Optimal Unemployment Insurance with Unobservable Savings*

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Abstract

This paper studies the optimal design of an unemployment insurance system in a repeated moral hazard environment where individual's can save and these savings are private information. This additional source of informational assymetry requires special treatment, here I apply the method developed in Werning (2000). Contrary to existing results in the literature I find that optimal unemployment benefits are not necessarily decreasing with unemployment duration. More importantly, numerical results show, however, that the optimal schedule is increasing but extremely flat, so that constant benefits may provide an excellent approximation to the optimal UI schedule.

1 Introduction

Unemployment insurance (UI) programs seek to insure workers against the income fluctuations due to uncertain job losses and uncertain unemployment duration. However, the insurance provided by these programs affects the incentives to find work, leading to higher unemployment and other efficiency costs. An important policy question is how to design UI programs that optimally trade-off the desired insurance benefits against the unavoidable

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efficiency costs. This paper is concerned with one aspect of this optimal trade-off and policy design: the optimal timing of benefits.

UI programs differ across countries in various ways but they all share one simple feature: benefits decrease with the duration of the unemployment spell. In most countries this is true because workers are entitled to a constant benefit for a certaint duration, after which they become ineligible for further benefits[1]. The decreasing pattern of benefits is widely believed to be a desirable property of these programs, promoting efficiency at a small loss in insurance. Previous formal results in the literature on optimal unemployment insurance have helped justify this view.

There are two important branches in the literature on optimal UI. The first branch of the literature takes a contract theory approach to address the optimal timing of benefits in repeated moral-hazard environments. The seminal work of Shavell and Weiss (1979) and the important extension by Hopenhayn and Nicolini (1997) are leading examples of this approach[2]. For tractability these papers assume that workers cannot save nor borrow so that consumption during an unemployment spell equals UI benefits[3]. They show that consumption while unemployed, and thus UI benefits, should decrease with unemployment duration.

The second branch of the literature takes on a more quantitative general equilibrium approach. For example, Hansen and İmrohoroğlu (1992), Alvarez and Veracierto (1999) and Wang and Williamson (1996) study UI policies in models that stress the role of worker's asset accumulation decisions as self-insurance. However, these papers do not solve for the unrestricted optimal unemployment contract. Instead, the focus is on the welfare properties of current policies or optimization of policy within a restricted class, i.e. constant replacement ratios. As a consequence, they do not address the optimal timing of UI benefits.

[1]For example, in the US UI benefits pay about 50-60% of previously earned wages for 26 to 39 weeks depending on the course of the economy (eligibility is often extended during recessions). Thus, the schedule is discontinuous as well as decreasing. This paper is not concerned with the discontinuity of the benefit schedule.

[2]Shavell and Weiss (1979) impose the additional restriction that consumption for employed workers equal labor earnings. Hopenhayn and Nicolini (1997) relax this assumption and interpret this as allowing for a variable employment tax that depends on the length of the previous unemployment spell. Other work includes Atkeson and Lucas (1995) and Zhao (1999).

[3]Shavell and Weiss (1979) contain a section that relaxes this assumption. However, in this section the essential the moral hazard problem is removed.

This paper attempts to bridge the gap between these literatures by combining aspects of both approaches. I study the optimal timing of UI benefits in a repeated moral-hazard model where workers can accumulate assets, by saving, and perhaps borrowing. In addition to the level of search effort I consider an additional source of informational asymmetry: asset accumulation decisions and consumption are the workers's own private information, they cannot be monitored by the government.

Making progress on these problems requires incorporating the large set of incentive constraints in a tractable way[4][5]. Here, I extend the recursive techniques used by Hopenhayn and Nicolini (1997) drawing on the general framework and results described in more detail in Werning (2000). I apply these ideas directly, motivating them only briefly, the reader is referred to that paper for details.

In this paper I consider two policy instruments available to the unemployment insurance agency. First, the agency must select a benefit schedule specifying the transfers to be received by the unemployed worker as a function of unemployment duration. Second, for the case where the worker can save but not borrow, we consider an additional policy instrument: the agency decides whether to implement a 'lending-scheme' which effectively removes the unemployed worker's borrow constraint[6]. Indeed, for the cases we study, optimal policy always implements the lending-scheme, removing the borrowing constraints faced by workers is optimal[7].

Including both instruments disentangles two distinct functions that an unemployment insurance program may play: insurance and liquidity. Previous work on the optimal timing of benefits necessarily entangled both aspects. Interestingly, lending-schemes are closely related to some recent policy proposals. For example, Feldstein and Altman's (1998) proposal for individual worker accounts can be viewed as lending-schemes coupled with some insur-

[4]Doepke and Townsend.

[5]Fudenberg, Holmstrom and Milgrom (1990) are an exception. They study a repeated moral hazard model where the agent can save and borrow. Their emphasis is, however, on the ex-post efficiency of optimal contracts. However, they do characterize the optimal contract for the case of exponential CARA utility and i.i.d. income. This result is related to a result obtained in this paper and will be discussed in more detail below.

[6]Of course, we must rule out Ponzi schemes.

[7]In repeated agency problems where the agent cannot save nor borrow the optimal allocation is such that the worker appears to be "savings-constrained" [Rogerson (1985)] suggesting why we find that it is not optimal here to exploit the agent's borrowing constraint.

ance features[8].

Without borrowing constraints there are many transfer schedules that implement the optimal allocation. In describing optimal policies I focus on the implementation that does not require taxes on labor earnings to depend on the length of the worker's previous unemployment spell. This normalization pins down the whole transfer schedule; my results regarding 'the' optimal benefit schedule focus on this particular normalization[9][10].

The results show that the implications for the timing of benefits change dramatically relative to the previous literature: optimal UI benefits are not necessarily decreasing with duration. Indeed, I find that optimal UI benefit schedules are typically increasing. On the other hand, a decreasing consumption schedule is a robust feature of the optimal allocation.

