The Distributional Consequences of Public School Choice[*]

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Abstract

School choice systems aspire to delink residential location and school assignments by allowing children to apply to schools outside of their neighborhood. However, choice programs also affect incentives to live in certain neighborhoods, and this feedback may undermine the goals of choice. We investigate this possibility by developing a model of public school and residential choice. School choice narrows the range between the highest and lowest quality schools compared to neighborhood assignment rules, and these changes in school quality are capitalized into equilibrium housing prices. This compressed distribution generates an ends-against-the-middle tradeoff with school choice compared to neighborhood assignment. Paradoxically, even when choice results in improvement in the lowest-performing schools, the lowest type residents need not benefit.

1 Introduction

A central fault line in debates about K-12 education involves how students access public schools. In most of the United States, students are assigned based on where they live, and given a families' neighborhood, there is no choice of public school. An alternative is school choice, in which pupils can apply to schools outside of their neighborhood and residence plays little or no role in determining access. Proponents argue that choice would result in a more equitable distribution of school access and lead to improvements in school productivity.[1] Despite these ambitious intentions, however, choice plans remain controversial. In recent years, there has been a backlash against choice in several districts and active discussions to return back to neighborhood-based assignment.[2]

The aim of this paper is to provide a simple model to explore how the link between school assignment rules, house prices, and the residential choices of families affect the distributional consequences of public school choice. Our model is motivated by empirical evidence showing that the housing market and residential choices reflect school assignment rules (see, e.g., Black (1999), Kane et al. (2006), Reback (2005), and Bayer et al. (2007)). By contrast to other recent work that emphasizes the connection between assignment rules and the incentives for schools to improve their quality (see, e.g., Hoxby (2003), MacLeod and Urquiola (2009), Barseghyan et al. (2014), and Hatfield et al. (2016)), we focus on the effect of outside options in nearby towns on locational decisions of families living in a town that adopts a school choice assignment rule.

For simplicity, we assume that each family has one child and consider a world of (primarily) one-dimensional types, which could be interpreted either as wealth or status (of the family) or ability (of the child) or some combination of them. We assume that the quality level of a school is determined by the average of the types of families/children who enroll in that school. With utility functions that provide incentives for assortative matching, students segregate by type. When a town with multiple school districts uses a neighborhood assignment rule, endogenous differentiation of housing prices and school qualities emerge in self-confirming fashion in equilibrium. At one extreme, a neighborhood known for highest quality schools will have the highest housing prices

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[1]Forms of this argument have been made by Friedman (1962), Chubb and Moe (1990) and Hoxby (2003), among others. These arguments have been central to recent policy efforts to expand choice (DeVos, 2017).

[2]For instance, a well-known advocate of Boston's 1970s busing plan recently called for a return to neighborhood assignment (Landsmark, 2009). Former Boston Mayor Thomas Menino encouraged the Boston school committee to adopt a plan that assigns pupils closer to home, and a plan restricting the amount of choice outside of neighborhoods was adopted in 2014 (for more details, see Pathak and Shi (2014)). Other districts have also severely scaled back their choice plans (see Pathak and Sönmez (2013) for details about Seattle).

and will attract only highest types, and thus will continue to have high quality schools. But at the other extreme, lowest types will locate in neighborhoods with low quality schools. As a consequence of these market forces, lowest types are relegated by self-selection and equilibrium pricing to subpar schools, and thus, the educational system can be expected to widen rather than narrow the inequality between initially high and low types.[3]

Our primary question is whether a town can improve outcomes for low types by adopting a school choice rule, whereby all families have equal access to all schools in that town. In practice, school rosters still tend to be somewhat differentiated by neighborhood within a town that adopts school choice for several reasons: some towns allow for residential preferences in school assignment (Abdulkadiroğlu and Sönmez (2003); Dur et al. (2014); Calsamiglia and Guell (2014)), families may have preferences for schools near them (Hastings et al. (2009) Abdulkadiroğlu et al. (2015)), and wealthier families tend to use more sophisticated strategies in school assignment lotteries (Pathak and Sönmez, 2008). There is even some evidence that the process of defining school boundaries can be captured by wealthy families - in the spirit of gerrymandering - with the consequence that school choice rules can even reinforce the incentives for school segregation by wealth within a particular town (Tannenbaum, 2013). Even when a school lottery is designed to eliminate residential preferences and other features that may favor wealthy families, segregated sorting may still result in an asymmetric equilibrium depending on the specific algorithm used for the assignment rule (Calsamiglia et al., 2014).

To make the strongest possible case for school choice, we abstract away from these practical details and assume that, in fact, all schools in a town that adopts school choice have exactly the same quality – equal to the average of types who locate in that town in equilibrium. We then ask how the adoption of a school choice rule by a particular town affects the locational choices of families in the resulting housing market equilibrium, with some families choosing to move to that town and others choosing to leave it.

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[3]These ideas have their roots in Tiebout (1956) and Schelling (1971, 1978), and have been explored extensively by (among many others) Benabou (1993, 1996), Durlauf (1996), and Loury (1977) in studies of intergenerational mobility, by Fernandez and Rogerson (1996) and Nechyba (2003a) in studies of the effects of different tax regimes for funding public schools, and by Epple and Romano (1998, 2003) and Nechyba (2000, 2003b) in studies of school vouchers. Epple and Sieg (1999) empirically examine the relationship between locational equilibrium and community income distribution, while Rothstein (2006) provides empirical evidence of the relationship between neighborhood sorting and school quality. Epple and Romano (2015) analyze efficient allocations in a multi-community model with peer effects.

The incentive for flight of high types from a town that adopts school choice has been discussed in the literature on the residential consequences of school desegregation or busing. For instance, Baum-Snow and Lutz (2011) attribute the decline in white public school enrollment in urban centers to court-ordered desegregation decrees, finding that migration to other districts plays a larger role than private school enrollment. In the context of our model, withholding the option of paying for a high quality school will drive high types to other towns that offer that option. But this same logic applies inexorably as well to predict flight of low types when a town adopts school choice. In fact, any model that predicts that school choice results in a narrowing of the range between highest quality and lowest quality schools in a town and allows for changes in school qualities to be capitalized into housing prices will generate a prediction that the adoption of school choice will produce incentives for types at both extremes to move. Yet to our knowledge, ours is the first paper to model how narrowing the gap between highest and lowest quality schools provides equilibrium incentives for flight of low types (in addition to high types) from the public schools in that town.

Our approach is also inspired by past studies of the effects of private school vouchers, especially Epple and Romano (1998) and Nechyba (2000). These papers develop ambitious models that include multi-dimensional student types, define school quality as a function of tax funding and average peer quality, and allow for tax regimes, housing prices, and residential choices of families to be determined endogenously in equilibrium, then typically use computational methods to assess the welfare implications of different voucher plans. Subsequent papers by these authors, Epple and Romano (2003) and Nechyba (2003b), consider the effects of public school choice in this framework. Epple and Romano (2003) provide an example in their concluding remarks (p. 273-274) where a public school choice rule induces exit by either low or high-income households, but do not conduct a formal analysis along those lines as the framework of that example is quite distinct from the models they analyze in the main section of the paper.

While we make a conscious decision to exclude many features in this earlier literature, our model is not a special case of any of these models for two important reasons. First, the models in the voucher literature typically assume that each family must purchase a house in a given town, and private schools provides the sole channel for flight from the public schools. Then private schools only attract high types, as enrollment in a private school then effectively requires a family to pay twice for schooling: first, paying for a public school in the form of housing costs and then paying a separate tuition to switch to private school. Second, some of the models, particularly Epple and Romano (2003), assume that there is a fixed price for houses attached to the lowest quality

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school in a town. But this is not an innocuous assumption, as it implies that changes in the quality of the worst school in the town are not capitalized into market prices, and thus improvements in the quality of the worst school are necessarily beneficial to low types. Although our model is superficially simpler than these earlier models, it allows for important effects that are excluded by the modeling choices in that literature.

Our results are also related to the literature on gentrification and the displacement hypothesis, which conjectures that neighborhood revitalization will result in higher prices that in turn cause low-income and minority residents to move (see, e.g., Vigdor (2002)), and a political economy literature on ends-against-the-middle in redistribution and public service provision (Epple and Romano, 1996).

The paper is organized as follows. Section 2 presents the model, the equilibrium concept, and describes a simple example. Section 3 provides a characterization of equilibrium and studies when school choice induces more flight than neighborhood assignment. Section 4 examines welfare and discussions some extensions, while Section 5 concludes. The Online appendix includes proofs of all results and details for additional examples. The Online appendix also develops a two-town model which endogenizes outside options.

2 The Model

2.1 Primitives

Each family has one child who will enroll in school as a student. We assume that there is a unitary actor for each household and refer interchangeably to families and students as decision makers. Each student has a two-dimensional type. The first dimension is binary and identifies "partisans" who derive a distinct benefit from living in town t. The second dimension is "student type," which is independent and identically distributed according to distribution $f(x)$ on [0, 1]. The density f is continuous and differentiable, and that $f(x) > 0$ for each x. To ease exposition, we frequently refer to the value of x as the one-dimensional type of a student, neglecting partisanship.

Each family i has a separable utility function that takes the type, x_i, the quality of school j chosen by the family, y_j, and the price of attending that school, p_j, as arguments. Since we study rules for assigning students to public schools which are freely provided, p_j is simply the cost of housing associated with school j with corresponding quality y_j. We write this utility function as

$$u(x_i, y_j, p_j) = \theta_{ij} + v(x_i, y_j) - p_j,$$

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where $\theta_{ij} = \theta > 0$ if family i is partisan to town t and school j is in town t, and $\theta_{ij} = 0$ otherwise. Although in principle families may vary continuously in their partisanship, we assume that it is binary to keep the model tractable.[4] As in Epple and Romano (1998), the quality of the school is simply equal to the average type of pupils attending the school, and so $y \in [0, 1]$; partisanship plays no direct role in determining school quality. Moreover, there is no capacity constraint at a school; a school can accommodate as many pupils as needed. A separable utility function facilitates interpretation of "marginal utility" and "marginal cost" of changes in school quality at given prices.

A critical assumption of the model involves properties of v, the value function for schooling.

Assumption 1 (Increasing Differences) *v is continuous, differentiable, strictly increasing in each argument, $v(0,0) = 0$, and there is a positive constant $\kappa > 0$ such that $\frac{\partial^2 v}{\partial x \partial y} > \kappa$ for each (x_i, y_j).*

Assumption 1 implies that v satisfies the property of strictly increasing differences in (x_i, y_j).[5] That is, if $x_i^H > x_i^L$ and $y_j^H > y_j^L$, then

$$v(x_i^H, y_j^H) - v(x_i^H, y_j^L) > v(x_i^L, y_j^H) - v(x_i^L, y_j^L).$$

This assumption follows much of the literature on local public economics (e.g., Epple and Romano (1998) and Rothstein (2006)). Since high types are willing to pay more for an increase in school quality than low types, this assumption induces assortative matching of students to schools.[6] The assumption that $v(0,0) = 0$ is simply a normalization.

Let measure m_t of families be town-t partisans and M_t be the measure of homogenous housing stock available in town t. We assume that $M_t \geq m_t$, so there are enough houses for all partisans to live in town t. There is also a continuum of non-partisans for each type x. Non-partisans are willing to live in town t under sufficiently favorable conditions, but reside outside the town if the town t price is too high.

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[4]Epple et al. (2017) study a model of income-targeted vouchers with families with two-dimensional types, where income is continuous and preference for religious instruction is binary. Appendix B presents Example 1, below, when partisanship and type are both continuously distributed.

[5]See, for example, van Zandt (2002).

[6]If the one-dimensional type in the model is initial wealth, then it is natural to use a slightly different formulation of utility, as is standard in the prior literature, namely $u(x_i, y_j, p_j) = h(x_i - p_j, y_j)$ for some function h. Then, as long as p_j, the price for attending school j, is an increasing function of the quality of that school, $h_{11} < 0$ and $h_{12} > 0$ are jointly sufficient for u to exhibit strictly increasing differences in (x, y). Since h_{ij} refers to the second derivative of h with respect to i and j, these sufficient conditions correspond to assumptions of decreasing marginal utility in net wealth and higher marginal utility for school quality as net wealth increases.

Assumption 2 (Competitive Outside Option) *There is a competitive market for schools outside of town t such that schools of quality y are available at competitive price $p(y)$ for each y.*

Assumption 2 implies that non-partisans have the option to reside outside town t and obtain school quality y for price $p(y)$. Assumption 2 is a reduced-form assumption that simplifies the analysis for the main section. The Appendix studies a two-town model without this assumption, where one town's outside option is given by the other town: as described in the appendix, the results are qualitatively similar to the results of the one-town model.[7]

In equilibrium, competitive prices $p(y)$ induce enrollment and residential choices by each student so that a school of quality y has associated housing price $p(y)$, and enrolls students with average type y. If schools of every quality level y are available in equilibrium, there must be perfect assortative matching with all students of type x enrolling at schools with quality $y = x$.[8] The next lemma illustrates how competitive pricing induces non-partisan students to chose schools that perfectly match their type.

Lemma 1 *The competitive pricing rule $p(y) = \int_{z=0}^{z=y} \frac{\partial v}{\partial y}(z,z)dz$ induces a non-partisan student of type x to choose a school of quality x.*

Lemma 1 identifies a unique pricing rule for self-sorting of all non-partisan types into homogeneous schools. Under Assumption 2, schools of every quality level y are available outside town t, so we denote the (outside option) value available in equilibrium to a partisan of town t with type x as

$$\pi(x) = v(x,x) - p(x).$$

Our last assumption is about the measure of non-partisans and their role in clearing the housing market. We assume that there are sufficiently many non-partisans so that there are no vacant houses in town t.

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[7]Epple et al. (2002) study a model of tracking and competition between public and private schools. One possible equilibrium of this model produces an outcome whereby only lowest and highest ability students attend public schools, as those highest ability students are attracted by the ability to enroll in the advanced track. This outcome is broadly similar to a Type 2 equilibrium of the two-town model that we describe and analyze in the Online appendix.

[8]The competitive market for public schools outside the given town is quite similar to the nature of private schools in Nechyba (2000, 2003b), where in equilibrium, each private school enrolls students of a single "ability" level, much as a school of quality y outside town t is chosen only by students of type y in our model. One important distinction is that students who opt for an outside option in our model do not also have to pay for a house in town t, whereas students who choose a private school in Nechyba (2003b) also have to reside in the original town and pay for a house there.

Assumption 3 (Housing Market Clearing) *The measure of non-partisan families of each type x is greater than the measure of housing stock M_t.*

In an allocation in which the measure of town-t partisans is less than the measure of houses, non-partisan families occupy the remaining houses in town t. These non-partisan families are indifferent between residing in town t and living outside town t and obtaining a school with quality equal to their type.

2.2 Equilibrium

Given our assumption of a continuum of non-partisans for each type x who can always access the outside option, we focus on the allocation of partisans in describing equilibrium.[9]

We define a neighborhood assignment rule as one where town t's houses are exogenously partitioned into equal-size districts $1, 2, ..., D$, where each district has one school, the measure of houses is $\frac{1}{D}$, house prices vary by district, and all students living in district d are assigned to the school in that district.

Definition 1 *A **Neighborhood School (NS) equilibrium** in town t consists of D districts each with enrollment $\frac{1}{D}$, prices $p_1, p_2, ..., p_D$ and sets of partisan types $T_1, T_2, ..., T_D$ enrolling in these districts with measures $m_{T_1}, m_{T_2}, ..., m_{T_D}$ and average types $y_1, y_2, ..., y_D$ such that $y_d = \mathbb{E}[x|x \in T_d]$ for each d and*

(1) $v(x, y_d) + \theta - p_d \geq \pi(x)$ for each d and each $x \in T_d$,

(2) $v(x, y_d) + \theta - p_d \leq \pi(x)$ for each d and each $x \notin T_d$,

(3) If $x \in T_d$, then $v(x, y_d) - p_d \geq v(x, y_k) - p_k$ for each $k \in \{1, 2, ..., D\}$,

(4) $m_{T_d} \leq \frac{1}{D}$ for each d where

(i) if $m_{T_d} = \frac{1}{D}$, then $p_d \geq p(y_d)$, and

(ii) if $m_{T_d} < \frac{1}{D}$, then $p_d = p(y_d)$.

The first two conditions of the definition are individual rationality constraints, which ensure partisan families live in district d in town t if and only if it yields higher utility than the option

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outside of town t. The third condition is an incentive compatibility constraint, which ensures that partisan families choose their most preferred district if they choose to live in town t. Finally, the last condition involves housing market clearing: partisan demand for housing in district d is no greater than the supply of housing in district d. If partisan housing demand is less than housing supply in district d, then house prices must be equal to the competitive price $p(y_d)$. When this occurs, non-partisans of type $x = y_d$ inhabit the remaining houses in district d.

We define a school choice rule as one where there is a lottery that assigns students to schools. We assume that there are no informational frictions or priorities in the lottery, so that all of the district's residents submit identical rank-order lists of schools in descending order of anticipated quality.[10] Under a school choice rule, all schools in town t therefore have equal quality levels and all houses have the same price. A School Choice equilibrium is a special case of a Neighborhood School equilibrium with only one district.

Definition 2 *A **School Choice (SC) equilibrium** in town t is a Neighborhood School equilibrium with $D = 1$. Let p_{SC} denote prices, T_{SC} denote partisan types residing in town t with measure m_{SC}, and y_{SC} denote school quality given by $y_{SC} = \mathbb{E}[x | x \in T_{SC}]$.*

Like the Neighborhood School equilibrium definition, non-partisans are used to exhaust housing supply if there are not enough partisans living in town t. If $m_{SC} = 1$, then partisans fill all available housing in town t and prices satisfy $p_{SC} \geq p(y_{SC})$, which discourages non-partisans from living in town t. If $m_{SC} < M_t$ then partisan demand does not exhaust town t's housing supply. In this case, the remaining houses in town t are filled by non-partisans of type $x = y_{SC}$, which in turn implies $p_{SC} = p(y_{SC})$. Since under school choice, there is one schooling quality in town t, condition (3) in the NS equilibrium definition is degenerate in the SC equilibrium definition.

In what follows, we find it convenient to describe Neighborhood School (and School Choice) equilibrium based on whether partisan types at either end of the interval of types reside in town t or whether no partisan types at the ends of the interval reside in town t. These two cases are defined as follows:

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[10]Xu (2016) develops an extension of the model in this paper where school choice takes place via deferred acceptance with residential priorities. Her model has two important differences: the quality of a school is exogenous and not a function of student types, and there is a capacity constraint at schools, so neighborhood priorities play a role in those cases. Xu (2016) shows that the possibility that some students are unassigned may lead some high types to opt for lower quality schools where they face lower risk of being unassigned.

