\tilde{c}_i)_{i=0,\overline{m}}$ *upstream dominates* another $c = (c_i)_{i=0,\overline{m}}$ if for every directed path connecting the initial seller to an intermediary, $0 = i_0, i_1, \dots, i_{\bar{s}} \leq m$ ($i_s \in N_{i_{s-1}}, s = \overline{1, \bar{s}}$), including the degenerate case with $\bar{s} = 0$, we have

$$\sum_{s=0}^{\bar{s}} \tilde{c}_{i_s} \geq \sum_{s=0}^{\bar{s}} c_{i_s}.$$

Intuitively, \tilde{c} upstream dominates c if costs are relatively more concentrated at the “top” under \tilde{c} . The next result shows that the downward redistribution of costs in the network entailed by the shift from \tilde{c} to c weakly benefits the initial seller. The idea is that if the bilateral monopoly outcome emerges in a transaction between seller i and partner k , then k “shares” a fraction $1 - p$ of i ’s cost (Remark 4). Hence higher downstream costs reduce intermediation rents in downstream exchanges, leaving more gains from trade for upstream players.

Theorem 3. *Let c and \tilde{c} be two distinct cost profiles in otherwise identical networks. If \tilde{c} upstream dominates c , then the initial seller’s limit profit is at least as high under c as under \tilde{c} .*

Remark 11. The notion of cost dominance and Theorem 3 admit straightforward generalizations to the setting with link specific costs discussed in Remark 2.

Theorem 3 has ramifications for the optimal cost distribution in the applications discussed in the introduction. In the case of bribery, lobbying, or illegal trade, an intermediary’s transaction cost represents the risk of getting caught performing the illegal or unethical activity and the potential penalties. Then crime can be discouraged by ratcheting up the severity of punishments at the top of the hierarchy. If monitoring expenses need to be budgeted across the network, the optimal targets for audits and investigations are higher ranking officials or heads of organized crime.

In the case of manufacturing, costs may capture both production expenditures and taxes. In order to implement efficient production, Theorem 3 (along with Proposition 4) suggests that costs have to be pushed downstream to the extent possible. In supply chains, retail sales taxes are more efficient than value-added taxes. Similarly, subsidies are more effective at earlier stages of production than at later ones. The result also sheds light on the optimal allocation of manufacturing processes in supply chains where each step of production is performed by a tier of specialized firms. If every step involves some essential processes that require specialization, as well as certain generic tasks (bells and whistles) that can be performed by downstream firms, then the generic features should be added as far along as possible in the production of the good.

The result is particularly relevant for situations in which a manufacturer contracts out specific steps in the production process. In that case trading paths describe competing suppliers and production technologies (as in the road network interpretation of the model). When there is flexibility in the order of production steps, the manufacturer prefers dealing with the costliest suppliers in the last stages. It is also more efficient to grant subsidies to the main producer rather than to contractors. However, if the manufacturer markets the

finished product directly to final consumers, then retail taxes should be reduced at the cost of higher taxes for contractors.

Xiao (2012) proves a related result in a setting in which a real estate developer chooses the order of bargaining with requisite landowners. Despite differences in modeling assumptions, our predictions are consistent. The developer should approach owners with smaller opportunity cost for their land earlier in the bargaining process.

11. RELATED LITERATURE

Rubinstein and Wolinsky (1987) introduced the first bargaining model of intermediation. They considered a stationary market with homogeneous populations of buyers, sellers, and middlemen. In a related model, Duffie et al. (2005) endogenized steady states and search intensities in the context of an over-the-counter market. In their setting, investors have stochastically changing types that determine whether they act as buyers or sellers. Investors may trade with one another, and also with market makers. Due to symmetry assumptions, the intrinsic intermediation network in either model takes the simple form of a complete tripartite graph (with links weighted by matching probabilities).

A growing body of recent research explores the effects of general network asymmetries on intermediation. We first discuss models which share our assumption that a single good is traded in the network. Goyal and Vega-Redondo (2007) propose a solution whereby the players essential for trade between the buyer and the seller split the proceeds equally. Their focus is on strategic network formation under the proposed payoff function.

Siedlarek (2012) considers a model in which traders along a randomly drawn path bargain over the division of surplus generated by the path. The multilateral nature of his bargaining protocol eliminates the hold-up problems we identify in our setting. One can show that trade is asymptotically efficient in Siedlarek's model. Our models also diverge on a second dimension. Siedlarek finds support for Goyal's and Vega-Redondo's assumption that inessential intermediaries do not earn any profits.

Gofman (2011) puts forward a solution concept building on the assumption that each intermediary receives an exogenous share of the gains from trade available in any exchange. Because of the exogenous division of surplus, his solution concept does not fully capture the effects of local competition. Farboodi (2014) employs a similar exogenous sharing rule along intermediation chains in the context of interbank lending.

Choi, Galeotti, and Goyal (2013) consider a static model in which all intermediaries post prices simultaneously and trade takes place along the least expensive path in the network. Efficient equilibria always exist in their setting. However, inefficient equilibria may arise due to coordination failures created by the simultaneity of actions. Condorelli and Galeotti (2012)

broach the issue of incomplete information regarding traders' valuations in the intermediation network.

Other studies of intermediation in networks analyze markets for multiple goods. Gale and Kariv (2007) consider a setting with several units of a homogeneous good, for which traders connected by a network have unit demand and heterogeneous values. Their main result is that trade is efficient if sellers have all the bargaining power and the network is sufficiently connected. Blume et al. (2009) study a static model in which every trade is intermediated by a single middleman. Intermediaries compete in posting bid prices for sellers and ask prices for buyers. In some networks multiple equilibria generating different divisions of profits may co-exist, but all equilibria are efficient.

Hatfield et al. (2013) develop a general equilibrium model of trading in networks via bilateral contracts. Their model embeds our framework as a special case, but does not explicitly specify a non-cooperative bargaining process. Each contract stipulates a buyer, a seller, and an implicit exchange, as well as the price. Traders simultaneously choose among the contracts available to them, taking prices as given. The simultaneity of buying and selling decisions, along with the price-taking assumption, eliminates hold-up problems and induces competition. As a consequence, competitive equilibria are efficient.

A related framework for intermediation in supply chains was provided by Ostrovsky (2008). In that model, intermediaries purchase inputs from upstream firms to utilize in a production activity whose outputs are then sold downstream. Ostrovsky proposes a cooperative solution concept based on the notion that a set of outstanding contracts is unstable if a chain of connected firms in the network is better off writing new contracts with one another and possibly dropping some existing arrangements.

In another static model, Nava (2010) studies the effects of quantity competition on the trade of a homogeneous good in a network. In his setting, the roles of buyers, sellers, and intermediaries are endogenously assigned to traders. Intermediation necessarily generates frictions and inefficiencies. However, trade is approximately efficient in certain large networks.

The effects of vertical and horizontal integration have also been examined by Hart and Tirole (1990) and Bolton and Whinston (1993) in setups with one or two suppliers and two downstream firms. However, their focus is on optimal integration structures for social welfare, competitiveness, and ex ante incentives for investment. Kranton and Minehart (2000) provide a network-based analysis of vertical integration in a two-sided setting where manufacturers decide whether to produce their own inputs or form relationships with potential suppliers.

12. CONCLUSION

This research investigates how competing paths of intermediation, which naturally call for a network formulation, determine the terms of trade between buyers and sellers. We introduced a general model of intermediation in networks in which a good is resold via sequential bilateral bargaining between linked traders. Even though a trader's resale value depends on intermediation rents that downstream neighbors earn in various subgames and on nondeterministic trading paths, the complexity becomes manageable when players are patient. Limit resale values are characterized by a recursive formula that does not invoke other equilibrium variables. The characterization of resale values reveals that local competition plays a crucial role in shaping the outcomes of intermediation. The seller's profit is not only a function of the number of middlemen along the path of trade, but also depends on the complete collection of competing paths of access to buyers available to each middleman.

