Investment Dynamics and Cyclical Redistribution

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Demand for durable goods and residential investment is strongly pro-cyclical. Workers employed in durable industries are imperfectly insured against these fluctuations, leading to distributional consequences during booms and busts. This paper studies the interaction between cyclical durable demand and redistribution of labor income. I explore this feedback loop within a heterogeneous agent New Keynesian (HANK) model with multiple sectors and lumpy durable adjustment. Crucially, lumpy adjustment at the micro level generates non-linearities at the macro level: the average marginal propensity to spend on durable goods varies with the size of income shocks. As a result, sectoral redistribution of labor incomes has aggregate effects. I find that the interaction between cyclical investment and redistribution amplifies the aggregate response of durable spending during booms and dampens it during recessions. The lumpy nature of durable adjustment entirely accounts for this non-linear effect.

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1 Introduction

Purchases of durable goods and residential investment are strongly pro-cyclical. Workers employed in the industries producing these goods are imperfectly insured against the fluctuations in durable expenditure, leading to distributional consequences during booms and busts (Guvenen et al. (2017)). Microeconomic evidence suggests that these income changes have a high pass-through to durable spending (Dynarski and Gruber (1997); Browning and Crossley (2009)). These two facts suggest the presence of a feedback loop that has not been explored in the literature: the cyclicality of durable expenditure induces a redistribution of labor income between sectors; in turn, this redistribution feeds back into the composition of spending and durable expenditure.

In this paper, I study this feedback loop and its role in the amplification of aggregate shocks. My starting point is a general equilibrium model of lumpy durable demand with incomplete markets (Berger and Vavra (2015)) and sticky prices. The main novelty lies in the presence of distributional effects. I recognize that consumption and investment goods are produced in different sectors. Redistribution stems from two features. First, durable spending is more elastic to aggregate shocks. This translates into a more cyclical demand for labor demand in durable industries. Second, labor mobility is restricted between sectors. Households employed in the durable sector fail to relocate and are disproportionately affected by these cyclical fluctuations.

Cyclical income inequality between sectors has aggregate effects in my model despite individuals having homogeneous preferences. The reason is that the response of aggregate durable spending to income shocks is non-linear in their size. This non-linearity is induced by infrequent and discontinuous adjustment of durables at the microeconomic level.\(^1\)\(^2\) I show that the average marginal propensity to spend (MPC) on durable goods increases with income changes. During expansions, households employed in the durable sector experience a disproportionate increase in income, and their MPCs on durables rise endogenously. As a result, cyclical income inequality amplifies the increase in durable spending during expansions. On the contrary, it dampens its decline during recessions.

My first contribution is to clarify the sources of the aggregate non-linearities in durables demand, and quantify their importance even without redistribution. I proceed in two steps. In the first step, I explore the theoretical determinants of this non-linearity using

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1 See Bertola and Caballero (1990) for a review on investment subject to lumpy adjustment. Fixed adjustment costs are essential to explain the micro behavior of durable investment (Eberly (1994); Attanasio (2000)), and its aggregate time series properties (Caballero (1990, 1993, 1994)).

2 In particular, this form of discontinuities has been shown to produce aggregate non-linearities in the context of capital investment (Caballero et al. (1997); Caballero and Engel (1999)) and price setting (Alvarez et al. (2017a,b)).
my model of lumpy adjustment. Non-convex adjustment costs lead to a standard inaction region for durable adjustment. Durable adjustment occurs along two margins: an extensive margin, which controls the propensity of households to exit their inaction region and pay the adjustment cost; and an intensive margin, which determines their MPC conditional on adjustment. The contribution of each of these margins to aggregate nonlinearities depends on two objects: the shape of the distribution of idiosyncratic characteristics at the edge of the inaction region; and the monotonicity and concavity of durable spending conditional on adjustment. Imposing restrictions that resemble those obtained in numerical simulations, I show that adjustment along the extensive margin amplifies expansionary income shocks as their magnitude increases: more and more households adjust their stock of durables. On the contrary, it dampens contractionary shocks. Lumpy and state-dependent adjustment plays a central role in this non-linear effect. Models of smooth or time-dependent adjustment, where only the intensive margin operates, would actually predict the opposite effect.

In the second step, I calibrate my model and use it to quantify the importance of these aggregate non-linearities. I simulate the response of durable investment to income shocks, varying their sign and magnitude. I find that the average MPC on durable goods increases with income changes. The degree of non-linearity is economically significant. For instance, for negative income shocks of the magnitude experienced by durable workers during the Great Recession, the response of durable investment is roughly 10-15% lower than would be predicted without non-linearities. The opposite is true for positive shocks of the same magnitude. Decomposing the responses into extensive and intensive margins, I find that the former dominates quantitatively. In other words, lumpy adjustment entirely accounts for these aggregate non-linearities.

My theoretical findings are in line with the existing evidence on the consumption response to transfers. Johnson et al. (2006) estimate negligible spending multipliers on durable goods in response to the 2001 stimulus payment, while Souleles (1999) and Parker et al. (2013) estimate strong responses for durable expenditure following springtime tax refunds and the 2008 stimulus payment, respectively. Parker et al. (2013) attribute this difference to the size of the transfers. Similarly, Fuster et al. (2018) and Christelis et al. (2019) provide survey evidence that the average MPC on durables increases with the size of (hypothetical) unexpected income changes. My model replicates this finding qualitatively and produces an increasing average MPC on durables.

3 In this case, MPCs are declining in wealth when markets are incomplete: expansionary shocks are dampened as their size increases, and contractionary shocks are amplified.

My second contribution is to explore the interaction between cyclical income inequality and lumpy durable investment. Durable spending is strongly pro-cyclical, which effectively redistributes labor income between sectors in general equilibrium. To assess the role of redistribution, I compare the aggregate response of durable investment in my model to that obtained in a counterfactual economy where fiscal policy undoes this redistribution and provides cross-sectoral insurance. I show that a cyclical income redistribution amplifies the effect of expansionary shocks and dampens contractionary shocks when the average MPC on durable investment is increasing with income changes.

An “earnings heterogeneity channel” (Auclert (2019); Werning (2015); Patterson (2019)) emerges endogenously in my setting. However, my model is set up so that there is no \textit{ex ante} heterogeneity in the average MPC on durables across sectors. Redistribution of income between sectors is neutral for first order deviations from the steady state. Instead, I focus on non-linear amplification effects away from the stationary equilibrium. Redistribution drives a wedge across sectors in the households’ propensity to adjust their stock of durables. In other words, MPCs on durable goods are heterogeneous \textit{ex post} across sectors. This heterogeneity ensures that cyclical income inequality is non-neutral, and amplifies the response of durable spending during expansions and attenuates it during recessions. I show that state-dependent adjustment plays a crucial role in this form of non-linear amplification.

Finally, I quantify the role of redistribution in general equilibrium. I embed my model of lumpy durable investment in a multi-sector heterogeneous agent New Keynesian framework (HANK). There are two sectors, which respectively produce a durable investment good, and a non-durable consumption good. There is no labor mobility between sectors, so that households employed in the durable sector are more exposed to aggregate shocks. Financial markets are incomplete, which limits risk sharing across households employed in different sectors.\footnote{Participation in financial markets is limited (Mankiw and Zeldes (1991); Heaton and Lucas (2000)), and households fail to hedge against (sector-specific) aggregate risk (Massa and Simonov (2006)).} Following Berger and Vavra (2015), I focus on productivity shocks. I find that income redistribution stimulates the response of durable spending by roughly 10% over the first year and a half after the occurrence of an expansionary shock, and increases the persistence of this response.

**Related literature.** My paper lies at the intersection of several strands of the literature on durable investment, heterogeneous agents, and redistribution.

First, my paper contributes to an extensive literature on lumpy durable investment.\footnote{Seminal contributions on the subject include Arrow (1968), Nickell (1974), Pindyck (1988), Bar-Ilan and Blinder (1987) and Grossman and Laroque (1990). Again, see Bertola and Caballero (1990) for a review.}
Durable adjustment is infrequent and discontinuous at the microeconomic level. In the closely related context of capital adjustment and price setting, these discontinuities tend to produce non-linearities at the macroeconomic level. In particular, Caballero et al. (1997) and Caballero and Engel (1999) find that firms’ capital investment responds proportionately more to large shocks than smaller ones when adjustment is lumpy. Building on this insight, I explore the aggregate non-linearities produced by a canonical model of lumpy durable investment with uninsured idiosyncratic risk and borrowing constraints. I find that expansionary income shocks are amplified as their size increases, while contractionary shocks are dampened. I trace this asymmetry back to the shape of the distribution of liquid assets around the durable adjustment thresholds, and the monotonicity of durable investment conditional on adjustment. I then explore the implications of this non-linearity in the presence of redistribution of labor income.

My quantitative model of durable demand builds directly on the one developed by Berger and Vavra (2015). My focus is complementary to theirs. They investigate the role of lumpy adjustment for the cyclical response to aggregate shocks. I extend their setting by adding redistributioinal effects. I recognize that durable investment goods and nondurable consumption goods are produced in different sectors. Households employed in the durable sector are more exposed to aggregate shocks, thus they claim a higher share of labor income during booms and a lower share during busts. In turn, this redistribution of labor income interacts with lumpy adjustment and the aggregate non-linearities it generates.

Models of lumpy adjustment typically produce state-contingent responses to aggregate shocks. In particular, Berger and Vavra (2015) find that the response of durable investment to aggregate shocks is pro-cyclical. This pro-cyclicality is consistent with the non-linearity that I document: an incremental income shock following a period of expansion is akin to a discrete aggregate shock in my model. I argue that the amplification is controlled in both cases by the same structural objects.

Second, my paper fits into a growing literature on heterogeneous agent New Keynesian (HANK) models. My approach reconciles two separate strands of this literature: one interested in the role of durable spending and one focused on redistribution of labor income. Falling into the first category, Guerrieri and Lorenzoni (2017) explore the effect of deleveraging shocks in the presence of durable investment with convex adjustment costs.

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7 On price setting, Alvarez et al. (2017b) show that the degree of monetary non-neutrality in menu costs models is non-linear in the size of monetary shocks, and Alvarez et al. (2017a) find empirical support for this property.

In a recent paper, McKay and Wieland (2019) quantify the effect of monetary policy and forward guidance in a model of lumpy durable investment. I introduce a role for cyclical income redistribution by assuming that labor markets are sector-specific. Households are either employed in the durable or non-durable sector and cannot relocate between them. Durable spending is strongly pro-cyclical, which induces a redistribution of labor income across sectors during booms and busts.

By introducing distributional concerns into a model of durable investment, I draw a connection to the existing literature on redistribution in heterogeneous agent models. Auclert (2019), Werning (2015) and Patterson (2019) explore the role of earnings heterogeneity for the marginal response to aggregate shocks in general equilibrium. My paper complements their work by exploring the non-linear effects of redistribution. In my model, redistribution is neutral for local deviations from the stationary equilibrium. This allows me to focus on higher-order effects. I find that redistribution amplifies expansionary shocks and dampens contractionary shocks. I show that lumpy investment is central to this non-linear amplification.

More broadly, my paper speaks to a literature on business cycle fluctuations in multi-sector economies. A first branch of this literature studies cyclical changes in the composition of spending and their implications for aggregate labor demand. In particular, Bils et al. (2013) and Jaimovich et al. (2019) investigate the interaction between cyclical spending on durable and luxury goods and the relatively low capital intensity in these sectors. A second strand of this literature focuses on real rigidities and amplification stemming from heterogeneous frequencies of price adjustment across sectors. I set up my model to abstract from these considerations, and isolate the role of redistribution and the non-linearities it produces.

**Layout.** I start by introducing a multi-sector heterogeneous agent New Keynesian (HANK) model with lumpy investment in Section 2. I discuss the sources of non-linearity inherent to this model in Section 3, and explore the implications of income redistribution in this setting. Section 5 describes the calibration of the model and the empirical targets. In Section 6, I quantify the degree of non-linearity in my calibrated model. I explore the general equilibrium interaction between income redistribution and aggregate non-linearities in Section 7. Section 8 concludes. The appendix contains the proofs, and complementary quantitative and empirical results.

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9 Alonso (2016) quantifies this effect in a heterogeneous agent model.

2 A Multi-Sector Model with Lumpy Investment

I introduce a multi-sector heterogeneous agent New Keynesian (HANK) model with lumpy durable investment. There are two sectors producing a consumption good and a durable investment good, respectively. Each household is employed in a given sector and is unable to relocate between them.\textsuperscript{11} The demand side of the economy builds on the canonical model of durable demand with incomplete markets from Berger and Vavra (2015). The supply side is standard. Prices are sticky à la Calvo (1983), which leads to sector-specific Phillips curves. I describe the environment below. For concision, I only include here the main expressions. Appendix A provides a full description of the full model.

2.1 Environment

Time is discrete, and there is no aggregate uncertainty.\textsuperscript{12} Periods are indexed by $t \in \{0, 1, \ldots\}$. The two goods are indexed by $h \in \mathcal{H} \equiv \{c, d\}$. Sector $h = c$ produces the non-durable consumption good, and sector $h = d$ produces the durable investment good.

**Households.** The economy is inhabited by a continuum of mass 1 of households. Each household is assigned permanently to a given sector $h$. Households are characterized by four idiosyncratic states: their financial asset holdings ($a$), their holdings of durable goods ($d$), their idiosyncratic labor supply shock ($\zeta$), and the sector they are employed in ($h$). The mass of households in each sector is denoted by $\mu \equiv \{\mu_h\}_h$.

Households consume durable and non-durable goods. Preferences are represented by

$$
\mathbb{E} \left[ \sum_{t \geq 0} \beta^t \frac{u(c_t, d_t)^{1-\sigma}}{1-\sigma} \right]
$$

with discount factor $\beta \in (0, 1)$, and inverse elasticity of substitution $\sigma > 0$. Intratemporal preferences exhibit constant elasticity of substitution:

$$
u(c, d) = \left[ \theta^{\frac{1}{\nu}} c^{\frac{\nu-1}{\nu}} + (1 - \theta)^{\frac{1}{\nu}} d^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}}
$$

with share parameter $\theta \in (0, 1)$ and elasticity of substitution $\nu > 0$.

\textsuperscript{11} There is evidence of some cyclical reallocation between durable and non-durable sectors over the cycle (Loungani and Rogerson (1989)). The magnitudes are relatively small, however, compared to the relative income changes between sectors. Durable employment is unevenly spread across the U.S., and geographic mobility is limited at the business cycle frequencies (Yagan (2014); Kaplan and Schulhofer-Wohl (2017)).

\textsuperscript{12} I focus on the effect of one-time, unanticipated but persistent shocks.
After observing their idiosyncratic labor income, households decide whether to adjust their stock of durable goods. Adjustment entails a non-convex cost $\Gamma_t$. Following Berger and Vavra (2015) and Kaplan et al. (2017), I assume that durable adjustment costs are proportional to the nominal value of the undepreciated stock of durable goods. If households do not adjust, they pay a maintenance cost, i.e. an investment required to repair or operate the existing stock of durables.\textsuperscript{13} This maintenance corresponds to a share $\iota \in [0, 1]$ of the current depreciation of their stock of capital. Summing up, the non-convex adjustment costs are

$$
\Gamma_t (d', d) = \begin{cases} 
(1 - \delta) P_t^d \gamma d & \text{if } d' \neq (1 - (1 - \iota) \delta) d \\
0 & \text{otherwise}
\end{cases}
$$

for some adjustment cost $\gamma \geq 0$, where $d'$ denotes the new stock of durables and $\delta \in (0, 1)$ denotes the depreciation rate. Here, $P_t \equiv \{ P_h^t \}_h$ denotes goods prices. I suppose that these adjustment costs take the form of services (real estate, moving, etc.) provided by the non-durable sector, while maintenance takes the form of investment goods (new windows, tires, etc.) purchased from the durable sector.\textsuperscript{14}

**Firms.** Firms have access to technologies with decreasing returns:

$$
y = F^h_l (l)
$$

for some concave $F^h_l : [0, 1] \to \mathbb{R}_+$ that allows for time-varying productivity.\textsuperscript{15} Here, $l$ denotes firms’ individual labor demand.

**Nominal rigidities.** Prices are flexible at the stationary equilibrium. Firms set prices to maximize profits, subject to technology and given wages. On the contrary, prices are sticky à la Calvo (1983) along the transition path. The idiosyncratic reset probability is denoted by $1 - \lambda^h$ in each sector, with $\lambda^h \in [0, 1]$. The (implicit) elasticity of substitution across varieties, within each sector, is denoted by $\varepsilon > 1$.

Households supply labor inelastically in their industry of employment. They are compensated in proportion to their idiosyncratic labor supply. Wages are flexible at the stationary equilibrium, but rigid along the transition path: $W^h_l = W^h$. Labor is demand-\textsuperscript{13} Maintenance is a standard feature of lumpy adjustment models (Bachmann et al. (2013); Berger and Vavra (2015)). It decreases the effective depreciation rate in the case of no adjustment. Fixing the depreciation rate, adjustment costs and idiosyncratic risk, a higher maintenance decreases the average frequency of adjustment.

\textsuperscript{14} This assumption is mostly innocuous. Spending on adjustment costs and maintenance are relatively acyclical in my calibrated model, compared to purchases of durable and non-durable goods.

\textsuperscript{15} Implicitly, I suppose that capital is firm-specific and fixed in the short-run.
determined in this case. As a result, earnings in the durable sector are more elastic to aggregate shocks in general equilibrium. The fiscal authority can potentially use lump sum taxes to provide cross-sectoral insurance against these aggregate shocks. Profits are redistributed symmetrically across households. Summing up, total individual (gross) incomes are

\[ e^h_t (\zeta) = \frac{1}{\mu^h_t} \left( \mathcal{Y}^h_t - T^h_t \right) + \pi_t, \]  

(2.2)

where \( \mathcal{Y}_t \equiv \{ \mathcal{Y}^h_t \}_h \) denotes the sector-specific wage bills, \( T_t \equiv \{ T^h_t \}_h \) denotes lump sum taxes and \( \pi_t \) denotes aggregate profits claimed by households. The idiosyncratic process for labor supply follows a Markov chain with transition kernel \( \Sigma \) on some set \( S \subset \mathbb{R}_{++} \), with \( E [ \zeta ] = 1 \) and independence across households.

**Policy.** The government can potentially use lump sum taxes \( T_t \) to provide cross-sectoral insurance. It maintains an exogeneous and constant debt level \( B > 0 \) and sets linear taxes on income \( \tau_t \) to balance its flow budget. Monetary policy implements a standard Taylor rule:

\[ i_t = \max \{ r + \phi^\tau (\Pi_t - 1), 0 \}, \]  

(2.3)

where \( \Pi_t \) denotes gross inflation\(^{16} \) and \( r \) denotes the nominal interest rate at the stationary equilibrium.

### 2.2 Households’ Optimization

The households’ problem can be formulated recursively. The durable adjustment choice solves:

\[ \mathcal{V}^h_t (a, d, \zeta) = \max_{A \in \{0, 1\}} \left\{ V^h_t (a, d, \zeta; A) \right\}, \]  

(2.4)

where \( A \in \{0, 1\} \) denotes the adjustment decision and \( V^h_t (\cdot; A) \) denotes the continuation values associated to each adjustment option.

The value associated with no adjustment is

\[ V^h_t (a, d, \zeta; 0) = \max_{\{c, d^*\}} \frac{u (c, d^*)^{1-\sigma}}{1-\sigma} + \beta E_t \left[ \mathcal{V}^h_{t+1} (a', d^*, \zeta') \mid \zeta \right] \]  

(2.5)

s.t. \( P^c_t c + P^d_t d^* \frac{\delta}{1-(1-l)} d^* + a' \leq (1 - \tau_t) e^h_t (\zeta) + (1 + r_{t-1}) a \)

\[ a' \geq 0, \]

\(^{16}\) See Appendix A.1 for the definition of the price index.
with \( d^* \equiv (1 - (1 - \iota) \delta) d \). Here, \( e_t^h(\zeta) \) denotes labor income, and \( r_t-1 \) denotes the nominal interest rate. Earnings are indexed by the idiosyncratic labor supply and the industry of employment.

