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DEDUCTION DILEMMAS:
THE TAIWAN ASSIGNMENT MECHANISM

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ABSTRACT

This paper analyzes the properties of the Taiwan mechanism, used for high school placement nationwide starting in 2014. In the Taiwan mechanism, points are deducted from an applicant's score with larger penalties for lower ranked choices. Deduction makes the mechanism a new hybrid between the well-known Boston and deferred acceptance mechanisms. Our analysis sheds light on why Taiwan's new mechanism has led to massive nationwide demonstrations and why it nonetheless still remains in use.

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1 Introduction

In June 2014, more than five hundred parents and teachers marched down Ketagalan Boulevard in Taipei the capital city of Taiwan. The protestors were marching against a new mechanism used to assign students to senior high schools. Fiercely protesting parents held placards stating, “fill out the preference form for us,” while others complained that the admissions process was akin to “gambling” (I-chia 2014). Due to this pressure and calls for his resignation, education Minister Chiang Wei-Ling subsequently issued a formal public apology for the new high school admissions system (CNA 2014a).

What are the protestors complaining about, and why is there so much turmoil associated with Taiwan’s new high school assignment mechanism? This paper investigates this question by studying the properties of Taiwan’s new assignment mechanism, a brand-new assignment mechanism (to our knowledge), which represents a generalization of widely-studied mechanisms based on the student-proposing deferred acceptance algorithm and the Boston or immediate acceptance mechanism.

Taiwan, like many other countries and regions, has recently launched a series of reforms to standardize and centralize its secondary school system. At the turn of the century, rising Taiwanese high schoolers took an admissions exam consisting of five subjects, with a total score of 700 points. Students submitted their ranking over schools, and those with a higher score chose first. For the next decade, an essay component was added to the admissions exam. Local districts were free to use other performance measures aside from exams (such as Chinese and English, music or sports) and to choose how to convert these measures to a total score.

In 2014, the Senior-High School Education Act established a *Comprehensive Assessment Program for Junior High School* students. This act changed how each of the five subjects on the admissions exam were scored, placing them into discrete categories: excellent, basic (pass), and needs more work (not pass). To separate high performing students, excellent was further split into A, A+, and A++. During 2014, more than 200,000 pupils took the Comprehensive Assessment Exam and applied to schools in their district of residence.

Aside from changing the admissions criteria, Taiwanese authorities also changed the assignment mechanism, introducing a **deduction** system. Loosely speaking, in Taiwan’s deduction system, a student’s priority is reduced based on the order in which preferences are ranked. Table 1 lists the schedule of points employed in 2014 across the largest districts in Taiwan. For instance, in Jibei, the largest district with over 60,000 applicants, a student’s score at her second choice is reduced by 1, the score at her third choice is reduced by 2, and so on. In Yunlin district, no points are deducted for the first four choices, and 2 points are

deducted from choices five through eight.¹ The deduction system has now been in place for the last five years, from 2014-2018.

There are many signs that the deduction system is one of the major reason for nationwide protests. For instance, an article in the China Post states (Wei 2014):

It is outrageous that the students have to have points deducted from their scores because they fill out the wrong slots; it is because of this that many students with A+ in all subjects eventually have to go to the same school with those who have achieved lower scores.

Despite calls to move towards different system where “students can choose the school they want according to their results,” senior Taiwanese leadership has decided to retain the deduction system and has only slightly modified the extent to which points are deducted from choices.

While economists have studied the properties of other school choice mechanisms, the deduction system represents a brand new class of matching mechanisms, which we term the **Taiwan mechanism**. Our goal in this paper is to analyze the properties of the Taiwan mechanism to provide insights into the developments in Taiwan.

To our knowledge, the only other paper to study the new system in Taiwan is Hsu (2014), who investigates properties of the new scoring system. Balinski and Sönmez (1999) and Abdulkadiroğlu and Sönmez (2003) initiated the formal research on the mechanism design approach to school choice. This paper is most closely related to Ergin and Sönmez (2006) and Pathak and Sönmez (2008). Both papers consider the equilibrium of the preference revelation game induced by the Boston mechanism under different assumptions on player sophistication. Since preference-driven priority mechanisms are a generalization of the Boston mechanism, the results contained in this paper can be seen as a generalization of those earlier papers. Motivated by Chinese college admissions, Chen and Kesten (2017) introduce application-rejection mechanisms with a parameter governing the permanency-execution period. Since it is possible to map a particular schedule of deduction points to the permanency-execution period, the results in this paper are related. Finally, there is a large literature on student assignment mechanisms. We refer interested readers to Pathak (2016) for a recent survey of the school choice literature and Sönmez and Ünver (2011) for a broader survey of the matching literature.

In the next section, after introducing the model and formal definitions, we examine the incentive properties of Taiwan mechanisms, showing that they can be compared in terms of manipulation based on a natural ordering on their deduction points. In Section 3, we analyze the equilibrium of the preference revelation game induced by the Taiwan mechanism. With

¹In some public documents, the deduction is described in terms of added points, where more points are added for higher ranked choices.

this characterization in hand, we turn to comparisons between the Taiwan mechanism and a mechanism based on the deferred acceptance algorithm. We briefly examine extensions of our result to the more general case when all schools do not have the same priorities. Finally, the last section concludes.

2 Model

2.1 Primitives

The ingredients of a school choice problem are a finite set of students and schools, each with a maximum capacity. Each student has a strict preference over all schools as well as remaining unassigned. Each student has priority score at each school.

Formally, a school choice problem consists of:

1. a set of students $I = \{i_1, \dots, i_n\}$,
2. a set of schools $S = \{s_1, \dots, s_m\}$,
3. a capacity vector $q = (q_{s_1}, \dots, q_{s_m})$,
4. a list of strict student preferences $P_I = (P_{i_1}, \dots, P_{i_n})$, and
5. a list of strict school priority score profiles $\pi = (\pi_{s_1}, \dots, \pi_{s_m})$, where π_s is school s 's priority scores over I .

