Durability, Deadline, and Election Effects in Bargaining*

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Abstract

One of the most robust empirical regularities in bargaining is the phenomenon called *the deadline effect*: the agreement is delayed until the very last minute before the deadline. In this paper we show that optimism about future bargaining power naturally generates a deadline effect. Optimism before elections generates a similar delay, which we refer to as *the election effect*. We further show that deadline and election effects are special cases of a more general phenomenon, *the durability effect,* which applies when the discipline on players' beliefs about bargaining prospects dramatically increases over a short period of time.

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1 Introduction

One of the most robust empirical regularities in bargaining is the phenomenon called *the deadline effect*: the agreement is delayed until the very last minute before the deadline (see

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e.g., Roth, Murnighan, and Schoumaker, 1988)). The labor negotiations are settled only at the "eleventh hour" before a strike starts and the litigants pursue costly negotiations only to reach an "agreement on the steps of the courthouse." These are all well-known by the practitioners of negotiation. Recently, general public also witnessed dramatic examples of deadline effect in the political arena. The Democratic and Republican leaders reached an agreement to raise the debt ceiling on July 31 2011 and passed a law only on August 2, 2011, under the threat of a US Treasury default on August 3, 2011. In "fiscal cliff" negotiations of late 2012, they reached an agreement on a new tax law late on the new year eve, in order to avert an across-the-board tax increase starting in the new year (the president signed the bill on January 2, 2013).

Deadlines are not the only sources of political gridlock: Elections appear to be another factor. Mayhew (1991) shows that the US Congresses between 1947-1990 enacted 25% fewer important laws on average when they convened in the two years before a presidential election compared to the two years after (see Section 4.4 for details). Binder (2000) notes as an example that House Republicans were reluctant to negotiate over tax cuts in late 1999, after President Bill Clinton vetoed their initial proposal, in the hopes of regaining the presidency. Their beliefs were in fact vindicated, and under the presidency of George W. Bush, Republicans managed to pass a sweeping tax cut legislation in 2001 shortly after the election.

One rationale for gridlock, proposed by many authors,[1] is *optimism*: players might be holding out since they both perceive there will be a better opportunity to strike deal. A major challenge for this rationale is to explain why optimism leads to gridlock at certain times, such as before deadlines or elections, but not at other times, such as shortly before deadlines or after elections. In this paper, we address this challenge by modeling explicitly players' beliefs about their bargaining prospects. We show that optimism about future bargaining power naturally generates a deadline effect. Optimism before elections generates a similar delay, which we refer to as *the election effect*. We also establish that deadline and election effects are special cases of a more general phenomenon, *the durability effect*, which applies when the discipline on players' beliefs dramatically increases over time.

Our model features two risk-neutral players, say Ann and Bob, who are negotiating in order to divide a dollar. Ann's bargaining power at time t, $\pi_t^{Ann} \in [0,1]$, determines Ann's share of the surplus from agreeing today rather than negotiating one more period. Bob's

[1]See Yildiz (2011) for a survey of the literature on bargaining with optimism and for the empirical evidence.

bargaining power is the residual, $\pi_t^{Bob} = 1 - \pi_t^{Ann}$. In sequential bargaining, the bargaining power corresponds to the probability of the player making a take-it-or-leave-it offer at time t. In the context of a congressional negotiation, in which players correspond to parties, a party's bargaining power can be thought of as capturing political factors such as the fraction of House and Senate members that support the party on the negotiated issue.

We take the bargaining power as an exogenous stochastic process, and characterize how it translates into endogenous bargaining outcomes. Our key ingredient is *optimism about the evolution of the bargaining power*. We assume that π_t^{Ann} and π_t^{Bob} are known at time t. This enables us to put a modicum of discipline on players' beliefs. However, we allow Ann and Bob to have optimistic beliefs about their own bargaining powers at a future time $t^* > t$. To fix ideas, suppose at time t Ann expects her bargaining power at time t^* to be $3/4$ on average. This implies Ann expects Bob's bargaining power at time t^* to be $1/4$ on average. But suppose Bob also expects his bargaining power at time t^* to be $3/4$. Observe that Bob is optimistic about his own bargaining power relative to Ann. Symmetrically, Ann is optimistic about her own bargaining power relative to Bob. We quantify players' optimism by the extent to which the sum of their expectations of their own bargaining powers exceeds 1. In this example, Ann and Bob's optimism about time t^* is measured by $3/4+3/4-1 = 1/2$.

Optimism about the bargaining power at the deadline leads to the deadline effect. The basic rationale is rather straightforward, as illustrated in the following example.

Example 1 (Deadline Effect)*.* Suppose that there is a firm deadline at time t^*, after which the negotiations end. First suppose the negotiations reach time t^*. If Ann and Bob do not agree at this time, then they each receive zero. Thus, the surplus from agreement is the whole negotiated dollar. Consequently, Ann and Bob agree on dividing the dollar according to their bargaining powers, giving $\pi_{t^*}^{Ann}$ to Ann and $\pi_{t^*}^{Bob}$ to Bob. Next consider an earlier time $t < t^*$ at which Ann and Bob have the optimistic beliefs described above. Suppose the time t value of receiving one dollar at time t^* is more than $2/3$ dollars. Then, Ann believes she can obtain more than $2/3 \times 3/4 = 1/2$ dollars simply by waiting until time t^*. Similarly, Bob believes he can obtain more than $1/2$ dollars by waiting until time t^*. Clearly, there is no division of the dollar at time t that can satisfy both players' optimistic expectations. When such optimism is maintained at each time $t < t^*$, the outcome exhibits the deadline effect: Ann and Bob wait until time t^* to reach an agreement.

This example provides a rationale for the deadline effect, based on optimism. In the last minute, the players agree because the cost of delay is very large. Moreover, the terms of a last minute agreement are greatly affected by the players' bargaining powers at that time.

Hence, any optimism about the bargaining powers in the last minute translates directly into optimism about the terms of a last minute agreement. This entices optimistic players to wait until the last minute to strike a deal. The delay may cost a half of the negotiated dollar.

Under optimism, a similar behavior emerges when the bargaining power becomes very durable at some time t^*. This is illustrated next.

Example 2 (Durability Effect). Imagine that there is no deadline but starting some time t^* the bargaining power becomes constant: $\pi_t^{Ann} = \pi_{t^*}^{Ann}$ and $\pi_t^{Bob} = \pi_{t^*}^{Bob}$ for each $t \geq t^*$ at all possible scenarios. Observe that at time t^* onwards, Ann and Bob both know that their bargaining powers remain constant. Then, the standard bargaining model without optimism implies they reach immediate agreement at time t^*, giving $\pi_{t^*}^{Ann}$ to Ann and $\pi_{t^*}^{Bob}$ to Bob. Suppose also that, the bargaining power is not constant before time t^*, which allows Ann and Bob to be optimistic about their bargaining powers at time t^*. If players' optimism at each time $t < t^*$ is sufficiently strong, as in the previous example, then the outcome exhibits a *durability effect*: Ann and Bob disagree before time t^*, and agree at time t^* with shares that reflect their bargaining powers.

In this example, players agree at time t^* since the bargaining power will not change in the future, dividing the dollar according to their current bargaining powers. Consequently, any earlier optimism about the bargaining powers at time t^* translates directly into optimism about the terms of an agreement at time t^*. If the bargaining power is not constant before time t^*, then optimistic players are enticed to wait until time t^* to strike a deal.

As an application, consider wage negotiations between a union and an employer. Suppose there is a pending labor law that will be enacted at time t^*. The terms of the law can affect the bargaining power between the union and the employer. Moreover, the bargaining power is more durable after time t^*—as it would take a long time to enact a new law—but less durable before time t^* since there could be many last minute changes in the law. If the players are optimistic that these changes will increase their own bargaining powers, then Example 2 predicts the agreement will be delayed until the law is enacted. More generally, the durability effect predicts bargaining delays in the run-up to major reforms that could affect (optimistic) players' bargaining powers in related individual negotiations.

A variant of this example can also explain the delays in the run-up to elections, as we illustrate next.

Example 3 (Election Effect). Imagine that the bargaining power remains constant at each time except t^*, that is, $\pi_t^i = \pi_0^i$ for all $t < t^*$ and $\pi_t^i = \pi_{t^*}^i$ for all $t \geq t^*$ at all possible scenarios. Time t^* can be thought of as an election that will reset the bargaining power to a new value. Suppose that before the election Ann and Bob are optimistic about their post-election bargaining powers. If players' optimism at each time $t < t^*$ is sufficiently strong, as in the previous examples, then the outcome exhibits a *deadline effect*: players disagree before the election, and agree after the election with shares that are equal to their post-election bargaining powers.

This example provides one explanation for delays in congressional bargaining in the run-up to presidential elections. In that context, bargaining power might change considerably depending on the outcome of the election. In particular, the party who wins the presidency will presumably have more bargaining power (e.g., due to the president's veto power). Moreover, parties' post-election bargaining powers are unlikely to change significantly for a considerable while (e.g., until the midterm election). If the parties are optimistic that their candidate will win the election, then Example 3 predicts that parties will disagree before the election, and agree after the election with terms that reflect relatively more the interests of the winning party—consistent with Republicans passing a tax cut legislation in 2001.

Examples 1-3 share two common features that are crucial in generating delays. First, at time t^*, the players' bargaining powers translate into their agreement shares. Intuitively, either the imminent deadline or the subsequent durability provides a discipline on how optimistic Ann and Bob can be about their bargaining prospects in the future. This in turn induces them to reach agreement with shares that reflect their current bargaining powers. Second, before time t^*, there is no such discipline on players' beliefs. In particular, either the lack of a deadline as in Example 1, or the volatility of the environment as in Example 2, or the upcoming election as in Example 3, allow Ann and Bob to have optimistic beliefs about their bargaining prospects in the future. When optimism is sufficiently large, there is no division that can meet both players' inflated expectations, which leads to delays.

The common principle behind delays is then *a dramatic increase of discipline on players' optimism.* Examples 1-2 illustrate that deadlines and durability play similar disciplining roles. To see that this is no coincidence, imagine a stochastic deadline that arrives on a small interval $(t^*, t^* + \varepsilon)$ with high probability p. Although the interval $(t^*, t^* + \varepsilon)$ is very short according to the calendar, it may be an eternity from the players' point of view: the value of a dollar at time $t^* + \varepsilon$ is only $1 - p$ dollars at time t^*. Assuming the bargaining power remains approximately constant in that short interval, the bargaining power remains nearly constant for eternity from the players' point of view. Hence, the durability and deadline effects established above are the two sides of the same coin. Example 3 further illustrates that discipline on optimism at a time is determined by "the weakest link" of durability following that time. Observe that, prior to the election, Ann and Bob perceive the bargaining power in the future to be durable most of the time except for the short election period, over which it will be highly non-durable. Nonetheless, this short period of non-durability is sufficient to eliminate the discipline on their beliefs.

When are the effects identified in Examples 1-3 more powerful? To address this question, we analyze the determinants of the length of disagreement before deadlines and elections. The severity of gridlock depends on—among other things—changes in the durability bargaining power. In particular, we obtain an additional *lame duck effect*, by which an upcoming election generates longer delays if the incumbent politician is not eligible to be reelected. This is because an election without an incumbent constitutes a greater drop in durability compared to an election in which the incumbent candidate has a non-trivial chance of being reelected. In Section 4.4, we present preliminary evidence from the legislative politics in the US that supports our election and lame duck effects. Specifically, we extend and confirm Mayhew's (1991) finding that the US Congresses that convene before presidential elections seem to enact fewer important laws. More interestingly, the evidence suggests that the election effect is especially powerful if the incumbent president is approaching the end of his/her two-term limit—consistent with our lame duck effect.

This paper makes two main contributions to the bargaining literature. First, we provide a model in which the bargaining power is exogenously specified as a continuous-time stochastic process. This enables us to capture durability by imposing a condition on the underlying stochastic process, and to capture optimism by allowing agents to have heterogeneous prior beliefs about the process. Second, we apply our model to establish the deadline, election, and durability effects—and their relationship—thereby providing a unified explanation for bargaining delays in seemingly distinct scenarios. We discuss our contribution relative to the literature in more detail in Section 6.

The rest of the paper is organized as follows. The next section introduces our general bargaining model and Section 3 characterizes the equilibrium. This section also presents a closed form solution when players have common priors. The heart of the paper is Section 4, which introduces a tractable canonical case with optimism and presents our main result, the durability effect, in this context. We then obtain the deadline and the election effects as slight variants of this analysis, formalizing the conceptual relationship between the three effects. We also establish comparative statics for when delays before deadlines and elections are more likely, and confront some of these results with data. Section 5 generalizes our results beyond the canonical case. As a caveat, we show that some intuitive notions of durability of bargaining power fail to provide discipline on players' beliefs. We then identify appropriate notions of durability that provide such discipline, which we use to present a generalized durability effect.

2 General Environment and Equilibrium

Consider two risk-neutral players, $i, j \in \{1, 2\}$, who negotiate over a continuum of times, $t \in \mathbb{R}_+ = [0, \infty)$, in order to pick some $x \in [0, 1]$. Player 1 and 2's preferences over x are respectively given by $u_1(x) = x$ and $u_2(x) = 1-x$. The players also discount the future at the common rate r, so their payoffs from striking a deal at time t are respectively given by $e^{-rt}x$ and $e^{-rt}(1-x)$. The players can strike a deal only at times on a grid $T = \{0, 1/n, 2/n, \ldots\}$, where n is a large integer.

Our first ingredient is a deadline, which we model as a continuous random variable over $\mathbb{R}_+$ denoted by d. If $t = d$, and if players have not agreed before time t, then negotiations end at time t with each player receiving 0. Let $F(\cdot)$ denote the cumulative distribution function and $f(\cdot)$ denote the density function corresponding to d. We assume $F(t) < 1$ for each $t \in \mathbb{R}_+$, so that the hazard rate, $\frac{f(t)}{1-F(t)}$, is well defined. Note that the hazard rate captures the probability that the deadline arrives at the next instant if it has not arrived by time t. We find it convenient to work with *the effective discount rate* that combines time discounting with the deadline hazard rate:

$$\hat{r}(t) = r + \frac{f(t)}{1-F(t)}. \tag{1}$$

We also define *the effective discount factor* between times t and s as

$$\delta_{t,s} = e^{-\int_t^s \hat{r}(\tilde{s})d\tilde{s}} = e^{-r(s-t)}\left(1-F(s)\right)/\left(1-F(t)\right). \tag{2}$$

Note that player i's expected payoff at time t from reaching an agreement at time s is simply given by $\delta_{t,s}u_i(x)$.

