Chronicle of a Dollarization Foretold:
Inflation and Exchange Rates Dynamics*

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We study the effects of an anticipated dollarization, announced today but planned to be implemented at some future date, in a simple open-economy model. Motivated by the profile of countries considering dollarization we make the following assumptions. First, the government faces a scarcity of dollars to pledge for the future conversion of domestic currency. Second, without dollarization monetary policy finances a deficit via seignorage. We focus on the pre-dollarization period. Our results are as follows. First, the announcement leads to a discrete devaluation on impact. Second, after this jump the devaluation rate also rises relative to the no dollarization benchmark. Finally, the devaluation and inflation rate may rises over time.

1 Introduction

"Dollarization" is a rare policy whereby a country abandons its local currency and adopts a foreign currency (typically the dollar) so as to rid themselves of high and persistent inflation. Such an extreme measure entails various costs studied in the economic literature and few countries have, to date, swallowed this pill.[1] However, in the last year, proposals for dollarization have gained traction in Argentina, spearheaded by a leading presidential candidate. Dollarization may be implemented immediately or announced for some future date. Recent proposals in Argentina initially pushed for immediate dollarization, but have shifted towards this latter possibility—due to lack of foreign reserves and the need to improve the fiscal outlook.

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[1]Panama adopted the dollar in 1904 as its main currency, see Goldfajn, Olivares, Frankel and Milesi-Ferretti (2001). Ecuador underwent dollarization in 2000-01, El Salvador did so in 2001, Zimbabwe in 2009.

What are the effects of announcing a dollarization that will not be implemented immediately, but instead carried out in future after, say, two years? How does the exchange rate react on impact and how does it evolve over time? How does the outcome depend on the amount of dollars pledged by the government for conversion, the fiscal situation during the transition, and the dollarization timeline?[2]

We explore these questions in a simple open-economy monetary economy model, similar to Calvo (1981) and our previous paper Caravello, Martinez-Bruera and Werning (2023), which focused on the dynamic post-dollarization. Since the focus of this paper is on the pre-dollarization period, we must be specific about monetary and fiscal policy during this phase. Countries suffering from high inflation, especially those considering dollarization, rely heavily on seignorage. Thus, we assume that, prior to implementing dollarization, monetary policy must finance a given fiscal deficit —a fiscal dominance regime, as in the seminal work by Cagan (1956) and Sargent and Wallace (1973).

In this context, we study the effect of announcing a dollarization to be implemented at some future date. A crucial parameter is the amount of dollars pledged by the government (likely through its Central Bank) to convert outgoing domestic currency (e.g. pesos) into dollars. Although our analysis allows for any value, the main interest is in realistic scenarios where dollars are scarce and these pledges fall short of the current real money balances.

We compare the effect of this announced dollarization to a benchmark "business as usual" policy without dollarization. Without dollarization the equilibrium involves a constant rate of inflation that is consistent with the needed constant seignorage. When multiple equilibrium inflation rates exist, we take the lowest one—the one on the "good side of the Laffer curve".[3] The exchange rate depreciates at this same constant rate. Figure 1 reproduces the standard "Laffer curve" diagram to represent the no dollarization benchmark.[4] The curve is given by $S(\pi) = m(\pi)\pi$ where $m(\pi)$ is money demand while $\bar{\tau}$ is the deficit level. The benchmark no-dollarization inflation rate is marked as π_G^* on

[2]For analytical concreteness, it is useful to consider a plan that converts all domestic currency at some fixed future date, using some given pledged amount of dollars. Obviously, one may consider variants where the amount of government dollars for conversion falls in a range and is somewhat elastic or where the timeline for conversion is also a range, e.g. voluntary after some date. In all these cases, a key constraint is that dollar conversion rate economic agents anticipate which is crucially affected by the expected dollar pledged for conversion; details aside, these are the essential considerations we capture: some time delay until dollarization conversion and some limited dollar pledge to do so.

[3]This is in line with current folk wisdom among most economists that, away from hyperinflations, the equilibrium is on the good side of the Laffer curve, but possibly at very high inflation rates. In fact, even the notion that a rational expectations equilibrium on the bad side of the Laffer curve is a good description of hyperinflations is not universally embraced (see Marcet and Nicolini, 2003).

[4]Figure 1 is dedicated to our teacher and friend, Juan Pablo Nicolini, and taught us this famous curve many times.

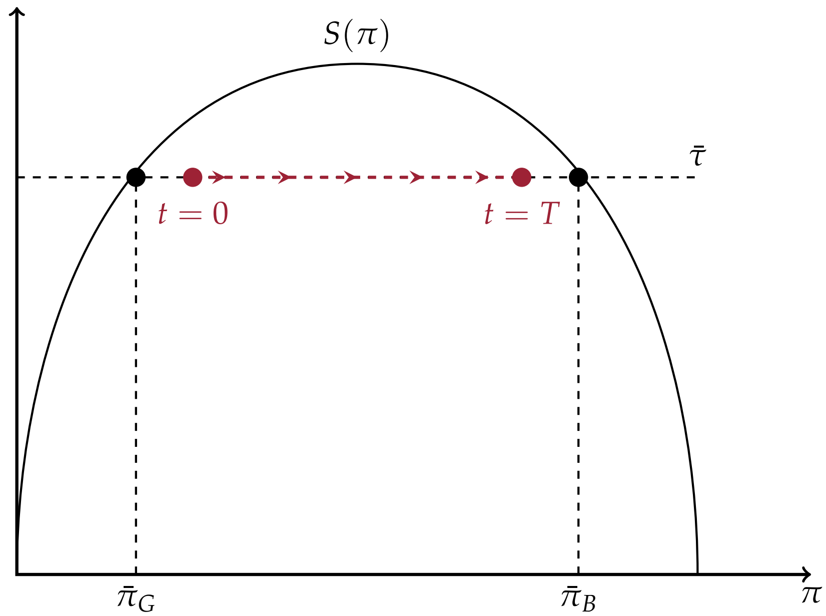

Figure 1: Laffer curve meets the deficit at two points. The benchmark no-dollarization equilibrium is the lowest of the two, π_G^*. An equilibrium path when dollarization is shown in red.

the good (left) side of the Laffer curve.

