Gifts of the Immigrants, Woes of the Natives: Lessons from the Age of Mass Migration

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ONLINE APPENDIX C

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1 Relationship Between N_L, H, and L

In what follows, I first obtain an expression that relates N_L to H and L. Next, I show that: $i)$ $\frac{\partial N_L}{\partial H} > 0$ and; $ii)$ $\frac{\partial N_L}{\partial L} > 0$ if $\gamma > 0$. Using $(B16)$ in $(B14)$, we get

$$Y_L = \frac{N_L L^\beta}{1-\beta}\left(\frac{r}{\eta_L \beta}\right)^{1-\beta}$$

Plugging this back in $(B4)$, we get

$$\psi^\beta L^{-\beta} = \left[1 + \frac{H^\gamma (1-\beta)^\gamma}{N_L^\gamma L^{\beta\gamma} \psi^{\gamma(1-\beta)}}\right]^{\frac{1-\gamma}{\gamma}} \tag{1}$$

where $\psi \equiv \frac{r}{\eta_L \beta}$. Rearranging, we obtain

$$\psi^\beta = \left[L^{\frac{\beta\gamma}{1-\gamma}} + \frac{H^\gamma (1-\beta)^\gamma}{N_L^\gamma \psi^{\gamma(1-\beta)}} L^{\frac{\beta\gamma^2}{1-\gamma}}\right]^{\frac{1-\gamma}{\gamma}}$$

After some algebra, it is possible to get expression $(B17)$ in online appendix B

$$N_L = \frac{L^{\frac{\beta\gamma}{1-\gamma}} H (1-\beta)}{\psi^{(1-\beta)}\left(\psi^{\frac{\beta\gamma}{1-\gamma}} - L^{\frac{\beta\gamma}{1-\gamma}}\right)^{\frac{1}{\gamma}}} \tag{2}$$

Then, using (2), we can derive exact expressions for $\frac{\partial N_L}{\partial H}$ and $\frac{\partial N_L}{\partial L}$, and show that (i) and (ii) above hold. First,

$$\begin{aligned} \frac{\partial N_L}{\partial H} &= \frac{L^{\frac{\beta\gamma}{1-\gamma}} (1-\beta)}{\psi^{(1-\beta)}\left(\psi^{\frac{\beta\gamma}{1-\gamma}} - L^{\frac{\beta\gamma}{1-\gamma}}\right)^{\frac{1}{\gamma}}} \\ &= \frac{N_L}{H} > 0 \quad \forall\gamma \text{ and } \forall N_L > 0 \end{aligned}$$

Second,

$$\begin{aligned} \frac{\partial N_L}{\partial L} &= \frac{\left(\frac{\beta\gamma}{1-\gamma}\right) L^{\frac{\beta\gamma}{1-\gamma}-1} H (1-\beta)}{\psi^{(1-\beta)}\left(\psi^{\frac{\beta\gamma}{1-\gamma}} - L^{\frac{\beta\gamma}{1-\gamma}}\right)^{\frac{1}{\gamma}}} + \frac{\beta}{1-\gamma} L^{\frac{\beta\gamma}{1-\gamma}-1} \frac{L^{\frac{\beta\gamma}{1-\gamma}} H (1-\beta)}{\psi^{(1-\beta)}\left(\psi^{\frac{\beta\gamma}{1-\gamma}} - L^{\frac{\beta\gamma}{1-\gamma}}\right)^{\frac{1+\gamma}{\gamma}}} \\ &= \phi\left(\gamma + \frac{L^{\frac{\beta\gamma}{1-\gamma}}}{\left(\psi^{\frac{\beta\gamma}{1-\gamma}} - L^{\frac{\beta\gamma}{1-\gamma}}\right)}\right) \end{aligned} \tag{3}$$

where

$$\phi \equiv \frac{L^{\frac{\beta\gamma}{1-\gamma}-1} H (1-\beta)\beta}{\psi^{(1-\beta)} \left(\psi^{\frac{\beta\gamma}{1-\gamma}} - L^{\frac{\beta\gamma}{1-\gamma}}\right)^{\frac{1}{\gamma}} (1-\gamma)} > 0$$

