An Empirical Equilibrium Model of a Decentralized Asset Market

Alessandro Gavazza

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Abstract

I estimate a search-and-bargaining model of a decentralized market to quantify the effects of trading frictions on asset allocations, asset prices and welfare, and to quantify the effects of intermediaries that facilitate trade. Using business-aircraft data, I find that, relative to the Walrasian benchmark, 18.3 percent of the assets are misallocated, prices are 19.2-percent lower, and the aggregate welfare losses equal 23.9 percent. Dealers play an important role in reducing trading frictions: In a market with no dealers, a larger fraction of assets would be misallocated, and prices would be higher. Overall, dealers reduce aggregate welfare, because their operations are costly and they impose a negative externality by decreasing the number of traders’ direct transactions.

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¶Department of Economics, London School of Economics. Houghton Street, London WC2A 2AE, United Kingdom. Email: a.gavazza@lse.ac.uk.
1 Introduction

How large are trading frictions? How do they affect the allocations and prices of assets, and the welfare of market participants? What is the role of intermediaries in reducing trading frictions? Do they increase welfare? This paper estimates a structural model of a decentralized market to provide quantitative answers to these questions.

Many assets trade in decentralized markets. Classic examples are financial assets such as bonds and derivatives, consumer durable goods such as cars and houses, and firms’ capital assets such as plants and equipment. The fundamental characteristics of decentralized markets are that agents must search for trading partners and that, once a buyer and a seller meet, they must bargain to determine a price. Moreover, in response to trading frictions, almost all decentralized markets have intermediaries. Indeed, starting with Demsetz (1968), trading frictions have been used to explain the existence of intermediaries. The key role of intermediaries in such markets is to improve allocations by reducing frictions, but, since their operations are costly, intermediaries’ effect on aggregate welfare may be ambiguous (Pagnotta and Philippon, 2012).

In this paper, I lay out a model of trading in decentralized markets with two-sided search and bilateral bargaining to study the effects of trading frictions on asset allocations, asset prices and welfare, as well as the effects of intermediaries on them. I then quantify the role of frictions and of dealers, estimating this model using data on business jet aircraft.

The theoretical framework combines elements from Rubinstein and Wolinsky (1987) and Duffie, Gärleanu and Pedersen (2005), extending them to capture key features of real asset markets, such as the heterogeneity of assets due to depreciation.¹ A flow of agents enters the market in every period, seeking to acquire an aircraft. They contact sellers at a rate that depends on the mass of aircraft for sale and on traders’ search ability, and they contact dealers at another rate that reflects dealers’ inventories and search ability. Dealers hold inventories because they do not meet buyers and sellers simultaneously (Grossman and Miller, 1988), and they extract surplus by shortening the time that buyers and sellers have to wait in order to trade (Rubinstein and Wolinsky, 1987).² When agents meet or meet dealers, they bargain over the terms of trade. Gains from trade arise from heterogeneous valuations of holding the

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¹In this way, the paper is one of the few papers to have trading frictions in a model with vintage capital; see, also, Hornstein, Krusell and Violante (2007).
²Thus, the paper focuses on dealers’ advantage over direct exchange in lowering the costs of searching; see Section 4.2 for empirical evidence on this. Intermediaries may also exist to ameliorate the consequences of asymmetric information (see, for example, Biglaiser, 1993). However, intermediaries play an important role in several markets in which asymmetries of information between parties are minimal.
aircraft: An owner wishes to sell when its valuation for the aircraft has dropped.

Equilibrium allocations and prices depend in an intuitive way on agents’ and dealers’ search abilities, bargaining powers and valuations. Specifically, while in a first-best Walrasian market with no delays, only high-valuation agents own aircraft and dealers are not active, trading frictions imply that some low-valuation agents own aircraft and dealers hold inventories, depressing asset prices, as well.

I estimate the model by using data on the secondary market of business jet aircraft—a typical decentralized market. The data are well-suited to studying the effects of search frictions and the role of intermediaries. In particular, they report the number of aircraft for sale and number of aircraft transactions in each month, and their ratio is informative on the magnitude of trading delays. Similarly, the data report dealers’ inventories and dealers’ transactions, and their ratio is informative on the role of dealers in reducing delays. In addition, the data report two series of prices: retail prices between final users of the aircraft; and wholesale prices between aircraft owners (as sellers) and dealers (as buyers). Their differences are useful in understanding how much dealers are able to command by supplying immediacy of trade (and, thus, sellers are willing to forego).

The estimation reveals that trading delays are non-trivial: On average, aircraft stay on the market approximately eight months before a seller is able to finalize a sale. The quantitative importance of these delays depends on how frequently agents seek to trade, determined by a drop in their valuations; this happens, on average, every five years. Moreover, the estimation implies that the dealers enjoy strong bargaining powers, capturing almost the entire surplus of transactions.

I use the estimated parameters to simulate two counterfactual scenarios. In the first one, I quantify the role of trading frictions on asset allocations, asset prices and aggregate welfare by computing a Walrasian market equilibrium, in which the highest-valuations agents always own all assets. The estimates imply that trading frictions generate moderate inefficiencies. Compared to the Walrasian benchmark, new-aircraft prices are 19.26-percent lower, and 18.3 percent of all business aircraft are misallocated: 13.2 percent are on the market for sale, whereas, for 5.1 percent of all aircraft, the trading frictions are larger than sellers’ expected gross gains from trade; thus, their low-valuation owners prefer to keep them rather than put them on the market for sale. Moreover, aggregate welfare in the estimated model is 23.98-percent lower than that in the Walrasian market. The decomposition of this welfare loss indicates that 13.38 percent of the welfare difference is due to the misallocation of assets to lower-valuation users; 10.32 percent is due to delaers’ costs, and only 0.28 percent is due
to the agents’ search costs.

In the second counterfactual, I examine the quantitative effect of dealers on the decentralized market equilibrium, comparing it to a decentralized market with no dealers. This comparison indicates that dealers have a sizable effect on asset allocations and asset prices: In a market without them, 20.6 percent of the assets would be misallocated, and prices would increase by 2.88 percent. Overall, the net effect of dealers is a welfare loss equal to 9.66 percent of aggregate welfare of the estimated model. The intuition for this result is that, while dealers improve the allocation of assets, their operations are costly. Moreover, each dealer imposes a negative externality by lowering other agents’ meeting rates and appropriates surplus that would be created by other agents (von Weizsaker, 1980; Mankiw and Whinston, 1986). Indeed, numerical solutions indicate that aggregate surplus is monotonically declining in the number of dealers. Thus, the optimal number of dealers is zero. Therefore, an interesting conclusion of this counterfactual analysis is that it may be incorrect to make aggregate welfare statements by looking exclusively at asset allocations and asset prices.

This paper makes three main contributions. First, it provides a framework suited to empirically analyzing decentralized asset markets. Search models have proved useful in understanding key features of labor markets, and, more recently, researchers have started to apply search models to financial markets. To my knowledge, this paper is the first to estimate a bilateral search model that investigates the microstructure of the market of a capital asset/durable good, quantifying the effects of trading delays and of intermediaries. Second, the empirical findings suggest that, even within a well-defined asset class such as business aircraft, trading frictions are a non-trivial impediment to the efficient allocation of assets and have significant effects on asset prices. Thus, the paper innovates on recent works that study the process of asset reallocations (Ramey and Shapiro, 1998, 2001; Maksimovic and Phillips, 2001; Eisfeldt and Rampini, 2006; Gavazza, 2011a and 2011b. For an empirical model of business transfers, see, also, Holmes and Schmitz, 1995) by quantifying its inefficiencies. This is a necessary step to understanding how asset markets work and, therefore, to designing any policy that affects them. Third, the paper contributes to the literature on the role of intermediaries by estimating their aggregate welfare effect. The empirical results highlight that intermediaries can reduce welfare since the costs of intermediation can be larger than the aggregate welfare gains from a more-efficient allocation. This is because intermediaries expend resources to capture the rents that trading frictions generate (see, also, Leslie and Sorensen, forthcoming).
The paper proceeds as follows. Section 2 reviews the literature. Section 3 presents some institutional details on the business-aircraft market. Section 4 introduces the data. Section 5 presents the theoretical model, and Section 6 estimates it. Section 7 performs the counterfactual analysis. Section 8 concludes. Appendix A reports on patterns of trade in the data, and Appendix B presents the analytical solution of the model with the assumption of no depreciation.

2 Related Literature

This paper contributes to the important literature that analyzes decentralized markets. The theoretical literature is vast. The most closely related papers examine bilateral search markets, in which both buyers and sellers search for a trading counterpart, and prices are determined through bilateral bargaining (Rubinstein and Wolinsky, 1985 and 1987; Gale, 1987; Mortensen and Wright, 2002; Duffie, Gărleanu and Pedersen, 2005 and 2007; Miao, 2006). The main focus of these theoretical papers is to investigate whether the equilibrium converges to the competitive outcome as frictions vanish. To my knowledge, this paper is the first to estimate a bilateral search model of a real asset market to quantify the effects of trading frictions—i.e., the focus of the theoretical literature.

The paper further contributes to the literature on intermediaries. Several papers investigate the role of brokers/dealers in financial markets and their inventory-management policies; for a survey, see O’Hara (1995). Spulber (1999) provides a thorough analysis of intermediaries between customers and suppliers. Weill (2007) presents a search-and-bargaining model to understand how intermediaries provide liquidity by accumulating inventories when selling pressure is great, and then dispose of those inventories after that selling pressure has subsided. Recent empirical analyses of non-financial intermediaries include Hall and Rust (2000), who focus on the inventory investment of a single steel wholesaler, and Gavazza (2011a), who focuses on the role of lessors in reallocating commercial aircraft. To my knowledge, this paper presents the first structural empirical analysis of the effects of intermediaries in a search-and-bargaining framework.

This paper is also related to the empirical literature on the structural estimation of search models. Most applications focus on labor markets. Eckstein and Wolpin (1990) pioneered this literature by estimating the model of Albrecht and Axell (1984). Eckstein and van der Berg (2006) provide an insightful survey of this literature. One key difference between the current paper and previous research is that this paper seeks to understand how search
frictions affect the level of asset prices and of asset allocations (and the role of intermediaries), while most papers that structurally estimate labor-market search models focus on how search frictions affect wage dispersion across workers (notable exceptions are Gautier and Teulings, 2006 and 2010). Search models have also been applied to housing markets, with Carrillo (2012) being the closest empirical paper.

Finally, this paper is related to a few that investigate aircraft transactions. Using data on commercial-aircraft transactions, Pulvino (1998) finds that airlines under financial pressure sell aircraft at a 14-percent discount, and that distressed airlines experience higher rates of asset sales than non-distressed airlines do. Using data on business jets similar to the data that I use in this paper, Gilligan (2004) finds empirical evidence consistent with the idea that leasing ameliorates the quality of used aircraft traded on secondary markets. Gavazza (2011b) empirically investigates whether trading frictions vary with the size of the asset market in commercial-aircraft markets. However, none of these papers quantifies the magnitudes and the welfare effects of trading frictions by estimating a structural model.

3 Business-Aircraft Markets

For several reasons, the business-aircraft market is an interesting context in which to investigate the effects of search frictions and the role of intermediaries.

First, used business aircraft trade in decentralized markets, organized around privately-negotiated transactions. Almost all buyers (and sellers) are wealthy individuals or corporations, that use the aircraft to fly their executives.\(^3\) To initiate a transaction, a prospective seller must contact multiple potential buyers or sell its aircraft to a dealer. For buyers, comparing two similar aircraft is costly since aircraft sales involve the material inspection of the aircraft, which could be in two different locations. Thus, aircraft markets share many features with other over-the-counter markets for financial assets (mortgage-backed securities, corporate bonds, bank loans, derivatives, etc.) and for real assets (real estate), in which trading involves material and opportunity costs (Duffie, Gârleanu and Pedersen, 2005 and 2007). Moreover, compared to financial markets and other equipment markets, business-aircraft markets are “thin”: Slightly more than 17,000 business jet aircraft were operated worldwide in December 2008. In thin markets, the search costs to find high-value buyers are

\(^3\)While business aircraft represents a small fraction of the aggregate corporate capital stock, all major corporations operate these assets, suggesting that they could be valuable in enhancing the productivity of top corporate executives.
usually large (Ramey and Shapiro, 2001; Gavazza, 2011b).

Second, intermediaries play an important role in mediating transactions. Most intermediaries operate as brokers who match buyers and sellers. Some larger intermediaries operate also as dealers, acquiring aircraft for inventories.

Finally, business aircraft are registered goods with all major “life” events (date of first flight, maintenance, scrappage, etc.) recorded, so detailed data are available. In the next section, I describe them.

4 Data

Patterns in the data suggest that trading delays are an important feature of aircraft markets. Moreover, the available data dictate some of the modeling choices of this paper. Hence, I describe the data before presenting the model. This description also introduces some of the identification issues that I discuss in more detail in Section 6.

4.1 Data Sources

I combine two distinct datasets. The first is an extensive database that tracks the history of business-aircraft transactions. The second reports the average values of several aircraft models, similar to “Blue Book” prices. I now describe each dataset in more detail.

