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When Does Labor Scarcity Encourage Innovation?

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This paper studies whether labor scarcity encourages technological advances, that is, technology adoption or innovation, for example, as claimed by Habakkuk in the context of nineteenth-century United States. I define technology as strongly labor saving if technological advances reduce the marginal product of labor and as strongly labor complementary if they increase it. I show that labor scarcity encourages technological advances if technology is strongly labor saving and will discourage them if technology is strongly labor complementary. I also show that technology can be strongly labor saving in plausible environments but not in many canonical macroeconomic models.

I. Introduction

There is widespread consensus that technological differences are a central determinant of productivity differences across firms, regions, and nations. Despite this consensus, determinants of technological progress and adoption of new technologies are poorly understood. A basic question concerns the relationship between factor endowments and technology, for example, whether the scarcity of a factor and the high factor prices that this leads to will induce technological progress. There is currently no comprehensive answer to this question, though a large literature develops conjectures on this topic. In his pioneering work *The*

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Theory of Wages, John Hicks was one of the first economists to consider this possibility and argued that "a change in the relative prices of the factors of production is itself a spur to invention, and to invention of a particular kind—directed to economizing the use of a factor which has become relatively expensive" (1932, 124).

Similarly, the famous *Habakkuk hypothesis* in economic history, proposed by H. J. Habakkuk (1962), claims that technological progress was more rapid in the nineteenth-century United States than in Britain because of labor scarcity in the former country, which acted as a powerful inducement for mechanization, for the adoption of labor-saving technologies, and more broadly for innovation.[1] For example, Habakkuk quotes from Pelling: "it was scarcity of labor 'which laid the foundation for the future continuous progress of American industry, by obliging manufacturers to take every opportunity of installing new types of labor-saving machinery'" (6). Habakkuk continues: "It seems obvious—it certainly seemed so to contemporaries—that the dearness and inelasticity of American, compared with British, labour gave the American entrepreneur . . . a greater inducement than his British counterpart to replace labour by machines" (17).

Robert Allen (2009) has more recently argued that the relatively high wages in eighteenth-century Britain were the main driver of the Industrial Revolution. For example, three of the most important eighteenth-century technologies, Hargreaves's spinning jenny and Arkwright's water frame and carding machine, reduced labor costs in cotton manufacturing significantly. They not only were invented in Britain but rapidly spread there, whereas their adoption was much slower in France and India. Allen (chap. 8) suggests that the reason was that these technologies were less profitable in France and India, where wages and thus savings in labor costs from their adoption were lower. Elvin (1972) similarly suggests that a sophisticated spinning wheel used for hemp in fourteenth-century China was later abandoned and was not used for cotton largely because cheap and abundant Chinese labor made it unprofitable.

Similar ideas are often suggested as possible reasons why high wages, for example, induced by minimum wages or other regulations, might have encouraged faster adoption of certain technologies, particularly those complementary to unskilled labor, in continental Europe (see, e.g., Beaudry and Collard 2002; Acemoglu 2003; Alesina and Zeira 2006). The so-called Porter hypothesis, which claims that tighter environmental regulations will spur faster innovation and increase produc-

[1] See Rothbarth (1946), Salter (1966), David (1975), Stewart (1977), and Mokyr (1990) for related ideas and discussions of the Habakkuk hypothesis.

tivity, is also related.[2] While this hypothesis plays a major role in various discussions of environmental policy, just like the Habakkuk hypothesis, its theoretical foundations are unclear.[3]

These conjectures seem plausible at first. Intuitions based on a downward-sloping demand curve suggest that if a factor becomes more expensive, the demand for it should decrease, and we may expect some of this adjustment to take place by technology substituting for tasks previously performed by that factor. It seems compelling, for example, that technologies such as the spinning jenny, the water frame, and the carding machine, which reduced the amount of labor required to produce a given quantity of cotton, should have been invented and adopted in places where the labor that they saved was more scarce and expensive. And yet, labor scarcity and high wages also reduce both the size of the workforce that may use the new technologies and the profitability of firms, and they could discourage technology adoption through both channels. In fact, labor scarcity and high wages discourage technological advances in the most commonly used macroeconomic models. Neoclassical growth models, when new technologies are embodied in capital goods, predict that labor scarcity and high wages slow down the adoption of new technologies.[4] Endogenous growth models also make the same prediction because lower employment discourages entry and the introduction of new technologies.[5]

This paper investigates the impact of labor scarcity on *technological advances* (i.e., innovation and adoption of technologies that increase the level of output in the economy) and offers a comprehensive answer

[2] See Porter (1991) and Porter and van der Linde (1995) for the formulation of this hypothesis. Jaffe et al. (1995) review early empirical evidence on this topic, and Newell, Jaffe, and Stavins (1999) provide evidence on the effects of energy prices on the direction of technological change. Recent work by Gans (2009) provides a theoretical explanation for the Porter hypothesis using the framework presented here.

[3] Related issues also arise in the context of the study of the implications of competition from Chinese imports on technological progress. Bloom, Draca, and Van Reenen (2009), e.g., provide evidence that Chinese competition has encouraged innovation and productivity growth among affected U.S. and European firms. One of the numerous impacts of Chinese competition is to reduce employment in the affected sectors. This creates a parallel between the aggregate impact of labor scarcity and the sectoral effects of Chinese competition. A priori, it is not clear whether we should expect more or less investment in innovation and technology in these sectors.

[4] See Ricardo (1951) for an early statement of this view. In particular, with a constant returns to scale production function $F(L, K)$, an increase in the price of L or a reduction in its supply will reduce equilibrium K, and to the extent that technology is embedded in capital, it will reduce technology adoption.

[5] In the first-generation models, such as Romer (1986, 1990), Segerstrom, Anant, and Dinopoulos (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992), it reduces the growth rate of technology and output, whereas in "semi-endogenous" growth models, such as Jones (1995), Young (1998), and Howitt (1999), it reduces their levels.

to this question. If technology is *strongly labor complementary*,[6] meaning that improvements in technology increase the marginal product of labor, then labor scarcity discourages technological advances (e.g., it makes such advances less likely or, in a dynamic framework, it slows down the pace of technological advances).[7] Conversely, if technology is *strongly labor saving*, meaning that improvements in technology reduce the marginal product of labor, then labor scarcity induces technological advances.[8]

The main result in this paper can be interpreted as both a positive and a negative one. On the positive side, it characterizes a wide range of economic environments in which labor scarcity can act as a force toward innovation and technology adoption, as claimed in various previous historical and economic analyses. On the negative side, it shows that this can be so only if new technology tends to reduce the marginal product of labor. This observation, in particular, implies that in most models used in the macroeconomics and growth literatures, where technological advances are assumed to increase the marginal product of labor, labor scarcity will discourage rather than induce technological advances.[9] It also implies that the relationship between labor scarcity and technological advances can vary over different epochs. It may well be that the technological advances of the late eighteenth and nineteenth centuries in Britain and the United States were strongly labor saving and did induce innovation and technology adoption, as envisaged by

[6] The word "strongly" is added since the terms "labor complementary" and "labor saving" are often used in several different contexts, not always satisfying the definitions here.

[7] More precisely, we need that aggregate output (or net output) can be expressed as a function $Y(L, Z, \theta)$, where L denotes labor, Z is a vector of other factors of production, and θ is a vector of technologies, and Y is *supermodular* in θ, so that changes in two components of the vector θ do not offset each other. We say that technology is strongly labor saving if an increase in θ reduces the marginal product of L in $Y(L, Z, \theta)$ and is strongly labor complementary if it increases this marginal product.

[8] Notably, in line with the directed technological change literature (e.g., Acemoglu 1998, 2002, 2007), an increase in the supply of a factor still induces a change in technology biased toward that factor, and thus labor scarcity makes technology biased against labor. In particular, recall that a change in technology is biased toward a factor if it increases the marginal product of this factor at given factor proportions. When technology is strongly labor complementary, labor scarcity discourages technological advances, and this is biased against labor. When technology is strongly labor saving, labor scarcity induces technological advances; in this case, because there is technology-labor substitutability, this again reduces the marginal product of labor and is thus biased against labor. As a consequence, even though changes in technology in response to an increase in the supply of a factor might induce or discourage technological advances, they will always be biased toward that factor.

[9] The fact that technological change has been the key driving force of the secular increase in wages also suggests that it may be more plausible to think of technology as strongly labor complementary rather than strongly labor saving. Nevertheless, it is possible for labor-saving technology to increase wages in the long run because past technological changes may increase wages whereas current technology adoption decisions, at the margin, reduce the marginal product of labor. This is illustrated by the dynamic model presented in Sec. V.A.

many contemporary commentators and more recently by H. J. Habakkuk and Robert Allen;[10] this may no longer be the case in industrialized economies or even anywhere around the world. It may also be that the relevant environmental technologies have a similar substitution property with carbon, so that an increase in the price of carbon may induce more rapid advances in environmental technology (though we will also see why the reasoning is different in this case). I further emphasize the differential effects of labor scarcity by considering a multisector economy and showing that labor scarcity may lead to technological advances in some industries while retarding them in others.

To illustrate the implications of these results, I consider several different environments and production functions and discuss when technology is strongly labor saving. An important class of models in which technological change can be strongly labor saving is developed by Champernowne (1963) and Zeira (1998, 2006) and is also related to the endogenous growth model of Hellwig and Irmen (2001). In these models, technological change takes the form of machines replacing tasks previously performed by labor. I show that there is indeed a tendency of technology to be strongly labor saving in these models.

Most of the analysis in this paper focuses on the implications of labor scarcity for technology choices. Nevertheless, these results can also be used to analyze the impact of an exogenous wage increase (e.g., due to a minimum wage or other labor market regulation) on technology choices because, in the context of a competitive labor market, such increases are equivalent to a decline in labor supply.[11] However, I also show the conditions under which the implications of labor scarcity and exogenous wage increases can be very different, particularly because the long-run relationship between labor supply and wages could be upward sloping owing to general equilibrium technology effects.

Even though the investigation here is motivated by technological change and the study of economic growth, the economic environment I use for most of the paper is static. A static framework is useful because it enables us to remove functional form restrictions that would be necessary to generate endogenous growth; it thus allows the appropriate level of generality to clarify the conditions for labor scarcity to encourage

[10] This is in fact what the Luddites, who thought that new technologies would reduce demand for their labor, feared (e.g., Mokyr 1990). Mantoux (1961) provides qualitative evidence consistent with this pattern in several industries. However, Sec. V.A shows that even when technology is strongly labor saving, technological advances may increase wages in the long run.

[11] The implications of exogenous wage increases in noncompetitive labor markets are more complex and depend on the specific aspects of labor market imperfections and institutions. For example, Acemoglu (2003) shows that wage push resulting from a minimum wage or other labor market regulations may encourage technology adoption when there is wage bargaining and rent sharing.

innovation and technology adoption. This framework is based on Acemoglu (2007) and is reviewed in Section II. The main results of this paper and some applications are presented in Section III. Section IV uses several familiar models to clarify when technology is strongly labor saving. Section V shows how the static framework can be easily extended to a dynamic setup and also discusses other extensions, including the application to a multisector economy. Section VI presents conclusions.

II. The Basic Environments

This section is based on and extends some of the results in Acemoglu (2007). Its inclusion is necessary for the development of the main results in Section III. Consider a static economy consisting of a unique final good and $N+1$ factors of production. The first factor of production is labor, denoted by L, and the rest are denoted by the vector $Z = (Z_1, \ldots, Z_N)$ and stand for land, capital, and other human or nonhuman factors. All agents' preferences are defined over the consumption of the final good. To start with, let us assume that all factors are supplied inelastically, with supplies denoted by $\bar{L} \in \mathbb{R}_+$ and $\bar{Z} \in \mathbb{R}_+^N$. Throughout I focus on comparative statics with respect to changes in the supply of labor while holding the supply of other factors, Z, constant at some level $\bar{Z}$ (though, clearly, mathematically there is nothing special about labor).[12] The economy consists of a continuum of firms (final good producers) denoted by the set $\mathcal{F}$, each with an identical production function. Without loss of any generality let us normalize the measure of $\mathcal{F}$, $|\mathcal{F}|$, to one. The price of the final good is also normalized to one.

I first describe technology choices in three different economic environments as follows:[13]

1. Economy D (for *decentralized*) is a decentralized competitive economy in which technologies are chosen by firms themselves. In this economy, technology choice can be interpreted as the choice of just another set of factors, and the entire analysis can be conducted in terms of technology adoption.
2. Economy E (for *externality*) is identical to economy D except for a technological externality as in Romer (1986).

[12] Endogenous responses of the supply of labor and other factors, such as capital, are discussed in Secs. V.D and C.

[13] A fourth one, economy O, with several technology suppliers and oligopolistic competition is discussed in the Appendix.

3. Economy M (for *monopoly*) will be the main environment used for much of the analysis in the remainder of the paper. In this economy, technologies are created and supplied by a profit-maximizing monopolist. In this environment, technological progress enables the creation of "better machines," which can then be sold to several firms in the final good sector. Thus, economy M incorporates Romer's (1990) insight that the central aspect distinguishing "technology" from other factors of production is the nonrivalry of ideas.

A. *Economy D: Decentralized Equilibrium*

In the first environment, economy D, all markets are competitive and technology is decided by each firm separately. This environment is introduced as a benchmark.

Each firm $i \in \mathcal{F}$ has access to a production function

$$y^i = G(L^i, Z^i, \theta^i), \tag{1}$$

where $L^i \in \mathbb{R}_+$, $Z^i \in \mathbb{R}_+^N$, and $\theta^i \in \Theta \subset \mathbb{R}^K$ is the measure of technology.[14] The function G is assumed to be twice continuously differentiable and increasing in (L^i, Z^i). The cost of technology $\theta \in \Theta$ in terms of final goods is $C(\theta)$. This cost can be interpreted as a one-time cost that firms pay (e.g., the cost of installing new machinery), and in that case, (1) can be interpreted as representing the net present discounted value of revenues. Throughout I assume that $C(\theta)$ is increasing in θ.