Similarly, the value associated with adjustment is

\[
V_t^h(a, d, \zeta; 1) = \max_{\{c, a', d', \zeta'\}} \frac{u(c, d')^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t \left[ V_{t+1}^h(a', d', \zeta') \bigg| \zeta \right] 
\]

subject to

\[
P_t^c c + P_t^d (d' - (1 - \delta) d) + \Gamma_t (d', d) + a' \\
\leq \left( 1 - \tau_t^h \right) e_t^h(\zeta) + (1 + r_{t-1}) a \\
a' \geq 0
\]

2.3 Earnings and Insurance

Households’ gross earnings (2.2) consist of labor income, aggregate profits, and transfers from the government. Wage bills are given by

\[
Y_t^h \equiv W_t^h \hat{\mu}_t^h, 
\]

where demands for labor \( \hat{\mu}_t \equiv \{\hat{\mu}_t^h\}_h \) satisfy

\[
P_t^h \left( \hat{\mu}_t^h \right) \equiv \Omega_t^h \left( \frac{1}{P_t^h} \right)^{-\varepsilon} Y_t^h 
\]

Here \( Y_t \) denotes aggregate demand for each good, and \( \Omega_t^h \) denotes the productivity distortion associated with price dispersion.\(^{17}\) Aggregate profits are

\[
\hat{\pi}_t \equiv \sum_h \left( P_t^h Y_t^h - Y_t^h \right) 
\]

Households claim constant profits from firms: \( \pi_t = \hat{\pi} \), where \( \hat{\pi} \) denotes aggregate profits at the stationary equilibrium.\(^{18}\) In the absence of insurance from fiscal policy, the strong pro-cyclical in the demand for durable investment \( Y_t^d \) translates into a high cyclical of labor incomes \( e_t^h(\zeta) \) in that sector, by (2.2) and (2.7)–(2.8). This leads to a redistribution of

\(^{17}\) See Appendix A.1 for the definition of this productivity distortion.

\(^{18}\) The comparative statics of interest is a persistent productivity shock. In the limit with rigid prices \( \lambda^h \rightarrow 1 \), this shock affects the allocation of revenues between wage bills and profits. Abstracting from heterogeneity in labor supply \( \zeta \), a productivity shock has no aggregate effect in this case if profits are rebated every period to households. For this reason, I assume that firms redistribute constant profits in each period, and save the rest by accumulating financial assets.
income between sectors.

I contrast two regimes to assess the role of redistribution. The first regime is one where fiscal policy is passive and labor income is endogenously redistributed between sectors. That is,

\[ T_t = 0 \quad (2.10) \]

for each period \( t \).

The second regime (denoted by \( * \)) corresponds to a counterfactual economy where fiscal policy undoes this redistribution of labor income using lump sum taxes. Specifically,

\[
\frac{\mathcal{Y}^d_t - \mu^d T^d_t, *}{\mathcal{Y}^c_t - \mu^c T^c_t, *} = \frac{\mathcal{Y}^d}{\mathcal{Y}^c'}
\]

with budget balance \( \sum_h \mu^h T^h_t, * = 0 \), where \( \mathcal{Y} \equiv \{ \mathcal{Y}^h \}_h \) denotes wage bills at the stationary equilibrium. This second regime rules out distributional effects and effectively corresponds to the case considered in the literature on lumpy durable investment.

### 2.4 Market Clearing

Markets for goods clear:

\[
Y^c_t = \sum_h \mu^h \int \left[ c^h_i (a, d, \zeta) + \Gamma_t \left( d'^{h,j}_t (a, d, \zeta), \bar{d} \right) \right] d\Lambda^h_t \quad (2.12)
\]

\[
Y^d_t = \sum_h \mu^h \int \left[ d'^{h,j}_t (a, d, \zeta) - (1 - \delta) \bar{d} \right] d\Lambda^h_t, \quad (2.13)
\]

where \( c^h_i \) and \( d'^{h,j}_t \) denote the solution to (2.5)–(2.6), with \( d'^{h,j}_t \equiv d^* \) when there is no adjustment. Here, \( \Lambda_t \equiv \{ \Lambda^h_t \}_h \) denotes the conditional distributions of idiosyncratic states within each sector.

At the stationary equilibrium, wages are flexible and the labor markets clear in the two sectors: \( \mu = \bar{\mu} \). Then,\(^{19}\)

\[
Y^h_t = F^h (\mu^h), \quad (2.14)
\]

where \( \mu^h \) denotes the exogeneous mass of households located in sector \( h \). However, the market clearing condition (2.14) typically does not hold along the transition path since

\(^{19}\)There is no price dispersion within each sector at the stationary equilibrium, so that \( \bar{\mu} \) denotes both aggregate and individual labor demands at the steady state.
wages are fixed. Labor demands are demand-determined:

$$\mu = Z_t \circ \hat{\mu}_t$$

(2.15)

with $Z^h \equiv 1$ at the stationary equilibrium, and $0 \leq Z^h_t < +\infty$ along the transition path, for each sector $h$ and period $t$. That is, there is rationing of labor incomes in response to aggregate shocks.

**Equilibrium.** An equilibrium in my economy consists of sequences of policy functions for consumption and assets, distributions of idiosyncratic states, and prices, incomes and taxes such that households and firms optimize, the government balances its budget (and potentially provides insurance) and monetary policy implements a Taylor rule. I provide a formal definition of an equilibrium in Appendix A.2. The economy is initially at its non-inflationary stationary equilibrium.

The comparative statics of interest is a one-time, unanticipated, and persistent aggregate productivity shock. I assume that this productivity shock is symmetric between the durable and non-durable sectors, so there is no redistribution of labor incomes in partial equilibrium. Instead, redistribution is induced in general equilibrium by the pro-cyclicality of durable investment. Financial markets are effectively incomplete with respect to this aggregate productivity shock.

### 3 Non-Linearity and Redistribution

I introduced a multi-sector model with lumpy investment in the previous section. Durable investment plays a dual role in my setting. First, its pro-cyclicality induces a redistribution of labor income between sectors. Second, its lumpiness implies that the average MPC on durable goods depends on the magnitude of income shocks. In this section, I discuss each of these roles, and I show that their interaction acts as a non-linear propagation channel. In Section 3.1, I explore how lumpy adjustment at the micro level shapes non-linearities at the macro level. In Section 3.3, I clarify the role of durable investment as a source of labor income redistribution. I then explore the interaction between these two roles and its aggregate implications in Section 3.4. I draw a connection to the existing literatures on redistribution with heterogeneous agents in Section 3.5. Finally, I briefly review supporting evidence in Section 4.
3.1 Lumpy Investment and Aggregate Non-Linearity

Durable adjustment is infrequent and discontinuous in my model. I am interested in the aggregate implications of these microeconomic discontinuities. For now, I abstract from redistribution, so I drop the sector index \( h \).

I assume that the economy is initially at its stationary equilibrium. I consider an exogeneous one-time, unanticipated change in aggregate income in period \( t = 0 \):

\[
Y_0 = (1 + \Delta) Y
\]

for some \( \Delta \in \mathbb{R} \), where \( Y \) denotes the aggregate wage bill at the stationary equilibrium. My focus is on the non-linearity of aggregate investment with respect to the size and sign of this income shock.

Aggregate durable investment in the first period as a function of the aggregate income shock is:

\[
I(\Delta) \equiv \int A(a, d, \zeta) \cdot (d^* (a, d, \zeta) - (1 - \delta) d) \, dA(a - \zeta \Delta Y, d, \zeta) \\
+ \int [1 - A(a, d, \zeta)] \, \delta d \, dA(a - \zeta \Delta Y, d, \zeta),
\]

where \( A(\cdot) \) denotes the durable adjustment hazard and \( d^*(\cdot) \) denotes the adjustment target that solves (2.6). The first integral in (3.1) captures investment by households who pay the fixed adjustment cost, while the second integral captures maintenance by those who do not.

The model described in Section 2 generates a standard inaction region for durable adjustment. That is, the adjustment hazard takes the form of a step function. In my calibrated model, upward adjustment is the most relevant margin at the stationary distribution. Downward changes account for roughly 1% of adjustments at the stationary equilibrium. They could potentially play a more important role in the presence of deleveraging shocks (Guerrieri and Lorenzoni (2017)).
focus on this case for illustration. The adjustment hazard satisfies

$$A(a, d, \zeta) = \begin{cases} 
1 & \text{if } a \geq \bar{a}(d, \zeta) \\
0 & \text{otherwise}
\end{cases}$$

(3.2)
or some threshold $\bar{a}(\cdot)$ that depends on durable holdings and idiosyncratic labor supply.

I am interested in the non-linear properties of the impulse response of aggregate durable investment to an income shock:\footnote{In the context of price setting, Caballero and Engel (2007) show that the aggregate response in the Caplin and Spulber (1987) model coincides with the average response across time at the firm-level, when the economy is stationary. In other words, my exercise captures the “average” degree of non-linearity over time in the household-level response of durable investment.}

$$\hat{I}(\Delta) \equiv \frac{I(\Delta)}{I} - 1,$$

(3.3)

where $I$ denotes aggregate investment at the stationary equilibrium.

Fixing the adjustment hazard, the impulse response given by (3.1) and (3.3) is determined by adjustment along two margins: the extensive margin, i.e. the households’ marginal propensity to adjust; and the intensive margin, i.e. their marginal propensity to invest conditional on adjustment. The following result decomposes the response of durable investment into these two margins. For the sake of exposition, I assume that the adjustment target $d^* (\cdot, d, \zeta)$ is smooth and increasing, and that the distribution of liquid assets conditional on durable holdings and labor supply admits a smooth density $d\Lambda(a|d, \zeta)$.$^{25}$ I define $\Lambda^* \equiv \text{marg}_{d\Lambda} \Lambda$. Finally, I consider discrete but plausibly small shocks.$^{26}$

**Proposition 1.** The response of durable investment to a positive income shock $\Delta > 0$ can be decomposed as follows:

$$\hat{I}(\Delta) \equiv \Sigma_1(\Delta) + \Sigma_2(\Delta) + \zeta(\Delta)$$

(3.4)

\footnote{For expositional purposes, I further assume that the conditional distribution $d\Lambda(\cdot|d, \zeta)$ has full support on $[0, a^*]$ with $a^* > \bar{A} \equiv \sup_{(d, \zeta)} \bar{a}(d, \zeta)$, so that some households adjust for each $(d, \zeta)$.}

\footnote{Specifically, I assume that $\Delta \in [-\sup_{(d, \zeta)} \frac{1}{\zeta} \frac{\partial a(d, \zeta)}{\partial d}, \inf_{(d, \zeta)} \frac{1}{\zeta} \frac{\partial a(d, \zeta)}{\partial d}]$ to avoid cases where either no household, or all households adjust. In the following examples, I assume some monotonicity properties around the durable adjustment threshold. These properties might not hold globally. I assume that $\Delta$ is sufficient small so that they apply.}
with

\[ \Sigma_1 (\Delta) \equiv \frac{1}{I} \int \left\{ \left[ \ddot{d} (d, \zeta) - (1 - \delta + i \delta) d \right] \int_{\ddot{a} + \zeta \Delta Y} \ddot{a} (d) \, d \Lambda (a|d, \zeta) \right. \\
+ \kappa (d, \zeta) \int_{\ddot{a} - \zeta \Delta Y} \ddot{a} (d) (a + \zeta \Delta Y - \ddot{a} (d, \zeta)) \, d \Lambda (a|d, \zeta) \right\} \, d \Lambda^* \]

\[ \Sigma_2 (\Delta) \equiv \frac{1}{I} \int \mathcal{A} (a, d, \zeta) \left[ d^* (a + \zeta \Delta Y, d, \zeta) - d^* (a, d, \zeta) \right] \, d \Lambda (a, d, \zeta) \]

for some \( \kappa (d, \zeta) > 0 \) and some residual \( \zeta (\Delta) \) that satisfies \( \lim_{\Delta \to 0} \frac{\zeta (\Delta)}{\Delta} = 0 \). The analogous decomposition for a negative income shock \( \Delta < 0 \) is provided in Appendix B.

**Proof.** See Appendix B.

The extensive margin consists of two terms. The first term captures *discontinuities* at the microeconomic level. Households who pay the fixed cost adjust their stock by a discrete amount.\(^{27}\) In turn, the mass of households who adjust depends on the shape of the distribution of liquid assets. The second term reflects heterogeneity among households who pay the fixed cost after the income shock: those that were initially further away from their adjustment threshold invest relatively less when adjusting their stock of durables. The intensive margin captures the decreasing propensity to invest conditional on adjustment, due to precautionary savings (Carroll and Kimball (1996); Bertola et al. (2005)). The residual plays a negligible role in my numerical simulations.

The following three examples clarify the contribution of each term in (3.4) to the impulse response of aggregate durable investment, and its non-linearity. For illustration, I focus on positive income shocks. The opposite case is symmetric. I collect all derivations in Appendix B.5.

**Example 1** (Extensive Margin I). I first focus on adjustment at the extensive margin. Specifically, I illustrate the role of the shape of the distribution of liquid assets, i.e. the first term in \( \Sigma_1 (\Delta) \). I suppose that the adjustment target \( d^* (\cdot) \) is constant at some level \( \ddot{d} > 0 \).\(^{28}\) In this case, there is no adjustment at the intensive margin, households adjust to a common level, and the residual is zero:

\[ \Sigma_2 (\Delta) = 0 \quad , \quad \kappa (d, \zeta) = 0 \quad \text{and} \quad \zeta (\Delta) = 0 \]

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\(^{27}\) See Baumol (1952), Tobin (1956) and Scarf (1960) for seminal contributions the subject.

\(^{28}\) That is, the durable adjustment cost satisfies \( \Gamma_l (d', d) = \mathbf{1}_{\{d'=d\}} \gamma + (1 - \mathbf{1}_{\{d'=d\}}) M \) with \( M \to +\infty \).
The impulse response (3.4) satisfies

\[ \hat{I}(\Delta) = \frac{1}{T} \int \left[ \bar{a}(d, \zeta) - (1 - \delta + i\delta) d \right] \int_{\bar{a}(\cdot) - \zeta \Delta}^{\bar{a}(\cdot)} d\Lambda(a|d, \zeta) d\Lambda^* \]  

(3.5)

The response of durable investment depends on the slope of the density of liquid assets around the thresholds. If \(d\Lambda(\cdot|d, \zeta)\) is uniform, then \(\hat{I}(\cdot)\) is linear in the size of the shock: there is neither amplification, nor dampening. Quantitatively, the relevant case is one where this density is \textit{decreasing} around the thresholds (Section 6.1). Then, the impulse response of aggregate durable investment \(\hat{I}(\cdot)\) is convex: more and more households exit their inaction region for expansionary shocks; less and less do so for contractionary shocks. In other words, positive income shocks are \textit{amplified} as their size increases, and negative shocks are \textit{dampened}. That is, the impulse response of aggregate durable investment \(\hat{I}(\Delta)\) is \textit{convex} in the income shock.

Figure 3.1 illustrates this non-linear amplification. The left panel depicts durable investment at the individual level (in blue), and the distribution of liquid assets (in red), together with the durable adjustment threshold \(\bar{a}(d, \zeta)\). I implicitly fix some level of durable holdings and labor supply. A positive income shock \(\Delta > 0\) shifts the density of cash-on-hand to the right, as shown in the right panel. When the density of liquid assets is decreasing at the adjustment threshold, positive income shocks are amplified: more and more households adjust at the extensive margin. The opposite happens for negative shocks.

\[ a \geq 0 \] acts as a reflection barrier. Hence the mass at \(a = 0\).

\[ a \] is set in discrete time. Therefore, there is a positive mass of households at the stationary equilibrium outside of the inaction region. These households adjust in the current period, which depletes their stock of liquid assets. I discuss the role of discrete time at the end of this section.
Example 2 (Extensive Margin II). I focus again on adjustment at the extensive margin, but I now highlight the role of the adjustment target. For illustration, I assume that the adjustment target \(d^* (\cdot)\) is linear, with \(\kappa (d, \zeta)\) denoting the corresponding slope. To abstract from the effect illustrated in the previous example, I suppose that the density \(d\Lambda (a|d, \zeta)\) is uniform over \([0, a^*]\) with \(\bar{a} (\cdot) < a^* < +\infty\). In this case, adjustment at the intensive margin is linear, and the residual is zero: \(\zeta (\Delta) = 0\). The impulse response (3.4) satisfies

\[
\hat{I} (\Delta) = \theta \Delta + \left[\frac{1}{2} \int \frac{1}{a^*} \int \kappa (d, \zeta) (\zeta Y)^2 d\Lambda^* \right] \Delta^2
\]

for some \(\theta > 0\). Again, this form of adjustment at the extensive margin contributes to a convex impulse response of aggregate durable investment \(\hat{I} (\cdot)\).

Figure 3.2 illustrates this case. As in the previous example, the left panel depicts durable investment at the individual level (in blue), and the distribution of liquid assets (in red), and the durable adjustment threshold. A positive income shock \(\Delta > 0\) shifts the density of cash-on-hand to the right, as shown in the right panel. Positive income shocks are amplified as their size increases: households who decide to adjust do so increasing amounts. On the contrary, the effect of negative shocks is dampened as their magnitude increases.

Figure 3.2: Extensive Margin II

Example 3 (Intensive Margin). Finally, I focus on the intensive margin, i.e. the one that would operate in a frictionless model. I assume that the adjustment target \(d^* (\cdot)\) is concave due to precautionary savings and binding borrowing constraints. By definition, the adjustment hazard and the thresholds satisfy \(A (a, d, \zeta) \equiv 1\) and \(\bar{a} (d, \zeta) \equiv 0\) in this case.
The residual is zero: $\zeta (\Delta) = 0$. Then, the impulse response is

$$\hat{I} (\Delta) = \frac{1}{\mathcal{T}} \int \left[ d^* (a + \zeta \Delta Y, d, \zeta) - d^* (a, d, \zeta) \right] d\Lambda (a, d, \zeta)$$

The response of durable investment $\hat{I} (\Delta)$ is concave, i.e. it inherits the shape of the durable adjustment target. The effect of positive income shocks is dampened as their size increases, while the effect of negative shocks is amplified.

### 3.2 Discussion

Before exploring the general equilibrium implications of the aggregate non-linearities documented above, a few remarks are in order. First, I re-state the discussion of Section 3.1 in terms of measurable statistics with testable implications. Second, I discuss the role state-dependent adjustment in my model. Third, I consider a continuous time version of my model to identify which margins operate when aggregate disturbances have continuous paths, i.e. shocks “build up” smoothly. Finally, I briefly elaborate on the determinants of the slope of the distribution of financial assets around the durable adjustment threshold.

**Marginal propensities to spend.** In Examples 1 and 2, durable spending reacts proportionately more (less) to large positive (negative) shocks. This non-linearity can be formulated in terms of observable statistics. Specifically, let

$$\overline{\text{MPC}}^d (\Delta) \equiv \int \frac{\partial}{\partial \Delta} \left[ d^{h', h} (a + \zeta \Delta Y, d, \zeta) - (1 - \delta) d \right] d\Lambda_0 = \frac{d}{d\Delta} \mathcal{T} (\Delta)$$

i.e. the average marginal propensity to spend (MPC) on durable goods.\(^{31}\) In Examples 1 and 2, $\mathcal{T} (\cdot)$ is convex in income changes $\Delta$. As a consequence, the average MPC on durable goods increases with income changes. As discussed in Section 4, this property is consistent with the evidence on the consumption response to tax rebates.

**Time-dependent adjustment.** Durable adjustment is lumpy and state-dependent (i.e. the adjustment hazard is endogeneous) in my model.\(^{32}\) This property plays a central role in the non-linear response of durable spending in my setting. In models with frictionless or time-dependent adjustment (à la Calvo (1983)), or even purely non-durable consumption,

---

\(^{31}\) Note that (3.7) corresponds to the average MPC on durables in each sector, which differs from individual MPCs. Individual MPC are not well-defined at the adjustment threshold (3.2).

\(^{32}\) See Alvarez et al. (2017a) for a formal definition of state- and time-dependent adjustment.
only the intensive margin operates. In this case, the average MPC decreases with income changes due to precautionary savings, as illustrated in Example 3. I elaborate on this point in Appendix B.2. The relative importance of these two effects, and the effective degree of non-linearity are quantitative questions. In Section 6, I use my calibrated model to implement the decomposition from Proposition 1. I find that adjustment at the extensive margin dominates. That is, the average MPC on durables increases with income changes.

Continuous time. My model is set in discrete time, following Berger and Vavra (2015).\(^{33}\) In Appendix B.3, I present a continuous time variant of my model where aggregate disturbances have continuous paths, i.e. shocks “build up” smoothly. This allows me to clarify the nature of the comparative statics in discrete time, and to identify which of the margins identified in Section 3.1 are specific to this comparative statics.

