Inattentive Economies

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Motivation

Rational inattention: uninformed, heuristic behavior = individually optimal given costs

What about the market mechanism? Can a planner do better by

1. manipulating attention via taxes or other means
2. "simplifying the world" to reduce propensity for mistakes

without removing inattention outright?

This paper: studies efficiency in an Arrow-Debreu market economy with rational inattention

Main Results

Key sufficient conditions for Welfare Theorems to extend:

1. **complete markets.** Required to net out *pecuniary externalities*
2. cognitive costs are **invariant** in a sense amounting to
   (i) free disposal of irrelevant or redundant information in state
   (ii) irrelevance of scale or perceptual distance

   cancels out *cognitive externality* arising with endogenous state variable

   satisfied by mutual information of Sims (2003)

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Taken together, results show how

- Markets *optimally manage information* only in rather special cases
- Optimal policy interventions operate via two distinct mechanisms, combating pecuniary or cognitive externalities

Related Literature

- **Efficiency in games and economies with cognitive frictions.** Hébert and La'O (2020); Colombo, Femminis, and Pavan (2014); Tirole (2015); Gabaix (2014); Gul, Pesendorfer, and Strzalecki (2017); Angeletos and La'O (2010, 2018).
- **Costs of information and stochastic choice.** Sims (2003, 2010); Caplin and Dean (2013, 2015); Caplin, Dean, and Leahy (2017); Denti (2020); Denti, Marinacci, and Rustichini (2020); Pomatto, Strack, and Tamuz (2019); Hébert and Woodford (2020a, 2020b)
- **Persuasion and contracting with inattentive agents.** Bloedel and Segal (2020); Lipnowski, Mathevet, and Wei (2019), Ravid (2020).

Outline

Notation

- N goods, J consumer types (each a continuum)
- State of nature $\theta \in \Theta$, distributed by $\pi \in \Delta(\Theta)$
- Fixed signal space Ω
- Payoff $u^j : \mathbb{R}_+^N \times \Theta \to \mathbb{R}$
- Endowment function $e^j : \theta \to \mathbb{R}_+^N$ for each type
- Price $p \in \mathbb{R}_+^N$, which will depend on state in equilibrium

Modeling Rational Inattention

- "Agent pays cost to obtain signal ω of random variable z"
- $z = (\theta, p) \in \Theta \times \mathbb{R}_+^N$ = state variable in consumer problem, distributed $\pi_z \in \Delta(\Theta \times \mathbb{R}_+^N)$
    - More general variant in paper: $z = Z(\theta, p, \text{taxes}, \ldots)$, where $Z(\cdot)$ is the primitive

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- Type-specific cost functionals $C^j : \Delta(\Omega \times (\Theta \times \mathbb{R}_+^N)) \to \mathbb{R}$
  - **Information acquisition**: cost to learn about z.
  - **Stochastic choice**: cost to make x contingent on z.

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  - Information acquisition: cost to learn about z.
  - Stochastic choice: cost to make x contingent on z.
- Key idea: "difficulty" of paying attention is endogenous to what others do. For instance, agents might struggle to
  - distinguish nearby prices (e.g., Hébert and Woodford, 2020)
  - continuously vary demand as a function of price (e.g., Morris and Yang, 2020)

Consumer Problem

Choose signal distributions $\phi \in \Delta(\Omega \times (\Theta \times \mathbb{R}_+^N))$ and consumption profiles $x : \Omega \to \mathbb{R}_+^N$ to maximize expected utility net of costs, subject to feasibility constraints ▸ Firm Problem

$$\max_{\phi(\cdot), x(\cdot)} \sum_{\omega, z} u(x(\omega), \theta)\, \phi(\omega, z) - C^j\left[\phi(\cdot)\right]$$

$$\text{s.t.} \sum_{\omega, z} (x(\omega) \cdot p)\, \phi(\omega, z) \leq \sum_z (e^j(\theta) \cdot p)\, \pi_z(z)$$

$$\sum_{\omega} \phi(\omega, z) = \pi_z(z), \ \forall z \in \Theta \times \mathbb{R}_+^N$$

Budget Constraint and Complete Markets

$$\sum_{\omega,z}(x(\omega)\cdot p)\,\phi(\omega,z) \leq \sum_{z}(e^{j}(\theta)\cdot p)\,\pi_z(z)$$

1. **Complete markets or insurance over ω and θ**: smooths marginal value of wealth, but does not eliminate mistakes
2. **Dynamic choice** with savings ("mistakes average out")
3. (Also sufficient) **Quasilinear** good

Open question: relaxing this and proving (conditions for) constrained efficiency as in Geanakoplos and Polemarchakis (1986)