Each final good producer maximizes profits; thus, it solves the following problem:

$$\max_{L^i, Z^i, \theta^i} \pi(L^i, Z^i, \theta^i) = G(L^i, Z^i, \theta^i) - w_L L^i - \sum_{j=1}^{N} w_{Zj} Z_j^i - C(\theta^i), \tag{2}$$

where w_L is the wage rate and w_{Zj} is the price of factor Z_j for $j = 1, \dots, N$, all taken as given by the firm. The vector of prices for Z is denoted by w_Z. Since there is a total supply $\bar{L}$ of labor and a total supply $\bar{Z}_j$ of Z_j, market clearing requires

$$\int_{i\in\mathcal{F}} L^i di \le \bar{L} \quad \text{and} \int_{i\in\mathcal{F}} Z_j^i di \le \bar{Z}_j \quad \text{for } j = 1, \dots, N, \tag{3}$$

each holding as an equality when the corresponding price is strictly positive.

An *equilibrium* in economy D is a set of decisions $\{L^i, Z^i, \theta^i\}_{i\in\mathcal{F}}$ and factor prices (w_L, w_Z) such that $\{L^i, Z^i, \theta^i\}_{i\in\mathcal{F}}$ solve (2) given prices (w_L, w_Z) and

[14] For most of the analysis, the reader may wish to think of θ as one-dimensional, though Sec. V.B explicitly uses the multidimensional formulation of technology. When θ is multidimensional, we will assume that G is supermodular in θ and Θ is a lattice (see, e.g., Topkis 1998), so that different components of θ move in the same direction.

(3) holds. I refer to any θ^i that is part of the set of equilibrium allocations, $\{L^i, Z^i, \theta^i\}_{i \in \mathcal{F}}$, as *equilibrium technology*.

In economy D, we assume that $G(L, Z, \theta) - C(\theta)$ is concave in (L, Z, θ). This is a restrictive assumption as it imposes concavity (strict concavity or constant returns to scale) jointly in the factors of production and technology. It is necessary for a competitive equilibrium in economy D to exist; the other economic environments considered below will relax this assumption.

PROPOSITION 1. Suppose that $G(L, Z, \theta) - C(\theta)$ is concave in (L, Z, θ). Then equilibrium technology θ^* in economy D is a solution to

$$\max_{\theta \in \Theta} G(\bar{L}, \bar{Z}, \theta) - C(\theta), \tag{4}$$

and any solution to this problem is an equilibrium technology.

Proposition 1 implies that to analyze equilibrium technology choices, we can simply focus on a simple maximization problem. An important implication of this proposition is that the equilibrium is a Pareto optimum (and vice versa). In particular, let us introduce the notation $Y(\bar{L}, \bar{Z}, \theta)$ to denote *net output* in the economy with factor supplies $\bar{L}$ and $\bar{Z}$ and technology θ. Clearly, in economy D,

$$Y(\bar{L}, \bar{Z}, \theta) \equiv G(\bar{L}, \bar{Z}, \theta) - C(\theta),$$

and equilibrium technology maximizes net output.

It is also straightforward to see that equilibrium factor prices are equal to the marginal products of the G function. That is, the wage rate is $w_L = \partial G(\bar{L}, \bar{Z}, \theta^*)/\partial L$, and the prices of other factors are given by $w_{Zj} = \partial G(\bar{L}, \bar{Z}, \theta^*)/\partial Z_j$ for $j = 1, \dots, N$, where θ^* is the equilibrium technology choice.

An important implication of (4) should be emphasized. Since equilibrium technology is a maximizer of $Y(\bar{L}, \bar{Z}, \theta)$, any induced small change in equilibrium technology, θ^*, cannot be construed as a *technological advance* since it will have no effect on net output at the starting factor proportions. In particular, assuming that Y is differentiable in L and θ and that the equilibrium technology θ^* is differentiable in $\bar{L}$, the change in net output in response to a change in the supply of labor, $\bar{L}$, can be written as

$$\frac{dY(\bar{L}, \bar{Z}, \theta^*)}{d\bar{L}} = \frac{\partial Y(\bar{L}, \bar{Z}, \theta^*)}{\partial \bar{L}} + \frac{\partial Y(\bar{L}, \bar{Z}, \theta^*)}{\partial \theta} \frac{\partial \theta^*}{\partial \bar{L}}, \tag{5}$$

where the second term is the induced technology effect. When this term is strictly negative, a decrease in labor supply (labor scarcity) will have induced a change in technology (increasing θ) that raises output. However, by the envelope theorem, this second term is equal to zero since θ^* is a solution to (4). Therefore, there is no effect on net output through induced technological changes[15] and no possibility of induced technological advances because of labor scarcity in this environment (at least for small changes in technology).[16] I next consider environments with externalities or market power, where there can be induced technological advances; that is, induced changes in technology can increase net output.

B. *Economy E: Decentralized Equilibrium with Externalities*

The discussion at the end of the previous subsection indicated why economy D does not enable a systematic study of the relationship between labor scarcity and technological advances (and, in fact, why there is no distinction between technology and other factors of production in this economy). A first approach to deal with this problem is to follow Romer (1986) and suppose that technology choices generate knowledge and thus create positive externalities on other firms. In particular, suppose that the output of producer i is now given by

$$y^i = G(L^i, Z^i, \theta^i, \bar{\theta}), \tag{6}$$

where $\bar{\theta}$ is some aggregate of the technology choices of all other firms in the economy. For simplicity, we can take $\bar{\theta}$ to be the average technology in the economy. In particular, if θ is a K-dimensional vector, then $\bar{\theta}_k = \int_{i\in\mathcal{F}}\theta^i_k di$ for each component of the vector (i.e., for $k = 1, 2, \ldots, K$). The remaining assumptions are the same as before. In particular, G is concave in L^i, Z^i, and θ^i and increasing in L^i, Z^i, and $\bar{\theta}$.

The maximization problem of each firm now becomes

$$\max_{L^i, Z^i, \theta_i} \pi(L^i, Z^i, \theta^i, \bar{\theta}) = G(L^i, Z^i, \theta^i, \bar{\theta}) - w_L L^i - \sum_{j=1}^{N} w_{Z_j} Z^i_j - C(\theta^i), \tag{7}$$

and under the same assumptions as above, each firm will hire the same amount of all factors, so in equilibrium, $L^i = \bar{L}$ and $Z^i = \bar{Z}$ for all $i \in \mathcal{F}$. Then the following proposition characterizes equilibrium technology.

PROPOSITION 2. Suppose that $G(L, Z, \theta, \bar{\theta})$ is concave in (L, Z, θ).

[15] This is unless one considers changes in technology that increase output gross of costs of technology, while leaving net output unchanged, as "technological advances," which does not seem entirely compelling.

[16] To see the intuition for why, with competitive technology adoption, there cannot be induced technological advances, consider the comparison between British and American technologies in the nineteenth century discussed by Habakkuk. In the context of this fully competitive economy D, it may have been the case that labor scarcity in the United States encouraged the adoption of certain capital-intensive technologies as Habakkuk hypothesized, but the adoption of these technologies cannot be considered as technological advances since their adoption in Britain, where labor was less scarce, would have reduced rather than increased net output; otherwise they would have been adopted in Britain as well.

Then, equilibrium technologies in economy E are given by the following fixed-point problem:

$$\theta^* \in \arg\max_{\theta\in\Theta} G(\bar{L}, \bar{Z}, \theta, \bar{\theta} = \theta^*) - C(\theta). \tag{8}$$

Even though this is a fixed-point problem, its structure is very similar to that of (4) and it can be used in the same way for our analysis (though in general multiple equilibria are possible in this case). However, crucially, the envelope theorem type reasoning no longer applies to the equivalent of equation (5). To see this, let us define net output again as $Y(\bar{L}, \bar{Z}, \theta) \equiv G(\bar{L}, \bar{Z}, \theta, \theta) - C(\theta)$. Then once again assuming differentiability, (5) applies, but now the second term in this expression is not equal to zero. In particular,

$$\frac{\partial Y(\bar{L}, \bar{Z}, \theta)}{\partial \theta} = \frac{\partial G(\bar{L}, \bar{Z}, \theta, \bar{\theta})}{\partial \theta} + \frac{\partial G(\bar{L}, \bar{Z}, \theta, \bar{\theta})}{\partial \bar{\theta}} - \frac{\partial C(\theta)}{\partial \theta}$$
$$= \frac{\partial G(\bar{L}, \bar{Z}, \theta, \bar{\theta})}{\partial \bar{\theta}},$$

which is positive by assumption. This implies that induced increases in θ will raise output and thus correspond to *induced technological advances.*

C. *Economy M: Monopoly Equilibrium*

The main environment used for the analysis in this paper features a monopolist supplying technologies to final good producers. There is a unique final good, and each firm has access to the production function

$$y^i = \alpha^{-\alpha}(1-\alpha)^{-1} G(L^i, Z^i, \theta)^{\alpha} q^i(\theta)^{1-\alpha}, \tag{9}$$

with $\alpha \in (0, 1)$. This expression is similar to (1) except that $G(L^i, Z^i, \theta)$ is now a subcomponent of the production function, which depends on technology θ. The subcomponent G needs to be combined with an intermediate good embodying technology θ. The quantity of this intermediate used by firm i is denoted by $q^i(\theta)$—conditioned on θ to emphasize that it embodies technology θ. This intermediate good is supplied by the monopolist. The term $\alpha^{-\alpha}(1-\alpha)^{-1}$ is included as a convenient normalization.

This production structure is similar to models of endogenous technology (e.g., Romer 1990; Grossman and Helpman 1991; Aghion and Howitt 1992) but is somewhat more general since it does not impose that technology necessarily takes a factor-augmenting form.

The monopolist can create (a single) technology $\theta \in \Theta$ at cost $C(\theta)$ from the technology menu (which is again assumed to be strictly increasing). In line with Romer's (1990) emphasis that technology has a "nonrivalrous" character and can thus be produced at relatively low cost once invented, I assume that once θ is created, the intermediate good embodying technology θ can be produced at a constant per-unit cost normalized to $1-\alpha$ unit of the final good (this is also a convenient normalization). The monopolist can then set a (linear) price per unit of the intermediate good of type θ, denoted by χ.

All factor markets are again competitive, and each firm takes the available technology, θ, and the price of the intermediate good embodying this technology, χ, as given and maximizes

$$\max_{L^i, Z^i, q^i(\theta)} \pi(L^i, Z^i, q^i(\theta)\,|\theta, \chi) = \alpha^{-\alpha}(1-\alpha)^{-1}G(L^i, Z^i, \theta)^{\alpha}q^i(\theta)^{1-\alpha} - w_L L^i - \sum_{j=1}^{N} w_{Zj}Z_j^i - \chi q^i(\theta), \tag{10}$$

which gives the following simple inverse demand for intermediates of type θ as a function of their price, χ, and the factor employment levels of the firm as

$$q^i(\chi, L^i, Z^i|\theta) = \alpha^{-1}G(L^i, Z^i, \theta)\chi^{-1/\alpha}. \tag{11}$$

The problem of the monopolist is to maximize its profits:

$$\max_{\theta,\chi,[q^i(\chi,L^i,Z^i|\theta)]_{i\in\mathcal{F}}} \Pi = [\chi - (1-\alpha)]\int_{i\in\mathcal{F}} q^i(\chi, L^i, Z^i|\theta)di - C(\theta) \tag{12}$$

subject to (11).

An *equilibrium* in economy M is now defined as a set of firm decisions $\{L^i, Z^i, q^i(\chi, L^i, Z^i|\theta)\}_{i\in\mathcal{F}}$, technology choice and pricing decisions by the technology monopolist (θ, χ), and factor prices (w_L, w_Z) such that $\{L^i, Z^i, q^i(\chi, L^i, Z^i|\theta)\}_{i\in\mathcal{F}}$ solve (10) given (w_L, w_Z) and (θ, χ), (3) holds, and (θ, χ) maximize (12) subject to (11).

This definition emphasizes that factor demands and technology are decided by different agents (the former by the final good producers, the latter by the technology monopolist), which is an important feature both theoretically and as a representation of how technology is determined in practice. Since factor demands and technology are decided by different agents, we no longer require concavity of $G(L^i, Z^i, \theta)$ in (L^i, Z^i, θ). Instead, it is sufficient that G is concave in (L^i, Z^i).[17]

To characterize the equilibrium, note that (11) defines a constant elasticity demand curve, so the profit-maximizing price of the monopolist is given by the standard monopoly markup over marginal cost and is equal to $\chi = 1$. Consequently,

$$q^i(\theta) = q^i(\chi = 1, \bar{L}, \bar{Z}|\theta) = \alpha^{-1}G(\bar{L}, \bar{Z}, \theta)$$

for all $i \in \mathcal{F}$. Substituting this into (12), we can express the maximization

[17] There is no loss of generality if G is taken to exhibit constant returns to scale in L and Z in the rest of the analysis.

problem of the monopolist as

$$\max_{\theta\in\Theta}\Pi(\theta) = G(\bar{L}, \bar{Z}, \theta) - C(\theta).$$

Thus we have established the following proposition.

PROPOSITION 3. Suppose that $G(L, Z, \theta)$ is concave in (L, Z) (for all $\theta \in \Theta$). Then any equilibrium technology θ^* in economy M is a solution to

$$\max_{\theta\in\Theta} G(\bar{L}, \bar{Z}, \theta) - C(\theta), \tag{13}$$

and any solution to this problem is an equilibrium technology.

This proposition shows that equilibrium technology in economy M is a solution to a problem identical to that in economy D, that of maximizing $G(\bar{L}, \bar{Z}, \theta) - C(\theta)$ as in (4) Naturally, the presence of the monopoly markup introduces distortions in the equilibrium. These distortions are the reason why equilibrium technology is not at the level that maximizes net output. In particular, let us use the fact that the profit-maximizing monopoly price is $\chi = 1$ and substitute (11) into the production function (9) and then subtract the cost of technology choice, $C(\theta)$, and the cost of production of the machines, $(1-\alpha)\alpha^{-1}G(L^i, Z^i, \theta)$, from gross output. This gives net output in this economy as

$$Y(\bar{L}, \bar{Z}, \theta) \equiv \frac{2-\alpha}{1-\alpha} G(\bar{L}, \bar{Z}, \theta) - C(\theta). \tag{14}$$

Clearly, the coefficient in front of $G(\bar{L}, \bar{Z}, \theta)$ is strictly greater than one. Recall also that C is strictly increasing in θ, and thus in any interior equilibrium θ^*, G must also be strictly increasing in θ. This implies that, as in economy E, $Y(\bar{L}, \bar{Z}, \theta)$ will be increasing in θ in the neighborhood of any equilibrium θ^*.