We compare this benchmark to a policy announcing dollarization. Assuming dollars are scarce, we first show that the exchange rate and price level undergo a discrete jump on impact: the exchange rate depreciates and the price level jumps up accordingly. Second, after this initial jump the exchange rate and price level paths behave smoothly, but with a positive rate of devaluation and inflation that we show is higher than that in our no-dollarization benchmark. Finally, unlike the no-dollarization benchmark, the rate of devaluation and inflation are not generally constant. Indeed, as long as dollars are scarce, but not extremely scarce, we show that the depreciation and inflation rate rises over time as dollarization nears. If, instead, dollars are extremely scarce, then inflation falls over time but remains above high levels, above π_B^* (the bad side of the Laffer curve) in the figure.

The equilibrium dynamics of dollarization are shown in solid red in Figure 1 for a situation of dollar scarcity, but not extreme scarcity. The initial discrete devaluation lowers real money balances, which is related to the fact that the initial inflation rate π_0 is above the no dollarization benchmark π_G^*. Over time inflation rises staying above π_G^*, but below π_B^*. At the dollarization conversion date T, the inflation rate π_T must equal the value of pledged dollars, our key policy parameter. A lower dollar pledge requires lower real money balances, which are associated with a higher π_T. Indeed, this condition pins down by the entire equilibrium path uniquely.

We also provide two comparative statics. If fewer dollars are pledged by the government or if one sets an earlier date for the implementation of dollarization, then the price level and inflation is everywhere higher. The discrete devaluation is greater on impact, as is the subsequent depreciation rate.

Some interesting possibilities emerge when dollar scarcity is more extreme. Indeed, there is a threshold level of dollars pledged by the government that implies a constant rate of inflation rate equal to π_B^*, the one associated with a steady state on the bad side of the Laffer curve. Unlike a situation without dollarization where this is one of multiple equilibria, usually not a focal point of analysis. Here, with dollarization, this constitutes the unique equilibrium. If dollars are still scarcer, so the government pledges less dollars than this threshold level, then inflation must lie above π_B^*. Interestingly, relative to the case with less extreme scarcity, the dynamics over time are now inverted: inflation starts falls over time, but is always at a very high level.

Finally, on the opposite side of the spectrum, if dollars are not scarce, but instead abundant and greater than real money balances, then the equilibrium path lies to the left of π_B^* and inflation rates fall over time. In this case, there is a discrete appreciation of the exchange rate on impact, and a rise in the price level, instead of a drop. Given the scarcity of dollars, this case does not seem plausible in practice.

It is natural to expect a devalued exchange rate upon dollarization in a situation of scarce dollars. However, our results go further and more precise than this. Indeed, a naive intuition would take the path for money supply as given and note that the conversion rate at $t = T$ must be low if the dollars pledged for conversion are low. However, the path for money supply is not exogenous and is affected by the announced dollarization, so solving for the equilibrium goes beyond the simple-minded intuition. Indeed, we find that money supply must grow faster after announcing dollarization, to finance the deficit, so the conversion exchange rate at $t = T$ is devalued more than one for one with dollar scarcity.

It is also natural to expect some earlier repercussions on the exchange rate before dollarization. After all, in a forward-looking model with rational expectations a depreciated future exchange rate typically affects the earlier path of the exchange rate. However, this intuition is incomplete without specifying the monetary policy regime. Our analysis based on the strict seignorage requirement provides a tight characterization, showing that there is both a jump on impact and a higher smooth rate of depreciation thereafter, with the rate of depreciation rising over time.

This paper complements Caravello, Martinez-Bruera and Werning (2023) which studied the post-dollarization dynamics. In contrast, here we consider an anticipated dollarization and study ex-ante dynamics for inflation and the exchange rate prior to the implementation of dollarization, but after its announcement.[5] Another important distinction is that post-dollarization dynamics are shaped by dollars available to households, whereas the ex ante effects we study here are shaped by the amount of dollars pledged by the government.

The rest of the paper is organized as follows. Section 2 lays out our simple model and defines the dollarization exercise. Section 3 provides our main analysis under the simplifying of no wealth effects on money demand. Section 4 completes the analysis discussing the other cases.

2 Dollar Promises in An Open Economy Model

We now describe very simple, stripped-down open-economy monetary model that is well tailored to the purposes of the present paper.

Basics. A small open economy trades with the world in a single consumption good at given world prices and has a constant endowment $y > 0$. Domestic prices, quoted in domestic currency before dollarization, are flexible and respond with a one-to-one passthrough from the nominal exchange rate. Households makes consumption, saving choices and holds money balances for its liquidity services.

Time is continuous and runs for eternity $t \geq 0$. In the dollarization scenario, at $t = 0$ it is announced that dollarization will be implemented at $t = T > 0$. We focus on outcomes before dollarization is implemented, for $t \in [0, T]$. We will compare this outcome to a benchmark regime without dollarization.

The present analysis is kept as simple as possible, but it could be extended to incorporate a non-traded sector, sticky prices, heterogeneous agents and to study the outcomes after dollarization as in Caravello, Martinez-Bruera and Werning (2023).

Exchange Rate and Prices. We normalize the foreign price to unity. Thus, the domestic price P_t is equal to the nominal exchange rate. At all times, the rate of depreciation is equal to the rate of inflation, $\pi_t = \dot{P}_t / P_t$.

[5] The effects after the implementation of dollarization for a pre-announced dollarization are the same as the unanticipated case studied we studied but with a potentially different initial stock of total dollars (public plus private).