Under the assumptions of Appendix B.3, the flow of durable spending satisfies

\[
i_t = (\bar{s} + \hat{m}\Delta) \Omega_0 - \mu (\bar{s} + \hat{m}\Delta)^2 (\exp(\delta t) - 1) \Omega_1
\]

for some \(\Omega_0, \Omega_1 > 0\). Here, \(\bar{s}, \hat{m} \geq 0\) control the savings rate, \(\Delta\) denotes the (flow) income shock and \(\mu\) denotes the slope of the density of the (conditional) stationary distribution of financial assets. Durable spending is given by two terms. The first one captures the first order impact of income changes: the larger the income shock, the larger the flow of households who exit their inaction region on impact \((t = 0)\). This term is linear in the size of the income shock. The second term captures the endogeneous response of the distribution of financial assets: the larger the income shock, the higher the stock of savings as time passes. When the slope of the density \(\mu\) is negative, the flow of households who exit their inaction region increases over time. Consequently, this second term is non-linear in the size of the income shock, and operates for \(t > 0\). This non-linear mechanism is the counterpart of the one discussed in discrete time (Example 1). The only difference is that it operates on impact \((t = 0)\) in discrete time, but takes time to build up in continuous time. Indeed, an income shock in discrete time is akin to an increase in \(\Delta\) over a discrete interval \([0, t]\) in continuous time.

\(^{33}\) Historically, the literature has favored continuous time models and adopted a more reduced form approach. Durables were the only explicit state variable, and the evolution of their stock absent adjustment was specified exogenously. See Bertola and Caballero (1990) for a review of this approach. A more recent strand of the literature has modelled jointly durables and financial assets accumulation subject to uninsured idiosyncratic shocks. Most of these models are set in discrete time, including Berger and Vavra (2015) in the context of durables, Wong (2019) and Kaplan et al. (2017) in the context of housing, and Bachmann et al. (2013) and Winberry (2019) in the context of capital investment. Exceptions include Achdou et al. (2017) (extensions) and McKay and Wieland (2019).
It should be noted that the continuous and discrete time cases differ in one important respect. The terms $\Omega_0, \Omega_1$ (Appendix B.2) do not depend on the adjustment threshold $d^*(\cdot, d)$ for $a > \bar{d}$ since households never effectively exit their inaction region in continuous time. In Section 6.1, I implement numerically the decomposition from Proposition 1.

Not surprisingly, I find that the terms involving $\kappa(d, \zeta)$ and $\Sigma_2(\Delta)$, and the residual $\zeta(\Delta)$ are small quantitatively. Instead, the extensive margin (setting $\kappa(d, \zeta) = 0$) dominates.

Shape of the density $d\Lambda(\cdot, d)$. The analysis of Section 3.1 identified a key role for the slope of the density of financial assets $\Lambda(\cdot|d, \zeta)$ around that adjustment threshold $\bar{a}(d, \zeta)$. Due to the importance of this object, I briefly elaborate on the determinants on this shape. For tractability, I build on the continuous time version of my model (Appendix B.3).

Since I focus on the properties of the stationary distribution, I temporarily abstract from aggregate income shocks. The process for idiosyncratic states $(a, D)^{34}$ is defined by:

(i) a law of motion conditional on no adjustment,\(^{35,36}\)

\[
\begin{align*}
da(t) &= s(a(t), D(t), \zeta(t)) \, dt \\
dD(t) &= -\delta D(t) \, dt
\end{align*}
\]

for some savings function $s(\cdot)$; (ii) an increasing adjustment threshold $\bar{a}(D)$ of the form (3.2); and (iii) an adjustment target $d^*(\cdot)$ for durables. Financial assets after adjustment satisfy the following budget constraint

\[a(t) = \bar{a}(D) - P^d[d^*(D) - D]\]  

(3.11)

For illustration, I assume that the adjustment target $d^*(\cdot)$ is constant at some level $\bar{d} > 0$, as in Example 1. Idiosyncratic income $\exp(\zeta(t))$ follows some diffusion process.

I am interested in the shape of the stationary distribution implied by the process described above. To gain some insights, I consider an initial distribution of idiosyncratic states $\Lambda$ that is uniform on $S \equiv \{(a, d) | a \leq \bar{a}(d)\}$, and I track its evolution over time while it converges to the stationary distribution. For illustration, I assume that the savings rate satisfies

\[s(a, D, \zeta) = \bar{s} + \bar{m} \exp(\zeta)\]  

(3.12)

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\(^{34}\) In the following, I denote durable holdings by $D$ instead of $d$ to avoid any confusion with differential operators or integrands.

\(^{35}\) I abstract from mass points at the borrowing constraint, by assuming below that savings are strictly positive.

\(^{36}\) For simplicity, I abstract from maintenance $(i = 0)$, and I assume that income shocks are independent over time.
for some $\bar{s}, \bar{m} > 0$, i.e. households have a constant propensity to save out of transitory income shocks. I proceed heuristically in the text to keep the exposition brief. The evolution of this distribution is characterized more formally in Appendix B.3.

The left panel of Figure 3.3 corresponds to the phase diagram associated to (3.9)-(3.10). Consider households who adjust over the interval $[0, t_0]$ with $t_0$ infinitesimally small. By assumption, these households share a common durable adjustment target $\bar{d} > 0$. They deplete their stock of financial assets (black arrow). After adjustment, households accumulate financial assets. I plot two possible paths associated to different realizations of the process $\{\zeta_t\}_{t \geq 0}$ (red curves). Eventually, households hit again the adjustment threshold $\bar{a}(\cdot)$ (blue line).

**Figure 3.3:** Density at the Adjustment Threshold

The right panel of Figure 3.3 depicts the evolution over time of two objects: the distribution of financial assets conditional on $d_t = \exp(-\delta t) \bar{d}$ and having adjusted over $[0, t_0]$; and the adjustment threshold $\bar{a}_t \equiv \bar{a}(\exp(-\delta t) \bar{d})$. In period $t = t_0$, this distribution is uniform over $[0, \bar{a}]$ with $\bar{a} \equiv \sup_{\bar{d}} \bar{a}(\bar{d}) - P^{\bar{d}}(\bar{d} - \bar{d})$, using (3.11) and by uniformity of the stationary distribution $\Lambda$. I consider two periods $t_1$ and $t_2$ with $t_0 < t_1 < t_2$. The mean and the variance of the distribution increase with time, from (3.9)–(3.10) and (3.12). On the contrary, the adjustment threshold decreases over time, since $\bar{a}(\cdot)$ is increasing, by assumption. At the top of the distribution of durables, i.e. for $t$ small, the slope of the density is decreasing at the adjustment threshold, and the distance between the durable adjustment threshold $\bar{a}_t$ and the mode of the conditional distribution is large. As $t$ increases, this distance shrinks.\(^{37,38}\)

\(^{37}\) In theory, this distance could potentially become negative for sufficiently low holdings of durable holdings, i.e. the slope of the density could become positive. In my calibrated model, I find this to be the case only for a negligible share of households.

\(^{38}\) In particular, the speed at which the threshold moves closer to the mode of the distribution is controlled
The predictions of my richer quantitative model are in-line with those described above (see Section 6.1). In particular, I document that: (i) the density of financial assets is decreasing around the adjustment threshold; but (ii) the distance between the adjustment threshold and the mode of the distribution is smaller at the bottom of the distribution of durable holdings.

### 3.3 Redistribution in General Equilibrium

I now explore the role of durable investment for the redistribution of labor income in general equilibrium. Keeping with the rest of the paper, the comparative statics is a one-time, unexpected, and persistent aggregate productivity shock. I assume that this productivity shock is not targeted toward any particular sector, so that redistribution of labor income is entirely induced by endogeneous changes in households’ spending. That is, \(A_t^h / A^h = \hat{A}_t\) for each sector \(h\), with

\[
\log (\hat{A}_t) = \rho^A \log (\hat{A}_{t-1}) + \psi_t
\]

for some persistence \(\rho^A \in (0, 1)\) and some innovation \(\psi_0 \in \mathbb{R}\) in the first period, with \(\psi_t \equiv 0\) for each consecutive period \(t \geq 1\).

Following a productivity shock, the pro-cyclicality of durable spending induces a redistribution of labor income. I impose some restrictions to avoid introducing additional sources of redistribution. In particular, I assume that technologies and price stickiness satisfy a certain degree of symmetry across sectors.

#### Assumption 1 (Technologies). Technologies are isoelastic and the elasticity is symmetric across sectors: \(F^h_t (l) = A^h_t l^\alpha\), for some \(\alpha \in (0, 1)\) and some vector of productivities \(A_t \equiv \{A^h_t\}_h\). Productivities at the stationary equilibrium \(A\) are such that \(W^c = W^d\), i.e. wages are symmetric across sectors.

#### Assumption 2 (Price setting). Price stickiness is symmetric across sectors: \(\lambda^h = \lambda \in [0, 1)\).

**Benchmark.** To highlight the role of durable investment for the redistribution of labor income, I first establish a benchmark where the two goods are non-durable. That is, there is full depreciation: \(\delta = 1\). The relative demand for the two goods is acyclical in this case, by homotheticity of the intratemporal preferences. Under Assumptions 1 and 2, there is

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39 This symmetry is somewhat restrictive empirically. In particular, the labor share is higher in the durable good sector, prices of durable goods tend to be more flexible (Bils and Klenow (2004)). These sources of heterogeneity and their role over the business cycle have been studied separately (Jaimovich et al. (2019); Pastén et al. (2018)). I choose to abstract from the sources of heterogeneity, to isolate the role of redistribution induced by cyclical durable investment.
no redistribution of labor income and thus no role for insurance from fiscal policy. I formalize this point in Proposition 2 (Appendix B.4). This benchmark case clarifies that, in my setting, durability is necessary for income redistribution to take place.\footnote{This result is not specific to productivity shocks: the same obtains for monetary policy shocks, deleveraging shocks or symmetric tax rebates across sectors.} Furthermore, it confirms that Assumptions 1 and 2 are as neutral as possible: the supply side does not induce any redistribution \textit{per se}. I maintain these two assumptions throughout the paper.

\textit{Redistribution.} Having established this benchmark, I now assume partial depreciation: $\delta \in (0, 1)$. Durable spending is more cyclical than non-durable spending, which induces a redistribution of labor income across sectors. This redistribution depends on the elasticity of durable investment with respect to income and price changes. In particular, these elasticities are governed by: the elasticity of substitution between durables and non-durables $\nu$; the adjustment cost $\gamma$; and the maintenance parameter $\iota$.\footnote{For instance, durable demand is inelastic to income changes as $\gamma \to +\infty$. Similarly, the interest rate elasticity of durable demand is zero as $\nu \to 0$.} In virtually every realistic calibration, durable investment is more cyclical than non-durable consumption. I assume that this is the case.

\textbf{Assumption 3 (Monotonicity).} The parameters characterizing the income fluctuations problem (2.4)–(2.6) are such that relative demand for durable investment $(d^h_t (\cdot) - (1 - \delta) d) / c^h_t (\cdot)$ and non-durable consumption $c^h_t (\cdot)$ increase with liquid assets.

To understand the pro-cyclicality of relative spending on durable goods, consider a representative-agent version of the model presented in Section 2.\footnote{Specifically, there is no idiosyncratic risk ($\Sigma \to_d 0$) and durable adjustment is frictionless ($\gamma = 0$).} In this case,

\[ \frac{d_t - (1 - \delta) d_{t-1}}{c_t} = \frac{1 - \theta}{\theta} \left( \frac{P^c_t}{P^d_t - (1 - \delta) P^d_{t+1} \theta + r_t} \right)^\nu - (1 - \delta) \frac{d_{t-1}}{c_t} \]

at optimum. A productivity shock of the form (3.13) affects the relative demand for the durable investment good through two channels. First, it raises incomes in general equilibrium. Non-durable consumption and the stock of durable consumption increase proportionately. Mechanically, the flow of durable investment is much more volatile. In the first place, this increase in incomes is induced by a fall in the real interest rate, when monetary policy is sufficiently responsive ($\phi > 1$). This decrease in the households’ user cost of durables tilts spending in favor of durable investment\footnote{This second channel is particularly powerful in typical models of investment. Existing estimate suggest that this effect is overstated (Hall (1977); Shapiro (1986); McKay and Wieland (2019)).}. This second effect is modulated...
by the elasticity of substitution between durables and non-durables ($\nu$).

### 3.4 Aggregate Amplification

In Sections 3.1 and 3.3, I clarified the dual role of lumpy durable investment in my model: its pro-cyclicality induces income redistribution; and micro discontinuities imply macro non-linearities. I now explore the interaction between these two properties, and the non-linear amplification it generates.

To assess the role of redistribution, I contrast the aggregate response of my economy under the two regimes (2.10)–(2.11) for fiscal policy. In the first regime, income redistribution takes place between sectors and fiscal policy provides no cross-sectoral insurance. In the second regime, fiscal policy undoes this redistribution and provides full (aggregate) insurance. For tractability, I focus on the general equilibrium response in period $t = 0$, keeping the sequence of labor incomes, aggregate profits, taxes, interest rates and sector-specific inflation $X_t \equiv (Y_t, \pi_t, \tau_t, T_t, r_t, \Pi_t)$ fixed for each period $t > 1$. I also abstract from changes in the relative price of durable goods\footnote{The relative price of durable goods is mostly acyclical in the data (Pistaferri (2016); Cantelmo and Melina (2018); McKay and Wieland (2019)).} by assuming that prices are rigid: $\lambda \equiv 1$. For illustration, I build on Example 2, where only the extensive margin operates, i.e. I implicitly assume that the role of precautionary savings is negligible. Again, I include the derivations in Appendix B.5.

**Example 4 (Amplification).** Consider an expansionary productivity shock: $\psi_0 < 0$.\footnote{Note that a positive productivity shock ($\psi_0 > 0$) is contractionary with rigid prices, since households claim constant profits from firms. The evidence on the effect of technology improvements on labor demand is mixed. References include Basu et al. (2006) and Alexopoulos (2011) (among others).} Labor demand increases in both sectors, and so does spending on both goods in partial and general equilibrium, by Assumption 3. However, durable spending is more cyclical so that $Y_d^d / Y_d^c > Y_c^c / Y_c$, i.e. labor income is redistributed in favor of the durable sector.

Now, suppose that fiscal policy undoes this redistribution by providing aggregate insurance, using lump sum taxes (2.11),

$$T_d^d = - \frac{\mu^c}{\mu^d} T_c^c > 0$$

These transfers reduce the dispersion in the distribution of labor incomes. Under the assumptions of Example 1, aggregate durable spending by households employed in each sector

$$Y_{d,h}^d \equiv \int \left[ d_{h}^d(a,d,\zeta) - (1 - \delta) \right] d\Lambda_h^d$$
is convex in (sectoral) incomes \(^46,47\) for each sector \(h\). By Jensen’s inequality, a lower dispersion due to fiscal policy depresses aggregate durable investment \(Y_0^d \equiv \sum_h \mu^h Y^{d,h}_0\). Put it differently, endogenous income redistribution between sectors amplifies the effect of an expansionary productivity shocks, compared to a benchmark with full aggregate insurance. Symmetrically, income redistribution dampens the effect of contractionary shocks.

![Figure 3.4: Impulse Response of Aggregate Durable Spending](image)

Lumpy and state-dependent adjustment is key to this non-linear amplification. Figure 3.4 contrasts the predictions of models with state- and time-dependent adjustment. As discussed in Section 3.2, (sectoral) durable spending is concave in (sectoral) incomes with time-dependent adjustment. Therefore, endogeneous income redistribution depresses aggregate demand. As a result, cyclical income inequality dampens the response of durable spending to expansionary shocks, and amplifies the response to contractionary shocks. A realistic micro-foundation of the durable adjustment hazard is thus crucial to understand the effect of income redistribution on aggregate durable spending.

### 3.5 Sufficient Statistics

The effect of income redistribution in my model can be understood through a sufficient statistics approach. Focusing on the first period \(t = 0\), let

\[
\hat{Y}_0^d (Y_0^h - \mu^h T_0^h) \equiv \int \left[ d_0^{h'} (a, d, \zeta) - (1 - \delta) d \right] d\Lambda_0
\]

\(^{46}\) Incomes are implicit to the time index.

\(^{47}\) Note that endogeneous changes in the nominal interest rate affect the durable adjustment threshold (3.2). I suppose that the density of the distribution of liquid financial assets is decreasing monotonically over the relevant range.
denote aggregate durable investment in each sector as a function of the aggregate wage bills $Y_0$ and transfers $T_0$.\footnote{Again, the dependance on incomes on the right-hand-side is implicit to the time index.} Similarly, define define $\text{MPC}_{0}^{d,h} (Y_0^h - \mu^h T_0^h)$ by analogy with (3.7), i.e. the average marginal propensity to spend (MPC) on durables in each sector. My focus is on non-linear effects, i.e. endogeneous changes in average MPCs.

Let lump sum taxes $T$ satisfy:

$$\frac{Y_0^d - \mu^d T_0^d}{Y_0^c - \mu^c T_0^c} = \frac{Y_0^d}{Y_0^c} + \omega \left( \frac{Y_0^d}{Y_0^c} - \frac{Y_0^d}{Y_0^c} \right)$$

where $Y$ denotes the wage bills at the stationary equilibrium. This formulation nests the two cases of interest: no insurance ($\omega = 0$); and full aggregate insurance ($\omega = 1$). Finally, fix some productivity shock $\psi_0$, and let $Y_0^h (\psi_0)$ denote the corresponding equilibrium wage bills under the regime with no insurance.

The shock $\psi_0$ induces a redistribution of labor income. I am interested in the effect of insurance from fiscal policy that undoes this redistribution. Let

$$Y_0^d (\omega; \psi_0) \equiv \sum_h \mu^h Y_0^{d,h} (Y_0^h (\psi_0) - \mu^h T_0^h (\omega; \psi_0))$$

(3.15)

denote aggregate demand for durable investment, where taxes $T_0 (\cdot)$ solve (3.14).

For the sake of exposition, I focus on the partial equilibrium effect of insurance from fiscal policy.\footnote{This explains why wage bills are not indexed by the degree of insurance $\omega$ in (3.15).} Starting from the no insurance case ($\omega = 0$), suppose that fiscal policy increases the degree of cross-sectoral insurance. Using (3.15),

$$\frac{d}{d\omega} Y_0^d (\omega; \psi_0) \bigg|_{\omega=0} = \sum_h \mu^h \frac{d}{d\omega} T_0^h (\omega; \psi_0) \bigg|_{\omega=0} \times \text{MPC}_{0}^{d,h} \left( Y_0^h (\cdot) - \mu^h T_0^h (\cdot) \right) \bigg|_{\omega=0}$$

(3.16)

After an expansionary shock ($\psi_0 < 0$), incomes expand proportionately more in the durable sector. Fiscal policy partly offsets this redistribution by taxing the households employed in the durable sector: $T_0^d (\cdot) = -\mu^c / \mu^d T_0^d (\cdot) > 0$, from (3.14). The effect of these transfers depends on the heterogeneity in average MPCs across sectors.

My model is set up so that there is no ex-ante heterogeneity in average MPCs on durables between sectors: $\text{MPC}_{0}^{d,c} = \text{MPC}_{0}^{d,d}$ at the stationary equilibrium.\footnote{The distribution of financial and durable holdings is symmetric across sectors at the stationary equilibrium, by Assumption 1.} In other

\footnote{I focus on one-time, unanticipated shocks. If households employed in the durable sector anticipated aggregate risk, they could in theory accumulate higher precautionary savings. There is only limited empirical support for this prediction (Skinner (1988)), which can partly be attributed to heterogeneity in}
words, the “earnings heterogeneity channel” studied by Auclert (2019) and Patterson (2019) has no aggregate effects in my model for first order deviations from the stationary equilibrium. As a consequence, the response of durable spending does not depend on the degree of insurance from fiscal policy for infinitesimal aggregate shocks:

\[
\left. \frac{d}{d\psi_0} Y_0^d (1; \psi_0) \right|_{\psi_0 = 0} = \left. \frac{d}{d\psi_0} Y_0^d (0; \psi_0) \right|_{\psi_0 = 0}
\]

However, MPCs are endogeneous in my model: as income redistribution increases, it gradually drives a wedge between \(\text{MPC}_{0}^{d,h}\) in the two sectors. Away from the stationary equilibrium, there is \textit{ex post} heterogeneity in MPCs on durables between sectors. In Examples 1 and 2, aggregate durable investment is a strictly convex function of (sectoral) incomes. That is, \(\text{MPC}_{0}^{d,h} (\cdot)\) is strictly increasing in income changes. After an expansionary productivity shock \((\psi_0 < 0)\), durable workers experience a larger increase in labor income, so

\[
\left. \text{MPC}_{0}^{d,d} (\cdot) \right|_{\omega = 0} > \left. \text{MPC}_{0}^{d,c} (\cdot) \right|_{\omega = 0} \Rightarrow \left. \frac{d}{d\omega} Y_0^d (\omega; \psi_0) \right|_{\omega = 0} < 0
\]

using (3.16). That is, insurance from fiscal policy mitigates the expansion. Put it differently, endogeneous redistribution of labor income \textit{amplifies} the response of aggregate durable spending after an expansionary shock, compared to a full aggregate insurance benchmark. Symmetrically, redistribution \textit{dampens} this response after a contractionary shock. Taken together, these observations explain why the response without insurance defines the \textit{upper envelope} of the response with insurance in the left panel of Figure 3.4.