Finally, it can be verified that equilibrium factor prices are given by $w_L = (1-\alpha)^{-1}\partial G(\bar{L}, \bar{Z}, \theta)/\partial L$ and $w_{Zj} = (1-\alpha)^{-1}\partial G(\bar{L}, \bar{Z}, \theta)/\partial Z_j$ and are also proportional to the derivatives of the net output function Y defined in (14). In what follows, I take economy M as the baseline.[18]

[18] Using this framework, Acemoglu (2007) investigates the question of (induced) equilibrium bias of technology, i.e., whether an increase in the supply of a factor, say labor L, will change technology θ in a way that is weakly or strongly equilibrium biased toward L. We say that there is *weak equilibrium bias* if the combined effect of induced changes in technology resulting from an increase in labor supply raises the marginal product of labor at the starting factor proportions (i.e., it "shifts out" the demand for labor). Similarly, there is *strong equilibrium bias* if this induced effect in technology is sufficiently large so as to outweigh the direct effect of the increase in L (which is always to reduce its marginal product). The results in that paper show that there is always weak equilibrium bias, meaning that any increase in the supply of a factor always induces a change in technology favoring that factor. Moreover, this effect can be strong enough so that there is strong equilibrium bias, in which case, in contrast to basic producer theory, endogenous tech-

III. Labor Scarcity and Technological Progress

This section presents the main results of the paper and a number of extensions and applications.

A. *Main Result*

Let us focus on economy M in this subsection and impose the following assumption to simplify the exposition.

ASSUMPTION 1. Let $\Theta = \mathbb{R}_+^K$. The function $C(\theta)$ is twice continuously differentiable, strictly increasing, and strictly convex in $\theta \in \Theta$, and for each $k = 1, 2, \dots, K$, we have

$$\lim_{\theta_k \to 0} \frac{\partial C(\theta)}{\partial \theta_k} = 0 \quad \text{and} \quad \lim_{\theta_k \to \infty} \frac{\partial C(\theta)}{\partial \theta_k} = \infty \quad \text{for all } \theta.$$

Moreover, $G(\bar{L}, \bar{Z}, \theta)$ is continuously differentiable in θ and L and concave in $\theta \in \Theta$ and satisfies

$$\lim_{\theta \to 0} \frac{\partial G(\bar{L}, \bar{Z}, \theta)}{\partial \theta_k} > 0 \quad \text{for all } \bar{L} \text{ and } \bar{Z}.$$

Recall that equilibrium technology, $\theta^*(\bar{L}, \bar{Z})$, is a solution to the maximization problem in (13). Assumption 1 then ensures that equilibrium technology $\theta^*(\bar{L}, \bar{Z})$ is uniquely determined and interior; that is, it satisfies

$$\frac{\partial G(\bar{L}, \bar{Z}, \theta^*(\bar{L}, \bar{Z}))}{\partial \theta_k} = \frac{\partial C(\theta^*(\bar{L}, \bar{Z}))}{\partial \theta_k} \quad \text{for } k = 1, 2, \dots, K.$$

Moreover, in this equilibrium, it must be the case that $\partial G(\bar{L}, \bar{Z}, \theta^*(\bar{L}, \bar{Z}))/\partial \theta_k > 0$ (for each $k = 1, 2, \dots, K$) as $C(\theta^*(\bar{L}, \bar{Z}))$ is strictly increasing from assumption 1. Since net output $Y(\bar{L}, \bar{Z}, \theta)$ is given by (14), this also implies that

$$\frac{\partial Y(\bar{L}, \bar{Z}, \theta^*(\bar{L}, \bar{Z}))}{\partial \theta_k} > 0 \quad \text{for } k = 1, 2, \dots, K. \tag{15}$$

In light of this, we say that there are *technological advances* if θ increases (meaning that each component of the vector θ increases or remains constant).

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nology choices in general equilibrium will lead to upward-sloping demand curves for factors. More specifically, there will be strong equilibrium bias if and only if the Hessian of the production function with respect to L and θ, $\nabla^2 F_{(L,\theta)(L,\theta)}$, is not negative semidefinite (see theorem 6 in the Appendix). Since, in economies M and D, L and θ are chosen by different agents, there is no presumption in general that $\nabla^2 F_{(L,\theta)(L,\theta)}$ needs to be negative semidefinite. Interestingly, these results about equilibrium bias imply almost nothing about the impact of labor scarcity on technological advances since a change in technology biased toward a factor could correspond to either a technological advance or a deterioration in technology.

The key concepts of strongly labor-saving (or more generally factor-saving) technology and strongly labor complementary technology are introduced in the next definition. Let $x = (x_1, \ldots, x_n)$ in $\mathbb{R}^n$. Then recall that a twice continuously differentiable function $f(x)$ is *supermodular* on X if and only if $\partial^2 f(x)/\partial x_i \partial x_{i'} \geq 0$ for all $x \in X$ and for all $i \neq i'$. In addition, a function $f(x, t)$ defined on $X \times \mathcal{T}$ (where $X \subset \mathbb{R}^n$ and $\mathcal{T} \subset \mathbb{R}^m$) has *increasing differences* in (x, t) if, for all $t'' > t$, $f(x, t'') - f(x, t)$ is nondecreasing in x and has *strictly increasing differences* in (x, t) if, for all $t'' > t$, $f(x, t'') - f(x, t)$ is increasing in x.[19] *Decreasing differences* and *strictly decreasing differences* are defined analogously by $f(x, t'') - f(x, t)$ being nonincreasing and decreasing, respectively. If f is differentiable and $\mathcal{T} \subset \mathbb{R}$, then increasing differences is equivalent to $\partial^2 f(x, t)/\partial x_i \partial t \geq 0$ for each i and decreasing differences is equivalent to $\partial^2 f(x, t)/\partial x_i \partial t \leq 0$ for each i.

DEFINITION 1. Technology is strongly labor saving at $(\bar{L}, \bar{Z}, \bar{\theta})$ if there exist neighborhoods $\mathcal{B}_L$, $\mathcal{B}_Z$, and $\mathcal{B}_\theta$ of $\bar{L}$, $\bar{Z}$, and $\bar{\theta}$ such that $G(L, Z, \theta)$ exhibits strict decreasing differences in (L, θ) on $\mathcal{B}_L \times \mathcal{B}_Z \times \mathcal{B}_\theta$. Conversely, technology is strongly labor complementary at $(\bar{L}, \bar{Z}, \bar{\theta})$ if there exist neighborhoods $\mathcal{B}_L$, $\mathcal{B}_Z$, and $\mathcal{B}_\theta$ of $\bar{L}$, $\bar{Z}$, and $\bar{\theta}$ such that $G(L, Z, \theta)$ exhibits strict increasing differences in (L, θ) on $\mathcal{B}_L \times \mathcal{B}_Z \times \mathcal{B}_\theta$. We say that technology is strongly labor saving (respectively, complementary) globally if it is strongly labor saving (complementary) for all $\bar{L}$, $\bar{Z}$, and $\theta \in \Theta$.

Intuitively, technology is strongly labor saving if technological advances reduce the marginal product of labor, and it is strongly labor complementary if technological advances increase the marginal product of labor. The next theorem gives a fairly complete characterization of when labor scarcity will induce technological advances.

THEOREM 1. Consider economy M and suppose that assumption 1 holds and $G(L, Z, \theta) - C(\theta)$ is supermodular in θ. Let the equilibrium technology be denoted by $\theta^*(\bar{L}, \bar{Z})$. Then labor scarcity will induce technological advances (increase θ), in the sense that $\partial \theta_k^*(\bar{L}, \bar{Z})/\partial \bar{L} < 0$ for each $k = 1, \ldots, K$ if technology is strongly labor saving at $(\bar{L}, \bar{Z}, \theta^*(\bar{L}, \bar{Z}))$, and will discourage technological advances, in the sense that $\partial \theta_k^*(\bar{L}, \bar{Z})/\partial \bar{L} > 0$ for each $k = 1, \ldots, K$, if technology is strongly labor complementary at $(\bar{L}, \bar{Z}, \theta^*(\bar{L}, \bar{Z}))$.

Proof. From assumption 1, G is increasing in θ in the neighborhood of $\theta^*(\bar{L}, \bar{Z})$. Equation (15) then implies that technological advances correspond to a change in technology from θ' to $\theta'' \geq \theta'$. From assumption 1, (13) is strictly concave and the solution $\theta^*(\bar{L}, \bar{Z})$ is strictly positive, unique, and, by the implicit function theorem, differentiable

[19] Throughout, "increasing" stands for "strictly increasing" and "decreasing" for "strictly decreasing."

in $\bar{L}$. Therefore, a small change in $\bar{L}$ will lead to a small change in each of $\theta_k^*(\bar{L}, \bar{Z})$ $(k = 1, \ldots, K)$. Since $G(L, Z, \theta) - C(\theta)$ is supermodular in θ by assumption, comparative statics are determined by whether G exhibits strict decreasing or increasing differences in L and θ in the neighborhood of $\bar{L}$, $\bar{Z}$, and $\theta^*(\bar{L}, \bar{Z})$. In particular, theorem 2.8.5 in Topkis (1998) implies that when technology is strongly labor saving, that is, when G exhibits strict decreasing differences in L and θ, $\partial\theta_k^*(\bar{L}, \bar{Z})/\partial\bar{L} < 0$ for each $k = 1, \ldots, K$. This yields the result for strongly labor-saving technology. Conversely, when G exhibits strict increasing differences in L and θ, $\partial\theta_k^*(\bar{L}, \bar{Z})/\partial\bar{L} > 0$ for each $k = 1, \ldots, K$, and labor scarcity reduces θ, establishing the desired result for strongly labor complementary technology. QED

Though simple, this theorem provides a fairly complete characterization of the conditions under which labor scarcity will lead to technological advances. The only cases that are not covered by the theorem are those in which G is not supermodular in θ and those in which G exhibits neither increasing differences nor decreasing differences in L and θ. Without supermodularity, the "direct effect" of labor scarcity on each technology component would be positive, but because of lack of supermodularity, the advance in one component may then induce an even larger deterioration in some other component; thus a precise result becomes impossible. When G exhibits neither increasing nor decreasing differences, then a change in labor supply $\bar{L}$ will affect different components of technology in different directions, and without making further parametric assumptions, we cannot reach an unambiguous conclusion about the overall effect. Clearly, when θ is single-dimensional, the supermodularity condition is automatically satisfied, and G exhibits either increasing or decreasing differences in the neighborhood of $\bar{L}$, $\bar{Z}$, and $\theta^*(\bar{L}, \bar{Z})$ (recall that when θ is single-dimensional, decreasing differences is equivalent to $\partial^2 G/\partial L\partial\theta \leq 0$ and increasing differences to $\partial^2 G/\partial L\partial\theta \geq 0$).

Another potential shortcoming of this analysis is that the environment is static. Although these results are stated for a static model, there are multiple ways of extending this framework to a dynamic environment, and the main forces will continue to apply in this case (see Sec. V.A for an illustration of this point using an extension to a growth model). The advantage of the static environment is that it enables us to develop these results at a fairly high level of generality without being forced to make functional form assumptions in order to ensure balanced growth or some other notion of a well-defined dynamic equilibrium.

B. Further Results

The results of theorem 1, which were stated under assumption 1 and for economy M, can be generalized to economy E, and they can also be extended to global results. The next theorem provides the analogue of theorem 1 for economy E, except that now equilibrium technology need not be unique (since the equilibrium is a solution to a fixed-point problem rather than to a maximization problem). As is well known (e.g., Milgrom and Roberts 1994; Topkis 1998), when there are multiple equilibria, we can typically provide unambiguous comparative statics only for "extremal equilibria." These extremal equilibria always exist in the present context given the assumptions we have imposed so far (supermodularity of G and the fact that Θ is a lattice), and they correspond to the smallest and greatest equilibrium technologies, θ^- and θ^+ (meaning that if there exists another equilibrium technology, $\tilde{\theta}$, we must have $\theta^+ \geq \tilde{\theta} \geq \theta^-$). In view of this, a technological advance now refers to an increase in the greatest and the smallest equilibrium technologies.

THEOREM 2. Consider economy E, and suppose that assumption 1 holds and also that $G(L, Z, \theta, \bar{\theta}) - C(\theta)$ is supermodular in θ and $\bar{\theta}$. Let θ^- and θ^+ denote the smallest and the greatest equilibrium technologies at $(\bar{L}, \bar{Z})$. Then if technology is strongly labor saving at $(\bar{L}, \bar{Z}, \theta^-)$ (respectively, at $(\bar{L}, \bar{Z}, \theta^+)$), labor scarcity will induce technological advances (in the sense that a small decrease in $\bar{L}$ will increase θ^- [respectively, θ^+]); if technology is strongly labor complementary at $(\bar{L}, \bar{Z}, \theta^-)$ (respectively, at $(\bar{L}, \bar{Z}, \theta^+)$), labor scarcity will discourage technological advances (in the sense that a small decrease in $\bar{L}$ will reduce θ^- [respectively, θ^+]).

Proof. See the Appendix.

We next present global versions of theorems 1 and 2, which hold without assumption 1 when technology is strongly labor saving or labor complementary globally. The statements again refer to the smallest and the greatest equilibria.

THEOREM 3. Consider economy M or E. Suppose that assumption 1 holds and $G(L, Z, \theta) - C(\theta)$ is supermodular in θ in economy M or $G(L, Z, \theta, \bar{\theta}) - C(\theta)$ is supermodular and increasing in $\bar{\theta}$ in economy E. If technology is strongly labor saving (respectively, labor complementary) globally, then labor scarcity will induce (respectively, discourage) technological advances in the sense of increasing (respectively, reducing) the smallest and the greatest equilibrium technologies, θ^- and θ^+.