A decomposition of the network into layers of intermediation power captures the endogenous structure of local monopolies and trading paths. We established that resale values decline exponentially as higher layers are reached. Thus layers measure the effective intermediation distance between traders. We also argued that layers delimit monopoly power from intermediation power. Intermediation rents are significant in transactions across layers, but not within. Only players who serve as gateways to lower layers earn substantial profits. Layers that build extensively on local monopolies exhibit rich sets of competing paths of access to powerful intermediaries in lower layers. In particular, any pair of same-layer intermediaries can reach the next layer down via non-overlapping chains of middlemen from their layer.

We showed that intermediation may create efficiency losses. Two types of inefficiency are possible: one resulting from hold-up problems, the other from incentives to employ intermediaries that offer competing paths to buyers. Furthermore, we found that trade does not always proceed along the shortest route between the seller and buyers.

We also provided comparative statics with respect to the network architecture and cost distribution. Specifically, we found that the seller's profit increases when new links are added, middlemen are eliminated, tiers are vertically integrated, or costs are transferred to downstream intermediaries. The effects of horizontal integration are ambiguous. We offered several interpretations of the model and discussed the implications of the analysis for various applications.

In future work, it would be useful to extend the analysis to markets where multiple units or commodities are traded at the same time. It would also be interesting to consider the case in which the seller maintains ownership of the good in the course of negotiations and intermediaries trade by consignment.

APPENDIX A. PROOFS

Proof of Proposition 1. Consider a stationary MPE of the bargaining game with no intermediation and discount factor δ . Let u_k denote the expected payoff of player $k \in N$ and π_j be the probability that the seller selects buyer $j = \overline{1, n}$ for bargaining (following a history along which the good has not yet been traded) in this equilibrium. Since $v_1 > c_0$, the general arguments from the proof of Theorem 1 establish that the seller reaches an agreement with conditional probability 1 with any buyer j such that $\pi_j > 0$. Furthermore, for all j with $\pi_j > 0$, we obtain the following payoff equations,

$$\begin{aligned} u_0 &= p(v_j - c_0 - \delta u_j) + (1 - p)\delta u_0 \\ u_j &= \pi_j(p\delta u_j + (1 - p)(v_j - c_0 - \delta u_0)). \end{aligned}$$

Expressing variables in terms of u_0 (≥ 0), we get

$$(A.1) \quad u_j = \frac{v_j - c_0}{\delta} - \frac{1 - \delta + \delta p}{\delta p} u_0$$

$$(A.2) \quad \pi_j = \frac{1 - \delta + \delta p}{\delta p} - \frac{(1 - \delta)(1 - p)(v_j - c_0)}{\delta p(v_j - c_0 - u_0)}.$$

The (equilibrium) condition $u_j \geq 0$ implies that $u_0 \leq p(v_j - c_0)/(1 - \delta + \delta p)$ if $\pi_j > 0$. It follows immediately that $\pi_j = 0$ whenever $u_0 \geq p(v_j - c_0)/(1 - \delta + \delta p)$ (if $u_0 = p(v_j - c_0)/(1 - \delta + \delta p)$ then the right-hand side of (A.2) becomes 0).

Moreover, $u_0 < p(v_j - c_0)/(1 - \delta + \delta p)$ implies that $\pi_j > 0$. Indeed, if $\pi_j = 0$ then $u_j = 0$. The seller has the option to choose buyer j as his bargaining partner in the first period, which leads to the incentive constraint $u_0 \geq p(v_j - c_0) + (1 - p)\delta u_0$, or equivalently $u_0 \geq p(v_j - c_0)/(1 - \delta + \delta p)$.

We established that selection probabilities in any MPE are described by the following functions of u_0

$$(A.3) \quad \tilde{\pi}_j(u_0) = \begin{cases} \frac{1 - \delta + \delta p}{\delta p} - \frac{(1 - \delta)(1 - p)(v_j - c_0)}{\delta p(v_j - c_0 - u_0)} & \text{if } u_0 < (v_j - c_0) \frac{p}{1 - \delta + \delta p} \\ 0 & \text{if } u_0 \geq (v_j - c_0) \frac{p}{1 - \delta + \delta p} \end{cases}.$$

One can easily check that each function $\tilde{\pi}_j$ is strictly decreasing over the interval $[0, p(v_j - c_0)/(1 - \delta + \delta p)]$ and continuous over $[0, p(v_1 - c_0)/(1 - \delta + \delta p)]$. Hence the expression $\sum_{j=1}^n \tilde{\pi}_j(u_0)$ is strictly decreasing and varies continuously for $u_0 \in [0, p(v_1 - c_0)/(1 - \delta + \delta p)]$. Note that

$$\sum_{j=1}^n \tilde{\pi}_j(0) = \frac{|\{j | v_j > c_0\}|}{\delta} > 1 \text{ and } \sum_{j=1}^n \tilde{\pi}_j\left(\frac{p(v_1 - c_0)}{1 - \delta + \delta p}\right) = 0.$$

Therefore, there exists a unique u_0 for which

$$(A.4) \quad \sum_{j=1}^n \tilde{\pi}_j(u_0) = 1.$$

This value pins down all other variables describing the equilibrium. Hence all MPEs are outcome equivalent. However, we cannot pin down equilibrium actions in subgames in which the seller selects a buyer j such that $\pi_j = 0$ and $v_j - c_0 \leq \delta u_0$. Behavior in these off the equilibrium path subgames does not affect expected payoffs. Conversely, we can track back through the necessary conditions for an MPE, starting with the solution to (A.4), to establish the existence of an MPE.

The limit MPE payoffs as $\delta \rightarrow 1$ for this special instance of the intermediation game are derived in the proof of Theorem 1. We next establish properties of MPEs for high δ . For every $\delta \in (0, 1)$, (A.3) implies that $\pi_1 \geq \pi_2 \geq \dots \geq \pi_n$.

Define $\Delta = \{\delta \in (0, 1) | \pi_1 = 1\}$. Assume first that $\sup \Delta = 1$. For $\delta \in \Delta$, the payoffs satisfy

$$\begin{aligned} u_0 &= p(v_1 - c_0 - \delta u_1) + (1 - p)\delta u_0 \\ u_1 &= p\delta u_1 + (1 - p)(v_1 - c_0 - \delta u_0) \\ u_j &= 0, j = \overline{2, n}. \end{aligned}$$

We immediately obtain that $u_0 = p(v_1 - c_0)$ and $u_1 = (1 - p)(v_1 - c_0)$. The seller's incentives then imply that $v_1 - c_0 - \delta(1 - p)(v_1 - c_0) \geq v_2 - c_0$ for all $\delta \in \Delta$. Since $\sup \Delta = 1$, it must be that $p(v_1 - c_0) \geq v_2 - c_0$.

Conversely, if $p(v_1 - c_0) \geq v_2 - c_0$ then $v_1 - c_0 - \delta(1 - p)(v_1 - c_0) \geq v_2 - c_0$ for all $\delta \in (0, 1)$, and one can construct an MPE in which $\pi_1 = 1$ for every $\delta \in (0, 1)$. By the first part of the proof, this constitutes the unique MPE for each δ . The arguments above establish that $\sup \Delta = 1$ if and only if $\pi_1 = 1$ for every $\delta \in (0, 1)$ if and only if $p(v_1 - c_0) \geq v_2 - c_0$.