Household Budget Constraints. The budget constraint

$$\dot{W}_t = y - c_t - m_t^d \pi_t + \tau_t$$

and

$$W_t \geq 0$$

for all $t \geq 0$, where y is the constant endowment, c_t is consumption, τ_t is a lump-sum transfer (or taxes if negative), $W_t = m_t^d + m_t^f$ denotes wealth, composed of foreign currency ("dollars") m_t^f and local domestic currency m_t^d where $m_t^d = M_t^d / P_t$. The term $m_t^d \pi_t$ is the "inflation tax"—a loss incurred for holding domestic currency instead of foreign currency. All households are identical and have some initial $m_0^f \geq 0$ and $M_m^d > 0$.

The budget constraint assumes, for simplicity, that the foreign price in dollars is constant and equal to unity and that foreign assets (dollars) pay no interest; we also assume there are no other domestic assets such as bonds, that pay interest. It is trivial to relax all of these assumptions, but it is not worthwhile since they are not influential in our analysis or results.

Household Preferences. Households have preferences summarized by the utility

$$\int_0^\infty e^{-\rho t} U(c_t, m_t)\, dt,$$

for $\rho > 0$ and U is concave utility function. In the no dollarization regime, liquidity services are provided by local currency: $m_t = m_t^d$ for all $t > 0$. If dollarization is announced, then

$$m_t = m_t^d \qquad t < T,$$
$$m_t = m_t^f \qquad t \geq T,$$

so that liquidity services pre-dollarization are provided by domestic currency, but provided by dollars post-dollarization are provided by dollars.[6]

[6] One might extend the analysis to allow dollars to provide some partial liquidity services. Announcing a delayed dollarization at T may also increase the liquidity services from dollar during $t \in [0, T)$ which would likely lower demand for local currency. We discuss some of the implications of this possibility in Section 3.2.

Fiscal Dominance and Seignorage. For $t < T$ we assume a situation of outright fiscal dominance, with some constant fiscal transfer $\tau_t = \bar{\tau}$ so that

$$\bar{\tau} = \frac{\dot{M}_t^d}{P_t} = \dot{m}_t^d + m_t^d \pi_t. \tag{1}$$

This condition amounts to a consolidated government budget constraint, due to the absence or non-issuance of government bonds. Transfers are covered by the real value of newly printed domestic currency. This seignorage equals the change in real money real balances $\dot{m}_t^d$ plus the inflation tax $m_t^d \pi_t$.

Dollarization. By definition, after dollarization domestic currency ceases to exist which requires fiscal balance to be achieved, so that for $t \geq T$

$$M_t^d = 0 \qquad \text{and} \qquad \tau_t = 0.$$

Dollarization at $t = T$ involves a conversion of pesos for dollars, with a retirement of the outstanding peso stock M_T. As our benchmark, we assume this conversion rate is endogenous and determined as follows. There is a given quantity $\bar{D}$ of dollars that the government is able to pledge to exchange or convert pesos for dollars at $t = T$; this is an important parameter in our analysis. The conversion or technical exchange rate at $t = T$ is then given by the ratio of outstanding pesos to dollars is $M_T / \bar{D}$. Recalling that the price equals the exchange rate this gives

$$P_T = \frac{M_T^d}{\bar{D}}$$

or equivalently that real domestic money balances must equal the pledged dollar amount $m_T^d = \bar{D}$.

Dollar Scarcity. When $\bar{D}$ is low we will say that dollars are scarce. When considering different values of $\bar{D}$ we either hold private dollars m_0^f fixed or we can hold the total dollars fixed

$$m_0^f + \bar{D} = NFA_0,$$

for some given net foreign asset position NFA_0. For the purposes of this paper, we prefer the latter exercise, because it better captures the fiscal strength of the government to pledge dollars for conversion, without changing the wealth or dollar availability for the country as a whole, which does not seem as directly relevant for our focus on the nominal exchange rate. In addition, in Caravello, Martinez-Bruera and Werning (2023) we previously studied the effects of different values of NFA_0, making a point that the scarcity of dollars in the country as a whole is an important determinant of outcomes post dollarization. Here we wish to focus on the scarcity of dollars the government has available for conversion and the effect that has prior to dollarization, after its announcement.

No Dollarization Regime. We consider two possibilities for the no dollarization benchmark. In the first scenario, we suppose that $\tau_t = \bar{\tau}$ for all $t > 0$. In the second scenario, we suppose that $\tau_t = 0$ for $t > T$. Although we treat the first case as our benchmark, it represents a pessimistic outlook and does not hold fiscal policy constant. This second case, instead, represents a "pure dollarization" exercise that holds the fiscal deficit covered by seignorage to be the same in both cases.

Equilibrium Definition. For given initial conditions M_0^d, m_0^f, and policy parameters $\bar{D}$, $\bar{\tau}$ and T, an equilibrium is a sequence $\{P_t, M_t^d, m_t^d, m_t^f, c_t\}$ such that (i) taking $\{P_t\}$ and $W_0 = m_0^f + M_0^d/P_0$ as given$\{m_t^d, m_t^f, c_t\}$ maximizes utility subject to the budget constraint; (ii) the international and money markets clear

$$m_t^f = m_0^f + \int_0^t (y - c_s)ds + \mathcal{I}_{t \geq T}\bar{D} \tag{2}$$

$$m_t^d = \frac{M_t^d}{P_t} \tag{3}$$

where $\mathcal{I}_{t \geq T} = 1$ if $t \geq T$ and 0 otherwise; and (iii) the government budget constraint holds

$$\bar{\tau} = \dot{m}_t^d + m_t^d \pi_t \qquad t < T. \tag{4}$$

and the conversion rate at $t = T$ is consistent with the government dollar pledge

$$m_T^d = \bar{D}. \tag{5}$$

By Walras' law, the equilibrium condition (2) is implied by the other equilibrium conditions,[7] so it can be dropped in the subsequent analysis.