Proof. I provide the proof for economy E (the proof for economy M is similar but more straightforward as the equilibrium is still a solution to a maximization problem). When G exhibits increasing differences in L and θ globally, the payoff of each firm i exhibits increasing differences in its own strategies and L. Then, given that G is supermodular in θ'

and θ, theorem 4.2.2 from Topkis (1998) implies that the greatest and smallest equilibria of this game are nondecreasing in $\bar{L}$ and assumption 1 again guarantees that equilibria are interior and thus must be increasing in $\bar{L}$. This establishes the second part of the theorem. The first part follows with the same argument, using $-\theta$ instead of θ, when technology is strongly labor saving globally. QED

This theorem shows that similar results hold for economy M or E (and the Appendix shows that they also extend to an oligopolistic setting). It can also be shown that similar changes in θ also hold in economy D. But for reasons already emphasized, increases in θ in economy D do not correspond to "technological advances" because in the neighborhood of an equilibrium in economy D, any change will reduce net output at given $\bar{L}$ and $\bar{Z}$, and small changes will have second-order effects in the neighborhood of $\bar{L}$ and $\bar{Z}$ because equilibrium technology maximizes output at these factor proportions.[20]

C. *Implications of Exogenous Wage Increases*

Let us define $F(L, Z, \theta) \equiv G(L, Z, \theta) - C(\theta)$ and let $\nabla^2 F_{(L,\theta)(L,\theta)}$ denote the Hessian of this function with respect to L and θ. The Appendix (in particular theorem 6) shows that if $\nabla^2 F_{(L,\theta)(L,\theta)}$ is negative semidefinite, the relationship between employment and the equilibrium wage, even in the presence of endogenous technology, is given by a decreasing function $w_L^*(L)$. As a consequence, we can equivalently talk of a decrease in labor supply (corresponding to labor becoming more "scarce") or an "exogenous wage increase," where a wage above the market-clearing level is imposed. In this light, we can generally think of equilibrium employment as $L^e = \min\{(w_L^*)^{-1}(w_L^e), \bar{L}\}$, where w_L^e is the equilibrium wage rate, either determined in competitive labor markets or imposed by regulation. Under these assumptions, all the results presented in this section continue to hold. This result is stated in the next corollary.

Corollary 1. Suppose that $\nabla^2 F_{(L,\theta)(L,\theta)}$ is negative semidefinite. Then under the same assumptions as in theorems 1–3, a minimum wage above the market-clearing wage level induces technological advances when technology is strongly labor saving and discourages technological advances when technology is strongly labor complementary.

Proof. Theorem 6 in the Appendix implies that when $\nabla^2 F_{(L,\theta)(L,\theta)}$ is negative semidefinite, a wage above the market-clearing level is equiv-

[20] Although the statement may not be true for noninfinitesimal changes, it is an immediate consequence of proposition 1 that any (induced) change in θ starting from θ^* cannot increase net output at $\bar{L}$ and $\bar{Z}$. The only reason why caution is necessary is that such a change, while reducing net output at $\bar{L}$ and $\bar{Z}$, may increase it at some other factor proportions.

alent to a decline in employment. Then the result in the corollary follows from theorems 1 and 3. QED

The close association between labor scarcity and exogenous wage increases in this result relies on the assumption that $\nabla^2 F_{(L,\theta)(L,\theta)}$ is negative semidefinite, so that the endogenous-technology demand curves are downward sloping (recall theorem 6). When this is not the case, exogenous wage increases can have richer effects; this is discussed in Section V.D.

While corollary 1 shows that exogenous wage increases can induce technological advances, it should be noted that even when this is the case, net output may decline because of the reduction in employment.[21] Nevertheless, when the effect of labor scarcity on technology is sufficiently pronounced, overall output may increase even though employment declines. Consider the following example, which both illustrates this possibility and also gives a simple instance in which technology is strongly labor saving.

EXAMPLE 1. Let us focus on economy M and suppose that $Z = (K, T)$ (where K denotes capital and T land), the G function takes the form

$$G(L, K, T, \theta) = 3[\theta K^{1/3} + (1-\theta)L^{1/3}]T^{2/3},$$

and the cost of technology creation is $C(\theta) = 3\theta^2/2$. Intuitively, θ here is a technology that shifts tasks away from labor toward capital (see Sec. IV.C). Let us normalize the supply of the nonlabor factors to $\bar{K} = \bar{T} = 1$ and denote labor supply by $\bar{L}$. Suppose that equilibrium wages are given by the marginal product of labor. The equilibrium technology is $\theta^*(\bar{L}) = 1 - \bar{L}^{1/3}$. The equilibrium wage, the marginal product of labor at $\bar{L}$ and technology θ, is then

$$w(\bar{L}, \theta) = (1-\theta)\bar{L}^{-2/3}.$$

To obtain the endogenous technology relationship between labor supply and wages, we substitute $\theta^*(\bar{L})$ into this wage expression and obtain

$$w(\bar{L}, \theta^*(L)) = \bar{L}^{-1/3},$$

which shows that there is a decreasing relationship between labor supply and wages.

Suppose that labor supply $\bar{L}$ is equal to 1/64. In that case, the equilibrium wage will be 4. Next consider a minimum wage at $\bar{w} = 5$. Since final good producers take prices as given, they have to be along their (endogenous-technology) labor demands; this implies that employment will fall to $L^e = 1/125$. Without the exogenous wage increase, technology

[21] Conversely, even if labor scarcity does not encourage technological advances, output per worker might increase because of the standard channel of diminishing returns to labor.

was $\theta^*(\bar{L}) = 3/4$, whereas after the minimum wage, we have $\theta^*(L^e) = 4/5$, which illustrates the induced technology adoption/innovation effects of exogenous wage increases.

Do such wage increases increase overall output? Recall that net output is equal to

$$Y(L, Z, \theta) \equiv \frac{2-\alpha}{1-\alpha} G(L, Z, \theta) - C(\theta),$$

where $1-\alpha$ is the share of intermediates in the final good production function (recall eq. [9]). It can be verified that for α close to zero, an exogenous wage increase reduces net output; however, for α sufficiently close to one, net output increases despite the decline in employment. Generalizing this example, one can verify that when $\Theta \subset \mathbb{R}$, an exogenous wage increase will increase output if the following conditions are satisfied: (1) technology is strongly labor saving, (2)

$$\frac{\partial G(\bar{L}, \bar{Z}, \theta)}{\partial L} \cdot \left| \frac{\partial^2 G(\bar{L}, \bar{Z}, \theta)}{\partial \theta^2} \right| < \left| \frac{\partial^2 G(\bar{L}, \bar{Z}, \theta)}{\partial L \partial \theta} \right| \cdot \frac{\partial G(\bar{L}, \bar{Z}, \theta)}{\partial \theta},$$

and (3) α is sufficiently close to one. These conditions can be easily generalized to cases in which θ is multidimensional.

D. *Applications*

In this subsection, we briefly discuss two applications: the implications of carbon taxes for "green technology" and the impacts of scarcity of skilled and unskilled labor.[22] It is straightforward to apply the framework developed so far to investigate the Porter hypothesis discussed in the introduction.[23] To do this, let us focus on economy M, with the only difference being that θ corresponds to "green technologies" and p, which represents carbon or "pollution," replaces L (for simplicity, we are ignoring nongreen technologies). Note, however, that p is not an input but part of the joint "output." Thus output is given by (9) with $G(Z^i, \theta)$ replacing $G(L^i, Z^i, \theta)$, and pollution is given as

$$p = \alpha^{-\alpha}(1-\alpha)^{-1} P(Z^i, \theta)^{\alpha} q^i(\theta)^{1-\alpha},$$

where the function $P(Z, \theta)$ is assumed to be decreasing in θ, capturing

[22] Gans (2009) also uses the framework developed in this paper to investigate the Porter hypothesis, and Acemoglu et al. (2010) develop a two-sector economy with directed technical change and dynamic environmental externalities to study the implications of environmental regulations on technological change and climate.

[23] It should be noted that what is being discussed here is a "sophisticated" Porter hypothesis. Porter's (1991) article implies that regulation on a single firm can increase that firm's profitability, which is not possible provided that firms are maximizing (net present discounted value of) profits. However, regulation or taxes on an industry can increase each firm's profitability, which is the "sophisticated" version of the hypothesis discussed here (without adding this qualifier in what follows to simplify the terminology).

the fact that θ is a vector of green technologies, and $\alpha^{-\alpha}(1-\alpha)^{-1}$ is again included as a normalization. We then assume that the cost of introducing technology θ, $C(\theta)$, is increasing, capturing the fact that more green technologies are more expensive. Final good producers pay a tax equal to τ units of final good on their production of p. It is then straightforward to see that, instead of (11), the demand for machines from the final good sector will be given by

$$q^i(\chi, Z^i|\theta) = \alpha^{-1}[G(Z^i, \theta)^\alpha - \tau P(Z^i, \theta)^\alpha]^{1/\alpha}\chi^{-1/\alpha},$$

where χ again denotes the per-unit price of machines embedding technology θ. This expression simply follows from the fact that the net revenue of the firm is now proportional to $[G(Z^i, \theta)^\alpha - \tau P(Z^i, \theta)^\alpha]^{1/\alpha}$. An equilibrium is defined in a similar fashion, except that θ^* will be a solution to

$$\max_{\theta\in\Theta}\, [G(Z^i, \theta)^\alpha - \tau P(Z^i, \theta)^\alpha]^{1/\alpha} - C(\theta).$$

Consider now an increase in environmental regulation, captured by a higher tax on pollution or carbon, that is, higher τ. Since P is decreasing in θ, this will clearly increase the marginal return to θ, and θ will increase. But this does not imply that environmental regulation will encourage technological advances as maintained by the Porter hypothesis. Recall that in this environment net output, ignoring environmental damages, is $Y(Z, \theta) \equiv (2-\alpha)/(1-\alpha)G(Z, \theta) - C(\theta)$. Since $C(\theta)$ is increasing in θ, with $\tau = 0$ an interior equilibrium θ^* (i.e., an equilibrium with $\theta^* > 0$) would have necessarily been at a point where $G(Z, \theta)$ is increasing in θ. Hence, a further increase in θ would have raised $Y(Z, \theta)$ and corresponded to a technological advance. This is no longer the case in the presence of the term $\tau P(Z^i, \theta)$ since an interior equilibrium θ^* might be at a point where $Y(Z, \theta)$ is decreasing in θ. In this case, further environmental regulation would encourage an increase in θ^*, but θ^* might already be too high. This is of course plausible: because pollution creates other negative effects, government regulation might set τ at such a level that green technologies may be adopted beyond the point where they contribute to output. The above discussion also reveals that there is one special case in which environmental regulation (higher tax τ) will necessarily correspond to a technological advance as in the Porter hypothesis: when we start with τ close to zero. In that case, our above argument ensures that any interior equilibrium θ^* must be in a region where $G(Z, \theta)$, and thus $Y(Z, \theta)$, is increasing in θ, so that the policy-induced change starting from θ^* would increase net output. Therefore, this model implies that the Porter hypothesis is valid whenever there is no or little environmental regulation to start with (and when θ^* is interior).[24]

The framework presented so far can also be applied to investigate the implications of an abundance of different types of labor. Suppose that the economy now consists of skilled labor, with supply $\bar{H}$, and unskilled labor, with supply $\bar{L}$, as well as nonlabor factors with supply vector $\bar{Z}$. Let us focus on economy M and assume that the function G in (A6) now takes the form $G(L, H, Z, \theta)$. The results derived so far can then be applied in a straightforward manner to changes in L or H. If only one of these is changed, then all the results derived so far apply with the relevant concepts being modified to strongly unskilled (or skilled) labor-saving (or complementary) technology. However, in many situations the vector of technologies, θ, likely includes components that are both strongly labor saving and strongly labor complementary. If so, one would need to put more structure in order to investigate whether scarcity of skilled labor and/or unskilled labor would induce technological advances. In particular, in specific episodes in which the most important technologies may be those related to skilled labor (e.g., as may have been the case with technologies replacing the labor of skilled artisans during the early phases of the Industrial Revolution [see Mantoux 1961] and with technologies complementing the skills of college graduates more recently), the relationship between the specific components of technology and skilled labor might determine whether abundance or scarcity of skilled labor will induce technological advances.

IV. When Is Technology Strongly Labor Saving?

In this section, I investigate the conditions under which, in a range of standard models, technology is strongly labor saving. The results show that it is possible to construct a rich set of economies in which this is the case, though this is difficult or impossible in the canonical models used in macroeconomics and economic growth literatures. In particular,

[24] Note that assumption 1 is important for the result that the Porter hypothesis is valid with no or little initial environmental regulation. Recall that this assumption imposes that G is increasing in θ at $\theta = 0$ and ensures that θ^* is interior. This combined with the assumption that C is also increasing in θ implies that when $\tau = 0$, net output will be increasing in θ in the neighborhood of the equilibrium technology θ^*. However, assumption 1 may be less plausible in the context of environmental technologies. For example, a "green" technology such as ethanol may not increase net output even when it is not being used at all. If assumption 1 is relaxed, then θ^* may be at zero when τ is small. In this case, it can be verified that, provided that C is still increasing, all the results presented so far continue to hold as weak rather than strict comparative statics. What this implies, in particular, is that if θ^* is at zero (rather than being interior), then an increase in τ may not affect it. If τ is raised sufficiently, this would again increase θ^*, but as the discussion in the text implies, change induced by a large τ may not correspond to a technological advance and may instead reduce net output.

when the aggregate production function, here corresponding to G, is Cobb-Douglas, technology cannot be strongly labor saving. Throughout, I simplify the discussion by focusing on economy M and a single-dimensional technology variable.