To understand these results note that when workers can save and borrow, UI benefits need not be decreasing for consumption to be decreasing. Indeed, permanent income reasoning suggests that consumption will fall over the course of an unemployment spell as the worker works down his assets[11]. Thus, there is no straightforward link between the consumption profile and the benefit profile. This suggests that the optimal timing of benefits depends on how the trade-off between insurance and incentives shifts with duration. In our stationary environment the trade-off shifts only due to wealth effects, and these typically play in a particular direction.

The optimal contract provides incentives by punishing workers who remain unemployed, reducing their consumption and their remaining lifetime utility. Thus, decreasing absolute risk-aversion preferences imply that insurance of absolute risks becomes more important with duration. Furthermore, if leisure is a normal good incentives will be easier to provide for any given effort level. Consequently, unless the optimal contract requires effort to rise sharply, insurance will increase with unemployment duration.

Numerical results show that the optimal UI benefit schedule is typically

[8]Enric Fernandes (2000) examines a general equilibrium life-cycle model to study the relative merits of two policies: unemployment insurance benfits vs. loosening the borrowing constraint to workers.

[9]This normalization is usually taken for granted in policy discussions regarding the UI benefit schedule.

[10]In Hopenhayn and Nicolini (1997) duration dependent labor taxes are important because in their model the worker cannot save nor borrow. Thus, restricting labor taxes to be constant restricts the set of attainable allocations. In contrast, when saving and borrowing are allowed, as they are here, it does not restrict the attainable allocations.

[11]This is true for any benefit schedule strictly below the wage.

increasing. Perhaps more importantly, the schedules are always extremely flat. This last result suggests that a constant benefit schedule may provide an excellent approximation to the unrestricted optimal UI schedule. We show that this is indeed the case.

Before describing the model in more detail one aspect of the solution method of Werning (2000) applied in this paper is worth mentioning here. The solution method make use of the first-order approach: the agent's first order necessary conditions replace the true incentive constraints. The advantage of this substitution is that the problem can then be stated recursively using a minimum of state variables. However, it is well know that the first-order approach may fail to uncover an optimal incentive compatible allocation [Mirrlees (1999) and Rogerson(1985)]. To ensure that the first-order apporach is warranted one should verify the allocation satisfies the true incentive compatibility constraints of the original problem. Fortunately, for the cases studied in this paper the first-order approach does identify incentive compatible allocations.[12]

The rest of the paper is organized as follows. Section 2 lays out the model, describes the first-best allocation and the problem faced by the agent. Section 3 states the second-best problem and presents a recursive representation of it. Section 4 uses this recursive representation to study two cases that can be solved analytically. Section 5 then presents numerical results for other cases. Section 6 concludes.

2 The Model

Consider the situation of a currently unemployed worker searching for a job. Our timing assumes that if a job found in period t then the agent is employed from t onwards – work commences immediately and jobs are permanent. Assuming employment is permanent simplifies the analysis and allowscomparisons with the previous literature which also adopts this assumption [e.g. Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997)]. Once em-

[12]Note, that for the first-order approach to fail it must be the case that the optimal response of agents are discontinuous with respect to some policy variable [see Mirrlees (1999)]. Such discontinuous responses are perhaps unlikely to characterize actual behavior. Thus restricting attention to the class of preferences where the first-order approach is valid may be of special interest. No doubt, understanding the optimal allocations for cases where the first-order approach would fail would highly complement this work.

ployed the worker produces w units of the consumption good each period.

The agent's preferences are represented by the expected discounted utility function,

$$E\left[\sum_{t=0}^{\infty}\beta^{t}u\left(c_{t},p_{t}\right)\right] \tag{1}$$

where c_t and p_t denote consumption and effort, respectively. I normalize effort so that it represents the probability of finding a job while unemployed: if the worker exerts effort p_t then with probability p_t he becomes employed at that time. To capture the disutility of work I assume that during employment the worker must exert effort $\bar{p}_e$. I assume that the function $u\left(c,p\right)$ is bounded, continuously differentiable and that $\lim_{c\rightarrow 0}u\left(c,p\right)=\infty$ and $\lim_{p\rightarrow 0}u\left(c,p\right)=\infty$ to ensure that consumption and search effort are strictly positive.

The unemployment insurance agency, which we shall refer to as the principal, is concerned with the cost of a transfer scheme defined as the expected discounted sum of transfers,[13]

$$E\left[\sum_{t=0}^{\infty}\beta^{t}\tau_{t}\right]$$

where τ_t represents transfers from the principal to the agent. These preferences can be justified as a reduced-form in a model with a continuum of agents with independent unemployment risk (justifying the risk neutrality) and where the principal has access to a technology for transferring goods over time with gross rate of return $\bar{R}=\beta^{-1}$.

It is instructive to characterize the first-best allocation – defined as the best allocation achievable without private information. The problem can be stated as choosing consumption and search effort to minimize the net cost $C^{*}\left(V\right)$ while delivering a lifetime utility level V to the agent:

$$\begin{aligned} C^{*}\left(V\right) \quad \equiv \quad & \min_{c_t,p_t} E\left[\sum_{t=0}^{\infty}\beta^{t}\left(c_{t}-y_{t}\right)\right] \\ & V=E\left[\sum_{t=0}^{\infty}\beta^{t}u\left(c_{t},e_{t}\right)\right] \end{aligned}$$

where $y_t=w$ if the agent is employed and $y_t=0$ if unemployed, at time t.

[13]Although we use the same discount factor, β, for the principal and the agent, the methods we use can accommodate the more general case, see Werning (2000).

