Motivation (I)

- Recent economic recession has reopened the debate on industrial policy.
- In October 2008, the US government bailed out GM and Chrysler. (Estimated cost, $82 Billion)
- Similar bailouts in Europe: Estimated cost €1.18 trillion in 2010, 9.6% of EU GDP.
- Many think that this was a success from a short-term perspective, because these interventions
  - protected employment, and
  - encouraged incumbents to undertake greater investments,
Motivation (II)

- More generally, what are the implications of “industrial policy” for R&D, reallocation, productivity growth, and welfare?
- Bailouts or support for incumbents could increase growth if there is insufficient entry or if they support incumbent R&D.
  - In fact, this is recently been articulated as an argument for industrial policy.
- They may reduce growth by
  - preventing the entry of more efficient firms and
  - slowing down the reallocation process.
- Reallocation potentially important, estimated sometimes to be responsible for up to 70-80% of US productivity growth.
Motivation (III)

- What’s the right framework?
  1. endogenous technology and R&D choices,
  2. rich from dynamics to allow for realistic reallocation and matched the data (and for selection effects),
  3. different types of policies (subsidies to operation vs R&D),
  4. general equilibrium structure (for the reallocation aspect),
  5. exit for less productive firms/products (so that the role of subsidies that directly or indirectly prevent exit can be studied).

- Starting point: Klette and Kortum’s (2004) model of micro innovation building up to macro structure.
Motivating Facts

- R&D intensity is independent of firm size.
- The size distribution of firms is highly skewed.
- Smaller firms have a lower probability of survival, but those that survive tend to grow faster than larger firms. Among larger firms, growth rates are unrelated to past growth or to firm size.
- Younger firms have a higher probability of exiting, but those that survive tend to grow faster than older firms.
- Gibrat’s law holds approximately (but not exactly): firm growth rate roughly independent of size, though notable deviations from this at the top and the bottom.
Model 1

- Representative household maximizes

\[ U = \max \int_{0}^{\infty} e^{-\rho t} \log C_t \, dt \]

- All expenses are in terms of labor. Hence \( C_t = Y_t \).
- The household owns all the firms including potential entrants. Therefore the total income is

\[ Y_t = w_t L + r_t A_t \]

where \( A \) is the total asset holdings and \( r_t \) is the rate of return on these assets.
Model II

- Final good production

\[ \ln Y_t = \int_0^1 \ln y_{jt} \, dj \]

- \( y_j \): quantity of intermediate \( j \)

- A fixed mass \( L \) of labor

\[ L_P + S_E + S_I = L \]

- \( L_P \): production
- \( S_E \): scientists working for outsiders
- \( S_I \): scientists working for incumbent firms.

- All workers receive \( w_t \)

- Normalize the price of the final good to 1.
A firm is defined as a **collection of product lines**.

**Figure 3: Example of a Firm**

- Quality level $A$
- Product line $j$
- Firm $f$
Profits II

- $n$ will denote the number of product lines that the firm operates.
- Each intermediate is produced with a linear technology
  
  $$y_{jt} = A_{jt} l_{jt}$$

- This implies that the marginal cost is
  
  $$w_t / A_{jt}$$

  where $w_t$ is the wage rate in the economy at time $t$.
- Innovations in each product line improves the productivity by $\lambda > 0$ such that
  
  $$A_{jt + \Delta t} = \begin{cases} 
  (1 + \lambda) A_{jt} & \text{if successful innovation} \\
  A_{jt} & \text{otherwise}
  \end{cases}$$
Bertrand competition \implies previous innovator will charge at least her marginal cost: \( \frac{(1 + \lambda) w_t}{A_{jt}} \).

Hence the latest innovator will charge the marginal cost of the previous innovator

\[
p_{jt} = \frac{(1 + \lambda) w_t}{A_{jt}}.
\]

Recall that the expenditure on each variety is \( Y_t \) (since \( P_t = 1 \)).

Then the profit is

\[
\pi_j = y_j (p_j - MC_j) = \frac{A_{jt} Y_t}{(1 + \lambda) w_t} \left( \frac{(1 + \lambda) w_t}{A_{jt}} - \frac{w_t}{A_{jt}} \right) = \pi Y_t
\]

where \( \pi \equiv \frac{\lambda}{1 + \lambda} \).
Innovation Technology I

- Innovations are undirected across product lines.
- Innovation technology

\[ X_i = \left( \frac{S_i}{\zeta} \right)^{1-\gamma} n^\gamma \]

where \( \gamma < 1 \), \( X_i \) is the innovation flow rate, \( S_i \) is the amount of R&D workers, \( n \) is the number of product lines to proxy for the firm specific (non-transferable, non-tradable) knowledge stock.
Alternatively, the cost of innovation:

\[ C(X, n) = \omega S_i \]

\[ = \zeta w n \left( \frac{X_i}{n} \right)^{\frac{1}{1-\gamma}} \]

\[ = \zeta w n x_i^{\frac{1}{1-\gamma}} \]

where \( x_i \equiv X_i / n \) is the innovation intensity (per product line).

- Let \( x \) denote the aggregate innovation rate in the economy.
- Innovation rate by entrants is \( x_e \).
- Aggregate innovation rate is

\[ \tau = x_i + x_e. \]
When a firm is successful in its current R&D investment, it innovates over a random product line \( j' \in [0, 1] \).

1. Then, the productivity in line \( j' \) increases from \( A_{j'} \) to \( (1 + \lambda)A_{j'} \).
2. The firm becomes the new monopoly producer in line \( j' \) and thereby increases the number of its production lines to \( n + 1 \).

At the same time, each of its \( n \) current production lines is subject to the creative destruction \( \tau \) by new entrants and other incumbents.

Therefore during a small time interval \( dt \),

1. the number of production units of a firm increases to \( n + 1 \) with probability \( X_i dt \), and
2. decreases to \( n - 1 \) with probability \( n \tau dt \).