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Definition 3 *In a **boundary equilibrium**, either type $x = 0$ or $x = 1$ resides in town t. In an **interior equilibrium**, types $x = 0$ and $x = 1$ do not reside in town t.*

2.3 Example

We next analyze a simple example to compare Neighborhood School and School Choice equilibrium.

Example 1 *Assume that $v(x, y) = xy$ and the distribution of types is Uniform on $(0, 1)$.*[11]

With $v(x, y) = xy$, the competitive price is given by

$$p(y) = y^2/2.$$

We first consider a SC equilibrium in town t with school quality y. A town-t partisan of type $y - s$ obtains utility $(y - s)(y) - y^2/2 + \theta$ in town t and utility $(y - s)^2/2$ from the outside option. Thus, town-t partisans with types below y enroll in town t if condition (1) holds:

$$(y - s)(y) - y^2/2 + \theta \geq (y - s)^2/2.$$

Similarly, a town-t partisan of type $y + s$ enrolls in town t if condition (1) holds:

$$(y + s)(y) - y^2/2 + \theta \geq (y + s)^2/2.$$

These equations have identical solutions:

$$s \leq \sqrt{2\theta}.$$

If all partisans live in town t, $s = 0.5$ and $y = 0.5$. Let θ_{SC} be a threshold under which all town-t partisans live in town t. If $\theta \geq \theta_{SC} = 1/8$, there is a SC equilibrium where all partisan types live in town t. If $\theta < \theta_{SC} = 1/8$, there is no SC equilibrium where all partisans live in town t. There is an interior SC equilibrium where partisans with types in $(0.5 - \sqrt{2\theta}, 0.5 + \sqrt{2\theta})$ live in town t.[12]

Next, consider a NS equilibrium with two districts, school qualities y_1 and y_2 and associated housing prices equal to the competitive prices: $p_1 = p(y_1) = y_1^2$ and $p_2 = p(y_2) = y_2^2$. A type-x

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[11]This function does not satisfy Assumption 1 because $\partial v / \partial x = 0$ when $y = 0$ and $\partial v / \partial y = 0$ when $x = 0$, but this does not affect the analysis.

[12]When $\theta < \frac{1}{8}$, in each interior SC equilibrium, partisans with types in the range $(s - \sqrt{2\theta}, s + \sqrt{2\theta})$ of types, for $s \in [\sqrt{2\theta}, 1 - \sqrt{2\theta}]$, reside in town t. In each interior NS equilibrium, partisans with types in the range $(b - \sqrt{8\theta}, b + \sqrt{8\theta})$, for $b \in [\sqrt{8\theta}, 1 - \sqrt{8\theta}]$, reside in town t.

partisan achieves utility $xy_2 - (y_2)^2/2 + \theta$ by enrolling in district 2 or utility $xy_1 - (y_1)^2/2 + \theta$ by enrolling in district 1. Comparing these values, a type-x student prefers district 2 if

$$xy_2 - (y_2)^2/2 \geq xy_1 - (y_1)^2/2$$

or $x \geq \frac{y_1+y_2}{2}$. Partisans with the lowest types either enroll in district 1 or take the outside option. Partisans with the highest types either enroll in district 2 or take the outside option. As in the SC equilibrium, an interval of partisan types around y_j prefer district j to the outside option for $j = 1, 2$.

We simplify analysis by assuming a symmetric NS equilibrium, where $y_2 = 1 - y_1$. Suppose that for some b, partisans of types $[0.5 - b, 0.5]$ enroll in district 1 while partisans of types $[0.5, 0.5 + b]$ enroll in district 2. Then $y_1 = 0.5 - b/2$ and $y_2 = 0.5 + b/2$. The incentive condition for a partisan of type $0.5 - b$ is

$$(0.5 - b)(1 - b)/2 - (1 - b)^2/2 + \theta = (0.5 - b)^2/2.$$

The incentive condition for a partisan of type $0.5 + b$ is

$$(0.5 + b)(1 + b)/2 - (1 + b)^2/2 + \theta = (0.5 + b)^2/2.$$

These equations have identical solutions

$$b = \sqrt{8\theta}.$$

The NS equilibrium also has two indifference conditions for a partisan type at the boundary between districts, $x = 1/2$. These conditions also yield the same restriction $b = \sqrt{8\theta}$.

Let θ_N be a threshold under which all town-t partisans live in town t. If all partisan types enroll in town t, then $b = 0.5$ and $y = 0.5$. If $\theta \geq \theta_N = 1/32$, there is a NS equilibrium where all partisan types in $(0, 0.5)$ live in district 1 while partisan types in $(0.5, 1)$ live in district 2 in town t. If $\theta < \theta_N = 1/32$, there is an interior NS equilibrium where partisan types in the range $(0.5 - \sqrt{8\theta}, 0.5)$ live in district 1 and partisan types in the range $(0.5, 0.5 + \sqrt{8\theta})$ live in district 2. Figure 1 depicts the school assignments in this example.

**School Choice
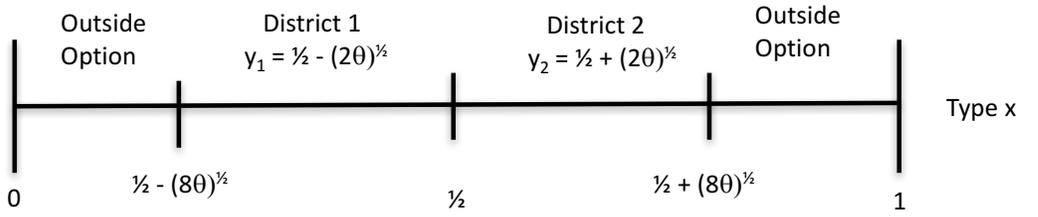Equilibrium**
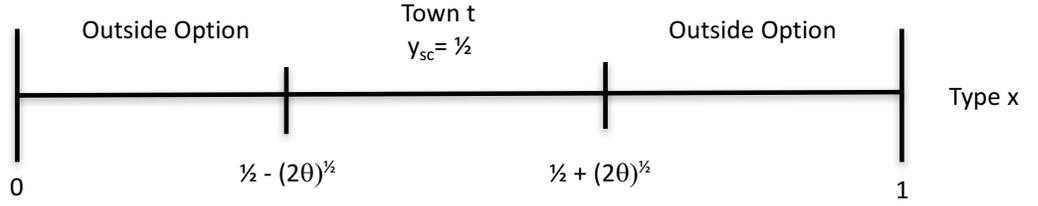
**Neighborhood
School
Equilibrium**

Figure 1. *School Assignments in Example 1 when $\theta \leq \frac{1}{32}$*

In this example, a school choice rule induces more "flight" from town t than a neighborhood assignment rule. If $\theta > \theta_{SC}$, there are SC and NS equilibria where all partisans live in town t. But, if $\theta_N < \theta < \theta_{SC}$, there is a NS equilibrium where all partisans live in town t, while in every SC equilibrium, only a subset of partisans live in town t. When $\theta < \theta_N$, the interval of types who reside in town t in any interior SC equilibrium is smaller than the interval in any interior NS equilibrium. We next to investigate the conditions under which more partisans always enroll in town t with neighborhood assignment than under school choice.

3 Equilibrium Analysis

3.1 Interval Characterization

In the model, students of all types have a preference for high quality schools. But since higher types are willing to pay more on the margin for increases in quality, competitive pricing induces assortative matching. Competitive pricing has the following implication: a student with type $x < y$ who selects a school of quality y "overpays" on the margin for school quality, while a student with type $x > y$ who selects a school of quality y values marginal school quality more than its cost, but forsakes additional gains by choosing a school with quality y.

To make these ideas precise, we define the **cost function** $C(x, y)$ as the "cost" for type x to

choose a district in town with quality y at competitive price $p(y)$ instead of an outside option with quality $y = x$ and competitive price $p(x)$. That is,

$$C(x,y) = [v(x,x) - p(x)] - [v(x,y) - p(y)] = p(y) - p(x) - (v(x,y) - v(x,x)).$$

Since Lemma 1 shows that $p(y) = \int_{z=0}^{z=y} \frac{\partial y}{\partial y}(z,z)dz$, we can express this cost $C(x,y)$ in integral form as

$$C(x,y) = \int_x^y \int_z^x \frac{\partial^2 v}{\partial x \partial y}(a,z)dadz.$$

In Example 1, $C(x,y) = (y-x)^2/2$. The assumption that v exhibits increasing differences in x and y ensures that the integrand in this formula is non-negative for all (x,y), which means that $C(x,y) \geq 0$ and also that $C(x,y)$ is decreasing in x for $x < y$ and increasing in x for $x > y$. We use these facts to show that the set of types in district d in town t form an interval in any NS or SC equilibrium.

Proposition 1 *In any Neighborhood School or School Choice equilibrium, the set of types in district d in town t, T_{N_d}, is an interval $[\underline{x}_{N_d}, \bar{x}_{N_d}]$, where $d = 1$ in a School Choice equilibrium.*

In either a Neighborhood School or School Choice equilibrium, the options for selecting schooling of quality y in town t are limited compared to the outside option, where each value of y is available at competitive price $p(y)$. Partisans of town t therefore face a tradeoff between their partisan interest of residing in town t and choosing a school with quality exactly equal to their type given competitive pricing for their types. Since $C(x,y)$ increases as x moves farther from y, each district in town t will only attract partisans with types close to the quality of that school, and thus an interval of partisan types containing the school's quality enroll in equilibrium. As a result, a NS equilibrium consists of ordered intervals, where lower type students choose lower quality schools within town t.[13]

If all partisan students enroll in town t, then districts can be ordered according to enrollment $\{[x_0 = 0, x_1], (x_1, x_2], ..., (x_D - 1, x_D = 1]\}$, where partisan students with types $x \in (x_{d-1}, x_d]$ choose district d in town t. When all partisans enroll in town t, then there is no gap between the intervals of types for district d and $d+1$. The marginal type at the boundary of the two intervals must be indifferent between the two districts so that partisan students with type x just below x_d will choose

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[13]Epple and Romano (2003) establish an analogous result in a model where school quality depends on expenditures and peer quality, and the residential choice problem is combined with voting over the tax schedule.

district d, while those with x just above x_d will choose district $d+1$. Following the terminology of Epple and Romano (2003), we call this incentive condition a boundary indifference condition:

$$\Delta p_{d+1} = p_{d+1} - p_d = v(x_d, y_{d+1}) - v(x_d, y_d),$$

for each d. Taken together, the $D-1$ boundary indifference conditions yield a general formula for the prices of all D districts in a NS equilibrium:

$$p_d = p_1 + \sum_{j=2}^{d} \Delta p_j. \tag{1}$$

The formula leaves one degree of freedom, which is the price in district 1. There is a unique choice of this price p_1 to meet the equilibrium conditions that all prices must be at least equal to competitive prices for schools of given quality, $p_d \geq p(y_d)$, and that at least one price is exactly equal to the competitive price to attract non-partisans to the remaining supply of houses in town t. In summary, there is a unique set of (potential) equilibrium prices for any partition of partisan types into intervals assigned to districts in town t. Given this set of equilibrium prices, it is straightforward to calculate the minimum value of θ required to attract the anticipated set of partisans to town t, which is equal to $-\min_x(v(x, y_d) - v(x, x) - p_d + p(x))$. This fact leads to the following corollary about existence of equilibria:

Corollary 1 *For any partition of $(0, 1)$ into D intervals $(0 = x_0, x_1), (x_1, x_2), ..., (x_{D-1}, x_D = 1)$ where $0 < x_1 < x_2 < ... < x_{D-1} < 1$, there is a cutoff θ^* such that $\theta \geq \theta^*$, there is a Neighborhood School equilibrium with D districts where partisan types in interval $d = (x_{d-1}, x_d)$ chose to live in district d in town t.*

3.2 Comparing Neighborhood School and School Choice Equilibrium

Our primary interest is comparing Neighborhood School and School Choice equilibrium. To facilitate this comparison, we focus on the neighborhood assignment rule with two districts ($D = 2$). We focus on this comparison because it eliminates the structural advantage whereby a neighborhood assignment rule provides a larger and larger menu of options as D increases: in the limit as D increases without bound, each partisan type can choose a district with school quality arbitrarily close to its ideal (possibly with some price distortion due to the boundary indifference conditions).[14]

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[14]Building on this insight, the working paper version, Avery and Pathak (2015), shows that, for any θ, there is a NS equilibrium where all partisan types enroll in town t so long as D is sufficiently large.

A Neighborhood School equilibrium with two districts imposes the additional incentive condition than a School Choice equilibrium that a student at the boundary between the two districts must be willing to enroll in town t rather than to choose the outside option. In Example 1, this additional constraint has no effect on equilibrium analysis because the cross-partial derivative $\frac{\partial^2 v}{\partial x \partial y}$ is constant in both x and y. With a Uniform distribution and utility function $v(x,y) = xy$, the value of the cost function is equal at the boundary, $C(x_j, y_j) = C(x_{j+1}, y_j)$. The incentive conditions therefore overlap for the highest and lowest types in each district.

Beyond Example 1, the assumption of increasing differences does not place any restriction on the relative magnitudes of the cross-partials $\frac{\partial^2 v}{\partial x \partial y}$ over the entire region of possible pairs (x,y). $C(x,y)$ is determined both by the distance from x to y and the magnitude of the cross-partial $\frac{\partial^2 v}{\partial x \partial y}$ in the range of pairs (a,b) where a and b are between x and y. If, for instance, this cross-partial is relatively large on the range with $1/2 < x < y < 3/4$, but close to zero everywhere else, then middle types near $1/2$ require the greatest partisan bonus to enroll in town t. For this reason, it is not possible to provide a universal ranking of θ_{SC} and θ_N, the minimum level of partisan bonus needed to support an interior School Choice and Neighborhood School equilibrium, respectively. The following proposition shows that to generalize the result that there is greater flight from town t under school choice, we need additional assumptions on the value function beyond increasing differences.

Proposition 2 *For any symmetric distribution of types, when $D = 2$, there is a continuous value function v with increasing differences in (x,y) such that $\theta_N > \theta_{SC}$ in a two-district Neighborhood School equilibrium where district 1 enrolls students with below-median types and district 2 enrolls students with above-median types.*

This Proposition shows that even with a symmetric distribution, it is possible to overturn the intuition behind Example 1, that it is easier to attract all types with a neighborhood assignment rule than a school choice rule. The proof of Proposition 2 is by construction. We develop an example in the Appendix where $\theta_N > \theta_{SC}$ with a Uniform(0,1) distribution of types; this example can be readily adapted to produce the same result for other distributions of types. The example relies on the discussion of the properties of $C(x,y)$ described above, specifically by constructing $v(x,y)$ to be linear - so that the cross-partial $\frac{\partial^2 v}{\partial x \partial y} = 0$ - over specific ranges of (x,y) values. That is, careful choice of the values of the cross-partial derivative produces a function $v(x,y)$ that prioritizes the importance of deviations from the middle of the distribution of types with the result that it

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actually becomes easier to attract the full set of partisan types with a school choice rule than with a neighborhood assignment rule.

3.3   When Does School Choice Induce More Flight?

We next turn to additional conditions on function v that restrict the relative magnitudes of its second-order cross partial derivatives. We also further make the assumption that the distribution of types is single-peaked. This assumption rules out cases where the level of partisanship required to sustain school choice is lower than that for neighborhood assignment because of bimodal distributions where the peaks are far from the center. Our main result identifies a sufficient condition on the value function for a school choice rule to induce more flight than a neighborhood assignment rule.

Theorem 1 *If the distribution of types is symmetric and single-peaked and the third-order partial derivatives of the value function, $\frac{\partial^3 v}{\partial x^2 y}$ and $\frac{\partial^3 v}{\partial x y^2}$, are either weakly positive or weakly negative for all pairs (x, y) then $\theta_N \leq \theta_{SC}$.*

When the third-order partial derivatives are weakly positive (for example, if $v(x, y) = x^\alpha y^\beta$ with $\alpha \geq 1$ and $\beta \geq 1$), the match between school quality and type has greatest effect on the value function for highest student types and school qualities, so it is hardest to attract these highest types to the town. Given equilibrium pricing, it is easier to attract these highest types for a neighborhood assignment rule than with a school choice rule, and so $\theta_N \leq \theta_{SC}$. When third-order partial derivatives are weakly negative, it is hardest to attract lowest-types to enroll, and the same argument implies that $\theta_N \leq \theta_{SC}$.

The proof illustrates two instructive features of the model. First, with weakly positive third-order partial derivatives, the highest district D has equilibrium price equal to the competitive price for its school quality y_D. Second, defining θ_D as the minimum partisan value required for type $x_D = 1$ to enroll in district D rather than choosing the outside option, $\theta_N = \theta_D$ (as discussed above, the highest types are the hardest to attract to enroll in this case.) These assumptions together yield the following comparative static result.

Corollary 2 *If the distribution of types is symmetric and single-peaked and the third-order partials of the value function are either weakly positive or weakly negative for all pairs (x, y), then θ_N is weakly decreasing in D.*

This result follows directly from the observation that $\theta_N = C(1, y_D)$ where $y_D = F^{-1}(\frac{D-1}{D})$. Since y_D is strictly increasing in D given our assumption of no point mass in the distribution, type $x_D = 1$ chooses an option closer to its ideal school quality as D increases and so $C(1, y_D)$ and thus θ_N is weakly decreasing in D. That is, when there are more districts, the value of partisanship needed to support a Neighborhood School equilibrium decreases. For that reason, the comparison between $D = 2$ and $D = 1$ (school choice) provides the best case for school choice.

4   Welfare Analysis

4.1   Comparing Equilibria

Corollary 1 shows that for the largest values of the partisan bonus ($\theta \geq \theta_N$), there is a Neighborhood School equilibrium where all partisan types live in town t. To proceed with a full welfare comparison across the two assignment rules, we need to extend equilibrium analysis to values of θ where either $\theta < \theta_N$ and $\theta < \theta_{SC}$ for each assignment rule.