Suppose now that $\sup \Delta < 1$, which is equivalent to $v_2 - c_0 > p(v_1 - c_0)$. Consider a buyer j with $v_j < v_2$. Since $\lim_{\delta \rightarrow 1} u_2 = 0$, we have $v_2 - c_0 - \delta u_2 > v_j - c_0 - \delta u_j$ for sufficiently high δ . Hence the seller prefers trading with buyer 2 rather than j , so $\pi_j = 0$ for high δ .

Therefore, if $v_1 = v_2$ then the seller bargains only with the highest value buyers when players are sufficiently patient. Equation (A.2) implies that all such buyers are selected as partners with equal probability.

If $v_1 > v_2$ then trade with any buyer other than player 1 generates a surplus of at most $v_2 - c_0 < v_1 - c_0$. Since $\lim_{\delta \rightarrow 1} u_0 + u_1 = v_1 - c_0$, it must be that $\lim_{\delta \rightarrow 1} \pi_1 = 1$. Since $\sup \Delta < 1$, we have $\pi_2 > 0$ for sufficiently high δ ; (A.2) implies that $\pi_j = \pi_2$ whenever $v_j = v_2$. For high δ , we argued that $\pi_j = 0$ if $v_j < v_2$. This completes the characterization of MPE outcomes for all parameter ranges. \square

Lemma 1. *Suppose that conditional on being the current owner, seller i selects player $k \in N_i$ for bargaining with positive probability in the MPE. Then in every subgame in which i has just chosen k as a bargaining partner, i expects a payoff of u_i^i . Moreover, resale values satisfy $u_i^i \leq \max(\delta u_k^k - c_i, 0) \leq u_k^k$.*

Proof. The first part follows from standard equilibrium mixing conditions. Suppose that conditional on being the current owner, seller i selects player $k \in N_i$ for bargaining with positive probability in the MPE. Then player i expects a payoff of u_i^i from bargaining with k . However, in the MPE player k never accepts a price above δu_k^k or offers a price above $\delta u_i^i + c_i$ to i . Moreover, player i 's continuation payoff in case of disagreement is δu_i^i . Hence $u_i^i \leq \max(\delta u_k^k - c_i, \delta u_i^i)$. Since $\delta \in (0, 1)$, it follows that $u_i^i \leq \max(\delta u_k^k - c_i, 0) \leq u_k^k$. \square

Lemma 2. *For any seller i and any subset of players $M \not\ni i$,*

$$u_i^i + \sum_{k \in M} u_k^i \leq \max(u_i^i, \delta \max_{k \in M} u_k^k - c_i).$$

For intuition, note that the left-hand side of the inequality above represents the total expected profits accruing to players from $M \cup \{i\}$ in subgame i . Consider a history of subgame i in which the good is traded within $M \cup \{i\}$ for a number of periods, until it reaches some intermediary $k \in M$, who resells it to a player outside M . Assume that the good does not return to M thereafter. The net contribution of transfers between pairs of players in $M \cup \{i\}$ to the sum of expected payoffs of $M \cup \{i\}$ in this scenario is zero. The only other contributions of the history to the sum are the costs of sellers from $M \cup \{i\}$ along the trading path and the price received by k . Seller k expects a price of $u_k^k + c_k$ and incurs the cost c_k at the time of the sale, which takes place at least one period into subgame i . Thus the conditional expected contribution of the history to the sum, discounted at the beginning of subgame i , does not exceed $\delta u_k^k - c_i$. The general argument is more involved, as it has to account for intermediation chains that exit and re-enter $M \cup \{i\}$ several times. The proof exploits the monotonicity of expected prices along equilibrium trading paths, which is a corollary of Lemma 1.

Proof. We prove the following claim by backward induction on i , for $i = n, n-1, \dots, 0$. For all $M \subset N \setminus \{i\}$,¹⁹

$$u_i^i + \sum_{k \in M} u_k^i \leq \max(u_i^i, \delta \max_{k \in M} u_k^k - c_i).$$

For buyers $i = \overline{m+1, n}$, the claim clearly holds (for arbitrary specifications of c_i) as $u_k^i = 0$ for all $k \neq i$. Assuming the claim is true for $n, \dots, i+1$, we prove it for i . Fix $M \subset N \setminus \{i\}$. Let $M' = \{h \in N \setminus \{i\} \mid u_h^h \leq \max_{k \in M} u_k^k\}$. We set out to show that

$$u_i^i + \sum_{k \in M'} u_k^i \leq \max(u_i^i, \delta \max_{k \in M} u_k^k - c_i).$$

The sum $u_i^i + \sum_{k \in M'} u_k^i$ represents the total payoffs of the players in $M' \cup \{i\}$ in subgame i . It may be evaluated as an expectation over contributions from several events unfolding in the first round of the subgame:

¹⁹By convention, let $c_i = 0$ for every buyer $i = \overline{m+1, n}$.

- (1) player i trades with some player $h \in M'$;
- (2) player i trades with some player $h \notin M'$;
- (3) player i does not reach an agreement with his selected bargaining partner.

In the first event, players i and h exchange the good for a payment. Besides the change in ownership, the net contribution of this transaction to the total payoffs of i and h is the cost c_i . The conditional continuation payoff of each player $k \in M'$ is given by δu_k^h . The sum of continuation payoffs of the players in $M' \cup \{i\}$ conditional on the transaction between i and h is thus

$$\delta \sum_{k \in M'} u_k^h - c_i.$$

By the induction hypothesis applied for player h ($h \in N_i \Rightarrow h > i$) and the set $M' \setminus \{h\}$, the expression above does not exceed $\delta \max(u_h^h, \delta \max_{k \in M' \setminus \{h\}} u_k^k - c_h) - c_i \leq \delta \max_{k \in M'} u_k^k - c_i = \delta \max_{k \in M} u_k^k - c_i$.

In the second event, player i trades with some $h \notin M'$. By definition, $u_h^h > \max_{k \in M'} u_k^k$. Since Lemma 1 implies that resale values are non-decreasing along every path on which trade takes place with positive probability, it must be that once h acquires the good, no player in M' ever purchases the good at a later stage. Thus conditional on i selling the good to h , the expected payoffs of all players in M' are 0. By Lemma 1, player i 's conditional expected payoff (when event (2) has positive probability) is u_i^i . Hence event (2) contributes to the sum of payoffs of the players in $M' \cup \{i\}$ with the amount u_i^i .

The third event contributes to the expectation with a term $\delta(u_i^i + \sum_{k \in M'} u_k^i)$.

Since $u_i^i + \sum_{k \in M'} u_k^i$ is evaluated as an expectation of contributions from events of one of the three types analyzed above, it must be that

$$u_i^i + \sum_{k \in M'} u_k^i \leq \max \left(u_i^i, \delta \max_{k \in M} u_k^k - c_i, \delta(u_i^i + \sum_{k \in M'} u_k^i) \right).$$

Then $u_i^i + \sum_{k \in M'} u_k^i \geq 0$, $u_i^i \geq 0$ and $\delta \in (0, 1)$ imply that

$$u_i^i + \sum_{k \in M'} u_k^i \leq \max(u_i^i, \delta \max_{k \in M} u_k^k - c_i).$$

As $M \subset M'$ and $u_k^i \geq 0$ for all $k \in M'$, the latter inequality leads to

$$u_i^i + \sum_{k \in M} u_k^i \leq \max(u_i^i, \delta \max_{k \in M} u_k^k - c_i),$$

which concludes the proof of the inductive step. \square

Proof of Theorem 1. We prove by backward induction on i , for $i = n, n-1, \dots, 0$, that u_i^i converges as δ goes to 1 to a limit r_i , which satisfies $r_i = v_i$ for $i = \overline{m+1, n}$ and $r_i = \max(p(r_{N_i}^I - c_i), r_{N_i}^H - c_i, 0)$ for $i = \overline{0, m}$. The base cases $i = n, \dots, m+1$ (corresponding

to buyers) are trivially verified. Assuming that the induction hypothesis holds for players $n, \dots, i+1$, we seek to prove it for seller i ($\leq m$).