A. *Cobb-Douglas Production Functions*

As a first example, suppose that the function G, and thus the aggregate production function of the economy, is Cobb-Douglas and also that the exponents of the Cobb-Douglas production function are fixed and cannot change as a result of technological change. In particular,

$$G(L, Z, \theta) = H(Z, \theta)L^{\beta},$$

where $\beta \in (0, 1)$ and $H: \mathbb{R}_{+}^{N} \to \mathbb{R}_{+}$. This implies that aggregate net output is given by

$$Y(L, Z, \theta) = \frac{2-\alpha}{1-\alpha} H(Z, \theta)(\theta L)^{\alpha} - C(\theta),$$

where $\alpha \in (0, 1)$ is the parameter of the production function in (9), measuring the elasticity of aggregate output to the subcomponent G. More generally, the function H can be chosen such that G exhibits constant returns to scale. The convention that θ corresponds to a technological advance implies that H is increasing in θ. It is then straightforward to verify that, provided that H is differentiable, the cross partial of G with respect to L and θ is

$$G_{L\theta}(L, Z, \theta) = \beta H_{\theta}(Z, \theta)L^{\beta-1} > 0.$$

Therefore, technology is always strongly labor complementary in this case, and labor scarcity or exogenous wage increases will necessarily discourage technological advances.

B. *Factor-Augmenting Technological Change*

Let us next turn to constant elasticity of substitution (CES) production functions (between labor and capital) with factor-augmenting technology, which are commonly used in the macroeconomics literature. To simplify the discussion, let us continue to focus on cases in which technology is represented by a single-dimensional variable, θ. Suppose also that there are two nonlabor factors of production, for example, capital K and land or entrepreneurial skill, T (i.e., $Z = (K, T)$). We need to distinguish between two cases, one in which θ "augments" capital and one in which θ "augments" labor. Let us start with the former. The G function can then be written as

$$G(L, K, T, \theta) = [(1-\eta)(\theta K)^{(\sigma-1)/\sigma} + \eta L^{(\sigma-1)/\sigma}]^{\gamma\sigma/(\sigma-1)} T^{1-\gamma}$$

for $\eta \in (0, 1)$ and $\gamma \in (0, 1)$. This production function exhibits constant returns to scale, but when $\gamma < 1$, there are decreasing returns to L and K with T held constant. Once again, net output is equal to the same expression multiplied by $(2 - \alpha)/(1 - \alpha)$ minus the cost of technology, $C(\theta)$.

Straightforward differentiation then gives

$$G_{L\theta}(L, K, T, \theta) = \frac{\gamma\sigma + 1 - \sigma}{\sigma}\gamma\eta(1 - \eta)K^{(\sigma-1)/\sigma}(\theta L)^{-(1/\sigma)} \times [(1 - \eta)(\theta K)^{(\sigma-1)/\sigma} + \eta L^{(\sigma-1)/\sigma}]^{[\gamma\sigma/(\sigma-1)]-2}T^{1-\gamma}.$$

This expression shows that technology will be strongly labor complementary (i.e., $G_{L\theta} > 0$) if either of the following two conditions is satisfied: (1) $\gamma = 1$ (constant returns to scale in L and K) or (2) $\sigma \leq 1$ (gross complements).

Therefore, for technology to be strongly labor saving, we would need both $\gamma < 1$ and $\sigma > 1$ (and in fact both of them sufficiently so) so that the following condition is satisfied:

$$1 - \gamma > \frac{1}{\sigma}. \tag{16}$$

This result can be generalized to any G featuring capital-augmenting technology (provided that it is also homothetic in L and K). In particular, for any such G, we can write $G(L, K, T, \theta) \equiv \tilde{G}(L, \theta K, T)$, where $\tilde{G}$ is homothetic in K and L given T. It can then be verified that (16) is again necessary and sufficient for technology to be strongly labor saving, with γ corresponding to the local degree of homogeneity of $\tilde{G}$ and σ corresponding to the local elasticity of substitution (both "local" qualifiers are added since these need not be constant).

This result shows that with capital-augmenting technology, constant returns to scale to labor and capital is sufficient to rule out strongly labor-saving technological progress. In addition, in this case we also need a high elasticity of substitution. Since θ is augmenting the other factor, Z, a high elasticity of substitution corresponds to technology "substituting" for tasks performed by labor. This intuition will exhibit itself somewhat differently next, when we turn to the CES production function with labor-augmenting technology.

With labor-augmenting technology, the G function takes the form

$$G(L, K, T, \theta) = [(1 - \eta)K^{(\sigma-1)/\sigma} + \eta(\theta L)^{(\sigma-1)/\sigma}]^{\gamma\sigma/(\sigma-1)}T^{1-\gamma}.$$

Straightforward differentiation now gives

$$G_{L\theta}(L, K, T, \theta) = \left[\gamma\eta(\theta L)^{(\sigma-1)/\sigma} + \frac{\sigma - 1}{\sigma}(1 - \eta)K^{(\sigma-1)/\sigma}\right] \times \gamma\eta(\theta L)^{-(1/\sigma)}[(1 - \eta)Z^{(\sigma-1)/\sigma} + \eta(\theta L)^{(\sigma-1)/\sigma}]^{[\gamma\sigma/(\sigma-1)]-2}T^{1-\gamma}.$$

Now define the labor share relative to capital as

$$s_L \equiv \frac{w_L L}{RK} = \frac{\eta(\theta L)^{(\sigma-1)/\sigma}}{(1-\eta)K^{(\sigma-1)/\sigma}} > 0,$$

where R is the marginal product (rental rate) of capital, and the condition that $G_{L\theta} < 0$ is equivalent to

$$s_L < \frac{1-\sigma}{\sigma\gamma}. \tag{17}$$

As with condition (16), (17) is more likely to be satisfied, and technological change is more likely to be strongly labor saving, when γ is smaller and thus there are strong decreasing returns. However, now technology can be strongly labor saving even when $\gamma = 1$. In particular, as $\sigma \to 0$ and the production function approaches the Leontief limit where $G = \min\{K, \theta L\}$, technology will necessarily be labor saving. In contrast, it can never be so when $\sigma \geq 1$, which is the opposite of the restriction on the elasticity of substitution in the case in which θ augments K. Intuitively, when θ augments K, a high degree of substitution between technology and labor requires a high elasticity of substitution, in particular, $\sigma > 1$. In contrast, when θ augments labor, a high degree of substitution between technology and labor corresponds to $\sigma < 1$.

This result can again be extended to labor-augmenting technology in general. Suppose again that $G(L, K, T, \theta) \equiv \tilde{G}(\theta L, K, T)$, with $\tilde{G}$ homothetic in L and θ given T. Then (17) characterizes strongly labor-saving technology with γ corresponding to the local degree of homogeneity of $\tilde{G}$ and σ corresponding to the local elasticity of substitution.

C. *Machines Replacing Labor*

Models in which technological change is caused or accompanied by machines replacing human labor have been proposed by Champernowne (1963), Zeira (1998, 2006), and Hellwig and Irmen (2001). Let us consider a setup building on and generalizing the paper by Zeira (1998), which also has a clear parallel to the seminal work by Dornbusch, Fischer, and Samuelson (1980) in international trade.

Let us start with a competitive economy and suppose that aggregate output is given by

$$y = \left[\int_0^1 y(\nu)^{(\varepsilon-1)/\varepsilon} d\nu\right]^{\varepsilon/(\varepsilon-1)},$$

where $y(\nu)$ denotes an intermediate good of type ν produced as

$$y(\nu) = \begin{cases} \dfrac{k(\nu)}{\eta(\nu)} & \text{if } \nu \text{ uses new technology} \\ \dfrac{l(\nu)}{\beta(\nu)} & \text{if } \nu \text{ uses old technology,} \end{cases}$$

ε is the elasticity of substitution between intermediates, and $k(\nu)$ and $l(\nu)$ denote capital and labor used in the production of intermediate good ν. I use capital as the other factor of production here to maximize similarity with Zeira (1998).

Firms are competitive and can choose which intermediate to produce with the new technology and which one with the old technology. Total labor supply is $\bar{L}$. For now, let us also suppose that capital is supplied inelastically, with total supply given by $\bar{K}$. Let the price of the final good be normalized to one and that of each intermediate good be $p(\nu)$. We write $n(\nu) = 1$ if ν is using the new technology. Clearly, $n(\nu) = 1$ whenever $R\eta(\nu) < w\beta(\nu)$, where w is the wage rate and R is the endogenously determined rate of return on capital. Let us define

$$\kappa(\nu) \equiv \frac{\eta(\nu)}{\beta(\nu)}$$

and assume that it is continuous and strictly increasing. In the competitive equilibrium, we will have θ^* such that $\theta^* = \kappa^{-1}(w/R)$, so that $n(\nu) = 1$ for all $\nu \leq \theta^*$. Since κ is increasing, its inverse is also increasing, so a higher wage to rental rate ratio encourages higher levels of θ^*. This effect is highlighted and exploited in Zeira (1998).

Let us now see that this is indeed related to technology being strongly labor saving. With the same reasoning, suppose that $n(\nu) = 1$ for all $\nu \leq \theta$ for some $\theta \in (0, 1)$ (since, clearly, in any equilibrium or optimal allocation, this type of "single crossing" must hold). Then, prices of intermediates must satisfy

$$p(\nu) = \begin{cases} \eta(\nu)R & \text{if } \nu \leq \theta \\ \beta(\nu)w & \text{if } \nu > \theta. \end{cases}$$

Therefore, the profit maximization problem of final good producers is

$$\max_{[y(\nu)]_{\nu\in[0,1]}} \left[\int_0^1 y(\nu)^{(\varepsilon-1)/\varepsilon} d\nu\right]^{\varepsilon/(\varepsilon-1)} - R\int_0^\theta \eta(\nu)y(\nu)d\nu - w\int_\theta^1 \beta(\nu)y(\nu)d\nu,$$

which gives the following simple solution:

$$y(\nu) = \begin{cases} [\eta(\nu)R]^{-\varepsilon}Y & \text{if } \nu \leq \theta \\ [\beta(\nu)w]^{-\varepsilon}Y & \text{if } \nu > \theta. \end{cases}$$

Now market clearing for capital implies

$$\int_0^\theta k(\nu)d\nu = \int_0^\theta \eta(\nu)y(\nu)d\nu = \int_0^\theta \eta(\nu)^{1-\varepsilon}R^{-\varepsilon}Yd\nu = \bar{K},$$

and similarly, market clearing for labor gives

$$\int_\theta^1 \beta(\nu)^{1-\varepsilon}w^{-\varepsilon}Yd\nu = \bar{L}.$$

Let us define

$$A(\theta) \equiv \int_0^\theta \eta(\nu)^{1-\varepsilon}d\nu \quad \text{and} \quad B(\theta) \equiv \int_\theta^1 \beta(\nu)^{1-\varepsilon}d\nu. \tag{18}$$

Then the market-clearing conditions can be expressed as

$$R^{1-\varepsilon} = \left[\frac{Y}{K}A(\theta)\right]^{(1-\varepsilon)/\varepsilon} \quad \text{and} \quad w^{1-\varepsilon} = \left[\frac{Y}{L}B(\theta)\right]^{(1-\varepsilon)/\varepsilon}. \tag{19}$$

Using (18) and (19), we can write aggregate output (and aggregate net output) as

$$Y = G(L, K, \theta) = [A(\theta)^{1/\varepsilon}K^{(\varepsilon-1)/\varepsilon} + B(\theta)^{1/\varepsilon}L^{(\varepsilon-1)/\varepsilon}]^{\varepsilon/(\varepsilon-1)}. \tag{20}$$

Equation (20) gives a simple expression for aggregate output as a function of the threshold task θ. It can be verified that Y exhibits decreasing differences in L and θ in the competitive equilibrium. In particular, equilibrium technology in this case will satisfy

$$\frac{\partial Y}{\partial \theta} = \frac{1}{\varepsilon - 1}[\eta(\theta^*)^{1-\varepsilon}A(\theta^*)^{(1-\varepsilon)/\varepsilon}K^{(\varepsilon-1)/\varepsilon} - \beta(\theta^*)^{1-\varepsilon}B(\theta^*)^{(1-\varepsilon)/\varepsilon}L^{(\varepsilon-1)/\varepsilon}]Y^{1/\varepsilon} = 0.$$

Since the term in brackets must be equal to zero, we must have

$$\frac{\partial^2 Y}{\partial\theta\partial L} = -\frac{1}{\varepsilon}\beta(\theta^*)^{1-\varepsilon}B(\theta^*)^{(1-\varepsilon)/\varepsilon}L^{-(1/\varepsilon)}Y^{1/\varepsilon} < 0.$$

This argument suggests why there is a close connection between machines replacing labor and technology being strongly labor saving. However, because we are in a fully competitive environment, $\partial Y/\partial\theta = 0$ in equilibrium (and hence induced changes in technology do not correspond to "technological advances").

Motivated by this, let us consider a version of the current environment corresponding to economy M and suppose that $G(L, K, \theta)$ is still given by (20), with cost $C(\theta)$, $\varepsilon > 1$, and $A(\theta)$ and $B(\theta)$ defined as in (18). The fact that $\alpha > 0$ in this economy ensures that an increase in θ indeed corresponds to a technological advance. Therefore, we have to check only whether technology is strongly labor saving or whether G exhibits decreasing differences in L and θ. Straightforward differentiation and some manipulation imply that $G_{L\theta}$ is proportional to

$$-\beta(\theta^*)^{1-\varepsilon}B(\theta^*)^{(1-\varepsilon)/\varepsilon}L^{(\varepsilon-1)/\varepsilon}G(L, K, \theta)^{1/\varepsilon} + (2-\alpha)C'(\theta^*)S_L,$$

with $S_L \equiv w_L L/[(2-\alpha)G(L, K, \theta)/(1-\alpha)]$ as the labor share of income. This expression will be negative when $C'(\theta^*)$ is small or when the labor share is small. But without specifying further functional forms, we cannot give primitive conditions for this to be the case.

Instead, technology is strongly labor-saving technology in a slight variation of this baseline model, where there is an additional factor of production, T, and decreasing returns to labor and capital. In particular, suppose that the G function takes the form

$$G(L, K, T, \theta) = [A(\theta)^{1/\varepsilon}K^{(\varepsilon-1)/\varepsilon} + B(\theta)^{1/\varepsilon}L^{(\varepsilon-1)/\varepsilon}]T^{1/\varepsilon}.$$

Then it can be verified that

$$G_{L\theta} = -\frac{\varepsilon-1}{\varepsilon^2}\beta(\theta^*)^{1-\varepsilon}B(\theta^*)^{(1-\varepsilon)/\varepsilon}L^{-(1/\varepsilon)}T^{1/\varepsilon} < 0,$$

so that technology is always strongly labor saving and a decrease in $\bar{L}$ will induce technological advances.