Aircraft Transactions—This database is compiled by AMSTAT, a producer of aviation-market information systems. It provides summary reports that track business-aircraft market transactions. For each month from January 1990 to December 2008 and for each model (e.g., Cessna Citation V or VI), the dataset reports information on the active fleet (e.g., the number of active aircraft; the number of new deliveries); information on aircraft for sale (e.g., the number of aircraft for sale, by owners and by dealers; the average vintage); and information on completed transactions (e.g., the total number of transactions; the number of retail-to-retail, retail-to-dealer, dealer-to-dealer and dealer-to-retail transactions). The dataset also reports the main characteristics of each aircraft model (e.g., average number of seats, maximum range, fuel consumption). I restrict the analysis to business jets, thus excluding turbo-propellers.

http://www.amstatcorp.com/pages/pr_stat.html. The website states: “AMSTAT’s customers are aircraft professionals, whose primary business is selling, buying, leasing and/or financing business aircraft, as well as providers of related services and equipment.”
Aircraft Prices—I obtained business-aircraft prices from the *Aircraft Bluebook Historical Value Reference*. This dataset is an unbalanced panel, reporting quarterly historic values of different vintages for the most popular business-aircraft models during the period 1990-2008. Two series are reported: average retail prices and average wholesale prices. Average retail prices report prices between final users of the aircraft, and average wholesale prices report average transaction prices between an aircraft owner (as a seller) and a dealer (as a buyer). All prices are based on the company’s experience in consulting, appraisal and fleet evaluation. All values are in U.S. dollars, and I have deflated them using the GDP Implicit Price Deflator, with 2005 as the base year.

It is important to note that the construction of the *Aircraft Bluebook Historical Value Reference* implies that the retail and wholesale price series are free of several biases. First, wholesale prices refer to sales to dealers—thus, before dealers could make any improvement to the aircraft. Second, the database reports historical retail and wholesale prices, even for those model-vintage pairs for which only one unit (i.e., one serial number) exists, suggesting that the price series are not affected by sellers’ selection based on aircraft quality (observable or unobservable to both trading parties) or on their valuations.

### 4.2 Data Description

Table 1 provides summary statistics of the main variables used in the empirical analysis. Panel A refers to the Aircraft Transactions Dataset, a sample containing 161 models (the definition of a model is quite fine in this dataset) comprising a total of 26,237 aircraft model-month observations. For each model, the stock of Active Aircraft equals approximately 90 units. In a given month, approximately ten out of these 90 aircraft are for sale: seven by their owners and three by dealers. The number of transactions is small relative to the number of aircraft for sale: On average, there are 0.58 Retail-to-Retail Transactions and 0.83 Dealer-to-Retail Transactions per model-month pair (I include lease transactions in Dealer-to-Retail Transactions). There are also a few Dealer-to-Dealer Transactions—0.28 per month, on average—suggesting that dealers smooth their inventories by trading with other intermediaries. The Total Number of Transactions, defined as the sum of Retail-to-Retail Transactions and Dealer-to-Retail Transactions,

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5The dataset is available at [http://www.aircraftbluebook.com](http://www.aircraftbluebook.com). The website describes the dataset as: “The Aircraft Bluebook Historical Value Reference is specifically designed for lease companies, bankers, aircraft dealers, or anyone who needs to know the pricing history of an individual aircraft.”

6The quantitative analysis in Section 6 does not exploit the cross-sectional differences across models.
Table 1: Summary statistics

<table>
<thead>
<tr>
<th>PANEL A: AIRCRAFT TRANSACTIONS</th>
<th>Obs.</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>MODELS OF AIRCRAFT</td>
<td>26,237</td>
<td>161</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ACTIVE AIRCRAFT</td>
<td>26,237</td>
<td>89.79</td>
<td>107.28</td>
<td>49</td>
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<tr>
<td>AIRCRAFT FOR SALE</td>
<td>26,237</td>
<td>10.91</td>
<td>15.11</td>
<td>6</td>
</tr>
<tr>
<td>–by OWNERS</td>
<td>26,237</td>
<td>7.75</td>
<td>11.94</td>
<td>4</td>
</tr>
<tr>
<td>–by DEALERS</td>
<td>26,237</td>
<td>3.16</td>
<td>4.78</td>
<td>2</td>
</tr>
<tr>
<td>AVERAGE AGE, AIRCRAFT FOR SALE</td>
<td>23,225</td>
<td>18.43</td>
<td>11.09</td>
<td>19</td>
</tr>
<tr>
<td>RETAIL-TO-RETAIL TRANSACTIONS</td>
<td>26,237</td>
<td>0.58</td>
<td>1.12</td>
<td>0</td>
</tr>
<tr>
<td>RETAIL-TO-DEALER TRANSACTIONS</td>
<td>26,237</td>
<td>0.83</td>
<td>1.48</td>
<td>0</td>
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<tr>
<td>DEALER-TO-DEALER TRANSACTIONS</td>
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<td>0.28</td>
<td>0.78</td>
<td>0</td>
</tr>
<tr>
<td>DEALER-TO-RETAIL TRANSACTIONS</td>
<td>26,237</td>
<td>0.83</td>
<td>1.50</td>
<td>0</td>
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<tr>
<td>TOTAL NUMBER OF TRANSACTIONS</td>
<td>26,237</td>
<td>1.42</td>
<td>2.29</td>
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<table>
<thead>
<tr>
<th>PANEL B: AIRCRAFT PRICES</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>MODELS OF AIRCRAFT</td>
<td>31,524</td>
<td>72</td>
<td></td>
<td></td>
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<tr>
<td>RETAIL PRICE (in $1,000)</td>
<td>31,524</td>
<td>7.607</td>
<td>8,534</td>
<td>4,343</td>
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<tr>
<td>WHOLESALE PRICE (in $1,000)</td>
<td>31,524</td>
<td>6.731</td>
<td>7,555</td>
<td>3,849</td>
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<tr>
<td>AGE (in Years)</td>
<td>31,524</td>
<td>14.43</td>
<td>9.69</td>
<td>13.25</td>
</tr>
</tbody>
</table>

Notes—This table provides summary statistics of the variables used in the empirical analysis. Panel A presents summary statistics for the Aircraft Transactions dataset. Each observation represents a model-month pair. Panel B presents summary statistics for the Blue Book prices dataset. Each observation represents a model-vintage-quarter tuple. Aircraft prices are in thousands of U.S. dollars and have been deflated using the GDP Implicit Price Deflator, with 2005 as the base year.

indicates that, on aggregate, approximately 1.5 aircraft per model trade in a given month and that a dealer is the seller to the final retail user in approximately 60 percent of these transactions.

Panel B provides summary statistics for the Aircraft Prices dataset. This sample contains 72 models (indicating that the definition of a model is coarser than in the Aircraft Transactions Dataset), comprising a total of 31,524 aircraft model-vintage-quarter observations. The average RETAIL PRICE of an aircraft in the sample is 7.6 million (year 2005) dollars, and the average WHOLESALE PRICE is 6.7 million (year 2005) dollars. Moreover, there is substantial variation in both prices, reflecting both across-model and across-vintage variation: The standard deviation of retail prices is 8.5 million dollars and of wholesale prices is 7.5 million dollars. Nonetheless, the difference between RETAIL PRICE and WHOLESALE PRICE is always positive within observations.

The two datasets provide a rich description of the business-aircraft market, providing
insights into the motives for trade in this market. Specifically, Appendix A reports on three related patterns that indicate that the main determinant of secondary-markets transactions is changes in owners’ valuations of the assets, against the alternative that these transactions are due to replacements of depreciated units with higher-quality ones (as in, for example, the car market; see Gavazza, Lizzeri, Roketskiy, 2013).

1. In asset markets in which replacement is the main motive for trade, comparative statics across different models imply that assets with faster depreciation have higher resale rates. The argument is as follows. Agents with heterogeneous willingness-to-pay for vertically-differentiated vintages wish to replace their durables when they depreciate, but trading frictions are an impediment to instantaneous (100 percent) trade. When there are transaction costs in the secondhand market, the volume of trade is higher when different vintages are more imperfect substitutes. A faster rate of depreciation increases the vertical differentiation between vintages, thereby increasing the volume of trade. Instead, when valuation shocks are the main motives for trade, resale rates are uncorrelated with assets’ depreciation rates. See Table 6 in Appendix A.

2. A similar cross-sectional comparative static of a framework in which replacement is the main motive for trade is that assets with greater depreciation should be traded earlier “in their life.” Since a faster rate of depreciation increases the vertical differentiation between vintages, owners replace assets with faster depreciation rates at younger ages than they replace assets with slower depreciation rates. Instead, when valuation shocks are the main motives for trade, the age of traded used assets is uncorrelated with assets’ depreciation rates. See Table 7 in Appendix A.

3. Many vintages are on the market simultaneously, including the youngest ones; see Figure 4 in Appendix A. This pattern contrasts with the evidence from other asset markets in which replacement purchases are the main motives for trade, such as the car market. In particular, in the car market, the fraction of cars sold is lowest for the youngest vintages (Porter and Sattler, 1999; Stolyarov, 2002). The reason is that, because of transaction costs, few households sell their cars one year after purchasing them.

Overall, these empirical patterns inform the model to focus on valuations’ shocks as the main motive for trade. Moreover, the data are well-suited to investigating the importance of frictions and the role of dealers. Specifically, three key patterns suggest that trading
delays may be non-trivial and that dealers reduce them. First, the difference between the number of Aircraft for Sale (on average, 11 aircraft per model-month) and the Total Number of Transactions (on average, 1.5 per model-month) means that aircraft stay on the market for several months before selling, indicating that trading delays are substantial. Second, the ratio between Retail-to-Dealer Transactions and Aircraft for Sale by Dealers is higher than the ratio between Total Number of Transactions and Aircraft for Sale, suggesting that dealers are faster than owners at turning aircraft over.\footnote{These differential trading patterns are difficult to explain through a model of “thin” markets—i.e., markets with a very small number of buyers and sellers wishing to trade at non-perfectly synchronized times. The differential patterns are also comparable between more- and less-popular aircraft.} Third, the difference between the Retail Price and the Wholesale Price is quite large (on average, 13 percent), corroborating that frictions are relevant in this market and that dealers are able to command a substantial markup by supplying immediacy of trade.

With all their advantages, however, the datasets pose some challenges. In my view, the main limitation is that both datasets provide aggregate statistics of the market for different models. This limitation implies that a model with rich heterogeneity of agents and dealers, while theoretically feasible, would not be identified with these aggregated data. For example, the aircraft-price dataset does not allow the identification of rich heterogeneity in asset valuations, which would be possible if transaction prices were available. Therefore, the model admits a parsimonious binary distribution of valuations, high and low. In Section 8, I discuss the implications of this limited heterogeneity for the interpretation of the empirical results. Similarly, the aircraft-transaction dataset reports only aggregate dealers’ inventories, limiting the possibilities of identifying heterogeneity across dealers in their inventory levels, or of identifying the effects of their market structure. Finally, an additional limitation is that the aircraft-transaction dataset does not report whether retail buyers or sellers hired a broker to search for trading counterparts, although these intermediaries are popular in business-aircraft markets.

5 Model

In this section, I lay out a model of a decentralized market with two-sided search to theoretically investigate the effects of search frictions on asset allocations, prices and welfare. The model combines elements from Rubinstein and Wolinsky (1987) and Duffie, Gărleanu
and Pedersen (2005), extending them to capture key features of real asset markets, such as the depreciation of assets.

I model frictions of reallocating assets explicitly. In particular, each agent contacts another agent randomly, and this is costly for two reasons: 1) There is an explicit cost $c_s$ of searching; and 2) there is a time cost in that all agents discount future values by the discount rate $\rho > 0$.

### 5.1 Assumptions

Time is continuous and the horizon infinite. A mass $\mu$ of risk-neutral agents enters the economy at every instant. All entrants have a valuation $z = z_h > 0$ for an aircraft. This valuation parameter is a Markov chain, switching from $z_h$ to $z_l < z_h$ with intensity $\lambda$; $z_l$ is an absorbing state. The valuation processes of any two agents are independent. Hence, the steady-state mass of high-valuations agents is equal to $\frac{\mu}{\lambda}$.

There is a constant flow $x < \mu$ of new aircraft entering the economy in every instant, and $x$ high-valuation agents entering the economy purchase them at the (endogenous) price $p^*$. Aircraft depreciate continuously over time and become worthless when they reach age $T$, but they could be endogenously scrapped earlier. Thus, the endogenous total mass $A$ of aircraft is, at most, equal to $xT$. I assume that $\frac{\mu}{\lambda} > xT$, implying that $\frac{\mu}{\lambda} > A$, which means that the “marginal” owner in a Walrasian market is a high-valuation agent. An aircraft of age $a$ generates an instantaneous flow of utility equal to $\pi(z, a)$ to its owner with valuation $z$. The function $\pi(z, a)$ satisfies the following: 1) It is increasing in the valuation $z$, $\pi(z_h, a) > \pi(z_l, a)$; 2) it is decreasing in the age $a$ of the asset, $\frac{\partial \pi(z,a)}{\partial a} < 0$; and 3) it exhibits negative complementarity between the valuation $z$ and the age $a$ of the asset, $\frac{\partial \pi(z_h,a)}{\partial a} < \frac{\partial \pi(z_l,a)}{\partial a}$.

