

Modeling Insurance Markets

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Modeling Competition Insurance Markets is Tough

- There is no well-agreed upon model of competitive insurance markets
 - Despite 50 years of research!
- Standard notions of pure strategy competitive equilibria break down
 - Preferences/Demand are related to cost
- Insurers can manipulate not only price but also the design of contracts to affect their own (and others) costs
 - Leads to unraveling!

- Akerlof (1970): Cars lose value the day after they're sold...
 - Argued that market for health insurance above age 65 does not exist because of adverse selection
 - Market unraveled because of adverse selection "death spiral"
- Problem with model: single contract traded, so competition only on price
 - Rothschild and Stiglitz (1976) + 1000+ other papers...
 - Compete on more than 1 dimension of the contract
 - Standard game-theoretic notions of (pure strategy) equilibria may not exist -> "Market unraveling"

- Clarify when the standard competitive model goes wrong (and hence we have to choose amongst competing game-theoretic models)
 - Clarify what we mean by “unraveling”
- Discuss 2 classes of “solutions” to non-existence
 - Miyazaki-Wilson-Spence (Reach the constrained pareto frontier)
 - Riley (1979) (Don't reach the frontier)
- Context: Binary insurance model with uni-dimensional type distribution

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- Agents vNM preferences

$$pu(c_L) + (1 - p)u(c_{NL})$$

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- First, insurers simultaneously offer a menu of consumption bundles
- Given the set of available consumption bundles,

$$A = \cup_j A_j$$

individuals choose the bundle that maximizes their utility

Definition

An allocation $A = \{c_L(p), c_{NL}(p)\}_{p \in \Psi}$ is a **Competitive Nash Equilibrium** if

- 1 A is incentive compatible

$$pu(c_L(p)) + (1-p)u(c_{NL}(p)) \geq pu(c_L(\tilde{p})) + (1-p)u(c_{NL}(\tilde{p})) \quad \forall p, \tilde{p}$$

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- 3 A has no profitable deviations [Next Slide]

No Profitable Deviations

For any other menu, $\hat{A} = \{\hat{c}_L(p), \hat{c}_{NL}(p)\}_{p \in \Psi}$, it must be that

$$\int_{p \in D(\hat{A})} [p(w - l - c_L(p)) + (1 - p)(w - c_{NL}(p))] dF(p) \leq 0$$

where

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- Require that the profits earned from these people are non-positive

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- **Rothschild and Stiglitz unraveling**

- Realize a Competitive Nash Equilibrium may not exist
- Market unravels a la Rothschild and Stiglitz when there does not exist a Competitive Nash Equilibrium

Theorem

The endowment, $\{(w - l, w)\}$, is a competitive equilibrium if and only if

$$\frac{p}{1-p} \frac{u'(w-l)}{u'(w)} \leq \frac{E[P|P \geq p]}{1 - E[P|P \geq p]} \quad \forall p \in \Psi \setminus \{1\} \quad (1)$$

where $\Psi \setminus \{1\}$ denotes the support of $F(p)$ excluding the point $p = 1$.

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 - Theorem extends Akerlof unraveling to set of all potential traded contracts, as opposed to single contract