For any student i , P_i is a strict preference relation over $S \cup \{\emptyset\}$ where \emptyset denote being unassigned option and $sP_i\emptyset$ means student i considers schools s as acceptable.² For any student i , let R_i denote the “at least as good as” relation induced by P_i . We denote the **rank** of school s under preference relation P_i with $r_s(P_i)$, i.e., $r_s(P_i) = |\{s' \in S \cup \{\emptyset\} | s'P_i s\}| + 1$. For any school s , the function $\pi_s : \{i_1, \dots, i_n\} \rightarrow \mathbb{R}$ is the priority score profile at school s where $\pi_s(i) > \pi_s(j)$ means that at school s , student i has higher priority than student j . We denote the priority order of school s over students implied by π_s with \succ_s^π , i.e. $i \succ_s^\pi j$ if and only if $\pi_s(i) > \pi_s(j)$. Let π_{max} be the maximum score possible.

We fix the set of students, the set of schools and the capacity vector throughout the paper; hence the pair (P, π) denotes a school choice problem (or simply a **problem**). The outcome of a school choice problem is a **matching**. Formally a matching $\mu : I \rightarrow S \cup \{\emptyset\}$ is a function such that $|\mu^{-1}(s)| \leq q_s$ for any school $s \in S$.

We refer to $\mu(i)$ as the assignment of student i under matching μ . With slight abuse of notation, we use $\mu(s)$ instead of $\mu^{-1}(s)$ in the rest of the paper.

²In the rest of the paper, we consider \emptyset as a “null” school such that $q_\emptyset = |I|$.

A matching μ is **individually rational** if there is no student i such that $\emptyset P_i \mu(i)$. A matching μ is **non-wasteful** if there is no student-school pair (i, s) such that $s P_i \mu(i)$ and $|\mu(s)| < q_s$. A matching μ is **fair** if there is no student-school pair (i, s) such that $s P_i \mu(i)$ and $\pi_s(i) > \pi_s(j)$ for some $j \in \mu(s)$.

A matching μ is **stable** if it is individually rational, non-wasteful, and fair. A matching is **student-optimal stable** if it is Pareto efficient for students.

A **mechanism**, denoted by φ , is a systematic procedure that selects a matching for each problem. We let $\varphi(P, \pi)$ denote the matching selected by mechanism φ in problem (P, π) and $\varphi(P, \pi)(i)$ denote the corresponding match of student i , and $\varphi(P, \pi)(s)$ denote the corresponding set of students assigned to school s . We say a mechanism φ satisfies a given property if $\varphi(P, \pi)$ satisfies that property for any (P, π) . For instance, a mechanism is stable if $\varphi(P, \pi)$ is stable for any (P, π) .

We say a mechanism φ is **vulnerable to manipulation** in (P, π) if there exists i and P'_i such that

$$\varphi(P'_i, P_{-i}, \pi)(i) P_i \varphi(P, \pi)(i).$$

A mechanism φ is **strategy-proof** if there is no problem (P, π) , student i and a preference order P'_i such that

$$\varphi(P'_i, P_{-i}, \pi)(i) P_i \varphi(P, \pi)(i),$$

where $P_{-i} = (P_j)_{j \neq i}$. In other words, a strategy-proof mechanism is not vulnerable to manipulation in any problem.

2.2 Taiwan Mechanisms

In this paper, we are interested in a new class of mechanisms which includes celebrated student-proposing deferred acceptance mechanism and the Boston or immediate acceptance mechanism. We call this class of mechanisms as **Taiwan Mechanisms**. To define a Taiwan mechanism, we first give the description of the two well-known member of this class of mechanisms.

For any given (P, π) , the student-proposing deferred acceptance (DA) mechanism selects its outcome through the following algorithm:

Step 1: Each student i applies to her best choice, possibly \emptyset , according to P_i . Each school s tentatively accepts the best q_s students among all applicants according to π_s and rejects the rest.

⋮

Step k: Each student i applies to her best choice which has not rejected her yet, possibly \emptyset , according to P_i . Each school s tentatively accepts the best q_s students among all

applicants according to π_s and rejects the rest.

The algorithm terminates when there are no more rejections and students are assigned to the choices they have applied in the last step.

For any given (P, π) , the Boston mechanism (BM) selects its outcome through the following algorithm:

Step 1: Each student i applies to her best choice, possibly \emptyset , according to P_i . Each school s permanently accepts the best q_s students among all applicants according to π_s and rejects the rest. Each accepted student and her assigned seat are removed.

⋮

Step k : Each remaining student i applies to her k^{th} choice, possibly \emptyset , according to P_i . Each school s permanently accepts the best students among all applicants according to π_s up to the number of its remaining seats and rejects the rest. Each accepted student and her assigned seat are removed.

The algorithm terminates when there are no more rejections.

A Taiwan mechanism can be implemented by deducting points and then applying DA to the resulting problem. More formally, we first define a **deduction rule** denoted by $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_{|S|+1}) \in \mathbb{R}_+^{|S|+1}$ such that $\lambda_1 = 0$ and $\lambda_k \leq \lambda_{k+1}$ for any $k \in \{1, 2, \dots, |S|\}$. We denote **Taiwan mechanism** associated with deduction rule λ by TM^λ . For a given problem (P, π) , the outcome of TM^λ is simply $DA(P, \hat{\pi}^\lambda)$ where for each student-school pair (i, s) ,

$$\hat{\pi}_s^\lambda(i) = \pi_s(i) - \lambda_{r_s(P_i)}.$$

When deduction points are zero, the associated Taiwan mechanism produces the same outcome as DA. When deduction points are very large, the associated Taiwan mechanism produces the same outcome as BM. That is, for any (P, π) , if $\lambda^1 = (0, 0, \dots, 0)$, then

$$TM^{\lambda^1}(P, \pi) = DA(P, \pi)$$

and if $\lambda^2 = (0, \pi_{max}, 2\pi_{max}, \dots, |S|\pi_{max})$, then

$$TM^{\lambda^2}(P, \pi) = BM(P, \pi).$$

The fact that the Taiwan mechanism can be implemented as DA with different inputs means that it inherits some properties of DA. Since DA mechanism is non-wasteful and individually rational, for any λ and (P, π) , $TM^\lambda(P, \pi)$ is non-wasteful and individually rational.

Through the choice of a deduction rule, it is possible for the class of Taiwan mechanisms to produce the same outcome as a large number of other mechanisms. Aside from DA and BM, the First Preference First (FPF) mechanisms used in England described by Pathak and Sönmez (2013) are in the class of Taiwan mechanism where the deduction schedule is allowed to depend on the school. FPF mechanisms were outlawed in more than 150 Local Education Agencies by English Parliament in 2007. Since the Chinese Parallel mechanisms described by Chen and Kesten (2017) span the DA and BM extremes, they can also be represented as a Taiwan mechanism. In particular, a deduction schedule would be related to the permanency-execution period in the Chinese parallel mechanism. If the choices within an execution period all have the same deduction points, and the deduction points in an earlier block are all sufficiently larger than those in a later block, then such a deduction schedule produces the same outcome as the Chinese Parallel mechanism.