The second line of the previous expression shows that, if $\gamma > 0$, $\frac{\partial N_L}{\partial L} > 0$. The intuition for this result is that, when Y_L and Y_H are gross substitutes in the production of the final good, then, the market size effect dominates over the price effect, inducing technological change that is biased towards Y_L. It is possible, instead, that if the two intermediate inputs are sufficiently complements, $\frac{\partial N_L}{\partial L} < 0$.[1]

2 Quantifying the Effects of L on Unskilled Wages

2.1 Perfect Substitutability Between Immigrants and Natives

Recall from ($B23$) in online appendix B that

$$w_L = \frac{\psi N_L}{L(1-\beta)}$$

Then,

$$\left(\frac{\partial w_L}{\partial L}\right)^{TOT} = \frac{\partial w_L}{\partial L} + \frac{\partial w_L}{\partial N_L}\frac{\partial N_L}{\partial L}$$

First, note that

$$\frac{\partial w_L}{\partial L} = -\frac{\psi N_L L^{-2}}{(1-\beta)} \tag{4}$$

Next,

$$\frac{\partial w_L}{\partial N_L} = \frac{\psi}{L(1-\beta)}$$

and, from (3),

$$\frac{\partial N_L}{\partial L} = \phi\left(\gamma + \frac{L^{\frac{\beta\gamma}{1-\gamma}}}{\left(\psi^{\frac{\beta\gamma}{1-\gamma}} - L^{\frac{\beta\gamma}{1-\gamma}}\right)}\right)$$

with $\phi \equiv \frac{L^{\frac{\beta\gamma}{1-\gamma}-1} H(1-\beta)\beta}{\psi^{(1-\beta)}\left(\psi^{\frac{\beta\gamma}{1-\gamma}} - L^{\frac{\beta\gamma}{1-\gamma}}\right)^{\frac{1}{\gamma}}(1-\gamma)}$. Thus,

$$\frac{\partial w_L}{\partial N_L}\frac{\partial N_L}{\partial L} = L^{-2}\tilde{\phi}\left[\gamma + \frac{L^{\frac{\beta\gamma}{1-\gamma}}}{\left(\psi^{\frac{\beta\gamma}{1-\gamma}} - L^{\frac{\beta\gamma}{1-\gamma}}\right)}\right] \tag{5}$$

[1] Note, however, that (3) is a sufficient, but not a necessary condition for N_L to be increasing in L.

where

$$\tilde{\phi} \equiv \frac{\beta\psi}{(1-\gamma)(1-\beta)} N_L > 0$$

Then, combining (4) with (5), we get

$$\begin{aligned}
\left(\frac{\partial w_L}{\partial L}\right)^{TOT} &= \left[-\frac{\psi N_L L^{-2}}{(1-\beta)} + L^{-2}\frac{\beta\psi}{(1-\gamma)(1-\beta)} N_L \left(\gamma + \frac{L^{\frac{\beta\gamma}{1-\gamma}}}{\left(\psi^{\frac{\beta\gamma}{1-\gamma}} - L^{\frac{\beta\gamma}{1-\gamma}}\right)}\right)\right] \\
&= \frac{\psi N_L L^{-2}}{(1-\beta)} \left[-1 + \frac{\beta}{(1-\gamma)} \left(\gamma + \frac{L^{\frac{\beta\gamma}{1-\gamma}}}{\left(\psi^{\frac{\beta\gamma}{1-\gamma}} - L^{\frac{\beta\gamma}{1-\gamma}}\right)}\right)\right]
\end{aligned}$$

Then, $\left(\frac{\partial w_L}{\partial L}\right)^{TOT} > 0$ whenever

$$\frac{L^{\frac{\beta\gamma}{1-\gamma}}}{\left(\psi^{\frac{\beta\gamma}{1-\gamma}} - L^{\frac{\beta\gamma}{1-\gamma}}\right)} > \frac{1-\gamma(1+\beta)}{\beta}$$

Since the left-hand side of the previous inequality is positive (when $N_L > 0$), it follows that

$$\gamma > \frac{1}{1+\beta} \implies \left(\frac{\partial w_L}{\partial L}\right)^{TOT} > 0 \tag{6}$$

The latter inequality provides a sufficient (but not a necessary) condition for when immigration can raise the unskilled wage. Intuitively, when Y_L and Y_H are sufficiently substitutable in the production of the final good, the directed technology effect (Acemoglu, 1998) will prevail over the (negative) substitution effect - a result consistent with the more general case considered in Acemoglu (2002).