Equilibrium using Money Demand. Appendix A shows that real money balances can be determined using a Frisch money demand where $m^*(\pi, \mu)$ is decreasing in inflation π

[7] By Walras' law, it can also be interpreted as being implied in equilibrium (see equilibrium definition below). Indeed, combining the agent budget constraint with market clearing in international markets (or the current account balance) which requires $\dot{m}_t^f = y - c_t$, implying $\dot{m}_t^d = -m_t^d \pi_t + \tau_t$. Then using that $m_t^d = M_t^d/P_t$ so that $\dot{m}_t = \dot{M}_t^d/P_t + m_t^d \pi_t$ gives the stated condition above.

and decreasing in μ the marginal utility of consumption. In particular

$$m_t = m^*(\pi_t, \mu_t) \qquad t < T$$

where μ_t is the marginal utility of income which satisfies $\mu_t = U_c(c_t, m_t) = e^{\rho t}\mu_0$. An equilibrium for $t \in [0, T)$ is fully determined once we know μ_0 and $\{\pi_t\}$.

For given μ_0 the money demand schedule is determined at each point in time and we can solve for $\{\pi_t\}$ using conditions (4) and (5). The appendix provides an equation to that μ_0 must satisfy to fully characterize an equilibrium.

Income Effects. In general, policy induces income effects that affect the marginal utility μ_0. For example, if we compare the no dollarization regime to the dollarization one, the latter induces negative income effects; this is related to the results in Caravello, Martinez-Bruera and Werning (2023) showing that dollarization leads to a drop in consumption, to accumulate dollars for use as money. Relatedly, comparative statics on T have income effects and affect μ_0. In contrast, comparative statics on $\bar{D}$ do not affect μ_0 because we hold initial wealth $m_0^f + \bar{D} = NFA_0$ constant, a fact that we shall exploit. Indeed, with dollarization and under additive separable utility one can solve for μ_0 as a function of $m_0^f + \bar{D}$ and T only.

If money demand is sensitive to income effects, then policy announcements shift the money demand schedule at any point in time. In particular, the announcement of dollarization will generally shift money demand down, although in practice this effect may be modest.

3 Induced Dynamics from Announcement

This section contains our main results. We start by providing a very and sharp graphical characterization in the case where there are no income effects for money demand. This also provides an approximation when income effects on money demand are small, which we conjecture is likely to be the case in most cases. The next subsection, however, provides results without relying on this assumption.

3.1 Stationary Money Demand: Graphical Solution

In this subsection we assume that the Frisch demand for money $m^*(\pi, \mu)$ does not vary with μ_t. A specific example is a quasi-linear utility function $u(c + h(m))$. We write

$$m_t^d = L(\pi_t)$$

for some decreasing money demand function L. This implies that money demand does not vary over time and it does not vary across equilibria; in particular, money demand is the same under no-dollarization as under dollarization. This is a useful starting point and may be considered a good approximation to the other cases we tackle in the next section. Stationary money demands form the basis of classical studies of seignorage, such as Sargent and Wallace (1973).

Solving the Equilibrium: A Simple Differential Equation. Define

$$S(\pi) \equiv L(\pi)\pi$$

This is often called a "Laffer curve" and depicted as in Figure 1. To study equilibrium dynamics, one can combine $\bar{\tau} = \dot{m}^d + S(\pi)$ with $m_t^d = L(\pi_t)$ to obtain

$$\bar{\tau} = m^{*\prime}(\pi)\dot{\pi} + S(\pi),$$

an equation for the dynamics of inflation. Returning to Figure 1, π_G^* and π_B^* are steady states where $S(\pi_G^*) = S(\pi_B^*) = \bar{\tau}$ so that $\dot{\pi} = 0$. Since $\dot{\pi} \geq 0$ if and only if $\bar{\tau} \leq S(\pi)$, the dynamics around steady state π_G^* are unstable, whereas π_B^* is stable.[8] This is essentially the approach developed by Sargent and Wallace (1973).

We pursue a small variant of this strategy which turns out to be preferable: a change of variables towards real money balances, rather than inflation. Thus, let $R(m^d)$ denote seignorage as a function of real money balances.[9] Then condition (4) becomes

$$\dot{m}_t^d = \bar{\tau} - R(m_t^d), \tag{6}$$

an ordinary differential equation (ODE) for $\{m_t^d\}_{t=0}^T$.

The equilibrium condition (5), requiring dollarization conversion to be feasible, then

[8]Dynamic instability under rational expectations around π_G^* should not be misinterpreted as making π_G^* more plausible than π_B^*—indeed, a strong case can be mounted to the contrary and π_G^* is usually selected by various small departures from rational expectations.

[9]Let ϕ denote the inverse of m^*, so that ϕ is decreasing. Then $R(m) = S(\phi(m))$.