We make the following two assumptions throughout the rest of the analysis. Since we are motivated primarily by policy developments in Taiwan, we assume that all schools share the same strict priority score profiles.

Assumption 1. *For all $s, s' \in S$ $\pi_s = \pi_{s'}$.*

In Taiwan, student priority score is determined by a combination of measures including test scores in Comprehensive Assessment Program for Junior High School Students and it applies to all schools. Examples of deduction rules are shown in Table 1. Appendix A examines whether our results generalize for situations other than Taiwan, where priorities may differ by school. We also assume that there is no ties in the deducted priority scores.

Assumption 2. *For any preference profile P , there is no ties in the deducted priority after applying the deducted rule λ to the problem (P, π) . That is, for $\forall P, s \in S$, $\hat{\pi}_s^\lambda(i) = \hat{\pi}_s^\lambda(j)$ implies that $i = j$.*

To understand the properties of Taiwan Mechanisms, we start with the following example:

Example 1. Suppose that there are four schools, $S = \{a, b, c, d\}$, each with one seat, and four students, i_1, i_2, i_3 , and i_4 . The original priority scores of students for each school $s \in S$ is: $\pi_s(i_1) = 100$, $\pi_s(i_2) = 50$, $\pi_s(i_3) = 11$ and $\pi_s(i_4) = 0$. The preferences of students are as follows:

P_{i_1} :	a	b	c	d	\emptyset
P_{i_2} :	a	b	c	d	\emptyset
P_{i_3} :	b	c	d	a	\emptyset
P_{i_4} :	c	a	d	b	\emptyset

We consider two different deduction rule: $\lambda^1 = (0, 41, 45, 51, 51)$ and $\lambda^2 = (0, 110, 220, 330, 330)$. The corresponding adjusted priority orders for π , $\hat{\pi}^{\lambda^1}$ and $\hat{\pi}^{\lambda^2}$ are:

	π				$\hat{\pi}^{\lambda^1}$				$\hat{\pi}^{\lambda^2}$			
<i>a</i> :	<i>i</i> ₁	<i>i</i> ₂	<i>i</i> ₃	<i>i</i> ₄	<i>i</i> ₁	<i>i</i> ₂	<i>i</i> ₃	<i>i</i> ₄	<i>i</i> ₁	<i>i</i> ₂	<i>i</i> ₄	<i>i</i> ₃
<i>b</i> :	<i>i</i> ₁	<i>i</i> ₂	<i>i</i> ₃	<i>i</i> ₄	<i>i</i> ₁	<i>i</i> ₃	<i>i</i> ₂	<i>i</i> ₄	<i>i</i> ₃	<i>i</i> ₁	<i>i</i> ₂	<i>i</i> ₄
<i>c</i> :	<i>i</i> ₁	<i>i</i> ₂	<i>i</i> ₃	<i>i</i> ₄	<i>i</i> ₁	<i>i</i> ₂	<i>i</i> ₄	<i>i</i> ₃	<i>i</i> ₄	<i>i</i> ₃	<i>i</i> ₁	<i>i</i> ₂
<i>d</i> :	<i>i</i> ₁	<i>i</i> ₂	<i>i</i> ₃	<i>i</i> ₄	<i>i</i> ₁	<i>i</i> ₂	<i>i</i> ₃	<i>i</i> ₄	<i>i</i> ₃	<i>i</i> ₄	<i>i</i> ₁	<i>i</i> ₂

The table orders applicants at schools from left to right, so that, e.g., $\pi_a(i_1) > \pi_a(i_2) > \pi_a(i_3) > \pi_a(i_4)$.

Under problem (P, π) , the matching produced by DA is:

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ a & b & c & d \end{pmatrix}.$$

The matching produced by the Taiwan mechanism with deduction λ^1 is:

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ a & c & b & d \end{pmatrix}.$$

The matching produced by BM is:

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ a & d & b & c \end{pmatrix}.$$

This last matching is identical to that produced by the Taiwan mechanism with deduction λ^2 .

2.3 Comparing Incentives Across Mechanisms

We have observed that in the limit when deduction points are very large, the Taiwan mechanism reduces to BM and when deduction points are all zero, it reduces to a serial dictatorship (equivalently DA). A serial dictatorship is a strategy-proof mechanism, while BM is highly manipulable. Does this comparison extend to intermediate values of the deduction rule?

Since BM is a highly manipulable mechanism, it is natural to expect that as we increase the deduction (and approach BM), the mechanism becomes more manipulable. To formalize this idea, we use the criteria to compare incentives across mechanisms developed by Pathak and Sönmez (2013). Recall that mechanism φ is vulnerable to manipulation at profile R if there is at least a student i who gets strictly better allocation by altering her report.

Definition. Mechanism ψ is **more manipulable** than φ if

(i) in any (P, π) such that φ is vulnerable to manipulation ψ is also vulnerable to manipulation, and

(ii) there exists some (P, π) such that ψ is vulnerable to manipulation but φ is not.

If only (i) is satisfied, then we say that ψ is **at least as manipulable as** φ .

Returning to Example 1, we consider a new deduction rule: $\lambda^3 = (0, 9, 20, 30, 30)$. Note that $\lambda_k^3 < \lambda_k^1$ for each $k > 1$. Then the adjusted priority orders induced by π , $\hat{\pi}^{\lambda^3}$, and $\hat{\pi}^{\lambda^1}$ are:

	π				$\hat{\pi}^{\lambda^3}$				$\hat{\pi}^{\lambda^1}$			
<i>a:</i>	i_1	i_2	i_3	i_4	i_1	i_2	i_4	i_3	i_1	i_2	i_3	i_4
<i>b:</i>	i_1	i_2	i_3	i_4	i_1	i_2	i_3	i_4	i_1	i_3	i_2	i_4
<i>c:</i>	i_1	i_2	i_3	i_4	i_1	i_2	i_3	i_4	i_1	i_2	i_4	i_3
<i>d:</i>	i_1	i_2	i_3	i_4	i_1	i_2	i_3	i_4	i_1	i_2	i_3	i_4

The outcome of the Taiwan Mechanism with deduction λ^1 , i.e. $TM^{\lambda^1}(P, \pi) = DA(P, \hat{\pi}^{\lambda^1})$, is:

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ a & c & b & d \end{pmatrix}.$$

If student i_2 instead reports b as the top choice, then i_2 obtains a better outcome than under truth-telling. She has a higher score than i_3 and i_4 under $\hat{\pi}_b^{\lambda^1}$ when she ranks b as top choice. Under TM^{λ^1} , student i_1 never applies to school b . Therefore, at school b , student i_2 is not rejected and is assigned to the more preferred school b when she ranks b as top choice.