2.2 Imperfect Substitutability Between Immigrants and Natives

When immigrants and unskilled natives are imperfect substitutes (see Section B.5.1), earnings of natives in the unskilled sector are given by[2]

$$w_U = \frac{\psi N_L}{(1-\beta)} \frac{L^{-\alpha}}{U^{1-\alpha}}$$

[2]See $(B30)$ in appendix B.

Then,

$$\left(\frac{\partial w_U}{\partial I}\right)^{TOT} = \left[-\alpha L^{-\alpha-1}\frac{\psi N_L}{(1-\beta)\,U^{1-\alpha}} + \frac{\psi}{(1-\beta)}\frac{L^{-\alpha}}{U^{1-\alpha}}\frac{\partial N_L}{\partial L}\right]\frac{\partial L}{\partial I}$$
$$= \chi\left[-\alpha N_L + \frac{L\partial N_L}{\partial L}\right]\frac{\partial L}{\partial I} \quad (7)$$

with $\chi \equiv \frac{\psi L^{-\alpha-1}}{(1-\beta)U^{1-\alpha}} > 0$. Note that

$$\frac{\partial N_L}{\partial L} = \frac{\beta N_L}{(1-\gamma)\,L}\left(\gamma + \frac{L^{\frac{\beta\gamma}{1-\gamma}}}{\left(\psi^{\frac{\beta\gamma}{1-\gamma}} - L^{\frac{\beta\gamma}{1-\gamma}}\right)}\right) \quad (8)$$

and so, (7) can be written as

$$\left(\frac{\partial w_U}{\partial I}\right)^{TOT} = \tilde{\chi}\left[-\alpha\,(1-\gamma) + \beta\gamma + \frac{\beta X}{(1-X)}\right]\frac{\partial L}{\partial I}$$

where $\tilde{\chi} \equiv \frac{\chi N_L}{1-\gamma}$ and $X \equiv \left(\frac{L}{\psi}\right)^{\frac{\gamma\beta}{1-\gamma}}$. Thus, $\left(\frac{\partial w_U}{\partial I}\right)^{TOT} > 0$ whenever

$$-\alpha\,(1-\gamma) + \beta\gamma + \frac{\beta X}{(1-X)} > 0$$

Or, whenever

$$X > \frac{(1-\gamma)\,\alpha - \beta\gamma}{(1-\gamma)\,\alpha - \beta\gamma + \beta\gamma} \quad (9)$$

And so, a sufficient condition for immigration to raise the wage of unskilled natives is that

$$\gamma > \frac{\alpha}{\alpha+\beta} \quad (10)$$

Note that when $\alpha = 1$, condition (10) coincides with (6). Moreover, for any $\alpha \in (0,1)$, (10) is satisfied for values of γ lower than those needed to satisfy (6). This is intuitive. On the one hand, if immigrants and natives are imperfect substitutes, the degree of competition induced by an immigration shock is lower than in the case of perfect substitutability. On the other, the capital response to immigration is larger the lower the degree of substitutability between immigrants and natives.

3 Quantifying the Effects of L on the Skill Premium

3.1 Perfect Substitutability Between Immigrants and Natives

In this paragraph I derive an explicit expression for the effects of changes in L on the skill premium. First, recall that

$$\omega = \left(\frac{1-\beta}{\psi}\right)\frac{\left(1-\psi^{\frac{\gamma\beta}{\gamma-1}}L^{-\frac{\gamma\beta}{\gamma-1}}\right)^{\frac{\gamma-1}{\gamma}}}{N_L\left(L\right)}L$$

As for w_L, also in this case, the effect of immigration on ω can be decomposed as

$$\left(\frac{\partial\omega}{\partial L}\right)^{TOT} = \frac{\partial\omega}{\partial L} + \frac{\partial\omega}{\partial N_L}\frac{\partial N_L}{\partial L}$$