The solution to this problem implies a constant consumption and search-effort level while unemployed and a constant consumption level once employed. If in addition the utility function is additively separable between c and p, $u(c,p) = U(c) - v(p)$, then the consumption levels during employment and unemployment are equal. In this way, the first-best allocation displays perfect insurance of consumption against employment risk and perfect consumption smoothing across time[14].

We are interested in cases where the first-best solution is not attainable due to informational asymmetries. I introduce two sources of informational asymmetries. The first is that the effort level is the agent's own private information and the planner has no way of monitoring this effort level. The second source is that the agent's asset accumulation and consumption are not observed by the principal. I turn next to the problem faced by the agent.

2.1 Agent's Problem

Let $s_t \in S$ denote the state of the agent's employment at time t: $s_t = e$ when the agent is employed and $s_t = u$ when unemployed. Let $S^t = S \times S \times ...S$ be the $t+1$ set product of S, with typical element $s^t = (s_0, s_1, ..., s_t) \in S^{t+1}$ recording the employment history up to and including time t. The employment history s^t is public information.

The principal announces a contract that specifies the sequence of transfers to the agent as a function of employment history: $\tau \equiv \{\tau_t(s^t)\}_{t=0}^{\infty}$. The principal has full commitment and the agent takes this contract as given.

The agent is subject to a standard intertemporal budget constraint,

$$k_{t+1}(s^t) + c_t(s^t) = y(s_t) + \tau_t(s^t) + k_t(s^{t-1})R, \tag{2}$$

where $k_t(s^{t-1})$ represents assets at the beginning of time t, given previous history s^{t-1}, which earn a gross rate of return equal to R. Here $y(e) = w$ and $y(u) = 0$. For simplicity I study the benchmark case where $R = \beta^{-1}$ so that, with the interpretation mentioned above, the agent has the same rate of return as the principal.

To incorporate the possibility of constraints on borrowing I assume that $k_{t+1}(s^t) \geq k_{\min}$ where $k_{\min} \in \mathbb{R}$ is a parameter primitive to the model. For the case where we do not wish to impose a binding borrowing constraint we

[14]The first-best equalizes the marginal utility of consumption at all dates. Without additive separability this may require higher or lower consumption while employed.

may set $k_{\min}$ to some negative number low enough to ensure the constraint does not bind.

The agent's problem is to maximize (1) subject to (2) and (**??**) by choice of a history contingent plan for consumption, assets and effort $\{c_t(s^t), k_{t+1}(s^t), p_t(s^{t-1})\}_{t=0}^{\infty}$ such that $p_t(s^{t-1}) = \bar{p}_e$ if $s_{t-1} = e$. This maximization yields the indirect utility function $V^*(\tau)$.

The first order conditions for this problem are:

$$\lambda_t\left(s^t\right) = u_c\left(c_t\left(s^t\right), p_t\left(s^{t-1}\right)\right)$$

for all s^t; for s^t with $s_{t-1} = u$ we must have,

$$\lambda_{t-1}\left(s^{t-1}\right) \geq p\lambda_t\left(s^{t-1}, e\right) + (1-p)\lambda_t\left(s^{t-1}, u\right)$$

with equality if $k_t(s^t) > k_{\min}$,

$$\begin{aligned} w_t\left(s^{t-1}, e\right) - w_t\left(s^{t-1}, u\right) &= p_t\left(s^{t-1}\right) u_p\left(c_t\left(s^{t-1}, e\right), p_t\left(s^{t-1}\right)\right) \\ &\quad + \left(1 - p_t\left(s^{t-1}\right)\right) u_p\left(c_t\left(s^{t-1}, u\right), p_t\left(s^{t-1}\right)\right) \end{aligned}$$

for s^t with $s_{t-1} = e$:

$$\lambda_{t-1}\left(s^{t-1}\right) \geq \lambda_t\left(s^{t-1}, e\right)$$

with equality if $k_t(s^t) > k_{\min}$; where,

$$w_t\left(s^t\right) \equiv u\left(c_t\left(s^{t-1}, e\right), p_t\left(s^{t-1}\right)\right) + \beta V_{t+1}\left(s^{t-1}, e\right)$$

and

$$V_{t+1}\left(s^t\right) \equiv E\left[\sum_{n=0}^{\infty} \beta^n u\left(c_{t+1+n}\left(s^{t+1+n}\right), p_{t+1+n}\left(s^{t+n}\right)\right) \middle| s^t\right]$$

Here $V_{t+1}(s^t)$ represents remaining expected lifetime utility from $t+1$ on after history s^t but before the realization of s_{t+1}.

3 Optimal Allocations

Consider the following second-best problem: choose $\tau \equiv \{\tau_t(s^t)\}$ to minimize the expected discounted cost to the planner of providing a certain lifetime utility level V to the agent subject to the incentive compatibility

constraints, i.e. the agent makes his own optimal decisions for search effort and consumption given τ.

The difficulty is incorporating the incentive constraints due to the agent's maximization in a tractable way. To circumvent this difficulty we apply the approach described in more in Werning (2000). Briefly, the idea is to use the first-order approach: substituting the agent's first-order conditions for the incentive constraints. The benefit of this substitution is that this modified problem has a recursive representation in a small number of variables: the promised expected discounted lifetime utility and the previous period's marginal utility. We describe the problem next directly in terms of this first-order recursive representation.