Let $\Delta = \{\delta \in (0, 1) \mid \delta \max_{k \in N_i} u_k^k \leq c_i\}$. If $\sup \Delta = 1$, then the induction hypothesis implies that $\max_{k \in N_i} \delta u_k^k$ converges to $r_{N_i}^I \leq c_i$ as $\delta \rightarrow 1$. Hence the maximum gains that i can create by trading with any of his neighbors vanish as $\delta \rightarrow 1$. It follows that u_i^i converges to 0 as $\delta \rightarrow 1$, so $\lim_{\delta \rightarrow 1} u_i^i = 0 = \max(p(r_{N_i}^I - c_i), r_{N_i}^{II} - c_i, 0)$.

For the rest of the proof, assume that $\sup \Delta < 1$ and restrict attention to $\delta > \sup \Delta$. Then $r_{N_i}^I \geq c_i$ and $\delta \max_{k \in N_i} u_k^k > c_i$. Fix $k^1 \in \arg \max_{k \in N_i} u_k^k$. By Lemma 1,

$$(A.5) \quad u_i^i \leq \max(\delta u_{k^1}^{k^1} - c_i, 0) = \delta u_{k^1}^{k^1} - c_i.$$

Then Lemma 2 implies that

$$(A.6) \quad u_i^i + \sum_{k \in N_i} u_k^i \leq \delta u_{k^1}^{k^1} - c_i.$$

If $\delta u_{k^1}^{k^1} - c_i \leq \delta u_{k^1}^i$ then the inequality above leads to

$$u_i^i + \sum_{k \in N_i} u_k^i \leq \delta u_{k^1}^{k^1} - c_i \leq \delta u_{k^1}^i,$$

which is possible only if $u_k^i = 0$ for all $k \in N_i \cup \{i\}$. However, if $u_{k^1}^i = 0$ then player i can select k^1 for bargaining and offer him an acceptable price arbitrarily close to $\delta u_{k^1}^{k^1}$. This deviation leads to a profit approaching $\delta u_{k^1}^{k^1} - c_i > 0$ in the event that i is chosen as the proposer, contradicting $u_i^i = 0$.

Hence $\delta u_{k^1}^{k^1} - c_i > \delta u_{k^1}^i$. Since player k^1 accepts any price z with $\delta u_{k^1}^{k^1} - z > \delta u_{k^1}^i$, player i can secure a positive profit by bargaining with k^1 and, when selected to make an offer to k^1 , proposing a price arbitrarily close to $\delta u_{k^1}^{k^1} - \delta u_{k^1}^i > c_i$. Thus $u_i^i > 0$.

Assume that the current seller i selects $k \in N_i$ for bargaining with positive probability. By Lemma 1, player i expects a payoff of u_i^i conditional on choosing k . Note that player k would never offer i a price higher than $\delta u_k^k + c_i$, and i 's continuation payoff in case of disagreement is δu_i^i . Since $u_i^i > 0$, it must be that k is willing to accept a price offer z from i with $z - c_i > u_i^i$. Then $\delta u_k^k - z \geq \delta u_k^i$, which leads to $\delta u_k^k - \delta u_k^i \geq z > u_i^i + c_i$. It follows that $\delta(u_i^i + u_k^i) < \delta u_k^k - c_i$. Standard arguments then imply that conditional on i selecting k as a bargaining partner, the two players trade with probability 1. When selected as the proposer, either player offers a price that makes the opponent indifferent between accepting and rejecting the offer. The equilibrium prices offered by i and k are $\delta u_k^k - \delta u_k^i$ and $\delta u_i^i + c_i$, respectively.

Let π_k denote the probability that seller i selects player $k \in N_i$ for bargaining in subgame i . The arguments above lead to the following equilibrium conditions for all $k \in N_i$,

$$(A.7) \quad u_i^i \geq p(\delta u_k^k - c_i - \delta u_k^i) + (1-p)\delta u_i^i, \text{ with equality if } \pi_k > 0$$

$$(A.8) \quad u_k^i = \pi_k (p\delta u_k^i + (1-p)(\delta u_k^k - c_i - \delta u_k^i)) + \sum_{h \in N_i \setminus \{k\}} \pi_h \delta u_k^h.$$

For all $h \in N_i$ with $\pi_h > 0$, seller i 's incentive constraints (A.7) lead to $\delta u_h^h - c_i - \delta u_h^i \geq \delta u_{k^1}^{k^1} - c_i - \delta u_{k^1}^i$, or equivalently,

$$(A.9) \quad u_{k^1}^{k^1} - u_h^h \leq u_{k^1}^i - u_h^i.$$

By Lemma 2, we have $u_h^h + u_{k^1}^h \leq \max(u_h^h, \delta u_{k^1}^{k^1} - c_h)$, which along with $\delta \in (0, 1)$ and $c_h \geq 0$, implies that $u_{k^1}^h \leq \max(0, u_{k^1}^{k^1} - u_h^h) = u_{k^1}^{k^1} - u_h^h$. Then (A.9) leads to

$$(A.10) \quad u_{k^1}^h \leq u_{k^1}^i - u_h^i \leq u_{k^1}^i.$$

Combining (A.10) with (A.8) for $k = k^1$, we obtain

$$u_{k^1}^i \leq \pi_{k^1} (p\delta u_{k^1}^i + (1-p)(\delta u_{k^1}^{k^1} - c_i - \delta u_{k^1}^i)) + \sum_{h \in N_i \setminus \{k^1\}} \pi_h \delta u_{k^1}^h.$$

Since $u_{k^1}^i \geq 0$ and $\delta u_{k^1}^{k^1} - c_i - \delta u_{k^1}^i \geq 0$ (A.5), it follows that

$$u_{k^1}^i \leq \frac{\pi_{k^1}}{1 - \delta \sum_{h \in N_i \setminus \{k^1\}} \pi_h} (p\delta u_{k^1}^i + (1-p)(\delta u_{k^1}^{k^1} - c_i - \delta u_{k^1}^i)) \leq p\delta u_{k^1}^i + (1-p)(\delta u_{k^1}^{k^1} - c_i - \delta u_{k^1}^i),$$

which leads to

$$(A.11) \quad u_{k^1}^i \leq \frac{(1-p)(\delta u_{k^1}^{k^1} - c_i - \delta u_{k^1}^i)}{1 - p\delta}.$$

By the incentive constraint (A.7) for $k = k^1$,

$$(A.12) \quad u_i^i \geq \frac{p(\delta u_{k^1}^{k^1} - c_i - \delta u_{k^1}^i)}{1 - (1-p)\delta}.$$

Substituting the bound on $u_{k^1}^i$ from (A.11) into (A.12) and collecting the u_i^i terms, we find that $u_i^i \geq p(\delta u_{k^1}^{k^1} - c_i)$. By the induction hypothesis, $u_{k^1}^{k^1}$ converges to $r_{N_i}^I$ as $\delta \rightarrow 1$. Therefore, $\liminf_{\delta \rightarrow 1} u_i^i \geq p(r_{N_i}^I - c_i)$.

