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# Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks: Comment

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In a recent issue of this journal, Stephen Morris and Hyun Song Shin (1998a) prove the uniqueness of an equilibrium in a model of self-fulfilling currency attacks, when speculators face uncertainty in their signals about macroeconomic fundamentals.

In Theorem 2 of their paper, Morris and Shin characterize the equilibrium as uncertainty approaches zero. They claim that the threshold of the fundamental state, up to which a currency attack will occur with probability one, is independent of the critical mass of capital needed for an attack to be successful. Hence, direct capital controls are less effective when speculators have fairly precise information about fundamentals.

Yet, their Theorem 2 holds only for a special case. This Comment gives the correct generalization and proves that for small uncertainty currency crises depend on the critical mass of capital needed for success. Thus, direct capital controls are effective even when fundamentals are fairly transparent to all market participants.

The reduced-game structure is given by the following assumptions: Fundamentals of the economy are characterized by some parameter $\theta$ unknown to agents. There is a continuum of agents receiving independently and identically distributed (i.i.d.) signals $x^i$ about the fundamentals. Each agent must decide whether or not to attack the currency, which is associated with transaction costs $t > 0$. If a proportion $a(\theta)$ of all traders attacks the currency, the attack is successful and each attacking agent obtains a reward $R(\theta) = e^* - f(\theta)$. $a$ and $R$ are continuous; there is a state $\underline{\theta}$ with $a(\theta) = 0$ for all $\theta \leq \underline{\theta}$; $a$ is strictly increasing above $\underline{\theta}$; and $a(\theta) < 1$ for all $\theta$. $R$ is strictly decreasing and

there is a state $\bar{\theta} > \underline{\theta}$ with $R(\bar{\theta}) = t$. $\theta$ is uniformly distributed over an interval, say [0, 1]. Given $\theta$, signals have a uniform distribution in $[\theta - \varepsilon, \theta + \varepsilon]$. Critical levels $\underline{\theta}$ and $\bar{\theta}$ must be at least $2\varepsilon$ away from the margins of the interval [0, 1].

In Section II of their paper, Morris and Shin show that a unique equilibrium switching point $x^*$ and a threshold $\theta^*$ exist, such that an agent attacks [does not attack] the currency if her signal $x^i$ is smaller [larger] than $x^*$, and that a successful speculative attack occurs with probability one [zero] if state $\theta$ is below [above] $\theta^*$.

In Section III, Theorem 2 states that "In the limit as $\varepsilon$ tends to zero, $\theta^*$ is given by the unique solution to the equation $f(\theta^*) = e^* - 2t$" (p. 594). As stated, this theorem holds only for the special case where $a(\theta^*) = \frac{1}{2}$. It should therefore be generalized as follows.

REVISED THEOREM: *In the limit as $\varepsilon$ tends to zero, $\theta^*$ approaches $\theta_0 \in (\underline{\theta}, \bar{\theta})$, given by the unique solution to the equation*

$$(1) \qquad (1 - a(\theta_0))R(\theta_0) = t.$$

The flaw in the proof by Morris and Shin (1998a, Appendix p. 597) occurs when, calculating $F'$, they ignore that $x^*$ and $\theta^*$ depend on $\varepsilon$.

PROOF OF THE REVISED THEOREM:

Given the assumptions above, equilibrium values $x^*(\varepsilon)$ and $\theta^*(\varepsilon)$ are characterized by the following two equations:

$$(2) \quad u(x^*, I_{x^*}) = \frac{1}{2\varepsilon} \int_{x^*-\varepsilon}^{\theta^*} R(\theta) \, d\theta - t = 0 \, ,$$

$$(3) \quad s(\theta^*, I_{x^*}) = \frac{x^* - \theta^* + \varepsilon}{2\varepsilon} = a(\theta^*).$$

$u(x^*, I_{x^*})$ is the expected payoff to an attacking

agent if all agents attack if and only if they get signals below $x^*$, and $s(\theta^*, I_{x^*})$ is the proportion of speculators who get signals below $x^*$ if the true state is $\theta^*$. Condition (3) can be rewritten as $(\theta^* - x^* + \varepsilon)/2\varepsilon = (1 - a(\theta^*))$.

Utilizing this result and recalling that $R' < 0$, condition (2) implies

$$(4) \quad \frac{1}{2\varepsilon} \int_{x^*-\varepsilon}^{\theta^*} R(\theta) \, d\theta = t$$

$$> \frac{1}{2\varepsilon} \int_{x^*-\varepsilon}^{\theta^*} R(\theta^*) \, d\theta$$

$$= (1 - a(\theta^*))R(\theta^*)$$

and

$$(5) \quad \frac{1}{2\varepsilon} \int_{x^*-\varepsilon}^{\theta^*} R(\theta) \, d\theta = t$$

$$< \frac{1}{2\varepsilon} \int_{x^*-\varepsilon}^{\theta^*} R(x^* - \varepsilon) \, d\theta$$

$$= (1 - a(\theta^*))R(x^* - \varepsilon).$$

Furthermore, (3) shows that $x^*$ and $\theta^*$ converge to each other as $\varepsilon \to 0$. Since $a$ and $R$ are continuous, inequalities (4) and (5) now imply that

$$(6) \quad \lim_{\varepsilon \to 0} (1 - a(\theta^*(\varepsilon)))R(\theta^*(\varepsilon)) = t.$$

Since $a' > 0$, $R' < 0$, $a(\underline{\theta}) = 0$, and $R(\bar{\theta}) = t$, equation $(1 - a(\theta))R(\theta) = t$ has a unique solution, given as $\theta_0 \in (\underline{\theta}, \bar{\theta})$, and $\lim_{\varepsilon \to 0} \theta^*(\varepsilon) = \theta_0$.

