Learning in Games and the Intepretation of Natural Experiments

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Online Appendix

Independent Priors

Suppose that $x_{it} = \chi + \gamma e_{it} + \omega_{it}$ where $\chi \in \{\underline{\chi}, \overline{\chi}\}$ and $\gamma \in \{\underline{\gamma}, \overline{\gamma}\}$. Suppose that the support of $F(\omega)$ includes $[-1 - \overline{\chi}, -\underline{\chi}]$ and in this range $F(\omega) = F_0 + f\omega$, where$F_0 > f > 0$. Finally suppose prior independence so that $p_{i1}(\gamma, \chi) = \tilde{p}_{i1}(\gamma)\tilde{p}_{i1}(\chi)$. Normalize so that the prior expected value of χ is zero, that is $\tilde{p}_{i1}(\overline{\chi})\overline{\chi} + \tilde{p}_{i1}(\underline{\chi})\underline{\chi} = 0$. Then the posterior for γ does not depend on the distribution of χ, specifically:

$$\tilde{p}_{i2}(D_{i1}) = \Pr(\overline{\gamma}|D_{i1}) = \left(\frac{1}{p_{i1} + (1 - p_{i1})/L(D_{i1})} \right) \tilde{p}_{i1}$$

where

$$L(0) = \frac{F_0 - \overline{\gamma} e_{i1}}{F_0 - \underline{\gamma} e_{i1}}, L(1) = \frac{1 - F_0 + \overline{\gamma} e_{i1}}{1 - F_0 + \underline{\gamma} e_{i1}}.$$

From Bayes law for the marginal of γ we have

$$\Pr(\gamma|D_{i1}) = \frac{\Pr(D_{i1}|\gamma)}{\sum_\gamma \Pr(D_{i1}|\gamma) p_{i1}(\gamma)} \tilde{p}_{i1}(\gamma),$$

which depends only on $\Pr(D_{i1}|\gamma)$ and $p_{i1}(\gamma)$. For the former we have

$$\Pr(D_{i1}|\gamma) = \sum_\chi \Pr(D_{i1}, \chi|\gamma) = \sum_\chi \Pr(D_{i1}|\gamma, \chi) \Pr(\chi|\gamma),$$

and applying independence

$$= \sum_\chi \Pr(D_{i1}|\gamma, \chi) \tilde{p}_{i1}(\chi).$$

As $\Pr(D_{i1}|\gamma, \chi)$ is linear in χ and $\sum_\chi \chi \tilde{p}_{i1}(\chi) = 0$ the result follows.