Next,

$$\begin{aligned}
\frac{\partial\omega}{\partial L} &= \left(\frac{1-\beta}{\psi}\right)\frac{\left(1-\psi^{\frac{\gamma\beta}{\gamma-1}}L^{-\frac{\gamma\beta}{\gamma-1}}\right)^{\frac{\gamma-1}{\gamma}}}{N_L} + \left(\frac{1-\beta}{\psi}\right)\beta\frac{\psi^{\frac{\gamma\beta}{\gamma-1}}L^{-\frac{\gamma\beta}{\gamma-1}}}{N_L}\left(1-\psi^{\frac{\gamma\beta}{\gamma-1}}L^{-\frac{\gamma\beta}{\gamma-1}}\right)^{\frac{\gamma-1}{\gamma}-1} \\
&= \left(\frac{1-\beta}{\psi}\right)\frac{\left(1-\psi^{\frac{\gamma\beta}{\gamma-1}}L^{-\frac{\gamma\beta}{\gamma-1}}\right)^{\frac{\gamma-1}{\gamma}}}{N_L}\left[\frac{1-\psi^{\frac{\gamma\beta}{\gamma-1}}L^{-\frac{\gamma\beta}{\gamma-1}}-\beta\psi^{\frac{\gamma\beta}{\gamma-1}}L^{-\frac{\gamma\beta}{\gamma-1}}}{\left(1-\psi^{\frac{\gamma\beta}{\gamma-1}}L^{-\frac{\gamma\beta}{\gamma-1}}\right)}\right] \\
&= \frac{\left(1-\beta\right)\left(1-\left(\frac{L}{\psi}\right)^{\frac{\beta\gamma}{1-\gamma}}\right)^{-\frac{1}{\gamma}}}{\psi N_L}\left[1-\left(1-\beta\right)\left(\frac{L}{\psi}\right)^{\frac{\beta\gamma}{1-\gamma}}\right] \qquad (11)
\end{aligned}$$

It can be shown that, in a BGP with $N_L > 0$,

$$1-\left(\frac{L}{\psi}\right)^{\frac{\beta\gamma}{1-\gamma}} > 0$$

and, since $\beta \in (0,1)$, $\frac{\partial\omega}{\partial L} > 0$. This is indeed consistent with the idea that an increase in immigration will lower w_L because of substitutability and increase w_H because of complementarity. Next, considering the indirect effect operating through changes in N_L, we have that

$$\frac{\partial\omega}{\partial N_L} = -\left(\frac{1-\beta}{\psi}\right)\frac{\left(1-\psi^{\frac{\gamma\beta}{\gamma-1}}L^{-\frac{\gamma\beta}{\gamma-1}}\right)^{\frac{\gamma-1}{\gamma}}}{N_L^2}L < 0$$

From (8) we know that

$$\frac{\partial N_L}{\partial L} = \frac{\beta N_L}{(1-\gamma) L} \left(\gamma + \frac{L^{\frac{\beta\gamma}{1-\gamma}}}{\left(\psi^{\frac{\beta\gamma}{1-\gamma}} - L^{\frac{\beta\gamma}{1-\gamma}} \right)} \right)$$

and so, we get

$$\begin{aligned} \frac{\partial \omega}{\partial N_L} \frac{\partial N_L}{\partial L} &= -\frac{(1-\beta)\beta}{\psi (1-\gamma) N_L} (1-X)^{\frac{\gamma-1}{\gamma}} \left(\gamma + \frac{X}{1-X} \right) \\ &= -\frac{(1-\beta)\beta}{\psi (1-\gamma) N_L} (1-X)^{-\frac{1}{\gamma}} (\gamma + X(1-\gamma)) \end{aligned} \tag{12}$$

with $X \equiv \left(\frac{L}{\psi}\right)^{\frac{\gamma\beta}{1-\gamma}}$, $X \in (0,1)$. Then, combining (11) with (12), we obtain

$$\begin{aligned} \left(\frac{\partial \omega}{\partial L}\right)^{TOT} &= \frac{(1-\beta)(1-X)^{-\frac{1}{\gamma}}}{\psi N_L} \left(1 - (1-\beta) X - \frac{\beta}{(1-\gamma)} (\gamma + X(1-\gamma)) \right) \\ &= \frac{(1-\beta)(1-X)^{-\frac{1}{\gamma}}}{\psi N_L} (1 - \gamma - \gamma\beta - (1-\gamma) X) \end{aligned}$$