Our key object is a player's *bargaining power*, denoted by π_t^i. As in the standard bargaining literature, we define the bargaining power as the probability that player i makes a take-it-or-leave-it offer in a sequential bargaining game. As we will see, the bargaining power is also equal to the fraction of the surplus from agreement a player gets in addition to her continuation value from delay. We take the bargaining power as an exogenously given process and explore how it translates into bargaining outcomes in equilibrium. The players do know the current bargaining power but they may have subjective, possibly optimistic, beliefs about the future bargaining powers.

Formally, consider a stochastic process, $(\pi_t^1)_{t\in\mathbb{R}_+}$, defined over a state space Ω. If the state of the world is $\omega \in \Omega$, then the bargaining power of player 1 at time t is $\pi_t^1(\omega)$, and the bargaining power of player 2 is the residual, $\pi_t^2(\omega) = 1 - \pi_t^1(\omega)$. We assume that $\pi_t^1(\omega)$ and $\pi_t^2(\omega)$ are publicly observable at time t. We write P^i for the probability distribution over the state space Ω that captures the belief of player i. We also write E_t^i for the expectation operator of player i at time t. We assume that the sample paths, $t \mapsto \pi_t^i(\omega)$ are piecewise continuous for almost all ω. We also assume the bargaining power process is stochastically independent from the deadline d.

Traditionally, one would assume that the parties have identical beliefs, i.e.,

$$P^1 = P^2. \tag{CPA}$$

In this paper, we depart from the (CPA) and assume instead that parties are *optimistic*. Note that players agree on the current bargaining power, π_t^1 (which they both observe). However, they can be optimistic about the future values of the bargaining power. To facilitate the analysis, we define *the optimism at time* t about time $s \geq t$ (as a function of ω):

$$y_{t,s} = E_t^1\left[\pi_s^1\right] + E_t^2\left[\pi_s^2\right] - 1. \tag{3}$$

Here, $y_{t,s}$ is the amount by which a player over-estimates her own bargaining power with respect to the other player, i.e.

$$y_{t,s} = E_t^i\left[\pi_s^i\right] - E_t^j\left[\pi_s^i\right]$$

for $i \neq j$. In general, $y_{t,s}$ can take any value in $[-1, 1]$. The common prior assumption corresponds to the case in which $y_{t,s} = 0$ for all times t and $s \geq t$. Optimism at a particular time t about a future time s is captured by $y_{t,s} > 0$.

Given the process $(\pi_t^1)_{t\in\mathbb{R}_+}$, the bargaining is modeled as follows. At each time $t \in T$, player i is recognized as the proposer with probability π_t^i. The recognized player offers some

$x \in [0, 1]$. If the other player accepts the offer, then the game ends, picking x. Otherwise, the game continues to the next period —unless there is a deadline arrival, in which case the game ends and players receive 0. We study the subgame perfect equilibrium of this game.

Remark 1 (Relationship with Existing Bargaining Models). If π_t^i is deterministic, then our game is identical to the standard random-proposer model (Binmore 1987; Merlo and Wilson 1995). Here we allow π_t^i to be stochastic so that the players can have subjective uncertainty about the recognition probability at a future time and can learn about that probability as time passes. Moreover, we take the bargaining power as a function of the real time, independently of how frequently players come together to negotiate. In fact we will often focus on the solution in the continuous time limit as $n \to \infty$. This approach is particularly useful to model the durability of the bargaining power. In contrast, the usual alternating offer bargaining models imply a highly non-durable bargaining power that shifts from one extreme to the other infinitely frequently.[2]

The closest work is Yildiz (2003), who also allows the players to have subjective beliefs about the recognition. With respect to Yildiz (2003), our main innovation is the introduction of publicly observable process π_t^i. The absence of π_t^i in Yildiz (2003) can be viewed as the alternative case that the players cannot observe their current and past bargaining power, allowing them to maintain optimistic beliefs about their current bargaining power. Since the players do observe who made offers in the past rounds, it can also be viewed as the special case in which π_t^i is restricted to be 0 or 1 and is a function of the grid.

Remark 2 (Uncertain and Soft Deadlines). We work with a stochastic deadline to capture the fact that bargaining deadlines in practice are often uncertain or soft. In the recent US debt ceiling negotiations, the deadline can be thought of as the time at which the Treasury will reach the statutory debt limit. In practice, this time is somewhat uncertain since Federal government expenses are not entirely predictable. Moreover, the deadline is also soft in the sense that the Treasury might use accounting maneuvers to continue making payments for a while after the statutory limit is reached. The final deadline at which the Treasury actually runs out of money can be quite uncertain, as captured by our formulation. For another example, consider legal negotiations, such as plea bargaining, in which the deadline can be

[2]In fact, bargaining power in those models also depends on subtle details of the bargaining procedure. Consider the alternating-offer bargaining game of Rubinstein (1982). If the real-time delays after the times at which party 1 makes an offer are K times as long as those after party 2 makes offers, then party 1 gets K times as much as party 2 gets in the continuous-time limit. In contrast, the parameter, K, would not affect the continuous time limit solution of our model (given our assumption that the sample paths are piecewise continuous almost surely).

thought of as the time at which the court will reach a judgement. This deadline is not only highly uncertain but also soft in the sense that in some cases the judgement can be appealed.

3 Characterization of Equilibrium

Let V_t^i denote the continuation value of player i at time t (as a function of ω) after π_t^1 is revealed but before the proposer at time t is recognized. By individual rationality, V_t^i is restricted to be in $[0,1]$. Given a subsequent negotiation time $s \in T$, we define

$$W_{t,s} \equiv \delta_{t,s}\left(E_t^1\left[V_s^1\right] + E_t^2\left[V_s^2\right]\right) \tag{4}$$

as the sum of players' perceived payoffs from delaying agreement (or waiting) until time s. Note that $W_{t,t+1/n}$ captures players' total perceived payoff from waiting until the next negotiation time. Hence, $1 - W_{t,t+1/n}$ captures players' perceived surplus from agreeing at time t.

First suppose $W_{t,t+1/n} < 1$, so that the surplus at time t is positive. Then, it is easy to check that the players reach an agreement with the proposer receiving the full surplus. Hence, player i's expected payoff before the proposer is recognized is given by:

$$V_t^i = \pi_t^i\left(1 - W_{t,t+1/n}\right) + \delta_{t,t+1/n}E_t^i\left[V_{t+1/n}^i\right].$$

In particular, players split the surplus according to their bargaining powers, π_t^i.

Next suppose the surplus is negative, that is, $W_{t+1/n} > 1$. In this case, there cannot be an agreement that satisfies both players' expectations, as the sum of their continuation values from delay exceeds 1. Hence, there will be disagreement at such t regardless of the proposer. Player i's continuation value is given by:

$$V_t^i = \delta_{t,t+1/n}E_t^i\left[V_{t+1/n}^i\right].$$

Finally, if $W_{t,t+1/n} = 1$, then the surplus is zero and the players are indifferent to agree. Our first result shows the equilibrium is characterized by combining the three cases. (The proof is similar to that of Theorem 1 in Yildiz (2003), and hence is omitted.)

Proposition 1 (Characterization and uniqueness). *For each time $t \in T$ and player $i \in \{1,2\}$, the player's continuation value in any subgame perfect equilibrium is characterized as* *the unique stochastic process* $V^i : \Omega \times T \to [0,1]$ *that solves the stochastic difference equation*

$$V_t^i = \pi_t^i \max\left\{0, 1 - W_{t,t+1/n}\right\} + \delta_{t,t+1/n} E_t^i\left[V_{t+1/n}^i\right]. \quad (5)$$

Throughout the paper, we study the unique subgame-perfect Nash equilibrium, by studying the stochastic difference equation (5). Our main objective is to understand when there is delay, i.e., when $W_{t,t+1/n} > 1$. In general, under optimism, it is hard to compute the solution to (5). We next present the benchmark case without optimism, in which the solution is rather straightforward.

Benchmark with Common Priors As a benchmark, consider the equilibrium under (CPA). In this case, it can be checked that players reach agreement at all times. In particular, players' continuation values sum to the size of the pie, $V_t^1 + V_t^2 = 1$, for each time t. Combining this observation with (4), the surplus, $1 - W_{t,t+1/n}$, is always positive and has a closed form solution given by $1 - \delta_{t,t+1/n}$. It follows that players' agreement shares, characterized by (5), also have closed form solutions.

Proposition 2 (Bargaining with Common Priors)**.** *Under (CPA), players reach agreement at each time (if they have not yet agreed). In the continuous-time limit (as* $n \to \infty$*), the agreement share of each player* $i \in \{1,2\}$ *is given by:*

$$\lim_{n\to\infty} V_t^i = E_t\left[\int_{s=t}^{\infty} e^{-\int_t^s \hat{r}(\tilde{s})d\tilde{s}} \hat{r}(s)\, \pi_s^i ds\right] \qquad (\forall t). \quad (6)$$

That is, when players have common priors, the overall bargaining power of a player is the expected discounted sum of his future instantaneous bargaining powers. The players weigh their bargaining power at different situations differently. First, as expected, they discount the future bargaining powers in the same way as they discount their future payoffs, using the effective discount factor $\delta_{t,s} = \exp\left(-\int_t^s \hat{r}(\tilde{s})\, d\tilde{s}\right)$. Second, they put higher weight to the bargaining power at times at which the effective discount rate $\hat{r}(t)$ is higher, that is, those times at which the deadline is more likely to arrive [cf. (1)]. Intuitively, the possibility that negotiations might end increases the cost of delay. This in turn raises the surplus from agreement, and thus, the weight of the players' bargaining power at these times [cf. (5)]. In particular, if there is a firm deadline, the players' bargaining power just before the deadline has a large impact on their agreement shares.

4 Durability, Deadline, and Election Effects in a Canonical Case

In this section, we present a tractable canonical case, which we refer to as the Poisson model, for which there is a closed form solution without the (CPA). In this context, we illustrate how players' optimism is disciplined by a combination of durability and deadlines. We then present our main result, the durability effect, and obtain the deadline and election effects as its applications. We also present preliminary empirical evidence consistent with the election effect and some of its comparative statics. We return to the general model in Section 5.

4.1 Baseline Poisson Model and Disciplining of Optimism

Consider a Poisson process with arrival rate λ. At each arrival, a new pair $(\pi^1, 1-\pi^1)$ of bargaining powers is drawn from a fixed distribution (independently from earlier values of the bargaining power). The bargaining powers remain constant as $(\pi_t^1, \pi_t^2) = (\pi^1, 1-\pi^1)$ until the next arrival. Let H^i denote the distribution of π^i according to player i. We assume H^i has full support over $[0,1]$ for simplicity. We let $\bar{\pi}^i = \int \pi dH^i(\pi)$ and assume players are optimistic about their expected bargaining powers conditional on an arrival, that is:

$$\bar{y} = \bar{\pi}^1 + \bar{\pi}^2 - 1 > 0.$$

We also assume that the deadline arrives with a constant hazard rate, so that the effective discount rate is constant, $\hat{r}(t) = \hat{r}$ for all t.

In this example, players have a perpetual tendency to be optimistic. However, this tendency is countered by *the durability of the bargaining power*. In particular, the bargaining power remains constant at its current level until some important event, such as a financial meltdown, occurs. As this happens, the balance of the power between the players is completely reset. The players agree on how often such events occur (common λ), but they are optimistic about these events' impact on the bargaining power.

More specifically, a player's expectation about his future bargaining power is a weighted average of his current bargaining power π_t^i and his perceived long-run bargaining power $\bar{\pi}^i$:

$$E_t^i\left[\pi_s^i\right] = e^{-\lambda(s-t)}\pi_t^i + \left(1-e^{-\lambda(s-t)}\right)\bar{\pi}^i. \tag{7}$$

The durability of the bargaining power is captured by the inverse measure, $1/\lambda$. When bargaining power is durable, so that $1/\lambda$ is large, the expectations reflect the current bargaining power. This in turn reduces optimism. Indeed, players' optimism is given by:

$$y_{t,s} = E_t^1\left[\pi_s^1\right] + E_t^2\left[\pi_s^2\right] - 1 = \left(1 - e^{-\lambda(s-t)}\right)\overline{y}. \tag{8}$$

Observe that the players do not have any optimism about the present, but as they consider increasingly distant future, optimism exponentially approaches $\overline{y}$. The durability controls the speed at which optimism grows.

We next characterize the equilibrium in the Poisson model. To state the result, we denote the product of the effective discount rate with the durability rate with

$$\rho = \hat{r}/\lambda,$$

and we refer to ρ as the *effective durability rate.*

Proposition 3 (Disciplining of Optimism in the Poisson Model)**.** *In the baseline Poisson model, players reach agreement at each time with agreement shares given by* $V_t^i = K_n\pi_t^i + L_n\bar{\pi}^i$ *for some positive constant* K_n *and* L_n*. In the continuous time limit, the agreement share of each player* $i \in \{1,2\}$ *is given by*

$$\lim_{n\to\infty} V_t^i \equiv K(\rho)\,\pi_t^i + L(\rho)\,\bar{\pi}^i \qquad (\forall t)\,, \tag{9}$$

with the corresponding constants

$$K(\rho) = \frac{\rho}{\rho + 1 + \overline{y}} \text{ and } L(\rho) = \frac{1}{\rho + 1 + \overline{y}}. \tag{10}$$

In particular, players reach agreement at all times with shares that are a weighted combination of their current bargaining powers and their expected values of long run bargaining powers. The weight on the current bargaining power, $K(\rho)$, is increasing in the effective durability rate, $\rho = \hat{r}/\lambda$, while the weight on the long-run bargaining power, $L(\rho)$, is decreasing in ρ. Observe also that, as $\rho \to \infty$, the weights satisfy $K(\rho) \to 1$ and $L(\rho) \to 0$. In particular, as the durability becomes very high, $1/\lambda \to \infty$, or the deadline becomes very likely to arrive, so that $\hat{r} \to \infty$, players' agreement shares approximate their current bargaining powers. Hence, Proposition 3 shows that players' optimism is disciplined by a combination of durability and deadlines.