A firm that loses all of its product lines exits the economy.
quality level

$q$

Firm $f$

product line

$j$

0

1
quality level

$q$

$\lambda$

$x$

Firm $f$

product line

$j$
Value Function I

- Relevant firm-level state variable: number of products in which the firm has the leading-edge technology, $n$.
- Then the value function of a firm as a function of $n$ is

$$rV_t(n) - \dot{V}_t(n) = \max_{x_i \geq 0} \left\{ n\pi_t - w_t\zeta n^{1-\gamma} \right\}$$

$$+ nx_i [V_t(n+1) - V_t(n)]$$

$$+ n\tau [V_t(n-1) - V_t(n)]$$

- This can be rewritten as

$$\rho v = \pi - \tau v + \max_{x_i \geq 0} \left\{ x_i v - \omega\zeta x_i^{1-\gamma} \right\}$$

where $v \equiv V_t(n)/nY_t$ is normalized per product value and $\omega \equiv w_t/Y_t$ is the labor share and constant in steady state.
First-order condition of R&D choice gives:

\[ x_i = \left( \frac{v}{\eta \zeta \omega} \right)^{\frac{1-\gamma}{\gamma}}. \]  

(1)

Or substituting it back:

\[ v = \frac{\pi - \zeta \omega x_i^{1-\gamma}}{\rho + \tau - x_i}. \]  

(2)
Value Function III

**Proposition**  Per-product line value of a firm $v$ can be expressed as a sum of production value $v_P$ and R&D option value $v_R$:

$$v = v_P + v_R$$

where

$$v_P = \frac{\pi}{\rho + \tau}$$

$$v_R = \frac{1}{(\rho + \tau)} \max_{x_i \geq 0} \left\{ x_i (v_R + v_P) - \omega \zeta x_i^{\frac{1}{1-\gamma}} \right\}.$$
Entry 1

- A mass of potential entrants.
- In order to generate 1 unit of arrival, entrants must hire a team of $\psi$ researchers, i.e., production function for entrant R&D is
  \[ x_e = \frac{S_E}{\psi}. \]
- The free-entry condition equates the value of a new entry $V_t(1)$ to the cost of innovation $\psi w_t$ such that
  \[ v = \omega \psi. \]
- Thus, together with (1) and (2):
  \[ x_e = \frac{\pi}{\omega \psi} - (1 - \gamma) \left( \frac{(1 - \gamma) \psi}{\zeta} \right)^{\frac{1-\gamma}{\gamma}} - \rho \quad \text{and} \quad x_i = \left( \frac{(1 - \gamma) \psi}{\zeta} \right)^{\frac{1-\gamma}{\gamma}}. \]
Labor Market Clearing I

- **Production workers**
  \[ L_P = \frac{Y_t}{A_j p_j} = \frac{1}{(1 + \lambda) \omega} \]

- **Incumbent R&D workers**
  \[ S_I = \zeta \left( \frac{(1 - \gamma) \psi}{\zeta} \right)^{\frac{1-\gamma}{\gamma}} \]

- **Entrant R&D workers**
  \[ S_E = \frac{\pi}{\omega} - \zeta \left( \frac{(1 - \gamma) \psi}{\zeta} \right)^{\frac{1-\gamma}{\gamma}} - \psi \rho \]
Therefore labor market clearing determines the normalized wage rate

\[ L = \frac{1}{(1 + \lambda)\omega} + \zeta \left( \frac{(1 - \gamma) \psi}{\zeta} \right)^{\frac{1-\gamma}{\gamma}} + \frac{\pi}{\omega} - \zeta \left( \frac{(1 - \gamma) \psi}{\zeta} \right)^{\frac{1-\gamma}{\gamma}} - \psi \rho \]

\[ \implies \]

\[ \omega = \frac{1}{L + \rho \psi} \]
Recall the final good production function

\[
\ln Y_t = \int_0^1 \ln y_{jt} \, dj
\]

\[
= \int_0^1 \ln A_{jt} l_{jt} \, dj
\]

\[
= \ln \frac{Y_t}{(1 + \lambda) w_t} + \int_0^1 \ln A_{jt} \, dj
\]

\[
= \ln \frac{L + \rho \psi}{1 + \lambda} + \int_0^1 \ln A_{jt} \, dj
\]
Define

\[ Q_t = \exp \left( \int_0^1 \ln A_{jt} \, dj \right) \]

\[ \ln Q_t = \int_0^1 \ln A_{jt} \, dj \]

Thus

\[ g = \frac{\dot{C}_t}{C_t} = \frac{\dot{Q}_t}{Q_t} \]
Moreover

\[
\ln Q_{t+\Delta t} = \int_0^1 [\tau \Delta t \ln (1 + \lambda) A_{jt} + (1 - \tau \Delta t) \ln A_{jt}] \, dj + o(\Delta t)
\]

\[
= \tau \Delta t \ln (1 + \lambda) + \ln Q_t + o(\Delta t)
\]

\[\iff\]

\[
g = \tau \ln (1 + \lambda)
\]

Hence

\[
g = \left[ \left( \frac{\lambda}{1 + \lambda} \right) \frac{L}{\psi} + \frac{1 - \gamma}{\gamma} \left( \frac{(1 - \gamma) \psi}{\zeta} \right)^{\frac{1 - \gamma}{\gamma}} - \frac{\rho}{1 + \lambda} \right] \ln (1 + \lambda)
\]
Moments

- Consider a firm with \( n \) product lines. The “approximate” growth rate is

\[
\begin{align*}
  n_{t+\Delta t} &= n_t + nx_i \Delta t - n \tau \Delta t \\
  \implies \quad \frac{\dot{n}_t}{n_t} &= x_i - \tau
  \end{align*}
\]