Define $c(\lambda, p)$ as the solution c to $\lambda = u_c(c, p)$, and define $\tilde{u}(\lambda, p) \equiv u(c(\lambda, p), p)$. Then we seek solutions to the Bellman equations with state variables (V, λ):

$$C^e(V, \lambda) = \min_{(V^e, \lambda^e) \in \Delta^e} \left[c(\lambda^e, \bar{p}_e) - w + \beta C^e(V, \lambda^e) \right]$$

$$\begin{aligned} \lambda &\geq \lambda^e \\ V &= \tilde{u}(\lambda^e, \bar{p}_e) + \beta V^e \end{aligned}$$

And,

$$\begin{aligned} C(V, \lambda) = \min_{\substack{p, \lambda^e, V^e \\ V^u, V^u}} \{ & \quad p \left[c(\lambda^e, p) - w + \beta C^e(V^e, \lambda^e) \right] \\ & + (1-p) \left[c(\lambda^u, p) + \beta C^u(V^u, \lambda^u) \right] \} \end{aligned}$$

$$\lambda \geq p\lambda^e + (1-p)\lambda^u$$

$$p\tilde{u}_p(\lambda^e, p) + (1-p)\tilde{u}_p(\lambda^u, p) = \left[\tilde{u}(\lambda^e, p) + \beta V^e \right] - \left[\tilde{u}(\lambda^u, p) + \beta V^u \right]$$

$$V = p\left[\tilde{u}(\lambda^e, p) + \beta V^e \right] + (1-p)\left[\tilde{u}(\lambda^u, p) + \beta V^u \right]$$

$$(V^s, \lambda^s) \in \Delta^s \text{ for } s = u, e$$

Note that we have dropped the transfer and asset variables as well as the budget constraint, these can be safely ignored and later solved as a residual,

see Werning (2000), Lemma 1. We have also written the problem for the case where saving is allowed but not borrowing, if borrowing is allowed the inequality in the Euler equation is replaced by an equality.

An additional restriction must be added for the above problem to be well posed. There are combinations of λ and V that are not feasible – for example, it may be impossible to give the agent very low lifetime utility if the previous period's marginal utility was very low because the agent would have saved. The set of feasible combinations (V, λ) for each s is the correct domain, Δ^s, for each function C^s. Of course, the domain restriction must be added as constraints on the minimization.

These domains can be found by iterating till convergence on a monotone set operator starting from an appropriate initial set, larger that Δ^e and Δ^u. In some special cases, as we shall see, the sets actually suggest themselves in more direct ways.

We can solve explicitly for $C^e(V, \lambda)$ and Δ^e,

$$\begin{aligned} C^e(V, \lambda) &= \frac{u^{-1}(V(1-\beta), e_e)}{1-\beta} \\ \Delta^e &= \left\{ (V, \lambda) \mid \frac{\tilde{u}(\lambda, \bar{e}_e)}{1-\beta} \leq V \right\} \end{aligned}$$

4 Optimal Unemployment Insurance

We first study two cases that can be solved analytically and then turn to a numerical calibration.

4.1 Exponential Utility and Monetary Cost of Effort

In the exponential case with monetary cost of effort,

$$u(c, a) = -\frac{1}{\gamma} \exp\{-\gamma(c + v(e))\},$$

the domain set can be found very easily. Note that,

$$u_c(c, e) = -\gamma u(c, e),$$

utility and marginal utility are linear functions of each other. We can be use this fact to compute the domains Δ^s directly as follows.

The Euler condition and the law of iterated expectations imply,

$$u_{c,t-1} \geq E_{t-1} u_{c,t+k}$$

Lifetime utility from period t on, before the realization of s_t is,

$$V_t = E_{t-1} \sum_{k=0}^{\infty} \beta^k u_{t+k} = \sum_{k=0}^{\infty} \beta^k E_{t-1} \left\{ -u_{c,t+k} \right\} \geq \frac{-1}{1-\beta} u_{c,t-1}.$$

It is easy to see that the above inequality is indeed the only restriction on V_t and λ_{t-1} – the search-effort incentive constraints in this case play no role in shaping the domain sets, contrary to the general case.. Thus the domain Δ^s is independent of s and given by:

$$\Delta^s \equiv \left\{ (V, \lambda) \mid V \geq \frac{-1}{1-\beta} \lambda \right\}.$$

If the agent can save the inequality above must be replaced by an equality.

The employment cost function has the form:

$$C^e (V, \lambda) = \frac{-1}{1-\beta} \log (-V) + \kappa^e$$

for some constant κ^e.

With these sets in hand we can write the problem as

$$\begin{aligned} C(V, \lambda) = \min \{ v(a) + \quad & p \left[-\log (\lambda^e) - w + \beta C^e (V^e, \lambda^e) \right] \\ & + (1-p) \left[-\log (\lambda^u) + \beta C^u (V^u, \lambda^u) \right] \} \end{aligned}$$

$$\lambda \geq p\lambda^e + (1-p)\lambda^u$$

$$V = p \left[-\lambda^e + \beta V^e \right] + (1-p) \left[-\lambda^u + \beta V^u \right]$$

$$\left[p\lambda^e + (1-p)\lambda^u \right] v'(p) = \left[-\lambda^e + \beta V^e \right] - \left[-\lambda^u + \beta V^u \right]$$

$$V^s \geq \frac{-1}{1-\beta} \lambda^s \text{ for } s = e, u$$

We will refer to the constraints in this problem as the Euler, promise-keeping, incentive-compatibility and domain constraints, respectively.

Lemma The domain constraints are binding.

Proof. See appendix.

We now simplify this problem significantly by using the domain constraints with equality. When the domain constraints hold with equality the Euler condition is automatically satisfied with equality whenever the promise keeping constraint holds. Hence, we can drop the Euler equation from the analysis and write the relevant cost function solely as a function of V; that is, abusing notation, define $C^s(V) \equiv C^s(V, -V(1-\beta))$:

$$\begin{aligned} C(V) = \log((1-\beta)) + \min_p \{ \quad & v(p) + p\left[-\log(-V^e) - w + \beta C^e(V^e)\right] \\ & + (1-p)\left[-\log(-V^u) + \beta C^u(V^u)\right]\} \end{aligned}$$

$$V = pV^e + (1-p)V^u$$

$$-V(1-\beta)v'(p) = V^e - V^u$$

It is easy to guess and verify that the cost function has the same functional form as the employment cost function with a different constant. The optimal effort level does not depend on V, and is thus constant over the unemployment spell.