To develop an intuition for this result, fix some time t^* and consider a prior time

$$t = t^* - \Delta/\hat{r}, \tag{11}$$

so that the discount factor between time t and t^* is given by $e^{-\Delta}$. This normalization is useful since Δ provides a measure of the payoff relevant distance between times t and t^*. Consider the sum of the perceived payoffs at time t from waiting until time t^* [cf. (4)], which we denote by $W(\Delta)$ to simplify the notation. The representation in (9) implies

$$W(\Delta) = W_{t,t^*} = e^{-\Delta}\left(1 + y_{t,t^*} K(\rho)\right). \tag{12}$$

Perceived payoffs are greater when players are more optimistic about their future bargaining powers and when the endogenous weight $K(\rho)$ that converts bargaining power to agreement shares is greater. Observe that if $W(\Delta) > 1$, then players disagree at time t since they are tempted by delaying the agreement until time t^*. Observe also that, for $\Delta = \hat{r}/n$, the surplus from agreement at time t is given by $1 - W(\Delta)$. In particular, a change in parameters that increases $W(\Delta)$ reduces the surplus, which in turn reduces the effect of current bargaining powers on agreement shares [cf. (5)]. Consequently, characterizing $W(\Delta)$ is useful to understand whether players reach agreement as well as the terms of their agreement.

Using (8) and the normalization in (11), we further have:

$$\begin{aligned} W(\Delta) &= e^{-\Delta}\left(1 + \left(1 - e^{-\lambda\Delta/\hat{r}}\right) K(\rho)\,\overline{y}\right) \\ &\simeq 1 + \left(\frac{K(\rho)\,\overline{y}}{\rho} - 1\right)\Delta, \end{aligned} \tag{13}$$

where the last line is a linear approximation around $\Delta = 0$. This expression illustrates how players' optimism is disciplined by durability and deadlines. Indeed, while the sum of perceived payoffs is increasing in optimism $\overline{y}$, it is decreasing in the effective durability rate $\rho = \hat{r}/\lambda$. This is true not only when K is fixed, but also when the equilibrium value of K in (10) is substituted in (13).

Intuitively, since durability reduces optimism about future bargaining power, it also naturally reduces optimism about future bargaining prospects. Deadlines provide a similar discipline by increasing the effective discount rate $\hat{r}$, which shortens the time intervals over which players face equivalent trade-offs. Formally, given a payoff-relevant time distance Δ, a greater discount rate $\hat{r}$ implies a smaller corresponding time distance $t^* - t = \Delta/\hat{r}$ [cf. (11)]. This in turn reduces the players' optimism at time t about their bargaining power at the nearby time t^*. Put differently, if the deadline arrives rapidly, then the bargaining power cannot change much before the deadline arrives. Hence, even though the bargaining power is not highly durable in the strict sense of the word, it is durable relative to the small amount of time left until the end of negotiations.

As this intuition suggests, the two sources of discipline also naturally interact: That is, a deadline that is likely to arrive around time t provides greater discipline when the bargaining power is more durable around time t. The combined disciplining effect is captured by the effective durability rate $\rho = \hat{r}/\lambda$. Equation (10) illustrates that ρ, along with the level of optimism $\overline{y}$, is sufficient to characterize the bargaining outcomes. As higher ρ disciplines players' optimism, it also increases the surplus from immediate agreement, which in turn increases $K(\rho)$. In the extreme, there is no room for optimism and players' agreement shares reflect their agreement shares, i.e., $\lim_{\rho\to\infty} K(\rho) = 1$. Similarly, greater optimism $\overline{y}$ reduces the surplus from immediate agreement, which in turn reduces $K(\rho)$.

A naive view could also posit that a sufficiently high level of optimism $\overline{y}$ combined with a lack of discipline, i.e., low ρ, reduces the surplus from agreement so much that players choose not to agree. This is not correct for the baseline Poisson model. Intuitively, given the stationarity of the environment, the behavior is independent of time. Since the players cannot disagree forever, this means that there is agreement at each time. Formally, the endogenously determined level of $K(\rho)$ is always sufficiently low to ensure $K(\rho)\overline{y} < \hat{r}/\lambda$, so that $W(\Delta) < 1$ regardless of the parameters [cf. (13)]. Hence, although optimism affects the way players split the pie, it need not generate delays. As we will see in the rest of this section, this result is overturned when the effective durability rate ρ changes over time.

4.2 Durability and Deadline Effects in the Poisson Model

We next use the Poisson model to present our main result, the durability effect, which also implies the deadline effect. To this end, modify the Poisson model so that the arrival rate λ and the effective discount rate $\hat{r}(t)$ are possibly different before and after some fixed t^*. For $t < t^*$, we have $(\hat{r}(t), \lambda) = (\hat{r}_0, \lambda_0)$, yielding the effective durability rate of $\rho_0 = \hat{r}_0/\lambda_0$. For $t \geq t^*$, we have $(\hat{r}(t), \lambda) = (\hat{r}_1, \lambda_1)$, yielding a higher effective durability rate of $\rho_1 = \hat{r}_1/\lambda_1$ as in Figure 1. Our next result shows that a sufficiently large increase in effective durability induces delays.

Proposition 4 (Durability Effect in the Poisson Model)**.** *Consider the modified Poisson*

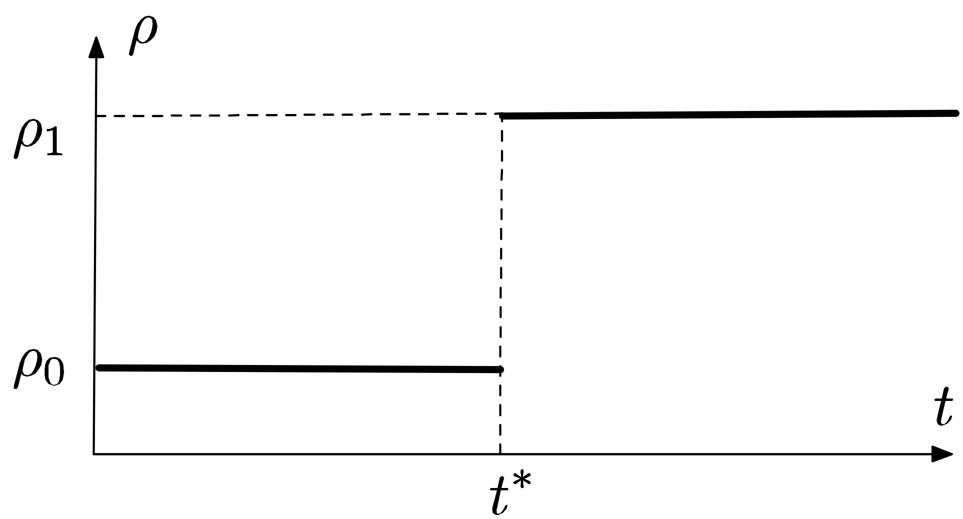

Figure 1: The effective durability rate in Section 4.2.

model with the assumption that ρ_1 *is sufficiently larger than* ρ_0*, so that:*[3]

$$\overline{y}K(\rho_1) > \rho_0, \tag{14}$$

where $K(\rho) = \frac{\rho}{\rho+1+\overline{y}}$*. Then, there exists* $\bar{\Delta} > 0$ *such that players disagree at each time* $\left(t^* - \bar{\Delta}/r_0, t^*\right)$ *and agree at time* t^* *and thereafter. The length of disagreement,* $\bar{\Delta}$*, is the unique solution to* $W\left(\bar{\Delta}\right) = 1$*, where*

$$W(\Delta) = e^{-\Delta}\left(1 + \left(1 - e^{-\Delta/\rho_0}\right) K(\rho_1)\,\overline{y}\right). \tag{15}$$

Durability Effect As a special case, suppose there is no deadline so that $\hat{r}_0 = \hat{r}_1 = r$, but the durability rate $1/\lambda$ increases sufficiently at time t^*, so that condition (14) holds. Then, Proposition 4 implies that there is a period of disagreement before t^*, generalizing the durability effect of Example 2.

Deadline Effect As another special case, suppose there is a stochastic deadline with exponential distribution $F(t) = 1 - e^{-\alpha(t-t^*)}$ starting at t^*. Put differently, the deadline arrives starting at time t^* with a constant hazard rate $\alpha = \frac{f(t)}{1-F(t)}$. Then the effective discount rate is given by:

$$\hat{r}(t) = \begin{cases} r+\alpha & \text{if } t \geq t^* \\ r & \text{otherwise} \end{cases}. \tag{16}$$

[3]Note that $\rho > K(\rho)\,\overline{y}$ for each ρ, so condition (14) requires ρ_1 to be larger than ρ_0.

The corresponding effective durability rates are given by $\rho_0 = r/\lambda_0$ and $\rho_1 = (r+\alpha)/\lambda_1$. Suppose the deadline arrival rate α is sufficiently large so that $K(\rho_1)\,\overline{y} > \rho_0$. Then, Proposition 4 implies the players wait for t^* to reach an agreement, generalizing the deadline effect of Example 1.

To prove Proposition 4, first note that the environment becomes stationary at t^* with effective durability rate ρ_1. Hence, the analysis for the baseline model applies starting at t^* after replacing ρ with ρ_1. Moreover, the sum of perceived payoffs at time $t^* - \Delta/\hat{r}_0$ is still given by (12). The only difference is that the level of optimism y_{t,t^*} is governed by a different set of parameters $\hat{r}_0, \lambda_0$ that apply prior to t^*. Consequently, $W(\Delta)$ is given by (15), which can be approximated around $\Delta = 0$ as:

$$W(\Delta) \simeq 1 + \left(\frac{K(\rho_1)\,\overline{y}}{\rho_0} - 1\right)\Delta.$$

Under condition (14), $W(\Delta) > 1$ for each sufficiently small Δ. Thus, there is a period of delay before t^*, and the length of delay is characterized by $W\left(\bar{\Delta}\right) = 1$.

Intuitively, the low effective durability prior to t^* implies there is little discipline on beliefs at time t, so that players can be optimistic about their bargaining prospects at time t^*. In contrast, a high effective durability following t^* implies there is considerable discipline on optimism at t^*—as reflected by a high $K(\rho_1)$. If the increase in effective durability is sufficiently large, then players wait until t^* to reach an agreement. Note also that the increase in effective durability rate $\rho = \hat{r}/\lambda$ can come from either an increase in the durability rate $1/\lambda$ or from an increase in the deadline arrival rate α. Hence, durability and deadline effects are two sides of the same coin.

How costly are the delays generated by deadline or durability effects? To get a sense of magnitudes, consider a disagreement time $t = t^* - \Delta/r$. The social cost of delaying the agreement until time t^*, as opposed to agreeing at time t, is given by $1 - e^{-\Delta}$. Using Eq. (15), and the condition for delay, $W(\Delta) \geq 1$, we can bound the social cost from above as:

$$1 - e^{-\Delta} \leq \frac{\bar{y}}{1+\bar{y}}. \tag{17}$$

Moreover, there are parameters under which the upper bound is attained (for instance, take $\lambda_0 \to 0, K(\rho_1) \to 1$ and consider $\Delta = \overline{\Delta}$). Hence, the social cost of delay, measured as a fraction of the total pie, is in the same ballpark as players' optimism about their long-run bargaining power, $\bar{y}$. When parties are highly optimistic, so that $\bar{y} = 1$, the cost can be as large as half of the total pie.

When is the deadline effect more prominent? Our next result establishes comparative statics for the length of disagreement, $\bar{\Delta}$. Note that $\bar{\Delta}$ also provides a measure of the maximum social cost of delay, $1 - e^{-\bar{\Delta}}$, which obtains at the earliest disagreement time $t = t^* - \bar{\Delta}/r$.

Observation 1. *Given the deadline described in* (16)*, the length of disagreement, $\bar{\Delta}$, is:*

(i) decreasing in the durability rate before the deadline arrival $1/\lambda_0$*, and increasing in the durability rate during deadline arrival* $1/\lambda_1$*,*

(ii) increasing in the deadline arrival rate α,

(iii) increasing in players' long-run optimism $\overline{y}$.

(iv) When the deadline is sufficiently firm (i.e., when α is sufficiently high), $\bar{\Delta}$ is decreasing in the discount rate r.

The first two parts follow from (15) after observing that $\rho_0 = r/\lambda_0$ and $\rho_1 = (r + \alpha)/\lambda_1$. The second part suggests there might be a silver lining to setting an uncertain or soft deadline in negotiations. An uncertain deadline, which we capture with low α, generates two main effects relative to a more deterministic deadline with high α. First, it leads to more "compromise" at time t^* by the player that has the higher bargaining power at that time, as captured by a lower weight $K(\rho_1)$. Depending on the context, this effect might be desirable in itself. Second, because players realize there will be compromise at time t^*, their optimism about bargaining powers at time t^* translates relatively less into optimism about their agreement shares, leading to a shorter period of disagreement.

The third part generates an additional testable prediction linking optimism to delays. Note from (15) that optimism has a direct effect that tends to increase the sum of players' perceived payoffs, $W(\Delta)$. However, optimism after time t^* also has an indirect effect that tends to reduce $W(\Delta)$ by reducing the weight $K(\rho_1)$. The net effect is governed by the product $K(\rho_1)\overline{y}$. By (10), the net effect is positive and optimism increases $W(\Delta)$. Intuitively, greater optimism leads to longer and costlier delays.

The last part considers the effect of the discount rate r, which captures players' cost of delay. Higher cost of delay generates a direct effect that tends to reduce $W(\Delta)$ (captured by $\rho_0 = r/\lambda_0$). However, higher r also generates an indirect effect that tends to increase $W(\Delta)$ by raising $K(\rho_1)$ (captured by $\rho_1 = (r + \alpha)/\lambda_1$). The net effect is in general ambiguous.