Equilibrium Concept

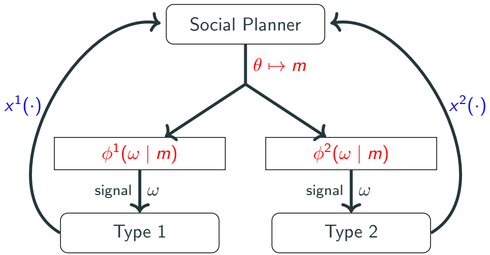
Definition (Equilibrium)

An equilibrium is an allocation of goods $(x^j(\omega))_{j=1}^J$ *and attention* $(\phi^j(\cdot))_{j=1}^J$ *and a price function* $P(\theta)$ *such that consumers optimize, markets clear,* $p = P(\theta)$*, and* $z = (\theta, P(\theta))$*.*

- Law of large numbers lets us write aggregates contingent on θ
- Equilibrium existence (+ "meaning" of market clearing) depends only on properties of demand aggregated across ω
- Simple extension (in paper): fixed point with different primitive definitions of $z = Z(\cdot)$.

Efficiency Concept ▸ Program

Pictured: two-type example of communication problem



- Planner chooses message $m = M(\theta)$, allocations $x^j(\omega)$, and attention $\phi^j(\omega, m)$ to maximize social surplus **including attention costs**
- Efficient allocation solves planner's problem for some Pareto weights
- Possible implementation = taxes on goods choice + attention, plus message that allows agents to condition on them

Outline

Alternative Representation of the Problem

- Let vector of state-contingent goods be $\overline{x} = (x(\theta))_{\theta \in \Theta}$; endowments be $\overline{e}^j = (e^j(\theta))_{\theta \in \Theta}$ for each j; prices be $\overline{p} = (P(\theta))_{\theta \in \Theta}$; and tracked objects by $\overline{z} = (\theta, P(\theta))_{\theta \in \Theta}$.
- Define utility functions $\overline{u}^j(\overline{x}; \overline{z})$ which optimize subject to costs of attention

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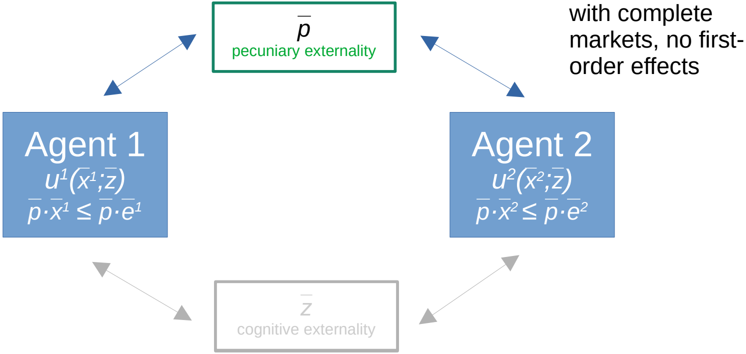

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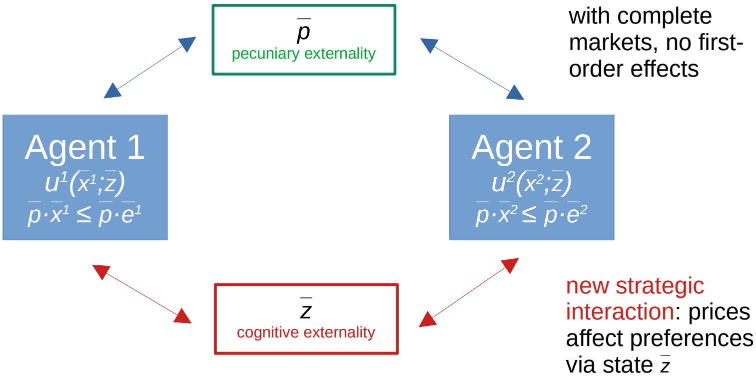

Intermediate Result: Efficiency in State-Tracking Variant

We first **shut off the cognitive externality** by studying "state-tracking" economies in which $z \equiv \theta$, and restricting the planner's message to $m \equiv \theta$.

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Theorem

Assume the primitive preferences represented by $u^j(\cdot, \theta)$ be locally non-satiated for all (j, θ). Then the market equilibrium of a state-tracking economy is efficient, or solves the (restricted) planner's program for some Pareto weights.