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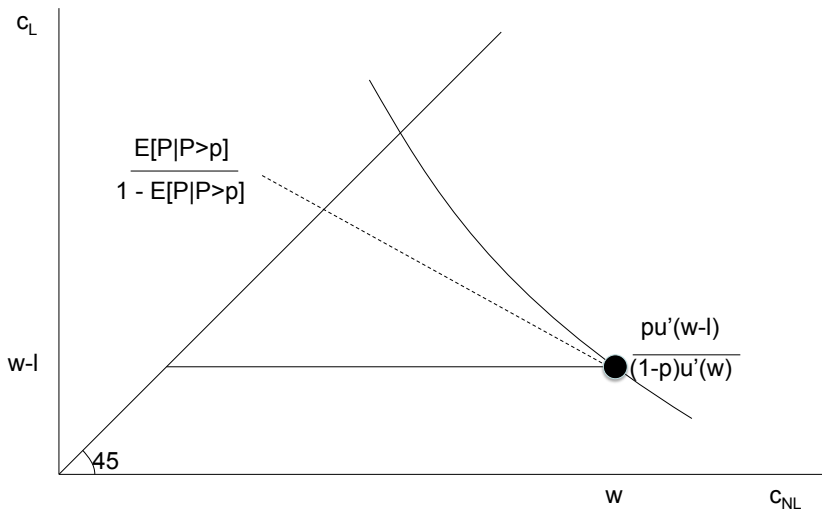
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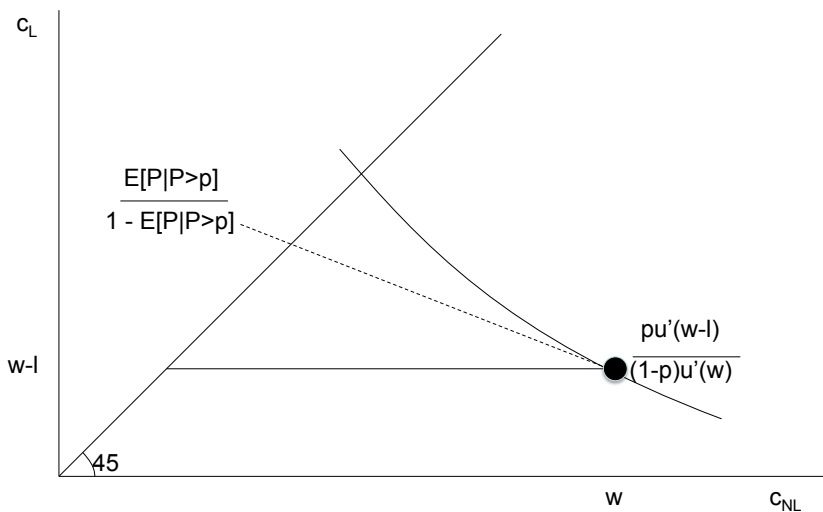
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 - No gains to trade -> no profitable deviations by insurance companies

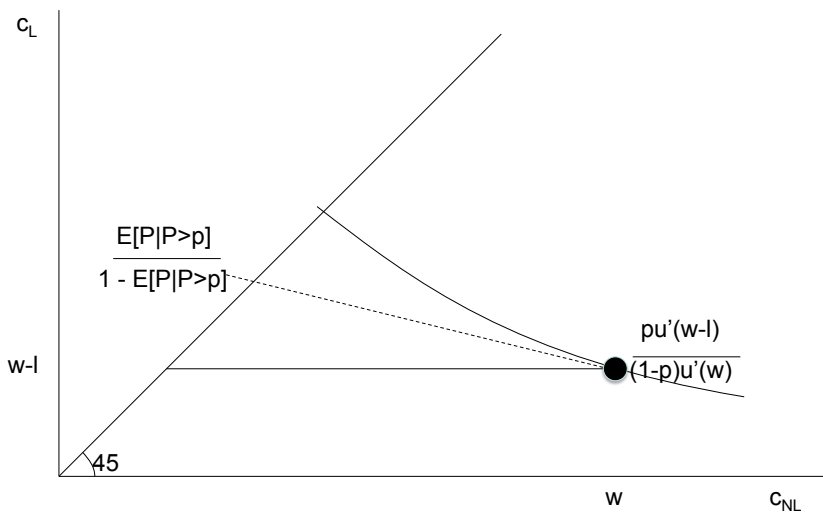
Akerlof Unraveling



Akerlof Unraveling (2)



Akerlof Unraveling (3)



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- Need full support of type distribution to get complete Akerlof unraveling
 - Can be relaxed with some transactions costs (see Chade and Schlee, 2013)

- When does a Competitive Nash Equilibrium exist?

