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ECONOMETRICS OF MATCHING‡

Regression Discontinuity in Serial Dictatorship: Achievement Effects at Chicago's Exam Schools†

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Many school and college admission systems use centralized mechanisms with lottery tie-breakers to allocate seats. Abdulkadiroğlu et al. (2015) show how lottery tiebreaking creates a stratified randomized trial, where the strata are preferences and priorities. In many settings, however, tiebreaking uses nonrandomly assigned criteria like a test score.

Non-lottery tiebreaking produces assignments that are correlated with applicants' potential outcomes, but the non-lottery scenario opens the door to regression discontinuity (RD) designs to measure school effects. This paper introduces a hybrid RD/propensity score empirical strategy that exploits the experiments embedded in serial dictatorship (SD), a mechanism widely used for college and selective K–12 admissions. The key to our analysis is an RD-SD propensity score that controls for the local probability of school assignment. We use the RD-SD propensity score to estimate effects of Chicago's exam schools on student achievement.

I. Characterizing Serial Dictatorship

Serial dictatorship with exam-score tie-breaking assigns applicants one at a time in the order of their exam scores to their most preferred schools with available seats. We assume (without loss of generality) that SD processes applicants in ascending order of exam scores, referred to here as the running variable and denoted by r_i for applicant i. SD assignments are characterized by a set of admissions cutoffs. Let $\mathbf{c} = (c_1, \ldots, c_S)$ denote admissions cutoffs, where c_s is the cutoff at school $s \in \{1, \ldots, S\}$. SD assigns applicant i her most preferred school for which $r_i < c_s$. With a continuum of applicants and school seats, these cutoffs are known to be constant, that is, fixed in repeated draws of the tiebreaker (Azevedo and Leshno 2017).

As in Abdulkadiroğlu et al. (2015), our goal is to learn about school effects using offers of school seats as instrumental variables for school attendance. Applicant type or preference order (denoted by θ_i) is a source of omitted variables bias (OVB) in such comparisons because applicants who rank schools differently tend to have different socioeconomic characteristics and therefore different outcomes. Type conditioning eliminates this source of OVB, but is unattractive when there are many types (5,776 applicants with nontrivial risk of an offer from Chicago's nine exam schools include 4,580 types). Our framework exploits the fact that the OVB induced by the correlation between type and offers is

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controlled by conditioning on a scalar function of type, the propensity score (Rosenbaum and Rubin 1983). This function is the conditional probability of assignment, $p_s(\theta) = E[D_{is} | \theta_i = \theta]$, where D_{is} indicates the SD-generated offer of a seat at school s to applicant i.

In general, $p_s(\theta)$ is an unrestricted function of type, so score conditioning would appear to have little advantage over full type conditioning. But the asymptotic approximation developed in Abdulkadiroğlu et al. (2015) yields a large market score for markets with lottery tiebreaking that is determined by only two statistics. Here, we derive a large market propensity score for SD mechanisms with nonrandom tiebreakers.

II. The RD-SD Propensity Score

We model assignment risk as being generated by draws from the running variable distribution, fixing the set of applicants and their preferences. Assume that running variables, R_i, are distributed over $[0,1]$, with cumulative distribution function F_R^i. Running variables for applicants i and j are independent, but, in contrast with the lottery case, not necessarily identically distributed. Note that r_i is the realized value of R_i.

The continuum economy RD-SD propensity score for any given treatment school s depends on at most two cutoffs. The first is the cutoff at s. The second, called the most informative disqualification and denoted by $MID_{\theta s}$, varies with type. The cutoff $MID_{\theta s}$ equals zero when s is type θ's first choice, but is otherwise the most forgiving (i.e., the maximum) cutoff among the schools type θ ranks ahead of s. $MID_{\theta s}$ captures the effect of truncation induced by disqualification at schools preferred to s on assignment risk at s: students who qualify at a school they prefer to s are never offered seats at s.

By the law of iterated expectations, the probability a type θ applicant has a running variable value below r_0 is $F_R(r_0|\theta) = E[F_R^i(r_0) | \theta_i = \theta]$, where $F_R^i(r_0)$ is F_R^i evaluated at r_0. Our first result, implied by a more general result in Abdulkadiroğlu et al. (2017), uses $F_R(r_0|\theta)$ to derive the RD-SD propensity score.