When the deadline is deterministic, the indirect effect is muted because $K(\rho_1) = 1$ regardless of r. Consequently, as long as the deadline is sufficiently firm, increasing r makes the deadline effect less prominent.

4.3 Election Effect in the Poisson Model

We next modify the Poisson model further to present the election effect. Suppose the effective discount rate is constant at a baseline rate, $\hat{r}(t) = \hat{r}_0$, throughout. Imagine that the durability rate of bargaining power is constant everywhere except for a short period of "election date" at which it is highly transient. In particular, suppose the arrival rate satisfies

$$\lambda = \begin{cases} \lambda_0 + \frac{\lambda^I}{\varepsilon} & \text{if } t^* - \varepsilon < t < t^* \\ \lambda_0 & \text{otherwise} \end{cases} \tag{18}$$

for some small $\varepsilon > 0$ and some positive constant λ^I with

$$\lambda^I/\varepsilon > \lambda_0 (1 + \rho_0) / \overline{y}. \tag{19}$$

Note that λ starts at λ_0, increases to $\lambda_0 + \frac{\lambda^I}{\varepsilon}$ over a period of $(t^* - \varepsilon, t^*)$ and switches back to the original level. Accordingly, the effective durability rate starts at $\rho_0 = \hat{r}_0/\lambda_0$, dips down to $\rho_1 = \hat{r}_0 / \left(\lambda_0 + \frac{\lambda^I}{\varepsilon}\right)$ over a period of $(t^* - \varepsilon, t^*)$ and switches back to the original level as in Figure 2. We will show that there is a long period of disagreement before t^* and provide comparative statics for the length of the delay.[4]

This example could capture political negotiations for which the baseline durability rate is likely to be high. In this context, the arrival rate λ_0 reflects rare events, such as extraordinary opening of a seat (e.g. due to a death), that change the positions of the existing officeholders. In contrast, the higher arrival rate $\lambda_0 + \frac{\lambda^I}{\varepsilon}$ reflects the impact of an election in which many offices are contested and can change hands. Our normalization ensures the conditional probability that the bargaining power changes due to the election is given by $1 - e^{-\lambda^I}$, which is independent of the length of the election ε. We refer to $e^{-\lambda^I}$ as *the incumbency effect*, as this loosely captures the probability that the incumbent candidate with political power remains in office (so that there is no change in bargaining power).

Proposition 5. *Consider the modified Poisson model with the arrival rate in* (18) *and con-*

[4]We consider only one election for simplicity. Our results extend to periodical elections, establishing a long period of gridlock at the end of each cycle.

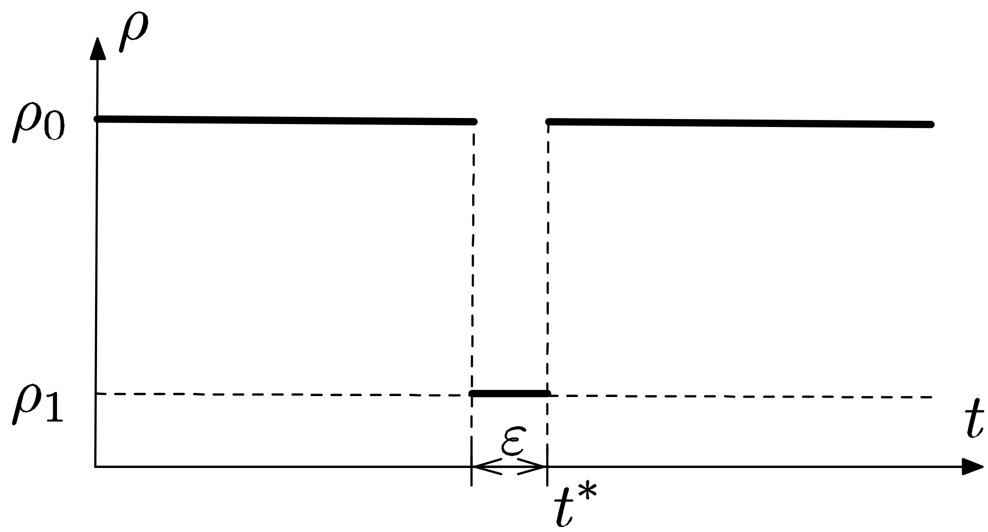

Figure 2: Effective durability rate in Section 4.3.

dition (19). *There exists* $\bar{\Delta} > 0$ *such that players disagree at each time* $\left(t^* - \bar{\Delta}/r_0, t^*\right)$ *and agree at time* t^* *and thereafter. Moreover, as* $\varepsilon \to 0$*, the length of disagreement,* $\bar{\Delta}$*, is the unique solution to* $W\left(\bar{\Delta}\right) = 1$*, where:*

$$W\left(\Delta\right) = e^{-\Delta}\left(1 + \left(1 - e^{-\Delta/\rho_0} e^{-\lambda^I}\right) K\left(\rho_0\right) \overline{y}\right). \tag{20}$$

In particular, a dip in durability due to an election generates a delay prior to the election, generalizing the election effect of Example 3. The result requires the parametric condition (19), which holds quite generally as long as the incumbent is replaced with positive probability, $\lambda^I > 0$, and the election length is short, that is, as $\varepsilon \to 0$.

The result that there is some delay before t^* follows from Proposition 4, since condition (19) implies $\overline{y}K\left(\rho_0\right) > \rho_1$. The additional content of Proposition 5 is to show that the delay extends to times $t \leq t^* - \varepsilon$, starting much before the election. Indeed, it is easy to check that as $\varepsilon \to 0$, the length of delay, $\bar{\Delta}$, remains strictly positive. Indeed, when the baseline bargaining power is highly durable, i.e., when $\rho_0 \to \infty$, the cost of delay due to the election effect is approximately

$$1 - e^{-\bar{\Delta}} \simeq \frac{(1 - e^{-\lambda^I})\overline{y}}{1 + (1 - e^{-\lambda^I})\overline{y}}, \tag{21}$$

by (20). As in deadline and durability effects, the social cost of delay due to the election effect can be as large as half of the total pie.

The proof of Proposition 5 is similar to that of the durability effect. The main difference is that optimism for time t^* at some prior time $t < t^* - \varepsilon$ is now calculated as

$$y_{t,t^*} = \left(1 - e^{-\lambda_0 (t^* - t)} e^{-\lambda^I}\right) \overline{y}$$

Here, $e^{-(t^*-t)\lambda_0}$ captures the baseline survival probability [cf. (8)] and $e^{-\lambda^I}$ is the incumbency effect. Using this expression, the sum of players' perceived payoffs from waiting at time $t^* - \Delta/\hat{r}_0$ (which is prior to $t^* - \varepsilon$ as $\varepsilon \to 0$) is now given by (20). This expression can be approximated around $\Delta = 0$ as

$$W(\Delta) \simeq 1 + \left(1 - e^{-\lambda^I}\right) \overline{y} K(\rho_0) > 1.$$

Thus, there is a strictly positive period of disagreement before the election, and the length of delay is characterized by $W\left(\bar{\Delta}\right) = 1$.

Intuitively, players are optimistic about their likelihood of "winning" the election. More specifically, they both believe the bargaining power will be reset during the election to a new value that is on average in their favor. Hence, there is little discipline on players' optimism in the run-up to an election. Indeed, note that $y_{t,t^*} \geq \left(1 - e^{-\lambda^I}\right) \overline{y} > 0$ for each $t \leq t^* - \varepsilon$, that is, players have significant optimism regardless of how soon the upcoming election will take place. In addition, there is more discipline on players' post-election beliefs, which implies their post-election agreement shares will reflect to some extent their bargaining powers, $K(\rho_0) > 0$. It follows that players disagree before the election in the hope that they will get a better deal after the election.

Hence, similar to the durability and deadline effects, the election effect also stems from an increase of discipline on players' beliefs. The election effect further illustrates the discipline at a time is determined by the "weakest link" of effective durability following that time. In particular, note that there is little discipline at time $t < t^* - \varepsilon$ for beliefs at time t^* despite the fact that the bargaining power is quite durable over most of the interval $[t, t^*]$. Put differently, if there is a period of transience, such as an election, durability in the rest of that period does not create much discipline.

We next establish comparative statics for the length of disagreement $\bar{\Delta}$, which also captures the maximum cost of delay, $1 - e^{-\bar{\Delta}}$. For the limit case of $\rho_0 \to \infty$, (21) already establishes that $\bar{\Delta}$ is an increasing function of the product $(1 - e^{-\lambda^I})\overline{y}$. Our result establishes these and other comparative statics more generally.

Observation 2. *The length of disagreement $\bar{\Delta}$ before an election (as $\varepsilon \to 0$) is:*

(i) decreasing in the incumbency effect, $e^{-\lambda^I}$*,*

(ii) increasing in players' long-run optimism $\overline{y}$*,*

(iii) increasing in the baseline durability rate $1/\lambda_0$*,*

(iv) increasing in the baseline effective discount rate $\hat{r}_0$*.*

The first part establishes that a decrease in the incumbency effect strengthens the election effect, thereby increasing the length of disagreement. This part follows from Eq. (20) since the incumbency effect lowers players' optimism about post-election bargaining powers.

When combined with political term limits, the first part also implies a *lame duck effect.* An incumbent politician approaching the end of his/her term limit—sometimes referred to as a lame duck—is not allowed to be reelected, which implies a zero incumbency effect, $e^{-\lambda^I} = 0$. In contrast, a similar incumbent who is eligible to be reelected is likely to be associated with a positive incumbency effect, $e^{-\lambda^I} > 0$. Observation 2 then suggests that lame duck incumbents (in the above sense) are likely to be associated with more frequent delays in the run-up to elections. In Section 4.4, we test this prediction in the context of legislative politics in the US, utilizing the two-term limit for the US presidency.

The second part of Observation 2 is similar to its analogue in the deadline effect. In this context, players are presumably more optimistic when the election is more closely contested (as this would make the outcome more uncertain). With this interpretation, the second part generates an additional testable prediction that delays are more likely before elections that are more closely contested.

To understand the third part, note that a higher durability rate $1/\lambda_0$ affects the length of disagreement in two ways. First, it lowers optimism (due to the term $e^{-\Delta/\rho_0}$), shortening the delay. More importantly, it increases the rate $K(\rho_0)$ at which post-election bargaining powers translate into agreement shares, increasing the delay. As we show in the appendix, the latter effect dominates. A high $1/\lambda_0$ could be thought of as capturing politically stable democracies in which most of the important changes to bargaining power happen during elections—as opposed to politically unstable democracies in which the bargaining power can also change in non-election times. Under this interpretation, the third part suggests the election effect is more prominent in politically stable democracies.

The last part follows as a corollary to the third part, since what matters for the length of disagreement is the effective durability rate $\rho_0 = r_0/\lambda_0$. To illustrate this result, consider the tax cut negotiations of September 1999 in the US, which took place before the elections in 2000. This was a highly visible political negotiation in which politicians presumably had high $\hat{r}_0$–since voters could punish them for the resulting stalemate.[5] A naive view could then suggest the election effect is less likely to generate delays in this case. Our analysis establishes the opposite result: When $\hat{r}_0$ is higher, the election effect induces longer and costlier delays. This is because, although higher $\hat{r}_0$ increases the cost of delaying agreement until the election, it also changes the post-election agreement terms, captured by a higher $K(\rho_0)$. Intuitively, the higher cost of delay forces the player that loses the election to accept the terms of the player that wins the election. In our model, the second effect dominates (since the subsequent election is very far, at $t = \infty$), leading to longer and costlier delays.

4.4 Election Effect in Legislative Politics in the US

We next analyze gridlock in legislative politics in the US, to test the election effect of Proposition 5 as well as the lame duck effect implied by Observation 2.

We test the election effect by replicating Mayhew's (1991) analysis mentioned in the introduction while also including more recent data. Specifically, we categorize the 33 US Congresses between 1947-2012 based on the time frame they fall within the 4-year presidential term. We measure legislative gridlock by the inverse of the number of important laws enacted by the Congress, as categorized by Mayhew (1991).[6] Figure 3 shows that, consistent with the election effect, the US Congresses enacted about 25% fewer important laws on average when they convened in the two years before a presidential election relative to the two years after. Hence, the empirical regularity identified in Mayhew (1991) continues to hold once we include more recent data—with about the same size and greater statistical significance.

To test the lame duck effect, we utilize the presidential term limit in the US. According to a constitutional amendment ratified in 1947, no person is allowed to be elected to the US presidency more than twice. Consequently, the lame duck effect discussed after Observation 2 suggests that an upcoming presidential election should generate more frequent legislative

[5] Binder (2003) finds that an increase in legislative gridlock in the US is indeed negatively correlated with public approval of the Congress. To give a more recent example, according to a Gallup Poll conducted in August 2011, shortly after the contentious debt-ceiling negotiations, only 21% of voters–a historic low–said yes to the question that most members of Congress deserved to be reelected (http://www.gallup.com/poll/148904/Record-Low-Say-Congress-Deserve-Election.aspx). That said, Binder (2003) does not find a statistically significant relationship between legislative gridlock and legislators' success in subsequent elections.

[6] Specifically, Mayhew has constructed a list of important enactments in the US based on contemporary judgements of journalists—who appraised the laws as they were passed, as well as retrospective judgements by policy specialists—who assigned importance to the laws in more recent writings. See Mayhew (1991) for details of the selection criteria. We have obtained the data set extended through 2012 from David Mayhew's website at http://davidmayhew.commons.yale.edu/datasets-divided-we-govern/.