For $k = k^1$, (A.7) can be rewritten as $u_i^i(1 - (1-p)\delta) + p\delta u_{k^1}^i \geq p(\delta u_{k^1}^{k^1} - c_i)$, which implies that (recall that k^1 is a function of δ)

$$\liminf_{\delta \rightarrow 1} (u_i^i + u_{k^1}^i) \geq \lim_{\delta \rightarrow 1} u_{k^1}^{k^1} - c_i = r_{N_i}^I - c_i.$$

Since (A.6) holds for every $\delta \in \Delta$, we obtain

$$\limsup_{\delta \rightarrow 1} \left(u_i^i + \sum_{k \in N_i} u_k^i \right) \leq r_{N_i}^I - c_i.$$

Consider a player $k^2 \in \arg \max_{k \in N_i \setminus \{k^1\}} u_k^k$ (k^2 is also a function of δ). The last two display equations imply that $\lim_{\delta \rightarrow 1} u_{k^2}^i = 0$. The incentive constraint (A.7) for $k = k^2$ leads to $u_i^i(1 - (1 - p)\delta) + p\delta u_{k^2}^i \geq p(\delta u_{k^2}^k - c_i)$. Since $\lim_{\delta \rightarrow 1} u_{k^2}^i = 0$ and, by the induction hypothesis, $\lim_{\delta \rightarrow 1} u_{k^2}^k = r_{N_i}^I$, we obtain $\liminf_{\delta \rightarrow 1} u_i^i \geq r_{N_i}^I - c_i$.

Thus far, we established that

$$(A.13) \quad \liminf_{\delta \rightarrow 1} u_i^i \geq \max(p(r_{N_i}^I - c_i), r_{N_i}^I - c_i).$$

We next prove by contradiction that

$$(A.14) \quad \limsup_{\delta \rightarrow 1} u_i^i \leq \max(p(r_{N_i}^I - c_i), r_{N_i}^I - c_i).$$

Suppose that (A.14) does not hold. Then there exists a sequence \mathcal{S} of δ approaching 1,²⁰ along which u_i^i converges to a limit greater than $\max(p(r_{N_i}^I - c_i), r_{N_i}^I - c_i)$. \mathcal{S} can be chosen to satisfy in addition one of the following properties:

- (i) there exists $k \in N_i$ such that for all $\delta \in \mathcal{S}$, current seller i selects k for bargaining with conditional probability 1 in the MPE;
- (ii) there exist $k \neq h \in N_i$ such that for all $\delta \in \mathcal{S}$, $u_k^k \geq u_h^h$ and current seller i bargains with positive conditional probability with both k and h .

In case (i), the analysis above establishes that for all $\delta \in \mathcal{S}$,

$$\begin{aligned} u_i^i &= p(\delta u_k^k - c_i - \delta u_k^i) + (1 - p)\delta u_i^i \\ u_k^k &= p\delta u_k^i + (1 - p)(\delta u_k^k - c_i - \delta u_k^i). \end{aligned}$$

The system of equations is immediately solved to obtain $u_i^i = p(\delta u_k^k - c_i)$. Then the induction hypothesis implies that $\lim_{\delta \in \mathcal{S}} u_i^i = p(r_k - c_i) \leq p(r_{N_i}^I - c_i)$.

In case (ii), it must be that $u_i^i = p(\delta u_h^h - c_i - \delta u_h^i) + (1 - p)\delta u_i^i$, which implies that

$$(A.15) \quad u_i^i(1 - (1 - p)\delta) \leq p(\delta u_h^h - c_i).$$

Note that $u_k^k \geq u_h^h$, along with the induction hypothesis, leads to $r_k = \lim_{\delta \rightarrow 1} u_k^k \geq \lim_{\delta \rightarrow 1} u_h^h = r_h$. In particular, $r_h \leq r_{N_i}^I$. Taking the limit for δ along \mathcal{S} in (A.15), we immediately find that $\lim_{\delta \in \mathcal{S}} u_i^i \leq r_{N_i}^I - c_i$.

In either case, we obtained a contradiction with the assumption that $\lim_{\delta \in \mathcal{S}} u_i^i > \max(p(r_{N_i}^I - c_i), r_{N_i}^I - c_i)$. Therefore, (A.14) holds, implying along with (A.13) that u_i^i converges as $\delta \rightarrow 1$ to $r_i = \max(p(r_{N_i}^I - c_i), r_{N_i}^I - c_i) = \max(p(r_{N_i}^I - c_i), r_{N_i}^I - c_i, 0)$ (recall that $r_{N_i}^I \geq c_i$ if $\sup \Delta < 1$). This completes the proof of the inductive step. \square

Proof of Proposition 2. We construct recursively a profile $(u_{k'}^k)_{k, k' \in N}$ that describes the payoffs in each subgame in an MPE of the intermediation game. For $k = \overline{m+1, n}$ we simply

²⁰To simplify notation, we write $\delta \in \mathcal{S}$ to represent the fact that δ appears in the sequence \mathcal{S} and use the shorthand $\lim_{\delta \rightarrow \mathcal{S} 1} f$ for the limit of a function $f : (0, 1) \rightarrow \mathbb{R}$ along the sequence \mathcal{S} .

set $u_k^k = v_k/\delta$ and $u_{k'}^k = 0$ for all $k' \neq k$ as discussed in Section 4. For $i \leq m$, having defined the variables $u_{k'}^k$ for all $k > i$ and $k' \in N$, we derive the payoffs $(u_{k'}^i)_{k' \in N}$ as follows.

If $\delta \max_{k \in N_i} u_k^k \leq c_i$, then let $u_k^i = 0$ for all k . MPEs for subgame i in which the current seller i never trades the good, generating the desired payoffs off the equilibrium path, are easily constructed.

Assume next that $\delta \max_{k \in N_i} u_k^k > c_i$. The proof of Theorem 1 shows that the payoffs $(u_k^i)_{k \in N_i \cup \{i\}}$ and the selection probabilities $(\pi_k)_{k \in N_i}$ describing MPE outcomes in the first round of subgame i solve the system of equations

$$(A.16) \quad u_k^i = \sum_{k \in N_i} \pi_k (p(\delta u_k^k - c_i - \delta u_k^i) + (1-p)\delta u_k^i)$$

$$(A.17) \quad u_k^i = \pi_k (p\delta u_k^i + (1-p)(\delta u_k^k - c_i - \delta u_k^i)) + \sum_{h \in N_i \setminus \{k\}} \pi_h \delta u_k^h, \forall k \in N_i,$$

where the variables $(u_k^h)_{k,h \in N_i}$ have been previously specified.

Consider an arbitrary vector $\pi = (\pi_k)_{k \in N_i}$ describing a probability distribution over N_i . Let f^π denote the function that takes any $u^i := (u_k^i)_{k \in N_i \cup \{i\}}$ (small abuse of notation) to $\mathbb{R}^{N_i \cup \{i\}}$ with component $k \in N_i \cup \{i\}$ defined by the right-hand side of the equation for u_k^i in the system (A.16)-(A.17). Then u^i solves the system for the given π if and only if it is a fixed point of f^π . One can easily check that f^π is a contraction of modulus δ with respect to the sup norm on $\mathbb{R}^{N_i \cup \{i\}}$. By the contraction mapping theorem, f^π has a unique fixed point. This means that the system of linear equations (A.16)-(A.17) with unknowns u^i is non-singular and can be solved using Cramer's rule. The components of the unique solution, which we denote by $\tilde{u}(\pi)$, are given by ratios of determinants that vary continuously in π .

For any $u^i \in \mathbb{R}^{N_i \cup \{i\}}$, let $\tilde{\pi}(u^i)$ denote the set of probability mass functions over N_i that are consistent with optimization by seller i , given the variables u^i and the previously determined resale values $(u_k^k)_{k \in N_i}$. That is, $\tilde{\pi}(u^i)$ contains all π such that $\pi_k > 0$ only if $k \in \arg \max_{h \in N_i} u_h^h - u_h^i$. Clearly, $\tilde{\pi}$ has a closed graph and is convex valued.