Proposition 3 *For any $\theta < \theta_{SC}$, there is a School Choice equilibrium. If there is a unique School Choice equilibrium, the distribution of types is symmetric and single-peaked and the third-order partials of the value function are either weakly positive or weakly negative for all pairs (x, y), then there is a two-district Neighborhood School equilibrium for the same value of θ such that a superset of the partisan types who enroll in town t under the school choice rule enroll under the neighborhood assignment rule.*

There are two separate parts to Proposition 3. The first part is an equilibrium existence result for the school choice rule. The second part is a related equilibrium existence result that extends Proposition 1 to the case of the partial enrollment of all partisan types in a unique school choice equilibrium.

As in Proposition 1, the equilibrium price in a School Choice equilibrium with $\theta < \theta_{SC}$ is simply the competitive price $p(y_{SC})$. With the exception of knife-edge cases such as Example 1, one district in a Neighborhood School equilibrium with two districts has a price equal to the competitive price, while the other district has an equilibrium price greater than the competitive price for its realized school quality (where this price is determined by the boundary indifference condition). As described above, there is competitive pricing in one of the two districts in a Neighborhood School equilibrium. Therefore, there is a clear ordering of school qualities in these equilibria with $y_1 \leq y_{SC} \leq y_2$.

Otherwise, if $y_{SC} > y_1$ and $y_{SC} > y_2$, and the highest partisan type living in town t under school choice would not live in town t in this neighborhood equilibrium (and similarly if $y_{SC} < y_1$ and $y_{SC} < y_2$, the lowest partisan type enrolling in town t under school choice would not live in town t in the Neighborhood School equilibrium).

These observations provide the foundation for an "Ends against the Middle" welfare comparison for the school assignment rules. We denote the equilibrium payoffs for partisan type t as $\pi_{SC}(x)$ and $\pi_N(x)$. The next result shows that the benefit to the school choice rule is greater at the center of the interval of types than at either end of the interval.

Corollary 3 *("Ends Against the Middle") For any partisan type enrolling in town t under school choice, $\pi_{SC}(x) - \pi_N(x)$ is increasing in x for types enrolling in district 1 and $\pi_{SC}(x) - \pi_N(x)$ is decreasing in x for types enrolling in district 2 under the neighborhood assignment rule*

Corollary 3 follows from simple differentiation. A partisan of type x enrolling under school choice receives payoff

$$\pi_{SC}(x) = v(x, y_{SC}) - p(y_{SC}) + \theta.$$

Proposition 3 implies that the same partisan type also enrolls in the corresponding Neighborhood School equilibrium, so

$$\pi_N(x) = v(x, y_N) - p_N + \theta.$$

Subtracting one from the other and then differentiating with respect to x gives

$$\frac{\partial}{\partial x}(\pi_{SC}(x) - \pi_N(x)) = \frac{\partial v}{\partial x}(x, y_{SC}) - \frac{\partial v}{\partial x}(x, y_N) = \int_{y_N}^{y_{SC}} \frac{\partial^2 v}{\partial x \partial y}(x, z)dz.$$

If partisan type x enrolls in town t under the school choice rule and enrolls in district 1 in town t under neighborhood assignment, then $y_N = y_1 \leq y_{SC}$ and so the integral is positive. Similarly, if partisan type x enrolls district 2 in town t under neighborhood assignment, then $y_N = y_2 \geq y_{SC}$ and so the integral is negative.

Corollary 3 shows that the school choice rule produces the greatest relative advantage (in terms of the absolute difference in payoffs) over the neighborhood assignment rule for the partisan type at the cutoff between enrolling in districts 1 and 2 in equilibrium under neighborhood assignment. Further, type $x = y_{SC}$ prefers school choice over neighborhood assignment because that type obtains its preferred school quality at competitive price in the School Choice equilibrium. Therefore, types near the middle among those enrolling under school choice rule must obtain a higher equilibrium payoff under school choice rule than under neighborhood assignment rule. Similarly, types near

the extreme in the district with a price equal to the competitive price for its school quality obtain a higher payoff under neighborhood assignment than under school choice. These types choose a school quality closer to their type under neighborhood assignment than with school choice, but they pay the competitive price for school quality in either case.

Table 1 illustrates these comparisons for the case of $\theta = \frac{1}{72}$ in Example 1. Here, partisans of types $[\frac{1}{6}, \frac{1}{2}]$ enroll in district 1 and partisans of types $[\frac{1}{2}, \frac{5}{6}]$ enroll in district 2 in a two-district Neighborhood School equilibrium, while partisans of types $[\frac{1}{3}, \frac{2}{3}]$ enroll in town t in the School Choice equilibrium.

Type x	NS Eqm. School	SC Eqm. School	NS Eqm. Utility	SC Eqm. Utility	Student Prefers
$[0, \frac{1}{6}]$	Outside Opt.	Outside Opt.	$x^2/2$	$x^2/2$	Indifferent
$[\frac{1}{6}, \frac{1}{3}]$	$y_1 = \frac{1}{3}$	Outside Opt.	$\frac{x}{3} - \frac{1}{24}$	$x^2/2$	Neighborhood
$[\frac{1}{3}, \frac{5}{12}]$	$y_1 = \frac{1}{3}$	$y_{SC} = \frac{1}{2}$	$\frac{x}{3} - \frac{1}{24}$	$\frac{x}{2} - \frac{1}{9}$	Neighborhood
$[\frac{5}{12}, \frac{1}{2}]$	$y_1 = \frac{1}{3}$	$y_{SC} = \frac{1}{2}$	$\frac{x}{3} - \frac{1}{24}$	$\frac{x}{2} - \frac{1}{9}$	School Choice
$[\frac{1}{2}, \frac{7}{12}]$	$y_2 = \frac{2}{3}$	$y_{SC} = \frac{1}{2}$	$\frac{2x}{3} - \frac{5}{24}$	$\frac{x}{2} - \frac{1}{9}$	School Choice
$[\frac{7}{12}, \frac{2}{3}]$	$y_2 = \frac{2}{3}$	$y_{SC} = \frac{1}{2}$	$\frac{2x}{3} - \frac{5}{24}$	$\frac{x}{2} - \frac{1}{9}$	Neighborhood
$[\frac{2}{3}, \frac{5}{6}]$	$y_2 = \frac{2}{3}$	Outside Opt.	$\frac{2x}{3} - \frac{5}{24}$	$x^2/2$	Neighborhood
$[\frac{5}{6}, 1]$	Outside Opt	Outside Opt.	$x^2/2$	$x^2/2$	Indifferent

Table 1. Welfare Comparison for Example 1 when $\theta = \frac{1}{18}$

(NS means Neighborhood School and SC means School Choice)

The "ends against the middle" element of comparisons of Neighborhood versus School Choice equilibria is evident in Table 1. This phenomenon was originally observed in Epple and Romano (1996)'s model of public good provision and voting. In that model, there is a voting equilibrium where the richest and poorest households prefer low rates of taxation while middle-income households prefer higher rates of taxation to fund a public good. The interests of the low and high income voters are aligned in Epple and Romano (1996) because the low-income voters have little ability to pay for essentials and place relatively low value on the public good, while the high-income voters plan to opt out and pay separately for a privately provided version of that public good. The "ends against the middle" result arises from broadly similar reasons in our model. Low types have relatively limited willingness to pay for school quality, while high types benefit from the creation of a low-cost low-quality school in a Neighborhood School equilibrium, because that implies provision

of a second high-cost high-quality public school that they can choose instead of the low-quality school.

So far, we have emphasized realized utility values as the basis for welfare comparisons. From the perspective of realized school quality, the neighborhood assignment rule offers a surprising advantage over school choice for the lowest types enrolling in district 1. As shown in Table 1, partisan types between $x = \frac{1}{6}$ and $\frac{1}{3}$ stay in town t in the neighborhood assignment rule, but choose the outside option under the school choice rule. As a result, these types achieve higher utility and also attend higher quality schools under the neighborhood assignment rule than under the school choice rule.

Welfare comparisons are more ambiguous for the next lowest range of partisan types, starting with the lowest partisan type that enrolls in town t under the school choice rule. These partisan types (e.g. from $x = \frac{1}{3}$ to $\frac{5}{12}$ in the example shown in Table 1) attend a higher quality school in town t under the school choice rule than the school they choose in a neighborhood assignment rule, but they achieve higher utility in the Neighborhood School equilibrium. Rather than base welfare comparisons solely on realized utility values, this result suggests that advocates of school choice might advance a paternalistic argument that it is beneficial to reduce the difference in quality between schools attended by high and low types and especially to increase the quality of schools attended by low types.

The welfare comparisons in Table 1 are also analogous to comparisons between unconditional and conditional cash transfers, particularly a conditional cash transfer that ties payments to school attendance of the children in a household. Like a conditional cash transfer, the adoption of a school choice rule offers a benefit to lowest types in exchange for taking a costly action – under school choice, the benefit is a higher quality school and the costly action is living in town t. One immediate parallel between the school choice rule and a conditional cash transfer is that lowest type households are excluded from each program in equilibrium, as the costly action required to qualify for the benefit is too expensive to be attractive to them. A second parallel is that the lowest range of households who choose to qualify for the benefit attain better education as a result, but would gain higher utility from an unconditional cash transfer (or from neighborhood assignment). As Das et al. (2005) summarize: "By imposing conditions, the policymaker provides incentives for households to take an action that they would not ordinarily take on their own (otherwise why have the condition in the first place?). But if that action is different from what households would have chosen on their own, their resulting welfare must be lower– by distorting the consumption choices

of households, conditional cash transfer programs reduce welfare compared with unconditional cash grants."

4.2 Aggregate Welfare

Given the assumption that $v(x, y)$ satisfies increasing differences in (x, y), assortative matching maximizes the average (realized) value of $v(x, y)$. A change from neighborhood assignment to school choice eliminates sorting of types into ordered intervals and thus represents a step away from assortative matching. Combining these observations, if all partisans enroll in town t under either assignment rule, neighborhood assignment should produce greater average values of $v(x, y)$ than a school choice rule. For instance, in Example 1, when all partisans enroll in town t under neighborhood assignment with two districts, then district 1 includes types $0 < x < \frac{1}{2}$, while district 2 includes types $\frac{1}{2} < x < 1$, so $y_1 = \frac{1}{4}$ and $y_2 = \frac{3}{4}$. In this case, the average value of $v(x, y)$ is 1/16 in district 1 and 9/16 in district 2, for an overall average of 5/16. By contrast, with school choice, $y_{SC} = \frac{1}{2}$ and so the average value of $v(x, y)$ is $1/2 * 1/2 = 1/4$.[15]

The apparent advantage of neighborhood assignment over school choice (in terms of aggregate utility) as a result of assortative matching can be overturned if not all partisans choose to live in town t. Example B.4 in the Appendix illustrates a case where the existence of the outside option makes high and low types effectively indifferent between the school choice and neighborhood assignment rules. Since middle types prefer the school choice rule, aggregate utility is higher under school choice than the neighborhood assignment rule.

4.3 Extensions

One of our primary goals in this paper was to develop a tractable and transparent model that links school assignment rules and residential sorting patterns. For this reason, we have excluded a number of factors by design that would otherwise have been natural to include in the model. We now discuss briefly the implications for several of these factors in the context of our model.

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[15]One complication with this comparison is that the average housing price in town t may differ across the two assignment rules. With $y_{SC} = \frac{1}{2}$ and $v(x, y) = xy$, the housing price in town t under school choice is 1/8. However, with neighborhood assignment, $y_1 = \frac{1}{4}$, and $y_2 = \frac{3}{4}$, the housing prices are $p_1 = 1/32$ and $p_2 = 9/32$, for an average price of 5/32. That is, both the average value of $v(x, y)$ and the average housing price are greater with neighborhood assignment than with school choice, but the net utility remains greater with neighborhood assignment than with school choice.

First, we assume that the only relevant characteristic of a house is the quality of the school associated with that house. If, in addition, houses have additional inherent qualities that can be ranked, then in equilibrium under either a neighborhood assignment rule, we would expect sorting by type according to the underlying quality of the houses in each district with highest types locating in the district with the nicest houses and lowest types in the district with the least attractive houses. However, we would still expect to see a reduction in housing price dispersion after a switch from neighborhood assignment to a school choice rule, which (depending partly on the nature of outside options) would likely result in the same qualitative patterns of flight as in the existing model, with both highest and lowest types moving to other towns under a school choice rule.

Second, we assume that the school choice process necessarily equalizes the qualities of all schools in the town. But differences in school quality could persist if there are frictions in the school choice process, either in the form of residential priorities in the school assignment mechanism, transportation costs, or behavioral responses by students in submitting their rankings to a school choice lottery. Alternately, if school quality is determined (at least partly) by exogenously fixed factors and not just by peer effects, then differences in school quality would result with or without frictions in the school choice process.

With persistent differences in school quality under a school choice rule, some high types might plan to enroll in town t if assigned to a top quality school, but to move (or choose private school) if assigned to a less desirable school. Adoption of this strategy by high types would likely yield systematic demographic differences in enrollment across schools, undoing to some degree the purpose of the school choice rule. Relatedly, Epple and Romano (2003) consider how a fixed transportation cost associated with exercising choice affects school access. They show that with this friction, a school choice rule can cause a decline in the quality of the worst school in town, as relatively high income students in that neighborhood will exercise choice, but the transportation cost discourages students from the lowest-income households from attending higher-quality choice schools.

Third, if housing prices are sticky and/or low-type families are immobile in their residential choices, then a school choice rule could, in fact, equalize the quality of schools in a town without displacing those low types. For example, families in public housing would likely remain in place and would (presumably) see no difference in their housing costs as a result of a change in school assignment rules. Even in this case, however, low-type families not living in public housing could still be displaced from the town by a school choice rule.

In sum, these three extensions tend to reduce but not eliminate the predicted negative effects of

a switch from neighborhood assignment to school choice rules for lowest type students, sometimes by suggesting that school choice outcomes will simply mimic neighborhood assignment. For example, with large transportation costs, residential and school sorting could still emerge in equilibrium under a school choice rule with low types ranking a nearby low quality school as their top choice to avoid large logistical costs of attending a distant high quality school.

On a separate point, it is also possible to question our foundational assumption that the function $v(x, y)$ exhibits increasing differences in (x, y) by distinguishing between parental and child utility from education. If we assume that x simply indicates wealth, then increasing differences in $v(x, y)$ indicate that willingness to pay for high-quality education increases with wealth, which in turn makes sense for parents whose buying power is limited by an exogenously fixed budget. However, these microfoundations for the utility functions of parents to exhibit increasing differences in (x, y) need not extend to their children - for example, perhaps children of all family types might benefit equally from high-quality education.[16] But since this logic suggests that parental utility functions would still exhibit increasing differences in (x, y), it would only alter the interpretation of our equilibrium results and not the equilibrium predictions themselves. Specifically, distinguishing between parent and child utility functions could serve to justify the paternalistic view that it is valuable to override the parents of preferences in order to improve the quality of schooling provided to disadvantaged children. Yet, the equilibrium prediction of our model, as highlighted by Figures 1 and 2, is that the lowest types whose school assignments are affected by the adoption of school choice enroll at *lower quality* schools when a given town adopts school choice than when it maintains a neighborhood assignment system.

Finally, our model also abstracts away from the potential productive effects of choice. Competition appears in the model through the outside option. If a student wishes to attend a school providing a given level of quality, that option is always available outside of town t, and pricing is determined competitively. We do not model any additional competitive effects because the theoretical and empirical literature on the productive effects of school choice has not reached a firm consensus. While Friedman (1962) conjectures that quasi-market forces may improve education quality, McMillan (2004) shows that choice expansions may cause rent-seeking public schools to reduce productivity. Empirical research has conflicting results (see, e.g., Hoxby (2003), Rothstein (2006), Rothstein (2007), Hoxby (2007), Card et al. (2010)). Abdulkadiroğlu et al. (2017) show that parental demand in a choice plan is driven by peer quality rather than school effectiveness.

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[16]We are grateful to Tim Van Zandt for suggesting this interpretation to us.

MacLeod and Urquiola (2015) develop a model that shows when student composition serves as a signal of effectiveness, it may result in an "anti-lemons" effect where schools invest in screening technologies over productivity improvements.

5   Conclusion

A common rationale for school choice is to improve the quality of school options for disadvantaged students. But, our analysis shows that feedback from residential choice can undercut this approach, for if a school choice plan succeeds in narrowing the range between the lowest and highest quality schools, that change should compress the distribution of house prices in that town, thereby providing incentives for the lowest and highest types to exit from the town's public schools.

Our analysis contributes to a recent literature on school choice mechanisms, which has focused on the best way to assign pupils to schools given their residential location in a centralized assignment scheme. In particular, some have argued that the goals of choice systems may not be undermined by flight via fine-tuning of socioeconomic or income-based criteria and cities have now experimented with complex school choice tie-breakers in an effort to achieve a stable balance (Kahlenberg, 2003, 2014). By incorporating feedback between residential and school choices, our model suggests that analysis of school assignments that does not account for the possibility of residential resorting may lead to an incomplete understanding about the distributional consequences of school choice.

A broader implication of our model is that systemic changes beyond the details of the school assignment system may be necessary to reduce inequalities in educational opportunities. One such approach addresses the residential choice problem directly by transferring low-income families to better neighborhoods. For instance, the US Department of Housing and Urban Development's Moving to Opportunity Program offered vouchers to low-income families to move to low-poverty neighborhoods. The evidence on the effects of this experiment on educational outcomes is mixed (Kling et al., 2007), though a recent literature suggests there may be some positive effects (Chetty et al. (2016)). A second approach involves directly influencing the quality of schools available to low-income families. There is growing evidence that some urban charter schools generate large achievement effects and more disadvantaged children benefit more (Abdulkadiroğlu et al. (2011); Angrist et al. (2013); Walters (2014)). Our model suggests that the general approach of attacking the roots of schooling inequities likely has more promise than efforts solely designed to change the rules by which students are assigned to schools.

A   Proofs

A.1   Proof of Lemma 1

Proof. Under Assumption 2, a non-partisan student of type x faces maximization problem:

$$\max_{y} u(x, y) = v(x, y) - p(y).$$

The first-order condition is

$$\frac{\partial v}{\partial y}(x, y) - p'(y) = 0.$$

For the first-order condition to hold at $x = y$, we have $p'(y) = \frac{\partial v}{\partial y}(y, y)$, which requires

$$p(y) = \int_{z=0}^{z=y} \frac{\partial v}{\partial y}(z, z)dz,$$

while the second-order condition for maximization follows from the fact that v satisfies increasing differences in x and y.[17] Given this pricing rule,

$$\frac{\partial u(x, y)}{\partial y} = \frac{\partial v}{\partial y}(x, y) - \frac{\partial v}{\partial y}(y, y),$$

which is strictly positive for $x > y$ and strictly negative for $x < y$ by the property of increasing differences for v. Thus, $u(x, y) = v(x, y) - p(y)$ is strictly increasing in y for $y < x$ and strictly decreasing in y for $y > x$, which verifies that $u(x, y)$ is maximized at $y = x$ given this pricing rule.

