

## Labor Economics, 14.661, Second Part, Problem Set 2

Please hand in questions 1, 2 and 4. This problem set is due on Wednesday, November 8, before class.

**Exercise 1** Consider an economy that consists of a large number  $n$  of workers and the same number of firms. The economy lasts for two periods. In the first period, workers choose the level of their human capital,  $h$  incurring a cost  $c(h)$  where  $c(\cdot)$  is differentiable, strictly convex and increasing and firms choose their capital stock at cost  $r$ . There is no discounting in this economy. In the second period, firms and workers are randomly matched 1-to-1 and switching is not allowed so if there is disagreement nothing is produced and both parties obtain zero return. If they agree to produce, output is equal to  $F(h, k)$ , where  $F$  is concave and  $k$  and  $h$  are complements in the production.

1. Assume that wages are determined by an asymmetric Nash bargain where the worker's bargaining power is  $\beta$ . Determine the equilibrium of this economy. Show that all firms will choose the same level of physical capital investment, all workers will choose the same level of human capital investment, and there will be underinvestment compared to the social optimum in both dimensions.
2. Now suppose that a fraction  $\lambda$  of firms have their cost of capital reduced to  $r' < r$ . Show that this creates "positive externalities" both on workers and other firms. Explain the intuition for this result.
3. Suppose  $F(h, k) = Ah^\alpha k^{1-\alpha}$ . At which value of  $\beta$  is output maximized. Explain.
4. Suppose there are many countries that differ in their labor market institutions thus have different  $\beta$ 's. Show that there will be an inverse U-shaped relationship between factor shares of capital and the level of income.
5. Show that a multiplicity of equilibria is possible with general technology  $F(h, k)$  (Hint: a diagrammatic answer is sufficient).
6. Now consider the human capital levels of the workers fixed, with a fraction  $\theta$  of the workers having a human capital level  $h_1$  and the rest with human capital level  $h_2 > h_1$ . First show that under the same assumption we have made so far that there is no switching partners, the equilibrium always involves all firms choosing the same level of physical capital.

Next assume that following the first random match between firms and workers, if either party does not want to form an employment relationship with their match partner, both parties will incur a cost  $c$ , but can then rematch with one of the workers that are still unmatched in the second period. In the first period, wages are given by Nash bargaining taking the second period values as the threat point. The worker's bargaining power  $\beta$  is common to both periods. Show that the above-characterized equilibrium is still an equilibrium, but for  $c$  sufficiently small, there also exists an equilibrium in which all firms no longer choose the same level of physical capital investment. Characterize this equilibrium.

[For bonus points] Show that for fixed  $c$ , as the gap between  $h_2$  and  $h_1$  increases, we can switch from an equilibrium in which firms all choose the same level of physical capital investment, to one in which they do not. In this extended model, are there positive or negative human capital externalities?

**Exercise 2** In this problem, you are asked to work through a model that combines signaling with productive aspects of schooling. There are two types of agents: "high" and "low" ability. Education ( $e$ ) is continuous and observed, but individual ability (and output) is not. The labor productivity for the "low" type is  $y_l(e) = \alpha_0$  and the cost of education is  $c_l(e) = 3e^2/2$ . For the "high" type, output and education costs are  $y_h(e) = \alpha_1 + \alpha_2 e$  and  $c_h(e) = e^2$ , respectively. Let  $\alpha_1 = \alpha_0$  for now.

1. Define Perfect Bayesian Equilibria (PBE) of this game (Hint: be specific about the actions of workers of different types and the actions of firms—which are wage offers as functions of publicly observable objects—at different points in time).
2. Solve for the PBE corresponding to the “Riley Equilibrium” (most efficient separating equilibrium) of this game. In particular, show that high type workers do not have an incentive to deviate from your proposed equilibrium strategies. Does the high types’ investment in education differ from what would have obtained in the perfect-information case? Why or why not?
3. Suppose again that  $c_l(e) = 3e^2/2$  and furthermore suppose that there is a compulsory schooling requirement of  $\underline{e}$ , where  $0 < \underline{e} < \alpha_2/4$ . Characterize the Riley Equilibrium. Does the high type invest in education more or less in this case than in part 2? Explain why.
4. Compute the observed return to schooling in parts 2 and 3, and discuss how they change as a result of the compulsory schooling requirement.
5. How does the observed return to schooling change if  $\alpha_1 - \alpha_0$  increases (starting from zero)? Explain the intuition for both the forces that tend to increase and decrease observed returns to schooling in this case.