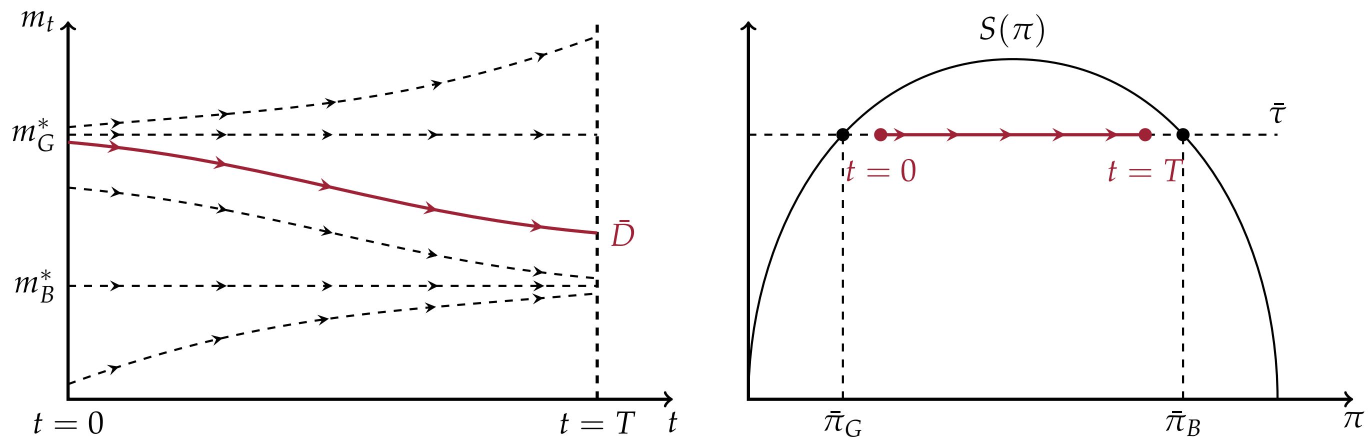

Figure 2: Equilibrium dynamics when dollars are scarce but not extremely scarce $\bar{D} \in (m_B^*, m_G^*)$.

provides the necessary boundary condition:

$$m_T^d = \bar{D},$$

so that real money balances reach the level of pledged dollars.

To solve an equilibrium: we can solve for the entire path $\{m_t^d\}$ starting from this boundary, working backwards. Given M_0 we can then compute $\{P_t\}$, $\{M_t^d\}$ and π_t uniquely.

Scarce Dollars, but not extremely scarce. Figure 2 illustrates the dynamics of paths solving the differential equation (6). Given $\bar{D}$ the equilibrium is then given by the path that also satisfies the boundary condition $m_T^d = \bar{D}$. The equilibrium path is shown in solid red for the case where $\bar{D} \in (m_B^*, m_G^*)$, a situation where dollars are scarce, but not extremely scarce.

With the aid of these figures, various properties can be deduced. First, with scarce dollars $\bar{D} < m_G^*$ real money must start below the no dollarization steady state $m_0 < m_G^*$. Since nominal balances M_0^d are given, this requires an upward jump in the exchange rate (devaluation) and an upward jump in price level P_0. Second, the entire path $\{m_t\}$ lies below m_G^* implying that the entire path for $\{\pi_t\}$ must lie above π_G^* so the devaluation and inflation rate is higher under dollarization. Finally, the path $\{m_t\}$ declines over time towards $\bar{D}$ implying that π_t rises over time. We collect these results in the next proposition.

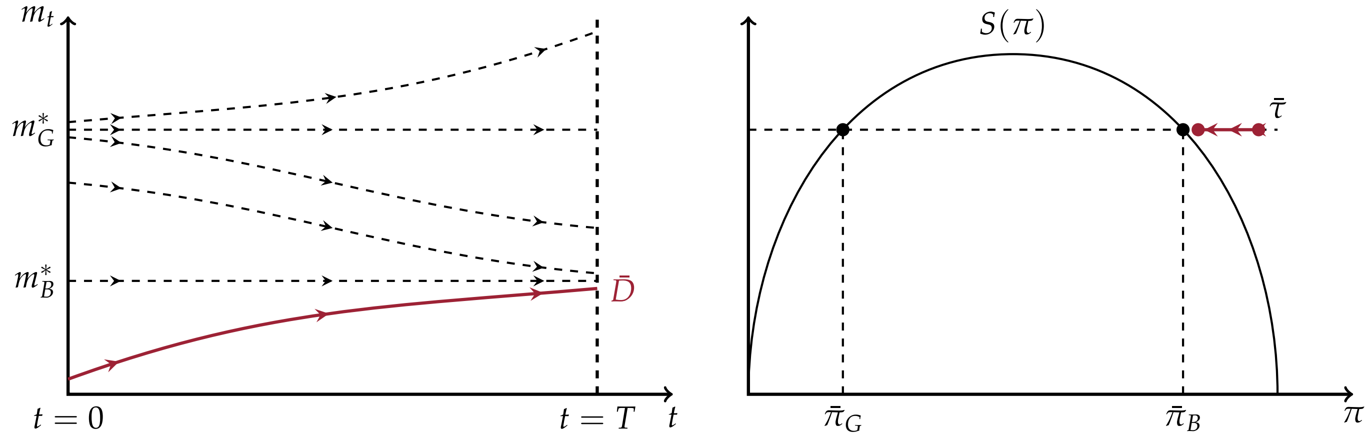

Figure 3: Equilibrium dynamics when dollars are extremely scarce $\bar{D} \in (0, m_B^*)$.

Proposition 1. *If dollars are scarce but not extremely scarce so that $\bar{D} \in (m_B^*, m_G^*)$, then announcing a future dollarization leads to*

1. *a discrete devaluation on impact ($t = 0$) and an upward jump in price level P_0;*
2. *a higher depreciation and inflation rate thereafter: $\pi_t > \pi_G^*$ for all $t \in (0, T)$;*
3. *depreciation and inflation rates that rise over time: π_t increasing in t.*

Intuitively, with scarce dollars, we are offering a bad exit deal for domestic currency holders at $t = T$. Earlier on, the lower value for money increases money issuances and inflation to meet the fixed seignorage needs.

Extremely Scarce Dollars. Figure 3 depicts the situation with greater dollar scarcity $\bar{D} < m_B^*$. The equilibrium path $\{m_t^d\}$ lies below m_B^* and rises over time, with inflation falling over time but always above the high steady state π_B^*. Inflation is high front-loaded.