4 Supporting Evidence

In this section, I briefly review supporting evidence on the interaction between the cyclicality of durable spending and income redistribution.

4.1 Redistribution

Purchases of durable goods and residential investment are strongly pro-cyclical in the data (Kydland and Prescott (1982); Baxter (1996)). Hall (2005) and Bils et al. (2013) document a substantial pass-through to durable employment during booms and busts. Using risk-aversion across sectors (Schulhofer-Wohl (2011)). Patterson (2019) finds that heterogeneity in MPCs across sectors plays a relatively minor role in the general equilibrium amplification of aggregate shocks.
administrative U.S. data, Guvenen et al. (2017) confirm that expansions and recessions have distributional consequences between durable and non-durable sectors.\footnote{They estimate a contemporaneous elasticity of individual income with respect to GDP of roughly 2 for durable industries.}

I illustrate the importance of this sectoral income redistribution in two contexts: the Great Recession; and in response to well-identified, exogeneous shocks. To account for cross-sectoral labor mobility and movements in and out of the labor force, I use longitudinal data from the Panel Study of Income Dynamics (PSID).\footnote{Other datasets commonly used for the study of income dynamics – such as the Current Population Survey (CPS), the Survey of Income and Program Participation (SIPP), or the National Longitudinal Survey of Youth (NLSY) – either have a shorter panel dimension, or their sample is not as representative of the U.S. population as the PSID’s.}

The PSID is a representative panel survey of U.S. households conducted annually during 1968-1997, and bi-annually since then. The sample consists of male household’s heads aged 24-65. To eliminate outliers, I exclude households with labor income lower than 5% of the annual average, and higher than the 95th percentile. I use (real) gross labor incomes as an empirical counterpart to labor earnings in my model.\footnote{I also report the responses for family income in Appendix C.3 to account for unemployment insurance and intra-household risk-sharing. The conclusions are very similar in this case.}

I describe the data, the sample selection, and the specifications in Appendix C.2.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure41.png}
\caption{Great Recession}
\end{figure}

The left panel of Figure 4.1 plots the time series of real spending during the Great Recession for the two categories of interest: durables and residential investment; and non-durables and services. Durable spending fell by roughly 25% over this period, compared to 5% for non-durable spending. The right panel plots the time series of mean labor in-
come for households employed in the corresponding sectors in 2008.\textsuperscript{55} The recession led to a substantial redistribution of labor income between sectors. The pass-through was incomplete, however. Labor incomes in durable sectors decreased by 13\% over two years, compared to a 4\% decline in non-durable sectors.

For robustness, I estimate the sector-specific response of labor incomes to well identified, exogeneous shocks. The leading example in my paper is an unanticipated productivity shocks. Fluctuations in measured productivity are potentially endogeneous, however.\textsuperscript{56} To address this concern, I focus instead on narratively-identified, exogeneous policy shocks. I use the series of exogeneous tax changes constructed by Romer and Romer (2010),\textsuperscript{57} which are sufficiently persistent to be aggregated at the low-frequency of the PSID data.

\textbf{Figure 4.2: Response to Exogeneous Tax Increase}

The left panel of Figure 4.2 plots the response of spending on durables and non-durables following a one-standard deviation contractionary Romer and Romer (2010) tax shock. Not suprisingly, durable spending is more elastic to these policy changes than non-durable spending.

To assess the importance of income redistribution in response to these policy changes,

\textsuperscript{55}Specifically, I allocate households to either the durable or non-durable sectors based on their industry of employment in 2008 (i.e. 2009 PSID wave). Fixing this cross-section, I compute mean labor income for each year.

\textsuperscript{56}See Chari et al. (2007) and Buera and Moll (2015), among others. Productivity shocks identified via Structural Vector Auto-Regression (SVAR) might not be exogeneous either (Ramey (2011)).

\textsuperscript{57}I prefer tax changes to government spending shocks (Ramey (2011)). Those are typically targeted toward a particular sector, which mechanically induces redistribution.
I specify the following moment condition:

\[ X_{t+s}^{h,j} - X_{t-1}^{h,j} = \alpha_s^{h,j} + \psi_{s}^{h} s_t^* + Z_{t-1}^{h} \theta_s^h + \eta_{t,s}^{h,j} \text{ for each } s \in \{0,1,\ldots,S\} \] (4.1)

together with the standard orthogonality condition. Here, \( h \in \{c,d\} \) denotes the sector of employment in the previous period \( t-1 \), \( j \) indexed individuals, and \( s \) indexes the horizon of the impulse response. The variable \( X_t \) denotes labor income (in log), \( s_t^* \) corresponds to the external instrument of Romer and Romer (2010), and \( Z_t^h \) denotes the set of control variables. The coefficients of interest are \( \left\{ \hat{\psi}_s^h \right\}_{h,s}, \) i.e. the cumulative impulse responses for each sector at various horizons. I specify (4.1) at the bi-annual frequency of the PSID. The set of control variables includes a sector-specific cubic time trends and 12 lags of the fiscal policy shock.

The right panel of Figure 4.2 plots these impulse responses, together with 90% confidence bands. The pattern is similar as in the Great Recession: durable investment is strongly pro-cyclical, which leads to a substantial redistribution of labor income.

### 4.2 Increasing MPCs

In my model, the average MPC out of an income shock depends on the magnitude of this shock (Section 3.1). This property ensures that cyclical income inequality has aggregate effects. The monotonicity in the average MPC depends on the relative strength of the margins identified in Section 3.1. The empirical evidence on the consumption response to tax shocks provides some guidance.

<table>
<thead>
<tr>
<th>Table 4.1: Average Marginal Propensity to Spend on Durables (1 quarter)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average amount</td>
</tr>
<tr>
<td>Average MPC ( (D) )</td>
</tr>
</tbody>
</table>

In the context of the 2001 stimulus payment, Johnson et al. (2006) estimate negligible spending multipliers on durable goods. On the contrary, Souleles (1999) finds that the response of spending to springtime tax refunds is almost entirely driven by durable expenditure (and purchases of vehicles in particular), while Parker et al. (2013) document

\[58\] Confidence intervals (90%) are bootstrapped (200 replications) to account for heteroskedasticity and serial correlation.
sizeable multipliers following the 2008 stimulus payment.\textsuperscript{59} Parker et al. (2013) attribute this difference to the size of the transfers.\textsuperscript{60} Table 4.1 lists the average transfer size and the average MPC on durable goods for these episodes. The average MPC is insignificant for a $500 average transfer, but accounts for roughly half of the spending response for the range $1,000 to $2,500. These findings is in-line with the survey evidence of Fuster et al. (2018) and Christelis et al. (2019), who document that the average MPC on durables increases with the size of (hypothetical) tax rebates. In Section 6.1, I find that my calibrated model produces an increasing average MPC to tax rebates, consistently with this empirical evidence.

5 Calibration

The remainder of the paper quantifies the mechanisms documented in Section 3. I first parametrize the model using a mix of external and internal calibration. Following Berger and Vavra (2015), I adopt a broad definition of durable goods that includes residential investment and consumer durables.

I calibrate five parameters internally: the discount factor \( \beta \), the preference parameter for non-durable goods \( \vartheta \), the durable adjustment costs \( \gamma \), the exogeneous supply of liquidity \( B \), and the relative productivity in the durable sector \( A^d \). I calibrate the remaining parameters externally, using standard values in the literature. I first review the external calibration, before discussing the targeted moments and the fitted parameters. Table 5.2 describes the parametrization. Data sources are listed in Appendix C.1.

5.1 External Calibration

I set the inverse elasticity of intertemporal substitution to \( \sigma = 4 \). This value, while large compared to typical calibrations, is commonly used in models with durable goods (Guerrieri and Lorenzoni (2017); McKay and Wieland (2019)), which predict a high elasticity of durable investment to interest rate changes. I choose a unitary elasticity of substitution between durables and non-durables \( \nu \rightarrow 1 \), following Berger and Vavra (2015).\textsuperscript{61} I choose

\textsuperscript{59} Parker et al. (2013) find that their estimates of MPCs on durable goods are less precise than for non-durable goods. The corresponding figure in Table 4.1 corresponds to the average between the lower and upper bounds that they obtained (p. 2531).
\textsuperscript{60} In particular (p. 2532): “For instance, some prior research finds that larger payments can skew the composition of spending towards durables, which is consistent with our findings given that the 2008 stimulus payments were on average about twice the size of the 2001 rebates.”
\textsuperscript{61} The literature has typically used a unitary elasticity, based on the estimates of Ogaki and Reinhart (1998) and Piazzesi and Schneider (2007). However, values below unity are sometimes used to dampen the
a maintenance parameter of $\iota = 0.5$, which lies between the estimates of Berger and Vavra (2015) ($\iota = 0.8$) and McKay and Wieland (2019) ($\iota = 0.35$).

The stock of durable goods depreciates at roughly 2% ($\delta = 0.018$). The income process (in log) follows an AR(1) process with persistence $\hat{\rho} = 0.975$ and standard deviation of innovations $\hat{\sigma} = 0.1$ to match the evidence of Floden and Lindé (2001). I set the mass of households in the durable sector to $\mu^d = 0.187$, based on data from the Current Employment Statistics (see Appendix C.1). Based on Assumptions 1 and 2, I assume a certain degree of symmetry between sectors. Technologies are isoelastic and the elasticity is symmetric across sectors. I normalize productivity to 1 in the non-durable sector. Labor receives roughly 2/3 of revenues at the stationary equilibrium ($\alpha = 0.3$). Similarly, price stickiness is symmetric across sectors. With imperfectly sticky prices ($\lambda < 1$), the cyclicality of durable spending induces substantial changes in the relative price of durable goods. This prediction is not verified in the data, however. Furthermore, models of lumpy investment typically predict an excessively high elasticity to changes in user cost. For this reason, I assume that prices are fixed $\lambda^h = 1$ for my general equilibrium exercise (Section 7). Note that the elasticity of substitution across varieties ($\varepsilon = 10$) and the coefficient in the monetary policy rule ($\varphi = 1.25$) are irrelevant in this case.

5.2 Internal Calibration

The remaining parameters ($\beta$, $\vartheta$, $\gamma$, $B$, $A^d$), together with the interest rate ($r$) and the price of the durable good ($P^d$) at the stationary equilibrium, are the implicit solution to seven restrictions: two equilibrium conditions, and five empirical moments. The moments I target, their values and the sources I use are listed in Table 5.1. Appendix A.4 describes the calibration strategy in more details.

I choose the discount factor $\beta$ to target a real interest rate at the stationary equilibrium of 2.5%, which corresponds to the average value since 1970. I set the preference parameter for non-durable goods $\vartheta$ to obtain a ratio of investment to consumption of roughly 18%. I adjust the durable adjustment parameter $\gamma$ to target an annual frequency of adjustment of 12%, as in Berger and Vavra (2015). I obtain $\gamma = 0.025$, which corresponds to an adjustment cost of 3.3% when expressed in terms of the numéraire. The exogeneous supply interest rate elasticity of durable demand (Barsky et al. (2016); McKay and Wieland (2019)).

See Pistaferri (2016), Cantelmo and Melina (2018) and McKay and Wieland (2019), among others.


In Table 5.1, “relative” refers to the ratio of the value for durable to that for non-durable.

This figure is slightly lower than the 5% typically used in the literature (Díaz and Luengo-Prado (2010); Berger and Vavra (2015)). However, the relative price of durables is larger than 1 in my model, which
of liquidity \((B)\) targets a ratio of liquidity to annual GDP of 1.4, following McKay et al. (2016). Finally, I choose the relative productivity in the durable goods sector \(A^d\) to target symmetric wages across sectors at the stationary equilibrium.

**Table 5.1: Targeted Moments**

<table>
<thead>
<tr>
<th>Target</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real interest rate ((A))</td>
<td>0.025</td>
<td>FRED and NIPA</td>
</tr>
<tr>
<td>Ratio of investment to consumption</td>
<td>0.18</td>
<td>NIPA</td>
</tr>
<tr>
<td>Frequency of durable adjustment ((A))</td>
<td>0.10</td>
<td>Berger and Vavra (2015)</td>
</tr>
<tr>
<td>Liquidity supply to GDP ((A))</td>
<td>1.4</td>
<td>McKay et al. (2016)</td>
</tr>
<tr>
<td>Relative wages</td>
<td>1</td>
<td>Normalization</td>
</tr>
</tbody>
</table>

6  **Lumpy Investment and Non-Linearity**

In this section, I assess the degree of aggregate non-linearities produced by my structural model. As discussed in Section 3.4, this non-linearity shapes the non-neutrality of income redistribution. In Section 6.1, I implement the decomposition from Proposition 1 in my calibrated model. In Section 6.2, I quantify this non-linearity by simulating the partial equilibrium response of durable spending to persistent income shocks. Finally, I relate my findings to the literature on state-contingent responses with lumpy adjustment in Section 6.3.

6.1 **Decomposition**

I start by examining the two sources of aggregate non-linearities identified in Section 3.1: the extensive and intensive margins of durable adjustment. Specifically, I implement the decomposition from Proposition 1 for a one-time, transitory shock \(\Delta\).

Figure 6.1 plots each of the three objects in (3.4) and (B.1). The total response (i.e. the sum of these three components) is non-linear: expansionary shocks are amplified as the size of the shock increases, while contractionary shocks are dampened. That is, the average MPC on durables increases with income changes. Adjustment at the extensive margin dominates quantitatively, and entirely accounts for this non-linear effect. In

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66 Liquidity includes: deposits, government-issued securities, corporate bonds and equitities and mutual fund shares.

67 This transitory shock is annualized with a persistence that delivers a half-life of 6 quarters.
Table 5.2: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Calibration</th>
<th>Source / Target</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Preferences</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.985</td>
<td>Internal calibration</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Elasticity of substitution</td>
<td>1</td>
<td>See text</td>
</tr>
<tr>
<td>$\vartheta$</td>
<td>Non-durable parameter</td>
<td>0.731</td>
<td>Internal calibration</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>EIS (inverse)</td>
<td>4</td>
<td>See text</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Elasticity of substitution (inverse)</td>
<td>10</td>
<td>Kaplan et al. (2018)</td>
</tr>
<tr>
<td><strong>Durable goods</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
<td>0.018</td>
<td>Berger and Vavra (2015)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Adjustment cost</td>
<td>0.025</td>
<td>Internal calibration</td>
</tr>
<tr>
<td>$\iota$</td>
<td>Maintenance parameter</td>
<td>0.5</td>
<td>See text</td>
</tr>
<tr>
<td><strong>Income process</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{\rho}$</td>
<td>Persistence</td>
<td>0.967</td>
<td>Floden and Lindé (2001)</td>
</tr>
<tr>
<td>$\tilde{\sigma}$</td>
<td>Standard deviation</td>
<td>0.13</td>
<td>Floden and Lindé (2001)</td>
</tr>
<tr>
<td><strong>Liquidity</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B$</td>
<td>Ratio of bond supply to GDP</td>
<td>1.490</td>
<td>Internal calibration</td>
</tr>
<tr>
<td><strong>Labor supply</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu^d$</td>
<td>Mass of households (durable)</td>
<td>0.187</td>
<td>CES</td>
</tr>
<tr>
<td><strong>Production</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A^d$</td>
<td>Relative productivity (durable)</td>
<td>0.490</td>
<td>Internal calibration</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Decreasing returns</td>
<td>0.3</td>
<td>Berger and Vavra (2015)</td>
</tr>
<tr>
<td><strong>Prices and policy</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Calvo parameter</td>
<td>1</td>
<td>See text</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Taylor rule coefficient (inflation)</td>
<td>1.25</td>
<td>Kaplan et al. (2018)</td>
</tr>
</tbody>
</table>
other words, lumpy and state-contingent adjustment is responsible for this aggregate nonlinearity, while precautionary savings plays a minor role. Figure A.1 in Appendix A.6 further decomposes the extensive margin of adjustment into each of the two terms in $\Sigma_1 (\Delta)$. Consistently with the discussion on continuous time (Section 3.2), I find that the term capturing changes in the size of durable purchases conditional on adjustment is quantitatively small. However, it accounts for a non-negligible share of the non-linearity in the response of durable spending for expansionary shocks.

**Figure 6.1: Decomposition from Proposition 1**

![Graph showing the decomposition of adjustment into extensive and intensive margins.](image)

As illustrated by Examples 1 and 2, the extensive margin of adjustment is controlled by two objects: the slope of the density of the distribution of liquid assets around the adjustment thresholds; and the slope of the durable adjustment target. In turn, the intensive margin is shaped by the concavity of the durable investment target. Figure A.2 (Appendix A.6) plots these objects for various percentiles of the distribution of durable goods (fixing idiosyncratic labor supply at its median level). The density of liquid assets is typically decreasing around the adjustment threshold and the durable adjustment target is increasing but concave. As anticipated in Section 3.2, the adjustment threshold gets closer to the mode of the distribution of liquid assets as the stock of durables decreases.

### 6.2 Persistent Income Shocks

I now consider the effect of an aggregate, persistent income shock. I abstract from redistribution for now. The wage bills satisfy: $Y^h_t / Y^h = \hat{Y}_t$, with $\hat{Y}_t = \psi_0 \rho t$ for some persistence $\rho \in (0, 1)$. I am interested in the non-linear properties of the aggregate response of durable
spending as I vary $\psi_0$. I set $\rho$ to obtain a half-life of 6 quarters and replicate the behavior of real filtered GDP since 1960.

**Impulse responses.** Figures 6.2 and 6.3 plot the cumulative response of spending over 4 quarters in terms of $\psi_0$, for expansionary and contractionary shocks.\textsuperscript{68,69} The left panels correspond to durable investment, and the right panels to non-durable consumption. The dashed lines extrapolate the response associated to $\psi_0 = -0.01$ and $\psi_0 = 0.01$, respectively.

I find that the response of durable spending to positive income shocks is amplified as their size increases. On the contrary, this response is dampened for negative income shocks. That is, durable spending is convex in income changes. This effect is economically significant. For instance, consider shocks of the magnitude experienced by durable workers during the Great Recession (Figure 4.1). Fixing this magnitude, the average MPC on durables is roughly 25% higher for expansionary shocks, compared to contractionary shocks. The response of non-durable consumption contrasts sharply with that of durable investment. Expansionary shocks are dampened due to precautionary savings, and contractionary shocks are amplified. In other words, lumpy and state-dependent durable adjustment not only undoes the effect of precautionary savings, but it also predicts that non-linearities actually operate in the opposite direction.

**Timing and redistribution.** The analysis of Section 3 assumed that income shocks were fully transitory. Households are non-Ricardian in my model due to borrowing constraints, so the timing of income shocks is actually relevant and affects the profile of the impulse response. In particular, the degree of non-linearity needs not be uniform over time. Figures 6.2 and 6.3 effectively obscure these dynamic considerations by aggregating over a sufficiently long time horizon. Figure A.4 in Appendix A.6 plots the dynamic impulse responses of durable spending to (positive) income shocks of various sizes. These impulse responses are normalized by the size of these shocks. A clear pattern emerges: as the size of income shocks gets larger, the normalized impulse response increases on impact, and becomes more persistent. However, it typically decreases for a few quarters in the medium term, as households accumulate savings to finance larger purchases in the following periods. The cumulative response of durable spending increases with the size of

\textsuperscript{68} I choose a horizon of 4 quarters for two reasons. First, the spending response to persistent income changes is typically hump-shaped in my calibration as discussed at the end of this section. A sufficiently long time horizon captures this delayed response. Second, averaging out over a longer horizons smoothes the impulse responses. The cumulative responses over two or more years are very similar.

\textsuperscript{69} I annualize these cumulative responses by diving them by the number of quarters over which I cumulate.
Figure 6.2: Impulse Response (4 quarters) – Contraction

Figure 6.3: Impulse Response (4 quarters) – Expansion
income shocks.\(^{70}\) Put it differently, the impulse response of durable spending (in level) is convex on impact and in the longer run, but concave for a few quarters after the shock.