- R&D spending/intensity

\[
\frac{R&D}{Sales} = \frac{\zeta wn x_i^{\frac{1}{1-\gamma}}}{n} = \zeta w x_i^{\frac{1}{1-\gamma}}
\]

- Both of these are independent of firm size.
Firm Size Distribution

- Firm size distribution: fraction of firms with \( n \) leading-edge products, \( \mu_n \), given by:

\[
\begin{align*}
\text{Outflow} & \quad \text{Inflow} \\
\text{entry\&exit:} & \quad \mu_1 \tau = x_e \\
1\text{-product:} & \quad (x_i + \tau) \mu_1 = \mu_2 2\tau + x_e \\
n\text{-product:} & \quad (x_i + \tau) n\mu_n = \mu_{n+1} (n + 1) \tau + \mu_{n-1} (n - 1) x_i
\end{align*}
\]

- This implies a geometric firm size distribution

\[
\begin{align*}
\mu_1 & = \frac{x_e}{\tau} \\
\mu_2 & = \frac{x_e}{2\tau^2} x_i \\
\mu_3 & = \frac{x_e x_i}{3\tau^3} \\
\ldots & = \ldots \\
\mu_n & = \frac{x_e x_i}{n\tau^n}
\end{align*}
\]
Firm Size Distribution

Figure 4: Firm Size Distribution
What’s Missing?

- And nice and tractable model, but:
  - no reallocation (all firms that previous in equilibrium are equally good at using all factors of production);
  - no endogenous exit of less productive firms;
  - limited heterogeneity (see next slide).

- All of these together imply very little room for endogenous selection which could be impacted by policy.

- We now consider a model that extended this framework to introduce these features.
Why Heterogeneity Matters

**1A: Transition Rates**

**1B: R&D Intensity**

**1C: Sales Growth**

**1D: Employment Growth**
Baseline Model: Preferences

- Simplified model (abstracting from heterogeneity and non-R&D growth).
- Infinite-horizon economy in continuous time.
- Representative household:
  \[ U = \int_{0}^{\infty} \exp(-\rho t) \frac{C(t)^{1-\theta} - 1}{1-\theta} dt. \]

- Inelastic labor supply, no occupational choice:
  - Unskilled for production: measure 1, earns \( w^u \)
  - Skilled for R&D: measure \( L \), earns \( w^s \).

- Hence the budget constraint is
  \[ C(t) + \dot{A}(t) \leq w^u(t) + w^s(t) \cdot L + r(t) \cdot A(t) \]

- Closed economy and no investment, resource constraint:
  \[ Y(t) = C(t). \]
Unique final good $Y$:

$$Y = \left( \int_{\mathcal{N}} y_j^\frac{\epsilon-1}{\epsilon} \, dj \right)^\frac{\epsilon}{\epsilon-1}.$$ 

- $\mathcal{N} \subset [0, 1]$ is the set of *active* product lines.
- The measure of $\mathcal{N}$ is less than 1 due to
  - exogenous destructive shock
  - obsolescence
Intermediate Good Technology

- As usual, each intermediate good is produced by a monopolist:
  \[ y_{j,f} = q_{j,f} l_{j,f}, \]
  \[ q_{j,f} : \] worker productivity, \( l_{j,f} : \) number of workers.
- Marginal cost:
  \[ MC_{j,f} = \frac{w^u}{q_{j,f}}. \]
- Fixed cost of production, \( \phi \) in terms of skilled labor.
- Total cost
  \[ TC_{j,f} (y_{j,f}) = w^s \phi + w^u \frac{y_{j,f}}{q_{j,f}}. \]
A firm is defined as a collection of product qualities as in Klette-Kortum:

\[ \text{Firm } f = Q_f \equiv \{ q^1_f, q^2_f, \ldots, q^n_f \} \]

\( n_f \equiv |Q_f| : \) is the number of product lines of firm \( f \).
Relative Quality

- Define *aggregate quality* as

\[ Q \equiv \left( \int_{\mathcal{N}} q_j^{\varepsilon-1} \, dj \right)^{\frac{1}{\varepsilon-1}}. \]

- In equilibrium,

\[ Y = C = Q, \]

- Define *relative quality*:

\[ \hat{q}_j \equiv \frac{q_j}{w^u}. \]
R&D and Innovation

- Innovations follow a “controlled” Poisson Process

\[ X_f = n_f^\gamma h_f^{1-\gamma}. \]

- \( X_f \): flow rate of innovation
- \( n_f \): number of product lines.
- \( h_f \): number of researchers (here taken to be regular workers allocated to research).

- This can be rewritten as per product innovation at the rate

\[ x_f \equiv \frac{X_f}{n_f} = \left( \frac{h_f}{n_f} \right)^{1-\gamma}. \]

- Cost of R&D as a function of per product innovation rate \( x_f \):

\[ w^s G(x_f) \equiv w^s n_f x_f^{\frac{1}{1-\gamma}}. \]
Innovation by Existing Firms

- Innovations are again *undirected* across product lines.
- Upon an innovation:
  1. firm \( f \) acquires another product line \( j \)
  2. if technology in \( j \) is active:
     \[
     q(j, t + \Delta t) = (1 + \lambda) q(j, t).
     \]
  3. if technology in \( j \) is not active, i.e., \( j \notin \mathcal{N} \), a new technology is drawn from the steady-state distribution of relative quality, \( F(\hat{q}) \).
Entry and Exit