We can solve the following problem and then construct the optimal contract.

$$\min_{p,\{\tilde{v}_i\}} v(p) + p\left[-\frac{1}{1-\beta}\log(\tilde{v}^e) - w + \beta\kappa^e\right] + (1-p)\left[-\frac{1}{1-\beta}\log(\tilde{v}^u) + \beta\kappa^u\right]$$

$$1 = p\tilde{v}^e + (1-p)\tilde{v}^u$$

$$(1-\beta)v'(p) = \tilde{v}^u - \tilde{v}^e$$

Then construct the optimal contract using:

$$\begin{aligned} c^s(V) &= \bar{c}^s - \log(-V) = \bar{c} - \log(-V^s) \\ V^s(V) &= \bar{v}^s V \\ p(V) &= \bar{p} \end{aligned}$$

How can the optimal consumption allocation be implemented by a transfers scheme? When the agent is allowed to save many transfer mechanisms may implement the same allocation – intertemporal reallocations can always be done through the principal or the agent. We need a some kind of meaningful normalization. It seems natural to look at the case where transfers are zero once employed, $\tau_e = 0$. We seek the sequence of transfers during unemployment, which we shall refer to as unemployment benefits, that implements the allocation subject to this restriction. In doing so we assume that the agent is not borrowing constrained. This is without loss in generality because the planner can always design a system whereby it lends to the agent, thus overcoming the borrowing constraint. However, this must be borne in mind when interpreting the results.

We use the following simple notation: let $\tau_t^u \equiv \tau_t(u, u, ..., u)$ denote the transfer to an unemployed worker with duration t, let $c_t^u \equiv c_t(u, u, ..., u)$ denote consumption of an unemployed agent with duration t, similarly for consumption of an employed and assets.

Proposition: with exponential utility and monetary cost of effort the optimal unemployment benefit is constant, i.e. $\tau_t^u = \bar{\tau}^u < 1$.

Proof: The agents budget constraint is,

$$k_{t+1}(s^t) + c_t(s^t) = y(s_t) + \tau_t(s^t) + k_t(s^{t-1})R.$$

If transfers are zero once employed, then periods of unemployment we have,

$$k_{t+1} + c_t^u = \tau_t^u + k_t R.$$

Once employed we must have,

$$w + rk_t = c_t^e$$

$$\Rightarrow k_t = \frac{\beta}{1-\beta}(c_t^e - w)$$

where $r = \beta^{-1} - 1$. This defines the required sequence of capital. Substituting this into the budget constraint during unemployment yields:

$$\tau_t^u = w - \left[\frac{\beta}{1-\beta}\left(c_t^e - c_{t+1}^e\right) + \left(c_t^e - c_t^u\right)\right]$$

Using the policy functions, the optimal sequence of consumption has the following form,

$$c_t^u = \rho - \delta t$$

$$c_t^e = \rho - \delta t + \alpha$$

for some constants ρ, δ, α

$$\tau_t^u = w - \left[\frac{\beta}{1-\beta}\delta + \alpha\right] \equiv \bar{\tau}^u$$

which completes the proof. ■

This result is related to a result by Fudenberg, Holmstrom and Milgrom (1990). Using very different techniques they show that with exponential preferences in a finitely repeated, i.i.d. moral-hazard problem (income in each period is independent of previous realizations and only depends on current effort) the optimal contract can be implemented by repeating a single 'static' insurance contract.

4.2 Constant Relative Risk Aversion with Constant Effort

Consider the standard following preferences

$$u(c,p) = \frac{1}{1-\sigma}\left[cv(e)\right]^{1-\sigma}. \tag{3}$$

The constant relative risk aversion specification of attitudes towards risk is surely more realistic than constant absolute risk aversion. The preferences between c and e in (3) are also widely used in the growth and macro literature because when e is interpreted as work time it implies that e does not increase with productivity: balanced growth is obtained.

To obtain an explicit characterization we add the requirement that the contract must implement some constant level of effort $\bar{e} = \bar{p}$ while unemployed. Note that we are not restricting the environment: the agent can still choose effort from the same set and have a non-constant effort path. We do restrict the contract to those that implement constant effort levels. Although this restriction is ad-hoc it is a useful benchmark to understand the results

that follow. The next section numerically computes the optimal contract for similar preferences without this restriction.

For any given level of effort we are forced to implement there is a one to one relationship between utility and marginal utility, u and λ. For the CRRA we find it more convenient to work with u instead of λ. The domain Δ^u for (V, v) for the unemployment state can be defined in terms of its frontier $\phi^u(u)$, so that $\Delta^u = \{(V,u) \mid V \geq \phi^u(u)\}$, which must be the lowest fix point of the Bellman operator

$$T[\phi^u](u) = \min_{p,\lambda_e,\lambda_u} \left\{ p\psi \frac{u^e}{1-\beta} + (1-p)\left[u^u + \beta v_u\right] \right\}$$

subject to,

$$\begin{aligned} u &\geq p(u_e)^{-\frac{\sigma}{1-\sigma}} + (1-p)(u^u)^{-\frac{\sigma}{1-\sigma}} \\ (1-\sigma)\frac{v'(\bar{p})}{v(\bar{p})}\left[pu^e + (1-p)u^u\right] &= \psi \frac{u^e}{1-\beta} - \left[u^u + \beta v_u\right] \\ v_u &\geq \phi^u(u^u) \end{aligned}$$

where ψ is a constant dependent on $\bar{p}$ and e_e. The Euler condition must bind; the frontier condition therefore must bind because otherwise it is feasible to increases u while lowering v

Exploiting the homogeneity in u, u^u and u^e notice that if $\phi^u(u) = \kappa u$ for some constant κ then $T[\phi^u](u) = \kappa' u$, for some constant κ' – the operator preserves the proportional functional form. The therefore must takes the simple form

$$\phi^u(\lambda) = \kappa^u u$$

for some constant κ^u.