By Kakutani's theorem, the correspondence $\pi \rightrightarrows \tilde{\pi}(\tilde{u}(\pi))$ has a fixed point π^* . Then $(u_k^i)_{k \in N_i \cup \{i\}} = \tilde{u}(\pi^*)$ and $(\pi_k)_{k \in N_i} = \pi^*$ satisfy the equilibrium constraints (A.7)-(A.8), as well as the system of equations (A.16)-(A.17). Also define $u_k^i = \sum_{h \in N_i} \pi_h u_k^h$ for $k \notin N_i \cup \{i\}$. One can easily specify strategies that constitute an MPE for subgame i and yield the desired payoffs provided that $u_k^i \geq 0$ for all $k \in N_i \cup \{i\}$. The rest of the proof demonstrates that the constructed values are indeed non-negative.

We prove that $u_k^i > 0$ by contradiction. Suppose that $u_k^i \leq 0$. Fix $k^1 \in \arg \max_{k \in N_i} u_k^k$. Since $\pi \in \tilde{\pi}(u^i)$, it must be that $u_h^h - u_h^i \geq u_{k^1}^k - u_{k^1}^i$ whenever $\pi_h > 0$. By construction, $(u_k^h)_{k \in N}$ constitute MPE payoffs for subgame h . Retracing the steps that establish (A.10) in

Theorem 1, we find that $u_{k^1}^h \leq u_{k^1}^i$ if $\pi_h > 0$. Then (A.17) implies that

$$(A.18) \quad u_{k^1}^i \leq \pi_{k^1} \left(p\delta u_{k^1}^i + (1-p)(\delta u_{k^1}^{k^1} - c_i - \delta u_{k^1}^i) \right) + \sum_{h \in N_i \setminus \{k^1\}} \pi_h \delta u_{k^1}^i.$$

Setting $k = k^1$ in (A.7), we obtain

$$(A.19) \quad u_i^i \geq p(\delta u_{k^1}^{k^1} - c_i - \delta u_{k^1}^i) + (1-p)\delta u_i^i.$$

Under the assumptions $u_i^i \leq 0$ and $\delta u_{k^1}^{k^1} > c_i$, we have

$$p\delta u_{k^1}^i + (1-p)(\delta u_{k^1}^{k^1} - c_i - \delta u_{k^1}^i) = \delta u_{k^1}^{k^1} - c_i - \left(p(\delta u_{k^1}^{k^1} - c_i - \delta u_{k^1}^i) + (1-p)\delta u_i^i \right) > 0.$$

Then (A.18) implies that

$$(A.20) \quad u_{k^1}^i \leq \frac{\pi_{k^1}}{1 - \delta \sum_{h \in N_i \setminus \{k^1\}} \pi_h} \left(p\delta u_{k^1}^i + (1-p)(\delta u_{k^1}^{k^1} - c_i - \delta u_{k^1}^i) \right) \leq p\delta u_{k^1}^i + (1-p)(\delta u_{k^1}^{k^1} - c_i - \delta u_{k^1}^i).$$

As in the proof of Theorem 1, inequalities (A.19) and (A.20) imply that $u_i^i \geq p(\delta u_{k^1}^{k^1} - c_i) > 0$, a contradiction.

We showed that $u_i^i > 0$. If $\pi_k > 0$, then $u_i^i = p(\delta u_k^k - c_i - \delta u_k^i) + (1-p)\delta u_i^i$, which implies that $\delta u_k^k - c_i - \delta u_k^i > \delta u_i^i$. It follows that $\delta u_k^k - c_i - \delta u_k^i > \delta u_k^i$ whenever $\pi_k > 0$. Then (A.17) leads to

$$u_k^i \geq \pi_k \delta u_k^i + \sum_{h \in N_i \setminus \{k\}} \pi_h \delta u_k^h$$

for all $k \in N_i$. Therefore, $u_k^i \geq \delta \sum_{h \in N_i \setminus \{k\}} \pi_h u_k^h / (1 - \delta \pi_k) \geq 0$ for all $k \in N_i$. \square

Proof of Theorem 2. We prove that

$$(A.21) \quad i \in \mathcal{L}_\ell \implies r_i \in \left[p^\ell v - \sum_{\ell'=0}^{\ell} p^{\ell-\ell'} \sum_{k \in \mathcal{L}_{\ell'}, i \leq k \leq m} c_k, p^\ell v \right]$$

by backward induction on i , for $i = n, n-1, \dots, 0$. The base cases $i = n, \dots, m+1$ (corresponding to buyers) are trivially verified. Assuming that the induction hypothesis holds for players $n, \dots, i+1$, we seek to prove it for seller i ($\leq m$). Suppose that $i \in \mathcal{L}_\ell$.

We first show that $r_i \leq p^\ell v$. By Theorem 1, $r_i = \max(p(r_h - c_i), r_{h'} - c_i, 0)$ for some $h, h' \in N_i$ with $r_h = r_{N_i}^I \geq r_{N_i}^{II} = r_{h'}$.²¹ Since $i \in \mathcal{L}_\ell$, the layer structure entails that $h, h' \in \bigcup_{\ell' \geq \ell-1} \mathcal{L}_{\ell'}$. Then the induction hypothesis for player h implies that $r_h \leq p^{\ell-1}v$.

Similarly, if $h' \notin \mathcal{L}_{\ell-1}$, then the induction hypothesis leads to $r_{h'} \leq p^\ell v$. Assume instead that $h' \in \mathcal{L}_{\ell-1}$. If $h \in \mathcal{L}_{\ell-1}$ as well, then the construction of layer $\ell-1$ implies that $i \in \mathcal{L}_{\ell-1}$, a contradiction. Hence $h \in \bigcup_{\ell' \geq \ell} \mathcal{L}_{\ell'}$. Then the induction hypothesis for player h leads to $r_h \leq r_h \leq p^\ell v$. In either case, $r_{h'} \leq p^\ell v$. Since $r_h \leq p^{\ell-1}v$, $r_{h'} \leq p^\ell v$ and $c_i, v \geq 0$, we obtain that $r_i = \max(p(r_h - c_i), r_{h'} - c_i, 0) \leq p^\ell v$.

²¹The argument only becomes simpler if $|N_i|=1$ and no such h' exists.

We next show that $r_i \geq p^\ell v - \sum_{\ell'=0}^{\ell} p^{\ell-\ell'} \sum_{k \in \mathcal{L}_{\ell'}, i \leq k \leq m} c_k$. Since $i \in \mathcal{L}_\ell$, it must be that i has out-links to either (exactly) one layer $\ell - 1$ player or (at least) two layer ℓ players. In the first case, let $\{h\} = N_i \cap \mathcal{L}_{\ell-1}$. By Theorem 1, $r_i \geq p(r_h - c_i)$. The induction hypothesis implies that $r_h \geq p^{\ell-1} v - \sum_{\ell'=0}^{\ell-1} p^{\ell-\ell'-1} \sum_{k \in \mathcal{L}_{\ell'}, h \leq k \leq m} c_k$. Therefore,

$$r_i \geq p^\ell v - p c_i - \sum_{\ell'=0}^{\ell-1} p^{\ell-\ell'} \sum_{k \in \mathcal{L}_{\ell'}, h \leq k \leq m} c_k \geq p^\ell v - \sum_{\ell'=0}^{\ell} p^{\ell-\ell'} \sum_{k \in \mathcal{L}_{\ell'}, i \leq k \leq m} c_k.$$

In the second case, N_i contains at least two players from layer ℓ . Hence there exists $h \in N_i \cap \mathcal{L}_\ell$ with $r_h \leq r_{N_i}^H$. By Theorem 1, $r_i \geq r_h - c_i$. The induction hypothesis implies that $r_h \geq p^\ell v - \sum_{\ell'=0}^{\ell} p^{\ell-\ell'} \sum_{k \in \mathcal{L}_{\ell'}, h \leq k \leq m} c_k$. It follows that

$$r_i \geq p^\ell v - c_i - \sum_{\ell'=0}^{\ell} p^{\ell-\ell'} \sum_{k \in \mathcal{L}_{\ell'}, h \leq k \leq m} c_k \geq p^\ell v - \sum_{\ell'=0}^{\ell} p^{\ell-\ell'} \sum_{k \in \mathcal{L}_{\ell'}, i \leq k \leq m} c_k.$$

This completes the proof of the inductive step. The last claim of the theorem is an immediate consequence of (A.21). \square

Proof of Proposition 3. Note that the first part of the theorem follows immediately from the second. Indeed, every player from layer ℓ not directly connected to layer $\ell - 1$ has two downstream neighbors in layer ℓ . If the second part of the result is true, the two neighbors provide non-overlapping paths of layer ℓ intermediaries to layer $\ell - 1$.