Each agent can own either zero or one aircraft. Agents can trade aircraft: A given agent wishing to trade (either a buyer or a seller) pays a flow cost $c_s$ while searching for a counterparty. While searching, he makes contact with other agents pairwise independently at Poisson arrival times with intensity $\gamma > 0$. Thus, given that matches are determined at

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9The main role of the search cost $c_s$ is to avoid swaps of assets. For example, a low-valuations agent with a young aircraft and a high-valuation agent with an old aircraft may want to swap their assets. However, a positive flow cost of search $c_s$ eliminates these transactions with small gains from trade.
random, the arrival rate $\gamma_s$ for a seller (the rate at which he meets buyers) is $\gamma_s = \gamma \mu_b$, and the arrival rate $\gamma_b(a)$ for a buyer (the rate at which he meets sellers of an age-$a$ aircraft) is $\gamma_b(a) = \gamma \mu_s(a)$, where $\mu_b$ and $\mu_s(a)$ are the endogenous equilibrium masses of buyers and sellers of an age-$a$ aircraft, respectively.\(^\text{10}\) I further assume that agents cannot both use an asset and search for a new one at the same time.\(^\text{11}\)

In addition, there is an endogenous mass $\mu_d$ of independent used-aircraft dealers that meets traders also through a search process. Each dealer has a flow cost equal to $k$ and has, at most, one unit of inventory. An agent wishing to trade meets dealers pairwise independently at Poisson arrival times with intensity $\gamma' > 0$. Thus, a buyer meets a dealer with a vintage-$a$ aircraft to sell at rate $\gamma_{bd}(a) = \gamma' \mu_{do}(a)$, and a dealer with an aircraft to sell meets a buyer at rate $\alpha_{ds} = \gamma' \mu_b$, where $\mu_{do}(a)$ is the endogenous mass of dealers with a vintage-$a$ aircraft to sell. The rates $\gamma_{bd}(a)$ and $\alpha_{ds}$ represent the sum of the intensity of buyers’ search for dealers and dealers’ search for buyers. A seller meets a dealer willing to buy an aircraft at rate $\gamma_{sd} = \gamma' \mu_{dn}$, and a dealer willing to buy an aircraft meets a seller of an aircraft of age $a$ at rate $\alpha_{db}(a) = \gamma' \mu_s(a)$, where $\mu_{dn}$ is the endogenous mass of dealers with an aircraft to sell. There is free entry into the dealers’ market.

Once a buyer and a seller meet, or one of them meets a dealer, parties negotiate a price to trade. I assume that a buyer and a seller negotiate a price according to generalized Nash bargaining, where $\theta_s \in [0, 1]$ denotes the bargaining power of the seller. Similarly, when an agent meets a dealer, they negotiate a price, and $\theta_d \in [0, 1]$ denotes the dealers’ bargaining power.\(^\text{12}\)

### 5.2 Solution

There are four types of agents in the model economy and two types of dealers: high- and low-valuation owners and non-owners, and dealers with and without an aircraft for sale. I denote their types $ho, lo, hn, ln, do, dn$, respectively.

\(^\text{10}\)Thus, the aggregate matching function exhibits increasing returns to scale. This assumption affects only the estimation of the parameters $\gamma$ and $\gamma'$ and not the remaining primitives of the model.

\(^\text{11}\)This assumption greatly simplifies the derivation of the equilibrium allocations and prices, and the quantitative and welfare implications of allowing agents to use the old aircraft and look for a new one should be small.

\(^\text{12}\)The model assumes symmetry of information about the quality of the asset. Several institutional features of aircraft markets support this assumption. First, the aviation authorities often regulate aircraft maintenance. Second, maintenance records are frequently available, and all parties can observe the entire history of owners of each aircraft. Finally, all transactions involve a thorough material inspection of the aircraft.
The owner of an age-$a$ aircraft with valuation $z$ can keep operating it, put it up for sale, or scrap it. In the first case, he enjoys the flow profit $\pi(z,a)$. In the second case, he meets potential trading partners at rate $\gamma_s$ and a dealer at rate $\gamma_{sd}$. In the last case, he becomes an agent with no aircraft. Owners prefer to sell their assets when their valuations are low and the assets are relatively young since the complementarity between valuation $z$ and age $a$ in the flow utility $\pi(z,a)$ means that the gains from trade between buyers and sellers are larger for younger aircraft. Moreover, high-valuation owners scrap their old aircraft, seeking to purchase newer ones. Similarly, an agent with no aircraft can meet active sellers of age-$a$ aircraft at rate $\gamma_b(a)$ and a dealer at rate $\gamma_{bd}(a)$, or he can exit the market. Non-owners choose to search when their valuation is high, and they purchase only relatively young assets. Dealers without inventories operate similarly to buyers, and dealers with inventories operate similarly to sellers, with the key difference that dealers do not enjoy utility from holding assets and their opportunity costs differ from buyers’ and sellers’; thus, their choice of which assets (i.e., vintages) to purchase differs.

I now formally derive the value functions for all agents of the economy and the transaction price at which trade occurs. These value functions allow me to pin down the equilibrium conditions and derive the endogenous distribution of owners’ valuations. This distribution describes how frictions generate allocative inefficiencies and affect aircraft prices.

5.2.1 Agents’ Value Functions

Let $U_{ho}(a)$ be the value function of an agent with valuation $z_h$ who owns an aircraft of age $a$ and is not seeking to sell it. $U_{ho}(a)$ satisfies:

$$pU_{ho}(a) = \pi(z_h,a) + \lambda (V_{lo}(a) - U_{ho}(a)) + U'_{ho}(a).$$

Equation (1) has the usual interpretation of an asset-pricing equation. An agent with valuation $z_h$ enjoys the flow utility $\pi(z_h,a)$ from an aircraft of age $a$. At any date, one possible event, at most, might happen to him: At rate $\lambda$, his valuation drops to $z_l$, in which case he chooses between continuing to operate the aircraft (enjoying the value $U_{lo}(a)$) and actively seeking to sell it (enjoying value $S_{lo}(a)$). Thus, the agent obtains a value $V_{lo}(a) = \max\{U_{lo}(a), S_{lo}(a)\}$ and a capital loss equal to $V_{lo}(a) - U_{ho}(a)$. Moreover, the aircraft depreciates continuously, so that the agent has a capital loss equal to $U'_{ho}(a)$.

Similarly, the value function $S_{ho}(a)$ of an agent with valuation $z_h$ who owns an aircraft
of age $a$ and is actively seeking to sell it satisfies:

$$
\rho S_{ho}(a) = \pi(z_h, a) - c_s + \lambda (V_{lo}(a) - S_{ho}(a)) + \gamma_s \max \{p(a) + V_{hn} - S_{ho}(a), 0\} + \\
\gamma_{sd} \max \{p_B(a) + V_{hn} - S_{ho}(a), 0\} + S'_{ho}(a).
$$

(2)

An agent with valuation $z_h$ enjoys the flow utility $\pi(z_h, a)$ from an aircraft of age $a$, but he pays the flow cost $c_s$ while actively seeking to sell it. At any date, one of three possible events, at most, might happen to him: 1) At rate $\lambda$, his valuation drops to $z_l$. In this case, the agent chooses between keeping the aircraft (enjoying the value $U_{lo}(a)$) and actively seeking to sell it (enjoying value $S_{lo}(a)$). Hence, the agent obtains a capital loss equal to $V_{lo}(a) - S_{ho}(a).$ 2) At rate $\gamma_s$, the agent meets a potential buyer and chooses between trading the aircraft or keeping it. If he trades it at price $p(a)$, he becomes a high-valuation non-owner with value $V_{hn}$, thus obtaining a capital gain equal to $p(a) + V_{hn} - S_{ho}(a).$ If he keeps it, he obtains a capital gain of zero. 3) At rate $\gamma_{sd}$, he meets a dealer and chooses between trading the aircraft or keeping it. If he trades it at price $p_B(a)$, he then chooses between actively searching for another aircraft or not, thus obtaining a capital gain equal to $p_B(a) + V_{hn} - S_{ho}(a).$ If he keeps it, he obtains a capital gain of zero. Moreover, the aircraft depreciates continuously, so that the agent has a capital loss equal to $S'_{ho}(a)$.

The value functions $U_{lo}(a)$ and $S_{lo}(a)$ of an agent with valuation $z_l$ who owns an aircraft of age $a$ satisfy the following Bellman equations, respectively:

$$
\rho U_{lo}(a) = \pi(z_l, a) + U'_{lo}(a),
$$

(3)

$$
\rho S_{lo}(a) = \pi(z_l, a) - c_s + \gamma_s \max \{p(a) + V_{hn} - S_{lo}(a), 0\} + \\
\gamma_{sd} \max \{p_B(a) + V_{hn} - S_{lo}(a), 0\} + S'_{lo}(a).
$$

(4)

The interpretation of equations (3) and (4) is now straightforward. An agent with valuation $z_l$ enjoys the flow utility $\pi(z_l, a)$ from an aircraft of age $a.$ If he does not seek to sell the aircraft, the only event that affects his utility is the depreciation of the aircraft, with capital loss $U'_{lo}(a).$ If he seeks to sell the aircraft, the search cost $c_s$ reduces his flow utility. Then, at any date, he meets a potential buyer at rate $\gamma_s,$ in which case he chooses between selling the aircraft at price $p(a)$—thus obtaining a capital gain equal to $p(a) + V_{hn} - S_{lo}(a)$—or keeping it—thus obtaining a capital gain of zero, where $V_{hn} = \max \{U_{hn}, S_{hn}\}.$ Similarly, he meets a dealer at rate $\gamma_{sd},$ in which case he chooses between trading the aircraft at price
\(p_B(a)\)—thus, obtaining a capital gain equal to \(p_B(a) + V_{ln} - S_{lo}(a)\)—or keeping it—thus obtaining a capital gain of zero. Moreover, the aircraft depreciates continuously, so that the agent has a capital loss equal to \(S_{ln}'(a)\).

The value functions of high- and low-valuation agents with no aircraft satisfy:

\[
\begin{align*}
\rho S_{hn} &= -c_s + \lambda (V_{ln} - S_{hn}) + \int \gamma_b(a) \max \{V_{ho}(a) - p(a) - S_{hn}, 0\} \, da + \\
\rho U_{hn} &= 0, \\
\rho S_{ln} &= -c_s + \int \gamma_b(a) \max \{V_{lo}(a) - p(a) - S_{ln}, 0\} \, da + \\
\rho U_{ln} &= 0.
\end{align*}
\]

Equation (5) says that a high-valuation agent with no aircraft who is paying the search cost \(c_s\) has a capital loss equal to \(V_{ln} - S_{hn}\) when, at rate \(\lambda\), his valuation drops from high to low; has a capital gain equal to \(\max \{V_{ho}(a) - p(a) - S_{hn}, 0\}\) when, at rate \(\gamma_b(a)\), he meets a seller of an aircraft of age \(a\); and has a capital gain equal to \(\max \{V_{ho}(a) - p_A(a) - S_{hn}, 0\}\) when, at rate \(\gamma_{bd}(a)\), he meets a dealer selling an aircraft of age \(a\), where \(V_{ho}(a) = \max \{U_{ho}(a), S_{hn}\}\).

Since the \(hn\)-agent does not know the age of the aircraft that the counterparty will have, he takes expectation over the possible capital gains that arise from the different vintages. Similarly, equation (7) says that a low-valuation agent with no aircraft, who is paying the search cost \(c_s\) to search for a counterparty, has an expected capital gain equal to \(\max \{V_{lo}(a) - p(a) - S_{ln}, 0\}\) when he meets a potential seller of an aircraft of age \(a\), and an expected capital gain equal to \(\max \{V_{lo}(a) - p_A(a) - S_{ln}, 0\}\) when he meets a dealer.

Equations (6) and (8) say that agents without aircraft who are not searching have a zero value.

### 5.2.2 Dealers’ Value Functions

As in Rubinstein and Wolinsky (1987), dealers can extract surplus by shortening the time that buyers and sellers have to wait in order to trade. They are capacity-constrained and cannot store more than one aircraft. Thus, the value functions \(J_{do}(a)\) and \(J_{dn}\) of dealers
with and without an aircraft for sale, respectively, satisfy:

\[
\begin{align*}
\rho J_{do} (a) &= \max \{-k + \alpha_{ds} (p_A (a) + J_{dn} - J_{do} (a)) + J'_{do} (a), \rho J_{dn}\}, \\
\rho J_{dn} &= -k + \int \alpha_{db} (a) \max \{J_{do} (a) - p_B (a) - J_{dn}, 0\} \, da.
\end{align*}
\]

Equation (9) says that a dealer who owns an aircraft of age \(a\) pays the flow cost \(k\) and chooses between two alternatives: 1) He can actively seek to sell it, meeting a potential buyer at rate \(\alpha_{ds}\). In this case, the dealer trades the aircraft at a negotiated price \(p_A (a)\), obtaining a capital gain equal to \(p_A (a) + J_{dn} - J_{do} (a)\). Moreover, the dealer has a capital loss equal to \(J'_{do} (a)\) because its aircraft depreciates continuously; or 2) he can scrap the aircraft, thereby becoming a dealer without aircraft for sale.