On the other hand, TM^{λ^3} produces:

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ a & b & c & d \end{pmatrix},$$

which is the same as $DA(P, \pi)$. One can easily see that no student can manipulate TM^{λ^3} in (P, π) .

This example illustrates how manipulation possibilities increase as we increase deduction points in the TM class. Our first proposition shows the pattern is more general than this specific example.

Proposition 1. *Under Assumptions 1 and 2, if $\lambda^1 > \lambda^2$, then TM with deduction rule λ^1 , i.e. TM^{λ^1} , is more manipulable than TM with deduction rule λ^2 , i.e. TM^{λ^2} .*

Proof. We first show that there exists at least one (P, π) such that no student can manipulate TM^{λ^2} but some student can manipulate TM^{λ^1} . Suppose $\lambda_k^1 = \lambda_k^2$ for all $k < \bar{k}$ and $\lambda_{\bar{k}}^1 > \lambda_{\bar{k}}^2$. Let $S = \{s_1, \dots, s_{\bar{k}}, \dots\}$, $I = \{i_1, \dots, i_{\bar{k}}, \dots\}$, and $q_s = 1$ for all $s \in S$. Student i_k has the k^{th} highest score under π . Student i_k prefers school s_k as top choice for all $k < \bar{k}$ and student i_k prefers \emptyset as top choice for all $k > \bar{k} + 1$. Preference of $i_{\bar{k}}$ is: $s_k P_{i_{\bar{k}}} s_{k+1}$ for all $k \geq 1$. School $s_{\bar{k}}$ is the only acceptable school for $i_{\bar{k}+1}$. Let $\pi_s(i_{\bar{k}}) - \lambda_{\bar{k}}^1 < \pi_s(i_{\bar{k}+1}) < \pi_s(i_{\bar{k}}) - \lambda_{\bar{k}}^1$. Then, $TM^{\lambda^1}(P, \pi)(i) = TM^{\lambda^2}(P, \pi)(i)$ for all $i \in I \setminus \{i_{\bar{k}}, i_{\bar{k}+1}\}$, $TM^{\lambda^2}(P, \pi)(i_{\bar{k}}) = TM^{\lambda^1}(P, \pi)(i_{\bar{k}+1}) = s_{\bar{k}+1}$. Then, student $i_{\bar{k}}$ can manipulate TM^{λ^1} by ranking $s_{\bar{k}}$ as top choice but no student can manipulate TM^{λ^2} .

Next, we show that TM^{λ^1} is at least as manipulable as TM^{λ^1} . We present two observations and three lemmas that we use in the proof.

Observation 1. *For any (P, π) , λ and $i \in I$, if $s P_i s'$ then $\hat{\pi}_s^\lambda(i) \geq \hat{\pi}_{s'}^\lambda(i)$.*

Observation 1 follows from the fact that $\lambda_k \leq \lambda_{k-1}$ for any λ .

Observation 2. *For any (P, π) , there exists a unique stable matching which is the outcome of the serial dictatorship (SD) mechanism under π and P . Hence, the unique stable matching is also Pareto efficient.*

Observation 2 follows from the fact that $\pi_s = \pi_{s'}$ for any $s, s' \in S$. With slight abuse of notation, we use $\pi(i)$ instead of $\pi_s(i)$ in the rest of the proof.

Lemma 1. *For an arbitrary (P, π) , let μ be the unique stable matching and ν be another matching such that $\nu \neq \mu$. Then, there exists a student i such that $\mu(i) P_i \nu(i)$ and $\mu(j) = \nu(j)$ for any student j with $\pi(j) > \pi(i)$.*

Proof. By Observation 2, μ is Pareto efficient and $\mu = SD(P, \pi)$. Since μ is Pareto efficient, $\nu \neq \mu$ implies that there exists a student i' such that $\mu(i') P_{i'} \nu(i')$. Without loss of generality, let i be the student with the highest priority score under π who prefers μ to ν . On the contrary, suppose there exists a student j with $\pi(j) > \pi(i)$ and $\mu(j) \neq \nu(j)$. Without loss of generality, let j be such a student with the highest priority score under π . Then, $\nu(j) P_j \mu(j)$. However, this contradicts with the fact that μ is the outcome of the SD mechanism and i is the student with highest score who prefers μ to ν . \square

We consider sequential version of the DA mechanism introduced by McVitie and Wilson (1970) in which students apply according to a predetermined order χ one at a time and in each step the student who has the highest rank in χ among the ones whose offer has not been tentatively accepted applies.

Lemma 2. *For arbitrary (P, π) , λ , and χ consider any step k of the sequential DA mechanism such that there is only one student i who has not been tentatively accepted by some*

school in $S \cup \{\emptyset\}$. If school s that i applies to in step k tentatively accepts her offer, then i is assigned to s when the sequential DA terminates.

Proof. Let $t_{\bar{k}} = \hat{\pi}_{\bar{s}}^{\lambda}(\bar{i})$ such that student \bar{i} applies in step \bar{k} of sequential DA to school $\bar{s} \in S \cup \{\emptyset\}$.

In step k of sequential DA, student i is tentatively accepted by school s if either the number of tentatively accepted students in step $k - 1$ by s is less than q_s or there exists a student j who is tentatively accepted in step $k - 1$ by s and $\hat{\pi}_s^{\lambda}(i) > \hat{\pi}_s^{\lambda}(j)$. If the prior case holds, then the mechanism terminates and the desired result follows. If the later case holds, by Observation 1 and the fact that in each future step at most one student is not tentatively assigned $t_{k'} < \hat{\pi}_s^{\lambda}(i)$ for any $k' > k$. Therefore, i will not be rejected by s . \square

Lemma 3. For arbitrary (P, π) and λ , let $\mu = DA(P, \pi)$ and $\nu = TM^{\lambda}(P, \pi) = DA(P, \hat{\pi}^{\lambda})$. If $\mu \neq \nu$, then there exists a student who can manipulate TM^{λ} in (P, π) .