PROPOSITION 1: *For all s and θ in any continuum economy, we have*

$$p_s(\theta) = (1 - F_R(MID_{\theta s}|\theta)) \times \max\left\{0, \frac{F_R(c_s|\theta) - F_R(MID_{\theta s}|\theta)}{1 - F_R(MID_{\theta s}|\theta)}\right\},$$

where we set $p_s(\theta) = 0$ when $MID_{\theta s} = 1$.

This proposition reflects the forces of qualification and disqualification that determine SD-generated assignment risk. Applicant i of type θ is assigned a school she prefers to s when $r_i < MID_{\theta s}$. Therefore, fraction $1 - F_R(MID_{\theta s}|\theta)$ of type θ applicants are considered for s. The second line is the probability of being assigned s conditional on not being assigned a more preferred choice, an event that occurs if and only if $MID_{\theta s} < r_i \leq c_s$, Applicants for whom $MID_{\theta s} > c_s$ are never seated at s because in this scenario those who fail to clear $MID_{\theta s}$ are surely disqualified at s as well. Proposition 1 generalizes Corollary 1 of Abdulkadiroğlu et al. (2015) to cover arbitrary distributions of R_i.

Control for the RD-SD propensity score eliminates OVB due to the association between type and potential outcomes. But Proposition 1 raises three empirical challenges not encountered under lottery tiebreaking. First, because $F_R(\cdot|\theta)$ depends on θ, the score in Proposition 1 need not have coarser support than θ. This is in spite of the fact applicants with different values of θ have the same $MID_{\theta s}$. Second, the conditional running variable distribution, $F_R(\cdot|\theta)$, is unknown and must be estimated for each θ. Third, while control for the propensity score eliminates confounding from type, conditional on $p_s(\theta)$, assignment is still correlated with potential outcomes because D_{is} is a function of r_i.

We tackle these three problems by focusing on applicants with running variable values in a δ–neighborhood of admissions cutoffs. Specifically, define the probability of an offer from school s for applicants in a neighborhood of r_0 as $p_s(\theta; r_0, \delta) = E[D_{is} | \theta_i = \theta, R_i \in (r_0 - \delta, r_0 + \delta)]$ for $\delta > 0$. For small enough δ, the restriction to applicants with admissions scores in $(r_0 - \delta, r_0 + \delta)$ eliminates OVB from the running variable, while conditioning on values of $p_s(\theta; r_0, \delta)$ eliminates confounding from differences in applicant preferences.

Our second theoretical result characterizes the local RD-SD propensity score as the limit of $p_s(\theta; r_0, \delta)$ as δ goes to 0.

PROPOSITION 2: *Suppose $F_R(r_0|\theta)$ is differentiable everywhere and that $c_s \neq c_{s'}$ for any $s \neq s'$. Then for all s, θ in a continuum economy,*

$$\lim_{\delta \to 0} p_s(\theta; r_0, \delta) = \begin{cases} 0 & \text{if } c_s < MID_{\theta s} \\ 0.5 & \text{if } MID_{\theta s} < c_s \end{cases}$$

for $r_0 = MID_{\theta s}$, or c_s, and

$$\lim_{\delta \to 0} p_s(\theta; r_0, \delta) = \begin{cases} 1 & \text{if } r_0 \in (MID_{\theta s}, c_s) \\ 0 & \text{otherwise} \end{cases}$$

for all $r_0 \neq MID_{\theta s}, c_s$.

The local propensity score in Proposition 2 is constant at 0.5 for applicants with nontrivial assignment risk, obviating the need to estimate $F_R(r_0|\theta)$. Proposition 2 also reveals the school-level RD-style experiments embedded in SD. In particular, consider type θ applicants to s with exam scores in nonoverlapping intervals around c_s and $MID_{\theta s}$. Proposition 2 says that offers to applicants in these intervals are approximately determined by a coin toss.