From the previous expression it then follows that $\left(\frac{\partial \omega}{\partial L}\right)^{TOT} > 0$ whenever

$$X < \frac{1-\gamma-\gamma\beta}{1-\gamma} \tag{13}$$

Since $X \in (0,1)$, it is easy to show that a sufficient condition for the skill premium to fall with immigration is that

$$\gamma > \frac{1}{1+\beta} \tag{14}$$

Note that, expressing (14) in terms of the derived elasticity of substitution, $\sigma \equiv (\varepsilon - 1)\beta + 1$, where $\varepsilon = \frac{1}{1-\gamma}$, we reach exactly the same condition as in Acemoglu (2002). That is, the skill premium falls following an increase in L, whenever $\sigma > 2$.

3.2 Imperfect Substitutability Between Immigrants and Natives

Let us now consider the case in which immigrants and unskilled natives are imperfect substitutes. As discussed in Section B.5.1, the skill premium is given by

$$\omega = \frac{w_H}{w_U} = \left(\frac{1-\beta}{\psi}\right) \frac{\left(1-\left(\frac{L}{\psi}\right)^{\frac{\gamma\beta}{1-\gamma}}\right)^{-\frac{1-\gamma}{\gamma}}}{N_L\left(L\right)} L^{\alpha} U^{1-\alpha}$$

We know from before that

$$\left(\frac{\partial \omega}{\partial L}\right)^{TOT} = \frac{\partial \omega}{\partial L} + \frac{\partial \omega}{\partial N_L}\frac{\partial N_L}{\partial L}$$

and so

$$\left(\frac{\partial \omega}{\partial I}\right)^{TOT} = \left[\frac{\partial \omega}{\partial L} + \frac{\partial \omega}{\partial N_L}\frac{\partial N_L}{\partial L}\right]\frac{\partial L}{\partial I}$$

where $\frac{\partial L}{\partial I} > 0$. Then,

$$\begin{aligned}
\frac{\partial \omega}{\partial L} &= \lambda \left(\beta \frac{\left(\frac{L}{\psi}\right)^{\frac{\gamma\beta}{1-\gamma}}}{1-\left(\frac{L}{\psi}\right)^{\frac{\gamma\beta}{1-\gamma}}} + \alpha\right) \\
&= \lambda\left(\alpha + \frac{X\beta}{1-X}\right) > 0 \qquad (15)
\end{aligned}$$

where $\lambda \equiv \frac{1-\beta}{\psi N_L} L^{\alpha-1} U^{1-\alpha}\left(1-X\right)^{-\frac{1-\gamma}{\gamma}} > 0$, and $X \equiv \left(\frac{L}{\psi}\right)^{\frac{\gamma\beta}{1-\gamma}}$ as before. Next,

$$\begin{aligned}
\frac{\partial \omega}{\partial N_L} &= -\left(\frac{1-\beta}{\psi}\right)\left(1-\left(\frac{L}{\psi}\right)^{\frac{\gamma\beta}{1-\gamma}}\right)^{-\frac{1-\gamma}{\gamma}} L^{\alpha} U^{1-\alpha} N_L^{-2} \\
&= -\left(\frac{1-\beta}{\psi}\right)\left(1-X\right)^{-\frac{1-\gamma}{\gamma}} L^{\alpha} U^{1-\alpha} N_L^{-2}
\end{aligned}$$

and, as we already saw many times,

$$\begin{aligned}
\frac{\partial N_L}{\partial L} &= \frac{\beta N_L}{\left(1-\gamma\right)L}\left(\gamma + \frac{L^{\frac{\beta\gamma}{1-\gamma}}}{\left(\psi^{\frac{\beta\gamma}{1-\gamma}} - L^{\frac{\beta\gamma}{1-\gamma}}\right)}\right) \\
&= \frac{\beta N_L}{\left(1-\gamma\right)L}\left(\gamma + \frac{X}{\left(1-X\right)}\right)
\end{aligned}$$