- Showing local non-satiation for $\overline{u}(\cdot)$ preferences uses complete markets over ω: agent always gains from more, no matter how severe their mistakes are.
- Second Welfare Theorem (see paper): requires convexity assumption

Cognitive Externalities and Informational Invariance

To achieve efficiency with $z = (\theta, p)$, need condition that zeros out cognitive externality

Definition (Informational Invariance for Cost Functional)

Let $z \in Z$ be a random variable and ω be some signal of z with joint density $\phi(\omega, z)$. Let $\tilde{z} = h(z) \in Z$ be a transformation of z and define $\tilde{\phi}(\omega, \tilde{z}) = \sum_{z \in Z} \phi(\omega, z)\mathbb{I}[h(z) = \tilde{z}]$ Then cost functional $C[\cdot]$ satisfies informational invariance if

(i) $C[\phi(\cdot)] \geq C[\tilde{\phi}(\cdot)]$ *always;*

(ii) $C[\phi(\cdot)] = C[\tilde{\phi}(\cdot)]$ *if and only if $\tilde{z}$ is a sufficient statistic for z about ω.*

- Implies invariance under compression (Caplin, Dean, and Leahy, 2017)
- Leading (non-trivial) example: mutual information

Main Result: Efficiency of Equilibrium

Theorem (First Welfare Theorem)

Assume locally non-satiated preferences and informational invariance for all agents' attention costs. Then the competitive equilibrium is efficient, or solves the planner's program for some Pareto weights.

Why does it work? A heuristic argument:

- $z = (\theta, p)$ is a superset of what agents really need to know
- Informational invariance ensures there is no cost to "throw away" extra information in z, and no benefit from "repackaging" information via rescalings, etc.
- Hence no room for the planner to send simpler messages and improve welfare

Proof: shows how to show decentralized economy is outcome-equivalent to the state-tracking economy; and how planner can restrict to messages $m \equiv \theta$ without loss of optimality.

Main Result: Implementation of Planner's Solution

Theorem (Second Welfare Theorem)

Assume locally non-satiated preferences, informational invariance, and convex attention costs. Then the solution to the planner's problem can be implemented as an equilibrium with transfers. In addition, one implementation of the planner's optimum sends the message $m = (\theta, P(\theta))$, where $P(\theta)$ is the price function in the decentralized implementation.

Proof: similar logic to that of First Welfare Theorem

Outline

Away from Informational Invariance

What can go wrong without invariance? And what interventions can improve welfare?

Example 1

- Cognitive externality: more volatile prices $\rightarrow$ harder to control mistakes
- **Corrective policy**: use taxes to make demand more elastic, prices less volatile

Example 2

- Cognitive externality: nearby prices $\rightarrow$ harder for consumers to distinguish
- **Corrective policy**: use taxes to pull prices farther apart

Are Markets Informationally Optimal?

> We must look at the price system as a mechanism for communicating information if we want to understand its real function. [...] The most significant fact about this system is the economy of knowledge with which it operates, or how little the individual participants need to know in order to be able to take the right action.
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"Economy of knowledge" in *laissez faire* is optimal in the special invariance (e.g., mutual information) + complete markets case... with two big caveats:

1. Planner is *indifferent* about saying (arbitrarily) more
2. In all other cases, prices may not have optimal information "content" or "format"

Conclusion

This paper: conditions for market economies with inattention to be efficient.

Efficiency is possible with *insurance over errors* and *invariance of cognitive costs* to what participants pay attention to

Otherwise planner wants to provide missing insurance and/or make stochastic environment "friendlier" for agents to learn, avoid mistakes

Firm Problem ▸ Back

Choose signal distributions $\phi \in \Delta(\Omega \times (\Theta \times \mathbb{R}_+^N))$ and production profiles $x : \Omega \to \mathbb{R}^N$ to maximize expected profits subject to "efficiency cost," which shows up in the feasibility constraint

$$\max_{\phi(\cdot),x(\cdot)} \sum_{\omega,z} (x(\omega) \cdot p)\, \phi(\omega, z)$$

$$\text{s.t. } F(x^j(\omega), \theta) + C^F[\phi(\cdot)] \leq 0, \forall(\omega, \theta) \text{ s.t. } \sum_z \phi(\omega, z)\, \pi(\theta \mid z) > 0$$

$$\sum_\omega \phi(\omega, z) = \pi_z(z), \forall z \in \Theta \times \mathbb{R}_+^N$$

Planner's Program ▸ Back

Let $\mathcal{M}$ be a message space (baseline: $\mathcal{M} = \Theta \times \mathbb{R}^N$)

$$\max_{\phi^j(\cdot), x^j(\cdot), M(\cdot)} \sum_{j=1}^{J} \mu^j \left(\sum_{\omega, m, \theta} u(x(\omega), \theta)\, \phi^j(\omega, m)\, \psi(\theta \mid m) - C^j \left[\phi^j(\cdot)\right] \right)$$

$$\text{s.t. } \sum_j \sum_{\omega, m} x^j(\omega) \phi^j(\omega, m)\, \psi(\theta \mid m) \leq \pi(\theta) \sum_j e^j(\theta)$$

$$\sum_\omega \phi^j(\omega, m) = \sum_\theta \psi(m, \theta), \forall m \in \mathcal{M}, j$$

$$\psi(m, \theta) = \begin{cases} \pi(\theta) & \text{if } m = M(\theta) \\ 0 & \text{else} \end{cases}$$