Similarly, equation (10) says that a dealer without aircraft pays the flow cost \(k\) while he actively seeks to purchase one. At rate \(\alpha_{db} (a)\), this dealer meets a potential seller of an aircraft of age \(a\), in which case the dealer decides whether or not to trade, thus enjoying a capital gain equal to \(\max \{J_{do} (a) - p_B (a) - J_{dn}, 0\}\). Since the \(dn\)-dealer does not know the age of the aircraft that the counterparty will have, he takes expectation over the possible capital gains that arise from the different vintages.

Dealers’ free entry requires that \(J_{dn} = 0\)—i.e., dealers’ expected capital gain is exactly equal to their fixed operating cost.

### 5.2.3 Prices

When a buyer and a seller meet and agree to trade, the negotiated price,

\[
p (a) = (1 - \theta_s) (S_{lo} (a) - V_{in}) + \theta_s (U_{ho} (a) - S_{hn}),
\]

is the solution to the following symmetric-information bargaining problem:

\[
\max_{p(a)} [U_{ho} (a) - p (a) - S_{hn}]^1 - \theta_s [p (a) + V_{in} - S_{lo} (a)]^{\theta_s}
\]

subject to: \(U_{ho} (a) - p (a) - S_{hn} \geq 0\) and \(p (a) + V_{in} - S_{lo} (a) \geq 0\).

Similarly, the ask and bid prices \(p_A (a)\) and \(p_B (a)\) satisfy:

\[
\begin{align*}
p_A (a) &= (1 - \theta_d) (J_{do} (a) - J_{dn}) + \theta_d (U_{ho} (a) - S_{hn}), \\
p_B (a) &= (1 - \theta_d) (J_{do} (a) - J_{dn}) + \theta_d (S_{lo} (a) - V_{in}).
\end{align*}
\]
Finally, the price $p^*$ of new assets is such that new entrants are indifferent between paying the price $p^*$ and searching on the secondary market:

$$U_{ho}(0) - p^* = S_{hn}.$$ 

Hence, new assets trade at a premium relative to all used assets because of differential frictions between the primary market and secondary markets.

### 5.2.4 Agents’ and Dealers’ Policies

We can simplify the value functions of all types of agents $\{ho, lo, hn, ln\}$ recognizing that gains from trade may arise only when high-valuation non-owners meet low-valuation owners. Moreover, since the flow payoff $\pi(z, a)$ exhibits complementarity between agents’ valuation $z$ and the age $a$ of the asset, the gains from trade are larger for younger assets. In turn, this implies that, depending on parameters, it is possible that not all assets trade in equilibrium. Indeed, Section 6 will show that not all assets trade in the estimated model, so I focus on this case.

Specifically, since $U_{ho}(a)$ is decreasing in $a$, there exists a cutoff age $a_{ho}^*$ such that $ho$-agents scrap their asset when it reaches age $a_{ho}^*$—i.e., $a_{ho}^*$ satisfies $U_{ho}(a_{ho}^*) = S_{hn}$. Moreover, a cutoff age $a_{hn}^*$ determines whether or not $hn$-agents purchase an age-$a$ asset. Trading frictions generate a wedge that implies that $a_{hn}^* \leq a_{ho}^*$. Thus, high-valuation owners’ value functions are:

$$V_{ho}(a) = \begin{cases} 
U_{ho}(a) & \text{for } a < a_{ho}^*, \\
S_{hn} & \text{for } a \geq a_{ho}^*.
\end{cases}$$

The policy of $lo$-owners satisfies a cutoff rule, as well. Specifically, since the flow utility $\pi(z_l, a)$ is decreasing in the age $a$ of the asset, whereas the flow search cost $c_s$ is constant, there exists a cutoff age $a_{l}^*$ such that $lo$-agents sell their aircraft if it is younger than $a_{l}^*$, but keep it if it is older—i.e., $U_{lo}(a) > S_{lo}(a)$ if $a < a_{l}^*$, $U_{lo}(a_{l}^*) = S_{lo}(a_{l}^*)$ and $U_{lo}(a) < S_{lo}(a)$ if $a > a_{l}^*$. Equilibrium requires that $lo$-agents sell assets that $hn$-agents are willing to purchase—thus, $a_{l}^* \leq a_{hn}^*$. Moreover, since $lo$-agents enjoy positive utility from these old assets, they keep them until they reach the scrappage age $T$. Thus, the low-valuation owners’ value functions are:

$$V_{lo}(a) = \begin{cases} 
S_{lo}(a) & \text{for } a < a_{l}^*, \\
U_{lo}(a) & \text{for } a \leq a_{l}^* < T, \\
V_{ln} & \text{for } a = T.
\end{cases}$$
Non-owners’ value functions are:

\[ V_{hn} = S_{hn}, \]
\[ V_{ln} = 0. \]

Similar arguments apply to dealers, as well. Their value function \( J_{do}(a) \) is decreasing in \( a \) because younger aircraft sell at greater margins, as the gains from trade are larger. Thus, since dealers’ fixed cost \( k \) is independent of the age of the asset, there exist two cutoff ages \( a_{dn}^* \) and \( a_{do}^* \) such that dealers do not purchase aircraft older than \( a_{dn}^* \) and scrap aircraft when they reach age \( a_{do}^* \). Since the asset purchase price is sunk at the time of scrapping it, but not at the time of buying it, \( a_{dn}^* < a_{do}^* \). Moreover, equilibrium requires that dealers sell assets that buyers are willing to purchase. Hence, \( a_{do}^* \leq a_{hn}^* \).

5.2.5 Distribution of Agents

Let \( \mu_i(a) \) be the masses of owners of age-\( a \) aircraft whose state is \( i \in \{ho, lo, do\} \), and let \( \mu_i \) be the masses of non-owners whose state is \( i \in \{hn, ln, dn\} \). The masses of owners evolve over time according to the following system of differential equations:

\[
\begin{align*}
\dot{\mu}_{ho}(a) &= (\gamma_b(a) \mu_{hn} + \gamma_{bd}(a) \mu_{hn}) - \lambda \mu_{ho}(a) \text{ for } a < a_{ho}^*, \quad (14) \\
\dot{\mu}_{lo}(a) &= \lambda 1(a < a_{ho}^*) \mu_{ho}(a) - \gamma_s 1(a < a_{l}^*) \mu_{lo}(a) + \gamma_{sd} 1(a < \min\{a_{l}^*, a_{dn}^*\}) \mu_{lo}(a) \text{ for } a < T, \quad (15) \\
\dot{\mu}_{do}(a) &= \alpha db(a) 1(a < a_{dn}^*) \mu_{dn} - \alpha ds \mu_{do}(a) \text{ for } a < a_{do}^*, \quad (16)
\end{align*}
\]

with initial conditions \( \mu_{ho}(0) = x \) and \( \mu_{lo}(0) = \mu_{do}(0) = 0 \), and terminal conditions \( \mu_{ho}(a) = 0 \) for \( a_{ho}^* \leq a < T \) and \( \mu_{do}(a) = 0 \) for \( a_{do}^* \leq a < T \). The notation \( 1(Y) \) represents an indicator function equal to one if the event \( Y \) is true, and zero otherwise.

The intuition for these equations is as follows. Equation (14) states that the mass of high-valuation agents with an age-\( a \) asset is the result of flows of three sets of agents: 1) the inflow of high-valuation non-owners that found a seller of an age-\( a \) aircraft—the term \( \gamma_b(a) \mu_{hn} \); 2) the inflow of high-valuation non-owners that found a dealer selling an age-\( a \) aircraft—the term \( \gamma_{bd}(a) \mu_{hn} \); and 3) the outflow of high-valuation age-\( a \) aircraft owners whose valuation just dropped—the term \( \lambda \mu_{ho}(a) \). Equation (14) already incorporates the equilibrium outcomes that low-valuation owners of an age-\( a \) aircraft are willing to sell it
rather than keep it (i.e., $\gamma_b(a) > 0$) and dealers have inventories of age-$a$ aircraft (i.e., $\gamma_{bd}(a) > 0$) only if high-valuation non-owners are willing to buy them (i.e., $a < a_{ho}^*$).

Equation (15) states that the mass of low-valuation agents with an age-$a$ asset is the result of flows of three sets of agents: 1) the inflow of high-valuation agents whose valuation just dropped. Since high-valuation agents scrap the oldest assets, this inflow applies only to owners of assets younger than $a_{ho}^*$—hence, the term $\lambda (a < a_{ho}^*) \mu_{ho}(a)$; 2) the outflow of low-valuation aircraft owners who prefer to sell their aircraft rather than keep it (i.e., an aircraft of age $a < a_t^*$) and who have found a buyer (this buyer is willing to purchase it because $a_t^* \leq a_{ho}^*$)—hence the term $\gamma_s 1(a < a_t^*) \mu_{lo}(a)$; and 3) the outflow of low-valuation aircraft owners that prefer to sell their aircraft rather than keep it (i.e., an aircraft of age $a < a_t^*$) and that found a dealer willing to purchase it (i.e., an aircraft of age $a < a_{dn}^*$). Hence the term $\gamma_{sd} 1(a < \min \{a_t^*, a_{dn}^*\}) \mu_{lo}(a)$. The intuition for equation (16) is similar.

Moreover, the masses of high-valuation non-owners and dealers without inventories evolve over time according to:

$$\dot{\mu}_{hn} = (\mu - x) + \mu_{ho}(a_{ho}^*) - \lambda \mu_{hn} - \mu_{hn} \int_0^{a_t^*} \gamma_b(a) \, da - \mu_{hn} \int_0^{a_{dn}^*} \gamma_{bd}(a) \, da,$$

$$\dot{\mu}_{dn} = \alpha_{ds} \int_0^{a_{do}^*} \mu_{do}(a) \, da - \mu_{dn} \int_0^{\min \{a_t^*, a_{dn}^*\}} \alpha_{db}(a) \, da + \mu_{do}(a_{dn}^*),$$

whereas, in each instant, the mass of $ln$-agents equals:

$$\mu_{ln} = \lambda \mu_{hn} + \gamma_s \int_0^{a_t^*} \mu_{lo}(a) \, da + \gamma_{sd} \int_0^{\min \{a_t^*, a_{dn}^*\}} \mu_{lo}(a) \, da + \mu_{lo}(T).$$

Steady state imposes the following constraints on the evolution of these masses: 1) The aggregate masses of owners $\int_0^T \mu_i(a) \, da$ for $i \in \{ho, lo, do\}$ and the aggregate masses $\mu_{hn}$ and $\mu_{dn}$ are constant over time; 2) the mass $\mu_{ln}$ of exiters equals the mass $\mu$ of new entrants; 3) the total mass of agents with high valuation $\mu_{hn} + \int_0^T \mu_{ho}(a) \, da$ equals $\frac{\mu}{\lambda}$; 4) the total mass of dealers $\mu_d$ equals $\int_0^T \mu_{do}(a) \, da + \mu_{dn}$; 5) the aggregate masses of assets sold and purchased by dealers equal the aggregate masses of assets purchased from and sold to dealers: $\mu_{hn} \int_0^{a_{do}^*} \gamma_{bd}(a) \, da = \alpha_{ds} \int_0^{a_{do}^*} \mu_{do}(a) \, da$ and $\mu_{dn} \int_0^{\min \{a_t^*, a_{dn}^*\}} \alpha_{db}(a) \, da = \gamma_{sd} \int_0^{a_{do}^*} \mu_{lo}(a) \, da$; and 6) dealers’ aggregate inventories do not change over time: $\mu_{hn} \int_0^{a_{do}^*} \gamma_{bd}(a) \, da = \mu_{dn} \int_0^{\min \{a_t^*, a_{dn}^*\}} \alpha_{db}(a) \, da$. In addition, equilibrium requires that the total mass of owners of age-$a$ aircraft $\mu_{ho}(a) + \mu_{lo}(a) + \mu_{do}(a)$ equals the mass $x$ of aircraft for $a \leq \min \{a_{do}^*, a_{ho}^*\}$, and $\mu_{ho}(a) + \mu_{lo}(a) + \mu_{do}(a) < x$ for $a > \min \{a_{do}^*, a_{ho}^*\}$.
All these steady-state equalities and the equilibrium condition $\mu_{ho}(a) + \mu_{lo}(a) + \mu_{do}(a) \leq x$ allow us to solve for the endogenous masses as a function of the exogenous parameters $x, T, \mu$ and $\lambda$, and the exogenous parameters of the matching functions.

Letting $\gamma$ increase, $\lim_{\gamma \to +\infty} \mu_{lo}(a)$ converges to 0: When frictions vanish, no low-valuation agent owns an aircraft. Similarly, $\lim_{\gamma \to +\infty} \mu_{do}(a)$ converges to 0. Thus, the masses $\int_0^T \mu_{lo}(a) \, da$ and $\int_0^T \mu_{do}(a) \, da$—i.e., the masses of low-valuation agents and dealers with inventories—are measures of assets inefficiently allocated in the economy. Based on the parameters estimated from the data, in Section 7.1, I compute these measures to quantify the effects of trading frictions on asset allocations.