To establish the second part, fix $\ell \geq 1$ and assume that $\mathcal{L}_\ell = \{i_1, i_2, \dots, i_{\bar{s}}\}$ with $i_1 < i_2 < \dots < i_{\bar{s}}$. Two directed paths in G (possibly degenerate, consisting of a single node) are called *independent* if they connect disjoint sets of layer ℓ players and end with nodes that are directly linked to layer $\ell - 1$. We say that players i_s and $i_{s'}$ *have independent paths* if there exist independent paths that originate from nodes i_s and $i_{s'}$, respectively.

We prove by backward induction on s , for $s = \bar{s}, \bar{s} - 1, \dots, 1$, that i_s and $i_{s'}$ have independent paths for all $s' > s$. For the induction base case $s = \bar{s}$, the claim is vacuously true.

Assuming that the induction hypothesis is true for all higher indices, we set out to prove it for s . Fix $s' > s$. If i_s is directly linked to a layer $\ell - 1$ node, then the degenerate path consisting of node i_s alone forms an independent pair with any layer ℓ path connecting node $i_{s'}$ to a player linked directly to layer $\ell - 1$. Player i_s does not belong to any such path because $i_s < i_{s'}$.

By the construction of \mathcal{L}_ℓ , if i_s is not directly linked to $\mathcal{L}_{\ell-1}$, then i_s must have at least two neighbors in \mathcal{L}_ℓ . Hence i_s is linked to some player $i_{s''} > i_s$ different from $i_{s'}$. The induction hypothesis applied for step $\min(s', s'') > s$ implies the existence of two independent paths originating from $i_{s'}$ and $i_{s''}$, respectively. If we append the link $(i_s, i_{s''})$ to the path originating from $i_{s''}$, we obtain a new path, starting with i_s , which forms an independent pair with the

path from $i_{s'}$. The constructed paths are indeed disjoint. Player i_s does not belong to the path of $i_{s'}$ because $i_s < i_{s'} \leq i$ for all nodes i along the latter path. This completes the proof of the inductive step. \square

Proof of Proposition 5. Suppose that a current seller $i \in \mathcal{L}_{\ell'}$ sells the good with positive conditional probability to some trader $k \in \mathcal{L}_{\ell}$ for a sequence \mathcal{S} of δ 's converging to 1. By Theorem 2, the limit resale values of players i and k are $p^{\ell'}v$ and $p^{\ell}v$, respectively. For $\delta \in \mathcal{S}$, since seller i chooses k as a bargaining partner with positive probability, i 's expected payoff conditional on selecting k must converge to $p^{\ell'}v$.

For every $\varepsilon > 0$, players i and k cannot reach an agreement at a price above $p^{\ell}v + \varepsilon$ for high δ , since k has a limit resale value of $p^{\ell}v$. Hence i 's limit payoff for $\delta \in \mathcal{S}$ conditional on bargaining with k cannot exceed $p^{\ell}v$. Since the conditional limit payoff was determined to be $p^{\ell'}v$, it follows that $\ell' \geq \ell$. As $k \in N_i \cap \mathcal{L}_{\ell}$, it must be that $\ell' \in \{\ell, \ell + 1\}$. This shows that for sufficiently high δ , player $k \in \mathcal{L}_{\ell}$ acquires the good in equilibrium only from sellers in layer ℓ or $\ell + 1$.

Since seller i 's limit resale value is $p^{\ell'}v$, the price offers that i receives from k converge to $p^{\ell'}v$ as $\delta \rightarrow 1$. For $\delta \in \mathcal{S}$, player i 's expected payoff from bargaining with k converges to $p^{\ell'}v$, so the price offers that i makes to k must converge to $p^{\ell'}v$ as well. Therefore, regardless of nature's draw of the proposer in the match (i, k) , player k purchases the good at a limit price of $p^{\ell'}v$. As k has a limit resale value of $p^{\ell}v$, his limit profit conditional on being selected by i as a trading partner is $(p^{\ell} - p^{\ell'})v$. The conclusion follows from the fact that i belongs to layer $\ell' \in \{\ell, \ell + 1\}$. \square

Proof of Proposition 6. By Theorem 1, the addition of a link (i, k) to a network weakly increases the (limit) resale value of player i and does not affect the resale value of any player $h > i$. Then a simple inductive argument invoking Theorem 1 proves that the resale values of all players $h \leq i$, including the initial seller, weakly increase when the link (i, k) is added.

We next prove that the elimination of a middleman weakly increases the initial seller's limit profit. Let $\tilde{G} = (\tilde{N}, (\tilde{N}_i), (\tilde{c}_i), (v_j))$ be the network obtained by eliminating middleman i from network $G = (N, (N_i)_{i=\overline{0,m}}, (c_i)_{i=\overline{0,m}}, (v_j)_{j=\overline{m+1,n}})$. Let $(r_k)_{k \in N}$ and $(\tilde{r}_k)_{k \in N \setminus \{i\}}$ denote the vectors of resale values in G and \tilde{G} , respectively.

Fix k such that $i \in N_k$. To elucidate the implications of k relying on i in G , note that the removal of the link (k, i) from G does not affect the resale value of any players with labels greater than k ; in particular, it does not change the resale values of players in N_k . Then, by Theorem 1, player k relies on i if and only if

$$(A.22) \quad (r_k =) \max(p(r_{N_k}^I - c_k), r_{N_k}^{II} - c_k, 0) > \max(p(r_{N_k \setminus \{i\}}^I - c_k), r_{N_k \setminus \{i\}}^{II} - c_k, 0),$$

and does not if and only if inequality is replaced by equality above.

We now show that if k relies on i then $r_k \leq r_i - c_k$. Indeed, if k relies on i then (A.22) implies that either $r_k = p(r_{N_k}^I - c_k) > p(r_{N_k \setminus \{i\}}^I - c_k)$ or $r_k = r_{N_k}^{II} - c_k > r_{N_k \setminus \{i\}}^{II} - c_k$. In the first case, we have $r_{N_k}^I > r_{N_k \setminus \{i\}}^I$, which means that $r_i = r_{N_k}^I$. Hence $r_k = p(r_i - c_k) \geq 0$, giving rise to $r_k \leq r_i - c_k$. In the second case, we obtain that $r_{N_k}^{II} > r_{N_k \setminus \{i\}}^{II}$, which is possible only if $r_i \geq r_{N_k}^{II}$. Then $r_k = r_{N_k}^{II} - c_k \leq r_i - c_k$. In both cases, we established that $r_k \leq r_i - c_k$.

We prove by reverse induction on k , for $k = n, n-1, \dots, i+1, i-1, \dots, 0$, that $r_k \leq \tilde{r}_k$. For the base cases $k = n, n-1, \dots, i+1$, it is obvious that $r_k = \tilde{r}_k$. Assuming that the induction hypothesis holds for all players different from i with labels greater than k , we aim to prove it for player $k < i$. Consider two cases:

- (i) $i \in N_k$ and k relies on i ;
- (ii) $i \notin N_k$ or $i \in N_k$ but k does not rely on i .