- A set of potential entrants invest in R&D.
- Exit happens in three ways:
  1. **Creative destruction.** Firm $f$ will lose each of its products at the rate $\tau > 0$ which will be determined endogenously in the economy.
  2. **Exogenous destructive shock** at the rate $\varphi$.
  3. **Obsolescence.** Relative quality decreases due to the increase in the wage rate, at some point leading to exit.
\[ \hat{q} = \frac{q}{w} \]
\[ \hat{q} = \frac{q}{w} \]
\[ \hat{q} = \frac{q}{w} \]
Without a fixed cost

\[ \hat{q} = \frac{q}{w} \]
With fixed cost $\phi > 0$

$$\hat{q} = \frac{q}{w}$$
Simplified Model

Innovation, Reallocation and Growth

September 26, 2013
Static Equilibrium

- Drop the time subscripts.
- Isoelastic demands imply the following monopoly price and quantity

\[ p_{j,f}^* = \left( \frac{\varepsilon}{\varepsilon - 1} \right) \frac{1}{\hat{q}_j} \text{ and } c_{j}^* = \left( \frac{\varepsilon - 1}{\varepsilon} \hat{q}_j \right)^\varepsilon Y \]

- In equilibrium,

\[ Y = C = Q \]

and

\[ w^u = \frac{\varepsilon - 1}{\varepsilon} Q. \]

Therefore the gross equilibrium (before fixed costs) profits from a product with relative quality \( \hat{q}_j \) are:

\[ \pi (\hat{q}_{j,f}) = \hat{q}_j^{\varepsilon - 1} \left( \frac{(\varepsilon - 1)^{\varepsilon - 1}}{\varepsilon^\varepsilon} \right) Y. \]
Dynamic Equilibrium

Let us also define *normalized values* as

\[ \tilde{V} \equiv \frac{V}{Y}, \quad \tilde{\pi}(\hat{q}_j, f) = \frac{\pi(\hat{q}_j, f)}{Y}, \quad \tilde{w}^u \equiv \frac{w^u}{Y} \quad \text{and} \quad \tilde{w}^s \equiv \frac{w^s}{Y}. \]
Dynamic Equilibrium (continued)

\[
\begin{align*}
\tau^* \tilde{V} (\hat{Q}_f) &= \\
&= \sum_{\hat{q}_{j,f} \in \hat{Q}_f} \left\{ \tilde{\pi} (\hat{q}_{jf}) - \tilde{w}^s \phi_j \right. \\
&\quad \left. + \tilde{\nu} \right. \\
&\quad + \tau \left[ \tilde{V} (\hat{Q}_f \setminus \{\hat{q}_{jf}\}) - \tilde{V} (\hat{Q}_f) \right] \\
&\quad - \tilde{w} G (x_f) \\
&\quad + x_f \left[ \mathbb{E}_{\hat{q}} \tilde{V} (\hat{Q}_f \cup (1 + \lambda) \hat{q}_{j',f}) - \tilde{V} (\hat{Q}_f) \right] \\
&\quad + \phi \left[ 0 - \tilde{V} (\hat{Q}_f) \right] \\
\end{align*}
\]

\(\tau\): creative destruction rate in the economy.
Dynamic Equilibrium (continued)

\[ r^* \tilde{V} (\hat{Q}_f) = \sum_{\hat{q}_{j,f} \in \hat{Q}_f} \left\{ \begin{array}{l} \tilde{\pi} (\hat{q}_{jf}) - \tilde{w}^s \phi_j \\ + \tau \left[ \tilde{V} (\hat{Q}_f \setminus \{\hat{q}_{jf}\}) - \tilde{V} (\hat{Q}_f) \right] \\ + \tau \left[ \tilde{V} (\hat{Q}_f \cup (1 + \lambda) \hat{q}_{j',f}) - \tilde{V} (\hat{Q}_f) \right] \\ + \phi \left[ 0 - \tilde{V} (\hat{Q}_f) \right] \end{array} \right\} + \right| \hat{Q}_f \left| \max_{x_f} \left\{ \begin{array}{l} x_f \left[ \mathbb{E}_{\hat{q}} \tilde{V} (\hat{Q}_f \cup (1 + \lambda) \hat{q}_{j',f}) - \tilde{V} (\hat{Q}_f) \right] \\ + \phi \left[ 0 - \tilde{V} (\hat{Q}_f) \right] \end{array} \right\} \right| \]

\( \tau \): creative destruction rate in the economy.
Franchise and R&D Option Values

Lemma  The normalized value can be written as the sum of franchise values:

\[ \tilde{V}(\hat{Q}_f) = \sum_{\hat{q} \in \hat{Q}_f} Y(\hat{q}), \]

where the franchise value of a product of relative quality \( \hat{q} \) is the solution to the differential equation (iff \( \hat{q} \geq \hat{q}_{\text{min}} \)):

\[ rY(\hat{q}) - \frac{\partial Y(\hat{q})}{\partial \hat{q}} \frac{\partial \hat{q}}{\partial w^u(t)} \frac{\partial w^u(t)}{\partial t} = \bar{\pi}(\hat{q}) - \bar{w}^u \phi + \Omega - (\tau + \varphi) Y(\hat{q}), \]

where \( \Omega \) is the R&D option value of holding a product line,

\[ \Omega \equiv \max_{x_f \geq 0} \left\{ -\bar{w}^s G(x_f) + x_f \left( E\hat{q} \bar{V}(\hat{Q}_f \cup (1 + \lambda) \hat{q}_j f) - \bar{V}(\hat{Q}_f) \right) \right\}, \]

Moreover, exit follows a cut-off rule: \( \hat{q}_{\text{min}} \equiv \pi^{-1}(\bar{w}^s \phi - \Omega) \).
Equilibrium Value Functions and R&D