6 Quantitative Analysis

The model does not admit an analytic solution for the asset allocations and asset prices as a function of all the model primitives, except in the special case when assets are homogeneous—i.e., $\frac{\partial \pi(z,a)}{\partial a} = 0$. Appendix B reports this special case. Hence, the goal of the quantitative analysis is to choose the parameters that best match moments of the data with the corresponding moments computed from the model’s numerical solution. Since the analytic solution of the model under the assumption that $\frac{\partial \pi(z,a)}{\partial a} = 0$, presented in Appendix B, provides an approximation of the general model, it helps to build intuition for the identification of the parameters.

6.1 Estimation and Identification

I estimate the model using the data on business aircraft described in Section 4, assuming that they are generated from the model’s steady state. I set the unit of time to be one quarter.

Unfortunately, the data lack some detailed information to identify all parameters. Therefore, I fix some values. Specifically, the discount rate $\rho$ is traditionally difficult to identify, and I fix it to $\rho = .015$. Moreover, I fix the useful lifetime of an aircraft to be equal to $T = 160$ quarters. I also fix the total mass of aircraft to be equal to the sample median $A = 9,687$, and I use a directory of aircraft dealers to “estimate” total dealers’ capacity to be equal to $\mu_d = 1000$. Furthermore, I assume that aircraft owners’ flow payoff equals:

$$\pi(z,a) = z\delta_1 e^{-\delta_2 a},$$

(17)
further imposing that \( \delta_1 = 1 \), as this parameter is not separately identified from the baseline valuation \( z_l \) (this is just a normalization).

I estimate the vector of parameters \( \psi = \{ \lambda, \gamma_s, \gamma_{sd}, \alpha_{ds}, z_h, z_l, \delta_2, c_s, \theta_s, \theta_d \} \) using a minimum-distance estimator that matches key moments of the data with the corresponding moments of the model. More precisely, for any value of these parameters, I solve the model of Section 5 to find agents’ and dealers’ policy functions and agents’ and dealers’ distributions \( \{ \mu_{ho}(a), \mu_{lo}(a), \mu_{do}(a), \mu_{hn}, \mu_{ln}, \mu_{dn} \} \) that are consistent with each other. Based on the model’s solution, I calculate the vector \( m(\psi) \) composed by two sets moments.

A. For transactions, I use the following moments:

1. The fraction of aircraft for sale, which, in the model, equals:

\[
\frac{\int_0^{a_l} \mu_{lo}(a) \, da + \int_0^{a_{do}} \mu_{do}(a) \, da}{A}.
\]  

(18)

2. The fraction of aircraft for sale by dealers (i.e., aggregate dealers’ inventories), which equals:

\[
\frac{\int_0^{a_{do}} \mu_{do}(a) \, da}{A}.
\]  

(19)

3. The fraction of retail-to-retail transactions to total aircraft, which equals:

\[
\frac{\gamma_s \int_0^{a_l} \mu_{lo}(a) \, da}{A}.
\]  

(20)

4. The fraction of dealer-to-retail transactions to total aircraft, which equals:

\[
\frac{\alpha_{ds} \int_0^{a_{do}} \mu_{do}(a) \, da}{A}.
\]  

(21)

5. The average age of aircraft for sale, which equals:

\[
\frac{\int_0^{a_l} a \mu_{lo}(a) \, da + \int_0^{a_{do}} a \mu_{do}(a) \, da}{\int_0^{a_l} \mu_{lo}(a) \, da + \int_0^{a_{do}} \mu_{do}(a) \, da}.
\]  

(22)

B. For prices \( p(a) \) and \( p_B(a) \), I use moments obtained from the following indirect inference procedure. Using the aircraft prices data, I estimate via non-linear least squares the
coefficients \( \{\beta_0; \beta_1; \beta_2; \beta_3; \beta_4; \beta_5\} \) of the following auxiliary equations:

\[
\begin{align*}
p(a) &= \beta_0 + \beta_1 e^{-\beta_2 a}, \\
p_B(a) &= \beta_3 + \beta_4 e^{-\beta_5 a}.
\end{align*}
\] (23) (24)

I simulate the model and use the simulated prices to estimate the corresponding coefficients \( \{\beta_{0,sim}; \beta_{1,sim}; \beta_{2,sim}; \beta_{3,sim}; \beta_{4,sim}; \beta_{5,sim}\} \) of equations (23) and (24). I then construct the following six moments:

\[
\begin{align*}
\beta_{0,sim} + \beta_{1,sim} - \beta_0 - \beta_1, \\
\beta_{1,sim} - \beta_1, \\
\beta_{2,sim} - \beta_2, \\
\beta_{3,sim} + \beta_{4,sim} - \beta_3 - \beta_4, \\
\beta_{4,sim} - \beta_4, \\
\beta_{5,sim} - \beta_5.
\end{align*}
\]

The minimum-distance estimator chooses the parameter vector \( \psi \) that minimizes the criterion function

\[
(m(\psi) - m_S)' \Omega (m(\psi) - m_S),
\]

where \( m(\psi) \) is the vector of moments computed from the model evaluated at \( \psi \), and \( m_S \) is the vector of corresponding sample moments. \( \Omega \) is a symmetric, positive-definite weighting matrix. In practice, I follow Hansen (1982), who shows that the optimal (two-step) estimator uses for \( \Omega (\tilde{\psi}) \) a consistent estimate of the inverse of the asymptotic variance-covariance matrix of the moments, obtained using a preliminary consistent estimate \( \tilde{\psi} \) of the parameter vector \( \psi \). Since I am combining two different datasets with two independent sampling processes, \( \Omega (\cdot) \) is block-diagonal.

Although the model is highly nonlinear, so that (almost) all parameters affect all outcomes, the identification of some parameters relies on some key moments in the data. Specifically, equations (18)-(21), along with the differential equations (14)-(16) that define the functions \( \mu_{ho}(a), \mu_{lo}(a) \) and \( \mu_{do}(a) \), identify the parameters \( \lambda, \gamma_s, \gamma_{sd} \) and \( \alpha_{ds} \). Table 1 reports that the fraction of aircraft for sale is non-trivial, thus indicating that trading delays are relevant. Therefore, the model can fit this key feature of the data very well. Similarly,
Table 1 reports that dealers are faster than sellers at trading aircraft, and the model captures this difference through the trading parameters $\gamma_s$, $\gamma_{sd}$ and $\alpha_{ds}$.

From the price equations (23) and (24), we can then recover the values of the parameters $\delta_2, z_h, z_l$ and $c_s$: The price differences across vintages identify the parameter $\delta_2$, whereas the level of prices identifies the parameters $z_h, z_l, c_s$. The separate identification of $z_h, z_l, c_s$ relies on the assumptions that the valuations $z_h$ and $z_l$ interact with the age of the aircraft in the flow utility (17), whereas the search cost $c_s$ does not.

Moreover, the average age of assets on the market—i.e., equation (22)—and the price intercepts $\beta_0$ and $\beta_3$ in equations (23) and (24) allow me to identify the bargaining parameters $\theta_s$ and $\theta_d$. To understand this, consider the case in which buyers and dealers enjoy all the bargaining power. In this case, at the time of the transaction, sellers will be indifferent between selling the asset or not. However, when deciding whether or not to put the asset on the market, sellers anticipate that they have to pay the flow cost $c_s$ while searching for a counterparty. This cost is sunk at the time of the negotiation, but ex-ante, when deciding to put their assets on the market, sellers will be unwilling to do so since they will be held up to their reservation value in the negotiation. This argument rules out $\theta_s = 0$ and $\theta_d = 1$ simultaneously. As either $\theta_s$ increases or $\theta_d$ decreases, sellers will be more willing to put their assets on the market, but only if it makes sense from an ex-ante point of view. Since the gains from trade are larger for younger assets, sellers are more likely to sell older aircraft if they enjoy greater bargaining power. Thus, the average age of assets on the market is informative about $\theta_s$ and $\theta_d$. Furthermore, equation (11) shows that the Nash bargaining model implies that one key component of prices is buyers’ value function $S_{hn}$. This value does not vary with the age of the asset and depends critically on the bargaining parameters: It is negative if sellers and dealers enjoy all bargaining power—i.e., $\theta_s = \theta_d = 1$—and increases with buyers’ bargaining powers $1-\theta_s$ and $1-\theta_d$. This constant value $S_{hn}$ affects the intercepts $\beta_0$ and $\beta_3$ in equations (23) and (24), and, thus, these intercepts contribute to identifying the bargaining parameters.  

Finally, from the estimates of the parameters $\lambda, \gamma_s, \gamma_{sd}, \alpha_{ds}$, together with $A, \mu_d$ and the steady-state condition $\frac{d\mu}{dt} = \mu_{hn} + \int_0^T \mu_{ho}(a) \, da$, I can recover $\mu, \gamma$ and $\gamma'$. From dealers’

\footnote{To my knowledge, this identification of the bargaining parameters is novel in the literature. Specifically, a few recent papers in the labor-search literature estimate workers’ and firms’ bargaining parameters; see Calvani, Postel-Vinay and Robin (2006) and Flinn (2006). Those papers use additional information on labor demand or on firms’ production function, together with observed wages, to infer workers’ bargaining parameter. In the current setting, it would correspond to having additional information on aircraft demand that could be used to infer agents’ valuations $z_h$ and $z_l$. Instead, in the absence of this information, I identify the bargaining parameters exploiting the vertical heterogeneity of the assets.}
6.1.1 Estimates

Table 2 reports estimates of the parameters. The top part of the table reports the parameters that are estimated directly, and the bottom of the panel reports the parameters $\mu, \gamma, \gamma'$ and $k$ that are derived from those estimated directly. I calculate asymptotic standard errors for the parameters estimated directly and 95-percent confidence intervals obtained by bootstrapping the data using 100 replications for the parameters $\mu, \gamma, \gamma'$ and $k$.

The magnitude of the parameter $\lambda$ indicates that, on average, valuations switch from high to low approximately every five years. The magnitude of the sum $\gamma_s + \gamma_{sd}$ indicates that an aircraft stays on the market approximately eight months before a low-valuation owner is able to sell it. The magnitude of the parameter $\alpha_{ds}$ indicates that, on average, it takes slightly more than four months for a dealer to find a buyer. On average, each dealer trades one aircraft every $\frac{1}{\int_0^a \alpha_{ds}(a)da} + \frac{1}{\alpha_{ds}} \simeq 6$ quarters. These parameters $\gamma_s, \gamma_{sd}$ and $\alpha_{ds}$ imply that trading delays are non-trivial in this market and that dealers play an important role in reducing them: their pairwise-meeting rate $\gamma'$ is almost four times the traders’ direct meeting rate $\gamma$. 

Notes—This table reports the estimates of the parameters. Asymptotic standard errors, in parentheses. 95-percent confidence intervals in brackets are obtained by bootstrapping the data using 100 replications.

free-entry condition $J_{dn} = 0$, I can further recover $k$.
The parameter $\delta_2$ indicates that aircraft depreciate by approximately seven percent every year, a decline comparable to that of larger commercial aircraft. Indeed, business aircraft older than thirty years are common. The valuations $z_h$ and $z_l$ indicate that the difference between buyers’ and sellers’ valuations are large—i.e., gains from trade are large. The parameter $c_s$ indicates that the flow search costs are small, implying that the average search costs of completing a transaction $\frac{c_s}{\gamma_s + \gamma_{sd}}$ are trivial: approximately $5,000.

The bargaining parameters $\theta_s$ and $\theta_d$ imply that buyers and sellers split the surplus quite equally, with buyers capturing a slightly large share. Dealers, however, capture almost all the surplus of their transactions.

Dealers’ bid-ask spread, together with their trading rates and their free-entry condition $J_{dn} = 0$, determine dealers’ fixed flow cost $k$. Table 2 reports that dealers’ cost $k$ equals $244,767$ per quarter, indicating that the costs of “market-making” are large.

### 6.1.2 Model Fit

Before considering some broader implications of our results, I examine the fit of the estimated model. Table 3 contains the moments calculated from the data, as well as from the numerical solutions of the model. To appreciate how different weighting of the moments affects the estimation, I report the numerical values of the moments calculated from the model using both the first-step parameters (in column (4)) and the second-step parameters (in column (5)).

Overall, Table 3 reports that the model evaluated at the first-step parameters matches the data quite precisely, with an average absolute percentage deviation between the empirical moments and the simulated moments of 3.8 percent. The weighting of the moments in the second step pushes the estimation to match the price moments more precisely than the transaction moments, since they have a relatively lower variance. The absolute percentage deviation between actual data and simulated data increases as a consequence of the differential weighting.