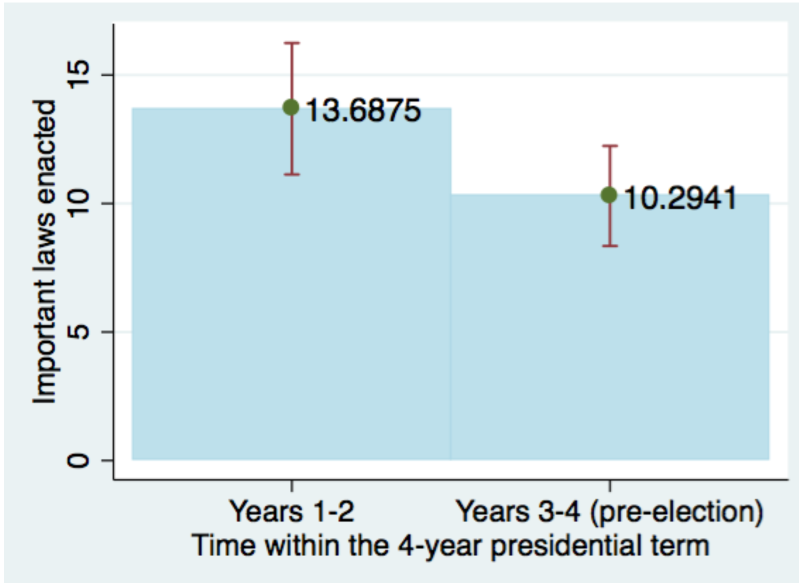

Figure 3: Average number of important laws enacted by the US Congresses between 1947-2012, as a function of the time within the 4-year presidential term. The capped lines illustrate the 5% confidence intervals around the mean.

gridlock when a US president is in his/her second term compared to the first term.

We test this prediction by categorizing the 30 US Congresses between 1953-2012 based on their time frame within the maximum 8-year presidential term. More specifically, we use "years 1-2" to label the US Congresses during the first two years of a first presidential term, "years 5-6" to label the US Congresses during the first two years of a second presidential term, and so on.[7] Figure 4 plots the average number of important laws enacted by the US Congresses in each category. An average congress in years 3-4 of a presidency enacted about 14% fewer laws than a congress in years 5-6, which suggests a positive yet somewhat weak election effect during the first term. In contrast, an average congress in years 7-8 enacted 37% fewer important laws than a congress in years 1-2, which suggests a stronger election effect during the second term—consistent with the lame duck effect.

Unfortunately, the evidence in Figure 4 (as well as Figure 3) is not statistically significant. This is expected, partly because we only have 30 data points in total divided across four

[7] We have excluded the congresses between 1947-1952, which convened during the presidency of Harry Truman. This is becuase, as the sitting president, Truman was exempted from the two-term limit ratified in 1947. We have coded the Kennedy-Johnson presidency in years 1963-1964 as "years 3-4" to ensure consistency with the electoral cycle. We have coded Lyndon Johnson's presidency of 1965-1968 as "years 1-2" and "years 3-4," since Johnson was eligible to be reelected in 1968. We have coded the Nixon–Ford presidency of 1973-1974 as "years 5-6" since Gerald Ford took office towards the end of this period. We have coded Ford's presidency of 1975-1976 as "years 3-4" since Ford was eligible to be reelected in 1976 (but not in 1980 had he won the election in 1976).

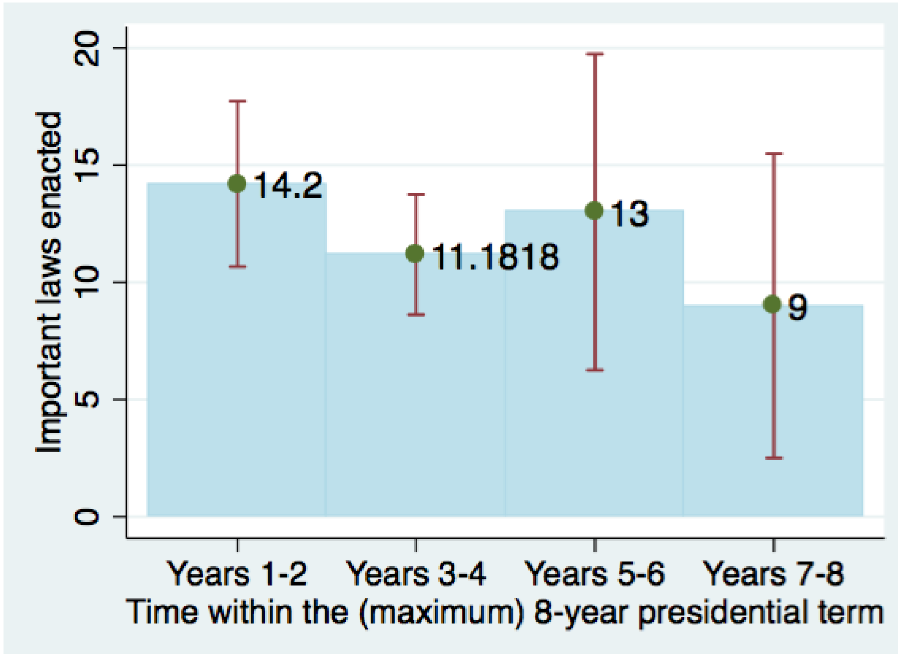

Figure 4: Average number of important laws enacted by the US Congresses between 1953-2012, as a function of the time within the maximum 8-year presidential term. The capped lines illustrate the 5% confidence intervals around the mean.

categories, and partly because there are many more factors excluded from our analysis that might also affect legislative gridlock (see Binder (2003) for a review). We therefore view our findings as preliminary evidence, which should be subject to closer empirical scrutiny as the relevant data becomes available.

5 General Result

We next generalize our main result, the durability effect, beyond the canonical case in the previous section. There are subtle difficulties in defining a general notion of durability that disciplines players' heterogeneous prior beliefs. After describing a main difficulty, we propose two notions of durability, one stronger than the other, that provide such discipline. We also show deadlines and durability play a common disciplining role, not only in the Poisson model, but also more generally. We then establish a generalized durability effect, and discuss how this result also naturally implies deadline and election effects.

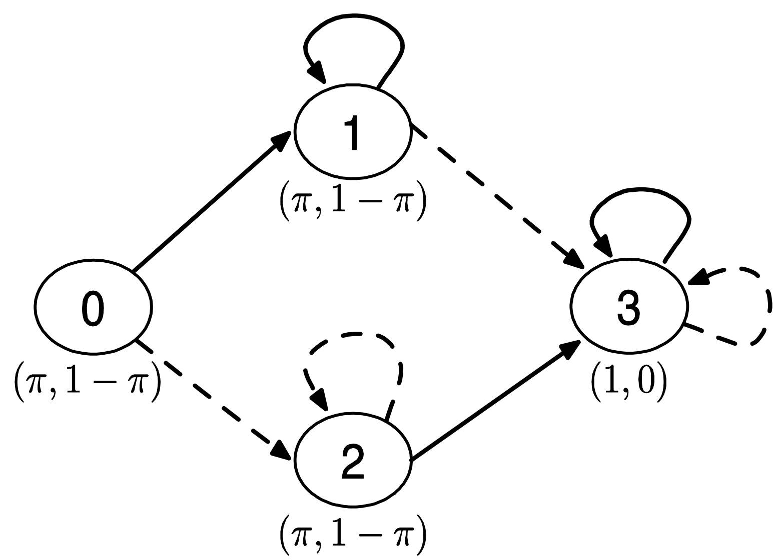

Figure 5: Markov chain in Example 4. The solid and the dashed arrows indicate the deterministic state transitions according to respectively players 1 and 2. The pair below each state is the vector of bargaining powers at that state.

5.1 Durability and Disciplining of Beliefs

Our first goal is to identify general notions of durability for the stochastic process, π_t^i, that induce discipline on players' beliefs and bargaining outcomes. With heterogeneous priors, this exercise is more subtle than it might first appear. The following example illustrates how players' bargaining prospects might deviate from their current bargaining powers, even though both players are certain that that the current bargaining power remains as is forever.

Example 4 (Higher order belief differences). The discount factor is $\delta \equiv e^{-r/n}$ at each t. The bargaining power is determined by the following Markov chain with states $0, 1, 2,$ and 3, plotted in Figure 5. The bargaining power of player 1 is π in states 0, 1, and 2, and 1 in state 3. At $t = 0$, the state is 0. According to player 1, state 0 transitions to state 1, and state 2 transitions to state 3, while states 1 and 3 are absorbing states. According to player 2, state 0 transitions to state 2, and state 1 transitions to state 3, while states 2 and 3 are absorbing states. Observe that each player is certain that the bargaining power of player 1 remains π throughout, as in the common-prior model. It is tempting to conclude that the players agree on division $(\pi, 1-\pi)$ as in Proposition 2. This is far from the case when δ is near 1: they agree to give almost everything to player 1 at the beginning. To see this, observe that at the "zero-probability" state 3, they agree on division $(1, 0)$. Now, at state 1, player 2 is certain that the next state is 3 and she will get 0. Then, at state 1, the agreement gives almost everything to player 1; he gets $V^1(1) = \pi / (1 - (1-\pi)\delta)$ by (5). Likewise, at state 2, he gets $V^1(2) = \delta + \pi(1-\delta)^2 / (1 - \delta\pi)$, leaving player 2 almost nothing: $V^2(2) = (1-\delta)(1-\pi) / (1 - \pi\delta)$. Finally, at state 0, he gets almost everything: $V^1(0) = \pi + \delta(1-\pi) V^1(1) - \delta\pi V^2(2)$.[8]

The main reason for such a stark divergence is as follows. The bargaining outcome depends not only on the beliefs about how bargaining power will evolve, but also on the beliefs about how the other players' beliefs will evolve, the beliefs about how those beliefs will evolve and so on. This is glossed over in common-prior models—thanks to the law of iterated expectations. Without a common prior, the individuals' beliefs about the bargaining power do not put any meaningful restriction on those higher-order beliefs. In the above example, although player 1 is certain that the bargaining power will remain unchanged, he is also certain that the other player will soon start worrying that she will no longer have any bargaining power in the future despite her current certainty that the bargaining powers will remain unchanged. This makes player 1 highly optimistic about her bargaining prospects in the future. On the other hand, although player 2 is certain that the bargaining power will remain unchanged, she is also certain that the other player will soon become highly optimistic about his bargaining power despite his current belief. This makes player 2 pessimistic about the future. Altogether, these effects lead to an agreement that gives nearly everything to player 1.

For a proper definition of durability that disciplines beliefs, one then needs to make sure not only that the players believe that the bargaining power is durable but also that all higher-order beliefs reflect such durability. Our first definition ensures this by requiring that *all* players' expectations about the future bargaining power remain close to the current bargaining power *throughout*.

Definition 1 (Durability)**.** For any interval $[t, s]$, the bargaining power is said to be *durable on* $[t, s]$ if there exists $D > 0$ such that for all $\left[\tilde{t}, \tilde{s}\right] \subset [t, s]$, for all i, and for all $\omega \in \Omega$,

$$|E^i_{\tilde{t}}[\pi^1_{\tilde{s}}](\omega) - \pi^1_{\tilde{t}}(\omega)| \leq (\tilde{s} - \tilde{t})/D. \tag{22}$$

The largest D with the above property is called *durability rate* and denoted by $D_{t,s}$.

Our notion of durability requires that a player's expectation of the future bargaining power is Lipschitz-continuous in time. The durability rate D determines how close the

[8] Observe formally that, as $\delta \to 1$, $V^1(1) \to 1$, $V^2(2) \to 0$, and $V^1(0) \to 1$.

expectations must be to the current bargaining power as a fraction of the time difference. The higher is D, the lower the expected deviation, and the more durable is the bargaining power. To find the durability rate in the baseline Poisson model, note that for any interval $\left[\tilde{t}, \tilde{s}\right] \subset [t, s]$, we have:

$$\left|E_{\tilde{t}}^{i}\left[\pi_{\tilde{s}}^{1}\right](\omega) - \pi_{\tilde{t}}^{1}(\omega)\right| = \left(1 - e^{-\lambda\left(\tilde{s}-\tilde{t}\right)}\right)\left(\bar{\pi}^{i} - \pi_{\tilde{t}}^{i}\right) \leq \left(\tilde{s} - \tilde{t}\right)\lambda\kappa,$$

for each i, where $\kappa = \max\left\{\bar{\pi}^{1}, 1 - \bar{\pi}^{1}, \bar{\pi}^{2}, 1 - \bar{\pi}^{2}\right\} \in (0, 1]$ is a constant. It is easy to check that D cannot be taken higher than $1/(\lambda\kappa)$, which implies $D_{t,s} = 1/(\lambda\kappa)$. In particular, the durability rate is always bounded below by $1/\lambda$, and it is equal to $1/\lambda$ if at least one player is extremely optimistic, i.e., $\bar{\pi}^{i} = 1$ for some i. The next result uses this notion of durability to establish a general bound on players' payoffs.

Lemma 1. *If the bargaining power is durable on $[t, s]$ for some $t, s \in T$, then*

$$\left(\pi_{t}^{i} - (s - t)/D_{t,s}\right)(1 - \delta_{t,s}) \leq V_{t}^{i} \leq \left(\pi_{t}^{i} + (s - t)/D_{t,s}\right)(1 - \delta_{t,s}) + \delta_{t,s}. \tag{23}$$

In particular, letting $\varepsilon = \max(\delta_{t,s}, (s - t)/D_{t,s})$, we also have

$$\left(\pi_{t}^{i} - \varepsilon\right)(1 - \varepsilon) \leq V_{t}^{i} \leq \left(\pi_{t}^{i} + \varepsilon\right)(1 - \varepsilon) + \varepsilon.$$

To understand this result, consider s chosen sufficiently large so that $\delta_{t,s}$ is below some desired ε. Then, the result says that the continuation values at t are close to bargaining powers at t, as long as the bargaining power is sufficiently durable over this interval, i.e., $(s - t)/D_{t,s}$ is also below ε. Intuitively, as in the Poisson model, durability of the bargaining power disciplines players' optimism. Indeed, our definition of durability implies the following bound on players' optimism:

$$y_{t,s} = E_{t}^{1}\left[\pi_{s}^{1}\right] + E_{t}^{2}\left[\pi_{s}^{2}\right] - 1 \leq 2(s - t)/D_{t,s}. \tag{24}$$

Moreover, durability also implies players' future bargaining powers are close to their current bargaining powers. The general solution (6) with common priors then suggests players' continuation values must also be close to their current bargaining powers. Under our notion of durability, a similar logic applies with heterogeneous priors and implies Lemma 1.