The intuition behind this result is illustrated in Figure 1. Given signal $x^*$, an agent attaches equal probability to the states within $(x^* - \varepsilon, x^* + \varepsilon)$. At any state $\theta$, individual signals are dispersed in $(\theta - \varepsilon, \theta + \varepsilon)$. If all agents pursue the strategy to attack if and only if their signal is below $x^*$, the resulting threshold state $\theta^*$ is determined by the requirement that the propor-
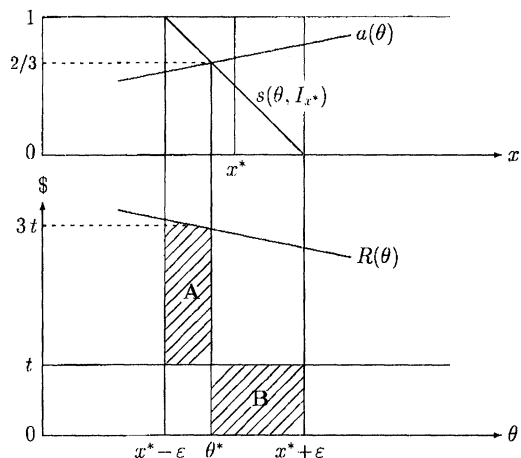


FIGURE 1. EQUILIBRIUM SWITCHING SIGNAL $x^*$ AND THRESHOLD $\theta^*$

tion of agents $s(\theta, I_{x^*})$ who are expected to receive signals below $x^*$ equal the proportion $a(\theta^*)$ needed for success [condition (3)]. Given the uniform distribution, only states within the left part, given by the fraction $(1 - a)$, of the interval $(x^* - \varepsilon, x^* + \varepsilon)$ lead to a distribution of signals that will result in an attack being successful. Thus, given signal $x^*$, an attack is successful with probability $1 - a$.

At the equilibrium switching point $x^*$ the expected payoff of an attack is zero, as stated by condition (2). As $x^*$ and $\theta^*$ converge towards each other for $\varepsilon \to 0$, equilibrium threshold $\theta^*$ has the property that the reward from a successful attack $R(\theta^*)$ weighted by the probability of success, $1 - a(\theta^*)$, equals the costs $t$ that have to be paid with certainty.

Figure 1 illustrates a case where the potential net gain from a successful attack $R(\theta^*) - t$ is about twice the costs $t$ associated with a failed attack. An agent getting signal $x^*$ is indifferent to attack, because expected gains and losses from, respectively, successful and failed attacks as given by the shaded areas A and B are of equal size.

In follow-up papers, Morris and Shin (1998b, 1999) have results similar to the Revised Theorem based on assuming a normal distribution. Heinemann and Gerhard Illing (1999) provide results based on a more general class of probability distributions.

According to the Revised Theorem, changes

in transaction costs $t$ induce changes of $\theta^*$ for $\varepsilon$ close to zero given by

$$(7) \qquad \frac{d\theta^*}{dt} = \frac{1}{(1-a)R' - Ra'}$$

$$= \frac{1}{t\dfrac{R'}{R} - Ra'} < 0.$$

A small change in transaction costs will have a large impact on the prevalence of speculation if it induces a small change in expected total rewards of all attacking agents at $\theta^*$. If transaction costs $t$ are small in comparison to $R(\theta^*)$, then $a(\theta^*) \approx 1$, and changing costs has a large impact if $a'(\theta^*)$ is close to zero. In contrast to the findings of Morris and Shin (1998a), the imposition of small transaction costs may have a large impact on the prevalence of speculation even if $R'$ is big, i.e., the consequences of speculative attacks are large. But, in general, it is true that the impact of transaction costs is larger the smaller are $R'$ and $a'$. Therefore, if the consequences of attacks *and* the hurdle for success do not vary much with the state of fundamentals, the imposition of a Tobin tax will lead to a substantial reduction in the probability of currency crises.

Morris and Shin (1998a p. 596) claim that for $\varepsilon \to 0$ a shift in the $a(\cdot)$ function has no effect on the threshold $\theta^*$, and thus, "aggregate wealth of speculators need not have a large impact on the incidence of currency attacks when the speculators have fairly precise information concerning the fundamentals." This statement, though, is wrong, as it relies on their Theorem 2 that is valid only for $a(\theta^*) = \frac{1}{2}$. The Revised Theorem shows that the $a(\cdot)$ function has a significant effect on $\theta^*$, even when $\varepsilon$ is close to zero. $a(\theta)$ is interpreted as the critical proportion of speculators' aggregate wealth that is needed for short sales to induce the government to abandon a currency peg in state $\theta$. Direct capital controls raise this critical mass. Hence, direct capital controls are effective even when fundamentals are fairly transparent to all market participants.

## REFERENCES

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