To further appreciate how the model fits the price data in a perhaps more-intuitive way, Figure 1 displays the empirical average retail prices by vintage along with their theoretical counterparts $p(a)$, showing that the model fits them very well.
Table 3: Model Fit

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Empirical Moment</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Empirical Value</td>
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<tr>
<td>Theoretical Moment</td>
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<tr>
<td>Simulated Value</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>First Step</td>
<td>0.1223</td>
<td>$\int_{0}^{A} \mu_p(a) da + \int_{0}^{a} d_0 \mu_p(a) da$</td>
<td>0.1266</td>
<td>0.1324</td>
<td></td>
</tr>
<tr>
<td>Second Step</td>
<td>0.0357</td>
<td>$\int_{0}^{A} \mu_p(a) da$</td>
<td>0.0362</td>
<td>0.0264</td>
<td></td>
</tr>
<tr>
<td>$E(\text{Aircraft for Sale by Dealers})$</td>
<td>0.0196</td>
<td>$\int_{0}^{A} \mu_p(a) da$</td>
<td>0.0188</td>
<td>0.0193</td>
<td></td>
</tr>
<tr>
<td>$E(\text{Retail-to-Retail Transactions})$</td>
<td>0.0279</td>
<td>$\int_{0}^{A} \mu_p(a) da$</td>
<td>0.0268</td>
<td>0.0186</td>
<td></td>
</tr>
<tr>
<td>$E(\text{Average Age, Aircraft for Sale})$</td>
<td>57.2148</td>
<td>$\int_{0}^{A} \mu_p(a) da + \int_{0}^{a} d_0 \mu_p(a) da$</td>
<td>61.3937</td>
<td>76.0232</td>
<td></td>
</tr>
</tbody>
</table>

Notes—This table reports the values of the empirical moments and of the simulated moments calculated at the estimated parameters. Column (4) reports the simulated moments calculated using the first-step estimates (not reported), and column (5) reports the simulated moments calculated using the second-step estimates reported in Table 2. The coefficients $\beta_0$, $\beta_1$ and $\beta_2$ are obtained from a non-linear least squares regression using Retail Prices; the coefficients $\beta_3$, $\beta_4$ and $\beta_5$ are obtained from a non-linear least squares regression using Wholesale Prices; the coefficients $\beta_0, sim$, $\beta_1, sim$ and $\beta_2, sim$ are obtained from a non-linear least squares regression using simulated retail-to-retail prices $p(a)$; the coefficients $\beta_3, sim$, $\beta_4, sim$ and $\beta_5, sim$ are obtained from a non-linear least squares regression using simulated retail-to-dealer prices $p_B(a)$. 

$\beta_0 + \beta_1$  
$\beta_1$  
$\beta_2$  
$\beta_3 + \beta_4$  
$\beta_4$  
$\beta_5$
7 Counterfactual Analyses

I now perform two counterfactuals to answer the questions of how frictions and how dealers affect asset allocations, prices and welfare. In both cases, I compute the equilibrium in these counterfactual scenarios using the parameter estimates reported in Table 2.

7.1 Quantifying the Effects of Frictions

The estimates of the parameters allow me to quantify the effect of trading frictions on the allocation and the prices of assets, and on welfare. Specifically, I compare asset allocations, asset prices and welfare with the Walrasian efficient benchmark, which is a special case of the model presented in Section 5 when trade is frictionless—i.e., $\gamma \to +\infty$ and $c_s = 0$.

In the Walrasian market, as soon as high-valuation agents enter the market, they immediately meet low-valuation sellers or dealers. Hence, in a frictionless market, no low-valuation agents and no dealers have aircraft—i.e., $\mu_{lo}^w(a) = \mu_{do}^w(a) = 0$ for any $a$. Thus, the sum $\int_0^T (\mu_{lo}(a) + \mu_{do}(a)) \, da$ measures the total mass of assets misallocated due to search frictions. Of these misallocated assets, the mass $\int_0^{a_i^*} \mu_{lo}(a) \, da + \int_0^{a_i^*} \mu_{do}(a) \, da$ is on the market for sale, and the mass $\int_0^T (\mu_{lo}(a) + \mu_{do}(a)) \, da - \int_0^{a_i^*} \mu_{lo}(a) \, da - \int_0^{a_i^*} \mu_{do}(a) \, da = \int_{a_i^*}^T \mu_{lo}(a) \, da$ corresponds to the mass of aircraft older than $a_i^*$ that low-valuation owners are keeping rather
than selling, since sellers’ expected share of the total gains from trade are lower than their transaction costs. Overall, the allocative costs of the trading frictions, captured by the parameters $\gamma_s$, $\gamma_{sd}$ and $\alpha_{ds}$, depend on how frequently agents seek to trade, captured by the parameter $\lambda$.

Walrasian prices are equal to:

\[
p^w(a) = \int_a^T e^{-\rho(t-a)} z_h \delta_1 e^{-\delta_2 t} dt
\]

Instead, in the estimated model with frictions, prices are equal to $p(a) = (1 - \theta_s) (S_{lo}(a) - V_n) + \theta_s (U_{ho}(a) - S_{hn})$, as in equation (11).

The top parts of columns (1) and (2) in Table 4 report the calculations of these magnitudes for the model and the Walrasian benchmark, respectively. The first row reports that trading frictions cause the misallocation of 1,681 aircraft, corresponding to 18.3 percent of all aircraft. The third row reports that 1,215 aircraft are on the market for sale, whereas, for the remaining $1,681 - 1,215 = 466$ aircraft, the trading costs of selling them are larger than the sellers’ expected gains from trade; thus, these aircraft are not even on the market for sale. Table 4 further shows that trading frictions decrease the price of new aircraft by approximately $p(0) - p^w(0) = -19.26$ percent (approximately $4,201,000) relative to the Walrasian benchmark. The percent decrease of a ten-year-old aircraft is similar.

The bottom part of Table 4 calculates the welfare effects of trading frictions, decomposing them into their three components:

1. **The welfare loss due to misallocation.**

   This is equal to the difference between the utility flows in the estimated model and in the Walrasian market and corresponds to $\int_0^T (\mu_{ho}(a) z_h + \mu_{lo}(a) z_i) \delta_1 e^{-\delta_2 a} da - xT \int_0^T z_h \delta_1 e^{-\delta_2 a} da$. Using the estimated parameters, this welfare loss equals $317,220,000 per quarter, or 13.38 percent of the total potential welfare $\int_0^T z_h \delta_1 e^{-\delta_2 a} da$ of the Walrasian market.

2. **The welfare loss due to buyers’ and sellers’ search costs.**

   This is equal to the costs incurred by buyers and sellers while searching for a counterparty and corresponds to $c_s \left( \int_0^a \mu_{lo}(a) da + \mu_{hn} \right)$. Using the estimated parameters,
Table 4: Comparison with Walrasian Market

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimated Model</td>
<td>Walrasian Market</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$c_s = 0$</td>
<td>$c_s = 0$</td>
<td>$c_s = 0$</td>
</tr>
<tr>
<td>$f_0^T (\mu_{lo} (a) + \mu_{do} (a)) da$</td>
<td>1,681</td>
<td>0</td>
<td>1,698</td>
</tr>
<tr>
<td></td>
<td>[1,646; 2,399]</td>
<td>[0;0]</td>
<td>[1,683; 2,480]</td>
</tr>
<tr>
<td>$\int_a^T (\mu_{lo} (a) + \mu_{do} (a)) da$</td>
<td>0.183</td>
<td>0</td>
<td>0.183</td>
</tr>
<tr>
<td></td>
<td>[0.179;0.256]</td>
<td>[0;0]</td>
<td>[0.182;0.264]</td>
</tr>
<tr>
<td>$f_0^a \mu_{lo} (a) da + \int_a^\alpha \mu_{do} (a) da$</td>
<td>1,215</td>
<td>0</td>
<td>1,218</td>
</tr>
<tr>
<td></td>
<td>[1,202;1,755]</td>
<td>[0;0]</td>
<td>[1,152;1,742]</td>
</tr>
<tr>
<td>$\int_a^\alpha \mu_{lo} (a) da + \int_a^\alpha \mu_{do} (a) da$</td>
<td>0.132</td>
<td>0</td>
<td>0.132</td>
</tr>
<tr>
<td></td>
<td>[0.130;0.187]</td>
<td>[0;0]</td>
<td>[0.125;0.184]</td>
</tr>
<tr>
<td>$p (0)$</td>
<td>17,611,151</td>
<td>21,812,709</td>
<td>17,648,470</td>
</tr>
<tr>
<td></td>
<td>[17,253;17,969] * 10^3</td>
<td>[21,532;22,145] * 10^3</td>
<td>[17,249;18,097] * 10^3</td>
</tr>
<tr>
<td>$p (10)$</td>
<td>8,437,910</td>
<td>11,008,194</td>
<td>8,475,461</td>
</tr>
<tr>
<td></td>
<td>[8,366;8,531] * 10^3</td>
<td>[10,633;11,221] * 10^3</td>
<td>[8,148;8,672] * 10^3</td>
</tr>
</tbody>
</table>

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Allocations</td>
<td>2.054 * 10^9</td>
<td>2.371 * 10^9</td>
<td>2.057 * 10^9</td>
</tr>
<tr>
<td>Search Costs</td>
<td>6.534 * 10^6</td>
<td>6.534 * 10^6</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>[4.817;14.182] * 10^6</td>
<td>[0;0]</td>
<td>[0;0]</td>
</tr>
<tr>
<td>Dealers Costs</td>
<td>244 * 10^6</td>
<td>244 * 10^6</td>
<td>255 * 10^6</td>
</tr>
<tr>
<td></td>
<td>[232.8;303.1] * 10^6</td>
<td>[0;0]</td>
<td>[206.5;311.6] * 10^6</td>
</tr>
<tr>
<td>Welfare</td>
<td>1.802 * 10^9</td>
<td>2.371 * 10^9</td>
<td>1.801 * 10^9</td>
</tr>
</tbody>
</table>

Notes—This table reports counterfactual allocations and prices. 95-percent confidence intervals in brackets are obtained by bootstrapping the data using 100 replications.
this welfare loss equals $6,534,000 per quarter, or 0.28 percent of total potential welfare of the Walrasian market.

3. The welfare loss due to dealers’ costs.

This is equal to the costs incurred by dealers to provide intermediation and corresponds to \( k\mu_d \). Using the estimated parameters, this welfare loss equals $244,770,000 per quarter, or 10.32 percent of total potential welfare of the Walrasian market.

Hence, the aggregate welfare loss due to trading frictions equals $568,520,000 per quarter, or 23.98 percent of welfare in the Walrasian market.

Two forces affect allocations, prices and welfare in the estimated model relative to the Walrasian market: trading delays and search costs. We can understand the role of search costs and assess their relative contribution by computing the equilibrium when \( c_s = 0 \); column (3) in Table 4 reports these calculations. In general, the elimination of search costs has the direct effect that parties’ outside options in bargaining change, thereby affecting agents’ policies and the negotiated prices. In addition, this reduction in search costs has general-equilibrium effects by affecting dealers’ margins. Thus, dealers’ free-entry condition implies that their mass changes, as well. Since dealers’ mass determines agents’ buying and selling rates, the elimination of search costs affects equilibrium allocations, further affecting equilibrium prices. In the specific case reported in Table 4, dealers’ margins increase, thereby increasing their mass \( \mu_d \) and their aggregate inventories. Overall, column (3) says that the mass of misallocated aircraft increases and that asset prices increase when \( c_s = 0 \).

The bottom part of column (3) calculates welfare, also decomposing it into its different components. It shows that aggregate welfare decreases relative to the case in which search costs are positive, indicating that inefficiencies may increase when search costs vanish—i.e., in contrast to the findings of a large search literature. However, the magnitude of this decrease is small, as the search costs are small.

Figure 2 plots the estimated and counterfactual prices, confirming that actual prices (the solid line) are always lower than Walrasian prices (the dashed line) and prices without search costs (the dash-dotted line). Moreover, the figure displays an additional interesting pattern: Walrasian prices decline at a faster rate than actual prices. The reason is that the willingness to pay for a marginally younger—i.e., better—aircraft is higher if there are no frictions.

Overall, these counterfactuals imply non-trivial effects on allocations, prices and welfare, illustrating clearly the importance of frictions in this decentralized market.
7.2 Quantifying the Effects of Dealers

The estimates of the parameters allow me to quantify the effects of dealers on asset allocation, asset prices and welfare. Specifically, I compare asset allocations and asset prices with the counterfactual of no dealers—i.e., $\gamma' = 0$ or $k \to +\infty$—and then compare aggregate welfare and its components: allocations, search costs and dealers’ costs.