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A.2   Proof of Proposition 1

Proof. We first show that a school choice equilibrium consist of an interval of types residing in town t. Suppose that school quality in town t in a school choice equilibrium is y_{SC} with price $p_{SC} = p(y_{SC})$. A partisan of type x enrolls in town t if $v(x, y_{SC}) - p(y_{SC}) + \theta \geq v(x, x) - p(x)$ or

$$\theta \geq [v(x, x) - v(x, y_{SC})] - [p(x) - p(y_{SC})].$$

Since $p(y) = \int_0^y \frac{\partial v}{\partial y}(z, z)dz$, the incentive condition for enrolling in town t in integral form is

$$\theta \geq \int_{y_{SC}}^{x} \left[\frac{\partial v}{\partial y}(x, z) - \frac{\partial v}{\partial y}(z, z) \right] dz.$$

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[17]Technically, this first-order condition would yield the result that $p(y) = \int_{z=0}^{z=y} \frac{\partial v}{\partial y}(z, z)dz + C$, where C is a constant, but since we assume that $v(0, 0) = 0$, we set this constant to 0.

If $x \geq y_{SC}$, the right-hand side of this equation is increasing in x by Assumption 1, so the condition holds for types in some range of types given by $[x, x_{SC}^H]$. Similarly, if $x \leq y_{SC}$, the right-hand side of the equation is decreasing in x, so the condition also holds for some range of types given by $[x_{SC}^L, x]$. Putting these ranges together, a range of types around y_{SC} will enroll in town t in a School Choice equilibrium.

Suppose that the school qualities in a D-district neighborhood equilibrium are $y_1, y_2, ..., y_D$ where $y_j > y_{j-1}$ and that the price for district j is $p_j = p(y_j) + \Delta_j$ where $\Delta_j \geq 0$. (At least one of the prices is the competitive price on the outside market, so $\Delta_j = 0$ for at least one j.) Following the analysis of the School Choice equilibrium above, a range of partisan types around y_j prefers district j to the outside option. Partisan type x prefers district k to district j with $k > j$ if $v(x, y_k) - p_k + \theta \geq v(x, y_j) - p_j + \theta$ which is equivalent to

$$v(x, y_k) - v(x, y_j) - [p(y_k) - p(y_j)] \geq \Delta_k - \Delta_j.$$

In integral form, the incentive condition for enrolling in district k rather than district j is then

$$\int_{y_j}^{y_k} \left[\frac{\partial v}{\partial y}(x, z) - \frac{\partial v}{\partial y}(z, z) \right] dz \geq \Delta_k - \Delta_j.$$

The integrand on the left-hand side of this equation is increasing in x, so there is some cutoff x_{jk} such that the condition holds iff $x \geq x_{jk}$. Thus, these pairwise comparisons between districts j and k with $j < k$ may adjust the lower bound of the range of types enrolling in district j and the upper bound of the range of types enrolling in district k. The end result remains the same as in the School Choice equilibrium: a range of partisan types enrolls in each district d in a Neighborhood School equilibrium. ∎

A.3 Proof of Proposition 2

Proof. We assume a Uniform distribution of types and proceed by construct the following counter-example. Define $v(x, y)$ as follows:

1. If $x \leq 0.25$ or $y \leq 0.25$, $v(x, y) = 0$,

2. If $0.25 \leq x, y \leq 0.75$ then $v(x, y) = 192(x - 0.25)(y - 0.25)^2$,

3. If $0.25 \leq x \leq 0.75, 0.75 \leq y \leq 1$, then $v(x, y) = 48x + 96y - 84$,

4. If $0.25 \leq y \leq 0.75$ and $0.75 \leq x \leq 1$, then $v(x, y) = 96(y - 0.25)^2 + 48x - 36$,

5. If $x \geq 0.75$ and $y \geq 0.75$, then $v(x, y) = 48x + 96y - 84$.

The competitive price function is defined in three pieces:

- First, for $x, y < 0.25$, $v(x, y) = 0$ so clearly $p(y) = 0$.

- Next, for $0.25 \leq x, y \leq 0.75$, $v(y, y) = 192(x - 0.25)(y - 0.25)^2$, $\frac{\partial v}{\partial y} = 384(x - 0.25)(y - 0.25)$ and so $p(y) = \int_{0.25}^{0.75} 384(z - 0.25)^2 dz = 128(y - 0.25)^3$, so $p(0.75) = 16$.

- Finally, for $x, y \geq 0.75$, $v(x, y) = 48x + 96y - 84$, $\frac{\partial v}{\partial y} = 96$, and so $p(y) = p(0.75) + \int_{0.75}^{y} 96 dz = 16 + (y - 0.75) = 96y - 56$.

The incentive condition for type $x = 1$ in a school choice equilibrium where all partisan types enroll in town t is

$$v(1, 0.5) - p(0.5) + \theta \geq v(1, 1) - p(1)$$

or $18 - 2 + \theta \geq 60 - 40$ or

$$\theta \geq 4.$$

The incentive condition for type $x = 0$ in a school choice equilibrium where all partisan types enroll in town t is

$$v(0, 0.5) - p(0.5) + \theta \geq v(0, 0) - p(0)$$

or $0 - 2 + \theta \geq 0 - 0$ or

$$\theta \geq 2.$$

Therefore, all partisan types will enroll in town t under school choice if

$$\theta \geq \theta_{SC} = \max(2, 4) = 4.$$

The minimum value of θ for all partisan types to enroll under neighborhood equilibrium is determined by the incentive conditions for the middle type $x = 1/2$. If partisan types from 0 to $1/2$ enroll in district 1 while partisan types from $1/2$ to 1 enroll in district 2, then $y_1 = 1/4$ and $y_2 = 3/4$ given the uniform distribution of types. For the moment, assume that the prices in the two districts are equal to the competitive prices on the outside market for schools of these qualities: $p_1 = p(1/4)$ and $p_2 = p(3/4)$.

Type $1/2$ prefers district 1 to the outside option if

$$v(1/2, 1/4) - p(1/4) + \theta \geq v(1/2, 1/2) - p(1/2)$$

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or $0 - 0 + \theta \geq 3 - 2$ or

$$\theta \geq 1.$$

Type $1/2$ prefers district 2 to the outside option if

$$v(1/2, 3/4) - p(3/4) + \theta \geq v(1/2, 1/2) - p(1/2)$$

or $12 - 16 + \theta \geq 3 - 2$ or

$$\theta \geq 5.$$

The boundary indifference condition for type $1/2$ requires p_1 to be above the competitive price $p(1/4) = 0$, so in fact $p_1 = 4$. Then partisan type $1/2$ requires a bonus $\theta \geq \theta_N = 5$ to enroll in either district 1 or district 2. Therefore, we have $\theta_N = 5$ and $\theta_{SC} = 4$ and thus

$$\theta_N > \theta_{SC}.$$

The construction of the counterexample is designed around the anticipated values $y_1 = 1/4$ and $y_2 = 3/4$ given a Uniform distribution of types and full enrollment of partisans in two equal-sized districts in town t. Specifically, $v(x, y)$ is constructed to be piecewise linear for $x, y \leq y_1$ and $x, y \geq y_2$. The distribution of types only affects these computations through the determination of $y_1 = \mathbb{E}(x | x \leq 0.5)$ and $y_2 = \mathbb{E}(x | x \geq 0.5)$. For any other distribution of types, a similar construction would apply, but with $v(x, y) = (x - y_1)(y - y_1)^2$ for values x, y with $y_1 \leq x, y \leq y_2$ and $v(x, y)$ again defined to be piecewise linear if $x, y \leq y_1$ or $x, y \geq y_2$. Thus, for any distribution of types, it is possible to construct $v(x, y)$ so that $\theta_N \geq \theta_{SC}$.

Each version of this counterexample is everywhere continuous and is differentiable except at boundary points $x, y = y_1$ or y_2. In addition, the second-order cross-partial derivative of v is not strictly positive since it is set to 0 when $x, y \leq y_1$ or $x, y \geq y_2$. But in each case, it is possible to make minor adjustments to the value function so that it is continuous and differentiable with strictly positive second-order cross-partials for all values (x, y); these minor adjustments will change the computations, but presumably will only infinitesimally, maintaining $\theta_N \geq \theta_{SC}$ with v everywhere continuous. ∎

A.4 Proof of Theorem 1

Proof. Assume that the distribution of types is symmetric and single-peaked on $[0, 1]$ and represent the cumulative distribution by F so that $F(0) = 0, F(0.5) = 0.5, F(1) = 1$, and $F(1-x) = 1 - F(x)$ for $x \in [0, 1]$.

Suppose that there are D districts of equal size in a Neighborhood School equilibrium. Write the associated cutoffs in ascending order $x_0 = 0 < x_1 < x_2 < ... < x_D = 1$, so that a student of type x chooses district d if $x_{d-1} \leq x \leq x_d$. By construction,

$$F(x_d) - F(x_{d-1}) = \frac{1}{D},$$

since each district enrolls the same number of students. Since F is symmetric about $1/2$,

$$x_d = 1 - x_{D-d},$$

for each $d \leq D$. If D is even, then types $x \leq 0.5$ enroll in districts 1 to $\frac{D}{2}$ and types $x \geq 0.5$ enroll in districts $\frac{D}{2} + 1$ to D with $x_{\frac{D}{2}} = 0.5$. If D is odd, then types x just below and just above 0.5 enroll in district $\frac{D+1}{2}$. In fact, district $\frac{D+1}{2}$ is symmetric about the median, so the average type in that district is $y_{\frac{D+1}{2}} = 0.5$.

Define $\Delta_d = v(x_d, y_{d+1}) - v(x_d, y_d)$, for values $d = 1, ..., D - 1$. In a Neighborhood School equilibrium, the boundary indifference condition implies that the difference in prices for adjacent districts d and $d + 1$ is

$$\Delta_d = p_{d+1} - p_d.$$

Define the price differential that would occur at competitive prices for these districts as

$$\Delta p_d = p(y_{d+1}) - p(y_d).$$

(The equilibrium price for each district must be greater or equal than the competitive price.)

The proof proceeds with two lemmas.

Lemma 2 *If the distribution of types is symmetric and single-peaked and the third-order partials of the value function are weakly positive for all pairs (x, y), then $p_D = p(y_D)$.*

In any Neighborhood School equilibrium, at least one district is priced at the competitive price on the outside market for its equilibrium quality. The lemma asserts that when the third-order partials are weakly positive, then the highest-quality district must be at the competitive price with $p_D = p(y_D)$; this district provides a reference price that, together with the indifference condition and the Δ_d values, determines the equilibrium prices for the other districts. (By symmetry, when the third-order partials are weakly negative, then the lowest-quality district must be the reference price district with $p_1 = p(y_1)$.)

The proof of the lemma proceeds through four claims.

Claim 1: *If $x_d \geq \frac{1}{2}$, then $y_{d+1} - x_d \geq x_d - y_d$.*

We first consider the special case where D is even (so $d = \frac{D}{2}$) and $x_d = \frac{1}{2}$. In this case, districts d and $d+1$ lie just above and below the median value. By symmetry, y_d and y_{d+1} are equidistant from the median at $x = \frac{1}{2}$, so we obtain our desired result

$$y_{d+1} - x_d = x_d - y_d.$$

Next, if $x_d > \frac{1}{2}$, all districts d to the right of the median have $y_d > \frac{1}{2}$ and $x_d > \frac{1}{2}$. Since f is symmetric and single-peaked, f is weakly decreasing in districts d through D. Suppose $f(x_d) = K$. Given that measure $\frac{1}{D}$ students enroll in each district and f is weakly decreasing, the minimum possible value for y_d occurs if $f(x) = K$ throughout district d (where $x_{d-1} \leq x \leq x_d$), in which case,

$$x_d = x_{d-1} + \frac{K}{D}$$

and

$$y_d^{\min} = x_d - \frac{K}{2D}.$$

Similarly, the minimum possible value for y_{d+1} occurs if $f(x) = K$ through district $d+1$, in which case,

$$x_{d+1} = x_d + \frac{K}{D}$$

and

$$y_{d+1}^{\min} = x_d + \frac{K}{2D}.$$

That is, $y_d \geq y_d^{\min} = x_d - \frac{K}{2D}$ and $y_{d+1} \geq y_{d+1}^{\min} = x_d + \frac{K}{2D}$, which implies the result. $\qquad\square$

Claim 2: *If $x_d \geq 1/2$, then $\Delta_d \leq \Delta p_d$.*

First, note that

$$\Delta p_d - \Delta_d = (p(y_{d+1}) - p(y_d)) - (p_{d+1} - p_d) = \int_{y_d}^{y_{d+1}} \frac{\partial v}{\partial y}(z, z)dz - \int_{y_d}^{y_{d+1}} \frac{\partial v}{\partial y}(x_d, z)dz. \qquad (2)$$

Since we assume the second-order cross-partial derivatives of v are non-negative, we rewrite equation (2) as the difference of two positive-valued integrals:

$$\Delta p_d - \Delta_d = \underbrace{\int_{x_d}^{y_{d+1}} \left[\frac{\partial v}{\partial y}(z, z) - \frac{\partial v}{\partial y}(x_d, z)\right] dz}_{>0} - \underbrace{\int_{y_d}^{x_d} \left[\frac{\partial v}{\partial y}(x_d, z) - \frac{\partial v}{\partial y}(z, z)\right] dz}_{>0}.$$

Applying the fundamental theorem of calculus, we rewrite each term as a double integral of a second-order mixed partial derivative of v:

$$\Delta p_d - \Delta_d = \int_{x_d}^{y_{d+1}} \int_{x_d}^{z} \frac{\partial^2 v}{\partial x \partial y}(a,z) da dz - \int_{y_d}^{x_d} \int_{z}^{x_d} \frac{\partial^2 v}{\partial x \partial y}(a,z) da dz. \tag{3}$$

From Claim 1, we know that for $x_d \geq 1/2$,

$$y_{d+1} - x_d \geq x_d - y_d.$$

Therefore, the first integral in equation (3) covers a (weakly) larger range of pairs (a, z) than the second integral in equation (3) and the arguments take on systematically higher values in the first integral than in the second. Since the third-order mixed partial derivatives of v are non-negative, the smallest value of the integrand in the first integral is at least as large as the largest value of the integrand in the second integral in equation (3). Taking these facts together, the first integral cannot be smaller than the second one, which shows

$$\Delta p_d \geq \Delta_d.$$

$\square$

Claim 3: If $x_d < \frac{1}{2}$, then $\Delta p_d + \Delta p_{D-d} \geq \Delta_d + \Delta_{D-d}$.

As in the proof of Claim 2,

$$\Delta p_d - \Delta_d = \underbrace{\int_{x_d}^{y_{d+1}} \int_{x_d}^{z} \frac{\partial^2 v}{\partial x \partial y}(a,z) da dz}_{=C} - \underbrace{\int_{y_d}^{x_d} \int_{z}^{x_d} \frac{\partial^2 v}{\partial x \partial y}(a,z) da dz}_{=D}. \tag{4}$$

With $x_d < \frac{1}{2}$, it is not necessarily true that $\Delta p_d \geq \Delta_d$ because in this case, the positive integral ranges over values closer to the center than the first integral in equation (4) and so covers a (weakly) smaller range of values than the second integral in equation (4). Instead, we apply a version of that argument to the combination of terms on opposite sites of the median, $\Delta p_d + \Delta p_{D-d}$ in comparison to $\Delta_d + \Delta_{D-d}$.

Using the analysis from Claim 2,

$$\Delta p_{D-d} - \Delta_{D-d} = \underbrace{\int_{x_{D-d}}^{y_{D-d+1}} \int_{x_{D-d}}^{z} \frac{\partial^2 v}{\partial x \partial y}(a,z) da dz}_{=A} - \underbrace{\int_{y_{D-d}}^{x_{D-d}} \int_{z}^{x_{D-d}} \frac{\partial^2 v}{\partial x \partial y}(a,z) da dz}_{=B}. \tag{5}$$

Note that

$$(\Delta p_d - \Delta_d) + (\Delta p_{D-d} - \Delta_{D-d}) = A - B + C - D.$$

We will show that the values of the integrands can be strictly ordered (because the ranges of (x, y) values are strictly ordered for these integrals) with highest values in Integral A, next highest in Integral B, third highest in Integral C, and lowest in Integral D.

To see why, first note that by symmetry, $x_d = 1 - x_{D-d}$ and $y_d = 1 - y_{D-d+1}$, for each d. As a result,

$$x_d - y_d = y_{D-d+1} - x_{D-d}.$$

Similarly,

$$y_{d+1} - x_d = x_{D-d} - y_{D-d}.$$

Therefore, Integrals A and D cover equal-sized triangles of (x, y) values, and Integrals B and C also cover equal-sized triangles of (x, y) values. In addition, from Claims 1 and 2, Integrals A and D cover larger-sized triangles of (x, y) values than Integrals B and C. This means that we can divide Integrals A and D into sub-integrals A_1, A_2 and D_1, D_2, where A_1, B, C, and D_1 all cover equal-sized ranges of (x, y) values. Since we can strictly order the integrands in these terms,

$$A_1 \geq B \geq C \geq D_1 \quad \text{and} \quad A_2 \geq D_2,$$

so

$$A - B + C - D = (A_1 - B) + (C - D_1) + (A_2 - D_2) \geq 0$$

because each term is weakly positive. $\qquad \square$

Claim 4: $p_D = p(y_D)$.

There are D different possible value for p_D:

1) $p(y_D)$,

2) $p(y_{D-1}) + \Delta_{D-1}$,

3) $p(y_{D_2}) + \Delta_{D-2} + \Delta_{D-1}$,
   $\vdots$

d) $p(y_{D-d+1}) + \sum_{j=D-d+1}^{D-1} \Delta_j$,
   $\vdots$

D) $p(y_1) + \sum_{j=1}^{D-1} \Delta_j$.

In equilibrium, the price for district D must be the maximum of these values. We show that $p(y_D)$ is greater than or equal to each of the other $D - 1$ possible prices.