Proof. First, we define an axiom, known as population monotonicity, that we use throughout the proof.³ A mechanism φ is population monotonic if for any (I, S, q, P, π) after removal of any student i the assignment of all remaining students are (weakly) improved, i.e. $\varphi(I \setminus \{i\}, S, q, P_{-i}, \pi|(I \setminus \{i\}))(j) R_j \varphi(I, S, q, P, \pi)(j)$ for all $j \in I \setminus \{i\}$ where $\pi|(I \setminus \{i\})$ is the restriction of π on students in $I \setminus \{i\}$.

By Lemma 1, there exists a student i such that $\mu(i) P_i \nu(i)$ and $\mu(j) = \nu(j)$ for any student j with $\pi(j) > \pi(i)$. Since TM is non-wasteful, there exists a student k such that $\nu(k) = \mu(i)$ and $\pi(i) > \pi(k) \geq \hat{\pi}_{\mu(i)}^{\lambda}(k)$. Under $(P, \hat{\pi}^{\lambda})$, we consider sequential DA for an order χ such that student i is the last student under χ . First note that, when it is i 's turn all seats at $\mu(i)$ are tentatively filled. Otherwise, i would be matched to $\mu(i)$ under ν . By the population monotonicity of (sequential) DA mechanism, when it is i 's turn to apply there exists at least one student j' who is tentatively accepted by $\mu(i)$ and $\pi(i) > \hat{\pi}_{\mu(i)}^{\lambda}(k) \geq \hat{\pi}_{\mu(i)}^{\lambda}(j')$. This follows from the fact that under the tentative matching attained just before i 's turn student k is assigned to weakly better school than $\nu(k) = \mu(i)$ and when the sequential DA terminates it selects matching nu . Hence, Lemma 2 implies that student i can get $\mu(i)$ by ranking it as top choice. \square

Now we are ready to give our proof for Proposition 1. Let μ be the student optimal stable matching and ν^1 and ν^2 be the outcomes of TM^{λ^1} and TM^{λ^2} , respectively. By Lemma 3, if $\nu^1 \neq \mu$, then there exists a student j who can manipulate TM^{λ^1} . We need to consider two more cases.

Case 1: $\nu^1 = \nu^2 = \mu$. Suppose i can get school s by manipulating TM^{λ^2} . For both $(P, \hat{\pi}^{\lambda^1})$ and $(P, \hat{\pi}^{\lambda^2})$, we consider the sequential DA mechanism for an order χ in which

³We also use population monotonicity in the proof of Proposition 3.

i applies last. Let $\bar{\nu}^1$ and $\bar{\nu}^2$ be the tentative allocations obtained just before i 's turn for $(P, \hat{\pi}^{\lambda^1})$ and $(P, \hat{\pi}^{\lambda^2})$, respectively. By the fact that $\nu^1 = \nu^2 = \mu$ and Lemma 2, $\bar{\nu}^1(s') = \bar{\nu}^2(s') = \mu(s')$ and for all $s' P_i \mu(i)$. Since i can get s by manipulating TM^{λ^2} , there exists a student $\bar{i} \in \bar{\nu}^2(s)$ such that $\hat{\pi}_s^{\lambda^2}(\bar{i}) < \pi(i)$. Then, by the fact that $\lambda^1 > \lambda^2$, $\hat{\pi}_s^{\lambda^1}(\bar{i}) < \pi(i)$. Hence, Lemma 2 implies that i can get s by ranking it as top choice under TM^{λ^1} .

Case 2: $\nu^2 \neq \nu^1 = \mu$. By Lemma 1, there exists a student \bar{i} such that $\mu(\bar{i}) P_i \nu^2(\bar{i})$ and $\mu(\bar{j}) = \nu^2(\bar{j})$ for any student \bar{j} with $\pi(\bar{j}) > \pi(\bar{i})$. Moreover, by non-wastefulness of TM , there exists a student k such that $\nu^2(k) = \mu(\bar{i})$ and $\pi(\bar{i}) > \pi(k) > \hat{\pi}_s^{\lambda^2}(\bar{i})$. Then, by the fact that $\lambda^1 > \lambda^2$, $\pi(k) > \hat{\pi}_s^{\lambda^2}(\bar{i}) > \hat{\pi}_s^{\lambda^1}(\bar{i})$. Then for λ^1 , we consider sequential DA mechanism for an order χ in which k applies last. Let $\bar{\nu}^1$ be the tentative allocation obtained just before k 's turn for λ^1 . By the fact that $\nu^1 = \mu$ and Lemma 2, $\bar{\nu}^1(s') = \mu(s')$ for all $s' P_k \mu(k)$. Hence, Lemma 2 implies that k can get s by ranking it as top choice under TM^{λ^1} . \square

Within the class of Taiwan mechanisms, BM involves very large deduction points. Therefore, Proposition 1 implies the following corollary.

Corollary 1. *BM is more manipulable than any other Taiwan mechanism.*

Proposition 1 relates to a statement of a high school principal in Tapei, who remarked (CNA 2014b):

as long as the deduction system exists, problems can not be solved.

That is, the greater the amount of deduction, the more manipulable the mechanism is, with manipulation only completely eliminated by DA, which sets all deduction points to zero.

There have been several changes to deduction schedules since the system was first launched in 2014. Table 1 reports on these changes for 15 Taiwanese districts. In most cases, districts have relaxed the deduction schedule from the first year. For instance, in Gaoxiong, each choice has a weakly smaller deduction in 2014 than in 2015. The same applies when we compare 2015 to the three years between 2016-2018. The two largest districts by number of applications, Jibei and Zhongtong, also changed their deduction schedules to reduce the amount of deduction. Proposition 1 implies that each of these changes have made the mechanism less manipulable. However, not all changes involves moves to less manipulable mechanisms. For instance, in Jinmen, there were no deductions in 2014, while in 2015-2018 there were deductions for each choice after the first.

3 Equilibrium Analysis

3.1 Characterization

Proposition 1 compares the incentives generated by the mechanisms in TM class. We now turn to analyzing the equilibrium properties of these mechanisms, by considering the Nash equilibrium of the (simultaneous) preference revelation game induced by a Taiwan mechanism, or hereafter the **Taiwan game** under complete information.