The analysis in this subsection therefore shows that models in which technological progress takes the form of machines replacing human labor create a natural tendency for strongly labor-saving technology. This result is intuitive since the process of machines replacing labor is closely connected to new technology substituting for and saving on labor.

V. Extensions and Further Results

In this section, I first discuss how the results presented so far can easily be extended to a dynamic framework. In addition to highlighting that the static model was adopted to communicate the main ideas in the clearest fashion, this analysis also shows that technology being strongly labor saving does not contradict the positive impact of secular technological changes on wages. Second, I consider an extension to a multisector economy in which labor scarcity and exogenous wage increases have different impacts on technology in different industries. Third, I briefly discuss how to incorporate endogenous factor supplies into this framework. Finally, I discuss how exogenous wage increases can lead to very different results than labor scarcity when the endogenous-technology demand curve for labor is upward sloping (in line with the conditions provided in theorem 6).

A. *Technological Change and Wage Increases*

Most studies of technological change use dynamic models. In contrast, the analysis in this paper so far has been carried out in a static model. This focus enabled me to isolate the impact of factor supplies on technological advances without introducing the additional functional form assumptions often imposed in dynamic models of economic growth. Nevertheless, it is useful to illustrate that the same insights apply in the context of a dynamic model. In addition, one objection to the plausibility of strongly labor-saving technology is that the growth process is accompanied by a steady increase in the wage rate, whereas strongly labor-saving technology implies that further technological advances will tend to reduce the marginal product of labor. I now provide a simple dynamic extension, which also shows that technological change can both be strongly labor saving and lead to increasing equilibrium wages.

For brevity, I use a slight variant of economy E and a simple demographic structure to communicate the main ideas, though the same results can be derived in the context of economy M (or the oligopolistic economy discussed in the Appendix). The form of the production function is motivated by the models in which machines replace labor such as those discussed in Section IV.C, though various different alternative formulations could also have been used to obtain similar results.

The economy is in discrete time and runs to infinite horizon. It is inhabited by one-period-lived individuals, each operating a firm. Therefore, each firm maximizes static profits. The total measures of individuals and firms are normalized to one. Suppose that there are three factors of production, labor, L, capital, K, and land or some other fixed factor, T. For simplicity, we focus on the case in which capital is also inelastically supplied (see subsec. C), so the supplies of the three factors are $\bar{L}$, $\bar{K}$, and $\bar{T}$. Past technology choices create an externality similar to that in economy E. In particular, suppose that all firms are competitive and the production function of each at time t is

$$\begin{aligned} y_t^i(L_t^i, K_t^i, T_t^i, \theta_t^i, \bar{A}_t) = \bar{A}_t[(\theta_t^i)^{1+\gamma}(K_t^i)^{(\varepsilon-1)/\varepsilon} \\ + (1-\theta_t^i)^{1+\gamma}(L_t^i)^{(\varepsilon-1)/\varepsilon}](T_t^i)^{1/\varepsilon}, \end{aligned} \tag{21}$$

where $\gamma < 0$ and $\varepsilon > 1$. This production function implies that higher θ will correspond to substituting capital for tasks previously performed by labor. Suppose that

$$\bar{A}_t = [1 + g(\bar{\theta}_{t-1})]\bar{A}_{t-1}, \tag{22}$$

where g is an increasing function and $\bar{\theta}_t \equiv \int_{i\in\mathcal{F}}\theta_t^i di$ is the average technology choice of firms at time t. This form of intertemporal technological externalities may result, for example, from the fact that past efforts to substitute machines or capital for labor advance, as well as build on, the knowledge stock of the economy.

A slightly modified version of proposition 2 applies in this environment and implies that equilibrium technology $\theta^*(\bar{L}, \bar{K})$ is given by the solution to

$$\frac{\partial Y_t^i(\bar{L}, \bar{K}, \bar{T}, \theta^*(\bar{L}, \bar{K}), \bar{A}_t)}{\partial \theta} = 0,$$

which implies that $\theta^*(\bar{L}, \bar{K})$ is uniquely determined and independent of $\bar{A}_t$ and $\bar{T}$. In particular,

$$\theta^*(\bar{L}, \bar{K}) = \frac{1}{1 + (\bar{K}/\bar{L})^{(\varepsilon-1)/\gamma\varepsilon}} \in (0, 1).$$

Since $\gamma < 0$ and $\varepsilon > 1$, $\theta^*(\bar{L}, \bar{K})$ is decreasing in $\bar{L}$, so labor scarcity increases $\theta^*(\bar{L}, \bar{K})$. Then (22) implies that a higher equilibrium level of $\theta^*(\bar{L}, \bar{K})$ will lead to faster growth of output and wages. This positive long-run association between output and wage growth occurs despite the fact that, at the margin, labor scarcity increases $\theta^*(\bar{L}, \bar{K})$ and substitutes for tasks previously performed by labor. It can be easily verified that an increase in $\bar{K}$ will also increase $\theta^*(\bar{L}, \bar{K})$. The immediate impact of this increase will be to reduce the level of wages, but this change will also increase the rate at which output and wages grow. This result highlights that in a dynamic framework with strongly labor-saving technology, the short-run and long-run impacts of technological advances on wages will typically differ. This analysis thus shows that in a dynamic economy, there is no tension between technological changes leading to a secular increase in wages and technology being strongly labor saving.[25]

B. *Technology Responses in a Multisector Economy*

The framework presented so far can be extended to a multisector economy to study how different sectors might respond to labor scarcity. To do this in the simplest possible way, let us suppose that the economy consists of S sectors, which are producing products that are perfect substitutes, and that each sector uses a different technology, $\theta_s \in \mathbb{R}^{K_s}$, and is supplied by a unique technology monopolist (with cost function $C_s(\theta_s)$). Let us also suppose that $Z = (Z_1, \ldots, Z_S)$ and Z_s is used only in

[25] Yet another alternative would be a dynamic competitive economy without externalities but one in which current advances in technology change the future level of technology. For example, we could assume that the cost function for technology creation/adoption for a firm at time t is $C(\theta_t, \theta_{t-1})$, which is increasing in θ_t and decreasing in θ_{t-1}, thus capturing the fact that past investments make future advances cheaper (one specific case would be $C(\theta_t - \theta_{t-1})$). While this is a reasonable specification, without a technology monopolist or externalities it does not change the conclusion that the choice of θ_t (for each t) would have already maximized net output, and thus local increases in θ_t cannot be considered "technological advances." In this case, naturally, the relevant measure of net output would be the discounted net present value of the future output stream. Then from the maximization problem of a decentralized firm with respect to the sequence $\{\theta_t\}$, it follows that a small change in any component of θ_t would have only a second-order effect on the net present discounted value of output.

sector s.[26] Imposing market clearing for nonlabor factors, we can then write the production function of each sector as in (9) with G replaced by $G_s(L_s, \bar{Z}_s, \theta_s)$, which is assumed to be concave in L_s and Z_s and strictly concave in L_s. An equilibrium is then defined in analogous fashion to the equilibrium in economy O in the Appendix, with the additional requirements that employment in each sector is consistent with profit maximization and the labor market clears. This implies that an equilibrium can be represented by w^*, $\theta^* = (\theta_1^*, \dots, \theta_S^*)$, and $L^* = (L_1^*, \dots, L_S^*)$ such that technology monopolists maximize profits, that is,

$$\theta_s^* \in \arg\max_{\theta_s} G_s(L_s^*, \bar{Z}_s, \theta_s) - C_s(\theta_s) \quad \text{(for each } s\text{)}; \tag{23}$$

final good producers in each sector maximize profits, which after solving out for profit-maximizing demand for machines we can write as

$$L_s^* \in \arg\max_{L_s} (1-\alpha)^{-1} G_s(L_s^*, \bar{Z}_s, \theta_s) - w^* L_s \quad \text{(for each } s\text{)}; \tag{24}$$

and the labor market clears, that is,

$$\sum_{s=1}^{S} L_s^* = \bar{L}. \tag{25}$$

Problems (23) and (24) can be combined and written as

$$(\theta_s^*, L_s^*) \in \arg\max_{\theta_s, L_s} G_s(L_s^*, \bar{Z}_s, \theta_s) - C_s(\theta_s) - (1-\alpha) w^* L_s \quad \text{(for each } s\text{)}, \tag{26}$$

provided that the right-hand side of (26) is concave in θ_s, L_s, and Z_s. Suppose that this is the case so that an equilibrium can be represented by (26) and (25).

Now consider the effect of a reduction in $\bar{L}$. Since each G_s is concave in L_s, each sector has a downward-sloping demand for labor. Then a reduction in $\bar{L}$ will increase the wage rate w^*, inducing lower employment in each sector. As a consequence, with the same reasoning as used so far, technological advances in sector s will be encouraged or discouraged depending on whether technology is strongly labor saving or labor complementary in that sector.

THEOREM 4. Consider the multisector economy discussed in this subsection. Suppose that, for each $s = 1, \dots, S$, $C_s(\theta_s)$ satisfies assumption 1, $G_s(L_s, Z_s, \theta_s) - C_s(\theta_s)$ is supermodular in θ_s, and the right-hand side of (26) is concave in θ_s, L_s, and Z_s. Then labor scarcity (lower $\bar{L}$)

[26] The perfect substitutes assumption can be relaxed, but one would then have to ensure that the indirect effects of technology choice in one sector working through relative prices do not overturn the consequences of the direct effects identified in this analysis. The assumption that there is no competition between sectors for nonlabor factors is made for simplicity and can also be relaxed.

will induce (discourage) technological advances in sector s if this sector's technology is strongly labor saving (labor complementary).

Proof. Consider the case in which technology is strongly labor complementary. Then, for each s, (26) is concave and supermodular in θ_s, L_s, and $-w^*$. Thus an increase in w^* will reduce θ_s^* and L_s^* (for each s). This implies that the left-hand side of (25) is decreasing in w^*, and therefore a reduction in $\bar{L}$ will increase w^* and reduce L_s^* and θ_s^* for each s. When technology is strongly labor saving, the same argument can be applied to $-\theta_s$ by noting that, for each s, (26) is concave and supermodular in $-\theta_s$, L_s, and $-w^*$. QED

The interesting implication of this theorem is that labor scarcity (or, equivalently, exogenous wage increases) need not have uniform effects in different sectors in the economy. They can encourage technological advances in some sectors while discouraging them in others. In the context of the implications of labor-intensive Chinese exports (discussed in n. 3 in the introduction), this implies that the consequent reduction in wages (and increase in labor abundance) may have differential effects across sectors, inducing technological advances in some while discouraging it in others.[27]

C. *Endogenous Factor Supplies*

To highlight the new results of the framework presented in this paper, the analysis so far has treated the supply of all factors as exogenous and has thus ignored both the response of labor supply to changes in wages and the adjustment of other factors, such as capital, to changes in factor supplies or labor market regulations that exogenously raise wages. Endogenous labor supply will be briefly discussed in the next subsection.

Here let us focus on the endogenous supply of other factors. For example, we can imagine a situation in which one of the other factors of production is capital that is infinitely elastically supplied. In this case, a change in labor supply will affect both technology and the supply of capital so that the rental rate of capital remains constant (since it is supplied with infinite elasticity). Consequently, the overall impact on technology will be a combination of the direct effect of labor supply and an indirect effect working through the induced changes in the capital stock of the economy. Although the details of the analysis are somewhat different in this case, the main results presented in Section III remain unchanged. In particular, those results were stated in terms of strongly labor-saving (complementary) technology with the supply of other factors held fixed. One can alternatively define notions of labor-

[27] This also implies that cross-industry comparisons might be partly driven by the impact on less affected industries.

saving (complementary) technology holding the price of capital constant. It is then straightforward to show that theorems 1–3 would apply with these modified definitions.

Endogenous supply (or accumulation) of capital could have richer effects in dynamic settings. For example, in the model considered in subsection A, an increase in $\bar{K}/\bar{L}$ raises $\theta^*(\bar{L}, \bar{K})$ and induces technological advances. In this case, we can have "feedback effects": labor scarcity induces technological advances that increase output, and this could increase the pace of capital accumulation, further encouraging technological advances.

D. *Exogenous Wage Increases versus Labor Scarcity*

Let us now suppose that the supply of labor is endogenous, given by a standard labor supply function $L^s(w_L)$. From the analysis leading to corollary 1, it is then clear that none of the results will be affected if the Hessian $\nabla^2 F_{(L,\theta)(L,\theta)}$ is negative semidefinite and $L^s(w_L)$ is increasing. In particular, in this case, we can study the impact of a shift in labor supply from $L^s(w_L)$ to $\tilde{L}^s(w_L)$, where $\tilde{L}^s(w_L) < L^s(w_L)$, or the impact of a binding minimum wage. Since $\nabla^2 F_{(L,\theta)(L,\theta)}$ is negative semidefinite, the endogenous-technology relationship between employment and wages is decreasing. Therefore, a leftward shift of the labor supply schedule from $L^s(w_L)$ to $\tilde{L}^s(w_L)$ will reduce employment and increase wages. The implications for technology are determined again from theorems 1–3 by whether technology is strongly labor saving or strongly labor complementary.

However, the close connection between exogenous wage increases and labor scarcity highlighted in corollary 1 is broken when $\nabla^2 F_{(L,\theta)(L,\theta)}$ is not negative semidefinite. In this case, the endogenous-technology demand curve is upward sloping, and thus a decrease in labor supply reduces wages, whereas an increase in labor supply increases wages. The implications of an upward-sloping endogenous-technology demand curve are particularly interesting when labor supply is endogenous. In this case, multiple equilibria, characterized by different levels of labor supply, technology, and wages, become possible as shown in figure 1. The next example illustrates this possibility using a simple extension of example 1.