Rothschild and Stiglitz Unraveling

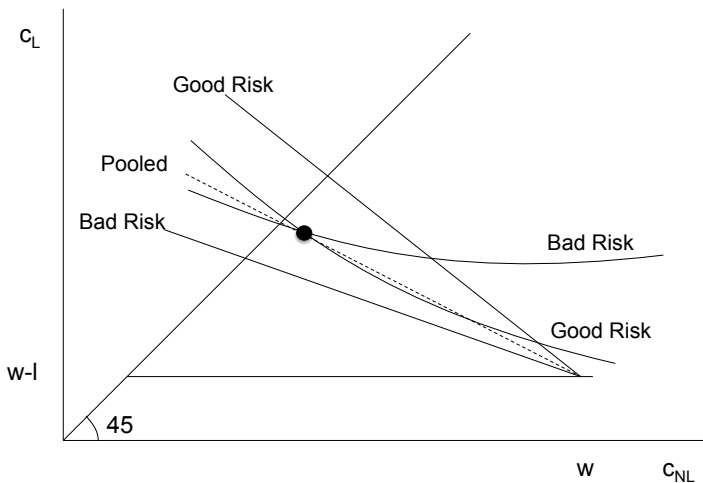
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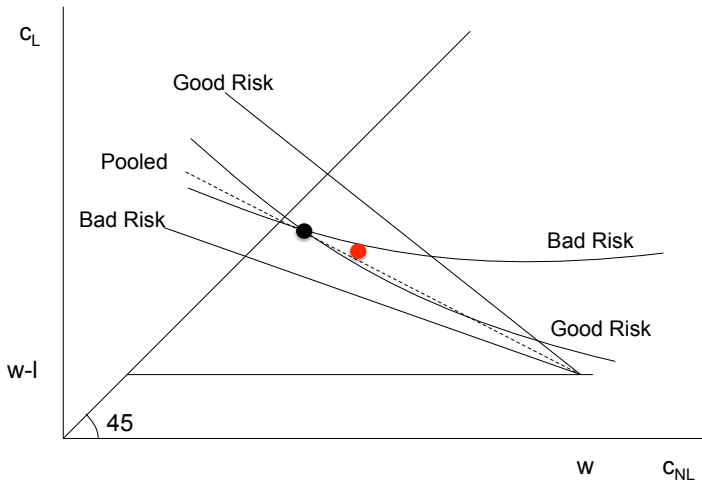
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- Key insight of Rothschild and Stiglitz (1976): Nash equilibriums can't sustain pooling of types

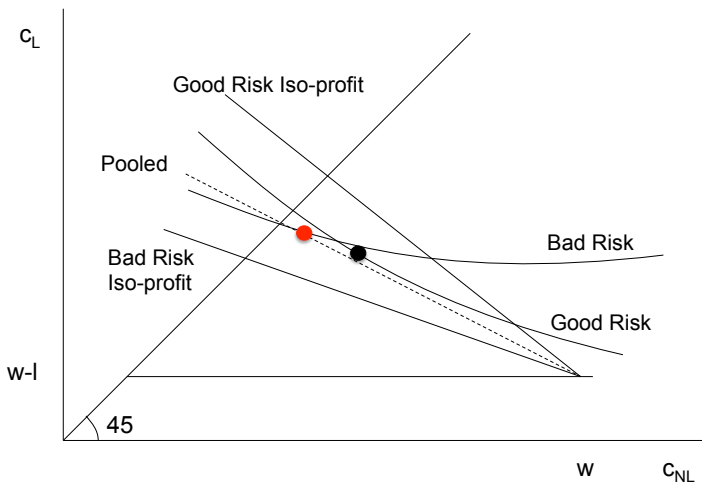
Rothschild and Stiglitz: No Pooling



Rothschild and Stiglitz: No Pooling (2)



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- Can approximate any distribution with distributions satisfying the regularity condition