Following Pathak and Sönmez (2008), we assume there are two types of students. Many families report confusion about the new Taiwanese mechanism, but some have spend considerable effort learning about the rules of the mechanism. Let \mathcal{N} and \mathcal{M} denote the set of sincere students and sophisticated students, respectively. For each $i \in \mathcal{N}$, the strategy space of student i is $\{P_i\}$, so i can only submit her true preference. We motivate this modeling choice by the fact that some participants may not understand how deductions change their incentives. In the case of BM, there is evidence of heterogeneous levels of sophistication (see, e.g., Pathak (2016) for examples). For each $j \in \mathcal{M}$, student j 's strategy space is all strict preferences over schools including being unassigned option.

Definition. Given a problem (P, π) and deduction rule λ , construct an **augmented priority score list** $\tilde{\pi}$ as:

- (i) For each school s , adjust each sincere student $i \in \mathcal{N}$ priority score according to $\tilde{\pi}_s(i) = \pi_s(i) - \lambda_{r_s(P_i)}$ (i.e., apply the deduction rule to sincere students for school s)
- (ii) For each school s , keep each sophisticated student $j \in \mathcal{M}$ priorities unchanged $\tilde{\pi}_s(j) = \pi_s(j)$.

Example 1 (cont). In Example 1 with deduction rule λ^1 , suppose student i_1 and i_3 are sincere and student i_2 and i_4 are sophisticated. Students i_1 and i_3 each have only one strategy corresponding to truth-telling: $abcd$ and $bcda$, respectively.⁴ Independent of the strategies played by the other students, i_1 gets school a in any equilibrium outcome. Moreover, student i_2 gets school b when she ranks it as first choice and i_3 gets school b when i_2 does not rank b as first choice. Hence, in any equilibrium i_2 ranks b as top choice and gets it. Similarly, when i_2 ranks b as top choice, student i_4 can get school c by only ranking it as top choice. Hence, there is a unique Nash equilibrium outcome:

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ a & b & d & c \end{pmatrix}.$$

⁴For the sake of expositional simplicity, we do not include the strategies in which \emptyset is not ranked as the last choice.

The augmented priority orders associated with $\tilde{\pi}$ are:

$a:$	i_1	i_2	i_3	i_4
$b:$	i_1	i_2	i_3	i_4
$c:$	i_1	i_2	i_4	i_3
$d:$	i_2	i_1	i_4	i_3

The only stable matching under $(P, \tilde{\pi})$ is

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ a & b & d & c \end{pmatrix},$$

which is the same as the unique equilibrium outcome.

The observation that the Nash equilibrium outcome is related to the stable matching of an augmented problem is more general than this example.

We first show that under problem $(P, \tilde{\pi})$ there exists a unique stable matching.

Proposition 2. *Under Assumptions 1 and 2, for arbitrary (P, π) , λ , \mathcal{M} and \mathcal{N} , let $\tilde{\pi}$ be the augmented priority score list. Under $(P, \tilde{\pi})$ there exists a unique stable matching and it is Pareto efficient.*

Proof. By using a recursive procedure, under $(P, \tilde{\pi})$, we show that one can always find a remaining student who has the highest priority among the remaining students at her top choice among the remaining schools and we remove that student and a seat from that school. This student will be matched to her top choice among the remaining ones in any stable matching and there will not exist a possible welfare improvement trade between the remaining students and this student.

Let i_k be the student who has the highest score in π among the students remaining in Step $k \geq 1$ of this procedure. We start with Step 1 in which all students and all seats are available. By our construction, i_1 has the highest priority at her top choice no matter she is sophisticated or sincere. Moreover, she will be assigned to her top choice in any stable matching. We remove i_1 and one seat from her top choice and consider the remaining students and schools in Step 2. Note that, i is assigned to her top choice in any stable matching.

Suppose our claim holds for the first $k - 1$ steps of this procedure. Now consider Step k . If i_k is a sophisticated student, then she has the highest priority among the remaining students at all remaining schools. Hence, the claim holds. Suppose i_k is a sincere student. Let s^1 be her top choice among the remaining schools. If the student with the highest priority for s^1 among the remaining ones prefers s^1 most, then we are done. Otherwise, we

consider the student $i^1 \neq i_k$ with the highest priority at s^1 among the remaining students and her most preferred school among the remaining ones denoted by $s^2 \neq s^1$. Note that, i_k cannot have higher priority than i^1 at school s^2 . If the student with the highest priority for s^2 among the remaining ones prefers s^2 most, then we are done. Otherwise, we consider the student $i^2 \notin \{i^1, i_k\}$ with the highest priority at s^2 among the remaining students and her most preferred school among the remaining ones denoted by $s^3 \notin \{s^1, s^2\}$. Note that, i_k and i^1 cannot have higher priority than i^2 at school s^3 . By finiteness, we will eventually find a student i and school s such that i has the highest priority at s among the remaining students and i prefers s most among the remaining schools. Hence, in any stable matching i is assigned to s and there cannot be a welfare improving cycle between the remaining students and i . \square

Next, we consider the preference revelation game under TM^λ for any λ .

Proposition 3. *Under Assumptions 1 and 2, for arbitrary (P, π) , λ , \mathcal{M} and \mathcal{N} , let $\tilde{\pi}$ be the augmented priority score list. Then, there exists a unique NE outcome of this game and it is Pareto efficient and equivalent to $DA(P, \tilde{\pi})$.*

Proof. Since under $(P, \tilde{\pi})$ there exists a unique stable matching (see Proposition 2), we will prove that there cannot be an NE outcome which is not stable.

On the contrary, let Q be an NE profile and the outcome of TM^λ under this strategy profile is μ , i.e., $TM^\lambda(Q, \pi) = \mu$, and μ is not stable under $(P, \tilde{\pi})$. Note that, $TM^\lambda(Q, \pi) = DA(Q, \hat{\pi}^\lambda)$ where $\hat{\pi}^\lambda$ is the implied by (Q, π) and deduction rule λ .

If matching μ is individually irrational, then there exists a student i who is assigned to an unacceptable school, i.e. $\emptyset P_i \mu(i)$. Then, since TM^λ is individually rational, ranking \emptyset as top choice is a profitable deviation for i .