Proposition

Equilibrium normalized value functions are:

\[
Y(\hat{q}) = \frac{\tilde{\pi}(\hat{q})}{r + \tau + \varphi + g(\varepsilon - 1)} \left[ 1 - \left( \frac{\hat{q}_{\text{min}}}{\hat{q}} \right)^{\frac{r + \tau + \varphi + g(\varepsilon - 1)}{g}} \right] \\
+ \frac{\Omega - \tilde{w}^s \phi}{r + \tau + \varphi} \left[ 1 - \left( \frac{\hat{q}_{\text{min}}}{\hat{q}} \right)^{\frac{r + \tau + \varphi}{g}} \right],
\]

and equilibrium R&D is

\[
x^*(\hat{q}) = x^* = \left[ \frac{(1 - \gamma) \mathbb{E}_\hat{q} Y(\hat{q})}{\tilde{w}^s} \right]^{\frac{1-\gamma}{\gamma}}.
\]
Entry

Entry by outsiders can now be determined by the free entry condition:

$$\max_{x^{entry} \geq 0} \left\{ -w^s \phi + x^{entry} \mathbb{E} V^{entry} (\hat{q}, \theta) - w^s G \left( x^{entry}, \theta^E \right) \right\} = 0$$

where $G \left( x^{entry}, \theta^E \right)$, as specified above, gives a number of skilled workers necessary for a firm to achieve an innovation rate of $x^{entry}$ (with productivity parameter $\theta^E$).

$X^{entry} \equiv m x^{entry}$ is the total entry rate where

- $m$ is the equilibrium measure of entrants, and
- $x^{entry}$ innovation rate per entrant.
Labor Market Clearing

- **Unskilled labor market clearing:**
  
  \[
  1 = \int_{N(t)} l_j (w^u) \, dj.
  \]

- **Skilled labor market clearing**

  \[
  L^s = \int_{N(t)} [\phi + h (w^s)] \, dj + m \left[ \phi + G \left( x^{entry}, \theta^E \right) \right].
  \]
Finally, we need to keep track of the distribution of relative quality $\rightarrow$ stationary equilibrium distribution of relative quality $F$.

This can be done by writing transition equations describing the density of relative quality.

These are more complicated than in Klette-Kortum because there is no strict Gibraltar’s law anymore.
Same preferences.

Introduce managerial quality affecting the rate of innovation of each firm.

Some firms start as more innovative than others, over time some of them lose their innovativeness.

- Young firms are potentially more innovative but also have a higher rate of failure.

Introduce non-R&D growth (so as not to potentially exaggerate the role of R&D and capture potential advantages of incumbents).
R&D and Innovation

- Innovations follow a controlled Poisson Process.
- Flow rate of innovation for leader and follower given by

$$X_f = (n_f \theta_f)^\gamma h_f^{1-\gamma}.$$ 

$n_f$: number of product lines.
$\theta_f$: firm type (management quality).
$h_f$: number of researchers.
Innovation Realizations

**With R&D**

- Innovations are *undirected* within the industry.
- After a successful innovation, innovation is realized in a random product line $j$. Then:
  1. firm $f$ acquires product line $j$
  2. technology in line $j$ improves

\[
q(j, t + \Delta t) = (1 + \lambda) q(j, t).
\]

**Without R&D**

- Firms receive a product line for free at the rate $\varrho$.
$\hat{q} \sim F(\hat{q})$

$q$

$m$-quality level

product line $j$

$\lambda$

$\xi$

Firm $f$

With R&D

Without R&D
Definition of a Firm

- A firm is again defined as a technology pair and a management quality pair
  \[
  \text{Firm } f \equiv \left( Q_f, \theta_f \right),
  \]
  where
  \[
  Q_f \equiv \{ q_1^f, q_2^f, \ldots, q_n^f \}.
  \]
- \( n_f \equiv |Q_f| \): is the number of product lines owned by firm \( f \).
Entry and Exit

- There is a measure of potential entrants.
- Successful innovators enter the market.
- At the time of initial entry, each firm draws a management quality $\theta$: 
  
  $$ \Pr \left( \theta = \theta^H \right) = \alpha $$
  
  $$ \Pr \left( \theta = \theta^L \right) = 1 - \alpha, $$

  where $\alpha \in (0, 1)$ and $\theta^H > \theta^L > 0$.

- Exit happens in three ways as in the baseline model.
Maturity Shock

- Over time, high-type firms become low-type at the rate $\nu > 0$:

  $$\theta^H \rightarrow \theta^L.$$  

- Convenient to capture the possibility of once-innovative firms now being inefficient (and the use of skilled labor).
Equilibrium

- Equilibrium definition and characterization similar to before (with more involved value functions and stationary transition equations).
Data: LBD, Census of Manufacturing and NSF R&D Data

- Sample from combined databases from 1987 to 1997.
- Longitudinal Business Database (LBD)
  - Annual business registry of the US from 1976 onwards.
  - Universe of establishments, so entry/exit can be modeled.
- Census of Manufacturers (CM)
  - Detailed data on inputs and outputs every five years.
- NSF R&D Survey.
  - Firm-level survey of R&D expenditure, scientists, etc.
  - Surveys with certainty firms conducting $1m or more of R&D.
- USPTO patent data matched to CM.
- Focus on “continuously innovative firms”:
  - I.e., either R&D expenditures or patenting in the five-year window surrounding observation conditional on existence.
Data Features and Estimation

- 17,055 observations from 9835 firms.
- Accounts for 98% of industrial R&D.
- Relative to the universal CM, our sample contains over 40% of employment and 65% of sales.
- “Important” small firms also included:
  - of the new entrants or very small firms that later grew to have more than 10,000 employees or more than $1 billion of sales in 1997, we capture, respectively, 94% at 80%.
- We use Simulated Method of Moments on this dataset to estimate the parameters the parameters of the model.
We target 21 moments to estimate 12 parameters.