We re-write the problem in terms of u instead of λ

$$\begin{aligned} C(V,u) = \min_{\{\lambda_i, V_i\}} \quad & p\left[(u^e)^{\frac{1}{1-\sigma}} - (1-\beta)w + \beta C^e(V^e, u^e)\right] + \\ & (1-p)\left[(u^u)^{\frac{1}{1-\sigma}} + \beta C^u(V^u, u^u)\right] \end{aligned}$$

$$
\begin{aligned}
u^{\frac{-1}{1-\sigma}} &\geq p\left(u^{e}\right)^{\frac{-1}{1-\sigma}}+(1-p)\left(u^{u}\right)^{\frac{-1}{1-\sigma}} \\
V &= p\left[u^{e}+\beta V^{e}\right]+(1-p)\left[u^{e}+\beta V^{e}\right] \\
(1-\sigma)\frac{v'\left(\bar{p}\right)}{v\left(\bar{p}\right)}\left[pu^{e}+(1-p)u^{u}\right] &= \left[u^{e}+\beta V^{e}\right]-\left[u^{e}+\beta V^{e}\right] \\
V^{s} &\geq \kappa^{s}u^{s} \text{ for } s=e,u
\end{aligned}
$$

Its easy to see that the domain constraints bind and that the solution takes the form:

$$C(V,u)=\gamma V^{\frac{1}{1-\sigma}}+\kappa.$$

Proposition: with CRRA the optimal unemployment scheme that implements a constant effort level is increasing, i.e. $\tau_t^u < \tau_{t+1}^u < 1$ for all t.
Proof: Just as before we have that:

$$\tau_t^u = w - \left[\frac{\beta}{1-\beta}\left(c_t^e - c_{t+1}^e\right)+\left(c_t^e - c_t^u\right)\right]$$

substituting the optimal consumption policies:

$$\tau_t^u = w + V_t\left[\frac{\beta}{1-\beta}\left(c^e v^u - c^e\right)+\left(c^u - c^e\right)\right]$$

Here V_t is falling over time, but the term $\left[\frac{\beta}{1-\beta}\left(c^e v^u - c^e\right)+\left(c^u - c^e\right)\right]$ must be negative if effort is positive. Hence, τ_t^u must be increasing. ■

5 Numerical Results

We now report some preliminary numerical results based on the following specification of preferences:

$$u\left(c,e\right)=\frac{c^{1-\sigma}}{1-\sigma}-\alpha\kappa v\left(e\right)$$

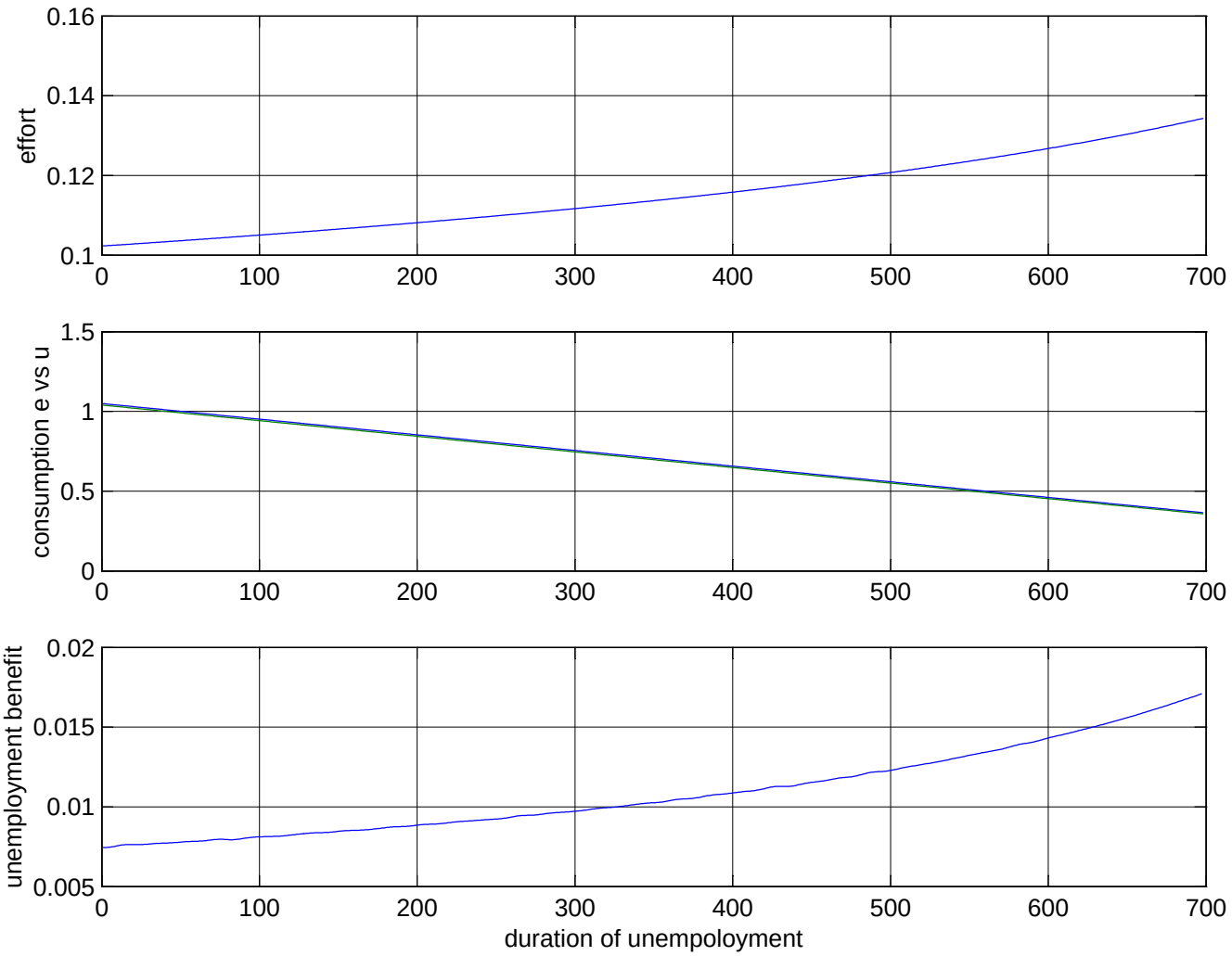
and we experiment with two specifications for the disutility of effort: $v\left(e\right)=e^{\gamma}/\gamma$ with $\gamma \geq 1$ (type A) or $v\left(e\right)=-\left(1-e\right)^{\gamma}/\gamma$ with $\gamma \leq 1$ (type A). For the case with $\sigma = 0$ this specification of preferences is a special case of the