Proposition 2. *If dollars are extremely scarce so that $\bar{D} \in (0, m_B^*)$ then announcing a future dollarization leads to*

1. *a discrete devaluation on impact ($t = 0$) and an upward jump in price level P_0;*
2. *a higher depreciation and inflation rate thereafter: $\pi_t > \pi_B^* > \pi_G^*$ for all $t \in (0, T)$;*
3. *depreciation and inflation rates that fall over time: π_t decreasing in t.*

The intuition for the result is similar to before. The extremely bad exit value pulls down the value of domestic currency, requiring high money issuances and inflation to satisfy the seignorage requirements. Why are the dynamics inverted relative to the previous case? This is related to the notion that the rational expectation dynamics are "unstable" at the good side of the Laffer curve, but "stable" on the bad side.

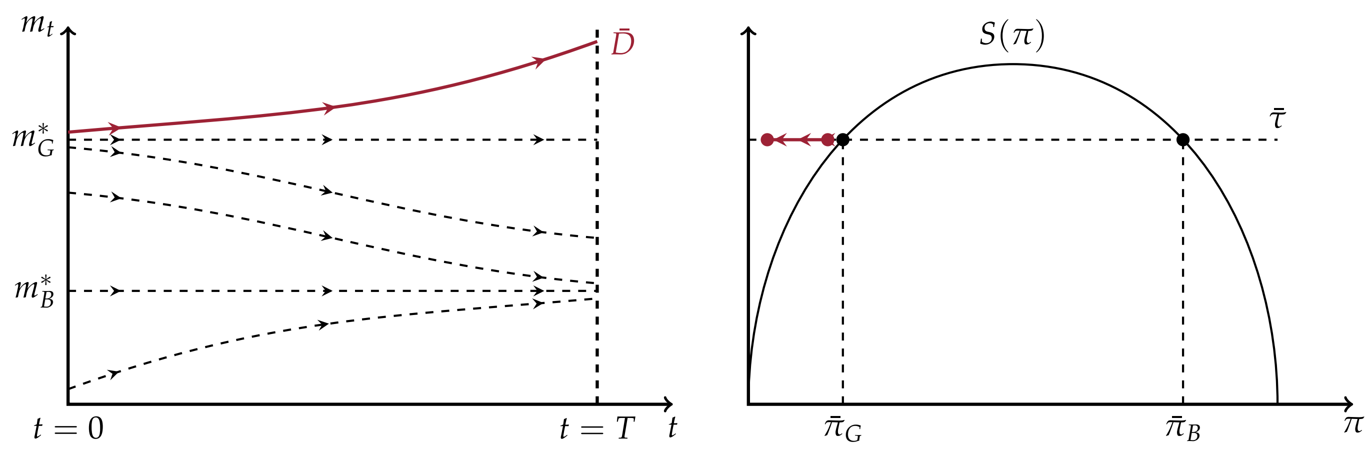

Figure 4: Equilibrium dynamics when dollars are abundant $\bar{D} > m^*_G$.

Abundant Dollars. We complete the picture with the most benign case, where dollars are abundant, so that $\bar{D} > m^*_G$. This situation is depicted in Figure 4. We see that $\{m_t\}$ lies above m^*_G and rises, so that inflation is below π^*_G and declines over time.

Proposition 3. *If dollars are abundant $\bar{D} > m^*_G$ then the announcement of a future dollarization leads to:*

1. *a discrete appreciation on impact ($t = 0$) and a downward jump in price level P_0;*
2. *a lower depreciation and inflation rate thereafter: $\pi_t < \pi^*_G$ for all $t \in (0, T)$;*
3. *depreciation and inflation rates that fall over time: π_t decreasing in t.*

Intuitively, with abundant dollars, we are offering a sweet exit deal for domestic currency holders at $t = T$. Earlier on, this higher exit value allows for lower money issuances and lower inflation to meet the same seignorage needs.

Two Interesting Knife-Edged Cases. Note that at the knife-edge case with $\bar{D} = m^*_G$. This case lies just between Proposition 1 and Proposition 3. Using the figures, one sees that the equilibrium outcomes before dollarization are just as in the no dollarization benchmark. In particular, inflation equals π^*_G and there is no jump in the exchange rate or price level on impact. A dollarization that pledges just the right amount of dollars for conversion has not impact on the equilibrium pre-dollarization.

Another noteworthy knife-edge case is $\bar{D} = m^*_B$. This case lies just between Proposition 1 and 2. Using the figures one sees that real money balances are constant at m^*_B, implying a constant inflation equal to π^*_B "bad side of the Laffer curve".

Comparative Statics of Dollarization Plans. Finally, we ask what are the effects of pledging fewer dollars or speeding up dollarization? That is, what are the comparative statics of $\bar{D}$ and T?

A lower dollar pledge $\bar{D}$ leads to a lower m_T and, thus, lower path $\{m_t^d\}$; this then implies a higher path $\{\pi_t\}$ and a higher jump for $P_0 = M_0^d/m_0^d$. Intuitively, a lower dollar pledge worsens the exit deal, lowering the conversion rate, which then reduces the exchange rate throughout.

Interestingly, the effect on the conversion rate itself is amplified by the endogenous rise in nominal money supply. To see this note that $\dot{M}_t^d = \bar{\tau} P_t$ but M_0^d is unchanged and we have argued that P_t rises at each point in time t with the drop in $\bar{D}$. This leads to a higher M_T^d which amplifies the effect of the lower $\bar{D}$ on $M_T^d/\bar{D}$.

What about the effects of speeding up dollarization, of lowering T? In this case, the effects depend on the dollar pledge $\bar{D}$. To see this, note that the endpoint $m_T = \bar{D}$ and $\pi_T = L^{-1}(\bar{D})$ do not vary with T. When dollars are scarce but not extremely scarce, then m_t falls over time and π_t rises over time. Lowering T then increases the devaluation rate as well as the jump on impact at $t = 0$. However, in the other two cases this logic is inverted since real money balances were increasing and inflation was deceasing with time.