Some of the moments are:

- Firm entry/exit into/from the economy by age and size.
- Firm size distribution.
- Firm growth by age and size.
- R&D intensity (R&D/Sales) by age and size.
- Share of entrant firms.
RESULTS
## Table 1. Parameter Estimates

<table>
<thead>
<tr>
<th>#</th>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$\varepsilon$</td>
<td>CES</td>
<td>1.701</td>
</tr>
<tr>
<td>2.</td>
<td>$\phi$</td>
<td>Fixed cost of operation</td>
<td>0.032</td>
</tr>
<tr>
<td>3.</td>
<td>$L^S$</td>
<td>Measure of high-skilled workers</td>
<td>0.078</td>
</tr>
<tr>
<td>4.</td>
<td>$\theta^H$</td>
<td>Innovative capacity of high-type firms</td>
<td>0.216</td>
</tr>
<tr>
<td>5.</td>
<td>$\theta^L$</td>
<td>Innovative capacity of low-type firms</td>
<td>0.070</td>
</tr>
<tr>
<td>6.</td>
<td>$\theta^E$</td>
<td>Innovative capacity of entrants</td>
<td>0.202</td>
</tr>
<tr>
<td>7.</td>
<td>$\alpha$</td>
<td>Probability of being high-type entrant</td>
<td>0.428</td>
</tr>
<tr>
<td>8.</td>
<td>$\nu$</td>
<td>Transition rate from high-type to low-type</td>
<td>0.095</td>
</tr>
<tr>
<td>9.</td>
<td>$\lambda$</td>
<td>Innovation step size</td>
<td>0.148</td>
</tr>
<tr>
<td>10.</td>
<td>$\gamma$</td>
<td>Innovation elasticity wrt knowledge stock</td>
<td>0.637</td>
</tr>
<tr>
<td>11.</td>
<td>$\varphi$</td>
<td>Exogenous destruction rate</td>
<td>0.016</td>
</tr>
<tr>
<td>12.</td>
<td>$\varrho$</td>
<td>Non-R&amp;D innovation arrival rate</td>
<td>0.012</td>
</tr>
</tbody>
</table>
### Table 2. Moment Matching

<table>
<thead>
<tr>
<th>#</th>
<th>Moments</th>
<th>model</th>
<th>data</th>
<th>#</th>
<th>Moments</th>
<th>model</th>
<th>data</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Firm Exit (small)</td>
<td>0.086</td>
<td>0.093</td>
<td>12.</td>
<td>Sales Gr. (small)</td>
<td>0.115</td>
<td>0.051</td>
</tr>
<tr>
<td>2.</td>
<td>Firm Exit (large)</td>
<td>0.060</td>
<td>0.041</td>
<td>13.</td>
<td>Sales Gr. (large)</td>
<td>-0.004</td>
<td>0.013</td>
</tr>
<tr>
<td>3.</td>
<td>Firm Exit (young)</td>
<td>0.078</td>
<td>0.102</td>
<td>14.</td>
<td>Sales Gr. (young)</td>
<td>0.070</td>
<td>0.071</td>
</tr>
<tr>
<td>4.</td>
<td>Firm Exit (old)</td>
<td>0.068</td>
<td>0.050</td>
<td>15.</td>
<td>Sales Gr. (old)</td>
<td>0.030</td>
<td>0.014</td>
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<tr>
<td>5.</td>
<td>Trans. large-small</td>
<td>0.024</td>
<td>0.008</td>
<td>16.</td>
<td>R&amp;D/Sales (small)</td>
<td>0.097</td>
<td>0.099</td>
</tr>
<tr>
<td>6.</td>
<td>Trans. small-large</td>
<td>0.019</td>
<td>0.019</td>
<td>17.</td>
<td>R&amp;D/Sales (large)</td>
<td>0.047</td>
<td>0.042</td>
</tr>
<tr>
<td>7.</td>
<td>Prob. small</td>
<td>0.539</td>
<td>0.715</td>
<td>18.</td>
<td>R&amp;D/Sales (young)</td>
<td>0.083</td>
<td>0.100</td>
</tr>
<tr>
<td>8.</td>
<td>Emp. Gr. (small)</td>
<td>0.063</td>
<td>0.051</td>
<td>19.</td>
<td>R&amp;D/Sales (old)</td>
<td>0.061</td>
<td>0.055</td>
</tr>
<tr>
<td>9.</td>
<td>Emp. Gr. (large)</td>
<td>-0.007</td>
<td>0.013</td>
<td>20.</td>
<td>5-year Ent. Share</td>
<td>0.363</td>
<td>0.393</td>
</tr>
<tr>
<td>10.</td>
<td>Emp. Gr. (young)</td>
<td>0.040</td>
<td>0.070</td>
<td>21.</td>
<td>Aggregate growth</td>
<td>0.022</td>
<td>0.022</td>
</tr>
<tr>
<td>11.</td>
<td>Emp. Gr. (old)</td>
<td>0.010</td>
<td>0.015</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2A: Transition Rates

2B: R&D Intensity

2C: Sales Growth

2D: Employment Growth
## Non-Targeted Moments

**Table 3: Non-targeted Moments**

<table>
<thead>
<tr>
<th>Moments</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corr(exit prob, R&amp;D intensity)</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td>Exit prob of low-R&amp;D-intensive firms</td>
<td>0.36</td>
<td>0.32</td>
</tr>
<tr>
<td>Exit prob of high-R&amp;D-intensive firms</td>
<td>0.37</td>
<td>0.34</td>
</tr>
<tr>
<td>Corr(R&amp;D growth, emp growth)</td>
<td>0.48</td>
<td>0.19</td>
</tr>
<tr>
<td>Share firm growth due to R&amp;D</td>
<td>0.77</td>
<td>0.73</td>
</tr>
<tr>
<td>Ratio of top 7.2% to bottom 92.8% income</td>
<td>13.4</td>
<td>9.3</td>
</tr>
</tbody>
</table>
Comparison to Micro Estimates

- Estimates of the elasticity of patents (innovation) to R&D expenditures (e.g., Griliches, 1990):
  - [0.3, 0.6]
  - This corresponds to $1 - \gamma$, so a range of [0.4, 0.7] for $\gamma$.
  - Our estimate is in the middle of this range.