For each value of d, we want to show that

$$p(y_D) \geq p(y_{D-d+1}) + \sum_{j=D-d+1}^{D-1} \Delta_j,$$

or equivalently

$$p(y_D) - p(y_d) - \sum_{j=d}^{D-1} \Delta_j \geq 0.$$

To simplify this expression, we write $p(y_D)$ as a telescoping sum of differences:

$$p(y_D) = p(y_d) + [p(y_{d+1}) - p(y_d)] + [p(y_{d+2}) - p(y_{d+1})] + ... + [p(y_D) - p(y_{D-1})]$$

or equivalently

$$p(y_D) = p(y_d) + \sum_{j=d}^{D-1} \Delta p_j.$$

The requirement that $p(y_D) - p(y_d) - \sum_{j=d}^{D-1} \Delta_j \geq 0$ is therefore equivalent to $\sum_{j=d}^{D-1}(\Delta p_j - \Delta_j) \geq 0$.

If $x_d \geq \frac{1}{2}$, then each pair of terms $\Delta p_d - \Delta_d$ is non-negative by Claim 2. Thus,

$$\sum_{j=D-d}^{D-1} (\Delta p_j - \Delta_j) \geq 0,$$

or

$$p(y_D) \geq p(y_d) + \sum_{j=D-d+1}^{D-1} \Delta_j,$$

as desired. If $x_d < \frac{1}{2}$, then we can apply a combination of Claims 2 and 3 to reach the same conclusion.

If D is even, then for each $d \leq \frac{D}{2}$,

$$\Delta p_d + \Delta p_{D-d} - \Delta_d - \Delta p_{D-d} \geq 0,$$

by Claim 3. Therefore,

$$\sum_{j=d}^{\frac{D}{2}} \Delta p_j + \Delta p_{D-j} - \Delta_j - \Delta p_{D-j} \geq 0.$$

Then we can rewrite

$$\sum_{j=d}^{D-1}(\Delta p_j - \Delta_j) = \sum_{j=d}^{\frac{D}{2}}(\Delta p_d + \Delta p_{D-d} - \Delta_d - \Delta p_{D-d}) + \sum_{j=\frac{D}{2}+1}^{D-d+1}(\Delta p_j - \Delta_j).$$

Similarly, if D is odd, we can rewrite

$$\sum_{j=d}^{D-1}(\Delta p_j - \Delta_j) = \sum_{j=d}^{\frac{D-1}{2}}(\Delta p_d + \Delta p_{D-d} - \Delta_d - \Delta p_{D-d}) + \sum_{j=\frac{D+1}{2}}^{D-d+1}(\Delta p_j - \Delta_j).$$

In each case, the first sum is non-negative by Claim 3 and the second sum is non-negative by Claim 2, so

$$\sum_{j=d}^{D-1}(\Delta p_j - \Delta_j) \geq 0,$$

as desired for Claim 4, which in turn establishes the Lemma. □

Lemma 3 *If the distribution of types is symmetric and single-peaked and the third-order partials of the value function are weakly positive for all pairs (x, y), then $\theta_N = \theta_D$, where θ_D is the partisan bonus required for types from x_D to $x = 1$ to prefer district D in town t at price $p(y_D)$ to the outside option.*

If θ is sufficiently large for type 1 to enroll in district D, then all types will enroll in the appropriate district rather than choose the outside option. Denote θ_d as the minimum value required for type x_d to enroll in town t under neighborhood assignment. (Boundary types always have the largest incentive to deviate from a Neighborhood School equilibrium.) For $d = 1, 2, ..., D - 1$, type x_d is indifferent between enrolling in district $d + 1$ and district d, and so θ_d is computed by finding the minimum partisan value required for x_d to enroll in either of these districts. For the extreme types, there is only one adjacent district, so θ_D is the minimum partisan value required for type $x_D = 1$ to enroll in district D and similarly, θ_0 is the minimum partisan value required for type $x_0 = 0$ to enroll in district 1.

For each d from $d = 1$ to $d = D$, we compare θ_{d-1} and θ_d by finding the separate partisan values required for these types to enroll in the same district d. (Note that this includes a comparison of θ_{D-1} and θ_D based on enrolling in district D and of θ_0 and θ_1 based on enrolling in district 1.) That is,

$$v(x_d, y_d) - p_d + \theta_d = v(x_d, x_d) - p(x_d)$$

and

$$v(x_{d-1}, y_d) - p_d + \theta_{d-1} = v(x_{d-1}, x_{d-1}) - p(x_{d-1}).$$

Solving these equations for θ_d and θ_{d-1} gives

$$\theta_d = v(x_d, x_d) - v(x_d, y_d) - p(x_d) + p_d \tag{6}$$

and

$$\theta_{d-1} = v(x_{d-1}, x_{d-1}) - v(x_{d-1}, y_d) - p(x_{d-1}) + p_d. \tag{7}$$

Subtracting equation (6) from equation (7),

$$\theta_d = \theta_{d-1} + [v(x_d, x_d) - v(x_d, y_d)] + [v(x_{d-1}, y_d) - v(x_{d-1}, x_{d-1})] - [p(x_d) - p(x_{d-1})].$$

In integral form,

$$\theta_d = \theta_{d-1} + \int_{y_d}^{x_d} \frac{\partial v}{\partial y}(x_d, z)dz + \int_{x_{d-1}}^{y_d} \frac{\partial v}{\partial y}(x_{d-1}, z)dz - \int_{x_{d-1}}^{x_d} \frac{\partial v}{\partial y}(z, z)dz,$$

or

$$\theta_d = \theta_{d-1} + \int_{y_d}^{x_d} \left[\frac{\partial v}{\partial y}(x_d, z) - \frac{\partial v}{\partial y}(z, z) \right]dz - \int_{x_{d-1}}^{y_d} \left[\frac{\partial v}{\partial y}(z, z) - \frac{\partial v}{\partial y}(x_{d-1}, z) \right]dz.$$

In double-integral form,

$$\theta_d = \theta_{d-1} + \underbrace{\int_{y_d}^{x_d} \int_z^{y_d} \frac{\partial^2 v}{\partial x \partial y}(a, z)dadz}_{=\text{Integral 1}} - \underbrace{\int_{x_{d-1}}^{y_d} \int_{x_{d-1}}^z \frac{\partial^2 v}{\partial x \partial y}(a, z)dadz}_{\text{Integral 2}} . \qquad (8)$$

We apply the following extended versions of Claims 1 to 4 from the proof of Lemma 2 to this equation to conclude that $\theta_D \geq \theta_d$ for each d.

Extended Version of Claim 1: If $d > \frac{D}{2}$ then $y_d \geq 0.5$. If $y_d = 0.5$, then district d is symmetric about $y_d = 0.5$, so

$$y_d - x_{d-1} = x_d - y_d.$$

If $y_d > 0.5$, then since the distribution of types is symmetric and single-peaked, the mean in the range of types from x_{d-1} to x_d (which is y_d) falls at or below the midpoint between x_{d-1} and x_d, so

$$x_d - y_d \geq y_d - x_{d-1}.$$

Extended Version of Claim 2: This result indicates that if $y_d \geq 0.5$ (i.e. $d > \frac{D}{2}$), Integral 1 in the double-integral equation (8) above covers a larger range of pairs (x, y) than Integral 2 in equation (8). Once again, the integrand in Integral 1 is always larger than the integrand in Integral 2. This shows that if $d > \frac{D}{2}$, then $\theta_d \geq \theta_{d-1}$. Iterating this reasoning, if $d > \frac{D}{2}$, then

$$\theta_D \geq \theta_d$$

Extended Version of Claim 3: If $d \leq \frac{D}{2}$, the integrand in the double-integral formula is larger for Integral 1 than for Integral 2, but Integral 2 covers a wider range of pairs of values than does Integral 1. Following the logic of Claim 3 above, we pair two sets of integrals based on the

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observation that $x_d - y_d = y_{D-d+1} - x_{D-d}$ and $y_d - x_{d-1} = x_{D-d+1} - y_{D-d+1}$ to produce the conclusion $\theta_{D-d+1} - \theta_{D-d} + \theta_d - \theta_{d-1} \geq 0$ or

$$\theta_d + \theta_{D-d+1} \geq \theta_{d-1} + \theta_{D-d}.$$

Extended Version of Claim 4: If $d \leq \frac{D}{2}$, then we write $\theta_D - \theta_d$ as a telescoping sum of first differences:

$$\theta_D - \theta_d = \sum_{j=d}^{D-1}(\theta_j - \theta_{j-1}).$$

<u>Case 1</u>: If D is even, then for each $j = d$ to $j = \frac{D}{2}$, we can pair the term $\theta_j - \theta_{j-1}$ with $\theta_{D-j+1} - \theta_{D-j}$. That is,

$$\theta_D - \theta_d = \sum_{j=d}^{\frac{D}{2}}[(\theta_j - \theta_{j-1}) + (\theta_{D-j+1} - \theta_{D-j})] + \sum_{j=D-d+2}^{D}(\theta_j - \theta_{j-1}).^{18}$$

By Claim 3, each term in the first sum is non-negative and by Claim 2, each term in the second sum is non-negative.

<u>Case 2</u>: If D is odd, then for each $j = d$ to $j = \frac{D-1}{2}$, we pair the term $\theta_j - \theta_{j-1}$ with $\theta_{D-j+1} - \theta_{D-j}$. That is,

$$\theta_D - \theta_d = \sum_{j=d}^{\frac{D-1}{2}}[(\theta_j - \theta_{j-1}) + (\theta_{D-j+1} - \theta_{D-j})] + \sum_{j=D-d+2}^{D}(\theta_j - \theta_{j-1}).$$

Once again, by Claim 3, each term in the first sum is non-negative and by Claim 2, each term in the second sum is non-negative. In either case, the conclusion is that

$$\theta_D - \theta_d \geq 0,$$

so $\theta_N = \theta_D$, as desired

To prove the theorem, given that $\theta_N = \theta_D$, it is straightforward to see that $\theta_N \leq \theta_{SC}$. Since $1 - y_D \geq y_D - x_{D-1}$ (from Claim 1 in the proof of Lemma 2) and the third-order mixed partial derivatives of v are positive, θ_D is the partisan bonus required for type $x = 1$ to enroll in district D with school quality y_D and price $p(y_D)$. Similarly, θ_{SC} is the partisan bonus required for type $x = 1$ to enroll under school choice with school quality $y_{SC} = 1/2$ and price $p(1/2)$. With two or more districts, $y_D > 1/2$, so a partisan of type 1 prefers district D in town t with full enrollment of partisan types to town t under school choice with full enrollment of partisan types, so $\theta_N = \theta_D \leq \theta_{SC}$. ∎

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[18]Note that if $d = 1$, $D - d + 2 > D$ and so this second sum is empty.

A.5 Proof of Proposition 3

Proof. We define the left-hand loss for a district with partisan types on the interval $[a, b]$ as $C(x, y(a, b))$, where $y(a, b) = \mathbb{E}(x | a \leq x \leq b)$. Similarly, we define the right-hand loss for that district as $C(b, y(a, b))$. Since $y(a, b)$ is increasing in each of a and b, the left-hand loss function for interval $[a, b]$ is increasing in b while the right-hand loss function is decreasing in a.[19]

Claim 1: *There exists a School Choice equilibrium.*

Define $h(y)$ to be the expected value of partisan types enrolling in town t when everyone anticipates school quality y and (competitive) price $p(y)$ in town t. This function h is continuous in y since v is continuous in x and y. If $\theta > 0$, then some types $x > 0$ enroll in town t if y is anticipated to be equal to 0, so $h(0) > 0$ and similarly $h(1) < 1$. Therefore, there is a fixed point of h between 0 and 1 and this fixed point corresponds to a school choice equilibrium. □

Claim 2: *If there is a unique School Choice equilibrium, then there is a two-district Neighborhood School equilibrium where a superset of the partisan types who enroll in town t under school choice enroll under neighborhood assignment.*

Define $L^{2R}(x, \theta)$ as the right-side loss function is district 2 when

(a) district 1 consists of types $(x, x_M(x, \theta))$ where $x_M(x, \theta)$ is chosen so that the left-hand loss function is equal to θ, and

(b) district 2 consists of types $(x_M(x, \theta), x_H(x, \theta))$ where $x_H(x, \theta)$ is chosen so that the left-hand side loss function in district 2 is equal to right-hand side loss function in district 1.

Here $x_M(x, \theta)$ and $x_H(x, \theta)$ are well defined since $C(x, b)$ is increasing in b for $b \geq x$.[20]

First, suppose that there is a boundary equilibrium where partisan types $[0, x_H^{SC} < 1]$ enroll in town t under school choice, and that this is the unique school choice equilibrium.[21] Then the left-hand loss on $[0, x_H^{SC}]$ is less than θ and the right-hand loss on $[0, x_H^{SC}]$ equals θ. We expect to find a

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[19]Since a change in a increases both the lower limit of the interval and $y(a, b)$, the effect of a change in a on the left-hand loss function is ambiguous in sign and similarly the effect of a change in b on the right-hand loss function is ambiguous in sign.

[20]If the left-hand loss on $[x, 1]$ is less than θ, then x is greater than the lowest partisan type enrolling under school choice and is not relevant to the analysis. If the left-hand loss on $[x_M(x, \theta), 1]$ is less than the right-hand loss on $[x, x_M(x, \theta)]$, then set $L^{2R}(x, \theta)$ as the right-hand loss on $[x_M(x, \theta), 1]$.

[21]Given the assumptions of symmetric single-peaked distribution and positive third-order mixed partial derivatives of v, the right side loss for any interval $[x, 1]$ is greater than the left-side loss for that interval, so there cannot be a boundary equilibrium where types from x to 1 enroll in town t under school choice.

similar boundary equilibrium under neighborhood equilibrium. To look for this, find the minimum value of z such that $L^{2R}(0, z) = \theta$, with intervals $[0, x_M(z)]$ in district 1 and $[x_M(z), x_H(z)]$ in district 2.[22]

If the left-hand loss in district 2 (which equals the right-hand loss in district 1 by construction) is greater than θ, then (1) the boundary school choice equilibrium with interval $[0, x_{SC}]$ has right-hand loss equal to θ and left-hand loss less than θ, and (2) the left-hand loss on $[x_M(z), x_H(z)]$ is greater than θ and the right-hand loss on this interval is equal to θ. By the Intermediate Value Theorem, there must be an interval $[x_1, x_2]$ where $x_{SC} < x_2 < x_H(z)$ with left-hand and right-hand losses each equal to θ. Then there is a second school choice equilibrium where types $[x_1, x_2]$ enroll in town t, which contradicts the assumption of a unique school choice equilibrium.

If the left-hand loss in district 1 is greater than θ (i.e. $z > \theta$), then there is one interval $[0, x_{SC}]$ with left-hand loss less than θ and right-hand loss equal to θ and another interval $[0, x_H(z)]$ with left-hand loss greater than θ and right-hand loss less than θ, so $x_{SC} < x_H(z)$. If we keep extending the right-hand limit of the interval, then there must be some point $x_{HH} > x_H(z)$ so that the interval $[0, x_{HH}(z)]$ has left-hand loss greater than θ and right-hand loss equal to θ. Therefore, there must be a value m between $x_H(z)$ and x_{HH} with left-hand and right-hand losses each equal to θ on $[0, m]$. But then this would be a second school choice equilibrium, which is a contradiction.

Therefore, if there is a unique school choice equilibrium and it is a boundary equilibrium where partisan types on $[0, x^{SC} < 1]$ enroll in town t under school choice, there is also a boundary equilibrium under neighborhood assignment with $[0, x_M]$ enrolling in district 1 and $[x_M, x_H]$ enrolling in district 2. The right hand loss in the interval $[x_M, x_H] = \theta$, so the right-hand loss for the interval $[0, x_H]$ must be greater than θ. If $x_H < x^{SC}$, then there is a value $x_2^{SC} < x_H < x^{SC}$ such that

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[22]If there is no such value of z, then there must be intervals $[0, a], [a, 1]$ where the right-hand loss on $[0, a]$ equals the left-hand loss on $[a, 1]$ and the right-hand loss on $[a, 1]$ is less than θ. Since the right-hand loss is greater than the left-hand loss on all intervals of form $[a, 1]$, the left-hand loss on $[a, 1]$, which equals the left-hand loss on $[0, a]$, is less than θ. If the left-hand loss on $[0, a]$ is also less than or equal to θ, then we have identified a neighborhood equilibrium where all partisan types enroll in town t. Alternately, if the left-hand loss on $[0, a]$ is greater than θ, then $a > x_{SC}$ since by assumption the left-hand loss on $[0, x_{SC}]$ is less than θ. Then the left-hand loss on $[0, 1]$ is larger than the left-hand loss on $[0, a]$ so must be greater than θ and the right-hand loss on $[0, 1]$ is even larger, so also is greater than θ. We can find a value b between a and 1 so that the right-hand loss on $[0, b]$ is equal to θ. Since right-hand loss on $[0, x^{SC}]$ and right-hand loss on $[0, b]$ are each equal to θ, while left-hand loss on $[0, x^{SC}]$ is less than θ and left-hand loss on $[0, b]$ is greater than θ, there must be some type d between a and b such that left-hand loss equals right-hand loss equals θ on $[0, d]$. This would indicate a second school choice equilibrium with enrollment of partisan types on $[0, d]$, but this contradicts the assumption of a unique school choice equilibrium.

the right-hand loss on the interval $[0, x_2^{SC}] = \theta$ and there is a second boundary equilibrium under school choice where types from 0 to x_2^{SC} enroll in town t. This is a contradiction.

Next suppose that there is an interior equilibrium under school choice where partisan types on the interval $[x_L^{SC}, x_H^{SC}]$ enroll in town t with $0 < x_L^{SC} < x_H^{SC} < 1$ and that this is the unique equilibrium under school choice. Then since there is no boundary equilibrium under school choice, the right-side loss for interval $[0, x_\theta(0)]$ (where $x_\theta(0)$ is defined to make the left-side loss on this interval equal to θ) must be less than θ. Then since the school choice equilibrium is assumed to be unique, the right-side loss on the interval $(x, x_\theta(x))$ must be less than θ for each $x < x_L^{SC}$ and greater than or equal to θ for $x \geq x_L^{SC}$.