While Lemma 1 establishes some discipline on continuation values, it is silent about how those continuation values are obtained. In particular, it is not clear whether players reach an agreement at time t (as in the case with common priors). Establishing this requires a slight strengthening of the notion of durability. To this end, write

$$L\left(c^L, \tilde{t}, \tilde{s}\right) = \left\{\omega \mid \left|\pi^1_{\tilde{s}'}(\omega) - \pi^1_t(\omega)\right| \leq c^L\left(\tilde{s}' - \tilde{t}\right), \forall \tilde{s}' \in \left[\tilde{t}, \tilde{s}\right]\right\}$$

for the set of states at which the bargaining power is Lipschitz-continuous in time over the interval $[\tilde{t}, \tilde{s}]$ with Lipschitz coefficient c^L.

Definition 2 (Strong durability)**.** For any interval $[t, s]$, the bargaining power is said to be *strongly durable on* $[t, s]$ if there exist $D^*, c^L, c^J \geq 0$ such that $c^L + c^J \leq 1/D^*$ and for all $\left[\tilde{t}, \tilde{s}\right] \subseteq [t, s]$, for all i, and for all $\omega \in \Omega$,

$$Pr^i_t\left(L\left(c^L, \tilde{t}, \tilde{s}\right)\right) \geq 1 - c^J(\tilde{s} - \tilde{t}). \tag{25}$$

The largest D^* with the above property is called *strong durability rate* and denoted by $D^*_{t,s}$.

That is, in each player's view, the bargaining power is Lipschitz-continuous with rate c^L, with the exception of some rare "jump" events which happen with a rate slower than c^J. To find the strong durability rate in the Poisson model, note that for any $\left[\tilde{t}, \tilde{s}\right] \subset [t, s]$, we have:

$$Pr^i_{\tilde{t}}(\pi^i_{\tilde{s}'} = \pi^i_{\tilde{t}}, \forall \tilde{s}' \in \left[\tilde{t}, \tilde{s}\right]) \geq e^{-\lambda\left(\tilde{s} - \tilde{t}\right)} \geq 1 - \left(\tilde{s} - \tilde{t}\right)\lambda.$$

Thus, condition (25) holds with $c^L = 0$ and $c^J = \lambda$. It can further be checked that $D^*_{t,s} = 1/\lambda$. In particular, the strong durability rate in the Poisson model is exactly $1/\lambda$, which is (weakly) smaller than the durability rate $1/(\lambda\kappa)$. This is a general feature: As the name suggests, strong durability over an interval $[t, s]$ implies (weak) durability, which in turn implies $D^*_{t,s} \leq D_{t,s}$.[9]

Strong durability disciplines players' optimism further in the sense that they both believe they reach an agreement with high probability after time t. To state this formally, let $t_a(\omega)$ denote the first date with agreement, namely the settlement date, at state ω. We use the convention that $t_a(\omega) = \infty$ when players fail to agree before the deadline. Let

$$A_s = \{\omega \mid t_a(\omega) \leq \min(d(\omega), s) |\} \tag{26}$$

[9] The difference $\left|\pi^1_s - \pi^1_t\right|$, which is bounded by 1, cannot exceed $c^L(s-t)$ on the event $L\left(c^L, t, s\right)$. Hence, under strong durability, we have:

$$|E^i_t[\pi^1_s] - \pi^i_t| \leq (1 - c^J(s-t))c^L(s-t) + c^J(s-t) \leq (c^L + c^J)(s-t),$$

implying weak durability.

denote the states in which players agree before the deadline and before time s.

Lemma 2. *If bargaining power is strongly durable on* $[t,s]$ *for some* $t, s \in T$*, then*

$$Pr_t^i\left(A_s\right) \geq \frac{\pi_t^i - 4\varepsilon^*}{\pi_t^i + 2\varepsilon^*}$$

for each i*, where* $\varepsilon^* = \max\left\{\delta_{t,s}, \left(s-t\right)/D_{t,s}^*\right\}$.

In words, when the bargaining power is strongly durable on $[t,s]$, where s is chosen appropriately to make $\delta_{t,s}$ small, then each player assigns a high probability that they will reach an agreement before s (and before the deadline arrives). There is a simple intuition for this result. By Lemma 1, the continuation value of a player i at time t is high. Then, she must assign a high probability $Pr_t^i(A_s)$ on reaching an agreement before s, since otherwise she gets at most a small payoff, $\delta^{t,s} \leq \varepsilon^*$. Unfortunately, this argument does not lead to a very high lower bound for $Pr_t^i(A_s)$. It leads to a lower bound for $Pr_t^1(A_s) + Pr_t^2(A_s)$ that is nearly 1. In the appendix, using more subtle arguments based on strong durability, we show that each of these probabilities is nearly 1.

5.2 Common Disciplining Role of Deadlines and Durability

Note that the required durability for Lemmas 1 and 2 depend not only on the rate of durability, $D_{t,s}$, but also on the length of the interval, $s-t$. In particular, the smaller we can choose the term $\varepsilon = \max\{\delta_{t,s}, (s-t)/D_{t,s}\}$, the tighter will be the bound established in Lemma 1 and the more beliefs will be disciplined. If we can choose s close to t while still making $\delta_{t,s}$ low, as would be the case with a deadline that arrives at a rapid rate, then a little bit of durability over a relatively short interval is sufficient to discipline players' bargaining prospects. Without a deadline, we have to choose s relatively large. In this case, disciplining requires a higher rate of durability over a longer interval. Hence, as in the Poisson model, deadlines and durability provide a similar discipline on players' beliefs.

Towards formalizing this relationship, let

$$T^{SD} = \{t \in T | \text{bargaining power is strongly durable on } [t,s] \text{ for some } s > t\}.$$

For any $t \in T^{SD}$ and any $s > t$, we define

$$\mathcal{E}\left(t,s\right) = \max\left\{\delta_{t,s}, (s-t)/D_{t,s}\right\}$$

as *the effective variability* over the interval $[t, s]$; we use the convention that $D_{t,s} = 0$ and $\mathcal{E}(t, s) = \infty$ whenever the bargaining power is not durable on $[t, s]$. We also define

$$\mathcal{E}(t) = \inf_{s>t} \mathcal{E}(t, s)$$

as *the minimum effective variability* following time $t \in T^{SD}$. We define $\mathcal{E}^*(t, s)$ and $\mathcal{E}^*(t)$ analogously as using the strong notion of durability. Observe that $\mathcal{E}(t)$ is weakly decreasing in durability rate $D_{t,s}$ and in the conditional arrival probability $(1 - F(s)) / (1 - F(t))$ of deadline for any $[t, s]$. Any increase in durability or in the arrival probability of deadline weakly decreases minimum effective variability $\mathcal{E}(t)$ and thereby puts a greater discipline on expectations in equilibrium, as the next result states. Let $A \equiv A_\infty$ denote the states in which players agree before the deadline [cf. Eq. (26)].

Proposition 6 (Disciplining of Beliefs)**.** *For any $t \in T^{SD}$ and any i,*

$$\begin{aligned} \left(\pi_t^i - \mathcal{E}(t)\right)(1 - \mathcal{E}(t)) &\leq V_t^i \leq \left(\pi_t^i + \mathcal{E}(t)\right)(1 - \mathcal{E}(t)) + \mathcal{E}(t), \\ Pr_t^i(A) &\geq \frac{\pi_t^i - 4\mathcal{E}^*(t)}{\pi_t^i + 2\mathcal{E}^*(t)}. \end{aligned}$$

This result, which is an immediate corollary to Lemmas 1 and 2, combines the disciplining roles of durability and deadlines. Whenever the durability rate of bargaining power or the arrival probability of deadline is higher on an interval $[t, s]$, we have lower effective variability, leading to tighter bounds on the bargaining prospects at time t in Proposition 6.

It is also instructive to consider the effective variability in the Poisson model, which is closely related the effective durability rate $\rho = \hat{r}/\lambda$ that played a central role in Section 4. The minimum effective variability in the baseline Poisson model is given by $\mathcal{E}(t) = \min_{s \geq t} \left\{e^{-\hat{r}(s-t)}, (s - t)\lambda\kappa\right\}$ and characterized as the unique positive solution to the equation $e^{-\rho\mathcal{E}(t)} = \mathcal{E}(t)\kappa$. In particular, $\mathcal{E}(t)$ is decreasing in ρ and it limits to 0 as $\rho \to \infty$. Hence, Proposition 6 provides a more general counterpart to Proposition 3.

5.3 Durability, Deadline, and Election Effects

We next present a generalized durability effect and discuss its implications for deadline and election effects. Given times $t, t^* \in T$, with $t < t^*$, we quantify the *effective optimism at t about t^** by

$$\mathcal{O}(t, t^*) \equiv 1 + y_{t,t^*} - 1/\delta_{t,t^*}. \tag{27}$$

Observe that effective optimism is increasing in optimism y_{t,t^*} about t^* and decreasing in the discount rate δ_{t,t^*} between times t and t^*. Intuitively, it provides a measure of players' tendency to delay agreement until time t^*.

Proposition 7 (General Result)**.** *For any $t^* \in T^{SD}$, there is disagreement at each $t < t^*$ with*

$$\mathcal{O}(t, t^*) > 4\mathcal{E}(t^*) . \tag{28}$$

Moreover, each player i assigns high probability on reaching an agreement after t^*:

$$Pr^i_{t^*}(A) \geq \frac{\pi^i_{t^*} - 4\mathcal{E}^*(t^*)}{\pi^i_{t^*} + 2\mathcal{E}^*(t^*)}.$$

In particular, the agreement is delayed beyond time t^* whenever the optimism about t^* exceeds four times the effective variability, $4\mathcal{E}(t^*)$, throughout $t \leq t^*$. Note also that, since low effective variability disciplines optimism, a high level of optimism about t^* requires a high level of effective variability before t^*. Indeed, it is easy to check—as we do in the appendix—that condition (28) implies:

$$\mathcal{E}(t, t^*) \geq \frac{16}{9}\mathcal{E}(t^*) . \tag{29}$$

Hence, the result requires an increase in effective durability at time t^* in the sense that the effective variability over the interval $[t, t^*]$ is substantially greater than the minimum effective variability following time t^*.

Intuitively, as we have formalized in Proposition 6, low effective variability (or high effective durability) after time t^* induces players to reach agreement with shares that are close to their immediate bargaining powers. Moreover, high effective variability before time t^* allows players to be optimistic about their bargaining prospects from waiting until time t^*. The combination of the two effects leads to possibly long delays before t^*. Note that, while the lack of discipline on beliefs before time t^* automatically translates into optimism (for a fixed $\overline{y} > 0$) in the Poisson model, optimism needs to be assumed explicitly as in (28) in the general model.

All in all, Proposition 7 establishes a *generalized durability effect*: if the bargaining power becomes durable starting at a key date, then players reach agreement after that date with high probability. Moreover, players receive payoffs that are close to their contemporaneous bargaining powers. If the players are sufficiently optimistic about these bargaining powers, then they fail to reach an agreement before then. Proposition 7 implies the deadline and election effects, as we explain next.

Deadline Effect Fix any small $\varepsilon > 0$ and any $t^* \in T^{SD}$, where bargaining power is strongly durable on some $[t^*, \bar{s}]$. Imagine that the deadline arrives between t^* and $s = t^* + \varepsilon D^*_{t^*,\bar{s}} \leq \bar{s}$ with high probability:

$$\frac{1 - F(s)}{1 - F(t^*)} \leq e^{r(s-t^*)}\varepsilon. \tag{30}$$

Then, Proposition 7 implies that the players delay the agreement at any $t < t^*$ with $\mathcal{O}(t, t^*) > 4\varepsilon$ and reach an agreement within interval $[t^*, s]$ with very high probability. This establishes the deadline effect in our general model.

To see this, observe from (1) and (30) that $\delta_{t^*,s} \leq \varepsilon$. Moreover, by definition, $(s - t^*)/D^*_{t^*,s} \leq (s - t^*)/D^*_{t^*,\bar{s}} = \varepsilon$. Hence, the effective variability $\mathcal{E}^*(t^*, s)$ on $[t^*, s]$ is bounded above by ε, showing that $\mathcal{E}(t^*) \leq \mathcal{E}^*(t^*) \leq \varepsilon$. Then, Proposition 7 implies that there is disagreement at any $t < t^*$ with $\mathcal{O}(t, t^*) > 4\varepsilon$. Moreover, the lower bound for the probability of reaching an agreement before the deadline is $1 - 6\varepsilon/(\pi^i_{t^*} + 2\varepsilon)$. Intuitively, the bargaining power cannot change much before the deadline arrives if the deadline arrives rapidly and the bargaining power is somewhat durable. This ensures that parties' payoffs before the arrival of deadline are close to their bargaining powers, which leads to ex-ante disagreement as in the durability effect.

There is a deep mathematical connection between the durability and the deadline effects. According to (1), arrival probability of a deadline increases the effective discount rate by an amount of $f(t)/(1 - F(t))$, the hazard rate of the deadline. This is equivalent to stretching the time at t by $1/r$ times the hazard rate, making the bargaining power more durable. The change in durability is proportional to the hazard rate of the deadline.

Election Effect Elections have the opposite effect on time: the changes that could take a long period of time happen in a short span of the time. Such an effect can be viewed as compressing the time and making the bargaining power less durable. This allows the players to form highly optimistic beliefs about their bargaining power after the election even right before the election. In Proposition 7, this corresponds to having a large $\mathcal{O}(t, t^*)$ when there is an election right before t^*. Since the bargaining power after the election is not affected, this means that the durability of the bargaining power jumps up at the end of the election period, enticing the optimistic parties to wait for the end of the election period for reaching an agreement. This version of the durability effect is called the election effect. The periods of compressed times such as election periods are important because they lead to a long period of disagreement in earlier times when the players are optimistic. This is because the overall durability on an interval is mainly determined by the times of least durability in the interval as we illustrated in the Poisson model.

6 Relation to the Literature

This paper makes two main contributions. First, we develop a model in which the bargaining power is a continuous-time stochastic process. Second, we apply our model to provide a unified explanation for deadline, election, and durability effects. We next discuss these contributions relative to the previous literature.