Suppose μ is wasteful or not fair. Then, there exists a school student pair (s, i) such that $s P_i \mu(i)$ and either $|\mu^{-1}(s)| < q_s$ or $\tilde{\pi}_s(i) > \tilde{\pi}_s(j)$ for some $j \in \mu^{-1}(s)$. Under both cases, we consider sequential version of DA under $(Q, \hat{\pi}^\lambda)$ such that i applies last. If the former case holds, then population monotonicity of DA and Lemma 2 imply that i can profitably deviate by ranking school s as top choice. By our construction if $i \in \mathcal{N}$ and $\tilde{\pi}_s(i) > \tilde{\pi}_s(j)$ then $\hat{\pi}_s^\lambda(i) > \hat{\pi}_s^\lambda(j)$. Hence, i cannot be a sincere student. If i is a sophisticated student, then the fact that DA is population monotonic and Lemma 2 imply that i can profitably deviate by ranking school s as top choice. \square

3.2 Becoming Sophisticated

Many families report confusion about the new Taiwanese mechanism, but some have spend considerable effort on learning about the rules of the mechanism. In the context of our

example, a student who becomes sophisticated becomes (weakly) better off, as we show next.

Example 1 (cont.) In Example 1, if i_2 and i_4 are sophisticated, under TM^{λ^1} the unique equilibrium outcome is

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ a & b & d & c \end{pmatrix}.$$

Student i_3 gets her third choice. If i_3 becomes sophisticated, then the augmented priority orders associated with $\tilde{\pi}$ are:

$a:$	i_1	i_2	i_3	i_4
$b:$	i_1	i_2	i_3	i_4
$c:$	i_1	i_2	i_3	i_4
$d:$	i_2	i_1	i_3	i_4

The unique equilibrium outcome when i_2 , i_3 and i_4 are sophisticated is

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ a & b & c & d \end{pmatrix}.$$

Hence, by becoming sophisticated, i_3 is better off.

This example illustrates a more general phenomenon summarized by our next proposition.

Proposition 4. *Under Assumptions 1 and 2, consider arbitrary (P, π) , λ , \mathcal{M} and \mathcal{N} . If a sincere student $i \in \mathcal{N}$ becomes sophisticated, then under the equilibrium outcome of TM^{λ} , i becomes (weakly) better off.*

Proof. Suppose that $\tilde{\pi}^1$ is the augmented priority score profile when i is sincere, and $\tilde{\pi}^2$ is the one when i becomes sophisticated. By definition, for all $s \in S$ $\tilde{\pi}_s^1(j) = \tilde{\pi}_s^2(j)$ for all $j \neq i$ and $\tilde{\pi}_s^1(i) \leq \tilde{\pi}_s^2(i)$. That is, i is improved under the associated priority orders when she becomes sophisticated. By Proposition 3, under both cases the unique equilibrium outcome is equivalent to $DA(P, \tilde{\pi}^1)$ and $DA(P, \tilde{\pi}^2)$, respectively. Since, DA respects improvement in priorities, $DA(P, \tilde{\pi}^2)(i) R_i DA(P, \tilde{\pi}^1)(i)$. \square

When one sincere student becomes sophisticated, she obtains a weakly better assignment. Does this imply that other sophisticated students obtain weakly worse assignments? The answer turns out to be no, as the following example illustrates.

Example 2. We slightly modify Example 1 by changing the preferences of i_2 and i_4 as follows: $aP_{i_2}bP_{i_2}dP_{i_2}c$ and $dP_{i_4}cP_{i_4}aP_{i_4}b$. Suppose initially only i_4 is sophisticated. Then, under TM^{λ^1} the unique equilibrium outcome is

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ a & d & b & c \end{pmatrix}.$$

Now consider the case where i_2 becomes sophisticated. Then, under TM^{λ^1} when i_2 and i_4 are sophisticated the unique equilibrium outcome is

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ a & b & c & d \end{pmatrix}.$$

Therefore, student i_4 is better off after i_2 becomes sophisticated. Note that, DA mechanism also selects the same outcome when all students report their true preferences.

3.3 Changing Mechanisms

Given that the Taiwan mechanism is manipulable, it is natural to consider what the consequences of adopting a non-manipulable mechanism. The most natural alternative in this setting is DA , which is strategy-proof for students (Dubins and Freedman 1981, Roth 1982) and increasingly in use (Pathak and Sönmez 2013, Pathak 2016). When schools share the same priority, DA reduces to a serial dictatorship. In a serial dictatorship, we process applicants in order and offer each applicant his or her most preferred school with seats remaining.

Example 1 (cont.) In Example 1, student i_3 is assigned a better school b under TM^{λ^1} than school c under the serial dictatorship mechanism.

This result is related to comments made by Education Minister Chiang Wei-ling, who remarked (Wei 2014):

no new policy would be carried out unless it would “benefit all students.”

Proposition 4 shows that sophisticated students may lobby against changing the mechanism and that the Taiwanese mechanism favors sophisticated students, since deduction only applies to sincere students. Indeed, it is consistent with the reluctance of Taiwanese authorities to abolish deduction rules in the face of massive condemnation and street protests.

However, it is not the case that all sophisticated students obtain a weakly better assignment under the Taiwan mechanism than under DA . This is a sharp contrast to earlier comparisons between BM and DA in Pathak and Sönmez (2008). Example 2 shows that the only sophisticated student i_4 may actually prefer DA over the Taiwan mechanism.

4 Conclusion

A new nationwide system to assign children to high school in Taiwan generated widespread turmoil and protests. In this paper, we have studied the incentive properties of this mechanism and characterized the equilibrium of the induced preference revelation game. Our

results show that any mechanism using deduction is manipulable, and that the scope for manipulation increases with the size of deduction. The Taiwanese mechanism has a unique equilibrium, which can be characterized in terms of a stable matching of an alternative economy. It is not possible to provide a strong comparison of the welfare of students in the Taiwan mechanism compared to strategy-proof alternative.

Our analysis provides a rationale for the reluctance of Taiwanese authorities to move to a strategy-proof alternative, illustrating a broader dynamic seen with manipulable mechanisms used in school choice elsewhere. Changes in market designs rarely involve Pareto improvements for all participants, and these participants may stand in the way of changing the mechanism.

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A Beyond Taiwan: Heterogeneous Priorities

In this appendix, we examine two examples of deduction mechanisms without assumption 1 and show that Proposition 1 and 3 no longer apply. The first example we consider shows that opportunities for manipulation need not increase with higher levels of deduction.