**Exercise 3** The economy lasts two periods. In period 1, an individual (parent) works, consumes  $c$ , saves  $s$  (to be left as bequest), decides how much education  $e$  to purchase on behalf of their offspring, and then dies at the end of the period. Utility of household  $i$  is given  $U(c_i, \hat{c}_i)$ , where  $\hat{c}_i$  is the consumption of the offspring, and  $U$  is increasing in both of its arguments and jointly concave. There is heterogeneity among children, so the cost of education,  $\theta_i e_i$ , varies across  $i$ . In the second period, individuals receive a wage  $w(e)$ , where  $w' > 0$  and  $w'' < 0$  as usual.

1. Consider the case in which credit markets are perfect: households can borrow and lend at the same interest rate  $r$ . Characterize the household’s decision problem. Show that the choice of education is independent of the form of the utility function.
2. Now assume a credit-market friction: households can lend at  $r$ , but cannot borrow going from period 1 to period 2. Write down the household’s decision problem, including this new constraint. Show how the education and consumption decisions are no longer separable.

3. One of your colleagues just ran the following regression:

$$\log(\text{Children's Income})_i = 0.35 \times \log(\text{Parents' Income})_i + \epsilon_i$$

where  $i$  denotes a household “dynasty.”

Interpret this regression. Provide at least two theories that might explain this relationship and relate them to the model in part 1 above. Discuss how you might go about discriminating among these competing theories.

4. Upon including a variety of covariates (such as parents’ education) in the regression, your colleague finds that the effect of parents’ income drops by more than half. He claims that this constitutes evidence against the idea that poor parents cannot finance human capital investment due to credit-market imperfections. Outline an argument in support of this view. Then discuss problems with his conclusion.

**Exercise 4** Consider the Galor-Zeira model of growth with imperfect credit markets in Lectures 3 and 4 with the following two modifications. First, the utility function is now

$$(1 - \delta)^{-(1-\delta)} \delta^{-\delta} c^{1-\delta} b^\delta$$

and second, unskilled agents receive a wage of  $w_u + \varepsilon$  where  $\varepsilon$  is a mean-zero random shock.

1. Suppose that  $\varepsilon$  is distributed with support  $[-\psi, \psi]$ , and show that if  $\psi$  is sufficiently close to 0, then as in the baseline model there are multiple steady states (that is, depending on their initial conditions some dynasties become high skilled and others become low skilled).
2. Why was it convenient to change the utility function from the log form used in the lecture to the one here?
3. Now suppose that  $\varepsilon$  is distributed with support  $[-\psi, \infty)$ , where  $\psi \leq w_u$ . Show that in this case there is a unique stationary distribution of wealth and no poverty trap. Explain why the results here are different from those in part 1? How rapid do you think converges to this stationary distribution will be?
4. How would the results be different if, in addition, the skilled wage is equal to  $w_s + v$ , where  $v$  is another mean-zero random shock? [Simply sketch the analysis and the structure of the equilibrium without repeating the full analysis of part 3].

**Exercise 5** Consider a society  $S$  consisting of  $M \times N$  students, and  $M$  schools, each of size  $N$ . Each student has a pre-schooling human capital  $e_i$ . The post-schooling human capital of a student  $i$  with pre-schooling human capital  $e_i$  attending school  $j$  is

$$h_i = G(e_i, e_{-i,j})$$

where  $e_{-i,j}$  is a vector of pre-schooling human capital of students other than  $i$  attending school  $j$ .

1. Show that if  $G$  is supermodular in its arguments, then to maximize the sum of post-schooling human capitals in society,  $\sum_{i \in S} h_i$ , the  $N$  students with the highest pre-schooling human capital should be allocated to one school, then the next  $N$  students with the highest pre-schooling human capital should be allocated to another school and so on.
2. Show that if parents maximize  $wh - p$ , where  $p$  is their cost of schooling, then the allocation that maximizes the sum of post-schooling human capitals can be decentralized by each school choosing some cost of attendance (tuition)  $p_j$  with the following “rationing mechanism”: if a student with higher pre-schooling human capital than the school’s maximum applies to a school that is already full, he replaces the current student with the highest pre-schooling human capital; if a student with lower pre-schooling capital than the school’s minimum applies, then he replaces the current student with the lowest pre-schooling human capital. [Hint: to simplify, you may want to establish this result focusing on schools consisting of two students].
3. How is the allocation that maximizes the sum of post-schooling human capitals affected when  $G$  is submodular? Can this allocation be decentralized in the same manner as in part 2? If not, why not?
4. A researcher tries to determine whether  $G$  is supermodular or not by running a regression of GPAs of college students on SATs of classmates. What might go wrong with this empirical approach? (Hint: you may want to discuss separately the issues of identification, the difficulty of determining who the peers are, and the difficulty of mapping GPAs into “human capital”).