By taking the bargaining power as a primitive process, our model enables us to capture durability naturally by imposing conditions on the underlying process—independent of how frequently players come together. In contrast, the basic alternating offer bargaining models, e.g., Rubinstein (1982), imply a highly non-durable bargaining power that shifts from one extreme to another infinitely frequently. To the best of our knowledge, ours is the first paper to study the durability of the bargaining power within the bargaining literature. In an independent work, Ambrus and Lu (2009) analyze a continuous-time bargaining model in which the players can make offers only at some Poisson arrivals. More recently, Ortner (2013) studies a continuous-time bargaining model in which the bargaining power is driven by a diffusion process. Both papers assume common priors.[10]

Our model also enables us to capture optimism about bargaining power by allowing the players to have heterogeneous beliefs about the underlying process. Yildiz (2003, 2004) analyze related models in which players are optimistic about their bargaining power—also modeled as players' probability of making take-it-or-leave-it offers. The main differences are that we allow for more general bargaining power processes, and we assume that players observe the current realizations of the bargaining power. The observability assumption naturally puts some discipline on optimism. Our analysis reveals that the extent of discipline crucially depends on the durability of the bargaining power. Unlike Yildiz (2003), we also allow deadlines to be stochastic, which seem to be the case in many real world bargaining situations (see Remark 2 in Section 2).

[10] Durability of the power also plays an explicit role in Dixit, Grossman, Gul (2000). But their setup is quite different since they focus on the efficient subgame perfect Nash equilibrium in a common-prior model of repeated allocation of a dollar. In their model, a Markov process determines who has the power to allocate, and the probability of having the power in the future depends on the current state.

Our paper is part of a theoretical literature that attempts to explain the deadline effect—which is commonly observed in negotiations in practice. The logic of Example 1 is the same as the logic of a two-period example in Yildiz (2003). Unfortunately, the delay in the example of Yildiz (2003) is highly fragile against stochastic deadlines: the players reach an immediate agreement in the continuous-time limit whenever the deadline is stochastic—with a density f. This is problematic because there is a significant amount of uncertainty about the deadline in real-life situations where the deadline effect is common. For example, in the case of pre-trial negotiations, it may take months for the court to announce its decision. In contrast, the deadline effect is robust against such uncertainty in our paper. Indeed, a part of our contribution is to shed light on how the size of the deadline effect relates to the uncertainty about the deadline and durability of the bargaining power.

Spier (1992) shows that, in a pre-trial negotiation with incomplete information, the settlement probability will be a U-shaped function of time, consistent with the deadline effect. Ma and Manove (1993) develop a model in which delay is not costly and a player can wait as much as she wants before making an offer. They show that the player waits until the deadline and makes a last minute take-it-or-leave it offer. Roth, Murnighan, and Schoumaker (1988) informally discuss a possible explanation based on the idea that there is no cost of delay except for a cost at the end due to a slight uncertainty about the deadline.[11] Our model provides an alternative explanation for the deadline effect based on players' optimism about their bargaining powers around the deadline.

Our paper is also part of a literature that analyzes the sources of gridlock in legislative politics (see Binder (2003) for a review). One strand of this literature emphasizes upcoming elections as a potential contributing factor to gridlock. In recent work, Ortner (2013) formalizes a mechanism for gridlock based on the idea that parties' agreement decisions might influence the outcome of the election. We formalize a different mechanism based on parties' optimism about the outcome of the election. In Section 4.3, we also delineate conditions under which this mechanism is stronger, generating additional testable predictions—including a lame duck effect. In Section 4.4, we also document new (yet preliminary) empirical evidence that points to particularly severe legislative gridlock in the US as the president approaches the end of his/her term limit—consistent with our lame duck effect.

Our durability effect—which implies the deadline and election effects as special cases—is also conceptually related to several papers that generate bargaining delays with optimism.

[11] Roth and Ockenfels (2002) consider a similar model in which the delay is motivated by the possibility that the last offer may not go through. They use this model to explain why the deadline effect is observed in e-Bay auctions but not in Amazon auctions.

In Example 2 discussed in the Introduction, at time t^*, Ann and Bob learn about their future bargaining powers, reaching an agreement that reflects that bargaining power. As a result, Ann waits in the hope that Bob will learn at time t^* and be persuaded to agree to her terms (and vice versa for Bob). This motive of waiting to persuade plays an implicit but a crucial role in our model due to durability. Such a motive has been explored in distinct environments in Yildiz (2004a), Thanassoulis (2010), Galasso (2012), and others. In multilateral bargaining, optimism can cause delay through other mechanisms (Ali, 2006) that do not play a role here.

7 Conclusion

In this paper, we have analyzed the conditions under which optimism about future bargaining power leads to costly gridlock and delays in negotiations. We established a general durability effect by which a dramatic increase in discipline on players' beliefs leads to ex-ante delays. We showed that deadlines and elections play a similar role in disciplining beliefs, and that they both lead to ex-ante delays in negotiations. The deadline effect is more prominent when players are more optimistic and when the deadline is less uncertain. For firm deadlines, the effect is also less prominent when players perceive a greater cost of delay. The election effect is more prominent when the incumbent candidate is more likely to be replaced—which also implies a lame duck effect for an incumbent that is not eligible to be reelected. The election effect is also more prominent when players are more optimistic and when the bargaining power is more durable during non-election times. Interestingly, the election effect is also more prominent—and gridlock is more likely—when players perceive a greater cost of delay.

We have also empirically analyzed legislative gridlock in the US, and presented preliminary evidence consistent with our election and lame duck effects. Specifically, the US Congresses that convene before a presidential election seem to enact fewer important laws, and especially so when the incumbent US president is approaching the end of his/her two-term limit. We leave a more complete empirical test of election and lame duck effects—as well as other predictions of our theory—for future work.

While we focused on deadlines and elections, durability may vary in a predictable way also in other situations, affecting the time and the terms of agreements according to our model. For example, a pending reform, such as a labor law or tort reform, might affect the bargaining power of parties in related negotiations, such as wage negotiations or pre-trial negotiations. The players' bargaining power is less durable before the law is enacted—as there could be last minute changes in the law—and it becomes more durable after the law is enacted—as it takes time to enact a new law. Our model then predicts that pending major reforms would cause delays in related individual negotiations. By the same token, one could expect delays in a wide range of individual non-political negotiations prior to the election, as the outcome of the election may provide precise information about the terms of the laws that will pass.

For clarity, we largely focused on a tractable model with a Poisson "reversion" process for the bargaining power—although we also established our main results more broadly in Section 5. While our Poisson model has many appealing properties, it also has some unappealing ones. For example, any change in the balance of power erases all the memory. Nevertheless, all of our insights could be captured in a more general setup. For example, one can take

$$\pi_t^i = \sum_{m=1}^{M} \alpha_m \pi_{t,m}^i$$

where each component $\pi_{t,m}^i$ of bargaining power is an independent Poisson process with time varying arrival rate $\lambda_m(t)$. Here, in the example of a political bargaining, each component m can be considered as a political office, and $\pi_{t,m}^i$ can be taken as the level of control party i has on that office. Such a process exhibits more appealing properties, e.g., the current bargaining power has an impact over the future bargaining power even after it changes. All of our results can be extended to this more general setup. The general setup also enables an exploration of additional issues, e.g., how various dimensions of bargaining power—and optimism about those dimensions—affect the terms as well as the timing of agreement. We leave the analysis of these issues for future work.

A Appendix: Omitted Proofs

Throughout the appendix, we let $\delta_t = \delta_{t,t+1/n}$ denote the one period discount factor to simplify the notation.

Proof of Proposition *2*. We conjecture that players agree at each time. Under this conjecture, the surplus at each time is given by $1 - W_{t,t+1/n} = 1 - \delta_t$. Moreover, the difference equation (5) has a closed form solution given by:

$$V_t^i = E_t \left[\sum_{k=tn}^{\infty} \delta_{t,k/n} \left(1 - \delta_{k/n}\right) \pi_{k/n}^i \right] \qquad (a.s.),$$

which verifies our conjecture by Proposition 1. Taking the limit of this expression as $n \to \infty$ implies Eq. (6), completing the proof. □

Proof of Proposition *3*. We claim that the agreement shares are given by

$$V_t^i = \frac{\rho_n}{\rho_n + 1 + \overline{y}} \pi_t^i + \frac{1}{\rho_n + 1 + \overline{y}} \bar{\pi}^i, \tag{31}$$

for each t and π_t^i, where

$$\rho_n = \left(1 - e^{-r/n}\right) / \left(e^{-r/n} - e^{-(r+\lambda)/n}\right).$$

Note that as $n \to \infty$, ρ_n approaches $\rho = \hat{r}/\lambda$, and the agreement shares approach the shares in (9) – (10). Hence, the proposition follows immediately from the claim.

To prove the claim, by Proposition 1, it suffices to verify that the conjecture in (31) is indeed a solution to (5). Check that, using (7),

$$E_t^i \left[V_{t+1/n}^i \right] = \frac{\rho_n e^{-\lambda/n} \pi_t^i + \left(\left(1 - e^{-\lambda/n}\right) \rho_n + 1 \right) \bar{\pi}^i}{\rho_n + 1 + \overline{y}}. \tag{32}$$

Since $\pi_t^1 + \pi_t^2 = 1$ and $\bar{\pi}^1 + \bar{\pi}^2 = \overline{y} + 1$, this further yields

$$\begin{aligned} W_{t,t+1/n} &= \delta_t \left(E_t^1 \left[V_{t+1}^1 \right] + E_t^2 \left[V_{t+1}^2 \right] \right) \\ &= e^{-\hat{r}/n} \left(1 + \frac{\left(1 - e^{-\lambda/n}\right) \rho_n \overline{y}}{\rho_n + 1 + \overline{y}} \right). \end{aligned}$$

In particular, the surplus is given by

$$1 - W_{t,t+1/n} = \left(1 - e^{-\hat{r}/n}\right) \frac{\rho_n + 1}{\rho_n + 1 + \overline{y}} > 0. \tag{33}$$

In order to verify (5), write $\left(1 - W_{t,t+1/n}\right) \pi_t^i + \delta_t E_t^i \left[V_{t+1/n}^i \right] =$

$$\begin{aligned} & \left(1 - e^{-\hat{r}/n}\right) \frac{\rho_n + 1}{\rho_n + 1 + \overline{y}} \pi_t^i + e^{-\hat{r}/n} \frac{\rho_n e^{-\lambda/n} \pi_t^i + \left(\left(1 - e^{-\lambda/n}\right) \rho_n + 1 \right) \bar{\pi}^i}{\rho_n + 1 + \overline{y}} \\ = & \left(\frac{\left(1 - e^{-\hat{r}/n}\right) (\rho_n + 1) + e^{-(\hat{r}+\lambda)/n} \rho_n}{\rho_n + 1 + \overline{y}} \right) \pi_t^i + \frac{e^{-\hat{r}/n} \left(\left(1 - e^{-\lambda/n}\right) \rho_n + 1 \right)}{\rho_n + 1 + \overline{y}} \bar{\pi}^i \\ = & \frac{\rho_n}{\rho_n + 1 + \overline{y}} \pi_t^i + \frac{1}{\rho_n + 1 + \overline{y}} \bar{\pi}^i. \end{aligned}$$

It follows that the shares are given by (31), completing the proof of the proposition. □

Proof of Observation 1. It remains to prove part (iv). Using Eq. (10) and $\rho_1 = (r+\alpha)/\lambda_1$, we have $\lim_{\alpha\to\infty} K(\rho_1) = 1$. Plugging this into Eq. (15), we obtain $\overline{\Delta}$ in the limit as $\alpha \to \infty$ as the solution to:

$$W\left(\overline{\Delta}\right) \equiv e^{-\bar{\Delta}}\left(1 + \left(1 - e^{-\bar{\Delta}\lambda/r}\right)\overline{y}\right) = 1.$$

Increasing r decreases $W(\Delta)$ for each Δ. Note also that, $\lim_{\Delta\to 0} W(\Delta) > 1$ by assumption (14). Combining these observations implies increasing r also decreases the solution to the equation $W\left(\overline{\Delta}\right) = 1$. By continuity of $W(\Delta)$ with respect to α [cf. Eq. (15)], there exists sufficiently large α such that $\bar{\Delta}$ is decreasing in the discount rate r, completing the proof. □

Proof of Observation 2. It remains to prove parts (iii)-(iv) of Observation 2. To prove these parts, it suffices to show that $\overline{\Delta}$ is increasing in $\rho_0 = \hat{r}_0/\lambda_0$. Rearrange (20) to write $\overline{\Delta}$ as the solution to:

$$f\left(\overline{\Delta}, \rho_0\right) = e^{\overline{\Delta}} - \left(1 - e^{-\overline{\Delta}/\rho_0} e^{-\lambda^I}\right) K(\rho_0)\,\overline{y} = 1,$$

where $\rho_0 = r/\lambda$ and $K(\rho_0) = \rho_0/(\rho_0 + 1 + \bar{y})$. Note that

$$\partial f/\partial\overline{\Delta} = e^{\overline{\Delta}} - e^{-\overline{\Delta}/\rho_0} e^{-\lambda^I} \frac{K(\rho_0)\,\overline{y}}{\rho_0} > 0,$$

where the inequality follows since $e^{\overline{\Delta}} > 1 > e^{-\overline{\Delta}/\rho_0} e^{-\lambda^I}$ and $1 > \frac{K(\rho_0)\overline{y}}{\rho_0}$. Using the implicit function theorem, we also have $\frac{d\overline{\Delta}(\rho_0)}{d\rho_0} = \frac{-\partial f/\partial\rho_0}{\partial f/\partial\overline{\Delta}}$. Thus, the length of delay is increasing in ρ_0 if and only if $-\partial f/\partial\rho_0$ is positive.