Example 2. Suppose that the G function takes a form similar to that in example 1 except for a slight variation in exponents. In particular, suppose that $G(L, K, T, \theta) = \frac{3}{2}[\theta K^{2/3} + 3(1-\theta)L^{2/3}]T^{1/3}$, and the cost of technology creation is $C(\theta) = \frac{3}{4}\theta^2$. It can now be verified that $\nabla^2 F_{(L,\theta)(L,\theta)}$ is no longer negative semidefinite (in contrast to example 1). Therefore, from theorem 6, we expect the endogenous-technology relationship between employment (labor supply) and the wage to be in-

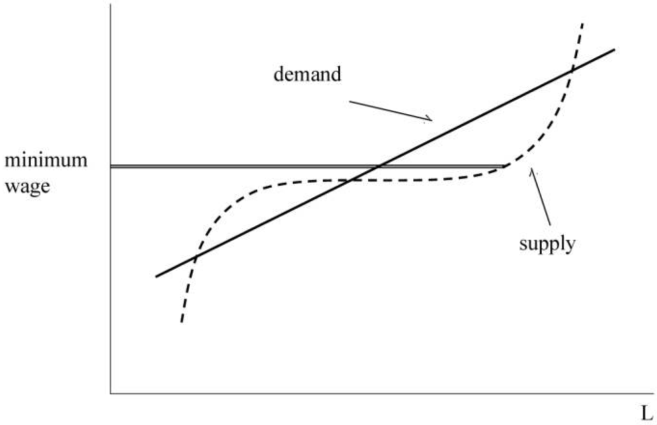

FIG. 1.—Multiple equilibria and the effects of exogenous wage increases

creasing. We will now see how this interacts with endogenous labor supply.

Let us again normalize the supply of the other factors to $\bar{K} = \bar{T} = 1$ and denote employment by L^e. Equilibrium technology then satisfies $\theta^*(L^e) = 1 - (L^e)^{2/3}$. The equilibrium wage is given by

$$w(L^e, \theta) = (1 - \theta)(L^e)^{-1/3}$$

for a given level of technology θ, and once we take into account the response of θ to employment L^e, we have

$$w(L^e, \theta^*(L^e)) = (L^e)^{1/3}, \tag{27}$$

which illustrates the potentially upward-sloping endogenous-technology relationship between employment and wages discussed briefly in note 18 (see also theorem 6 in the Appendix). Now suppose that labor supply is also responsive to wages and takes the form $L^s(w) = 6w^2 - 11w + 6$. Now combining this supply relationship with (27), we find that there are three equilibrium wages, with different levels of labor supply and technology, $w = 1$, 2, and 3. Moreover, technology is most advanced and labor supply is highest at $w = 3$.

Next consider a minimum wage between 2 and 3. This will typically destroy the first two equilibria. Thus the implications of exogenous wage increases could be very different (see also fig. 1). The minimum wage indeed destroys the equilibria at $w = 1$ and $w = 2$, but depending on the exact price determination procedure, other equilibria, including an extreme no-activity equilibrium with zero employment, may also emerge. When we are in economy M, such a no-activity equilibrium does not exist because the monopolist acts as a "Stackelberg leader" and chooses the technology anticipating employment.[28]

VI. Conclusion

This paper studied the conditions under which the scarcity of a factor encourages technological progress (innovation or adoption of technologies increasing output). Despite a large literature on endogenous technological change and technology adoption, we do not yet have a comprehensive theoretical or empirical understanding of the determinants of innovation, technological progress, and technology adoption. Most important, how factor proportions, for example, abundance or scarcity of labor, affect technology is poorly understood.

In standard endogenous growth models, which feature a strong scale effect, an increase in the supply of a factor encourages technological progress. In contrast, the famous Habakkuk hypothesis claims that technological progress was more rapid in the nineteenth-century United States than in Britain because of labor scarcity in the former country. Related ideas are often suggested as possible reasons for why high wages might have encouraged more rapid adoption of certain technologies in continental Europe than in the United States over the past several decades. The Porter hypothesis in the context of green technologies has a related logic and suggests that environmental regulations can be a powerful inducement to technological progress.

This paper characterizes the conditions under which factor scarcity can induce technological advances (innovation or adoption of more productive technologies). The main result of the paper shows that labor scarcity induces technological advances if technology is strongly labor saving, meaning that technological advances reduce the marginal product of labor. In contrast, labor scarcity discourages technological advances if technology is strongly labor complementary, meaning that technological advances increase the marginal product of labor. I also show that, under some further conditions, an increase in wage levels above the competitive equilibrium has effects similar to labor scarcity. In addition, I provide examples of environments in which technology can be strongly labor saving and showed that such a result is not possible in certain canonical models. These results clarify the conditions under which labor scarcity and high wages are likely to encourage innovation and adoption of more productive technologies. Notably, these conditions do not hold in most commonly used macroeconomic and growth models, which may be one reason why the positive effects of labor scar-

[28] However, in economy O, such a no-activity equilibrium may arise if a high level of minimum wage is imposed.

city on technology, though conjectured and discussed often, have not appeared prominently in the growth literature.

Although technology tends to be strongly labor complementary (rather than labor saving) in many commonly used models, this does not imply that it is so in reality. Whether labor scarcity and high wages may induce innovation and technology adoption in practice is thus an open empirical question and is likely to depend on the specific application (time period, institutional framework, the industry in question, etc.). Existing evidence suggests that this is a possibility but is not conclusive. For example, Newell et al. (1999) show an effect of changes in energy prices on the direction of innovation and on the energy efficiency of household durables, and Popp (2002) provides similar evidence using patents. Acemoglu and Finkelstein (2008) show that the Prospective Payment System reform of Medicare in the United States, which increased the labor costs of hospitals with a significant share of Medicare patients, appears to have induced significant technology adoption in the affected hospitals. In a different context, Lewis (2005) shows that the skill mix in U.S. metropolitan areas appears to have an important effect on the choice of technology of manufacturing firms. Further research could shed more systematic light on the empirical conditions under which we may expect greater factor prices and factor scarcity to be an inducement, rather than a deterrent, to technology adoption and innovation.

Appendix

Weak and Strong Equilibrium Bias Results

We say that there is *weak equilibrium bias* if the combined effect of induced changes in technology resulting from an increase in labor supply raises the marginal product of labor at the starting factor proportions (i.e., it "shifts out" the demand for labor). We say that there is *strong equilibrium bias* if this induced effect in technology is sufficiently large so as to outweigh the direct effect of the increase in L (which is always to reduce its marginal product). Mathematically, there is weak equilibrium bias at some $(\bar{L}, \bar{Z})$ if

$$\sum_{k=1}^{K} \frac{\partial w_L}{\partial \theta_k} \frac{\partial \theta_k^*}{\partial L} \geq 0,$$

where w_L is the wage evaluated at $(\bar{L}, \bar{Z})$ and θ^* stands for $\theta^*(\bar{L}, \bar{Z})$. Similarly, there is strong equilibrium bias at $(\bar{L}, \bar{Z})$ if

$$\frac{dw_L}{dL} = \frac{\partial w_L}{\partial L} + \sum_{k=1}^{K} \frac{\partial w_L}{\partial \theta_k} \frac{\partial \theta_k^*}{\partial L} > 0,$$

where dw_L/dL denotes the total derivative, and $\partial w_L/\partial L$ denotes the partial derivative holding $\theta = \theta^*(\bar{L}, \bar{Z})$.

The following results are adapted from Acemoglu (2007). They apply to economies D and M as stated and also apply to economy E or O with the additional condition that $\nabla_L \theta^*$ exists. But importantly, the conditions for strong bias in theorem 6 cannot be true in economy D. Recall that $F(L, Z, \theta) \equiv G(L, Z, \theta) - C(\theta)$.

THEOREM 5. Let the equilibrium technology at factor supplies $(\bar{L}, \bar{Z})$ be $\theta^*(\bar{L}, \bar{Z})$ and suppose that assumption 1 holds. Then there is *weak absolute equilibrium bias* at all $(\bar{L}, \bar{Z})$, that is,

$$\sum_{k=1}^{K} \frac{\partial w_L}{\partial \theta_k} \frac{\partial \theta_k^*}{\partial L} \geq 0 \quad \text{for all } (\bar{L}, \bar{Z}),$$

with strict inequality if $\partial \theta_k^* / \partial L \neq 0$ for some $k = 1, \ldots, K$.

Proof. The proof follows from the implicit function theorem. For a matrix (vector) v, let v' denote its transpose. Define Δw_L as the change in w_L resulting from the induced change in θ (at given factor proportions):

$$\begin{aligned} \Delta w_L &\equiv \sum_{j=1}^{K} \frac{\partial w_L}{\partial \theta_j} \frac{\partial \theta_j^*}{\partial L} \\ &= [\nabla_\theta w_L]'[\nabla_L \theta^*] \qquad \text{(A1)} \\ &= [\nabla_{\theta L}^2 F]'[\nabla_L \theta^*], \end{aligned}$$

where $[\nabla_\theta w_L]$ is a $K \times 1$ vector of changes in w_L in response to each component of $\theta \in \Theta \subset \mathbb{R}^K$ and $[\nabla_L \theta^*]$ is the gradient of θ with respect to L, that is, a $K \times 1$ vector of changes in each component of θ in response to the change in $\bar{Z}$. The vector $[\nabla_L \theta^*]$ is well defined following from the implicit function theorem given assumption 1. The second line above uses the fact that w_L is the derivative of the F function, so $[\nabla_{\theta L}^2 F]$ is also the $K \times 1$ vector of changes in w_L in response to each component of θ. Thus

$$[\nabla_L \theta^*]' = -[\nabla_{\theta L}^2 F]'[\nabla_{\theta\theta}^2 F]^{-1}, \qquad \text{(A2)}$$

where $\nabla_{\theta\theta}^2 F$ is the $K \times K$ Hessian of G with respect to θ. The fact that θ^* is a solution to the maximization problem (13) implies that $\nabla_{\theta\theta}^2 F$ is negative semidefinite. That $\nabla_L \theta^*$ exists then implies that $\nabla_{\theta\theta}^2 F$ is nonsingular and thus negative definite. Since it is a Hessian, it is also symmetric. Therefore, its inverse $[\nabla_{\theta\theta}^2 F]^{-1}$ is also symmetric and negative definite. Substituting (A2) in (A1), we obtain

$$\Delta w_L = -[\nabla_{\theta L}^2 F]'[\nabla_{\theta\theta}^2 F]^{-1}[\nabla_{\theta L}^2 F] \geq 0,$$

which establishes the weak inequality.

By the definition of a negative definite matrix B, $x'Bx < 0$ for all $x \neq 0$, so to establish the strict inequality, it suffices that one component of $\nabla_L \theta^*$ is nonzero, that is, $\partial \theta_j^* / \partial L \neq 0$ for one $j = 1, \ldots, K$. QED

THEOREM 6. Let the equilibrium technology at factor supplies $(\bar{L}, \bar{Z})$ be $\theta^*(\bar{L}, \bar{Z})$ and suppose that assumption 1 holds. Then there is *strong absolute equilibrium bias* at $(\bar{L}, \bar{Z})$, meaning that

$$\frac{dw_L}{dL} = \frac{\partial w_L}{\partial L} + \sum_{k=1}^{K} \frac{\partial w_L}{\partial \theta_k} \frac{\partial \theta_k^*}{\partial L} > 0$$

if and only if $F(L, Z, \theta)$'s Hessian in (L, θ), $\nabla^2 F_{(L,\theta)(L,\theta)}$, evaluated at $(\bar{L}, \bar{Z}, \theta^*(\bar{L}, \bar{Z}))$, is not negative semidefinite at $(\bar{L}, \bar{Z})$.

Proof. Once again note that $w_L = \partial F/\partial L$. Then the overall change in the price of factor L is

$$\frac{dw_L}{dL} = \frac{\partial^2 F}{\partial L^2} - [\nabla^2_{\theta L}F]'[\nabla^2_{\theta\theta}F]^{-1}[\nabla^2_{\theta L}F]. \tag{A3}$$

From the maximization problem of final good producers, $\partial^2 F/\partial L^2 \leq 0$, and from the maximization problem of technology suppliers, $\nabla^2_{\theta\theta}F$ is negative definite and symmetric (which implies that its inverse $[\nabla^2_{\theta\theta}F]^{-1}$ is also negative definite and symmetric). Lemma 1 in Acemoglu (2007) shows that for an $(n-1) \times (n-1)$ symmetric negative definite matrix Q with inverse denoted by Q^{-1}, scalar b, and $(n-1) \times 1$ column vector v, the $n \times n$ matrix

$$B = \begin{pmatrix} Q & v \\ v' & b \end{pmatrix}$$

is negative semidefinite *if and only if* $b - v'Q^{-1}v \leq 0$. Let us now apply this lemma with $B = [\nabla^2 F_{(L,\theta)(L,\theta)}]$, $b = \partial^2 F/\partial L^2$, $Q = [\nabla^2_{\theta\theta}F]$, and $v = [\nabla^2_{\theta L}F]$, so that (A3) evaluated at $(\bar{L}, \bar{Z}, \theta^*(\bar{L}, \bar{Z}))$ is equal to $b - v'Q^{-1}v$. This lemma implies that if $\nabla^2 F_{(L,\theta)(L,\theta)}$ is not negative semidefinite at $(\bar{L}, \bar{Z}, \theta^*(\bar{L}, \bar{Z}))$, then $b - v'Q^{-1}v > 0$, so that $dw_L/dL > 0$ and there is strong bias at $(\bar{L}, \bar{Z}, \theta^*(\bar{L}, \bar{Z}))$.

Conversely, again from lemma 1 in Acemoglu (2007), if $\nabla^2 F_{(L,\theta)(L,\theta)}$ is negative semidefinite at $(\bar{L})$, then $b - v'Q^{-1}v \leq 0$ and $dw_L/dL \leq 0$, so that there is no strong bias at $(\bar{L}, \bar{Z})$. QED

Proof of Theorem 2[29]

Let

$$\phi(\bar{L}, \theta) \equiv \arg\max_{\theta' \in \Theta} G(\bar{L}, \bar{Z}, \theta', \bar{\theta} = \theta) - C(\theta'),$$

where I have dropped the dependence on Z to simplify notation. Assumption 1 ensures that $\phi(L, \theta)$ is single valued. Recall that here $\phi(L, \theta) \in \Theta \subset \mathbb{R}^K$, and I will use $\phi_k(L, \theta)$ to denote its kth component and θ_k for the kth component of θ. I write $\theta > \theta'$ for $\theta \geq \theta'$ with a strict inequality for at least one component and $\theta \not> \theta'$ to denote the opposite of this (i.e., that not all components of θ are greater than those of θ' with at least one strict inequality).