Finally, what are the effects of a lower demand for money? This could occur if before T dollars become a partial substitute for domestic currency, lowering the demand for domestic currency.

Proposition 4. *Suppose dollarization is announced to be implemented at T with pledged dollars $\bar{D}$ for conversion. Then*

1. A decrease in $\bar{D}$ increases the depreciation on impact and the depreciation rate. The conversion rate $M_T \bar{D}^{-1}$ rises more than proportionally with $\bar{D}^{-1}$.

2. (a) if $D \in (m_B^*, m_G^*)$ then a decrease in T leads to a higher depreciation on impact at $t = 0$ and a higher depreciation rate for $t \in (0, T)$; (b) conversely, if $D \notin (m_B^*, m_G^*)$ then a decrease in T lowers depreciation on impact $t = 0$ and a lower depreciation rate for $t \in (0, T)$.

3. Under no dollarization or dollarization (for any given T and $\bar{D}$) a drop in the money demand schedule $L(m)$ implies a higher equilibrium exchange rate and price path $\{P_t\}$.

As we show in Section 3.2, the first part of this result extends to the case with income effects.

Fiscal Reform Without Dollarization. Our no-dollarization benchmark took a relatively pessimistic view and assumed a constant deficit $\bar{\tau}$ forever. In contrast, our dollarization regime had a fiscal reform at T that reduced the deficit to zero. One motivation for these different fiscal paths is the idea that dollarization may offer a commitment device to forego seignorage and force fiscal discipline.

Thus, our previous analysis was not an "apples to apples" comparison of monetary regimes. We now consider a relatively optimistic variant, where a fiscal reform is also undertaken in the no-dollarization regime at T. Thus, the two fiscal deficit paths are the same. This comparison turns out to be simpler and our main conclusion are reinforced for a wider set of dollar pledges.

Proposition 5. *Consider a no dollarization regime with the a fiscal reform at T so that $\tau_t = 0$ for $t \geq T$. Then Propositions 1–2 continue to hold. However, for a range of abundant dollars $\bar{D} \in (m_G^*, L(0))$, announcing a future dollarization leads to*

1. *a discrete depreciation on impact ($t = 0$) and a upward jump in price level P_0;*
2. *a higher depreciation and inflation rate thereafter: $\pi_t < \pi_t^{ND} < \pi_G^*$ for all $t \in (0, T)$;*
3. *depreciation and inflation rates that fall over time: π_t decreasing in t.*

This result reverses parts 1-2 in Proposition 3. Even with relatively abundant dollars, dollarization leads to a devaluation and inflation. Part 3 remains unchanged: the devaluation and inflation rate falls over time.

The proof of this result is as follows. The equilibrium under no dollarization has no inflation for $t \geq T$. Thus, we solve (6) backwards in time from $t = T$ to $t = 0$ using the boundary condition $m_T = L(0)$. This implies that π_t is increasing in T—if fiscal reform is further out in the future, inflation is higher. Indeed, for any t we have that $\pi_t \in [0, \pi_G^*)$ and $\pi_t \to \pi_G^*$ as $T \to \infty$. The dollarization equilibrium now coincides with the no dollarization one when $D = L(0) < m_G^*$ (previously, it required $D = m_G^*$). However, for any $D > L(0)$ the initial price level and inflation is higher under dollarization than under no dollarization, this follows from the monotonicity in $\bar{D}$ as in Proposition 4.

3.2 Money Demand Shifts and Income Effects

We now consider cases where money demand shifts when dollarization is announced. This amounts to two things. First, income effects. Second, outright shifts in domestic money demand due to dollarization.

We no longer abstract from income effects. We adopt an additive separable specification of utility

$$U(c, m) = u(c) + U_m(m).$$

Dollarization creates a negative income effect that reduces money demand; money demand may also shifts over time. The proof of the next result is contained in Appendix B.

Proposition 6. *Suppose dollarization is announced to be implemented at T with pledged dollars $\bar{D}$ for conversion. Then*

1. A decrease in $\bar{D}$ increases the depreciation on impact and the depreciation rate. The conversion rate $M_T\bar{D}^{-1}$ rises more than proportionally with $\bar{D}^{-1}$.

2. There exists $\hat{D} \leq m_G^*$ such if $\bar{D} < \hat{D}$ then the there is a discrete devaluation on impact ($t = 0$) with an upward jump in price level P_0 and a higher depreciation and inflation rate thereafter: $\pi_t > \pi_G^*$ for all $t \in (0, T)$.

Similar results obtain if we consider outright shift in money demand for domestic currency. This may capture, in reduced form, an increasing use of the dollar as a substitute for domestic currency.

4 Conclusions

When dollars pledged for conversion are scarce, announcing dollarization may harbor low inflation in the long run once dollarization is implemented, but it exacerbates the problems in the short run with a discrete devaluation on impact and higher rates of inflation and devaluation thereafter.

A Solving the Equilibrium Pre-Dollarization

In this section we derive the household first order conditions and use them to characterize the equilibrium in a simple manner. We focus on the pre-dollarization phase, and this focus allows us to solve things quite simply.

The problem at $t = 0$ can be written as

$$\max_{c,W,m} \int_0^T e^{-\rho t} U(c_t, m_t) dt + e^{-\rho t} V(W_T)$$

subject to

$$W_T = W_0 + T\bar{\tau} + \int_0^T (y - c_t - m_t \pi_t)\, dt \tag{7}$$

where the value function V encapsulate the continuation utility from $t \geq T$ after dollarization and is given by

$$V(W_T) \equiv \max_{c,m} \int_0^\infty e^{-\rho s} u(c_{T+s}, m_{T+s}) ds$$

with $m_T = W_T$, $\dot{m}_t = y - c_t$ and $m_t \geq 0$. a value function V. The problem post dollarization (behind the value function) was studied in our previous work Caravello, Martinez-Bruera and Werning (2023) (indeed, for a richer model) and is also related to dynamic studied in Calvo (1981) (see also Vegh, 2013).