Next note that

$$\begin{aligned}
\frac{-\partial f/\partial\rho_0}{\bar{y}} &= \frac{\partial\left(1 - e^{-\overline{\Delta}/\rho_0} e^{-\lambda^I}\right)}{\partial\rho_0} K(\rho_0) + \left(1 - e^{-\overline{\Delta}/\rho_0} e^{-\lambda^I}\right)\frac{\partial K(\rho_0)}{\partial\rho_0} \\
&= -\frac{\overline{\Delta}}{\rho_0^2} e^{-\overline{\Delta}/\rho_0} e^{-\lambda^I} K(\rho_0) + \left(1 - e^{-\overline{\Delta}/\rho_0} e^{-\lambda^I}\right)\left(\frac{K(\rho_0)}{\rho_0}\right)^2 (1+\bar{y})\,.
\end{aligned}$$

Rearranging terms further, we have:

$$\begin{aligned}
\frac{-\partial f/\partial\rho_0}{\bar{y}\overline{\Delta}K(\rho_0)/\rho_0^2} &= -e^{-\overline{\Delta}/\rho_0} e^{-\lambda^I} + \frac{\left(1 - e^{-\overline{\Delta}/\rho_0} e^{-\lambda^I}\right) K(\rho_0)}{\overline{\Delta}}(1+\bar{y}) \\
&= -e^{-\overline{\Delta}/\rho_0} e^{-\lambda^I} + \frac{e^{\overline{\Delta}} - 1}{\overline{\Delta}}\left(1 + \frac{1}{\bar{y}}\right)
\end{aligned}$$

where the second line uses $f\left(\overline{\Delta},\rho_0\right)=1$. Since $e^{-\overline{\Delta}/\rho_0}e^{-\lambda^I}<1$ and $\frac{e^{\overline{\Delta}}-1}{\overline{\Delta}}\geq 1$, we obtain $-\partial f/\partial\rho_0>0$, completing the proof. □

Proof of Lemma 1. We establish the following stronger result, which implies Lemma 1 as a corollary by taking $\bar{t}=t\in T$.

Lemma 3. *If the bargaining power is durable on $[\bar{t},\bar{s}]$ for some $\bar{s}\in T$, then for each $t\in T$ with $\bar{t}\leq t\leq\bar{s}$ and for each i,*

$$\left(\pi_t^i-(\bar{s}-t)/D_{\bar{t},\bar{s}}\right)(1-\delta_{t,\bar{s}})\leq V_t^i\leq\left(\pi_t^i+(\bar{s}-t)/D_{\bar{t},\bar{s}}\right)(1-\delta_{t,\bar{s}})+\delta_{t,\bar{s}}.$$

We prove this lemma by induction. We denote the durability rate over this interval with $D=D_{\bar{t},\bar{s}}$ to simplify notation. For $t=\bar{s}$, the inequalities in the lemma are trivially satisfied because $\delta_{t,\bar{s}}=1$. Towards an induction, assume that for each i,

$$\begin{aligned}&\left(\pi_{t+1/n}^i-(\bar{s}-(t+1/n))/D\right)\left(1-\delta_{t+1/n,\bar{s}}\right)\leq V_{t+1/n}^i\\&\quad\leq\left(\pi_{t+1/n}^i+(\bar{s}-(t+1/n))/D\right)\left(1-\delta_{t+1/n,\bar{s}}\right)+\delta_{t+1/n,\bar{s}}.\end{aligned}$$

By the weak durability assumption, $E_t^i[\pi_{t+1/n}^i]\in[\pi_t^i-1/(Dn),\pi_t^i+1/(Dn)]$. Hence,

$$\begin{aligned}LHS^i&\equiv\left(\pi_t^i-(\bar{s}-t)/D\right)\left(1-\delta_{t+1/n,\bar{s}}\right)\leq E_t^i\left[V_{t+1/n}^i\right]\\&\leq E_t\left[\left(\pi_t^i+(\bar{s}-t)/D\right)\right]\left(1-\delta_{t+1/n,\bar{s}}\right)+\delta_{t+1/n,\bar{s}}\equiv RHS^i.\end{aligned}\tag{34}$$

Recall that

$$V_t^i=\pi_t^i\max\left\{1-\delta_tE_t^j\left[V_{t+1/n}^j\right],\delta_tE_t^i\left[V_{t+1/n}^i\right]\right\}+\left(1-\pi_t^i\right)\delta_tE_t^i\left[V_{t+1/n}^i\right].$$

Combining the last two statements we obtain the necessary bounds. To find an upper bound, we write:

$$\begin{aligned}V_t^i&\leq\pi_t^i\max\left\{1-\delta_tLHS^j,\delta_tRHS^i\right\}+\left(1-\pi_t^i\right)\delta_tRHS^i\\&=\pi_t^i(1-\delta_t)+\delta_tRHS^i\\&=\left(\pi_t^i+(\bar{s}-t)/D\right)(1-\delta_{t,\bar{s}})+\delta_{t,\bar{s}}-(\bar{s}-t)(1-\delta_t)/D\\&\leq\left(\pi_t^i+(\bar{s}-t)/D\right)(1-\delta_{t,\bar{s}})+\delta_{t,\bar{s}}.\end{aligned}$$

Here, the first inequality is by (34), the next equality is by $RHS^i+LHS^j=1$, and the last equality and inequality are by simple algebra. Similarly,

$$\begin{aligned} V_t^i &\geq \pi_t^i \max\left\{1-\delta_t RHS^j, \delta_t LHS^i\right\} + \left(1-\pi_t^i\right)\delta_t LHS^i = \pi_t^i\left(1-\delta_t\right) + \delta_t LHS^i \\ &= \left(\pi_t^i - (\bar{s}-t)/D\right)\left(1-\delta_{t,\bar{s}}\right) + (\bar{s}-t)/D(1-\delta_t) \\ &\geq \left(\pi_t^i - (\bar{s}-t)/D\right)\left(1-\delta_{t,\bar{s}}\right). \end{aligned}$$

□

Proof of Lemma *2*. Recall that $L\left(c^L, t, s\right)$ is the set of states in which the bargaining power "does not jump" between t and s. By the strong durability assumption, $Pr_t^i\left(\Omega \setminus L\left(c^L, t, s\right)\right) \leq c^J\left(s-t\right)$. We will find a lower bound for $Pr_t^i(A_s \cap L\left(c^L, t, s\right))$. Towards this end, we write $\Omega^A = \{\omega \quad | t_a\left(\omega\right) < \infty\}$ for the set of states with agreement. Note that

$$\begin{aligned} E_t^i[V_t^i] = E_t^i[V_t^i \mid \Omega^A] &= \left[\begin{array}{c} E_t^i[V_t^i|A_s \cap L\left(c^L,t,s\right)]Pr(A_s \cap L\left(c^L,t,s\right))+ \\ E_t^i[V_t^i|A_s \setminus L\left(c^L,t,s\right)]Pr(A_s \setminus L\left(c^L,t,s\right)) + E_t^i[V_t^i|\Omega^A \backslash A_s]Pr(\Omega^A \backslash A_s) \end{array}\right] \\ &\leq \left[\begin{array}{c} E_t^i[V_t^i|A_s \cap L\left(c^L,t,s\right)]Pr(A_s \cap L\left(c^L,t,s\right)) \\ +c^J(s-t) + E_t^i[V_t^i|\Omega^A \backslash A_s]Pr(\Omega^A \backslash A_s) \end{array}\right] \\ &\leq \left[\begin{array}{c} E_t^i[V_t^i|A_s \cap L\left(c^L,t,s\right)]Pr\left(A_s \cap L\left(c^L,t,s\right)\right) \\ +c^J(s-t) + \delta_{t,s} \end{array}\right] \end{aligned}$$

where the first inequality follows since $V_t^i \in [0,1]$ and

$$Pr_t^i(A_s \setminus L\left(c^L,t,s\right)) \leq Pr_t^i(\Omega \setminus L\left(c^L,t,s\right)) \leq c^J(s-t),$$

and the last inequality follows since:

$$E_t[V_t^i \,|\Omega^A \backslash A_s]Pr(\Omega^A \backslash A_s) \leq e^{-r(s-t)}\frac{1-F(s)}{1-F(t)} = \delta_{t,s}.$$

Combining this inequality with Lemma 3, we obtain:

$$E_t^i[V_t^i|A_s \cap L\left(c^L,t,s\right)]Pr\left(A_s \cap L\left(c^L,t,s\right)\right) \geq (\pi_t^i - (s-t)/D^*)(1-\delta_{t,s}) - c^J(s-t) - \delta_{t,s}, \quad (35)$$

where we denote the strong durability rate over this interval with $D^* = D_{t,s}^*$ to simplify notation.

To establish a lower bound for $Pr(A_s \cap L\left(c^L,t,s\right))$, we will find an upper bound for $E_t^i[V_t^i|A_s \cap$

$L\left(c^L,t,s\right)]$. Note that, we have

$$\begin{aligned}
E_t^i[V_t^i|A_s\cap L\left(c^L,t,s\right)] &= E_t^i[\delta_{t,t_a}V_{t_a}^i|A_s\cap L\left(c^L,t,s\right)] \\
&\leq E_t^i[\delta_{t,t_a}\left(\left(\pi_{t_a}^i+(s-t_a)/D^*\right)(1-\delta_{t_a,s})+\delta_{t_a,s}\right)|A_s\cap L\left(c^L,t,s\right)] \\
&= E_t^i[\left(\pi_{t_a}^i+(s-t_a)/D^*\right)(\delta_{t,t_a}-\delta_{t,s})|A_s\cap L\left(c^L,t,s\right)]+\delta_{t,s} \\
&\leq E_t^i[\left(\pi_{t_a}^i+(s-t_a)/D^*\right)(1-\delta_{t,s})|A_s\cap L\left(c^L,t,s\right)]+\delta_{t,s} \\
&\leq E_t^i[\left(\pi_t^i+c^L(t_a-t)+(s-t_a)/D^*\right)(1-\delta_{t,s})|A_s\cap L\left(c^L,t,s\right)]+\delta_{t,s} \\
&\leq E_t^i[\left(\pi_t^i+(s-t)/D^*\right)(1-\delta_{t,s})|A_s\cap L\left(c^L,t,s\right)]+\delta_{t,s} \\
&= \left(\pi_t^i+(s-t)/D^*\right)(1-\delta_{t,s})+\delta_{t,s}.
\end{aligned}$$

Here, the first inequality uses Lemma 3 to bound $V_{t_a}^i$ from above, the next equality follows by simple algebra using the fact that $\delta_{t_1,t_2}\delta_{t_2,t_3}=\delta_{t_1,t_3}$, the second inequality follows since $\delta_{t,t_a}\leq 1$, the third inequality follows by definition of $L\left(c^L,t,s\right)$, and the last inequality follows by simple algebra and the fact that $c^L\leq 1/D^*$. Then, using (35), we obtain

$$\begin{aligned}
Pr_t^i(A_s\cap L\left(c^L,t,s\right)) &\geq \frac{(\pi_t^i-(s-t)/D)(1-\delta_{t,s})-c^J(s-t)-\delta_{t,s}}{\left[\pi_t^i+(s-t)/D\right](1-\delta_{t,s})+\delta_{t,s}} \\
&\geq \frac{\pi_t^i-\varepsilon^*-(\pi_t^i-(s-t)/D)\delta_{t,s}-\varepsilon^*-\varepsilon^*}{\pi_t^i+\varepsilon^*+\varepsilon^*} \\
&\geq \frac{\pi_t^i-4\varepsilon^*}{\pi_t^i+2\varepsilon^*}.
\end{aligned}$$

Here, the inequalities after the first one follow since $(s-t)/D^*\leq\varepsilon^*$, $\delta_{t,s}\leq\varepsilon^*$ and $\pi_t^i\leq 1$. Since $Pr_t^i(A_s)\geq Pr_t^i(A_s\cap L\left(c^L,t,s\right))$, this completes the proof of the proposition. □

Proof of Proposition 7. The second part follows from Proposition 6. To prove the first part, note that Proposition 6 also implies

$$V_{t^*}^i\geq\left(\pi_{t^*}^i-\mathcal{E}\left(t^*\right)\right)\left(1-\mathcal{E}\left(t^*\right)\right)$$

for each i. Then, for each time $t<t^*$, we have

$$\begin{aligned}
\delta_{t,t^*}\left(E_t^1[V_{t^*}^1]+E_t^2[V_{t^*}^2]\right) &\geq \delta_{t,t^*}(1-\mathcal{E}\left(t^*\right))\left(E_t^1\left[\left(\pi_{t^*}^1-\mathcal{E}\left(t^*\right)\right)\right]+E_t^2\left[\left(\pi_{t^*}^2-\mathcal{E}\left(t^*\right)\right)\right]\right) \\
&= \delta_{t,t^*}\left[(1-\mathcal{E}\left(t^*\right))(1+y_{t,t^*}-2\mathcal{E}\left(t^*\right))\right] \\
&> \delta_{t,t^*}\left[1+y_{t,t^*}-4\mathcal{E}\left(t^*\right)\right]>1.
\end{aligned}$$

In the last line, the first inequality uses $y_{t,t^*} \leq 1$ and the second inequality follows from the assumption that $\mathcal{O}(t, t^*) > 4\mathcal{E}(t^*)$. Thus, players disagree at each $t < t^*$.

We also show condition (28) implies the inequality in (29). Suppose, to reach a contradiction, $\mathcal{E}(t, t^*) < \frac{16}{9}\mathcal{E}(t^*)$, which also implies $\delta_{t,t^*} < \frac{16}{9}\mathcal{E}(t^*)$ and $(t^* - t)/D_{t,t^*} < \frac{16}{9}\mathcal{E}(t^*)$. Using also the bound on optimism, $y_{t,t^*} \leq 2(s - t)/D_{t,t^*}$, these inequalities further imply

$$\mathcal{O}(t, t^*) = 1 + y_{t,t^*} - 1/\delta_{t,t^*} < 1 + \frac{32}{9}\mathcal{E}(t^*) - \frac{9}{16}\frac{1}{\mathcal{E}(t^*)}.$$

Combining this with the condition, $\mathcal{O}(t, t^*) \geq 4\mathcal{E}(t^*)$, and rearranging terms, we obtain

$$1 > \frac{4}{9}\mathcal{E}(t^*) + \frac{9}{16}\frac{1}{\mathcal{E}(t^*)} \geq 2\sqrt{\frac{4}{9}\frac{9}{16}},$$

where the last line inequality follows from the arithmetic-geometric mean inequality. This yields a contradiction and completes the proof. □

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