- Use IV estimates from R&D tax credits.
  - US spending about $2 billion with large cross-state over-time variation.
  - Literature estimates:
    \[
    \log(R&D_{i,t}) = \alpha_i + \beta_t + \gamma \log(R&D\_Cost\_of\_Capital_{i,t})
    \]
    - Bloom, Griffith and Van Reenen (2002) find -1.088 (0.024) on a cross-country panel. Similar estimates from Hall (1993), Baily and Lawrence (1995) and Mumuneas and Nadiri (1996).
    - In the model, \( \ln R&D = \frac{\gamma-1}{\gamma} \ln (c_{R&D}) + \text{constant} \).
    - So approximately $\gamma \approx 0.5$, close to our estimate of $\gamma = 0.637$. 

Daron Acemoglu (MIT)  Innovation, Reallocation and Growth  September 26, 2013
### Baseline Results

#### Table 4. Baseline Model

<table>
<thead>
<tr>
<th>$x^{entry}$</th>
<th>$x^l$</th>
<th>$x^h$</th>
<th>$m$</th>
<th>$\Phi^l$</th>
<th>$\Phi^h$</th>
<th>$\hat{q}_{l,\text{min}}$</th>
<th>$\hat{q}_{h,\text{min}}$</th>
<th>$g$</th>
<th>Wel</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.46</td>
<td>2.80</td>
<td>9.58</td>
<td>73.6</td>
<td>71.16</td>
<td>24.53</td>
<td>13.90</td>
<td>0.00</td>
<td>2.24</td>
<td>100</td>
</tr>
</tbody>
</table>

Note: All numbers except wage ratio and welfare are in percentage terms.

- $g$: growth rate
- $x^{out}$: entry rate
- $x^{low}$: low-type invn rate
- $x^{high}$: high-type invn rate
- $\Phi^{low}$: fraction of low p. lines
- $\Phi^{high}$: fraction of high p. lines
- $\hat{q}_{l,\text{min}}$: low-type cutoff quality
- $\hat{q}_{h,\text{min}}$: high-type cutoff quality
- wel: welfare in cons equiv.
• Explains why very little obsolescence of high-type products.
Policy Analysis: Subsidy to Incumbent R&D

Table 4. Baseline Model

<table>
<thead>
<tr>
<th>$x^{entry}$</th>
<th>$x^l$</th>
<th>$x^h$</th>
<th>$m$</th>
<th>$\Phi^l$</th>
<th>$\Phi^h$</th>
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<td>13.90</td>
<td>0.00</td>
<td>2.24</td>
<td>100</td>
</tr>
</tbody>
</table>

- Use 1% and 5% of GDP, resp., to subsidize incumbents R&D:

Table 5A. Incumbent R&D Subsidy ($s_i = 15\%$)

<table>
<thead>
<tr>
<th>$x^{entry}$</th>
<th>$x^l$</th>
<th>$x^h$</th>
<th>$m$</th>
<th>$\Phi^l$</th>
<th>$\Phi^h$</th>
<th>$\hat{q}_{l,\text{min}}$</th>
<th>$\hat{q}_{h,\text{min}}$</th>
<th>$g$</th>
<th>Wel</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.46</td>
<td>3.05</td>
<td>10.56</td>
<td>68.1</td>
<td>70.74</td>
<td>24.96</td>
<td>13.40</td>
<td>0.00</td>
<td>2.23</td>
<td>99.86</td>
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</tbody>
</table>

Table 5B. Incumbent R&D Subsidy ($s_i = 39\%$)

<table>
<thead>
<tr>
<th>$x^{entry}$</th>
<th>$x^l$</th>
<th>$x^h$</th>
<th>$m$</th>
<th>$\Phi^l$</th>
<th>$\Phi^h$</th>
<th>$\hat{q}_{l,\text{min}}$</th>
<th>$\hat{q}_{h,\text{min}}$</th>
<th>$g$</th>
<th>Wel</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.46</td>
<td>3.61</td>
<td>13.04</td>
<td>49.8</td>
<td>69.58</td>
<td>25.97</td>
<td>13.15</td>
<td>0.00</td>
<td>2.16</td>
<td>98.48</td>
</tr>
</tbody>
</table>
Policy Analysis: Subsidy to the Operation of Incumbents

**Table 4. Baseline Model**

<table>
<thead>
<tr>
<th>$x^{entry}$</th>
<th>$x^l$</th>
<th>$x^h$</th>
<th>$m$</th>
<th>$\Phi^l$</th>
<th>$\Phi^h$</th>
<th>$\hat{q}_{l,min}$</th>
<th>$\hat{q}_{h,min}$</th>
<th>$g$</th>
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<td>9.58</td>
<td>73.6</td>
<td>71.16</td>
<td>24.53</td>
<td>13.90</td>
<td>0.00</td>
<td>2.24</td>
<td>100</td>
</tr>
</tbody>
</table>

- Use 1% of GDP to subsidize operation costs of incumbents:

**Table 6. Operation Subsidy ($s_o = 6\%$)**

<table>
<thead>
<tr>
<th>$x^{entry}$</th>
<th>$x^l$</th>
<th>$x^h$</th>
<th>$m$</th>
<th>$\Phi^l$</th>
<th>$\Phi^h$</th>
<th>$\hat{q}_{l,min}$</th>
<th>$\hat{q}_{h,min}$</th>
<th>$g$</th>
<th>Wel</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.46</td>
<td>2.80</td>
<td>9.59</td>
<td>73.7</td>
<td>71.30</td>
<td>24.52</td>
<td>11.74</td>
<td>0.00</td>
<td>2.22</td>
<td>99.82</td>
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</table>

- Now an important negative selection effect.
Policy Analysis: Entry Subsidy and Selection