**Figure 6.4:** Persistent Income Shocks – Timing and Redistribution

For reference, the left panel of Figure 6.4 schematizes this pattern and depicts the impulse response of durable investment \(\hat{I}(\Delta)\) normalized by the size of the income change \(\Delta\) for two possible values of this shock. The timing of the impulse response depends on the magnitude of the shock. Accordingly, income redistribution should have an uneven effect across time. For illustration, suppose that sector-specific wage bills are given by:

\[
\frac{Y^d_t}{Y^d} = \left(\frac{Y_t}{Y}\right)^\psi \quad \text{and} \quad \sum_h \mu^h Y^h_t = Y_t
\]  

(6.1)

for some elasticity \(\psi \geq 1,^{71}\) where \(Y^{(h)}\) denote wage bills at the stationary equilibrium. That is, durable workers are more exposed to aggregate income shocks. As depicted in the right panel, income redistribution should amplify the response of durable spending on impact and increase its persistence, since the response of durable spending is convex in the size of income shocks over the corresponding range. On the contrary, income redistribution should dampen the response of durable spending for a few quarters after the occurrence of the shock. The general equilibrium responses I obtain in Section 7 confirm these predictions.

\(^{70}\) Note that the cumulative response of spending would necessarily be linear (for sufficiently long horizons) in the size of income shocks if households consumed a single good. Indeed, the increase in income is eventually spent. This need not be the case with multiple goods, as in my setting.

\(^{71}\) Guvenen et al. (2017) estimate an elasticity of \(\psi \simeq 2\) for durable industries using administrative U.S. data.
6.3 State-Contingency

I conclude this section by drawing a connection between the non-linearity that I focus on, and another property of models of lumpy adjustment: state-contingency.

Using a version of the income fluctuations problem (2.4)–(2.6) with aggregate risk, Berger and Vavra (2015) find that the response of durable investment to (income) changes is pro-cyclical, i.e. the effect of shocks is amplified during expansions compared to contractions. For illustration, I compute the response of aggregate investment to a one-time, transitory income shock following a period of expansion or contraction. The initial boom or bust results from a persistent, unanticipated income change. Specifically, aggregate incomes satisfy \( \frac{Y_t}{Y} = \hat{Y}_t \), with \( \hat{Y}_t = \psi_0 \rho^t \). The persistence \( \rho \) calibrated as in Section 6.2, and I vary the initial income shock \( \psi_0 \). A transitory, unanticipated income shock takes place after 6 quarters. It takes the form of an exogeneous $1,000 transfer, i.e. the average 2008 stimulus payment. Table 6.1 reports the average (cumulative) MPC on durable goods over a year, for various magnitudes of the initial expansion or recession (\( \psi_0 \)). The cumulative response of aggregate durable spending is larger after a boom than a bust. The effect is relatively small in this example however, since the income shock is fully transitory, i.e. the present discounted value of income changes is small.

Table 6.1: Average Marginal Propensity to Spend on Durables (4 quarters)

<table>
<thead>
<tr>
<th>Initial state (( \psi_0 ))</th>
<th>-4%</th>
<th>-2%</th>
<th>0%</th>
<th>2%</th>
<th>4%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average MPC ($1,000)</td>
<td>0.2192</td>
<td>0.2225</td>
<td>0.2265</td>
<td>0.2292</td>
<td>0.2319</td>
</tr>
</tbody>
</table>

I now argue that this state-contingent amplification can be understood as a manifestation of the form of non-linearity that I am interested in. For expositional purposes, I suppose there is a single source of aggregate disturbance \( \{ \xi_t \} \), which follows a Markov process of order 1. I am interested in the impulse response of aggregate durable investment, conditional on the state of the economy. Specifically, this response is parametrized by two aggregate states: the exogeneous disturbance (\( \xi_t \)) and the distribution of idiosyncratic states (\( \Lambda_t \)). The impulse response to an innovation \( z \) in \( \xi_t \) in period \( t \) is

\[
R(z; \xi_t, \Lambda_t) = I(\xi_t + z, \Lambda_t) - I(\xi_t, \Lambda_t),
\]

(6.2)

where \( I(\cdot) \) denotes aggregate durable investment (3.1). In the context of Section 6.2, \( \{ \xi_t \} \)

\footnote{It should be noted that average MPCs on durables is substantially smaller in my model (Table 6.1), than in the data (Table 4.1).}
corresponds to a persistent sequence of transfers. In this case, the impulse response $R(\cdot)$ corresponds to the average MPC on durables following a transitory income shock.

There are three possible sources of state-dependency: the aggregate disturbance $\xi_t$ itself; the (marginal) distribution of liquid assets $\text{marg}_L \Lambda_t$; or the (marginal) distribution of durable holdings $\text{marg}_d \Lambda_t$.\footnote{Obviously, the joint distribution $\Lambda_t$ is the relevant state variable. I focus on the marginal distributions for the sake of the argument.} The distribution of durable holdings is unlikely to be responsible for the pro-cyclicality of the response of durable investment: durable holdings are already high at the peak of a boom, which should actually mitigate the response of durable expenditure. I thus focus on the two other states in the following.

By definition, dependence on the aggregate disturbance $\xi_t$ corresponds to the non-linearity documented in Figures 6.2 and 6.3. Holding the distribution of idiosyncratic states at its stationary level $\Lambda$ and fixing some innovation $z > 0$,

$$R(z; \xi_t', \Lambda) > R(z; \xi_t, \Lambda) \quad \forall \xi_t' > \xi_t \iff \mathcal{I}(\cdot, \Lambda) \text{ is strictly convex using (6.2).}$$

Similarly, dependence on the (marginal) distribution of financial assets is intrinsically related to the form of non-linearity I am interested in. I illustrate this point using the continuous time version of my model presented in Appendix B.3. Specifically, I am interested in the response to an (unanticipated) income shock $\Delta$ over an interval $[t, t']$ with $t' > t > 0$ after a sequence of (unanticipated) shocks $\Delta^*$ over the interval $[0, t)$. These initial shocks shift the distribution of financial assets in period $t$. Under the assumptions of Appendix B.3, the flow of durable investment in period $t$ satisfies

$$i_t(\Delta) = (\bar{s} + \hat{m} \Delta) \Omega_0 - \mu (\bar{s} + \hat{m} \Delta) (\bar{s} + \hat{m} \Delta^*) (\exp(\delta t) - 1) \Omega_1$$  \hspace{1cm} (6.3)$$

for some $\Omega_0, \Omega_1 > 0$. Here, $\mu$ denotes the slope of the density of financial assets at the adjustment threshold (stationary equilibrium), and $\bar{s}, \hat{m} \geq 0$ control the savings rate. In particular, durable spending (6.3) is linear in $\Delta$, but contingent on the size of the initial income change and the duration over which it occurs ($\Delta^*, t$). Fixing some horizon $t$, the impulse response increases after a period of expansion ($\Delta^* > 0$), but decreases after a period of recession ($\Delta^* < 0$) whenever the slope $\mu^*$ is negative (Section 6.1).

Note that (6.3) coincides with the dynamic impulse response to an income shock (3.8) when parametrized with $\Delta^* = \Delta$. In particular, the degrees of non-linearity and state-contingency are determined by the same structural objects. The very reason why large shocks generate non-linear responses is that they induce changes the distribution of finan-


cial assets (Section 3.2), i.e. they endogenously affect the aggregate state. In other words, the dynamic impulse response to an income shock is non-linear in the size of the shock \( \Delta \) if and only if the impulse response to a shock at the same horizon exhibits state-contingency.

**Redistribution.** My focus is on the role of income redistribution for the response of durable spending to aggregate shocks. I showed that this effect depends on the non-linearity of the impulse response of durable spending to income shocks. And I argued that this non-linearity is intrinsically related to the state-contingency of the response of durable spending. At this point, it is worth noting that income redistribution could directly affect the degree of state-contingency.

To illustrate this point, I suppose that sector-specific wage bills satisfy (6.1). That is, durable workers are more exposed to the initial expansion or recession that precedes the income shock. Figure A.5 in Appendix A.6 plots the state-contingent impulse response of durable spending to a one-time, unanticipated $1,000 income, 6-quarter into a persistent contraction or expansion of 4% of GDP. I repeat this exercise with income redistribution \((\psi > 1)\) and without \((\psi = 1)\). In the former case, I calibrate the elasticity of labor income in the durable sector using the estimates of Guvenen et al. (2017) based on U.S. administrative data.\(^\text{74}\) I find that the initial redistribution reduces the degree of state-contingency. While state-contingency reflects the sign of the slope of the (conditional) density of financial assets around the adjustment threshold, the effect of income redistribution on state-contingency reflects the convexity of this slope. A casual inspection of this density in my calibrated model (Figure A.2 in Appendix A.6) suggests that this slope is concave, which is consistent with a dampening of the degree of state-contingency.

7 General Equilibrium

In the previous sections, I showed that lumpy investment at the microeconomic level produces non-linear responses to aggregate income shocks. I now explore the implications of these non-linearities in presence of cyclical income redistribution between sectors.

I simulate the response of my general equilibrium model (Section 2) to aggregate disturbances. Following Berger and Vavra (2015), I focus on productivity shocks. Income redistribution between sectors takes place endogenously in my model, as durable investment responds more strongly than non-durable consumption.

The left panel of Figure 7.1 plots the general equilibrium responses of durable invest-

\(^\text{74}\) I set \(\psi = 2\), which corresponds to the average income elasticity in construction and durable manufacturing, weighted by relative employment in these industries (Section C.1).
ment and non-durable consumption to an persistent, expansionary productivity shock of 5% with a half-life of 6 quarters (as in Section 6.2). As expected, the impulse response of durable spending is substantially larger than for non-durable consumption, and the response builds up gradually. The right panel plots the response of durable investment under the two regimes (2.10) and (2.11) for fiscal policy, i.e. with and without endogeneous labor income redistribution. As anticipated in Section 6.2, income redistribution has an uneven effect over time. I find that redistribution amplifies the cumulative response of durable spending by roughly 9% over the first year and a half after the shock, dampens this response by 21% over the next year, and then amplifies it again by 45% over the following two years.\footnote{The cumulative response over the full sample is larger with income redistribution, which is consistent with the analysis of Section 3.} That is, cyclical income inequality boosts the short-term response of durable spending during expansions and increases the persistence of this response.

\textbf{Figure 7.1: General Equilibrium}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{general-equilibrium.png}
\caption{General Equilibrium Consumption and Investment\hspace{1cm} Durable Spending}
\end{figure}

\begin{itemize}
\item \textbf{Durables} \hspace{1cm} \textbf{Non-Durables} \hspace{1cm} \textbf{No Redistribution} \hspace{1cm} \textbf{Redistribution}
\end{itemize}

Figure A.6 in Appendix A.6 plots the response of non-durable consumption under the same two regimes for fiscal policy. There is no discernible amplification in this case, despite the complementarity between durables and non-durables in households’ preferences. This finding confirms that lumpy and state-contingent adjustment is key to understand the role of cyclical income redistribution for the response of durable spending.
8 Conclusion

In this paper, I study the implications of cyclical income inequality for the dynamics of aggregate durable investment. I explore this question using a multi-sector heterogeneous agent (HANK) model with lumpy durable investment. Durable demand plays a dual role in my setting. First, its pro-cyclicality induces a redistribution of labor income between durable and non-durable sectors. Second, lumpy durable adjustment at the micro level produces non-linearities at the macro level: the average MPC on durable goods increases with income changes. As a result, income redistribution has aggregate effects.

I clarify the source of these macro non-linearities analytically, and I confirm in numerical exercises that the response of durable investment is non-linear in income changes. Finally, I simulate the response of durable investment to productivity shocks in general equilibrium. I find that cyclical income inequality affects both the short-term response of durable spending and the persistence of this response.
References


A Quantitative Appendix

In this Appendix, I present the full model and describe the approach used to simulate and calibrate the model. Section A.1 describes the full model. Section A.2 provides a formal definition of an equilibrium. Section A.3 provides the algorithm used to solve for the stationary equilibrium and the transition dynamics. I discuss the calibration strategy in Section A.4. Finally, Section A.5 provides details about the numerical implementation.

A.1 Environment

For concision, Section 2 provided a partial description of the environment. For reference, I now present the full model.

Timing. Periods are indexed by $t \in \{0, 1, \ldots \}$. There is no aggregate risk. I focus on one-time, unanticipated but persistent shocks. Each period effectively consists of two sub-periods, indexed by $t.0$ (−) and $t.1$ (+). Households make their adjustment decisions at $t.0$, and their consumption, saving and investment decisions at $t.1$. In period $t.0$, households are indexed by their financial asset holdings ($a$), their holdings of durable goods ($d$), their idiosyncratic labor supply shock ($\zeta$) and the sector they are employed in ($h$). In period $t.1$, households are in addition indexed by their adjustment choice ($A$) during the previous sub-period. The conditional distributions of idiosyncratic states within each sector, at the beginning of each sub-period, are denoted by $\Lambda_{t-1}$ and $\Lambda_t^+$. All agents have perfect foresight, and all (aggregate) information is revealed at the beginning of the first sub-period at $t = 0$.

Households. The households’ value function in period $t.0$ satisfies:

$$V_{t.0}^h(a, d, \zeta) = \max_{A \in \{0, 1\}} \left\{ V_{t.0}^h(a, d, \zeta; A) \right\} \quad (A.1)$$

The adjustment choice satisfies:

$$A_{t.0}^h(a, d, \zeta) \equiv \begin{cases} 1 & \text{if } V_{t.0}^h(a, d, \zeta; 1) > V_{t.0}^h(a, d, \zeta; 0) \\ 0 & \text{otherwise} \end{cases} \quad (A.2)$$

The continuation value functions in period $t.1$ associated to adjustment and no adjustment

---

This additional state variable is for notational convenience. Financial assets are a sufficient statistics for this adjustment.
write:

\[
V^h_i (a, d, \zeta; 0) = \max_{\{c, a\}} \frac{u(c, d^*)}{1-\sigma} \beta E_t \left[ V^h_{i+1} (a', d^*, \zeta') \right] \quad (\text{A.3})
\]

s.t. \( P^c_i c + P^d_i d \delta + a' \leq (1-\tau_i) e^h_i (\zeta) + (1+ r_{t-1}) a \)

\[ a' \geq 0 \]

\[
V^h_i (a, d, \zeta; 1) = \max_{\{c, a\}} \frac{u(c, d^*)}{1-\sigma} \beta E_t \left[ V^h_{i+1} (a', d', \zeta') \right] \quad (\text{A.4})
\]

s.t. \( P^c_i c + P^d_i (d' - (1-\delta) d) + \gamma P^d_i (1-\delta) d + a' \leq (1-\tau_i) e^h_i (\zeta) + (1+ r_{t-1}) a \)

\[ a' \geq 0 \]

with \( d^* \equiv [1-(1-i) \delta] d \), where \( e^h_i (\zeta) \equiv \frac{1}{\mu^h} \zeta (\Omega^h_i - T^h_i) + \pi_t \) denotes incomes and \( \mu \) denotes the mass of households in each sector. The distributions in period \( t.0 \) and \( t.1 \) evolve as follows:

\[
\Lambda^h_{i+1} (a, d, \zeta, A) = \Lambda^h_{i-1} (a, d, \zeta) \times \begin{cases} A^h_i (a, d, \zeta) & \text{if } A = 1 \\ 1 - A^h_i (a, d, \zeta) & \text{otherwise} \end{cases} \quad (\text{A.5})
\]

and

\[
\Lambda^h_{i-1} (a', d', \zeta') = \sum_A \Omega^h_i (a', d'; A) \sum A^h_{i+1} (a, d, \zeta, A) \Sigma \left( \log (\zeta') | \zeta \right) \quad (\text{A.6})
\]

with \( \Omega^h_i (a^*, d^*; A) \equiv \{ (a, d, \zeta) | a^h_{i+1} (\cdot; A) = a^*, d^h_{i+1} (\cdot; A) = d^* \} \), where \( \Sigma \) denotes the transition kernel characterizing the income process and \( a^h_{i+1} \) and \( d^h_{i+1} \) denote the solution to \((\text{A.1})-(\text{A.4})\), with \( d^h_{i+1} \equiv d^* \) when no adjustment. I define \( e^h_i \) similarly.

**Firms.** Prices are sticky along the transition path:

\[
\left( P^h_i \right)^{1-\epsilon} = \lambda^h \left( P^h_{i-1} \right)^{1-\epsilon} + \left( 1 - \lambda^h \right) \left( P^h_{i} \right)^{1-\epsilon} \quad (\text{A.7})
\]

with initial condition \( P^h_{-1} \equiv P^h \), i.e. prices at the stationary equilibrium. Reset prices \( P^*_i \)
satisfy:
\[
P_{t}^{*,h} = \left[ \frac{1}{1 - \alpha} \frac{\varepsilon}{\varepsilon - 1} \frac{G_{t}^{h}}{H_{t}^{h}} \right]^{\frac{1}{1 - \alpha}} (A.8)
\]
in each sector \(h\), with
\[
G_{t}^{h} = W^{h} \left[ \left( \frac{1}{\tilde{P}_{t}^{h}} \right)^{-\varepsilon} \frac{Y_{t}^{h}}{A^{h}} \right]^{\frac{1}{1 - \alpha}} + \lambda_{t}^{h} \frac{1}{1 + r_{t}} G_{t+1}^{h} (A.9)
\]
\[
H_{t}^{h} = \left( \frac{1}{\tilde{P}_{t}^{h}} \right)^{-\varepsilon} Y_{t}^{h} + \lambda_{t}^{h} \frac{1}{1 + r_{t}} H_{t+1}^{h} (A.10)
\]
for each \(t \in \{0, 1, \ldots, T - 1\}\), and
\[
G_{T}^{h} = \frac{1 + r}{1 + r - \lambda_{T}} W^{h} \left[ \left( \frac{1}{\tilde{P}_{T}^{h}} \right)^{-\varepsilon} \frac{Y_{T}^{h}}{A^{h}} \right]^{\frac{1}{1 - \alpha}}
\]
\[
H_{T}^{h} = \frac{1 + r}{1 + r - \lambda_{T}} \left( \frac{1}{\tilde{P}_{T}^{h}} \right)^{-\varepsilon} Y_{T}^{h}
\]
for each sector \(h\), where \(W\) denotes nominal wages at the stationary equilibrium, and \((Y, P, r)\) denotes aggregate demands for each good, prices and the nominal interest rate at the steady state.

**Policy.** Depending on the regime of interest (insurance, or not), lump sum taxes satisfy:
\[
T_{t} = 0 \quad \text{or} \quad \frac{Y_{t}^{d} - \mu^{d} T_{t}^{d}}{Y_{t}^{c} - \mu^{c} T_{t}^{c}} = \frac{Y_{t}^{d}}{Y_{t}^{c}} (A.11)
\]
with \(\sum_{h} \mu^{h} T_{t}^{h} \equiv 0\), where \(Y\) denotes wages bills at the stationary equilibrium The government’s flow budget constraint is
\[
\tau_{t} \sum_{h} \mu^{h} \int e_{t}^{h} (\zeta) d\Lambda_{t}^{h} = -r_{t-1} B (A.12)
\]
Monetary policy implements a Taylor rule:
\[
i_{t} = \max \{ r + \phi^{T} (\Pi_{t} - 1), 0 \} (A.13)
\]
Here, $\Pi_t - 1 \equiv \Delta \log (\hat{P}_t)$ denotes the inflation rate, where

$$\hat{P}_t \equiv \left[ \varrho \left( P^c_t \right)^{1-\nu} + (1 - \varrho) \left( P^d_t \right)^{1-\nu} \right]^{\frac{1}{1-\nu}} \quad (A.14)$$

denotes the CES ideal price index.

**Market clearing.** Markets for goods clear:

$$Y^c_t = \sum_h \mu^h \int \left[ c^h_i (a, d, \zeta) + \Gamma_t \left( d^{i,h'}_t (a, d, \zeta), d \right) \right] d\Lambda^{h_i}_{t} \quad (A.15)$$

$$Y^d_t = \sum_h \mu^h \int \left[ d^{h',a} (a, d, \zeta) - (1 - \delta) d \right] d\Lambda^{h_i}_{t} \quad (A.16)$$

The TFP component associated to price dispersion is

$$\left( \Omega^h_t \right)^{\frac{1}{1-\alpha}} \equiv \lambda^h \left( \Omega^h_{t-1} \right)^{\frac{1}{1-\alpha}} + \left( 1 - \lambda^h \right) \left( P^{h,\star}_t \right)^{-\frac{\epsilon}{1-\alpha}} \quad (A.17)$$

with $\Omega^h_{t-1} \equiv (P^h)^{-\epsilon}$. Demands for labor satisfy

$$\hat{\mu}^h_t \equiv \hat{\Omega}^h_t \left[ \frac{1}{A^h} \left( \frac{1}{P^h_t} \right)^{-\epsilon} Y^h_t \right]^{\frac{1}{1-\alpha}} \quad (A.18)$$

with $\hat{\Omega}^h_t \equiv (\Omega^h_t)^{\frac{1}{1-\alpha}}$. Wage bills and aggregate profits write

$$Y^h_t = W^h \hat{\mu}^h_t \quad (A.19)$$

$$\hat{\pi}_t = \sum_h \left( P^h_t Y^h_t - Y^h_t \right), \quad (A.20)$$

Households claim constant profits from firms: $\pi_t = \hat{\pi}$, where $\hat{\pi}$ denotes aggregate profits at the stationary equilibrium. In turn,

$$\mu = Z_t \circ \hat{\mu}_t \quad (A.21)$$

with $Z^h \equiv 1$ at the stationary equilibrium, and $0 \leq Z^h_t < +\infty$ along the transition path, for each sector $h$ and period $t$.  