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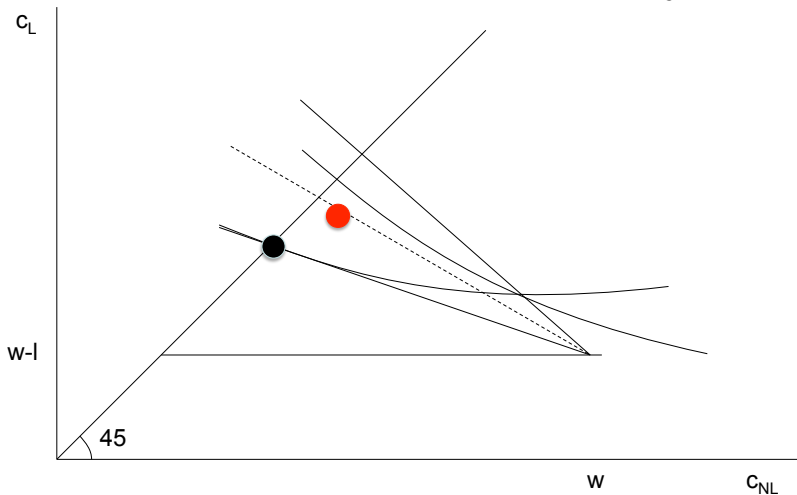
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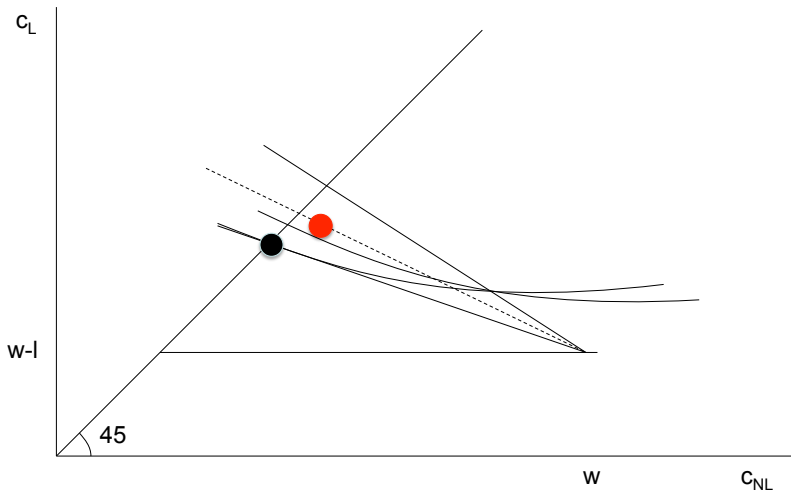
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 - We know Akerlof unraveling condition cannot hold
 - Follow Riley (1979) – shows there's an incentive to pool types \rightarrow breaks potential for Nash equilibrium existence

Generic No Equilibrium (Riley)



Generic No Equilibrium (Riley) (2)



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- We don't have a model of insurance markets!
 - Generically, the standard Nash model generically fails to make predictions precisely when there are theoretical gains to trade

Solutions to the Non-Existence Problem

- Two classes of models in response to non-existence
- Consider 2-stage games:
- Stage 1: firms post menu of contracts
- Stage 2: Assumption depends on equilibrium notion:
 - Miyazaki-Wilson-Spence: Firms can drop unprofitable contracts
 - Formalized as dynamic game in Netzer and Scheuer (2013)
 - Riley: Firms can add contracts
 - Formalized as dynamic game in Mimra and Wambach (2011)
- Then, individuals choose insurance contracts

- Reaching the Pareto frontier requires allowing some contracts to run deficits/surplus
 - Riley shows that individuals generically are willing to “buy off” worse risks’ incentive constraints
- Miyazaki Wilson Spence allows for this if the good types want to subsidize the bad types
 - If you try to steal my profitable contract, I drop the corresponding negative profit contract and you get dumped on!
- **MWS equilibrium maximizes welfare of best risk type by making suitable compensations to all other risk types to relax IC constraint**

- Predicts “fully separating” contracts with no cross-subsidization across types
 - IC constraint + zero profit constraints determine equilibrium
- Why no cross-subsidization?
 - If cross-subsidization, then firms can add contracts.
 - But, firms forecast this response and therefore no one offers these subsidizing contracts
- Predicts no trade if full support type distribution

- Walrasian:
 - Bisin and Gotardi (2006)
 - Allow for trading of choice externalities -> reach efficient frontier/MWS equilibrium (pretty unrealistic setup...)
 - Azevedo and Gottlieb (2014)? -> reach inefficient Riley equilibria
- Search / limited capacity / etc.
 - Guerrieri and Shimer (2010) -> reach inefficient Riley equilibria

Empirical Question?

- Need theory of a mapping from type distributions to outcomes
 - Standard model works if prediction is no trade
 - Hendren (2013) shows this happens for those with “pre-existing conditions” in LTC, life, and disability insurance
 - But, standard model fails when market desires cross-subsidization
 - Key debate: can competition deliver cross-subsidization?
 - Should be empirical question!?
- In short, insurance markets are fun because no one agrees about how to model them!