If $L^{2R}(0, \theta) \geq \theta$, then there is a value $\theta^0 \leq \theta$ such that $L^{2R}(x, \theta_0) = \theta$, which produces a candidate neighborhood equilibrium with intervals $[x_L = 0, x_M = x_{\theta^0(0)}]$ of types enrolling in district 1 and $[x_M, x_H]$ enrolling in district 2. If instead, $L^{2R}(0, \theta) < \theta$, then either there is a value $x_L > 0$ such that $L^{2R}(x_L, \theta) = \theta$, or $L^{2R}(x, \theta) < \theta$ for each x. In the first case, where $L^{2R}(x_L, \theta) = \theta$, there is a candidate equilibrium with intervals $[x_L, x_M]$ and $[x_M, x_H]$ enrolling in the two districts where the left-hand loss in district 1 and the right-hand loss in district 2 are equal to θ while the left-hand loss in district 2 equals the right-hand loss in district 1 by construction. In the second case, there is a candidate equilibrium with intervals $[x_L, x_M]$ and $[x_M, 1]$ enrolling in the two districts where the left-hand loss in district 1 equals θ, the right-hand loss in district 2 is less than θ and the left-hand loss in district 2 equals the right-hand loss in district 1 by construction.

Each of these candidate neighborhood equilibria (one for the case where $L^{2R}(0, \theta) \geq \theta$, one for the case where $L^{2R}(x, \theta) \leq \theta$ for each x and a third one for the case where $L^{2R}(0, \theta) \leq \theta$ and there exists x_L such that $L^{2R}(x_L, \theta) = \theta$) is an actual neighborhood equilibrium (with competitive pricing) if the right-hand loss in district 1, which equals the left-hand loss in district 2, is less than θ. If instead, the right-hand loss in district 1 is greater than θ in any of these candidate equilibria, then there must be some value of Δ such that the left-hand loss equals θ and the right-hand loss is less than θ on the interval $[x_M + \Delta, x_H]$. By the Intermediate Value Theorem, there must be an interval $[a_2, b_2]$ with $x_L < a < x_M + \Delta$ such that left-hand loss and right-hand loss each equal θ, so that there is a school choice equilibrium with types from a_2 to b_2 enrolling in town t. But since the right-hand loss on the interval $[x, x_\theta(0)]$ is less than θ there must be another interval $[a_1, b_1]$ where $0 < a_1 < x_L$ where left-hand loss and right-hand loss are each equal to θ, producing another school choice equilibrium.[23] This is a contradiction, so given the assumption of a unique school

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[23]In fact, there at least three school choice equilibria in this case because the left-hand loss is less than the right-hand

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choice equilibrium, the candidate equilibria in each of these three cases is an actual neighborhood equilibrium.

In the case of a unique interior school choice equilibrium, we have identified a neighborhood equilibrium where partisan types on $[x_L, x_M]$ enroll in district 1, partisan types on the interval $[x_M, x_H]$ enroll in district 2, the left-hand loss in district 1 and the right-hand loss in district 2 equal θ and right-hand loss in district 1 and left-hand loss in district 2 are less than θ. We now want to show that all types who enroll in the unique interior school choice equilibrium also enroll in the associated neighborhood equilibrium that we have identified.

Since the left-hand loss on the interval $[x_L, x_M]$ is equal to θ, the left-hand loss on $[x_L, x_H]$ must be greater than θ. Since the left-hand loss on the interval $[x_L, x_H]$ is greater than θ and the left-hand loss on the interval $[x_M, x_H]$ is less than θ, there must be a type x_{LM} with $x_L < x_{LM} < x_M$ such that the left-hand loss on the interval $[x_{LM}, x_H]$ is equal to θ. Since the right-hand loss on the interval $[x_M, x_H]$ equals θ and the right-hand loss and $x_{LM} < x_M$, the right-hand loss on the interval $[x_{LM}, x_H]$ must be greater than θ. In sum, we have identified intervals $[x_L, x_M]$ and $[x_{LM}, x_H]$ with left-hand loss equal to θ in each interval, right-hand loss less than θ for the first interval and right-hand loss greater than θ in the second interval. Since $x_L < x_{LM}$, there must be an interval with starting point between x_L and x_{LM} where left-hand loss and right-hand loss are each equal to θ – this interval must be exactly the range of types enrolling in the school choice equilibrium, so $x_L < x_{SC} < x_{LM} < x_M$, so every type enrolling in the (interior) school choice equilibrium also enrolls in the neighborhood equilibrium. ∎

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loss for any interval $[x, 1]$.

B Additional Examples (Online Appendix)

Example 2 discusses a variant of Example 1 where both partisanship and type are continuously distributed. Example 3 demonstrates that it is possible to have a unique boundary equilibrium under school choice. Example 4 shows that it is possible to have a unique interior School Choice equilibrium (neither of which is true for all values of θ in Example 1). Example 5 shows that our assumption of increasing differences does not always imply that average utility is higher under neighborhood assignment.

B.1 Example 2: Continuous Partisanship

Suppose that $v(x, y) = xy$ and that both types and the partisan bonus are drawn from two independent continuous distributions. Furthermore, suppose that type x is distributed $U(0,1)$ as in Example 1.

School Choice Equilibrium: There is a school choice equilibrium where a partisan with bonus θ_j enrolls in town t if $\frac{1}{2} - \sqrt{2\theta_j} \leq x_j \leq \frac{1}{2} + \sqrt{2\theta_j}$. Each decision rule is symmetric about $\frac{1}{2}$, so with x distributed uniformly on $(0,1)$, the school quality is $1/2$ and the equilibrium price is $1/8$, just as in Example 1.

Neighborhood School Equilibrium: There is also a neighborhood equilibrium with two districts and equilibrium school qualities $y_1 = \frac{1}{4}$ and $y_2 = \frac{3}{4}$. Here a partisan with bonus θ_j enrolls in town t if $\frac{1}{2} - \sqrt{2\theta_j} \leq x_j \leq \frac{1}{2} + \sqrt{2\theta_j}$. Each decision rule is symmetric about $\frac{1}{2}$, so with x distributed uniformly on $(0, 1)$, the school quality is still $1/2$ and the equilibrium price is $1/8$, just as in Example 1.

In this extended example, as in Example 1, for each partisan bonus, a wider range of type x's choose town t under neighborhood assignment than under school choice. One difference here, however, is that for the smallest values of θ_j, the set of type x's enrolling under neighborhood assignment does not necessarily subsume the set of types x's enrolling under school choice, as the neighborhood ranges may be near $\frac{1}{4}$ and $\frac{3}{4}$ while the school choice range is near to $\frac{1}{2}$.

A special feature of this case is that partial derivatives of the value function are the same everywhere. So it is also possible to construct equilibrium, which moves the center of the school choice interval. For instance, if range is $[0.4, 0.6]$ for SC, there will be any SC equilibrium with length 0.5. However, given a SC interval, we can construct a Neighborhood equilibrium with middle of SC interval and have that more people stay in town t under the Neighborhood equilibrium than

under School Choice, consistent with Theorem 1.

B.2 Example 3: Unique Boundary Equilibrium under School Choice

Suppose that the distribution of types is $U(0,1)$ as in Example 1 and that $v(x,y) = xy^2$. Now $\frac{\partial^2 v}{\partial x \partial y} = 2y$, which is constant in x and strictly increasing in y. With a Uniform distribution of types, the school quality for any interval of types $[a,b]$ is exactly in the middle of the range of types at $\frac{a+b}{2}$. Further, since $\frac{\partial^3 v}{\partial x^2 \partial y} = 0$ and $\frac{\partial^3 v}{\partial x \partial y^2} > 0$, if types in a range $[a,b]$ enroll in a school, the right-hand loss function at type b is greater than the left-hand loss function at type a. Thus, any school choice equilibrium is a boundary equilibrium where types from $x = 0$ to $x = x_{SC}$ enroll in town t, with school quality $y_{SC} = x_{SC}/2$ and price $p_{SC} = p(y_{SC}) = (2/3)y_{SC}^3 = x_{SC}^3/16$.

In equilibrium, type x_{SC} must be exactly indifferent between enrolling in town t and taking the outside option. That is $x_{SC}(x_{SC}/2))^2 - p(y_{SC}) + \theta = \pi(x_{SC})$, or $x_{SC}^3/4 - (2/3)(x_{SC}^3)(1/8) + \theta = (1/3)(x_{SC}^3)$ or $\theta = x_{SC}^3(1/3 + 1/12 - 1/4) = x_{SC}^3/6$. Solving for x_{SC} as a function of θ, $x_{SC} = (6\theta)^{1/3}$ identifies a unique school choice equilibrium for this case where types on $[0, x_{SC}]$ enroll in town t. For example, with $\theta = \frac{1}{48}, x_{SC} = \frac{1}{2}$, so types $[0, 1/2]$ enroll in town t with $y_{SC} = 1/4$ and

$$p_{SC} = p(y_{SC} = (2/3)(1/4)^3 = 1/96.$$

For similar reasons, we anticipate a boundary equilibrium with neighborhood assignment where types $[x_L = 0, x_M]$ enroll in district 1 and types $[x_M, x_H]$ enroll in district 2 in town t. Since the left-hand loss is greater than the right-hand loss in each district (and the price in each district should equal the competitive price), the equilibrium conditions are that (1) the right-hand loss in district 2 equals θ; (2) the left-hand loss in district 2 equals the right-hand loss in district 1; (3) the left endpoint in district 1 is equal to 0. These equilibrium conditions yield polynomial equations that are not easily solvable in closed form, so instead we used numerical methods to approximate the equilibrium with $\theta = 1/48$: $x_M \approx .471$ and $x_H \approx .8158$. Thus with $\theta = 1/2$, types from 0 to 1/2 enroll in town t under school choice, while types from 0 to about 0.8158 enroll in equilibrium with a neighborhood assignment rule. Consistent with Proposition 3, any type enrolling under school choice will also enroll in equilibrium under a neighborhood assignment rule.

B.3 Example 4: Unique Interior Equilibrium under School Choice

Suppose that $v(x) = xy$ as in Example 1, but that the distribution of types is triangular on $(0,1): f(x) = 4x$ if $x \le 1/2$ and $f(x) = 4(1-x)$ if $x \ge 1/2$. With this distribution function,

the expected value of types on an interval $[a, b]$ is greater than $\frac{a+b}{2}$ if $b < 1 - a$. This rules out the possibility of a boundary equilibrium (which would require a lower loss at the boundary rather than the other end of the interval of types enrolling in the town) under school choice. An interior equilibrium requires the school quality to be exactly equal to $\frac{a+b}{2}$, which is only possible if the interval of types is symmetric about $1/2 : [1/2 - \Delta, 1/2 + \Delta]$, with school quality $y_{SC} = 1/2$ and $p_{SC} = p(1/2) = 1/8$. In equilibrium, type $1/2 - \Delta$ must be indifferent between enrolling in town t and taking the outside option: $(1/2 - \Delta)(1/2) - 1/8 + \theta = (1/2 - \Delta)^2/2$, with solution $\Delta = (2\theta)^{1/2}$. For example, with $\theta = 0.02$, $2\theta^{1/2} = 0.2$, so partisan types in the range $[0.3, 0.7]$ enroll in the unique school choice equilibrium with $y_{SC} = 1/2$ and $p_{SC} = p(1/2) = 1/8$.

By the same logic, there is also an interior equilibrium with neighborhood assignment whenever $\theta < \theta_N$ in this example, again with $x = 1/2$ as the midpoint of enrollment in town t. With two districts and neighborhood assignment, a symmetric equilibrium has types in the range $[1/2 - \Delta, 1/2]$ enroll in district 1 and types in the range $[1/2, 1/2 + \Delta]$ enroll in district 2. With a triangular distribution, the expected value of types on the range $[a, b]$ with $a < b \leq 1/2$ is $\frac{2(b^3 - a^3)}{3(b^2 - a^2)}$. Using this formula with $a = 1/2 - \Delta$ and $b = 1/2$, we find $y_1 = \frac{3 - 6\Delta + 4\Delta^2}{6 - 6\Delta}$. Type $x = 1/2 - \Delta$ is indifferent between enrolling in district 1 in town t and taking the outside option if $(1/2 - \Delta)y_1 - 2y_1^2/2 + \theta = (1/2 - \Delta)^2/2$. Substituting $y_1 = \frac{3 - 6\Delta + 4\Delta^2}{6 - 6\Delta}$ and $\theta = .02$, this indifference condition produces a quartic equation in $\Delta : 8\Delta^4 - 24\Delta^3 + 15.12\Delta^2 + 5.76\Delta - 2.88 = 0$. Only one of the four roots of this equation, partisan types from $[0.158872, 1/2]$ enroll in district 1 while partisan types in the range $[0.5, 0.841128]$ enroll in district 2 in town t with $y_1 \approx 0.358872, y_2 \approx 0.641128$. Once again, any type enrolling in the school choice equilibrium also enrolls in this neighborhood assignment equilibrium.

B.4 Example 5: Aggregate Welfare Can be Higher Under School Choice

Suppose that there are three types $x = 0$, $\frac{1}{2}$, and 1 with associated probabilities $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$ that $v(x, y) = xy$, and that $\theta = 1/32$.

In a Neighborhood School equilibrium with two equal sized districts, low types enroll in district 1, high types enroll in district 2 and middle types divide equally between the two districts. Then, as in Example 1, $y_1 = \frac{1}{4}$, $y_2 = \frac{3}{4}$, $p_1 = 1/32$ and $p_2 = 9/32$. Further, the value of θ is exactly high enough so that each type is indifferent between enrolling in town t and choosing the outside option. (We assume that all types break ties by remaining in town t.)

With a school choice rule, the value of θ is not high enough for all types to enroll in town t. But there is an equilibrium where high and low types choose the outside option, while middle types enroll in town t with $y_{SC} = \frac{1}{2}$ and $p_{SC} = 1/8$. Thus, high and low types are indifferent between the two assignment rules, as they get utility equal to the value of the outside option in each case. However, middle types strictly prefer the School Choice equilibrium.

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C   Two-Town Model (Online Appendix)

C.1   Setup

We now alter the analysis to consider a general equilibrium version of the model with two towns, A and B. We also assume that an equal number of partisans are attached to each town, there are no non-partisans, and that each family must choose a house in either town A or town B. In the two-town model, outside options are determined endogenously in equilibrium in contrast to Assumption 2 in the main text.

As before, we assume that the utility function for each family is given by

$$u(x_i, y_j, p_j) = \theta_{ij} + v(x_i, y_j) - p_j,$$

where $\theta_{ij} = \theta > 0$ if family i is partisan to town t and school j is in town t, and $\theta_{ij} = 0$ if family i is partisan to town t and school j is not in town t. We assume a continuum of partisan families of measure 1 for each town and that partisans of both types have identical distributions for student type $f(x)$ on $[0, 1]$, maintaining all other properties assumed for f and v from the one-town model.

Each town has D equal-size districts, which we label as $A_1, A_2, ..., A_D$ for town A and $B_1, B_2, ..., B_D$ for town B. Districts are ordered in ascending school quality: $y_{t_1} \leq y_{t_2} \leq ... \leq y_{t_D}$ for each town $t \in \{A, B\}$. We denote the sets of town-A and town-B partisans choosing district d in town t as α_{t_d} and β_{t_d}, respectively, and denote an assignment of town-A partisans to districts by $\alpha = \{\alpha_{A_1}, \alpha_{A_2}, ..., \alpha_{A_D}, \alpha_{B_1}, \alpha_{B_2}, ..., \alpha_{B_D}\}$ and an assignment of town-B partisans to districts by $\beta = \{\beta_{A_1}, \beta_{A_2}, ..., \beta_{A_D}, \beta_{B_1}, \beta_{B_2}, ..., \beta_{B_D}\}$.

Definition 1 *A **two-town equilibrium** is an allocation of families to schools, α and β, associated average types in each district $\{y_{A_1}, y_{A_2}, ..., y_{A_D}, y_{B_1}, y_{B_2}, ..., y_{B_D}\}$ and prices $(p_{A_1}, p_{A_2}, ..., p_{A_D}, p_{B_1}, p_{B_2}, ..., p_{B_D})$ where*

(1) Each student maximizes utility $u(x_i, y_d, p_d)$ with the choice of school district d,

(2) Each district d enrolls $\frac{1}{D}$ students,

(3) If town $t \in \{A, B\}$ uses a school choice rule, then $y_{t_1} = y_{t_2} = ... = y_{t_D} = \mathbb{E}[x|$ enroll in town $t]$.

A focal outcome in this model is one where all partisans of a given town reside in that town. We define this as a non-integrated equilibrium. Our first result is on the existence of a symmetric non-integrated equilibrium where partisans make the same housing decision in each town.

Definition 2 *In a **non-integrated equilibrium**, all town-A partisans live in town A and all town-B partisans live in town B. In an **integrated equilibrium**, either some town-A partisans live in town B or some town-B partisans live in town A.*

Proposition 1 *If both towns use the same assignment rule, then there is a symmetric non-integrated equilibrium with cutoffs $\{x_0 = 0, x_1, x_2, ..., x_{D-1}, x_D = 1\}$, where students of type $x \in [x_{d-1}, x_d]$ enroll in district d of their partisan town.*

This result is immediate whether both towns use neighborhood assignment or school choice. In both cases, the options and prices for schooling in two towns are identical, so clearly town-A partisans will choose to live in town A and town-B partisans will choose to live in town B. With a neighborhood schooling rule in both towns, (1) the type cutoffs are determined by the capacities in each district and the implicit equation $F(x_d) = \frac{d}{D}$ for each d, (2) the school qualities equal the conditional expectation $y_{A_d} = y_{B_d} = y_d = \mathbb{E}[x|x_{d-1} < x < x_d]$ for each d, and (3) price increments between districts in towns A and B are determined by the boundary indifference conditions

$$p_d - p_{d-1} = v(x_d, y_d) - v(x_d, y_{d-1})$$

for districts $d = 2, ..., D$. Then by construction, given the property of increasing differences of v in x and y, any choice of price for district 1, $p_{A_1} = p_{B_1} = p_1$ induces the precise sorting of students to districts as stated in the proposition. The resulting symmetric non-integrated equilibrium is stable for either assignment rule if θ is strictly greater than 0, in the sense that a small change in locational choices will not induce any partisan to switch towns at the cost of θ.[24]

We use the non-integrated equilibrium with neighborhood school assignment in each town as the baseline outcome for comparisons to the results when one town adopts school choice primarily because it is the unique symmetric equilibrium when both towns use neighborhood assignment rules and all districts are the same size. Furthermore, there is perfect sorting of partisans within each town in this non-integrated neighborhood school equilibrium, so the adoption of a school choice rule necessarily reduces inequalities in school assignment if families do not move.