Figure 1 depicts the cutoffs that determine assignment risk for 373 applicants to King High School for whom $MID_{\theta s}$ is the cutoff at (more selective) Brooks High School. The Brooks cutoff is indicated with a left vertical line; King applicants with MID equal to the Brooks cutoff are never seated at King when they qualify at more highly ranked Brooks. The King cutoff is indicated with the right vertical line; applicants with running variable values above this are likewise never seated at King. Applicant with values between the King and Brooks cutoffs are offered seats at King. Dots in the figure identify average offer rates as a function of the running variable. A consequence of Proposition 2 is that marginal applicants at King include two groups: applicants with running variable values near the King cutoff, and a group well away from the King cutoff, near the Brooks cutoff instead. Exam school effects might differ for these two groups.

The fact that offers are randomized while enrollment remains a choice motivates our two-stage least squares (2SLS) estimation strategy using offer dummies to instrument enrollment. Many King offers are declined; this can be seen in the enrollment rates plotted with triangles in Figure 1. No applicant not offered a seat at King

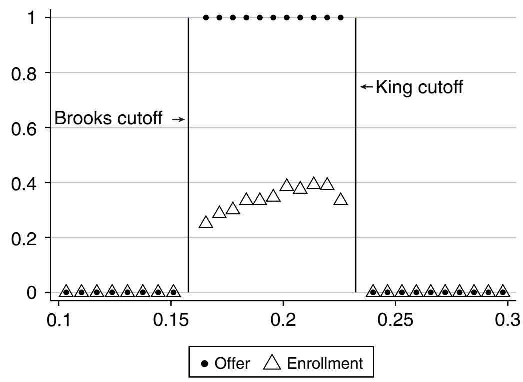

Figure 1. Offers and Enrollment at King

Note: Offers and enrollment for 373 applicants to King with MID given by the Brooks cutoff.

enrolls there, while the King first-stage, that is, the offer take-up rate between MID and the cutoff, averages around 0.35.

We're often interested in an overall school sector effect, rather than the effect of enrollment at specific schools. Under SD, the risk of receiving an exam school offer *somewhere* is determined by the cutoff at the least selective school an applicant ranks. Formally, let S_θ be the set of exam schools that type θ ranks and define the qualifying cutoff to be the most forgiving cutoff among schools in S_θ: $QC_\theta = \max_{s \in S_\theta} c_s$. An indicator for any exam school offer can then be coded as $D_i = \mathbf{1}[r_i < QC_{\theta_i}] = \sum_s D_{is}$, where the second equality reminds us that, because SD is a single-offer system, the any-offer dummy equals the sum of all single offer dummies.

As in Proposition 2, the any-offer propensity score is derived after first defining a local assignment probability around value r_0: $q_s(\theta; r_0, \delta) = E[D_i \,|\, \theta_i = \theta, R_i \in (r_0 - \delta, r_0 + \delta)]$. Using this notation, we have the following result.

PROPOSITION 3: *If F_R is differentiable everywhere and $c_s \neq c_{s'}$ for any $s \neq s'$, then for all θ in a continuum economy and for any $r_0 \in [0, 1]$,*

$$\lim_{\delta \to 0} q(\theta; r_0, \delta) = \begin{cases} 0 & \text{if } r_0 > QC_\theta \\ 0.5 & \text{if } r_0 = QC_\theta . \\ 1 & \text{if } r_0 < QC_\theta \end{cases}$$

Proposition 3 reflects the simplified nature of the risk behind D_i: applicants with a running variable value below their qualifying cutoff are sure to get an offer somewhere, though they may do better than the school that determines qualification. The limiting score treats qualification as random for those with values near the cutoff; this local risk is again a coin toss.

III. Empirical Strategies and Estimates

Proposition 2 and 3 provide a foundation for identification strategies that capture the causal effect of enrollment at Chicago's exam schools on achievement, as measured by tenth grade PLAN and eleventh grade ACT tests. Chicago students apply for exam school seats in eighth grade, hoping to enroll in ninth grade.

In our sample period (2011–2012), Chicago Public Schools (CPS) operated nine exam schools. Applicants rank up to six schools. Exam schools prioritize applicants using a common composite index formed from an admissions test, GPA, and grade 7 standardized test scores. This composite is the running variable.

The CPS exam school assignment mechanism incorporates place-based affirmative action, in which applicant addresses are classified into one of four tiers by the socioeconomic status of the census tract in which they live. Schools divide 70 percent of their seats equally between applicants from each of the four tiers, with each quarter treated as a subschool that assigns priority to one tier. The remaining 30 percent, said to be merit seats, are assigned without priorities.