If dealers are not active, buyers’ and sellers’ meeting rates change, thus changing their distributions over the states. Specifically, the distributions of agents evolves according to:

\[
\begin{align*}
\dot{\mu}_{ho}(a) &= \gamma_b(a) \mu_{hn} - \lambda \mu_{ho}(a) \quad \text{for } a < a_{ho}^{**}, \\
\dot{\mu}_{lo}(a) &= \lambda_1 (a < a_{ho}^{**}) \mu_{ho}(a) - \gamma_s 1 (a < a_{l}^{**}) \mu_{lo}(a) \quad \text{for } a < T, \\
\dot{\mu}_{hn} &= (\mu - x) + \mu_{ho}(a_{ho}^{**}) - \lambda \mu_{hn} - \mu_{hn} \int_0^{a_{l}^{**}} \gamma_b(a) da,
\end{align*}
\]

with initial conditions $\mu_{ho}(0) = x$ and $\mu_{lo}(0) = 0$ and terminal conditions $\mu_{ho}(a) = 0$ for $a_{ho}^{**} < a < T$. Moreover, in each instant, the mass of $ln$-agents equals:

\[
\mu_{ln} = \lambda \mu_{hn} + \gamma_s \int_0^{a_{l}^{**}} \mu_{lo}(a) da + \mu_{lo}(T).
\]
Table 5: Comparison with a Market with No Dealers

<table>
<thead>
<tr>
<th></th>
<th>Estimated Model</th>
<th>No Dealer Market</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\int_0^T \mu_{lo}(a) , da + \int_0^{a^*<em>0} \mu</em>{do}(a) , da$</td>
<td>1,681</td>
</tr>
<tr>
<td></td>
<td>$\int_0^T \mu_{lo}(a) , da + \int_0^{a^*<em>0} \mu</em>{do}(a) , da$</td>
<td>[1,646;2,399]</td>
</tr>
<tr>
<td></td>
<td>$\int_0^{a^<em><em>l} \mu</em>{lo}(a) , da + \int_0^{a^</em><em>0} \mu</em>{do}(a) , da$</td>
<td>0.183</td>
</tr>
<tr>
<td></td>
<td>$\int_0^{a^<em><em>l} \mu</em>{lo}(a) , da + \int_0^{a^</em><em>0} \mu</em>{do}(a) , da$</td>
<td>[0.179;0.256]</td>
</tr>
<tr>
<td></td>
<td>$\int_0^{a^<em><em>l} \mu</em>{lo}(a) , da + \int_0^{a^</em><em>0} \mu</em>{do}(a) , da$</td>
<td>1.215</td>
</tr>
<tr>
<td></td>
<td>$\int_0^{a^<em><em>l} \mu</em>{lo}(a) , da + \int_0^{a^</em><em>0} \mu</em>{do}(a) , da$</td>
<td>[1,202;1,755]</td>
</tr>
<tr>
<td></td>
<td>$\int_0^{a^<em><em>l} \mu</em>{lo}(a) , da + \int_0^{a^</em><em>0} \mu</em>{do}(a) , da$</td>
<td>0.132</td>
</tr>
<tr>
<td></td>
<td>$\int_0^{a^<em><em>l} \mu</em>{lo}(a) , da + \int_0^{a^</em><em>0} \mu</em>{do}(a) , da$</td>
<td>[0.130;0.187]</td>
</tr>
<tr>
<td></td>
<td>$\int_0^{a^<em><em>l} \mu</em>{lo}(a) , da + \int_0^{a^</em><em>0} \mu</em>{do}(a) , da$</td>
<td>17,611,151</td>
</tr>
<tr>
<td></td>
<td>$\int_0^{a^<em><em>l} \mu</em>{lo}(a) , da + \int_0^{a^</em><em>0} \mu</em>{do}(a) , da$</td>
<td>[17,253;17,969] * 10^3</td>
</tr>
<tr>
<td></td>
<td>$\int_0^{a^<em><em>l} \mu</em>{lo}(a) , da + \int_0^{a^</em><em>0} \mu</em>{do}(a) , da$</td>
<td>8,437,910</td>
</tr>
<tr>
<td></td>
<td>$\int_0^{a^<em><em>l} \mu</em>{lo}(a) , da + \int_0^{a^</em><em>0} \mu</em>{do}(a) , da$</td>
<td>[8,366;8,531] * 10^3</td>
</tr>
<tr>
<td>Allocations</td>
<td>2.054 * 10^9</td>
<td>1.984 * 10^9</td>
</tr>
<tr>
<td></td>
<td>[1.942;2.060] * 10^9</td>
<td>[1.859;1.990] * 10^9</td>
</tr>
<tr>
<td>Search Costs</td>
<td>6.534 * 10^6</td>
<td>7.817 * 10^6</td>
</tr>
<tr>
<td>Dealers Costs</td>
<td>244 * 10^6</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>[232.8;303.1] * 10^6</td>
<td>[0;0]</td>
</tr>
<tr>
<td>Welfare</td>
<td>1.802 * 10^9</td>
<td>1.977 * 10^9</td>
</tr>
</tbody>
</table>

Notes—This table reports counterfactual allocations and prices in a market without dealers. 95-percent confidence intervals in brackets are obtained by bootstrapping the data using 100 replications.

The cutoff ages $a_{ho}^{**}$ and $a_{l}^{**}$ may differ from $a_{ho}^{*}$ and $a_{l}^{*}$ defined in Section 5.2.5, as agents’ value functions and, thus, policies depend on whether or not dealers are active. Moreover, the counterfactual steady-state distribution of agents have to be consistent with the counterfactual trading probabilities $\gamma_b(a)$ and $\gamma_s$, which are also key components of the counterfactual prices.\(^{14}\)

Table 5 reports the calculations of these magnitudes for the model and the counterfactual market with no dealers, and Figure 3 plots the prices. The table and the figure show interesting features. The first row of the table reports that the number of misallocated

\(^{14}\)Thus, the counterfactual uses the following parameters obtained from the estimated model: the valuation switching rate $\lambda$; the bilateral meeting rate $\gamma$; the search cost $c_s$; the valuations $z_2$ and $z_1$; depreciation rate $\delta_2$; and the mass of entrants $\mu$. It obtains the counterfactual equilibrium: policies $a_{ho}^{**}$, $a_{ho}^{**}$, $a_{l}^{**}$; masses $\mu_{ho}(a)$, $\mu_{lo}(a)$, $\mu_{hn}$, $\mu_{ln}$; trading probabilities $\gamma_b(a)$ and $\gamma_s$; and retail prices $p^{nd}(a)$. Hence, the counterfactual equilibrium allows agents to reoptimize their choices.
assets increases to 2,013 units, or 20.6 percent of all aircraft. This 2.3-percentage-point increase in misallocated assets corresponds to a 12.5-percent increase in misallocated assets relative to the estimated model. The third row reports that 2,000 of these 2,013 misallocated aircraft would be on the market for sale in a market without dealers. This mass is larger than in the estimated model for two reasons: 1) the longer trading delays increase the mass of assets on the market for a given volume of trade, and 2) low-valuation owners are more likely to sell their older assets—i.e., their cutoff increases: \( a_t^{**} > a_t^* \)—since they capture a greater share of the surplus: sellers’ bargaining power is now always equal to \( \theta_s = 0.3756 \), whereas it was equal to \( 1 - \theta_d = 1 - 0.9732 = 0.0268 \) when they sold to a dealer. This second effect dominates the opposing effect that total search costs are greater because delays are longer, since the flow cost \( c_s \) is small.

We can decompose the welfare difference between the estimated model and the counterfactual market with no dealers into these three components:

1. The difference in misallocation.

   This is equal to the difference between the utility flows in the estimated model and in the counterfactual market without dealers. Using the estimated parameters, dealers improve the allocation, and this welfare gain equals $69,261,000 per quarter, corre-

---

**Fig. 3:** Actual (solid line) and counterfactual (dashed line) aircraft prices if there were no dealers as a function of aircraft age.
sponding to 3.84 percent of aggregate welfare of the estimated model.

2. The difference in buyers’ and sellers’ search costs.

This is equal to the costs incurred by buyers and sellers while searching for a counterparty, in the estimated model and in the counterfactual market without dealers. Without dealers, traders search longer, and this welfare loss equals $1,282,600 per quarter, corresponding to 0.07 percent of aggregate welfare of the estimated model.

3. The difference in dealers’ costs.

This is equal to the costs incurred by dealers to provide intermediation, and this corresponds to \( k\mu_d \). Using the estimated parameters, this welfare loss equals $244,443,693 per quarter, corresponding to 13.57 percent of aggregate welfare of the estimated model.

Thus, the net effect is a welfare loss of $174,220,000 per quarter, or 9.66 percent of aggregate welfare of the estimated model. The intuition for this result is that, while dealers improve the allocation of assets, their operations are costly. Moreover, each dealer imposes a negative externality by lowering other agents’ meeting rates, and each appropriates surplus that would be created by other agents (von Weizsaker, 1980; Mankiw and Whinston, 1986). Indeed, numerical solutions indicate that aggregate surplus is monotonically declining in the number of dealers: The optimal number of dealers is zero.

Table 5 further shows that prices of new aircraft increase by approximately $500,000, a 2.88-percent increase. Thus, the effect on allocations is opposite than the effect on prices. The reason is the following: Without dealers, the volume of trade is lower. Therefore, at any point in time, the mass of high-valuation agents seeking to buy an aircraft is larger—i.e., \( \mu_{hn}^{nd} > \mu_{hn} \). Hence, it is easier for sellers to find a high-valuation buyer—i.e., \( \gamma_{ps}^{nd} > \gamma_s \)—thereby increasing their value function \( V_{lo}(a) \) and their outside option in bargaining. This effect dominates the opposite effects of higher search costs and slower trade on asset prices.

Overall, this counterfactual analysis highlights that it may be difficult to infer from changes in asset allocations and in asset prices the direction and the magnitude of the change in aggregate welfare.

8 Concluding Remarks

This paper provides a framework to empirically analyze decentralized asset markets. I apply this framework to investigate the effects of trading frictions and of intermediaries on
asset allocations, asset prices and welfare in the business-aircraft market. The empirical application makes clear the data requirements to estimate such a model and how the parameters are identified. The model fits the data well, and the quantitative analyses deliver a sense of the magnitudes involved, allowing me to assess which forces dominate and their welfare effects. Most notably, the estimated model implies that trading frictions generate moderate allocative inefficiencies in such markets: Compared to the Walrasian benchmark, 18.3 percent of all business aircraft are misallocated, and aircraft prices are 19.2-percent lower. Moreover, dealers play an important role in reducing frictions: In a market with no dealers, 20.6 percent of the assets would be misallocated, and prices would increase by 2.88 percent. However, the net effect of dealers is a welfare loss equal to 9.66 percent of aggregate welfare of the estimated model, because dealers’ operations are costly, and they impose a negative externality by decreasing traders’ direct transactions.

At the same time, the model has some limitations. First, the supply $x$ of new assets is exogenous. This modeling assumption is common in many other models that focus on secondary markets and on trading frictions. A second limitation is that information on buyers is unavailable: The mass of new buyers $\mu$ is exogenous, and their meeting rates are derived from the model. Hence, the identification of trading frictions relies exclusively on the sellers’ side of the market. This limitation is common to many equilibrium search models, and part of the point of this paper is to show that, under the same assumptions, it is possible to quantify the effect of trading delays on allocations, prices and welfare. A third limitation is that there are no aggregate shocks. In reality, the business-aircraft market, like most other asset markets, exhibits fluctuations in prices and trading volume, corresponding to fluctuations in supply and demand. The key advantage of a stationary framework is that the numerical solution of the model is quite fast to compute, thereby facilitating the estimation and the counterfactual analyses. Nonetheless, a potential concern is how sensitive the estimation results are to this stationarity assumption. The fact that the estimates of the parameters are quite precise (the standard errors are small; see Section 6.1.1) may suggest that aggregate shocks do not greatly affect the empirical moments and, thus, the results. Fourth, an additional limitation, quite common among search models, is that price is the only characteristic over which parties bargain, and no other characteristics, such as financing, are taken into consideration.

Finally, in my view, perhaps the main limitation is that the model allows for limited heterogeneity in agents’ valuations and dealers’ inventories. As explained in Section 4, this limitation stems from the aggregate nature of the data, which makes a model with richer
heterogeneity difficult to identify. One important consequence of agents’ limited heterogeneity is that the marginal buyer and the marginal seller in the estimated model and in the Walrasian market are the same. Instead, if agents’ heterogeneity is richer, the marginal buyers and the marginal sellers differ in a model with or without frictions. More precisely, there exists a unique buyers’ threshold in the valuation distribution such that an agent that does not currently own an aircraft and whose valuation jumps above the threshold chooses to acquire one. Similarly, there exists a unique sellers’ threshold valuation such that an agent that currently owns an aircraft and whose valuation falls below the threshold chooses to sell it. When there are trading frictions, buyers’ threshold is higher than sellers’ since frictions create a wedge that prevents sellers from selling and buyers from buying. Instead, in a frictionless Walrasian market, these thresholds converge, and the marginal buyer and the marginal seller coincide. This argument suggests that the mass of misallocated young assets may be larger than the estimated one, since the latter does not take into account the misallocation of assets owned by agents between the thresholds who are not searching to sell or buy an aircraft, but would do so in a frictionless market. However, this argument also suggests that the mass of misallocated old assets may be smaller than the estimated one, since the latter does not take into account the transactions of assets between agents with more extremal valuations than the model’s binary valuations. The net effect of these two approximations on the total mass of misallocated assets and on aggregate welfare is ambiguous. Moreover, the previous argument implies that Walrasian prices may be lower than the counterfactual values calculated in Section 7.1 since the marginal owner in a Walrasian market has a lower valuation than in the counterfactual market of Section 7.1.
APPENDICES

A Motives for Trade

The goal of this Appendix is to document patterns of trade in the data. These patterns are inconsistent with the idea that most secondary-market transactions are due to replacements of depreciated units with higher-quality ones. Instead, they favor the idea that the main motive for trade in the secondary market is the change in owners’ valuations of the assets.