54
A.2 Definition of Equilibrium

Definition 1. An equilibrium in my economy consists of a sequence policy functions for consumption and assets \(\{c_t(\cdot), a_t'(\cdot), d_t'(\cdot)\}_{t \geq 0}\), a sequence of distributions of idiosyncratic states \(\{\Lambda_{t-1}^-, \Lambda_t^+\}_t\), sequences for the nominal interest rates \(\{r_t\}_t\), sectoral price indices \(\{P_t\}_t\), reset prices \(\{P^*_t\}_t\), wages, wage bills and profits \(\{W_t, Y_t, \pi_t\}_t\), the TFP component associated to price dispersion \(\{\Omega_{t-1}\}_t\) and linear taxes \(\{\tau_t\}_t\) such that: (a) the policy functions solve (A.1)–(A.4), given interest rates, prices and wage bills; (b) reset prices satisfy (A.8)–(A.10), given outputs and wages; (c) the distributions of idiosyncratic states evolve according to (A.5)–(A.6); (d) sectoral prices and the TFP component associated to price dispersion satisfy (A.7) and (A.17); (e) monetary policy implements the Taylor rule (A.13)–(A.14), given prices; (f) fiscal policy sets linear taxes to satisfy its flow budget constraint (A.12); and (g) the market clearing conditions for good (A.15)–(A.16) and labor (A.21) hold.

A.3 Numerical Solution

A.3.1 Stationary Equilibrium

I solve numerically for the stationary equilibrium by iterating on the nominal interest rate \(r\), the (relative) price of the durable good \(P^d\), the vector of wage bills, profits\(^77\) and linear taxes \(X \equiv (Y, \pi, \tau)\). The price of the non-durable good is used as a numéraire \(P^c \equiv 1\) at the stationary equilibrium, and the TFP component associated to price dispersion satisfies \(\Omega^h \equiv (P^h)^{-\varepsilon}\). I proceed in three steps.

Step 0 (Initial conditions). I choose a set of initial conditions for \((r, P^d)\) and \(X\). The procedure described in Section A.4 provides a good guess.

Step 1 (Households). Let

\[
\hat{V}^h(a, d, \zeta) \equiv \mathbb{E}\left[ V^h(a, d, \zeta') \middle| \zeta \right] \tag{A.22}
\]

for each sector \(h\). Starting from a guess \(\hat{V}_{-1}\) for this value function, I first solve the problems (A.3)–(A.4) to obtain policy functions \((c_0, a'_0, d'_0)\),\(^78\) and the value functions associated to each adjustment choice. I obtain the adjustment flows \((A_0)\) from (A.2), and the expected

---

77 Profits are non-zero at the stationary equilibrium, due to decreasing returns.
78 See Section A.5 for details.
value functions $\hat{V}_0^h$ from (A.1) and (A.22). If

$$\|\hat{V}_0 - \hat{V}_{-1}\|_{+\infty} < \epsilon^V$$

for some tolerance $\epsilon^V > 0$, I set $\hat{V} \equiv \hat{V}_0$ and $x \equiv x_0$, for each policy function $x \in \{c, a', d', A\}$. Otherwise, I repeat this procedure after updating $V_{-1}$ with $V_0$.

Starting with a guess for the initial distribution $\Lambda_{-1}^-$, I iterate on (A.5)–(A.6) using the policy functions obtained previously to obtain distributions $\Lambda_0^-$ and $\Lambda_0^+$. If

$$\|\Lambda_0^- - \Lambda_{-1}^-\|_{+\infty} < \epsilon^\Lambda$$

for some tolerance $\epsilon^\Lambda > 0$, I set $\Lambda^- \equiv \Lambda_0^-$ and $\Lambda^+ \equiv \Lambda_0^+$. Otherwise, I repeat this procedure after updating $\Lambda_{-1}^-$ with $\Lambda_0^-$. Finally, aggregate demands $Y$ for each good are obtained from (A.15)–(A.16) using the policy functions $(c, d')$ and the distribution $\Lambda^+$. Similarly, aggregate savings is defined as

$$S \equiv \sum_h \mu^h \int a'(h, a, d, \zeta) d\Lambda_{h,+}$$

(A.23)

**Step 2** (Firms and market clearing). From (A.7)–(A.10), (A.18) and (A.21), I obtain the wages that insure labor market clearing in each sector:

$$\bar{\Pi}^h = A^h \left(1 - \alpha^h\right) \frac{\varepsilon - 1}{\varepsilon} \left(\hat{\mu}^h\right)^{-\alpha}$$

Similarly, demand for labor in each sector satisfies

$$Y^h = A^h \left(\hat{\mu}^h\right)^{1-\alpha}$$

(A.24)

since $\Omega^h \equiv (p^h)^{-\varepsilon}$ at the stationary equilibrium.

I then compute the associated wage bills and aggregate profits:

$$\bar{\bar{Y}}^h = \bar{\Pi}^h \hat{\mu}^h$$

$$\bar{\tau} = \sum_h \left(p^h Y^h - \bar{\bar{Y}}^h\right)$$

Finally, I obtain the linear tax $\tau$ from (A.12).

---

79 To each set of initial conditions $X$ corresponds a set of values for the same variables that satisfy the (stationary) equilibrium restrictions. I denote these variables with a “bar”.

56
Step 3 (Convergence). Finally, I check whether the vector of prices \((r, P^d)\) and wage bills, profits and linear taxes \(X\) insure market clearing. If,
\[
|S| < \epsilon^S \quad \text{and} \quad |\mu^d - \bar{\mu}^d| < \epsilon^d
\]
for some tolerances \(\epsilon^S, \epsilon^d > 0\), with aggregate savings given by (A.23) and labor demand given by (A.24), then the policy functions \((c, a', d', A)\), the distributions \((\Lambda^-, \Lambda^+)\), the vector of prices \((r, P^d)\), and wage bills, profits and linear taxes \(X\) form a (stationary) equilibrium.\(^{80}\) Otherwise, I update the vector of prices \((r, P^d)\) to reduce excess demand and I repeat Step 1 onward. In this case, I also update the vector of wage bills, profits and linear taxes \(X\) using a weighted average of the initial guess \(X\), and the values computed above \(\bar{X}\).

A.3.2 Transition Dynamics

The comparative statics of interest is a one-time, unanticipated innovation in aggregate productivity. The shock is symmetric across sectors. Specifically,
\[
\log \left( A^h_t \right) = \rho^A \log \left( A^h_{t-1} \right) + \psi_t
\]
for each sector \(h\), with \(\psi_0 \in \mathbb{R}\) and \(\psi_t \equiv 0\) for each period \(t \geq 1\). I solve numerically for the transition dynamics by iterating on the sequence for \(X_t \equiv (r_t, P_t, \mathcal{Y}_t, \tau_t, \tau_t, T_t)\), where \(T_t\) corresponds to the lump sum taxes that satisfy (A.11).\(^{81}\) I proceed in four steps.

Step 0 (Initial and terminal conditions). I set \(X_t = X\), for each period \(t\), as an initial guess, where \(X\) denotes the vector at the stationary equilibrium. Similarly, fix \(\Lambda^-_{-1} \equiv \Lambda^-\) and \(\mathcal{V}_{T+1} \equiv \mathcal{V}\) as initial and terminal conditions for the distribution of idiosyncratic states and the value function, where \(T\) denotes the horizon over which I compute the impulse responses.

Step 1 (Households). First, I iterate backward on the functional equation (A.1)–(A.4), for each \(t \in \{0, \ldots, T\}\), to obtain a sequence of policy functions \(\{c_t, a'_t, d'_t, A_t\}_t\). Then, I iterate forward on the transition kernel (A.5)–(A.6) using these policy functions, for \(t \in \{0, \ldots, T\}\), to obtain a sequence of distributions \(\{\Lambda^-_{t-1}, \Lambda^+_{t}\}_t\). Finally, I compute the

\(^{80}\) Note that the market clearing conditions for labor in the non-durable sector is implied by (A.21) and (A.23) when \(S = 0\).

\(^{81}\) In the case of ful (aggregate) insurance, I omit the “star” subscripts.
sequence of aggregate demands for each goods \( \{ Y^c_td_t \} \) using (A.15)–(A.16).

**Step 2** (Firms and market clearing). I iterate backward on (A.8)–(A.10), for \( t \in \{ 0, \ldots , T \} \), to obtain the sequence of reset prices \( \{ P_t \} \). Then, I iterate forward on (A.7) and (A.17) to obtain a new sequence of sectoral price indices \( \{ \bar{P}_t \} \). Labor demands are computed from (A.18), using the sequence \( \{ Y^c_t , Y^d_t \} \) from Step 1 and the initial sequence of prices \( \{ P_t \} \). Finally, I compute new sequences for the wage bills \( \{ \bar{Y}_t \} \) and aggregated profits \( \{ \bar{\pi}_t \} \) from (A.19)–(A.20).

**Step 3** (Policy). I obtain new sequences for lump sum taxes \( \{ \bar{T}_t \} \) and linear taxes \( \{ \bar{\tau}_t \} \) from (A.11)–(A.12). Similarly, I compute the new sequence of nominal interest rate \( \{ \bar{r}_t \} \) set by the monetary policy rule (A.13), given the price index (A.14).

**Step 4** (Convergence). Finally, I check whether the sequence of endogenous variables \( X_t \equiv (r_t, P_t, \mathbf{Y}_t, \pi_t, \tau_t, T_t) \) insures market clearing. If,

\[
\| \bar{x} - x \|_{\infty} < \epsilon^x
\]

for some tolerance \( \epsilon^x > 0 \), for each sequence \( x \in \{ r, P, \mathbf{Y}, \pi, \tau, T \} \), then the policy functions \( \{ c_t, a'_t, d'_t, A_t \} \), the distributions \( \{ \Lambda_{t}^{-}, \Lambda_{t}^{+} \} \), and the prices and incomes \( \{ X_t \} \) form an equilibrium. Otherwise, I update the sequence \( \{ X_t \} \) using a weighted average of the initial sequences \( \{ X_t \} \) and the new sequences \( \{ \bar{X}_t \} \) and I repeat Step 1 onward.

**A.4 Calibration Strategy**

The internal calibration consists of solving for the vectors of parameters \( \{ \beta, \theta, \gamma, B, A^d \} \) — i.e. the discount factor, the preference parameter on non-durables, the (non-convex) durable adjustment cost, the supply of bonds, and the relative productivity in the durable sector — and prices \( \{ r, P^d \} \) — i.e. the real interest rate and the relative price of the durable good at the stationary equilibrium — that solve the following restrictions: the two market clearing conditions for labor (A.21); and the five empirical moments listed in Table 5.1. I proceed in two steps. First, I use the empirical targets and the restrictions in the model to pin down the parameters \( \{ A^d, B \} \) and the prices \( \{ r, P^d \} \). Then, I iterate over the remaining

---

82 Again, to each set of initial conditions \( X \) corresponds a set of values for the same variables that satisfy the equilibrium restrictions. I denote these variables with a "bar".

83 To ensure convergence of the algorithm, the corresponding weights decrease exponentially in the period \( t \).
parameters to insure market clearing, and match the rest of the targets.

Step 1. The first empirical moment in Table 5.1 directly pins down \( r \). From (A.18) and (A.21),

\[
\mu_t^h \equiv \left( \frac{1}{A_h^t} \gamma_t^h \right)^{1-\alpha}
\]

(A.25)

for each sector \( h \), at the stationary equilibrium. The second moment in Table 5.1 and (A.25) thus pin down \( A^d \). Similarly, from (A.8),

\[
\frac{\bar{w}_t^h}{\bar{p}_t^h} = A^h \left( 1 - \alpha^h \right) \frac{\epsilon - 1}{\epsilon} \left( \mu^h \right)^{-\alpha}
\]

(A.26)

for each sector \( h \). The last moment in Table 5.1 and (A.26) pin down \( \bar{p}_d \), given the numéraire \( \bar{p}_c \equiv 1 \) and the vector of productivities \( A \) previously solved for. Aggregate output \( \bar{Y} \) is defined as

\[
\bar{Y} \equiv \sum_h \bar{p}_t^h \bar{Y}_t^h \quad \text{with} \quad \bar{Y}_t^h = A^h \left( \mu^h \right)^{1-\alpha}
\]

(A.27)

for each sector \( h \), by definition of the production technologies. The fourth moment in Table 5.1 and (A.27) pin down \( B \), given the vectors of prices \( \bar{P} \) and productivities \( A \) obtained above.

Step 2. Finally, I iterate on the remaining parameters \( (\beta, \vartheta, \gamma) \) to satisfy the rest of the restrictions. Specifically, I adjust \( \beta \) until aggregate savings (A.23) are zero. Similarly, I use \( \vartheta \) to clear the market for durable goods (A.21), with labor demand given by (A.16) and (A.18). Finally, I vary \( \gamma \) until the aggregate frequency of adjustment

\[
\mathcal{A}_T \equiv \sum_h \mu^h \int A^h(a,d,\zeta) \ d\bar{\Lambda}^h
\]

matches the third moment in Table 5.1.

A.5 Numerical Implementation

I describe below the computation of the impulse responses in Section 6, and the numerical implementation of the algorithms described in Appendices A.3.1 and A.3.2.

Grids and transition. The value functions are approximated on discrete grids for financial assets and durable goods, consisting of 100 points each. The distribution of financial
assets and durable goods is discretized on grids consisting of 300 points. I interpolate
policy functions linearly between grid points, and use a generalization of Young (2010)’s
non-stochastic stimulation method with multiple assets when using the policy functions
to iterate on the distribution of assets. For accuracy, the grid for financial assets is more
dense in the neighborhood of the borrowing constraint. Finally, I discretize the income
process on a 7-point grid using the method of Rouwenhorst (1995). I set \( T = 25 \) (quarters)
when solving for the transition dynamics in partial equilibrium (Section 6), and \( T = 100 \)
in general equilibrium (Section 7).

**Numerical solution.** The households’ problems (A.3)–(A.4) are non-convex due to fixed
adjustment costs. To solve for the optimal policy, I first perform a grid search on a fine
grid to locate a candidate for the global maximum. I then use the gradient-free, simplex
algorithm (Nelder-Meade) implemented in Matlab’s fminsearch to solve for a local optimum,
using this candidate as an initial condition. I use a tolerance of \( 10^{-5} \) to solve for the
value function at the stationary equilibrium, and one of \( 10^{-17} \) for the stationary distribu-
tion (Step 1 in Appendix A.3.1).

**Impulse responses.** The computation of the responses to income shocks in Sections 6.2
and 6.3 follows Step 1 in Appendix A.3.2. In particular, let \( I_t (\{ Y_t \}_t) \) denote aggregate
investment (A.16) in terms of the sequence of wage bills. The other parameters entering
the households’ income fluctuations problem and the distribution of idiosyncratic states
are implicitly fixed at their stationary equilibrium level. The impulse responses associated
to two sequences \( \{ Y_t, Y'_t \}_t \) defined as follows:

\[
\mathcal{R}_t (\cdot) \equiv I_t (\{ Y'_s \}_s) - I_t (\{ Y_s \}_s)
\]

In Section 6.2, I am interested in on the non-linear response to income shocks. This corre-
sponds to \( Y_s = \bar{Y} \) for each period \( s \), and some mean-reverting sequence \( \{ Y'_s \}_s \). In Section
6.3, I focus instead on the state-contingency. In this case, I fix some sequence \( \{ Y_s \}_s \)
and consider a perturbation of the form \( Y'_s \equiv Y_s + \eta_s \), for some mean-reverting sequence
\( \{ \eta_s \}_s \).

**Distance to threshold.** In Appendix A.6, I report the distribution of distances to the ad-
justment threshold \( a - \bar{a} (\cdot) \) at the stationary equilibrium. As mentioned above, the grids I
use for the financial assets is more dense in the neighborhood of the borrowing constraint.
Using this grid to compute the distribution of distances to threshold would mechanically
over-weight households at the bottom of the wealth distribution. To address this issue, I re-interpolate the policy functions and the distribution of idiosyncratic states on a linearly-spaced grids consisting of 500 points.

**Decomposition.** I implement the decomposition of Proposition 1 in Section 6.1. I compute the extensive margin $\Sigma_1 (\Delta)$ in three steps. First, I approximate:

$$\hat{d} (a, d, \zeta) \equiv \bar{d} (d, \zeta) - (1 - \delta + i\delta) d + \kappa (d, \zeta) (a - \bar{a}(\cdot))$$

Specifically, I filter the adjustment target using a rolling average, interpolate the resulting vector on a fine grid around the adjustment threshold, estimate the slope by ordinary least squares and extrapolate on the grid for financial assets. Second, I recover the density $d\Lambda (a, d, \zeta)$ using the (discretized) distribution of idiosyncratic states and the non-regular grid for financial assets. Finally, I integrate numerically over the range $[\bar{a}(\cdot) - \zeta \Delta Y, \bar{a}(\cdot)]$ (for positive income shocks). I repeat this computation using actual investment instead of $\hat{d} (a, d, \zeta)$, and define the residual $\zeta (\Delta)$ as the difference between the resulting response and $\Sigma_1 (\Delta)$. The computation of the intensive margin $\Sigma_2 (\Delta)$ is straightforward.

### A.6 Complementary Numerical Results

I provide below some numerical results that were omitted from the main text.

*Extensive margin.* In Section 6.1, I implement the decomposition from Proposition 1. In Figure A.1, I further decompose the extensive margin of adjustment into the two terms in $\Sigma_1 (\Delta)$ in (3.4) and (B.1). I denote by $\Sigma_1^* (\cdot)$ the first term in $\Sigma_1 (\cdot)$, i.e. with $\kappa (d, \zeta) = 0$.

**Figure A.1:** Decomposition from Proposition 1

![Graph showing Decomposition from Proposition 1](image)

<table>
<thead>
<tr>
<th>Extensive 1: $\Sigma_1^* (\cdot) / \mathcal{I}$</th>
<th>Extensive 2: $(\Sigma_1 (\cdot) - \Sigma_1^* (\cdot)) / \mathcal{I}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Graph" /></td>
<td><img src="image" alt="Graph" /></td>
</tr>
</tbody>
</table>
Sources of non-linearities. Figure A.2 plots the density of the distribution of liquid assets (normalized by quarterly GDP) and the durable investment target for various percentiles of the distribution of durable goods (fixing idiosyncratic labor supply at its median level). The dashed lines denote the durable adjustment thresholds.

Figure A.2: Sources of Non-Linearity (median labor supply)

Distance to threshold. Figure A.3 plots the distribution of distances to the adjustment threshold $a - \bar{a}(\cdot)$ (normalized by quarterly GDP) at the stationary equilibrium. A large fraction of households lies relatively closely to their adjustment threshold.

Figure A.3: Distribution of Distance to Adjustment threshold

Timing. Figure A.4 plots the impulse response of durable spending to (positive) income changes of various sizes, normalized by the size of shocks. The response of durable spending (in level) is convex on impact in the size of income shocks, i.e. the normalized response
is higher for larger income shocks. This impulse response is then concave for a few quarters \((t_0 \text{ to } t_1)\), and convex in the medium term \((after \ t_1)\).

**Figure A.4:** Impulse Responses to Earnings Shocks

State-contingency. Figure A.5 plots the state-contingent impulse response of durable spending (with and without income redistribution) to a one-time, unanticipated $1,000 income, 6-quarter into a persistent contraction or expansion of 4% of GDP. Absent redistribution, durable spending is more responsive after an expansion compared to a recession. With redistribution, the impulse response exhibits limited state-contingency.

**Figure A.5:** State-Contingency
General equilibrium. In Section 7, I show that income redistribution affects the magnitude and the timing of the general equilibrium response of durable investment to productivity shocks. Figure A.6 plots the corresponding responses for non-durable consumption. There is no discernible pattern in this case, which highlights the role of lumpy and state-contingent adjustment for the response of durable spending.

Figure A.6: Non-Durable Spending

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**B Omitted Proofs, Results and Derivations**

**B.1 Decomposition**

I provide a decomposition of the response of durable investment in income shocks in Section 3.1. Expression (3.4) in Proposition 1 applies to positive income shocks. I first provide the analogous decomposition for negative income shocks, before proving these results.