In practice, applicants from a given tier are almost always offered either a merit seat or one of the seats prioritizing their tier. We can therefore analyze Chicago's assignment system as a serial dictatorship in which each school is split into five subschools.[1] Applicants to school s are treated as if they apply to both the subschool containing merit seats and the subschool containing seats reserved for their tier. Our notation for empirical models below ignores tiers; empirically, each school indexed by s is a school-tier combination.

We use Propositions 2 and 3 to classify applicants by risk for school-specific and any-school offers, and to find students in the neighborhood of each school's cutoff. The realized CPS allocation for school year 2011–2012 is used to compute these cutoffs. These in turn determine $MID_{\theta_i s}$ for each applicant i.

Individual school offer dummies, D_{is}, indicate $r_i \in [MID_{\theta_i s}, c_s]$. Given a cutoff-specific bandwidth δ_s, the estimated local RD-SD propensity score for each school-specific offer to an applicant of type θ_i, $\hat{p}_{is}$, is computed as follows:

$$\hat{p}_{is} = \begin{cases} 0.5 & \text{if } MID_{\theta_i s} < c_s \text{ and} \\ & \quad r_i \in (c_s - \delta_s, c_s + \delta_s) \text{ or} \\ & \quad r_i \in (MID_{\theta_i s} - \delta_s, MID_{\theta_i s} + \delta_s) \\ 1 & \text{if } MID_{\theta_i s} < c_s \text{ and} \\ & \quad r_i \in (MID_{\theta_i s} + \delta_s, c_s - \delta_s) \\ 0 & \text{if } MID_{\theta_i s} > c_s \text{ or} \\ & \quad r_i \notin (MID_{\theta_i s} - \delta_s, c_s + \delta_s) \end{cases}$$

Because each school offer changes exam enrollment to a different degree, we use the collection of school-specific offers to construct over-identified 2SLS estimates of the effects of any exam school enrollment, indicated by C_i. For outcome variable Y_i, the 2SLS first and second stages can be written

$$\text{(1)} \quad C_i = \sum_s \gamma_{1s} D_{is} + \sum_s \sum_p \eta_{1ps} \mathbf{1}_{\{\hat{p}_{is}=p\}} + h(r_i) + \nu_i;$$

$$Y_i = \gamma_2 C_i + \sum_s \sum_p \eta_{2ps} \mathbf{1}_{\{\hat{p}_{is}=p\}} + h(r_i) + \epsilon_i,$$

where $h(r_i)$ is a running variable control described below. The η_{1ps} and η_{2ps} coefficients control for the propensity score associated with each offer dummy in the first and second stages. The sample consists of applicants for whom $\hat{p}_{is} \in (0,1)$ for at least one s.

[1] Specifically, each school is split into five sub-schools as follows: 30 percent merit, and equal-size tier 1, tier 2, tier 3, and tier 4 subschools each with size 17.5 percent. An applicant from a given tier first ranks the merit subschool and then the tier subschool corresponding to their tier. Since CPS assigns merit seats before reserve seats, this version of the Chicago assignment mechanism matches 99.7 percent of the assigns from their DA-based system. Dur, Pathak, and Sonmez (2016) present more details on Chicago's assignment system.

The score for any offer, D_i, denoted $\hat{q}_i$, is computed as

$$\hat{q}_i = \begin{cases} 0.5 & \text{if } r_i \in [QC_i - \delta_s, QC_i + \delta_s] \\ 1 & \text{if } r_i < QC_i - \delta_s \\ 0 & \text{if } r_i > QC_i + \delta_s \end{cases},$$

where s is the school that determines applicant i's qualifying cutoff, QC_i.

Using a single any-offer instrument and the propensity controls suggested by Proposition 3 generates the following just-identified 2SLS setup:

$$(2) \quad C_i = \gamma_1 D_i + \sum_s \alpha_{1s} \mathbf{1}_{\{s = s_\theta^q\}} + h(r_i) + \nu_i;$$

$$Y_i = \gamma_2 C_i + \sum_s \alpha_{2s} \mathbf{1}_{\{s = s_\theta^q\}} + h(r_i) + \epsilon_i,$$

where s_θ^q identifies the school that determines the qualifying cutoff for type θ. This model omits score controls because the estimation sample is limited to applicants with $\hat{q}_i = 0.5$. These applicants have running variable values in the bandwidth around their qualifying cutoff.