**Table 4. Baseline Model**

<table>
<thead>
<tr>
<th>$x^{\text{entry}}$</th>
<th>$x^l$</th>
<th>$x^h$</th>
<th>$m$</th>
<th>$\Phi^l$</th>
<th>$\Phi^h$</th>
<th>$\hat{q}_{l,\text{min}}$</th>
<th>$\hat{q}_{h,\text{min}}$</th>
<th>$g$</th>
<th>$\text{Wel}$</th>
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</thead>
<tbody>
<tr>
<td>8.46</td>
<td>2.80</td>
<td>9.58</td>
<td>73.6</td>
<td>71.16</td>
<td>24.53</td>
<td>13.90</td>
<td>0.00</td>
<td>2.24</td>
<td>100</td>
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</tbody>
</table>

- Use 1% of GDP to subsidize entry:

**Table 7. Entry Subsidy ($s_e = 5\%$)**

<table>
<thead>
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<th>$x^h$</th>
<th>$m$</th>
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<th>$\Phi^h$</th>
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<th>$\hat{q}_{h,\text{min}}$</th>
<th>$g$</th>
<th>$\text{Wel}$</th>
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<td>9.30</td>
<td>75.3</td>
<td>71.16</td>
<td>24.41</td>
<td>15.91</td>
<td>0.00</td>
<td>2.26</td>
<td>100.15</td>
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</table>
Understanding the Selection Effect

Figure 4. Policy effect on Productivity Distributions

A. high type

B. low type
Social Planner’s Allocation

### Table 4. Baseline Model

<table>
<thead>
<tr>
<th>$x^{\text{entry}}$</th>
<th>$x^l$</th>
<th>$x^h$</th>
<th>$m$</th>
<th>$\Phi^l$</th>
<th>$\Phi^h$</th>
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</table>

### Table 8. Social Planner

<table>
<thead>
<tr>
<th>$x^{\text{entry}}$</th>
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<th>$m$</th>
<th>$\Phi^l$</th>
<th>$\Phi^h$</th>
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<td>80.9</td>
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<td>27.76</td>
<td>118.6</td>
<td>1.02</td>
<td>3.80</td>
<td>106.5</td>
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</tbody>
</table>

- What would the social planner do (taking equilibrium markups as given)?
Optimal Policy (I)

Table 4. Baseline Model

<table>
<thead>
<tr>
<th>$x^{entry}$</th>
<th>$x^l$</th>
<th>$x^h$</th>
<th>$m$</th>
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<th>$\Phi^h$</th>
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<th>$\hat{q}_{h,\min}$</th>
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<td>73.6</td>
<td>71.16</td>
<td>24.53</td>
<td>13.90</td>
<td>0.00</td>
<td>2.24</td>
<td>100</td>
</tr>
</tbody>
</table>

- Optimal mix of incumbent R&D subsidy, operation subsidy and entry subsidy:

Table 9. Optimal Policy Analysis and Welfare

| Incumbent & Entry Policies ($s_i = 17\%, s_o = -246\%, s_e = 6\%$) |
|-----------------------------------------------|-------|-------|------|----------|----------|---------------------|---------------------|------|-----|
| $x^{entry}$ | $x^l$ | $x^h$ | $m$  | $\Phi^l$ | $\Phi^h$ | $\hat{q}_{l,\min}$ | $\hat{q}_{h,\min}$ | $g$  | Wel |
| 8.46        | 3.04  | 10.21 | 75.5 | 62.19    | 25.53    | 96.28               | 55.88               | 3.12 | 104.6 |
Optimal Policy (II)

Table 4. Baseline Model

<table>
<thead>
<tr>
<th>$x^{entry}$</th>
<th>$x^l$</th>
<th>$x^h$</th>
<th>$m$</th>
<th>$\Phi^l$</th>
<th>$\Phi^h$</th>
<th>$\hat{q}_{l,\min}$</th>
<th>$\hat{q}_{h,\min}$</th>
<th>$g$</th>
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<tbody>
<tr>
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<td>2.80</td>
<td>9.58</td>
<td>73.6</td>
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<td>24.53</td>
<td>13.90</td>
<td>0.00</td>
<td>2.24</td>
<td>100</td>
</tr>
</tbody>
</table>

Optimal mix of incumbent R&D subsidy and operation subsidy:

Table 9. Optimal Policy Analysis and Welfare

Incumbent Policies ($s_i = 12\%, s_o = -264\%$)

<table>
<thead>
<tr>
<th>$x^{entry}$</th>
<th>$x^l$</th>
<th>$x^h$</th>
<th>$m$</th>
<th>$\Phi^l$</th>
<th>$\Phi^h$</th>
<th>$\hat{q}_{l,\min}$</th>
<th>$\hat{q}_{h,\min}$</th>
<th>$g$</th>
<th>Wel</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.46</td>
<td>3.04</td>
<td>10.21</td>
<td>75.3</td>
<td>62.31</td>
<td>25.53</td>
<td>91.38</td>
<td>54.85</td>
<td>3.11</td>
<td>104.6</td>
</tr>
</tbody>
</table>
Summing up

- Industrial policy directed at incumbents has negative effects on innovation and productivity growth—though small.
- Subsidy to entrants has small positive effects.
- But not because R&D incentives are right in the laissez-faire equilibrium.
- The social planner can greatly improve over the equilibrium.
- Similar gains can also be achieved by using taxes on the continued operation of incumbents (plus small R&D subsidies).
  - This is useful for encouraging the exit of inefficient incumbents who are trapping skilled labor that can be more productively used by entrants and high-type incumbents.
Robustness

- These results are qualitatively and in fact quantitatively quite robust.
- The remain largely unchanged if:
  - We impose $\gamma = 0.5$.
  - We impose $\varrho = 0$.
  - We make the entry margin much less elastic.