Consider the case of strongly labor complementary technology and focus on the smallest equilibrium corresponding to labor supply $\bar{L}$, denoted by $\theta^-(\bar{L})$. Since G is supermodular in θ and $\bar{\theta}$, ϕ is increasing in θ (Topkis 1998, theorem 2.8.2), and thus ϕ has a smallest fixed point and such a smallest equilibrium indeed exists (theorem 2.7.1). Since $\theta^-(\bar{L})$ is the smallest equilibrium, we have

$$\theta^-(\bar{L}) = \phi(\bar{L}, \theta^-(\bar{L})), \tag{A4}$$

and ϕ does not have a smaller fixed point. Assumption 1 ensures that (A4) holds with equality for some $\theta^-(\bar{L}) > 0$. Since technology is strongly labor complementary at $(\bar{L}, \bar{Z})$ and ϕ is continuous (and $\theta^-(\bar{L}) > 0$), there exists a real number $\delta > 0$ such that ϕ is (strictly) increasing in L on

[29] This proof generalizes the argument in proposition 4 of Acemoglu and Wolitzky (2010).

$$[\bar{L}-\delta, \bar{L}+\delta] \times [\theta^-(\bar{L})-\delta, \theta^-\bar{L}+\delta],$$

where $\theta+\delta$ stands for the vector with δ added to each component. The function $\phi(\bar{L}, \theta)$ has a smallest fixed point on

$$[\bar{L}-\delta, \bar{L}+\delta] \times [\theta^-(\bar{L})-\delta, \theta^-\bar{L}+\delta]$$

(Topkis 1998, theorem 2.5.1). Consider the function

$$\Phi_\delta(L) \equiv \min_{\{\theta \geq 0: \theta \not> \theta^-(\bar{L})-\delta\}} \sum_{k=1}^{K} |\phi_k(L, \theta)-\theta_k|,$$

which is well defined and continuous in L by Berge's maximum theorem (since the set of θ such that $\theta \geq 0$ and $\theta \not> \theta^-(\bar{L})-\delta$ is a compact set). Moreover, there exists $\varepsilon > 0$ such that $\Phi_\delta(\bar{L}) \geq \varepsilon$ since otherwise there would exist $\theta \not> \theta^-(\bar{L})-\delta$ such that $\theta = \phi(\bar{L}, \theta)$, but this would contradict the fact that $\theta^-(\bar{L})$ is the smallest equilibrium (since at least one component of such a θ would be strictly smaller than the corresponding component of $\theta^-(\bar{L})$). As $\Phi_\delta(L)$ is continuous, for any $\varepsilon' > 0$ there exists $\delta' > 0$ such that, for any $L \in (\bar{L}-\delta', \bar{L}+\delta')$,

$$|\Phi_\delta(L)-\Phi_\delta(\bar{L})| < \varepsilon'.$$

Choose $\varepsilon' = \varepsilon$ and denote the corresponding δ' by $\hat{\delta}$, and let $\tilde{\delta} = \min\{\hat{\delta}, \delta\}$. Then for any $L \in (\bar{L}-\tilde{\delta}, \bar{L}+\tilde{\delta})$, $\Phi_{\tilde{\delta}}(L) > 0$, which implies $\theta^-(L) > \theta^-(\bar{L})-\tilde{\delta}$.

Let us next establish that for any $L \in (\bar{L}, \bar{L}+\tilde{\delta})$, $\theta^-(L) > \theta^-(\bar{L})$. To obtain a contradiction, suppose that this is not the case. This implies that there exists $\hat{L} \in (\bar{L}, \bar{L}+\tilde{\delta})$ such that $\theta^-(\bar{L})-\tilde{\delta} \leq \theta^-(\hat{L}) \leq \theta^-(\bar{L})$. The first inequality follows from the relationship that we have just established (that $\theta^-(L) > \theta^-(L_0)-\tilde{\delta}$ for all $L \in (\bar{L}-\tilde{\delta}, \bar{L}+\tilde{\delta})$). To obtain the second inequality, note first that, by hypothesis, $\theta^-(L) \not> \theta^-(\bar{L})$ and, second, that on

$$[\bar{L}-\tilde{\delta}, \bar{L}+\tilde{\delta}] \times [\theta^-(\bar{L})-\tilde{\delta}, \theta^-\bar{L}+\tilde{\delta}],$$

ϕ has increasing differences in θ and L and is supermodular in θ, and thus its set of fixed points for any L is a lattice, and the smallest fixed point is increasing in $L \in [\bar{L}-\tilde{\delta}, \bar{L}+\tilde{\delta}]$ (Topkis 1998, theorem 2.5.2). Thus if $\theta^-(\hat{L}) \not> \theta^-(\bar{L})$, we must have $\theta^-(\hat{L}) \leq \theta^-(\bar{L})$. Moreover, since (each component of) $\phi(L, \theta)$ is (strictly) increasing in L on $[\bar{L}-\tilde{\delta}, \bar{L}+\tilde{\delta}] \times [\theta^-(\bar{L})-\tilde{\delta}, \theta^-(\bar{L})+\tilde{\delta}]$,

$$\theta^-(\hat{L}) = \phi(\hat{L}, \theta^-(\hat{L})) > \phi(\bar{L}, \theta^-(\hat{L})). \tag{A5}$$

Since $\phi(\bar{L}, \theta)$ is continuous in θ and $\phi(\bar{L}) \geq 0$ for all θ, Brouwer's fixed-point theorem implies that ϕ has a fixed point θ^* in $[0, \theta^-(\hat{L})]$. Moreover, $\theta^* < \theta^-(\hat{L}) \leq \theta^-(\bar{L})$, where the first inequality follows from (A5) and the second by hypothesis. This contradicts the fact that $\theta^-(\bar{L})$ is the smallest fixed point of ϕ and establishes that $\theta^-(L) > \theta^-(\bar{L})$ for any $L \in (\bar{L}, \bar{L}+\tilde{\delta})$, $\theta^-(L) > \theta^-(\bar{L})$.

The proof that for any $L \in (\bar{L}-\tilde{\delta}, \bar{L})$, $\theta^-(L) < \theta^-(\bar{L})$ is analogous, and thus we have established that when technology is strongly labor complementary, the smallest equilibrium technology increases when L increases in the neighborhood of $(\bar{L}, \bar{Z})$. The proofs for strongly labor-saving technology and the greatest equilibrium are also analogous. QED

Economy O—Oligopoly Equilibrium

It is also straightforward to extend the environment in the previous section so that technologies are supplied by a number of competing (oligopolistic) firms rather than a monopolist. Let θ be the vector $\theta \equiv (\theta_1, \ldots, \theta_S)$, and suppose that output is now given by

$$y^i = \alpha^{-\alpha}(1-\alpha)^{-1}G(L^i, Z^i, \theta)^\alpha \sum_{s=1}^{S} q_s^i(\theta_s)^{1-\alpha}, \tag{A6}$$

where $\theta_s \in \Theta_s \subset \mathbb{R}^{K_s}$ is a technology supplied by technology producer $s = 1, \ldots, S$, and $q_s^i(\theta_s)$ is the quantity of intermediate good (or machine) embodying technology θ_s, supplied by technology producer s, used by final good producer i.[30] Factor markets are again competitive, and a maximization problem similar to (10) gives the inverse demand functions for intermediates as

$$q_s^i(\chi_s, L^i, Z^i|\theta) = \alpha^{-1}G(L^i, Z^i, \theta)\chi_s^{-1/\alpha}, \tag{A7}$$

where χ_s is the price charged for the intermediate good embodying technology θ_s by oligopolist $s = 1, \ldots, S$.

Let the cost of creating technology θ_s be $C_s(\theta_s)$ for $s = 1, \ldots, S$. The cost of producing each unit of any intermediate good is again normalized to $1 - \alpha$. An *equilibrium* in economy O is a set of firm decisions $\{L^i, Z^i, [q_s^i(\chi_s, L^i, Z^i|\theta)]_{s=1}^S\}_{i\in\mathcal{F}}$, technology choices $(\theta_1, \ldots, \theta_S)$, and factor prices (w_L, w_L) such that

$$\{L^i, Z^i, [q_s^i(\chi_s, L^i, Z^i|\theta)]_{s=1}^S\}_{i\in\mathcal{F}}$$

maximize firm profits given (w_L, w_L) and the technology vector $(\theta_1, \ldots, \theta_S)$, (3) holds, and the technology choice and pricing decisions for technology producer $s = 1, \ldots, S$, (θ_s, χ_s), maximize its profits subject to (A7).

The profit maximization problem of each technology producer is similar to (12) and implies a profit-maximizing price for intermediate goods equal to $\chi_s = 1$ for any $\theta_s \in \Theta_s$ and each $s = 1, \ldots, S$. Consequently, with the same steps as in the previous section, each technology producer will solve the problem

$$\max_{\theta_s\in\Theta_s} \Pi_s(\theta_s) = G(\bar{L}, \bar{Z}, \theta_1, \ldots, \theta_s, \ldots, \theta_S) - C_s(\theta_s).$$

This argument establishes the following proposition.

PROPOSITION 4. Suppose that $G(L, Z, \theta_1, \ldots, \theta_s, \ldots, \theta_S)$ is concave in L and Z. Then any equilibrium technology in economy O is a vector $(\theta_1^*, \ldots, \theta_S^*)$ such that θ_s^* is a solution to

[30] Equation (A6) implicitly imposes that technology θ_s will affect productivity even if firm i chooses $q_s^i(\theta) = 0$. This can be relaxed by writing

$$y^i = \alpha^{-\alpha}(1-\alpha)^{-1}G(L^i, Z^i, \tilde{\theta}^i)^\alpha \sum_{s=1}^{S} q_s^i(\theta_s)^{1-\alpha},$$

where $\tilde{\theta}^i \equiv (\tilde{\theta}_1, \ldots, \tilde{\theta}_S)$, with $\tilde{\theta}_s \equiv \mathbf{1}(q_s^i(\theta_s > 0))\theta_s$, so that the firm does not benefit from the technologies that it does not purchase. Let θ_{-s} be equal to θ with the sth element set equal to zero. Then, provided that $G(\bar{L}, \bar{Z}, \theta) - G(\bar{L}, \bar{Z}, \theta_{-s})$ is not too large, in particular, if $G(\bar{L}, \bar{Z}, \theta) - G(\bar{L}, \bar{Z}, \theta_{-s}) \le \alpha(1-\alpha)G(\bar{L}, \bar{Z}, \theta)/(S-1)$, then the analysis here applies. This latter condition ensures that no oligopolist would like to deviate and "hold up" final good producers by charging a very high price.

$$\max_{\theta_s \in \Theta_s} G(\bar{L}, \bar{Z}, \theta_1^*, \ldots, \theta_s, \ldots, \theta_S^*) - C_s(\theta_s)$$

for each $s = 1, \ldots, S$, and any such vector gives an equilibrium technology.

This proposition shows that the equilibrium corresponds to a Nash equilibrium, and thus, as in economy E, it is given by a fixed-point problem. Nevertheless, this has little effect on the results below, and all the results stated in this paper hold for this oligopolistic environment.[31]

THEOREM 7. Consider economy O. Suppose that assumption 1 holds and $G(L, Z, \theta) - C_s(\theta_s)$ is supermodular (for each s), and denote the smallest and the greatest equilibria by θ^- and θ^+.

1. If technology is strongly labor saving at $(\bar{L}, \bar{Z}, \theta^-)$ (respectively, at $(\bar{L}, \bar{Z}, \theta^+)$), labor scarcity will induce technological advances (in the sense that a small decrease in $\bar{L}$ will increase θ^- [respectively, θ^+]); if technology is strongly labor complementary at $(\bar{L}, \bar{Z}, \theta^-)$ (respectively, at $(\bar{L}, \bar{Z}, \theta^+)$), labor scarcity will discourage technological advances (in the sense that a small decrease in $\bar{L}$ will reduce θ^- [respectively, θ^+]).
2. If technology is strongly labor saving globally, then labor scarcity will induce technological advances (in the sense of increasing θ^- and θ^+). If technology is strongly labor complementary globally, then labor scarcity will discourage technological advances (in the sense of reducing θ^- and θ^+).

Proof. The proof of the first part is analogous to the proof of theorem 2. For the second part, note that in economy O, the equilibrium is given by proposition 4 and corresponds to a Nash equilibrium of a game among the S oligopolist technology suppliers. The profit function of oligopolist s is

$$G(\bar{L}, \bar{Z}, \theta_1, \ldots, \theta_s, \ldots, \theta_S) - C_s(\theta_s)$$

and is supermodular in θ, and thus this is a supermodular game. In addition, when G exhibits strict increasing differences in L and θ globally, the payoff of each oligopolist exhibits strict increasing differences in its own strategies and L. Then theorem 4.2.2 from Topkis (1998) implies that the greatest and smallest equilibria of this game are nondecreasing in $\bar{L}$ and assumption 1 ensures that they are increasing. This establishes the result when technology is strongly labor complementary globally. The result when technology is strongly labor saving globally follows with the same argument, using $-\theta$ instead of θ. QED

[31] It is also worth noting that the special case in which $\partial^2 G/\partial\theta_s\partial\theta_{s'} = 0$ for all s and s' is identical to the product variety models of Romer (1990) and Grossman and Helpman (1991), and in this case, the equilibrium can again be represented as a solution to a single maximization problem, i.e., that of maximizing $G(\bar{L}, \bar{Z}, \theta_1, \ldots, \theta_s, \ldots, \theta_S) - \sum_{s=1}^{S} C_s(\theta_s)$. Finally, note also that, with a slight modification, this environment can also embed monopolistic competition, where the number of firms is endogenous and is determined by the zero profit condition (the technology choice of nonactive firms will be equal to zero in this case), and the equilibrium problem will be

$$\max_{\theta_s \in \Theta_s} G(\bar{L}, \bar{Z}, \theta_1^*, \ldots, \theta_s, \ldots, \theta_{S'}^*, 0, \ldots, 0) - C_s(\theta_s)$$

for $1 \leq s \leq S'$, with S' being determined endogenously in equilibrium.

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