CRRA studied in the previous section. With $v(e) = -(1-e)^{\gamma}/\gamma$ and $\gamma \to 0$ and $\sigma = 1/2$, the preference specification is as in Hopenhayn and Nicolini (1997).

A period was calibrated to a week so that $\beta = .999$, an annual interest rate of 4%. The wage was normalized to 1. We experimented with various values of σ and γ. The value of α was calibrated in each case so that the level of effort implied a hazard rate around 10%, consistent with Meyer (1990).

The results are show in figures 1, 2 and 3. In all three panels the x-axis is unemployment duration measured in weeks. The first panel shows the effort level, which is generally rising. The second panel shows the consumption profile for employment and unemployment. The bottom panel shows the implied unemployment benefit.

Figure 1 has $\sigma = .5$, with $v(a)$ of type B and $\gamma = 0$ as in Hopenhayn and Nicolini (1997). Note that the implied UI benefits are extremely low. This is presumably due to the low risk aversion and the relatively high moral hazard problem. Figure 2 and 3 experiments with higher levels of σ : $\sigma = 3$, and $v(e)$ of type A with $\gamma = 4$, and $\sigma = 6$ and $\gamma = 10$. The UI benefits are now higher. In all three cases the UI benefits rise with the unemployment spell.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0.12
0.1
0
100
200
300
400
500
600
700
consumption e vs u
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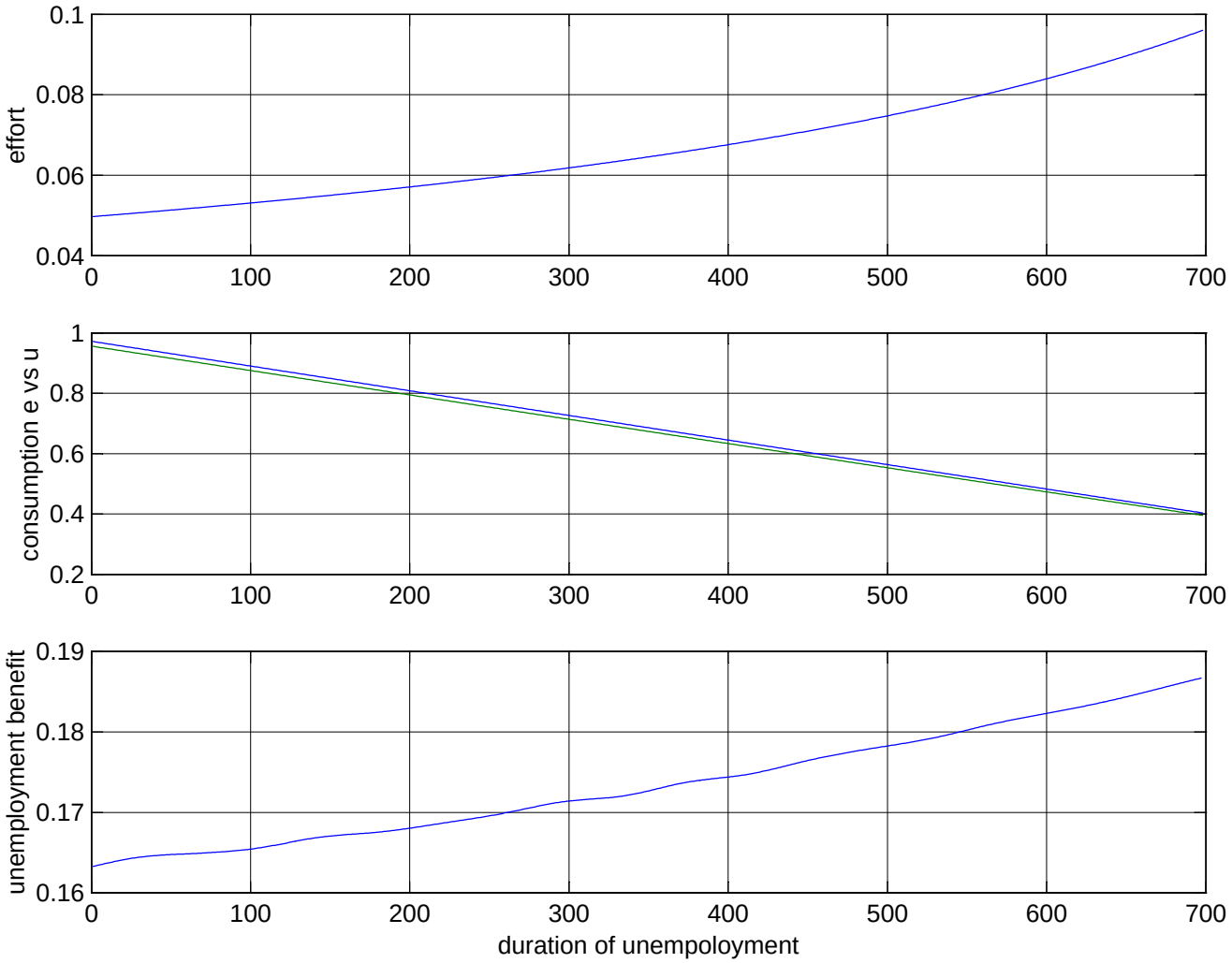
1
0.5
0
unemployment benefit
0.02
0.015
0.01
0.005