**Proposition 1 (cont’d).** The response of durable investment to a negative income shock \( \Delta < 0 \) can be decomposed as follows:

\[
\hat{I} (\Delta) \equiv \Sigma_1 (\Delta) + \Sigma_2 (\Delta) + \zeta (\Delta) \tag{B.1}
\]

with

\[
\Sigma_1 (\Delta) \equiv \frac{1}{\mathcal{I}} \int \left\{ [\tilde{d} (d, \zeta) - (1 - \delta + \delta \zeta) d] \int_{\bar{a} (\cdot) - \zeta \Delta}^{\bar{a} (\cdot)} d \Lambda (a | d, \zeta) + \kappa (d, \zeta) \int_{\bar{a} (\cdot) - \zeta \Delta}^{\bar{a} (\cdot)} (a - \bar{a} (d, \zeta)) d \Lambda (a | d, \zeta) \right\} d \Lambda^* \]
\[ \Sigma_2' (\Delta) \equiv \frac{1}{\mathcal{I}} \int \int_{a(\cdot)-\zeta \Delta \mathcal{Y}}^{+\infty} [d^* (a + \zeta \Delta \mathcal{Y}, d, \zeta) - d^* (a, d, \zeta)] d\Lambda (a|d, \zeta) d\Lambda^* \]

for some \( \kappa (d, \zeta) > 0 \) and some residual \( \zeta' (\Delta) \) that satisfies \( \lim_{\Delta \to 0} \frac{\zeta' (\Delta)}{\Delta} = 0. \)

**Proof of Proposition 1.** From (3.1)–(3.3),

\[
\hat{I} (\Delta) \equiv \frac{1}{\mathcal{I}} \left[ \int \int_{a(d, \zeta)}^{+\infty} (d^* (a, d, \zeta) - (1 - \delta) d) d\Lambda (a - \zeta \Delta \mathcal{Y} | d, \zeta) d\Lambda^* \\
+ \int \int_{0}^{\hat{a}(d, \zeta)} \delta d\Lambda (a - \zeta \Delta \mathcal{Y}, d, \zeta) d\Lambda^* - \mathcal{I} \right]
\]

where \( \Lambda^* \equiv \text{marg}_{d, \zeta} \Lambda \) denotes the marginal distribution of durable holdings and productivity. By definition, aggregate durable investment at the stationary equilibrium satisfies: \( \mathcal{I} \equiv \hat{I} (0) \). Thus,

\[
\hat{I} (\Delta) \equiv \frac{1}{\mathcal{I}} \left[ \int \int_{a(d, \zeta)}^{+\infty} (d^* (a, d, \zeta) - (1 - \delta) d) d\Lambda (a - \zeta \Delta \mathcal{Y} | d, \zeta) d\Lambda^* \\
\underbrace{\equiv \Omega_1} \\
+ \int \int_{0}^{\hat{a}(d, \zeta)} \delta d\Lambda (a - \zeta \Delta \mathcal{Y} | d, \zeta) d\Lambda^* \equiv \Omega_2 \right] \quad (B.2)
\]

where \( \hat{\Lambda} (a - \zeta \Delta \mathcal{Y} | d, \zeta) \equiv \Lambda (a - \zeta \Delta \mathcal{Y} | d, \zeta) - \Lambda (a|d, \zeta) \), with an abuse of notation.

Using the change of variable \( a' \equiv a - \zeta \Delta \mathcal{Y} \), the first term in (B.2) is

\[
\Omega_1 = \int_{a(d, \zeta) - \zeta \Delta \mathcal{Y}}^{+\infty} (d^* (a + \zeta \Delta \mathcal{Y}, d, \zeta) - (1 - \delta) d) d\Lambda (a|d, \zeta) \\
- \int_{a(d, \zeta)}^{+\infty} (d^* (a, d, \zeta) - (1 - \delta) d) d\Lambda (a|d, \zeta)
\]

Then,

\[
\Omega_1 = \int_{a(d, \zeta)}^{+\infty} (d^* (a + \zeta \Delta \mathcal{Y}, d, \zeta) - d^* (a, d, \zeta)) d\Lambda (a|d, \zeta) d\Lambda^* \\
+ \int_{a(d, \zeta) - \zeta \Delta \mathcal{Y}}^{a(d, \zeta)} \left( d^* (a + \zeta \Delta \mathcal{Y}, d, \zeta) - (1 - \delta) d \right) d\Lambda (a|d, \zeta) \quad (B.3)
\]
The first integral in (B.3) corresponds to the intensive margin of adjustment in Proposition 1. The second integral contributes to the extensive margin, together with the term \( \Omega_2 \) in (B.2).

This second integral can be decomposed as follows:

\[
\hat{\Omega}_1 = \int_{\bar{a}(d, \xi)}^{\tilde{a}(d, \xi)} \left( d^* (a + \xi \Delta Y, d, \xi) - \bar{d}(d, \xi) \right) d\Lambda(a|d, \xi) \\
+ (\bar{d}(d, \xi) - (1 - \delta) d) \int_{\bar{a}(d, \xi)}^{\tilde{a}(d, \xi)} d\Lambda(a|d, \xi)
\]

where \( \bar{d}(d, \xi) \equiv d^* (\bar{a}(d, \xi), d, \xi) \) denotes investment at the threshold. By assumption, \( d^*(\cdot) \) is smooth. Define \( \kappa(d, \xi) \equiv \frac{d}{da} d^*(a, d, \xi) \Big|_{a=\bar{a}(d, \xi)} > 0 \). Then,

\[
\hat{\Omega}_1 \equiv \kappa(d, \xi) \int_{\bar{a}(d, \xi)}^{\tilde{a}(d, \xi)} \left( a + \xi \Delta Y - \bar{a}(d, \xi) \right) d\Lambda(a|d, \xi) + \xi(\Delta, d, \xi) \\
+ (\bar{d}(d, \xi) - (1 - \delta) d) \int_{\bar{a}(d, \xi)}^{\tilde{a}(d, \xi)} d\Lambda(a|d, \xi)
\]

(B.4)

where \( \xi(\Delta, d, \xi) \) is defined residually.

Again, using a change of variable, the second term in (B.2) corresponds to

\[
\Omega_2 = \delta d\left[ \int_{0}^{\bar{a}(d, \xi) - \xi \Delta Y} d\Lambda(a|d, \xi) - \int_{0}^{\tilde{a}(d, \xi)} d\Lambda(a|d, \xi) \right] \\
= -\delta d \int_{\bar{a}(d, \xi) - \xi \Delta Y}^{\tilde{a}(d, \xi)} d\Lambda(a|d, \xi)
\]

(B.5)

since \( \Lambda(\cdot|d, \xi) \) has positive support, from the household’s income fluctuations problem (2.5)–(2.6).

Note that the previous expressions apply to both positive income shock \( \Delta > 0 \) and negative income shocks \( \Delta < 0 \). However, the attribution of each term to the extensive and intensive margins differs between these two cases. I thus treat them separately.

**Case 1:** \( \Delta < 0 \). The expression (3.4) in the text follows by collecting the terms from (B.2)–(B.4) and (B.5), and by definition of the hazard (3.2) and the marginal distribution \( \Lambda^* \). Finally, \( \xi(\Delta) \) is a second-order term, by Taylor’s Theorem.
Case 2: $\Delta < 0$. Note that

$$\Sigma_2 (\Delta) = \frac{1}{\bar{\Lambda}} \int \left\{ \int_{\bar{a}(\cdot)}^{\bar{a}(\cdot)-\zeta \Delta Y} [d^* (a + \zeta \Delta Y, d, \zeta) - d^* (a, d, \zeta)] d\Lambda (a | d, \zeta) \\
+ \int_{\bar{a}(\cdot)-\zeta \Delta Y}^{+\infty} [d^* (a + \zeta \Delta Y, d, \zeta) - d^* (a, d, \zeta)] d\Lambda (a | d, \zeta) \right\} d\Lambda^*$$

Equivalently,

$$\Sigma_2 (\Delta) = \frac{1}{\bar{\Lambda}} \int \left\{ \int_{\bar{a}(\cdot)}^{\bar{a}(\cdot)-\zeta \Delta Y} [d^* (a + \zeta \Delta Y, d, \zeta) - \bar{d} (d, \zeta)] d\Lambda (a | d, \zeta) \\
+ \int_{\bar{a}(\cdot)-\zeta \Delta Y}^{+\infty} [\bar{d} (d, \zeta) - d^* (a, d, \zeta)] d\Lambda (a | d, \zeta) \\
+ \int_{\bar{a}(\cdot)-\zeta \Delta Y}^{+\infty} [d^* (a + \zeta \Delta Y, d, \zeta) - d^* (a, d, \zeta)] d\Lambda (a | d, \zeta) \right\} d\Lambda^*$$

(B.6)

Then,

$$\Sigma_2 (\Delta) = \frac{1}{\bar{\Lambda}} \int \left\{ \kappa (d, \zeta) \int_{\bar{a}(\cdot)}^{\bar{a}(\cdot)-\zeta \Delta Y} \zeta \Delta Y d\Lambda (a | d, \zeta) \\
+ \int_{\bar{a}(\cdot)-\zeta \Delta Y}^{+\infty} [d^* (a + \zeta \Delta Y, d, \zeta) - d^* (a, d, \zeta)] d\Lambda (a | d, \zeta) \right\} d\Lambda^* + \xi (\Delta)$$

(B.7)

for some second-order term $\xi (\Delta)$, by definition of $\kappa (d, \zeta)$ and $\bar{d} (d, \zeta)$. Therefore, summing up $\Sigma_1 (\cdot)$ and $\Sigma_2 (\cdot)$ in the expression (3.4) in the text:

$$\Sigma_1 (\Delta) + \Sigma_2 (\Delta) = \frac{1}{\bar{\Lambda}} \int \left\{ [\bar{d} (d, \zeta) - (1 - \delta + i\delta) d] \int_{\bar{a}(\cdot)-\zeta \Delta Y}^{\bar{a}(\cdot)} d\Lambda (a | d, \zeta) \\
\kappa (d, \zeta) \int_{\bar{a}(\cdot)-\zeta \Delta Y}^{\bar{a}(\cdot)} (a - \bar{a} (d, \zeta)) d\Lambda (a | d, \zeta) \\
+ \int_{\bar{a}(\cdot)-\zeta \Delta Y}^{+\infty} [d^* (a + \zeta \Delta Y, d, \zeta) - d^* (a, d, \zeta)] d\Lambda (a | d, \zeta) \right\} d\Lambda^* + \xi (\Delta)$$

(B.8)

Finally, the expression (B.1) is obtained from (B.8), collecting the terms appropriately.  

\[ \Box \]

### B.2 Time-Dependent Adjustment

State-dependent (or lumpy) adjustment is key for the the non-linear effect of income shocks discussed in Section 3.1. To illustrate this point, I consider an alternative formulation where adjustment is instead time-dependent à la Calvo (1983).
In this case, the households’ problem is

\[
V^h_t (a, d, \zeta; 0) = \max_{\{c,a',d\}} \frac{u(c, d^*)^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t \left[ V^h_{t+1} \left( a', d^*, \zeta', A_{t+1} \right) \right] \tag{B.9}
\]

\[
\text{s.t. } P^c_t c + P^d_t d' \delta (1 - \delta) d^* + a' \leq (1 - \tau_t) e^h_t (\zeta) + (1 + r_{t-1}) a
\]

\[
a' \geq 0
\]

with \( d^* \equiv (1 - (1 - \iota) \delta) d \), and

\[
V^h_t (a, d, \zeta; 1) = \max_{\{c,a',d'\}} \frac{u(c, d')^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t \left[ V^h_{t+1} \left( a', d', \zeta', A_{t+1} \right) \right] \tag{B.10}
\]

\[
\text{s.t. } P^c_t c + P^d_t (d' - (1 - \delta) d) + a' \leq (1 - \tau^h_t) e^h_t (\zeta) + (1 + r_{t-1}) a
\]

\[
a' \geq 0
\]

with \( \{A_t\}_{t \geq 0} \) following a Poisson process with intensity \( \theta \in [0, 1] \), with independence across households and sectors. Note that the case \( \theta \equiv 1 \), i.e. frictionless adjustment, coincides with the case \( \Gamma_t (d', d) \equiv 0 \) in the original formulation with lumpy adjustment.

Aggregate durable spending in period \( t = 0 \) is

\[
\mathcal{I} (\Delta) = \theta \int [d^* (a, d, \zeta) - (1 - \delta) d] \Delta (a - \zeta \Delta Y, d, \zeta)
\]

\[
+ (1 - \delta) \Delta \delta d \Delta (a - \zeta \Delta Y, d, \zeta)
\] \tag{B.11}

where \( d^* (\cdot) \) solves the problem with adjustment (B.10) at \( t = 0 \). Thus, the impulse response of durable spending is

\[
\hat{\mathcal{I}} (\Delta) = \frac{1}{\mathcal{I}} \theta \int [d^* (a + \zeta \Delta Y, d, \zeta) - d^* (a + \zeta \Delta Y, d, \zeta)] \Delta (a, d, \zeta)
\] \tag{B.12}

since \( \mathcal{I} \equiv \mathcal{I} (0) \). Therefore, the impulse response \( \hat{\mathcal{I}} (\cdot) \) inherits the shape of durable spending \( d^* (\cdot; d, \zeta) \) as a function of liquid, financial assets.

Figure B.1 plots durable spending \( d^* (\cdot) \) in terms of financial assets, for various levels of durable holdings and fixing productivity at its median level. Durable spending is concave due to precautionary savings (Carroll and Kimball (1996)). As a result, aggregate durable spending is \textit{concave} in income changes. This stands is sharp contrast with the case of
lumpy adjustment, where the durable spending is convex in income changes (see Sections 3.1 and 6.1).

**Figure B.1: Durable Spending - Calvo (median labor supply)**

— 25th percentile — 50th percentile — 75th percentile (durable holdings)

### B.3 Continuous Time

My model is set in discrete time. In this appendix, I consider instead a continuous time version. To simplify the expressions, I abstract from maintenance and uninsured idiosyncratic income risk in the following.

**Dynamic system.** For the sake of illustration, I assume that income shocks \( \{ \Delta_t \}_{t \geq 0} \) are perceived as transitory, even if I am interested in persistent sequences of income changes. As a consequence, households’ optimal savings policy conditional on no adjustment is given by some time-invariant \( s(a, D, \Delta) \). \(^84\) The process for idiosyncratic states \((a, D)\) is defined by: (i) a law of motion conditional on no adjustment, \(^85\)

\[
\begin{align*}
da(t) &= s(a(t), D(t), \Delta(t)) \, dt \quad \text{(B.13)} \\
dD(t) &= -\delta D(t) \, dt \quad \text{(B.14)}
\end{align*}
\]

(ii) an adjustment threshold \( \bar{a}(D) \) of the form (3.2); and (iii) an adjustment target \( d^*(D) \) for durables. \(^86\) Financial assets after adjustment satisfy the following budget constraint (3.11).

---

\(^84\) In the following, I denote durable holdings by \( D \) instead of \( d \) to avoid any confusion with differential operators or integrands.

\(^85\) I abstract from mass points at the borrowing constraint, by assuming that there is no uninsured idiosyncratic income risk and that the savings rate is strictly positive (see Assumption 5 below).

\(^86\) Formally, \( \bar{a}(\cdot) \) is a jump reflecting barrier, and the adjustment target \( d^*(\cdot) \) corresponds to the reset point when hitting this barrier. Note that \( a = \bar{a}(D) \) when hitting the barrier, so that \( d^*(\cdot) \) can be expressed in terms of \( D \) only.
The economy is initially at its stationary equilibrium. I impose the following restrictions to simplify the analysis.

**Assumption 4.** The adjustment threshold satisfies \( \bar{a} (\cdot) = \bar{a} > 0 \).

**Assumption 5.** The savings function satisfies

\[
s (a, D, \Delta) = \bar{s} + \hat{m} \Delta \tag{B.15}
\]

for some \( \bar{s} > 0 \) and some \( \hat{m} > 0 \).

**Assumption 6.** Households who adjust borrow up to their constraint,

\[
d^* (D) = D + \frac{1}{\bar{ho} d} \bar{a} (D)
\]

The first restriction holds in my (discrete time) calibrated model. The second assumption stipulates the households have a constant marginal propensity to spend out of marginal, transitory income changes,\(^{87}\) and that the savings rate does not depend on asset holdings. Finally, the third restriction allows me simplify the expressions when characterizing the evolution of the distribution of idiosyncratic states in the interior of the state-space \((a > 0)\). This restriction also typically holds in my (discrete time) calibrated model.

**Comparative statics.** I consider a sequence of persistent (but unanticipated) income changes. That is, \( \Delta_t \equiv \Delta \in \mathbb{R} \) for all \( t \in [0, T] \) and some horizon \( T > 0 \).

**Investment.** Cumulative aggregate durable investment satisfies\(^{88}\)

\[
I (t') - I (t) \equiv \int_0^{+\infty} \int_t^{t'} (d^* (D) - D) d\Lambda_t \left( \hat{a} (\bar{a}, D; \hat{t}, t), \exp \left( \delta (\hat{t} - t) \right) D \right) d\hat{t}
\]

\[
+ \varsigma (t', t) \tag{B.16}
\]

for some term \( \varsigma (t', t) \) with \( \lim_{s \to 0} \frac{\varsigma(t+s,t)}{s} = 0 \). Here, I define the threshold \( \hat{a} (\cdot) \) such that

\[
a - \hat{a} (a, D; t', t) \equiv \int_t^{t'} s \left( \hat{a} (\hat{t}) \right) \exp \left( -\delta (\hat{t} - t') \right) D, \Delta) d\hat{t}, \tag{B.17}
\]

\(^{87}\) However, spending need not be linear in income shocks (even conditional on no adjustment) when income changes over a discrete interval \([0, T]\). See the discussion at the end of this section.

\(^{88}\) The second integral is over \( \hat{t} \).
The process \( \{\tilde{a}(\bar{t})\}_{\bar{t} \geq t} \) in (B.17) is characterized by
\[
d\tilde{a}(\bar{t}) = s(\tilde{a}(\bar{t}), \exp(-\delta(\bar{t} - t)) D, \Lambda) \, d\bar{t}
\]
with initial condition \( \tilde{a}(0) \equiv \tilde{a}(a, D; t', t) \). Figure B.2 corresponds to the phase diagram associated to the system (B.13)–(B.14) and the barrier \( \bar{a}(\cdot) \) expression (B.19) is the continuous time counterpart of (3.1) in the text.

Here, \( \hat{a} \) evolves as follows
\[
\hat{a} \text{ evolves as follows } D \text{ and } a \equiv a^*(D; \bar{t}, t) \text{ with } a^*(D; \bar{t}, t) \equiv \hat{a}(\bar{a}, \exp(-\delta(\bar{t} - t)) D; \bar{t}, t) \text{ and Assumptions 4 and 5},
\]
\[
I(t') - I(t) = \int_{0}^{+\infty} \int_{a^*(D, t')}^{\bar{a}} \left( d^*(\hat{D}(a, D)) - \hat{D}(a, D) \right) d\Lambda_t(a, D) \text{ Density}
\]
\[
+ \zeta(t', t)
\]
Here, \( \hat{D}(a, D) \equiv \exp(-\delta(\hat{A}(a, d) - t)) D \), where \( \hat{A}(a, d) \) denotes the stopping time associated to (B.13)–(B.14) and the barrier \( \bar{a}(\cdot) \). By definition, \( \hat{D}(\bar{a}, D) \equiv D \). Note that expression (B.19) is the continuous time counterpart of (3.1) in the text.

From (B.19), the durable investment flow satisfies
\[
i_t \equiv \frac{d}{dt}I(t) = -\int_{0}^{+\infty} \frac{d}{dt^'}a^*(D; t', t) \bigg|_{t'=t} [d^*(D) - D] \lambda_t(\bar{a}, D) \, dD
\]
where \( \lambda_t \) denotes the density associated to \( \Lambda_t \), using \( a^*(D; t, t) \equiv \bar{a} \) and \( \hat{D}(a^*(D; t, t), D) \equiv D \). Using (B.17)–(B.18) and Assumption 5,
\[
\frac{\partial}{\partial t'}a^*(D; t', t) = -(\bar{s} + \bar{m}\Delta)
\]

Distribution. Using Assumptions 4 and 6, the joint distribution of idiosyncratic states evolves as follows
\[
\Lambda_{t'}(A, D) - \Lambda_{t}(A, D) \equiv \int_{D} \int_{0}^{\hat{D}(A, D; t', t)} d\Lambda_{t}(a, D)
\]
\[
- \int_{0}^{\hat{D}(A, D; t', t)} d\Lambda_{t}(a, D)
\]
\[
+ \int_{0}^{\hat{D}(D)} \int_{\bar{a}(\bar{a}, D; t', t)} d\Lambda_{t}(a, D) + \zeta(A, D; t', t)
\]

89 Implicitly, this process is indexed by \((t, t')\) as well.

90 I use \( A \) and \( D \) as arguments for the distribution \( \Lambda_t \) and \( a \) and \( D \) as integrands. This is a slight abuse of notation, since \( A \) is used in the text to denote that adjustment hazard.
when $0 \leq a < \bar{a}$, for some term $\xi(A, D; t', t)$ with $\lim_{s \to 0} \frac{\xi(A, D; t' + s, t)}{s} = 0$. The three terms on the RHS of (B.22) capture households’ inflows and outflows into the relevant region of the state space.