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[24]There may also be equilibria other than the non-integrated outcome when both towns use the same school assignment rule. For example, if both towns use a school choice rule, there could be an equilibrium where one town has higher school quality than the other and town-A partisans and town-B partisans of highest types both choose the higher quality school. One complication is that if town A has the higher quality school in this case, then partisans of town B must forego θ to attend that school, while partisans of town A gain θ by choosing it, so any equilibrium other than the no mixing equilibrium involves asymmetric decision rules for partisans of town A and partisans of town B.

C.2 School Choice in Town A and Neighborhood Schools in Town B

Suppose that town A uses the school choice rule and town B uses the neighborhood assignment rule. At this point, we specialize in the analysis to the case with $D = 2$ districts in town B. To simplify notation, we denote the equilibrium school quality and price (for each district) in town A as y_A and p_A and the corresponding values in town B as y_{B_1} and p_{B_1} for district 1 and y_{B_2} and p_{B_2} for district 2, using the convention that $y_{B_1} \leq y_{B_2}$.

Much of the intuition from the one town model carries over to the two-town model. In particular, Proposition 2 indicates that when town A adopts school choice, partisan enrollment takes the form of intervals in each district. Furthermore, the range of types of town-A partisans enrolling in town A subsumes the range of types of town-B partisans who enroll in town A.

Proposition 2 *In any equilibrium where town A uses school choice and town B uses neighborhood assignment, an interval $[x_A^L, x_A^H]$ for town-A partisans and an interval $[x_B^L, x_B^H]$ of town-B partisans enroll in town A, where $x_A^L \leq x_B^L \leq x_B^H \leq x_A^H$.*[25]

We distinguish between three types of equilibria according to the ordering of school quality y_A in town A relative to school qualities in the two districts in town B, y_{B1} and y_{B2}.

1. In a **Type 1 equilibrium**, $y_A > y_{B_2} > y_{B_1}$;

2. In a **Type 2 equilibrium**, $y_{B_2} > y_A > y_{B_1}$;

3. In a **Type 3 equilibrium**, $y_{B_2} > y_{B_1} > y_A$.[26]

For relatively large values of θ, partisans of each town have a strong incentive to enroll in that town. In the limiting case, partisans of town B with types above the median enroll in district B_2, partisans of town B with types below the median enroll in district B_1 and all partisans of town A enroll in town A, producing a unique equilibrium which happens to be of Type 2. For relatively small values of θ, however, this logic need not hold and it is possible that there can be equilibria (one or more) of each type for a given value of θ.

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[25]In a non-integrated equilibrium, since all town-A partisans and no town-B partisans enroll in town A, $x_A^L = 0$ and $x_A^H = 1$. In this case, we set $x_B^L = x_B^H = y_{SC}$ and the result holds. It is natural to set $x_B^L = x_B^H = y_{SC}$ because the first town-B partisans to enroll in town A will be those of types nearest to y_{SC}.

[26]We make an explicit choice to use strict rather than weak inequalities in these definitions. As long as $\theta > 0$, $y_A = y_{B_1}$ or $y_A = y_{B_2}$ is only possible in an equilibrium where only town-A partisans enroll in town A, but this in turn implies the trivial equilibrium with $y_A = y_{B_1} = y_{B_2} = 0.5$. Similarly, if $y_{B_1} = y_{B_2}$, then we have identified an equilibrium where the two districts in B are identical and so it is as if B has adopted school choice rather than a neighborhood assignment rule.

C.3   Example

Example 2 *Suppose that the distribution of types is Uniform on (0, 1) for partisans of each town and that the utility function is $u(x, y) = xy$.*

We consider several possibilities in turn. We leave out detailed computations for equilibria of Type 3. Given the symmetry of the example (and the fact that third-order cross partials of $u(x, y) = xy$ are equal to 0), Type 1 and Type 3 equilibria are essentially mirror images of each other.

Case 1: Non-Integrated Equilibrium

In a non-integrated equilibrium, town-B partisans are partitioned into districts with types $[0, 1/2]$ in district 1 and types $[1/2, 1]$ in district 2 so that $y_1 = 1/4$ and $y_2 = 3/4$, while all town-A partisans choose town A so that $y_A = 1/2$. We work backwards from the equilibrium conditions to identify equilibrium prices and restrictions on θ to construct a non-integrated equilibrium. A marginal town-B partisan at $x = 1/2$ must be indifferent between districts 1 and 2. Thus,

$$\frac{1}{2}y_1 - p_1 = \frac{1}{2}y_2 - p_2,$$

or equivalently $p_2 - p_1 = 1/4$.

Given $p_2 - p_1 = 1/4$, partisans of either town with $x < 1/2$ prefer district 1 to 2 in town B. The incentive condition for town-A partisans with $x < 1/2$ to choose A is $x/2 + \theta - p_A \geq x/4 - p_1$, or $\theta \geq p_A - p_1$ at $x = 0$ where the condition is most binding. Similarly, the incentive condition for partisans of town B with $x < 1/2$ to choose B_1 is $x/4 + \theta - p_1 \geq x/2 - p_A$, or $\theta \geq 1/8 - p_A + p_1$ at $x = 1/2$ where the condition is most binding. Thus, the smallest value for which both conditions hold jointly is $\theta_{NI} \equiv 1/16$, and hence $p_A - p_1 = 1/16$. (A similar argument shows that the incentive conditions for partisans with types $x > 1/2$ also hold simultaneously at $\theta = 1/16$ when $p_2 - p_A = 3/16$).

In sum, there is a Non-Integrated Equilibrium if $\theta \geq 1/16$.

Case 2: Integrated Equilibrium of Type 2

For values of $\theta < 1/16$, we simplify computations by looking for an integrated equilibrium with symmetric cutoffs x_A^L and $x_A^H = 1 - x_A^L$. Given the constraints that $1/4$ of all students must enroll in each district in town B (and half of all students must enroll in town A),

$$x_B^L = \frac{1}{2} - x_A^L$$

and

$$x_B^H = 3/2 - x_A^H = \frac{1}{2} + x_A^L.$$

Thus, under the assumption that $x_A^H = 1 - x_A^L$, equilibrium assignments can be described as a function of x_A^L alone. Furthermore, by Proposition 2, $x_B^L \geq x_A^L$, which implies that x_A^L must be less than or equal to 1/4. We provide detailed computations in Appendix D.1 to show that there is a unique equilibrium of this form for each value $\theta < \theta_{NI}$, and further that x_A^L is decreasing in θ, so that fewer partisans of town A choose to live in town B as θ increases.

Case 3: Integrated Equilibrium of Type 1

The analysis for equilibria of Type 1 and Type 3 are much simpler than that of an equilibrium of Type 2 because one of the boundary indifference conditions is between districts B_1 and B_2. A town-B partisan with type x is indifferent between enrolling in these two districts if

$$v(x, y_{B_1}) - p_{B_1} + \theta = v(x, y_{B_2}) - p_{B_2} + \theta.$$

Similarly, a town-A partisan with type x is indifferent between enrolling in these two districts if

$$v(x, y_{B_1}) - p_{B_1} = v(x, y_{B_2}) - p_{B_2}.$$

That is, the equilibrium indifference conditions are the same and so the cutoff determining whether types enroll in district B_2 or in district B_1 must be the same for partisans of each town. In a Type 1 equilibrium, lowest types enroll in B_1. Since enrollment in B_1 must equal measure 1/2 in equilibrium, the cutoff between districts B_1 and B_2 must be the 25th percentile, $x_{0.25}$, for partisans of each town.

Given this observation, there are only two degrees of freedom in the enrollment pattern, specifically the values x_A and x_B which are the cutoffs distinguishing between town A and district B_2 for partisans of the two towns, respectively. Then the two equilibrium conditions are that partisans of town A with type x_A and also partisans of town B with type x_B are indifferent between living in A and in B_2. These conditions are the following:

$$v(x_A, y_A) - p_A + \theta = v(x_A, y_{B_2}) - p_{B_2}$$
$$v(x_{B_2}, y_{B_2}) - p_{B_2} + \theta = v(x_B, y_A) - p_A.$$

Combining these equations, we have a single condition

$$2\theta = v(x_{B_2}, y_{B_2}) - v(x_{B_2}, y_A) - v(x_A, y_{B_2}) - v(x_A, y_A).$$

With a Uniform distribution of types, the market clearing condition in town A requires $x_B = 1 - x_A$, and with $v(x, y) = xy$, this condition can be simplified to $(x_{B2} - x_A)(y_{B2} - y_A) = 2\theta$, or $(1 - 2x_A)(y_{B2} - y_A) = 2\theta$. Town-B partisans with types between 0.25 and $x_B = 1 - x_A$ enroll in B_2. Similarly, partisans of town A with types between 0.25 and x_A enroll in district B_2. Therefore, $y_{B2} = 2x_A^2 - 2x_A + 7/8$ and $y_A = 0.5 + x_A - x_A^2$. Substituting these values in the market clearing condition gives $\theta = 3x_A^3 - 4.5x_A^2 + 15x_A/8 - 3/16$. This cubic equation also has a unique solution in the relevant range for $\theta < 3/64$.[27]

Comparisons of Equilibria for a Single Value of θ

We compare the equilibria of each type for the particular value $\theta = 37/2000$.[28] Table C1 lists the enrollment patterns for partisans of each town in the three equilibria corresponding to Types 1, 2, and 3. There is considerable overlap in the enrollment patterns for partisans of the two towns. In each equilibrium, lowest types enroll in the district with lowest school quality, while highest types enroll in the district with highest school quality, regardless of partisanship. Furthermore, many "middle" types enroll in the district with middling school quality in each equilibrium, regardless of partisanship, though the definition of a "middle" type varies endogenously across the three types of equilibria. Of these three distinct equilibria with $\theta = 37/2000$, the Type 2 equilibrium is closest in nature to the equilibrium from Example 1 in the one town model; in the Type 2 equilibrium, the choice by town A to adopt school choice induces a change in school quality towards the middle, but induces flight of lowest and highest partisan types.

	Town-A Partisan			Town-B Partisan		
Equilibrium	District B_1	District B_2	Town A	District B_1	District B_2	Town A
Type 1	$0 \leq x \leq 1/4$	$1/4 \leq x \leq 0.55$	$x \geq 0.55$	$0 \leq x \leq 1/4$	$1/4 \leq x \leq 0.45$	$x \geq 0.45$
Type 2	$0 \leq x \leq 0.2$	$x \geq 0.8$	$0.2 \leq x \leq 0.8$	$0 \leq x \leq 0.3$	$x \geq 0.7$	$0.3 \leq x \leq 0.7$
Type 3	$0.55 \leq x \leq 3/4$	$x \geq 3/4$	$x \leq 0.55$	$0.45 \leq x \leq 3/4$	$x \geq 3/4$	$x \leq 0.45$

[27]The value $\theta = 3/64$ yields a knife-edge equilibrium of this sort where $x_A = 1/4$, meaning that no partisans of town A enroll in district B_2. In this equilibrium, partisans of town A with type 1/4 are exactly indifferent between all three options: enrolling in A, B_2, or B_1. For $\theta < 3/64$, there is a unique Type 1 equilibrium in this example, where x_A is strictly decreasing in θ with $x_A = 0.5$ at $\theta = 0$ and $x_A = 1/4$ at $\theta - 3/64$.

[28]The interior cutoffs with values $(0.2, 0.8)$ and $(0.3, 0.7)$ are exact for the Type 2 equilibrium. The interior cutoffs with values 0.45 and 0.55 for the Type 1 and the Type 3 equilibria are approximate to three decimal places.

Equilibrium	Quality y_{B_1}	Quality y_{B_2}	Quality y_A
Type 1	1/8	1/2	11/16
Type 2	13/100	87/100	1/2
Type 3	1/2	7/8	5/16

Table C1. Comparison of Equilibria in Example 2

C.4 General Properties of the Two-Town Model

We next ask whether we can generalize the insights of this example. Proposition 3 shows that a non-integrated equilibrium of Type 2 exists when partisanship is not too large.

Proposition 3 *There exists a value θ_{NI} such that there is a non-integrated equilibrium of the two-town model where town A adopts school choice and town B adopts a neighborhood assignment rule iff $\theta \geq \theta_{NI}$ and there is an integrated equilibrium of Type 2 for each $\theta < \theta_{NI}$.*

Our proof of Proposition 3 relies on a fixed point argument. Intuitively, if $\theta < \theta_{NI}$, then there are incentives for highest and/or lowest type town-A partisans to trade places with marginal type town-B partisans. But as trades of these sorts occur in equilibrium, then the identities of marginal type families change and specifically the marginal low-type town-A partisans increases, when the marginal low-type town-B partisans decreases. Thus, for each θ with $0 < \theta < \theta_{NI}$, there must be a critical point (with $x_A^L < x_B^L$ and associated values for x_A^H and x_B^H) where the pair of values of marginal types (x_A^L, x_B^L) yields exactly equal utility gains (excluding prices) for each of these two marginal types to choose town A rather than district 1 in town B, thereby producing an integrated equilibrium.

Corollary 4 *In an integrated equilibrium of Type 2 where town A uses school choice and town B uses neighborhood assignment, the lowest-type partisans of each town enroll in schools with lower qualities and highest-type partisans of each town enroll in schools with higher qualities than they would in a non-integrated equilibrium with neighborhood assignment in both towns.*

Corollary 4 follows from the observation that any type x student will choose the same district within town B whether that student is partisan to town A or to town B. In a Type 2 equilibrium, highest and lowest type students (regardless of partisanship) enroll in town B in an integrated equilibrium. Since partisans of each town with x close to 0 enroll in district B_1 while partisans of each town with x close to 1 enroll in district B_2, the quality of these districts must be spread farther

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than in the non-integrated equilibrium. Thus, if $\theta < \theta_{NI}$, town A's adoption of school choice rule only increases inequality of educational opportunities in a Type 2 equilibrium (as measured by the spread between the highest and lowest quality schools chosen by partisans of town A.)

Proposition 4 extends Proposition 3 to confirm the existence of Type 1 and Type 3 equilibria for relatively small values of θ. As suggested by our analysis of the example above, the proof of Proposition 4 is much simpler than that of Proposition 3 because we know in advance that $x_{0.25}$ is the marginal type between B_1 and B_2 for partisans of either town in a Type 1 equilibrium and that $x_{0.75}$ is the marginal type between B_1 and B_2 for partisans of either town in a Type 3 equilibrium. The proof follows by another fixed point argument.

Proposition 4 *There exists values θ_1 and θ_3 such that there an integrated equilibrium of Type j if $\theta < \theta_j$ for $j = 1, 3$.*

C.5 Welfare Analysis for the Two-Town Model

Welfare analysis in the two-town model shares features of the one-town model but is complicated both by multiplicity of equilibria and by the fact that outside options are generated endogenously rather than fixed exogenously. In the one-town model, when a student enrolls in town t in equilibrium 1 but takes the outside option in equilibrium 2, by revealed preference, that student must prefer equilibrium 1 since the same outside option is available in both cases. However, this is not the case in the two-town model, for a change from neighborhood assignment to school choice in town A, likely improves outside options in town B for some town-A partisans but degrades them for others.

Given these complexities, we focus our welfare analysis on lowest types, in particular lowest type town-A partisans, which adopts school choice, since proponents of school choice typically argue it is beneficial for lower-type families. Suppose that town A has a neighborhood rule with two districts, A_1 and A_2 in the base case. We will consider what happens when town A offers school choice.

Equilibrium	District	School Quality
Base Case	A_1	$\mathbb{E}[x\|0 < x < 0.5]$
Type 1	B_1	$\mathbb{E}[x\|0 < x < 0.25]$
Type 2	B_1	$\mathbb{E}[x\|0 < x < 0.25] < y_{B_1} < \mathbb{E}[x\|0 < x < 0.5]$
Type 3	A	$\mathbb{E}[x\|0 < x < 0.25] < y_A < \mathbb{E}[x\|0 < x < 0.5]$

Table C2. Equilibrium School Qualities for Lowest-Type Town-A Partisans

In any integrated equilibrium where town A offers school choice and town B uses a neighborhood assignment rule, the district with the lowest school quality enrolls the lowest types of partisans of each town. From a purely mechanical standpoint, in any integrated equilibrium where district B_1 has lowest school quality, that school quality is lower than in the baseline case of a non-integrated equilibrium where only partisans of town B enroll in district B_1. For instance, in Example 2, school quality is $y_{B_1} = 0.25$ in the base case, but this school quality falls to 1/8 (the minimum possible) in an integrated equilibrium of Type 1 and to 13/100 in an integrated equilibrium of Type 2. The only exception is in a Type 3 equilibrium, where town A has the lowest quality schools and those schools have higher quality than district B_1 in the base case. In Example 2, lowest types enroll in schools with quality 1/4 in the base case and with quality 5/16 in a Type 3 equilibrium where town A offers school choice.

As a general observation, the value of the lowest quality school in town A increases when town A adopts school choice, whether the equilibrium is Type 1, 2, or 3. Yet, in similar fashion to the one town model, this increase in quality of schools in district $A1$ need not directly affect lowest type partisans of town A because they move to town B in either a Type 1 or a Type 2 equilibrium. Lowest-type partisans do attend higher quality schools in a Type 3 equilibrium when town A adopts school choice than in the base case. However, this only occurs because property values throughout town A are uniformly lower than in town B in a Type 3 equilibrium.

There is a degree of freedom in the description of any equilibrium in the two town model, namely the price of houses in the district with lowest school quality. One seemingly natural rule would be to set the price in this district to the competitive price for schools of this quality in the one town model. Given this assumption, it is possible to have a clean comparison of realized utility values for lowest-type town A partisans in the base (non-integrated) case and in a Type 3 equilibrium. In the two equilibria, lowest type town A partisans pay the competitive price to live in town A. Therefore, as in the one town model, they achieve higher utility when attending a lower quality school in the base case than in an integrated equilibrium of Type 3. Also, as in the one town model, it is possible to take the paternalistic point of view that the Type 3 equilibrium is preferable to the base case for these lowest type partisans of town A.

By contrast, comparisons of the base case equilibrium to an integrated equilibrium of either Type 1 or Type 2 are not as clear from the perspective of lowest-type partisans of town A because they move to town B in an integrated equilibrium of Type 1 or 2. Since there is no consistent outside option, we cannot use revealed preference to compare realized utility values across a pair of

equilibria. What we can say is that lowest-type partisans of town A attend unambiguously lower quality schools in a Type 1 or 2 equilibrium after the adoption of school choice by town A than beforehand and also lose the benefit of the partisan bonus after moving to town B.