More specifically, Hendel and Lizzeri (1999), Porter and Sattler (1999) and Stolyarov (2002) develop models in which agents with heterogeneous willingness-to-pay for vertically-differentiated vintages wish to replace their durables after they depreciate, but transaction costs are an impediment to instantaneous trade. In these models, a faster rate of depreciation increases the vertical differentiation between vintages, thereby leading to more-frequent replacements. Thus, these theoretical arguments lead to the following implications:

1. Assets with faster depreciation should have higher resale rates.

To investigate whether the data are consistent with this prediction, I proceed in two steps: 1) I obtain an annual Depreciation Rate of each aircraft-model $i$ in year $t$ in the data by fitting the following regression on the different vintages $a$ within model $i$ in year $t$:

$$\log p(a) = \varphi_{0it} + \varphi_{1it} a,$$

where $p(a)$ is the Retail Price of the vintage-$a$ aircraft. The absolute value of $\varphi_{1it}$ (note that it is negative) is the estimate of the Depreciation Rate.

2) I construct the variable Volume of Trade of aircraft-model $i$ and year $t$ as the ratio of the Total Number of Transactions to the Active Aircraft and investigate whether it is correlated with the Depreciation Rate.

Table 6 reports the results of several specifications. Columns (1)-(4) report coefficients of OLS regressions that pool all model-year pairs; columns (5)-(8) report coefficients of random-effects panel regressions; and columns (9)-(12) report coefficients of fixed-effects panel regressions. The Depreciation Rate has no significant effect on the
Table 6: The Effect of Depreciation on Volume of Trade

<table>
<thead>
<tr>
<th>Depreciation Rate</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
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<td></td>
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<td>.1532</td>
<td>.0659</td>
<td>.1733</td>
<td>-.2453</td>
<td>-.0058</td>
<td>-.1800</td>
<td>.0243</td>
<td>-.2867</td>
<td>-.0843</td>
<td>-.2043</td>
<td>-.0173</td>
</tr>
<tr>
<td></td>
<td>(.1758)</td>
<td>(.1887)</td>
<td>(.1784)</td>
<td>(.1965)</td>
<td>(.1420)</td>
<td>(.1458)</td>
<td>(.1310)</td>
<td>(.1462)</td>
<td>(.1534)</td>
<td>(.1372)</td>
<td>(.1380)</td>
<td>(.1324)</td>
</tr>
<tr>
<td>Log(Active Aircraft)</td>
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<td>.0094</td>
<td>.0274</td>
<td>.0206</td>
<td>.0355</td>
<td>.0356</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.0044)</td>
<td>(.0044)</td>
<td>(.0038)</td>
<td>(.0046)</td>
<td>(.0044)</td>
<td>(.0051)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
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<td>861</td>
<td>861</td>
<td>861</td>
<td>861</td>
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</tbody>
</table>

Notes—Standard errors in parentheses are clustered at the model level.
Volume of Trade in any of the specifications (1)-(12), suggesting that replacement is not the main motive for trade.\footnote{Interestingly, specifications (3), (4), (7), (8), (11) and (12) indicate that the “thickness” of the market, proxied by the variable $\text{Log(Active Aircraft)}$, is correlated with the Volume of Trade; this is consistent with the model of Section 5 with increasing returns in the meetings between buyers and sellers, as well as with evidence from other capital assets (Gavazza, 2011b).}

2. Assets with faster depreciation should trade earlier “in their life.”

To investigate whether the data are consistent with this prediction, I investigate whether the Average Age, Aircraft for Sale of aircraft-model $i$ in year $t$ is correlated with the Depreciation Rate.

Table 7 reports the results of several specifications.\footnote{The number of observations in Table 7 is lower than those in Table 6 because Average Age, Aircraft for Sale is missing in years in which there are no aircraft for sale.} Columns (1)-(4) report coefficients of OLS regressions that pool all model-year pairs; columns (5)-(8) report coefficients of random-effects panel regressions; and columns (9)-(12) report coefficients of fixed-effects panel regressions. Overall, specifications (1)-(12) indicate that the Depreciation Rate has no effect on the Average Age, Aircraft for Sale, reinforcing the idea that replacement is not the main motive for trade in the business-aircraft market.\footnote{The negative coefficient of $\text{Log(Active Aircraft)}$ in the more-saturated specifications (8) and (12) is due to the fact that out-of-production aircraft are progressively scrapped as they age, thereby simultaneously decreasing Active Aircraft and increasing the average age of the stock of aircraft.}

3. The fraction of aircraft sold should be lowest for the youngest vintages.

The reason is that, because of transaction costs, few households sell their assets one year after purchasing them.

The Aircraft Transaction database reports the Average Age, Aircraft for Sale, and I use it to investigate this prediction. However, the data are not ideally suited to investigate it because, as described in Section 4, they are aggregated at the model-quarter level. Hence, this aggregation could mask some important age heterogeneity between different aircraft for sale within each model-quarter pair. Nonetheless, I provide suggestive evidence by restricting the analysis to model-quarter pairs for which Aircraft for Sale equals one: For this restricted sample, there is obviously no age heterogeneity within a model-year pair. Figure 4 plots the histogram of the age of aircraft for sale of this restricted sample, showing that the youngest vintages are
Table 7: The Effect of Depreciation on the Age of Aircraft for Sale

<table>
<thead>
<tr>
<th>Depreciation Rate</th>
<th>(1)</th>
<th>(2)</th>
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<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
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<th>(10)</th>
<th>(11)</th>
<th>(12)</th>
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<tr>
<td></td>
<td>38.445</td>
<td>49.005</td>
<td>40.098</td>
<td>51.949</td>
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<td>−37.154</td>
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<td>Log(Active Aircraft)</td>
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<td>−1.390</td>
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<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>(1.124)</td>
<td>(1.139)</td>
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<td>(.235)</td>
<td>(.529)</td>
<td>(.247)</td>
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</tbody>
</table>

Notes—Standard errors in parentheses are clustered at the model level.
frequently for sale. This pattern contradicts the evidence from other asset markets in which replacement purchases are the main motives for trade, such as the car market, in which the fraction of cars sold is lowest for the youngest vintages (Porter and Sattler, 1999; Stolyarov, 2002). Thus, while the sample restriction recommends some caution in putting too much weight on this evidence, Figure 4 further corroborates that replacement is not the main motive for trade.

**Fig. 4:** Histogram of the age of aircraft for sale.
B The case $\delta_2 = 0$

When assets are not depreciating—i.e., $\delta_2 = 0$—we can obtain the analytical solution of the model (Duffie, Gârleanu and Pedersen, 2005 and 2007). Since $\delta_2$ is estimated to be small, this restricted version of the model provides a reasonable approximation to the more-general model, thus providing a clean intuition for the identification of the parameters.

Since assets are not depreciating, we can assume that the mass of assets is fixed and equal to $A$; thus, $x = 0$ and $T \to \infty$. Moreover, without loss of generality, we can set $\delta_1 = 0$ and $\delta_0 = 1$.

The steady-state evolution of the masses $\mu_i$ determines the equilibrium allocation. Up to terms in $o(\epsilon)$, in equilibrium, these masses evolve from time $t$ to time $t + \epsilon$ according to:

$$
\begin{align*}
\mu_{lo}(t + \epsilon) &= \lambda \epsilon \mu_{ho}(t) + (1 - \gamma_s \epsilon - \gamma_{sd} \epsilon) \mu_{lo}(t), \\
\mu_{ho}(t + \epsilon) &= \gamma_b \epsilon \mu_{hn}(t) + \gamma_{bd} \epsilon \mu_{ln}(t) + (1 - \lambda \epsilon) \mu_{ho}(t), \\
\mu_{hn}(t + \epsilon) &= \epsilon \mu + (1 - \lambda \epsilon - \gamma_b \epsilon - \gamma_{bd} \epsilon) \mu_{hn}(t), \\
\mu_{ln}(t + \epsilon) &= \lambda \epsilon \mu_{hn}(t) + \gamma_s \epsilon \mu_{to}(t) + \gamma_{sd} \epsilon \mu_{lo}(t), \\
\mu_{do}(t + \epsilon) &= \alpha_{ds} \epsilon \mu_{dn}(t) + (1 - \alpha_{ds} \epsilon) \mu_{do}(t), \\
\mu_{dn}(t + \epsilon) &= \alpha_{ds} \epsilon \mu_{do}(t) + (1 - \alpha_{db} \epsilon) \mu_{dn}(t).
\end{align*}
$$

The intuition for these equations is similar to the intuition reported in Section 5.2.5. For example, the first equation states that the mass of low-valuation agents with an asset results from the flows of three sets of agents: 1) the inflow of high-valuation owners whose valuation just dropped—the term $\lambda \epsilon \mu_{ho}(t)$; 2) the outflow of low-valuation owners who found a dealer—the term $\gamma_{sd} \epsilon \mu_{lo}(t)$; and 3) the mass of low-valuation owners who have not found a buyer—the term $(1 - \gamma_s \epsilon) \mu_{lo}(t)$.

Steady state implies that the total mass of agents with high valuation $\mu_{hn} + \mu_{ho}$ is equal to $\frac{A}{\lambda}$; that the total mass of owners $\mu_{ho} + \mu_{lo} + \mu_{do}$ is equal to the mass of assets $A$; and that the total mass of dealers $\mu_d$ is equal to $\mu_{do} + \mu_{dn}$. Hence, $\mu_{hn} + A - \mu_{to} - \mu_{do} = \frac{A}{\lambda}$. In turn, since $A < \frac{A}{\lambda}$, this implies that $\mu_{hn} > \mu_{to} + \mu_{do}$—i.e., sellers are the “short” side of the market. Moreover, the masses of assets sold and purchased by dealers must be equal to the mass of assets purchased and sold to dealers: $\alpha_{ds} \mu_{do} = \gamma_{bd} \mu_{hn}$ and $\alpha_{db} \mu_{dn} = \gamma_{sd} \mu_{lo}$. Furthermore, steady state requires $\alpha_{ds} \mu_{do} = \gamma_{bd} \mu_{hn}$ and $\alpha_{db} \mu_{dn} = \gamma_{sd} \mu_{lo}$ since dealers’ aggregate inventories do not change over time.
Rearranging and taking the limit for $\epsilon \to 0$, the masses of agents are:

\[
\begin{align*}
\mu_{lo} &= \frac{\lambda}{\gamma_s + \gamma_{sd}}, \\
\mu_{ho} &= \frac{\gamma_b \mu_{hn} + \gamma_{bd} \mu_{hn}}{\lambda}, \\
\mu_{hn} &= \frac{\mu}{\lambda + \gamma_b + \gamma_{bd}}, \\
\mu_{ln} &= \lambda \mu_{hn} + \gamma_s \mu_{lo} + \gamma_{sd} \mu_{lo} = \mu, \\
\mu_{do} &= \frac{\alpha_{db}}{\alpha_{ds}} \mu_{dn} = \frac{\gamma_{sd}}{\alpha_{ds}} \mu_{lo}.
\end{align*}
\]

We can now calculate the moments (18)-(21) that we seek to match to their empirical analogs in the quantitative analysis. Specifically, the fraction of aircraft for sale is equal to:

\[
\frac{\mu_{lo} + \mu_{do}}{A} = \frac{\lambda (\alpha_{ds} + \gamma_{sd})}{\lambda \alpha_{ds} + \lambda \gamma_{sd} + \gamma_s \alpha_{ds} + \alpha_{ds} \gamma_{sd}}.
\]

The fraction of aircraft for sale by dealers (or dealers’ inventories) is equal to:

\[
\frac{\mu_{do}}{A} = \frac{\lambda \gamma_{sd}}{\lambda \alpha_{ds} + \lambda \gamma_{sd} + \gamma_s \alpha_{ds} + \alpha_{ds} \gamma_{sd}}.
\]

The fraction of retail-to-retail transactions to total aircraft is equal to:

\[
\frac{\gamma_s \mu_{lo}}{A} = \frac{\lambda \gamma_s \alpha_{ds}}{\lambda \alpha_{ds} + \lambda \gamma_{sd} + \gamma_s \alpha_{ds} + \alpha_{ds} \gamma_{sd}}.
\]

The fraction of dealer-to-retail transactions to total aircraft is equal to:

\[
\frac{\alpha_{ds} \mu_{do}}{A} = \frac{\lambda \gamma_{sd} \alpha_{ds}}{\lambda \alpha_{ds} + \lambda \gamma_{sd} + \gamma_s \alpha_{ds} + \alpha_{ds} \gamma_{sd}}.
\]

The equilibrium prices are determined by the solution to the following system of equa-
The first four equations are the value functions of agents; the fifth and the sixth equations are the value functions of dealers; and the last three equations are the negotiated prices.
References