In case (i), since k relies on i , we have $r_k \leq r_i - c_k$. Then Theorem 1 leads to

$$\begin{aligned} r_k &\leq r_i - c_k = \max(p(r_{N_i}^I - c_i) - c_k, r_{N_i}^{II} - c_i - c_k, -c_k) \\ &\leq \max(p(r_{N_i}^I - c_k - c_i), r_{N_i}^{II} - c_k - c_i, 0) = \max(p(\tilde{r}_{N_i}^I - \tilde{c}_k), \tilde{r}_{N_i}^{II} - \tilde{c}_k, 0) \\ &\leq \max(p(\tilde{r}_{\tilde{N}_k}^I - \tilde{c}_k), \tilde{r}_{\tilde{N}_k}^{II} - \tilde{c}_k, 0) = \tilde{r}_k. \end{aligned}$$

The sequence of equalities and inequalities above uses the following conditions: $c_k \geq 0, r_h = \tilde{r}_h, \forall h \in N_i \subset \tilde{N}_k$ and $\tilde{c}_k = c_k + c_i$.

In case (ii), either $i \notin N_k$ or $i \in N_k$ and k does not rely on i implies that $r_k = \max(p(r_{N_k \setminus \{i\}}^I - c_k), r_{N_k \setminus \{i\}}^{II} - c_k, 0)$. By the induction hypothesis, $r_h \leq \tilde{r}_h$ for all $h \in N_k \setminus \{i\}$. Theorem 1 then leads to

$$r_k = \max(p(r_{N_k \setminus \{i\}}^I - c_k), r_{N_k \setminus \{i\}}^{II} - c_k, 0) \leq \max(p(\tilde{r}_{N_k \setminus \{i\}}^I - c_k), \tilde{r}_{N_k \setminus \{i\}}^{II} - c_k, 0) = \tilde{r}_k.$$

This completes the proof of the induction step. Step $k = 0$ yields the desired inequality, $r_0 \leq \tilde{r}_0$. \square

Proof of Proposition 7.1. Consider the network \tilde{G} obtained from the vertical integration of tiers τ and $\tau + 1$ in a tier network G . Let (N_i, c_i, r_i) and $(\tilde{N}_i, \tilde{c}_i, \tilde{r}_i)$ denote the sets of downstream neighbors, the transaction costs, and the limit resale values of seller i in G and \tilde{G} (when defined), respectively.

Let i be a seller from tier τ in G . Suppose that $k \in \arg \max_{h \in N_i} r_h$. Player k is an intermediary belonging to tier $\tau + 1$ in G . Note that the pattern of connections among traders situated downstream of tiers $\tau + 2$ in G and $\tau + 1$ in \tilde{G} is identical in G and \tilde{G} , respectively. Then $r_h = \tilde{r}_h$ for all $h \in \tilde{N}_i$, as \tilde{N}_i is a subset of tier $\tau + 2$ in G as well as tier $\tau + 1$ in \tilde{G} .

By Theorem 1, $r_i = \max(p(r_{N_i}^I - c_i), r_{N_i}^{II} - c_i, 0) \leq \max(r_k - c_i, 0)$. Therefore,

$$\begin{aligned}
r_i &\leq \max(r_k - c_i, 0) \\
&= \max(p(r_{N_k}^I - c_k) - c_i, r_{N_k}^{II} - c_i - c_k, 0) \\
&\leq \max(p(r_{N_k}^I - c_i - c_k), r_{N_k}^{II} - c_i - c_k, 0) \\
&\leq \max(p(\tilde{r}_{N_i}^I - \tilde{c}_i), \tilde{r}_{N_i}^{II} - \tilde{c}_i, 0) \\
&= \tilde{r}_i,
\end{aligned}$$

where the last inequality uses the following consequences of vertical integration: $\tilde{c}_i = c_i + c_k$, $N_k \subset \bigcup_{h \in N_i} N_h = \tilde{N}_i$, and $r_h = \tilde{r}_h$ for all $h \in \tilde{N}_i$.

We established that $\tilde{r}_i \geq r_i$ for every seller i from tier τ in G . Since the connections among tiers 0 through τ and the costs of tiers 0 through $\tau - 1$ are identical in G and \tilde{G} , a simple inductive argument invoking Theorem 1 shows that $\tilde{r}_i \geq r_i$ for every trader i in tiers $0, \dots, \tau - 1$. In particular, $\tilde{r}_0 \geq r_0$. \square

Proof of Theorem 3. It is useful to generalize the concept of cost domination as follows. For $\kappa \geq 0$, a cost pattern \tilde{c} κ -dominates another c at some node $i \in N$ if for every path of sellers originating at i , $i = i_0, i_1, \dots, i_{\bar{s}} \leq m$ ($i_s \in N_{i_{s-1}}$, $s = \overline{1, \bar{s}}$), including the case $\bar{s} = 0$,

$$(A.23) \quad \kappa + \sum_{s=0}^{\bar{s}} \tilde{c}_{i_s} \geq \sum_{s=0}^{\bar{s}} c_{i_s}.$$

The condition above is vacuously satisfied if i is a buyer since there are no seller paths originating at i in that case.

We prove by backward induction on i , for $i = n, n - 1, \dots, 0$, that for any c and \tilde{c} such that \tilde{c} κ -dominates c at node i for $\kappa \geq 0$, we have $r_i + \kappa \geq \tilde{r}_i$, where r_i and \tilde{r}_i denote player i 's limit resale values in the intermediation game under cost structures c and \tilde{c} , respectively. The result to prove is a special case of the claim above, for $i = 0$ and $\kappa = 0$.

For buyers i , we have $r_i = \tilde{r}_i = v_i$, which along with $\kappa \geq 0$ proves the base cases $i = n, \dots, m + 1$. Assuming that the induction hypothesis holds for players $n, \dots, i + 1$, we seek to prove it for seller i ($\leq m$). Suppose that \tilde{c} κ -dominates c at i , and let r and \tilde{r} denote the vectors of resale values under c and \tilde{c} , respectively.

Fix $k \in N_i$. Considering all seller paths starting with the link (i, k) in (A.23), we can check that \tilde{c} ($\kappa + \tilde{c}_i - c_i$)-dominates c at k . The domination inequality (A.23) for the degenerate path consisting of the single node i becomes $\kappa + \tilde{c}_i - c_i \geq 0$. The induction hypothesis applied to $k > i$ implies that $r_k + \kappa + \tilde{c}_i - c_i \geq \tilde{r}_k$, or equivalently, $r_k - c_i + \kappa \geq \tilde{r}_k - \tilde{c}_i$. Since the latter inequality holds for all $k \in N_i$, it must be that $r_{N_i}^I - c_i + \kappa \geq \tilde{r}_{N_i}^I - \tilde{c}_i$ and

$r_{N_i}^H - c_i + \kappa \geq \tilde{r}_{N_i}^H - \tilde{c}_i$. Then Theorem 1 leads to

$$\begin{aligned} \tilde{r}_i &= \max(p(\tilde{r}_{N_i}^I - \tilde{c}_i), \tilde{r}_{N_i}^H - \tilde{c}_i, 0) \\ &\leq \max(p(r_{N_i}^I - c_i + \kappa), r_{N_i}^H - c_i + \kappa, 0) \\ &\leq \max(p(r_{N_i}^I - c_i), r_{N_i}^H - c_i, 0) + \kappa \\ &= r_i + \kappa, \end{aligned}$$

where the last inequality relies on $\kappa \geq 0$. This completes the proof of the inductive step. \square

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