The latter two expressions imply that

$$\begin{aligned}\frac{\partial\omega}{\partial N_L}\frac{\partial N_L}{\partial L} &= -\frac{U^{1-\alpha}}{L^{1-\alpha}N_L}\frac{(1-\beta)\beta}{\psi(1-\gamma)}(1-X)^{-\frac{1-\gamma}{\gamma}-1}(\gamma(1-X)+X) \\ &= -\frac{U^{1-\alpha}}{L^{1-\alpha}N_L}\frac{(1-\beta)\beta}{\psi(1-\gamma)}(1-X)^{-\frac{1}{\gamma}}(\gamma+X(1-\gamma)) \end{aligned} \tag{16}$$

Finally, combining (15) and (16), we get

$$\begin{aligned}\left(\frac{\partial\omega}{\partial L}\right)^{TOT} &= \frac{1-\beta}{\psi N_L}\frac{U^{1-\alpha}}{L^{1-\alpha}}(1-X)^{-\frac{1}{\gamma}}(\alpha(1-X)+X\beta) - \frac{U^{1-\alpha}}{L^{1-\alpha}N_L}\frac{(1-\beta)\beta}{\psi(1-\gamma)}(1-X)^{-\frac{1}{\gamma}}(\gamma+X(1-\gamma)) \\ &= \frac{1-\beta}{\psi N_L}\frac{U^{1-\alpha}}{L^{1-\alpha}}(1-X)^{-\frac{1}{\gamma}}\left[\alpha(1-X)+X\beta-\frac{\beta}{1-\gamma}(\gamma+X(1-\gamma))\right] \\ &= \xi\left[\alpha(1-\gamma)-\beta\gamma-\alpha(1-\gamma)X\right]\end{aligned}$$

where $\xi \equiv \frac{1-\beta}{\psi(1-\gamma)N_L}\frac{U^{1-\alpha}}{L^{1-\alpha}}(1-X)^{-\frac{1}{\gamma}} > 0$. Hence, it follows that $\left(\frac{\partial\omega}{\partial L}\right)^{TOT} > 0$ whenever

$$\alpha(1-\gamma)-\beta\gamma-\alpha(1-\gamma)X > 0$$

Or, whenever

$$X < \frac{\alpha(1-\gamma)-\beta\gamma}{\alpha(1-\gamma)} \tag{17}$$

Note that, when $\alpha = 1$, (17) coincides with (13) that we derived for the case of perfect substitutability between immigrants and natives. As we have done many times at this point, we can derive a sufficient condition, relating γ to α and β, such that immigration lowers the skill premium. In particular, if

$$\gamma > \frac{\alpha}{\alpha+\beta} \tag{18}$$

an inflow of (unskilled) immigrants will lower income inequality among natives. As for wages of unskilled natives, also in this case, the range of values of γ for which immigration compresses the income gap between high and low skilled workers is larger than when $\alpha = 1$.

4 Natives' Occupational Choice

In this section, I derive the expression for the number of native whites in the unskilled sector before the immigration shock, i.e. ($B35$) in online appendix B. Start from ($B34$), and combine it with ($B17$). Remembering that $L = I + U$, and that $H = 1 - U$, ($B34$) becomes

$$\frac{\psi^{\beta}(1-U)(I+U)^{\frac{\beta\gamma}{1-\gamma}}}{\left[\psi^{\frac{\beta\gamma}{1-\gamma}}-(I+U)^{\frac{\beta\gamma}{1-\gamma}}\right]^{\frac{1}{\gamma}}}=\left(1-\left(\frac{I+U}{\psi}\right)^{\frac{\gamma\beta}{1-\gamma}}\right)^{\frac{\gamma-1}{\gamma}}(I+U)$$

$\Longrightarrow$

$$(1-U)\left(\frac{I+U}{\psi}\right)^{\frac{\beta\gamma}{1-\gamma}}=\left(1-\left(\frac{I+U}{\psi}\right)^{\frac{\gamma\beta}{1-\gamma}}\right)(I+U)$$

And, after some further rearrangements, it is possible to obtain

$$(I+U)^{\frac{\gamma(1+\beta)-1}{1-\gamma}}=\frac{\psi^{\frac{\gamma\beta}{1-\gamma}}}{1+I}\Longrightarrow U=\frac{\psi^{\frac{\gamma\beta}{\gamma(1+\beta)-1}}}{(1+I)^{\frac{1-\gamma}{\gamma(1+\beta)-1}}}-I$$

verifying $(B35)$.