duration of unempoloyment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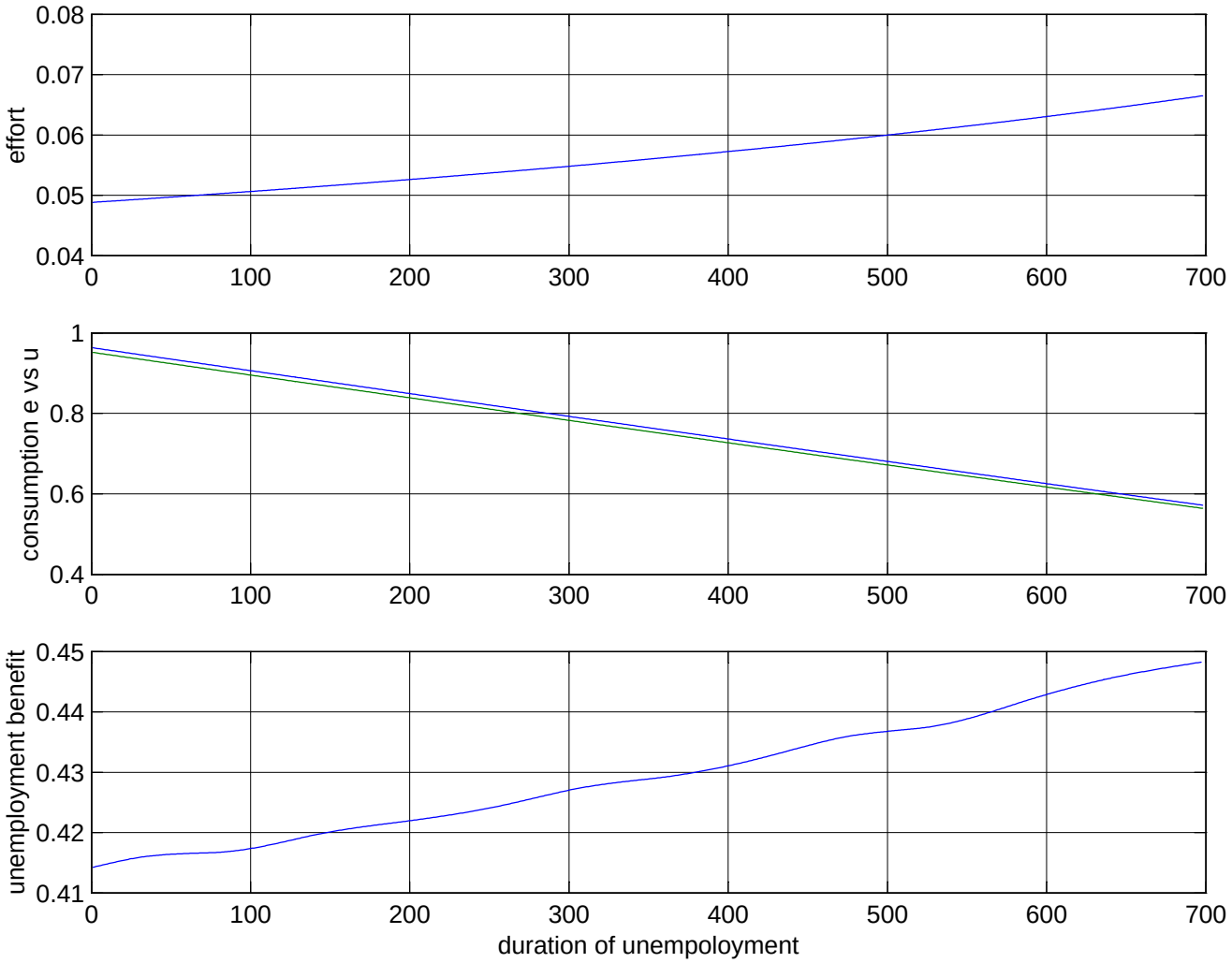
0.06
0.04
effort
0
100
200
300
400
500
600
700
1
0.8
0.6
0.4
0.2
consumption e vs u
0
100
200
300
400
500
600
700
0.19
0.18
0.17
0.16
unemployment benefit
0
100
200
300
400
500
600
700
duration of unempoloyment


effort
0.08
0.07
0.06
0.05
0.04
0
100
200
300
400
500
600
700
consumption e vs u
1
0.8
0.6
0.4
unemployment benefit
0.45
0.44
0.43
0.42
0.41
duration of unempoloyment

6 Conclusions

The results of this paper cast some doubts on the widely held view that decreasing UI benefits are a desirable feature of UI policy. Some caution is required, however, in interpreting this result. As mentioned above, to the extent that the unemployed worker is borrowing constrained the transfer system found above should be supplemented with a "lending-scheme": the government should allow the agent to borrow from it. Total net-transfers, including lending borrowing withdrawals and paybacks, may well be decreasing in this case.

The point of this paper was to focus on the simplest moral hazard setting with savings. Extensions or other models may provide forces for decreasing benefits. For example, if the wage decreases with unemployment duration due to human capital depreciation a constant benefit may have very adverse incentives after a long spell (Ljungqvist and Sargent (1998)). Another extension would be to include unobservable heterogeneity in the search or work disutility.

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