D Additional Details and Proofs for Two-Town Model (Online Appendix)

D.1 Calculations for Example 2

Suppose that town-A partisans enroll in district B_1 if $x \in [0, x_A^L]$, enroll in town A if $x \in (x_A^L, 1-x_A^L)$, and enroll in district B_2 if x in $[1-x_A^L, 1]$. Then, given that each district enrolls an equal number of students and the types of partisans of both towns are distributed according to $U(0,1)$, this means that the cutoffs for town-B partisans are given by $x_B^L = \frac{1}{2} - x_A^L$ and $x_B^H = \frac{1}{2} + x_A^L$. That is, town-B partisans enroll in district B_1 if $x \in [0, \frac{1}{2} - x_A^L]$, enroll in town A if $x \in (\frac{1}{2} - x_A^L, \frac{1}{2} + x_A^L)$, and enroll in district B_2 if $x \in [\frac{1}{2} + x_A^L, 1]$.

Given these choices, the average type of town-A partisans is $\frac{x_A^L}{2}$ in district B_1, $\frac{1}{2}$ in town A and $(1 - x_A^L/2)$ in district B_2. Similarly, the average type of town-B partisans is $\frac{1}{4} - x_A^L/2$ in district B_1, $\frac{1}{2}$ in town A and $(\frac{3}{4} + \frac{x_A^L}{2})$ in district B_2. Taking weighted averages, we have

$$
\begin{aligned}
y_{B_1} &= \left[x_A^L \frac{x_A^L}{2} + (\frac{1}{2} - x_A^L)(\frac{1}{4} - \frac{x_A^L}{2}) \right] / \left[\frac{1}{2} \right] &&= 2(x_A^L)^2 - x_A^L + \frac{1}{4}, \\
y_{B_2} &= \left[x_A^L \frac{(1 - x_A^L)}{2} + (\frac{1}{2} - x_A^L)(\frac{3}{4} + x_A^L) \right] / \left[\frac{1}{2} \right] &&= \frac{3}{4} + x_A^L - 2(x_A^L)^2.
\end{aligned}
$$

In equilibrium,

(1) Town-A partisans with $x = x_A^L$ obtain equal utility from A and B_1.

(2) Town-B partisans with $x = x_B^L$ obtain equal utility from A and B_1.

(3) Town-A partisans with $x = 1 - x_A^L$ obtain equal utility from A and B_2.

(4) Town-B partisans with $x = 1 - x_B^L$ obtain equal utility from A and B_2.

Given $v(x, y) = xy$, these conditions can be represented as

$$x_A^L y_A - p_A + \theta = x_A^L y_{B_1} - p_{B_1}, \tag{9}$$

$$x_B^L y_A - p_A = x_B^L y_{B_1} - p_{B_1} + \theta, \tag{10}$$

$$x_A^H y_A - p_A + \theta = x_A^H y_{B_2} - p_{B_2}, \tag{11}$$

$$x_B^H y_A - p_A = x_B^H y_{B_2} - p_{B_2} + \theta. \tag{12}$$

Solving for $p_A - p_{B_1}$ in (9) and (10) gives $(x_A^L - x_B^L)(y_A - y_{B1}) + 2\theta = 0$, or equivalently $2\theta = (\frac{1}{2} - 2x_A^L)(y_A - y_{B_1})$ after substituting $x_B^L = \frac{1}{2} - x_A^L$. Then substituting $y_A = \frac{1}{2}$ and

$y_{B_1} = \frac{1}{4} - x_A^L/2$, we have $y_A - y_{B_1} = \frac{1}{4} + x_A^L/2$, and thus $2\theta = \frac{1}{8} - 3(x_A^L)^2 + 4(x_A^L)^3)$. Solving (11) and (12) for θ as a function of x_A^L yields the identical equation.

Based on these computations, there is an equilibrium of the given form whenever $2\theta = 1/8 - 3(x_A^L)2 + 4(x_A^L)^3$ or equivalently $\theta = 1/16 - (3/2)(x_A^L)^2 + 2(x_A^L)^3$, and $x_A^L \leq 1/4$ so that $x_A^L \geq x_B^L$. This is a cubic equation for θ as a function of x_A^L, so is not naturally conducive to an analytic solution with x_A^L as a function of θ. However, we can identify some of the properties of $x_A^L(\theta)$ by studying comparative statics of this equation with θ as a function of x_A^L.

Differentiating $\theta(x_A^L) = 1/16 - (3/2)(x_A^L)^2 + 2(x_A^L)^3$ with respect to x_A^L gives $d\theta/dx_A^L = 6(x_A^L)^2 - 3x_A^L < 0$ for $x_A^L < \frac{1}{2}$. So θ is declining as a function of x_A^L over the relevant range of values of x_A^L from 0 to 1/4 and further $\theta(x_A^L = 0) = 1/16$, corresponding to the cutoff $\theta_{NI} = 1/16$ for a non-integrated equilibrium, and $\theta(x_A^L = 1/4) = 0$, corresponding to an integrated equilibrium where partisans of both towns follow identical decision rules. That is, there is a one-to-one relationship between θ and x_A^L for x_A^L between 0 and 1/4, and therefore a unique equilibrium of this form for each value $\theta < \theta_{NI}$. Substituting $x_A^L = 0.2$ into the equations above yields $\theta(x_A^L = 0.2) = 37/2000$, $y_{B_1} = 0.13$ and $y_{B_2} = 0.87$ – the values used in the example in the text.

D.2   Proof of Proposition 2

Proof. Suppose that a town-B partisan of type x_h enrolls in district d in town B where $y_d > y_A$. Then since this student prefers district d in town B to enrolling in town A,

$$v(x_h, y_d) + \theta - p_d \geq v(x_h, y_A) - p_A,$$

or equivalently,

$$\theta \geq p_d - p_A + v(x_h, y_d) - v(x_h, y_A).$$

By the property of increasing differences of v, the difference $v(x, y_d) - v(x, y_A)$ is strictly increasing in x given $y_d > y_A$, so any partisan of town B with $x' > x_h$ strictly prefers district d in town B to enrolling in town A and will not enroll in town A. By similar reasoning, if type x_l enrolls in a district in town B with school quality less than y_A, then town-B partisans of type $x'' < x_l$ also will not enroll in town A. Thus, the set of town-B partisans who enroll in town A must be an interval of types $[x_B^L, x_B^H]$. An essentially identical argument extends this result to show that the set of town-A partisans who enroll in town A is an interval of types $[x_A^L, x_A^H]$.

Since town-A partisans receive a bonus for enrolling in town A, while town-B partisans receive a bonus for enrolling in town B, if a town-B partisan of type x enrolls in town A, then a town-A

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partisan of type x will also enroll in town A in equilibrium. This shows that $x_A^L \leq x_B^L \leq y_A$, $x_A^H \geq x_B^H$. A town-B partisan of type $x < x_A^L$ enrolls in a school in town B, so $v(x, y_d) + \theta - p_d \geq v(x, y_A) - p_A$ for some district d in town B. We can rewrite this inequality as

$$v(x, y_d) - v(x, y_A) \geq p_d - p_A - \theta.$$

But if $y_d \geq y_A$, then this inequality would hold for all types greater than x (by the property of increasing differences for v), and so none of them would enroll in town B.[29] Thus, town-B partisans with types below x_L^A enroll in districts in town B with qualities less than y_A. By a similar argument, town-B partisans with types above x_A^L enroll in districts in town B with qualities greater than y_A, with analogous properties holding for town-A partisans. ∎

D.3 Proof of Proposition 3

Proof. Suppose that there are two districts of equal size in each town, that there is measure 1 each of town-A partisans and of town-B partisans (so that each district has capacity equal to measure $1/2$), and that the distribution of types is identical for partisans of each town. By Proposition 2, when town A uses a school choice rule and town B uses a neighborhood school assignment rule, in any equilibrium, an interval of partisans of type A $[x_A^L, x_A^H]$ and an interval of town-B partisans $[x_B^L, x_B^H]$ enroll in town A, where $x_A^L \leq x_B^L \leq x_B^H \leq x_A^H$ and these cutoffs are determined endogenously in equilibrium.

Given these enrollment constraints, the choice of x_A^L implicitly determines the choice of x_B^L given the enrollment constraint $F(x_A^L) + F(x_B^L) = 1/2$. Then since $x_A^L \leq x_B^L$, x_A^L takes possible values on $[0, x_{0.25}]$, where $x_{0.25}$ is defined by $F(x_{0.25}) = 1/4$. Similarly, x_A^H takes possible values on $[x_{0.75}, 1]$ where $F(x_{0.75}) = 3/4$ and x_B^H is an implicit function of x_A^H according to the equation $(1 - F(x_A^H)) + (1 - F(x_B^H)) = 1/2$ or equivalently $F(x_A^H) + F(x_B^H) = 3/2$.

Define

$$\lambda_L(x_A^L, x_A^H) = [v(x_B^L, y_A) - v(x_A^L, y_A)] - [v(x_B^L, y_1) - v(x_A^L, y_1)] - 2\theta$$

and

$$\lambda_H(x_A^L, x_A^H) = [v(x_A^H, y_2) - v(x_B^H, y_2)] - [v(x_A^H, y_A) - v(x_B^H, y_A)] - 2\theta,$$

The arguments to λ_L and λ_H exploit the fact that x_B^L (x_B^H) can be written in terms of x_A^L (x_A^H), and the value of y_A depends on x_A^L and x_A^H.

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[29]We assume that town-B partisans enroll in town B in case of a tie in utility between the most preferred district in town B and the most preferred district in town A.

There is no integration at the bottom if $x_A^L = 0$ and no integration at the top if $x_A^H = 1$. Given school qualities $y_1 < y_A < y_2$ and prices p_A, p_1, and p_2, there is no integration at the bottom if $v(x_A^L = 0, y_A) + \theta - p_A \geq v(x_A^L, y_1) - p_1$ and $v(x_B^L, y_{B_1}) + \theta - p_1 \geq v(x_B^L, y_A) - p_A$, so that marginal (boundary) types of partisans of each town each prefer not to integrate. Combining these two equations to eliminate the prices gives the condition $\lambda_L(x_A^L = 0, x_A^H) \leq 0$ as a necessary condition for an equilibrium with non-integration at the bottom. If there is integration at the bottom, then both incentive conditions must hold with equality so that $\lambda_L(x_A^L = 0, x_A^H) = 0$ is a necessary condition for an equilibrium with $x_A^L > 0$. Similarly, $\lambda_H(x_A^L, x_A^H = 1) \leq 0$ is a necessary condition for equilibrium with no integration at the top and $\lambda_H(x_A^L, x_A^H) = 0$ is necessary for an equilibrium with integration at the top.

Holding x_A^L fixed, increased integration at the top, as represented by a reduction in x_A^H, yields an increase in y_2 and a decline in y_A. That is, y_A is strictly increasing and y_2 is strictly decreasing in x_A^H, while x_A^L, x_B^L, and y_1 are constant in x_A^H. By increasing differences of v in both arguments, $\lambda_L(x_A^L, x_A^H)$ is strictly increasing in x_A^H, so it takes its maximum value at $x_A^H = 1$ for each value of x_A^L. Thus, for each x_A^L, there is at most one value of x_A^H such that $v(x_A^L, x_A^H) = 0$. Further, when $x_A^L = x_{0.25}$ (its maximum possible value), then $x_A^L = x_B^L$ and $\lambda_H(x_{0.25}, x_A^H) = -2\theta$ for each value of x_A^H. Since v is continuous and $\lambda_L(x_{0.25}, 1) < 0$, then either (1) there exists some value $\bar{x} < x_{0.25}$ such that $\lambda_L(\bar{x}, 1) = 0$ and $\lambda_L(x_A^L, 1) < 0$ for $x_A^L > \bar{x}$ or (2) $\lambda_L(x_A^L, 1) < 0$ for all $x_A^L \leq x_{0.25}$.

In case (1), by construction, there exists a uniquely defined function $\varphi(x_A^L)$ for $\underline{x} \leq x_A^L \leq \bar{x}$ such that $\lambda_L(x_A^L, \varphi(x_A^L)) = 0$. From above, we know $\varphi(x) = x_{0.75}$ and $\varphi(x) = 1$ Furthermore, since $\lambda_L(\bar{x}, 1) = 0$, then $\lambda_L(\bar{x}, x_{0.75}) < 0$ since λ_L is strictly increasing in its second argument. Then since v is continuous, there either

(1A) exists a value $\underline{x} < \bar{x}$ such that $\lambda_L(x, x_{0.75}) = 0$ and $\lambda_L(x_A^L, x_{0.75}) < 0$ for each x_A^L such that $\underline{x} < x_A^L < \bar{x}$, or

(1B) $\lambda_L(x_A^L, x_{0.75}) < 0$ for each $x_A^L < \bar{x}$.

When $x_A^H = x_{0.75}$ (its minimum possible value), then $x_A^H = x_B^H$ and so $\lambda_H(x_A^L, x_{0.75}) = -2\theta$ for each value of x_A^L. So, in particular, in Case (1A), $\lambda_H(x, x_{0.75}) = \lambda_H(x, \varphi(x)) = -2\theta$. Then, since v (and therefore λ_H) is continuous in each argument, there either exists x_A^L between $\underline{x}$ and $\bar{x}$ so that $\lambda_H(x_A^L, \varphi(x_A^L)) = 0$, in which case there is an equilibrium with integration at top and bottom at $[x_A^L, x_A^H = \varphi(x_A^L)]$ or $\lambda_H(\bar{x}, \varphi(\bar{x}) = 1) \leq 0$, in which case there is an equilibrium with integration at the bottom and non-integration at the top at $[x_A^L = \bar{x}, x_A^H = 1]$.

Similarly, in Case (1B), there exists a uniquely defined function $\varphi(x_A^L)$ for each $x_A^L \leq \bar{x}$ such that $\lambda_L(x_A^L, \varphi(x_A^L)) = 0$. The distinction between Case (1A) and Case (1B) is that since the range $(0, \bar{x})$ of relevant values of x_A^L includes 0, it is now possible to find an equilibrium with non-integration at the bottom. Since $\lambda_H(0, x_{0.75}) = -2\theta < 0$, either there exists a value x_A^H between $x_{0.75}$ and $\varphi(x_A^L = 0)$ such that $\lambda_H(0, x_A^H) = 0$, in which case there is an equilibrium with integration at the top and non integration at the bottom at $(0, x_A^H)$ or $\lambda_H(0, \varphi(0)) < 0$, in which case the logic from (1A) implies that there exists an equilibrium.

In case (2), $\lambda_L(x_A^L, 1) < 0$ for all $x_A^L \leq x_{0.25}$, so in fact $\lambda_L(x_A^L, x_A^H) < 0$ in all cases. This rules out the possibility of an equilibrium with integration at the bottom, so assume that $x_A^L = 0$ and look for an equilibrium with non integration at the bottom. Since $\lambda_H(0, x_{0.75}) = -2\theta < 0$, either $\lambda_H(0, 1) \leq 0$, in which case there is an equilibrium with non-integration at top or bottom, or there exists some value x_A^H between $x_{0.75}$ and 1 such that $\lambda_H(0, x_A^H) = 0$ in which case there is an equilibrium at $(0, x_A^H)$ with integration at the top and no integration at the bottom. ∎

D.4 Proof of Proposition 4

Proof. In a Type 1 integrated equilibrium, $y_A > y_{B_1} > y_{B_2} = \mathbb{E}[x | 0 < x < 0.25]$. Since we know that $x_{0.25}$ is the type-cutoff between district B_1 and B_2 in this equilibrium, the only remaining parameters to identify are the cutoffs for partisans of each town between district B_2 and town A. As described in the text, denote x_A as the cutoff for town-A partisans and x_B be the cutoff for town-B partisans. The boundary indifference condition for partisans of type A is

$$v(x_A, y_A) - p_A + \theta = v(x_A, y_{B_2}) - p_{B_2}. \tag{13}$$

Similarly, the boundary indifference condition for partisans of type B is

$$v(x_B, y_A) - p_A = v(x_B, y_{B_2} - p_{B_2}) + \theta. \tag{14}$$

Subtracting equation (14) from (13) gives

$$2\theta + v(x_A, y_A) - v(x_A, y_{B_2}) = v(x_B, y_A) - v(x_B, y_{B_2}).$$

Since we know $y_A > y_{B_2}$, we can convert this to integral form:

$$2\theta + \int_{y_{B_2}}^{y_A} \frac{\partial v}{\partial y}(x_A, z)dz = \int_{y_{B_2}}^{y_A} \frac{\partial v}{\partial y}(x_B, z)dz.$$

This equation can only hold if $x_A < x_B$ given that $\theta > 0$, so we can rewrite it in double integral form:

$$2\theta = \int_{x_A}^{x_B} \int_{y_{B_2}}^{y_A} \frac{\partial^2 v}{\partial x \partial y}(a, z)\, da\, dz.^{30} \tag{15}$$

The market clearing condition here requires

$$1 - F(x_B) + 1 - F(x_A) = 1,$$

or

$$F(x_B) = 1 - F(x_A).$$

Consider two extreme possibilities: $x_A = x_{0.25}$ and $x_B = x_{0.75}$, where no town-A partisans enroll in district B_2 and $x_A = x_B = 1/2$, where the same number of partisans of each town enroll in district B_2. If $x_A = x_B = 1/2$, the right-hand side of the last equation is 0, so less than the left-hand side which is equal to 2θ. Next define

$$\theta^{*1} = 0.5 \int_{x_{0.25}}^{x_{0.75}} \int_{y_{B2}}^{y_A} \frac{\partial^2 v}{\partial x \partial y}(a, z)\, da\, dz,$$

where y_A and y_{B_2} take the appropriate values corresponding to $x_A = x_{0.25}$ and $x_B = x_{0.75}$. By construction, if $x_A = x_{0.25}$ and $x_B = x_{0.75}$, the right-hand side of the equation (15) is greater than the left-hand side if $\theta < \theta^{*1}$. Then by the Intermediate Value Theorem, there is a value of x_A between $x_{0.25}$ and $1/2$ where the combined boundary indifference condition holds with equality. This value will support an equilibrium of Type 1 where price p_{B_1} is set to the competitive price for y_{B_1} from the one-town model and the price increments between districts B_1 and B_2 as well as B_2 and A are determined by the boundary indifference conditions.

An essentially identical argument proves the existence of a Type 3 integrated equilibrium for θ below a cutoff θ^{*3}. ∎

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[30]We note that y_A and y_{B_2} are in fact functions of x_A and x_B and allow for this in the analysis described below.