**Figure B.2:** Density at Adjustment Threshold

Figure B.2 corresponds to the phase diagram associated to the system (B.13)–(B.14). Inflows in (B.22) are depicted in green, and outflows in red. The first term in (B.22) captures the inflows due to the depreciation of durables. This corresponds to households with an initial stock of durables below $\hat{d}(D; t' - t) \equiv \exp(\delta(t' - t))D$ and sufficiently low financial assets. The second term captures the outflows due to accumulation of financial assets. Note that the reset level of financial assets is $a^\star(\cdot) = 0$ in either case, by Assumption 6. Therefore, outflows occur only when the adjustment target $d^\star(\cdot)$ exceeds $D$. Hence $d^\star(\hat{D}(D)) \equiv D$, and $\hat{D}(D) \leq D$ is well-defined by Assumption 6. The third term captures inflows from households whose initial holdings of financial assets exceeded $A$ but was depleted after they adjusted their stock of durables. Inflows only occur when the adjustment target $d^\star(\cdot)$ is below $D$.

From (B.22), the evolution of the density of idiosyncratic states is characterized by

$$
\partial_t \lambda_t (A, D) = \partial_D \left( \frac{\partial}{\partial t'} \hat{d}(D; t' - t) \bigg|_{t' = t} \lambda_t (A, D) \right) + \partial_A \left( \frac{\partial}{\partial t'} \hat{a}(A, D; t', t) \bigg|_{t' = t} \lambda_t (A, D) \right)
$$

(B.23)

Note that (B.23) coincides with the Fokker-Planck equation associated to the system (B.13)–(B.14), abstracting from the barrier $\bar{a}$. The reason is that the third term on the RHS of (B.22) does not contribute to the density for $a > 0$, by Assumption 6.
when \(0 < \mathcal{A} < \bar{a}\) and for sufficiently small \(t\), using the definition of \(\hat{a}\) (·) and \(\hat{d}\) (·). Using (B.17)–(B.18), the definitions of \(a^\star\) (·) and \(\hat{d}\) (·), and Assumption 5,

\[
\frac{\partial}{\partial t} \hat{d} (D; t', t) \bigg|_{t'=t} = \delta D , \quad \frac{\partial}{\partial t} a^\star (D; t', t) \bigg|_{t'=t} = - (\bar{s} + \hat{m} \Delta) \tag{B.24}
\]

Finally, the evolution of investment is entirely characterized by (B.20)–(B.21) and (B.23)–(B.24).

**Impulse response.** I am interested in the response of cumulative investment, starting in period \(t = 0\).\(^{92}\)

From (B.20)–(B.21) and (B.23)–(B.24),

\[
i_t (\Delta) \equiv (\bar{s} + \hat{m} \Delta) \int_0^{+\infty} [d^\star (D) - D] \lambda_t (\bar{a}, D) dD \tag{B.25}
\]

Note that

\[
\frac{\partial}{\partial \Delta} i_t (\Delta) \bigg|_{t=0} = \hat{m} \int_0^{+\infty} [d^\star (D) - D] \lambda (\bar{a}, D) dD , \tag{B.26}
\]

For illustration, I suppose that the stationary distribution satisfies

\[
\partial_A \lambda (\mathcal{A}, \mathcal{D}) = \partial_A \lambda (\mathcal{A}', \mathcal{D}') \equiv \mu < 0 \quad \text{and} \quad \lambda (\mathcal{A}, \mathcal{D}) = \lambda (\mathcal{A}, \mathcal{D}')
\]

for each \(\mathcal{A}, \mathcal{A}' \geq 0\), \(\mathcal{D}, \mathcal{D}' \geq 0\), i.e. the distribution has constant slope with respect to financial holdings and is uniform with respect to durable holdings. Then, \(\lambda_t\) inherits these properties, from (B.22) and using (B.17)–(B.18) and Assumption 5. Therefore,

\[
\partial_t \lambda_t (\mathcal{A}, \mathcal{D}) = \delta \lambda_t (\mathcal{A}, \mathcal{D}) - (\bar{s} + \hat{m} \Delta) \mu
\]

using (B.23)–(B.24) and Assumption 5. Then,

\[
\lambda_t (\mathcal{A}, \mathcal{D}) = \lambda (\mathcal{A}, \mathcal{D}) - \mu (\bar{s} + \hat{m} \Delta) (\exp (\delta t) - 1) \tag{B.27}
\]

Finally, cumulative investment satisfies

\[
i_t (\Delta) \equiv (\bar{s} + \hat{m} \Delta) \int_0^{+\infty} [d^\star (D) - D] [\lambda (\bar{a}, \mathcal{D}) - \mu (\bar{s} + \hat{m} \Delta) (\exp (\delta t) - 1)] dD \tag{B.28}
\]

using (B.25) and (B.27). As in discrete time, the impulse response of investment is convex in income changes for any \(t > 0\), whenever the density \(\lambda\) at the stationary equilibrium is

\(^{92}\) That is, I parametrize the above expressions with 0 for \(t\) and \(t\) for \(t'\).
decreasing \((\mu < 0)\). In other words, the average marginal propensity to spend on durables increases with income changes. I elaborate on these results in Section 3.1.

**State-contingency.** The form of non-linearity discussed above is intrinsically related to another property of models of lumpy adjustment: state-contingency.

I am interested in the response to an (unanticipated) income shock \(\Delta\) over an interval \([t, t']\) with \(t' > t\) after a sequence of (unanticipated) shocks \(\Delta^*\) over the interval \([0, t]\). By analogy with (B.20)–(B.21) and (B.27),

\[
i_t(\Delta) = (\bar{s} + \hat{m}\Delta) \int_0^{+\infty} [d^* (D) - D] [\lambda (A, D) - \mu (\bar{s} + \hat{m}\Delta^*) (\exp (\delta t) - 1)] dD \quad (B.29)
\]

Note that (B.28) and (B.29) coincide when \(\Delta = \Delta^*\), by definition. By construction, the slope of the density \(\lambda\) at the stationary equilibrium \((\mu)\) is responsible for both the non-linearity that I focus on, and the state-contingency of impulse responses. I elaborate on this point in Section 6.3. Also, note that the impulse response of investment is linear in \(\Delta\), but contingent on the size of the initial income change and the period over which it occurs \((\Delta^*, t)\). In particular,

\[
\frac{\partial^2}{\partial t \partial \Delta} i_t(\Delta) = -\delta \exp (\delta t) \mu (\hat{m})^2 \Delta^* \int_0^{+\infty} [d^* (D) - D] dD
\]

when \(\bar{s} \to 0\), i.e. the economy before the income shocks is stationary. The impulse response increases over time after a period of expansion \((\Delta^* > 0)\), but decreases after a period of recession \((\Delta^* < 0)\), whenever the slope of the density of liquid assets at the adjustment threshold \(\mu\) is negative. I elaborate on these results in Section 6.3.

**B.4 Benchmark Case**

In Section 3.3, I discuss a benchmark where the two goods are non-durable: \(\delta = 1\), i.e. full depreciation. In this case, no redistribution of labor income takes place across sectors and there is no role for insurance from fiscal policy. For concision, no formal statement is provided in the text. I do so in this section instead.

**Proposition 2.** Let Assumptions 1-2 hold. Consider a persistent, unanticipated productivity shock of the form (3.13), for any \(\psi_0 \in \mathbb{R}\). Then, taxes under the insurance regime (2.11) satisfy:

\[T^* = 0\]
Consequently, the response of durable investment satisfies:

\[
\frac{Y^d_0}{Y^d} = \frac{Y^d_0}{Y^d,*}
\]

for each period \( t \geq 0 \), where \( Y^d_0 \) denotes aggregate durable investment (2.13) under the regimes (2.10) and (2.11).

**Proof of Proposition 2.** First, I guess and verify that outputs in the economy without insurance \((T \equiv 0)\) satisfy:

\[
\frac{Y^c_t}{Y^c} = \frac{Y^d_t}{Y^d}
\]

(B.30)

By homotheticity,

\[
c_t^h (a, d, \zeta) = c^*_t e_t^h (a, d, \zeta) \quad \text{and} \quad d^h_t (a, d, \zeta) = d^*_t e_t^h (a, d, \zeta)
\]

with full depreciation. Here, \((c^*_t, d^*_t)\) denotes the (dual) cost-minimizing bundle that achieves \( u (c, d') \geq 1 \), and \( e_t^h (a, d, \zeta) \) solves the following income fluctuations problem\(^93\):

\[
V^h_t (a, \zeta) = \max \left\{ V^h_{t+1} (a', \zeta') \right\} \frac{e^{1-\sigma}}{1 - \sigma} + \beta \mathbb{E}_t \left( V^h_{t+1} (a', \zeta') \right) |
\]

s.t. \( \hat{P}_t e + a' \leq (1 - \tau_t) e_t^h (\zeta) + (1 + r_{t-1}) a \)

\( a' \geq 0 \)

where \( \hat{P}_t \) denotes the CES ideal price index (A.14). In particular, \((c^*_t, d^*_t)\) is homogeneous of degree 0 in prices \( P_t \).

From the firm’s price setting (A.8)–(A.10), the law of motion of price indices (A.7), the guess (B.30), and by Assumptions 1-2,

\[
\frac{p^c_t}{p^d_t} = \frac{p^d_t}{p^d}
\]

(B.32)

so that \((c^*_t, d^*_t) = (c^*, d^*)\), i.e. the relative demand is unchanged compared to the stationary equilibrium. Thus, the guess (B.30) is verified, from the market clearing conditions (A.15)–(A.16). The remaining equilibrium restrictions are satisfied.

Furthermore, note that:

\[
\frac{Y^c_t}{Y^c} = \frac{Y^d_t}{Y^d}
\]

(B.33)

\(^93\) Note that the state variable \( d \) is redundant when \( \delta = 1 \), and households do not incur a fixed costs when adjusting their consumption of the good \( h = d \).
since wages are rigid, using the definitions of labor demands (A.17)–(A.18), and the wage bills (A.19). Then,

$$T^*_t = 0$$

(B.34)

under the regime with insurance, for each period \( t \geq 0 \), by definition of taxes (A.11) and given the wage bills (B.33).

**B.5 Omitted Derivations**

*Derivations for Example 1.* The impulse response satisfies:

$$\hat{I} (\Delta) = \frac{1}{T} \int \left[ \hat{d} (d, \zeta) - (1 - \delta + i\delta) d \right] \Psi (\Delta) \, d\Lambda^*$$

(B.35)

with

$$\Psi (\Delta) \equiv \int_{a(\cdot) - \zeta \Delta Y}^{a(\cdot)} \, d\Lambda (a | d, \zeta)$$

(B.36)

Let

$$\eta (\Delta, \Delta') \equiv \Psi (\Delta') + \Psi (\Delta) - 2 \Psi \left( \frac{\Delta + \Delta'}{2} \right)$$

(B.37)

with \( \Delta' > \Delta \) without loss of generality.

From (B.36),

$$\eta (\Delta, \Delta') = 2 \int_{a(\cdot) - \zeta \Delta Y}^{a(\cdot)} \, d\Lambda (a | d, \zeta) + \int_{a(\cdot) - \zeta \Delta Y}^{a(\cdot) - \zeta \Delta Y} \, d\Lambda (a | d, \zeta) - 2 \int_{a(\cdot), - \zeta \Delta Y}^{a(\cdot), - \zeta \Delta Y} \, d\Lambda (a | d, \zeta)$$

Thus,

$$\eta (\Delta, \Delta') = \int_{a(\cdot) - \zeta \Delta Y}^{a(\cdot) - \zeta \Delta Y} \, d\Lambda (a | d, \zeta) - \int_{a(\cdot), - \zeta \Delta Y}^{a(\cdot), - \zeta \Delta Y} \, d\Lambda (a | d, \zeta)$$

Using the change of variable \( a' \equiv a + \zeta \frac{\Delta' - \Delta}{2} Y \) for the first integral,

$$\eta (\Delta, \Delta') = \int_{a(\cdot), - \zeta \Delta Y}^{a(\cdot), - \zeta \Delta Y} \, d\Lambda (a | d, \zeta)$$

(B.38)

where \( \hat{\Lambda} (a | d, \zeta) \equiv \Lambda \left( a - \zeta \frac{\Delta' - \Delta}{2} Y \right) \) \( d\zeta \) \( \Lambda (a | d, \zeta) \), with an abuse of notation. Then, \( \eta (\Delta, \Delta') > 0 \) since the density \( d\Lambda (a | d, \zeta) \) is decreasing at the adjustment threshold \( a (\cdot) \), by assumption. The convexity of the impulse response follows from (B.35)–(B.38).
Derivations for Example 2. From (3.4) and by definition of the adjustment threshold (3.2), the extensive margin satisfies:

\[ \Sigma_1 (\Delta) = \frac{1}{I} \int a^* \left\{ \left[ \bar{a} (d, \zeta) - (1 - \delta + i \delta) d \right] \zeta \Delta Y \right. \\
\left. + \kappa (d, \zeta) \int \bar{a}(d, \zeta) - \zeta \Delta Y (a + \zeta \Delta Y - \bar{a} (d, \zeta)) da \right. \\
\left. + \int \bar{a}^* (d^* (a + \zeta \Delta Y, d, \zeta) - d^* (a, d, \zeta)) da \right\} d \Lambda^* \]  

(B.39)
since the (conditional) distribution of liquid assets is uniform on \([0, a^*] \) and shocks are sufficiently small, by assumption. By linearity of the adjustment target,

\[ \Sigma_1 (\Delta) = \theta \Delta + \frac{1}{I} \int a^* \kappa (d, \zeta) \int \bar{a}(d, \zeta) - \zeta \Delta Y (a + \zeta \Delta Y - \bar{a} (d, \zeta)) d a d \Lambda^* \]  

(B.40)
where \( \kappa (d, \zeta) \) denotes the slope of the adjustment target \( d^* (\cdot) \), with

\[ \theta \equiv \frac{1}{I} \int \left[ \bar{d} (d, \zeta) - (1 - \delta + i \delta) d + \kappa (d, \zeta) (a^* - \bar{a} (d, \zeta)) \right] \zeta Y d \Lambda^* \]

Integrating the second term in (B.40),

\[ \Sigma_1 (\Delta) = \theta \Delta + \left[ \frac{1}{2} \frac{1}{I} \int a^* \kappa (d, \zeta) \left( \zeta Y \right)^2 d \Lambda^* \right] \Delta^2 \]  

(B.41)
The convexity of the impulse response immediately follows from (B.41), since \( \kappa (d, \zeta) > 0 \) by assumption.

Derivations for Example 4. Consider an expansionary productivity shock \( \psi_0 < 0 \). The decrease in productivity increases the demand for labor. In partial equilibrium, incomes increase proportionately across sectors. From (A.12), distortionary taxes decrease \( \tau_0 < \tau \) which further contributes to an increase in liquid assets for households. Iterating on the market clearing conditions (2.12)–(2.13) and using Assumption 3,

\[ \frac{\mathcal{Y}_0^d}{\mathcal{Y}_0^d} > \frac{\mathcal{Y}_0^c}{\mathcal{Y}_0^c} > 1 \]  

(B.42)
in general equilibrium, since prices are rigid \( (\lambda \equiv 1) \).

Starting from this equilibrium, suppose now that fiscal policy provides full aggregate

\[ \text{Again, note that a positive productivity shock } (\psi_0 > 0) \text{ is contractionary in this setting.} \]
insurance. From (A.11) and (B.42), taxes satisfy:

\[ T_d^0 = -\frac{\mu_c}{\mu_d} T_c^0 > 0 \]  

(B.43)

Under the assumptions of Example 2, aggregate (sectoral) durable investment

\[ \gamma_{d,h}^0 \equiv \int \left[ d_{0,h}^{h'}(a,d,\zeta) - (1 - \delta) d \right] d\Lambda_{h}^0 \]

is \textit{convex} in (sectoral) incomes\(^{95}\), for each sector \(h\). Then, the redistribution from fiscal policy (B.43) reduces aggregate durable investment in partial equilibrium, using (B.42). Aggregate non-durable consumption is unchanged, since the role of precautionary savings is negligible, by assumption. Therefore, incomes in durable sector sector while those in the non-durable sector are unchanged in partial equilibrium, from the market clearing conditions (2.12)–(2.13) and the definition of incomes (A.18)–(A.20). Again, iterating on the market clearing conditions (2.12)–(2.13) and using Assumption 3, aggregate durable investment is lower than in the economy without insurance (\(T \equiv 0\)).

\section{C \hspace{1em} Empirical Appendix}

\subsection{C.1 \hspace{1em} Aggregate Series}

The series for the nominal interest rate, household consumption and investment expenditures, and employment are listed in Table C.1.

I define durable spending as the sum of household’s expenditures on durable goods and residential investment. Non-durable spending consists of the sum of households’ expenditures on non-durable goods and services, minus housing and financial / insurance services. I define durable-employment as the sum of employment in: construction, durable manufacturing, wholesale trade and durable retail for durables, and repair and maintenance. Non-durable employment consists of the sum of employment in: non-durable manufacturing, wholesale trade and durable retail for durables, information, professional and business services, leisure and hospitality, and other services (except repair and maintenance). I exclude wholesale trade, retail trade (for data availability reasons) from either employment category. I exclude financial services and public administration.

\(^{95}\) Incomes are implicit to the time index.
Table C.1: Data Sources

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<th>Series</th>
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<th>Source</th>
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<td>Fed Board H.15</td>
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<td>Effective Fed funds rate</td>
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<td>Deflator</td>
<td>Yes</td>
<td>NIPA 1.1.4.</td>
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<tr>
<td>PCE</td>
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<td>Personal Consumption Expenditures</td>
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<td>(lines 4-6)</td>
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<tr>
<td>Residential Investment</td>
<td>Yes</td>
<td>NIPA 1.1.3</td>
</tr>
<tr>
<td>Real</td>
<td>Yes</td>
<td>(lines 14)</td>
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<tr>
<td>Employment</td>
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<td>Population</td>
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<td>NIPA 2.1</td>
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<tr>
<td>Total</td>
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C.2 PSID Data

In Section 4, I provide supporting evidence on the interaction between cyclical durable investment and redistribution. In particular, I confirm that labor income decreases proportionally more in durable sectors, compared to non-durable sectors, following a contractionary tax shock. To account for cross-sectoral labor mobility and movements in and out of the labor force, I use longitudinal data from the Panel Study of Income Dynamics (PSID).

Industry classification. Following Berger and Vavra (2015), I adopt a broad definition of durable goods when calibrating the quantitative model in Section 5. This definition includes both consumer durables, and residential investment. Consistently, I classify industries as either durable, or non-durable when using the PSID. Durable industries consists of construction, and durable manufacturing. Non-durable industries include non-durable manufacturing, and all services except public administration and the military, and finance. Financial activities and real estate are inter-related, making their classification ambiguous. Public administration and the military spending / employment have no immediate counterpart in my model. Excluding these two industries is conservative with respect to the mechanism I am interested in since spending and employment are less cyclical for these two industries than for private industries.

Income data. My preferred measure of incomes corresponds to (pre-tax) labor income, deflated using a price index for total consumption expenditure. I also use family money

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96 Financial activities and real estate are inter-related, making their classification ambiguous. Public administration and the military spending / employment have no immediate counterpart in my model. Excluding these two industries is conservative with respect to the mechanism I am interested in since spending and employment are less cyclical for these two industries than for private industries.
income to account for unemployment insurance and intra-household risk sharing.

Sample selection. I use the bi-annual PSID waves from 1968 to 2015. The sample consists of male household’s heads aged 24-65. To eliminate outliers, I exclude households with labor income lower than 5% of the annual average, and higher than the 95th percentile. I use PSID longitudinal weights.

C.3 Complementary Empirical Evidence

The measure of income that I use in the text is (real) labor income from the PSID. For robustness, I also use family income in to account for unemployment insurance and intra-household risk sharing. Figure C.1 plots the corresponding series. The pattern is very similar: aggregate shocks lead to a redistribution of income between durable and non-durable workers.

Figure C.1: Family Income (PSID)

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97 The industry classification in the PSID changed in 2017. I do not exploit this latest wage to avoid measurement issues.