Letting μ denote the multiplier on (7) the first-order conditions for $t < T$ are

$$e^{-\rho T} V'(W_T) = \mu_0$$

and

$$e^{-\rho t} U_c(c_t, m_t) = \mu_0,$$
$$e^{-\rho t} U_m(c_t, m_t) = \pi_t \mu_0.$$

These last two equations define a Frisch demand system

$$c_t = c^*(\pi_t, \mu_0, t)$$
$$m_t = m^*(\pi_t, \mu_0, t)$$

for some functions c^* and m^* that are decreasing in marginal utility μ and decreasing over time t.[10] In addition, $m^*(\pi, \mu_0, t)$ is decreasing in π; and c^* is increasing in π if $U_{cm} \geq 0$ and decreasing if $U_{cm} \leq 0$.

Given these relations, we are left to determine μ and $\{\pi_t\}$. Or equivalently, μ and $\{m_t\}$ since π_t and m_t are in a one to one relation, given μ at any time t.

With U is additively separable then c^* does not depend on π. Using the FOC for consumption and the terminal condition, we find the following equation that pins down μ_0:

$$\mu_0 = e^{-\rho T} V' \left(NFA_0 + \int_0^T (y - c^*(\mu_0, t)) dt \right), \tag{8}$$

Due to an envelope argument, we have that after announcing dollarization $V'(W_T)$ is higher for all W_T, since by the arguments spelled out in Caravello, Martinez-Bruera and Werning (2023) at time T we will be below the steady-state level of dollar holdings. This induces high marginal utility of having dollars, which after T provide liquidity services. Therefore, after announcing dollarization $V'(W_T)$ increases due to this effect, which implies that μ_0 will be above the pre steady-state μ_{0^-}.

[10]The downward trend in consumption and money holdings is not robust, it is due to several of our simplifying assumptions. The presence of positive subjective discounting combined with the absence of interest on foreign assets, the absence of liquidity and precautionary motives. Below we remove this unimportant aspect by taking the limit of no discounting.

Note also that the dependence of c^* and m^* on t vanishes as $\rho \to 0$. In that case, the equation for $\mu_0$simpifies to:

$$\mu = V'\left(NFA_0 + T(y - c^*(\mu))\right), \tag{9}$$

which is independent of $\{\pi_t\}$, but dependent on T. We denote money demand by $L(\pi) = m^*(\pi; \mu)$ suppressing the dependence on μ.

B Proof of Proposition 6

Part 1. We first show monotonicity of $\{m_t\}$ with respect to $\bar{D}$. The path $\{m_t\}$ is characterized as the solution to:

$$\dot{m} = \bar{\tau} - e^{-\rho t}\frac{U_m(m)m}{\mu} = f(t, m)$$

with terminal condition $m_T = \bar{D}$. We proceed by contradiction. Assume that there exists t_0 and t_1, with $t_1 > t_0$,such that $m_{t_0} > m_{t_1}$but $m'_{t_0} < m'_{t_1}$ i.e there is a crossing between the trajectories m'_t and m_t. Then, by continuity of the solution, there exists a $\tilde{t}$ such that $m_{\tilde{t}} = m'_{\tilde{t}} = \tilde{m}$. Now consider the same system starting at time $\tilde{t}$ and $\tilde{m}$. Since there is a crossing between $\{m_t\}$ and $\{m'_t\}$, that implies that the solution to the ODE with those initial conditions is not unique. But this contradicts the Picard–Lindelöf theorem: our assumptions imply that$f$ is Lipschitz locally around $(\tilde{t}, \tilde{m})$.

The decrease in the depreciation rate follows directly from the fact that comparative statics in $\bar{D}$ do not affect μ_0, so the money demand schedule is unaffected. Since $D' > D$ implies $m'_t \geq m_t$ for all t, then it also implies $\pi'_t \leq \pi_t$ for all t. The more than proportional pass-though comes from the fact that, from the seigniorage equation, $M_T = M_0 + \frac{1}{\bar{\tau}}\int_0^T P_t dt$, so if the initial jump and also the depreciation rate are higher for D compared to D', we have $P'_t \leq P_t$. The fact that it is more than proportional then follows from the same argument in the text.

Part 2. For part 2 of the proposition, consider the figure with the Laffer curve corresponding to t. Define $\pi^*_G(t)$ and $\pi^*_B(t)$ as the two solutions to

$$\bar{\tau} = S(\pi, T)$$

pick $\hat{\pi} > \pi^*_B(0)$. Let $\hat{D} = L(\pi^*_B(0), 0)$. Then, we know that over time the Laffer curve will shrink towards zero. Thus, we never leave the region in which $\pi_t > \pi^*_B(0) > \pi^*_B(t)$. But we know from our analysis that in that region π_t grows over time. Since $\pi^*_B(0) > \pi^*_G(0)$

this shows the last part of the proposition. In order to see that the price level jumps downwards, note that without considering income effects nor discounting, we had that at m_G^* there was no jump. Once we add both income effects and discounting, money demand is lower for all t. That implies that the right hand side of the ODE, $f(t, m)$ is now higher than its counterpart in Section 3 for all t, m, i.e $f(t, m) > \bar{\tau} - R(m)$. Thus, if we started at $\tilde{D} = m_G^*$, now we would have $f(t, \tilde{m}_t) > \bar{\tau} - R(\tilde{m}_t)$. Since the solution for that initial condition in Section 3 featured $\dot{m} = 0$, we would now have $\dot{\tilde{m}} > 0$ along the transition, which implies that $m_0 < m_G^*$ when solving backwards. Finally, due to monotonicity, if this holds for $\tilde{D} = m_G^*$, then $m_0 < m_G^*$ also holds for all $\bar{D} < \hat{D}$.

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