We report 2SLS estimates using two specifications of the running variable control function, $h(r)$: $h_1(r) = \phi_0 r + \sum_s \phi_s \max\{0, r - c_s\}$, and $h_2(r) = \sum_{k=0}^{4} \phi_k r^k$. The first specifies a piecewise linear function of the running variable, with slope changes at each cutoff (in practice, these are school and tier specific). This control function is motivated by commonly employed RD implementation strategies using local linear control for the running variable with slope changes at the cutoff. The second control function specifies a quartic with polynomial coefficients that are fixed throughout the support of the running variable.

All models are estimated in a sample of applicants with running variable values in a set of cutoff-specific bandwidths. This is motivated by the limiting argument behind Propositions 2 and 3. Within these bandwidths, propensity scores are fixed at 0.5 for applicants with nontrivial risk of assignment; no further controls should therefore be necessary. Not surprisingly, however, and as in other RD applications, bandwidths are large enough to require control for running variable effects; this is accomplished here by including the control function, $h(r)$. We also add a set of four tier dummies to the running variable controls. These improve both precision and covariate balance for parsimonious specifications of $h(r)$. Finally, because bandwidths are cutoff-specific, the risk of any exam school offer varies (in our finite sample) with the identity of the qualifying cutoff. Just-identified models, therefore, include qualifying-cutoff fixed effects (α_{1s} and α_{2s}).

The any-offer first stage for exam school enrollment is about 0.38. First stages for individual school offers range from 0.26–0.69. The 2SLS estimates from both over-identified and just-identified models, and for different choices of the running variable control, consistently suggest exam schools have no effect on student achievement. These results can be seen in panel A of Table 1, which reports over-identified estimates based on Proposition 2 in the first two rows for two choices of $h(r)$. Exam school effects on math range from -0.11 to -0.16, while effects on reading are very close to zero. Just-identified estimates using a single any-offer instrument, reported in the fourth

Table 1—2SLS Estimates Exam School Effects

Instrument	PLAN Math (1)	PLAN Reading (2)	ACT Math (3)	ACT Reading (4)
Panel A. Propensity score conditioning				
School-specific offers	−0.137	−0.003	−0.114	0.058
	(0.096)	(0.096)	(0.087)	(0.093)
School-specific offers (polynomial controls)	−0.163	−0.022	−0.151	0.036
	(0.095)	(0.094)	(0.085)	(0.088)
Observations	5,387	4,969	5,275	4,624
Any offer	−0.035	−0.094	−0.220	0.050
	(0.131)	(0.133)	(0.133)	(0.137)
Observations	4,815	4,301	4,639	3,816
Panel B. Full type conditioning				
School-specific offers	−0.053	0.108	−0.018	0.301
	(0.216)	(0.228)	(0.202)	(0.218)
Observations	1,747	1,532	1,657	1,489
Any offer	−0.109	0.275	−0.194	0.511
	(0.260)	(0.271)	(0.277)	(0.295)
Observations	1,406	1,162	1,269	1,056

Notes: This table reports 2SLS estimates of exam school enrollment effects for four outcomes. Panel A shows estimates using propensity score controls; panel B reports 2SLS estimates controlling for preferences and tier. Models using school-specific offers are over-identified. The school-specific specification uses a uniform kernel, while the any offer specification uses the edge kernel. Estimates in this table were computed using the Imbens-Kalyanaraman bandwidth.

row of the table, are similar though less precise. For example, the ACT math standard error increases from 0.087 to 0.133 between row 1 and row 3. Barrow, Sartain, and de la Torre (2016) similarly find no evidence of achievement gains at Chicago exam schools.

The payoff to propensity score control for applicant risk can be seen in panel B of Table 1. This panel reports estimates of models (1) and (2) that include a full set of controls for applicant type, as in Abdulkadiroglu, Angrist, and Pathak (2014). The panel A sample is limited to applicants with nontrivial assignment risk, that is, with local propensity scores of 0.5. Full type control reduces the panel A sample by about two-thirds because, within type, there is no treatment variation. Consequently, panel B shows results that are far less conclusive. We see, for example, large positive and negative effects. None of these are significantly different from zero, since the standard errors here are more than twice those in panel A.

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