Example 3. There are five schools $S = \{a, b, c, d, e\}$ and five students $I = \{i_1, i_2, i_3, i_4, i_5\}$. Let $q_s = 1$ for all $s \in S$. The priority scores of students for schools are as follow: $\pi_a(i_1) = 100$, $\pi_a(i_3) = 98$, $\pi_a(i_4) = 97$, $\pi_b(i_2) = 99$, $\pi_b(i_3) = 98$, $\pi_c(i_3) = 90$, $\pi_c(i_4) = 86$, $\pi_c(i_5) = 100$, $\pi_d(i_4) = 100$, $\pi_d(i_5) = 90$, and $\pi_e(i_3) = 100$. The preference of students are as follows:

P_{i_1} :	a	\emptyset			
P_{i_2} :	b	\emptyset			
P_{i_3} :	a	b	c	e	\emptyset
P_{i_4} :	a	c	d	\emptyset	
P_{i_5} :	d	c	\emptyset		

We consider two deduction rules: $\lambda = (0, 1, 2, 2, 2, 2)$ and $\lambda' = (0, 2, 7, 7, 7, 7)$. Given λ , P and π , the implied priority scores profile π^λ is: $\pi_a^\lambda(i_1) = 100$, $\pi_a^\lambda(i_3) = 98$, $\pi_a^\lambda(i_4) = 97$, $\pi_b^\lambda(i_2) = 99$, $\pi_b^\lambda(i_3) = 97$, $\pi_c^\lambda(i_3) = 88$, $\pi_c^\lambda(i_4) = 85$, $\pi_c^\lambda(i_5) = 99$, $\pi_d^\lambda(i_4) = 98$, $\pi_d^\lambda(i_5) = 90$, and $\pi_e^\lambda(i_3) = 98$. Given λ' , P and π , the implied priority scores profile $\pi^{\lambda'}$ is: $\pi_a^{\lambda'}(i_1) = 100$, $\pi_a^{\lambda'}(i_3) = 98$, $\pi_a^{\lambda'}(i_4) = 97$, $\pi_b^{\lambda'}(i_2) = 99$, $\pi_b^{\lambda'}(i_3) = 96$, $\pi_c^{\lambda'}(i_3) = 83$, $\pi_c^{\lambda'}(i_4) = 84$, $\pi_c^{\lambda'}(i_5) = 98$, $\pi_d^{\lambda'}(i_4) = 93$, $\pi_d^{\lambda'}(i_5) = 90$, and $\pi_e^{\lambda'}(i_3) = 93$.

In this example, there is an Ergin (2002) cycle under $\hat{\pi}^\lambda$ and π and such a cycle does not exist under $\hat{\pi}^{\lambda'}$.

In this problem, TM^λ selects:

$$\mu = \begin{pmatrix} i_1 & i_2 & i_3 & i_4 & i_5 \\ a & b & e & d & c \end{pmatrix}.$$

In this problem, $TM^{\lambda'}$ selects:

$$\nu = \begin{pmatrix} i_1 & i_2 & i_3 & i_4 & i_5 \\ a & b & e & c & d \end{pmatrix}.$$

Under TM^λ , student i_4 can manipulate her preferences by ranking s_3 at the top and the outcome is

$$\nu = \begin{pmatrix} i_1 & i_2 & i_3 & i_4 & i_5 \\ a & b & e & c & d \end{pmatrix}.$$

On the other hand, no student can benefit from manipulation under $TM^{\lambda'}$.

Next, we show that under Taiwan mechanism, it is possible for there to be a unique Nash equilibrium outcome in weakly undominated strategies, but that outcome is not stable under augmented priorities. This example slightly modifies the previous one.

Example 4. There are five schools $S = \{a, b, c, d, e\}$ and six students $I = \{i_1, i_2, i_3, i_4, i_5, i_6\}$. Let $q_s = 1$ for all $s \in S$. Suppose all students are strategic. The priority scores of students for schools are as follow: $\pi_a(i_1) = 100$, $\pi_a(i_3) = 99.5$, $\pi_a(i_4) = 97$, $\pi_b(i_2) = 99$, $\pi_b(i_3) = 98.5$, $\pi_c(i_3) = 90$, $\pi_c(i_4) = 86$, $\pi_c(i_5) = 100$, $\pi_d(i_4) = 100$, $\pi_d(i_5) = 90$, $\pi_e(i_3) = 100$, and $\pi_e(i_6) = 99.5$. The preference of students are as follows:

P_{i_1} :	a	\emptyset			
P_{i_2} :	b	\emptyset			
P_{i_3} :	a	b	c	e	\emptyset
P_{i_4} :	a	c	d	\emptyset	
P_{i_5} :	d	c	\emptyset		
P_{i_6} :	e	\emptyset			

We consider the following deduction rule: $\lambda = (0, 1, 2, 2, 2, 2)$. Given λ , π and P , the implied priority scores profile π^λ is: $\pi_a^\lambda(i_1) = 100$, $\pi_a^\lambda(i_3) = 99.5$, $\pi_a^\lambda(i_4) = 97$, $\pi_b^\lambda(i_2) = 99$, $\pi_b^\lambda(i_3) = 97.5$, $\pi_c^\lambda(i_3) = 88$, $\pi_c^\lambda(i_4) = 85$, $\pi_c^\lambda(i_5) = 99$, $\pi_d^\lambda(i_4) = 98$, $\pi_d^\lambda(i_5) = 90$, $\pi_e^\lambda(i_3) = 98$ and $\pi_e^\lambda(i_6) = 99.5$.

Note that, i_1 and i_2 can obtain their top choices by submitting their true preferences and they may not be assigned to their top choices if they do not rank them as top choice. Hence, in any weakly undominated strategy i_1 and i_2 rank their true top choice at the top. Independent of the other students ranking, student i_4 and i_5 can get one of their top two choices by submitting their true preferences. Moreover, submitting something else is weakly dominated by their true preference profile.

Similarly, if i_6 ranks an unacceptable school at the top she may be assigned to it and ranking \emptyset as top choice is weakly dominated by her true preference profile. That is, for all students except i_3 submitting true preference is weakly undominated strategy. For such a strategy profile i_3 's best response is ranking e as top choice. Otherwise she will be unassigned. The corresponding equilibrium outcome is:

$$\mu = \begin{pmatrix} i_1 & i_2 & i_3 & i_4 & i_5 & i_6 \\ a & b & e & c & d & \emptyset \end{pmatrix}.$$