JOURNAL OF ECONOMIC THEORY 61, 206–229 (1993)

Finite Bubbles with Short Sale Constraints and Asymmetric Information*

FRANKLIN ALLEN, STEPHEN MORRIS, AND ANDREW POSTLEWAITE†

University of Pennsylvania, Philadelphia, Pennsylvania 19104

Received May 25, 1992; revised September 23, 1992

We present a finite period general equilibrium model of an exchange economy with asymmetric information. We say that a rational expectations equilibrium exhibits a strong bubble if the price is higher than the dividend with probability one. We show that necessary conditions for a strong bubble to occur are that (1) each agent must have private information in the period and state in which the bubble occurs, (2) each agent must be short sale constrained at some period in the future with positive probability, and (3) agents' trades are not common knowledge. We present examples of bubbles when the necessary conditions are satisfied. *Journal of Economic Literature* Classification Numbers: C72, D52, D82, D84, G12, G14.

1. INTRODUCTION

What determines stock prices? Are they determined by expectations about future dividends so that stocks trade at their "fundamental value," or are they "bubbles" which are determined by crowd psychology, fads, or some other arbitrary factor? These questions are central to the issue of whether stock markets allocate resources efficiently or not.

There is no wide agreement on how the empirical evidence on stock prices should be interpreted. There have been a number of extreme historical episodes, including the Dutch tulipmania (1634–1637), the Mississippi bubble (1719–1720), and the South Sea bubble (1720) where asset prices rose very quickly and then dramatically collapsed. Some authors have argued that these examples provide evidence of bubbles (see, e.g., Kindleberger [13]) while others have argued that assets traded at their fundamental values during these episodes (see, e.g., Garber [8]). More

* This paper is an extension of an earlier paper "Rational Expectations and Stock Market Bubbles" by two of the authors.

† This research was prompted by a conversation with Michael Waldman. We thank Allan Kleidon and Rody Manuelli for helpful comments. Responsibility for any remaining errors rest with the authors. Financial support from the NSF is gratefully acknowledged.

0022-0531/93 $5.00

recent evidence has been presented which suggests that the volatility of stock prices is too large to be explained by variations in underlying dividend streams (see, e.g., Leroy and Porter [19] and Shiller [24]). However, the interpretation of these studies has also been challenged by a number of authors who claim the econometric techniques used are flawed (see, e.g., Flavin [7], Kleidon [14, 15], Marsh and Merton [20], and West [28]).

In addition to the empirical debate on the existence of bubbles, there has been an extensive theoretical debate on whether bubbles are consistent with rational behavior. An important strand of the literature has developed models of bubbles where at least some agents are irrational (see, e.g., De Long *et al.* [6], Shleifer and Summers [26], and Shiller [25]).

Another strand has adopted the more traditional approach of assuming all agents are rational. An argument that is often used in this framework to support the position that stock prices reflect fundamental values relies on backward induction. Suppose that at time T an asset is known to have a final payoff P_T. Then at time $T-1$ it must be worth the discounted present value of P_T, otherwise there would be an arbitrage opportunity. By extending this argument backward appropriately, it can be seen that at any point in time the value of a stock must be equal to the present discounted value of its future dividends.

One case for which this argument fails is when there is an infinite horizon so that there is no final payoff. There is a large literature on the possibility of assets trading above their fundamental value in this case, starting with Samuelson's [23] overlapping generations model explaining the existence of fiat money (Camerer [5] contains a survey of these and other theories of bubbles). It has also been shown that bubbles can still exist when there is a finite horizon provided there are an infinite number of trading opportunities (Allen and Gorton [2] and Bhattacharya and Lipman [4]).

The purpose of this paper is to show that even if there is a finite number of trading opportunities the backward induction argument may fail in rational expectations equilibrium. We present an example in which the market price of a security is above the present value of its future dividends even though every agent is rational and knows the dividends with certainty. The reason is that agents do not know other agents' beliefs. Because of private information it is not common knowledge that everybody believes the stock price will fall. Everybody realizes the stock is overpriced but each person thinks he may be able to sell it at a higher price to somebody else before the true value becomes publicly known. An important feature of the example that is discussed below is that in the event of a bubble, every agent will know that the bubble will "burst," that is that prices will fall, although they will be uncertain of the exact time at which this will occur. An aspect of much informal discussion of real world bubbles is that all, or nearly all, participants in the market believe prices will eventually fall. For all agents in a rational expectations equilibrium to know that the price will fall, there must be asymmetric information (specifically that it not be common knowledge that the price will fall). Thus the asymmetry of information in our model is necessary to understand such bubbles.

Section 2 provides a more detailed discussion of the question of bubbles and fundamental values and Section 3 outlines the model. Before presenting the example in Section 5, we make clear what drives the example by giving necessary conditions for such a bubble in Section 4. It is necessary that there are short sale constraints and that at the state and time where the bubble occurs, every agent either is short sale constrained or will be short sale constrained at some possible contingency in the future. It is also necessary that private information at that state is not fully revealed and, in particular, it is not common knowledge that the asset is overvalued. Last, it is necessary that other agents' trades are not common knowledge—agents know their own trades and prices, but not the individual trades of other agents.

A number of recent papers have emphasized the importance of the common knowledge assumption in related contexts. Abel and Mailath [1] show that it is possible that a project can obtain financing even though everybody believes it has a negative net present value. Kraus and Smith [17] demonstrate that there can be a change in asset prices, even though there is no new information about security payoffs, because some agent's beliefs about other agents' beliefs change. Jackson and Peck [11] present an infinite horizon overlapping generations model where rational agents attempt to deduce "market psychology" by examining the past movements of prices and show that bubbles can exist.

2. Fundamental Values and Bubbles

The term "bubble" is often used to designate the situation in which the price of an asset is higher than would be warranted given the "fundamentals" of the asset. Much of the discussion of bubbles and fundamentals implicitly presumes that an asset has an exogenously given fundamental value. The idea that an asset would have such a fundamental value is understandable given the simple economic models that provide much of our intuition. For example, in models in which there is a single good that is valued identically by all agents there will typically be no ambiguity as to what is meant by the asset's fundamental value.

The notion of an exogenous fundamental value for each asset becomes problematic in richer models in which there is trade among agents, however. The difficulty can be understood by considering sunspot equilibria of simple exchange economies. Consider a simple one-period exchange economy with at least two Walrasian equilibria. Now consider a discrete-time dynamic economy with the property that in each period there is a given set of agents with preferences and endowments as in the original economy. The agents live for one period only and are replaced by identical agents in the next period. Last, suppose there are sunspots—that is, in each period there is a random variable whose realization will be common knowledge, but is of no economic relevance in the sense that the technologically feasible sets, endowments, and utility functions are independent of the realization of the random variable. There will then be an equilibrium of the dynamic economy of the following sort: for some event with probability greater than zero but less than one, the outcome of the dynamic economy is one of the static Walrasian equilibria and for the complement of the event it is a different Walrasian equilibrium. Thus, in different periods, the dynamic economy will be identical in every economically relevant respect, yet the prices of a specific asset can differ in the two periods.

We would argue that whichever of the two events arises, the prices reflect the fundamentals of the economy. This is essentially a semantic statement of what we mean by "the fundamentals of the economy." The difference between those periods in which the dynamic economy is in one state rather than another is not evidence that prices deviate from fundamental values in one state or another, but that the fundamental value of an asset depends upon the particular equilibrium for the economy. We take fundamental value to mean the value of an asset in normal use as opposed to some value it may have as a speculative instrument. The sunspot example demonstrates that the value in normal use is not unique; any model with nontrivial multiple equilibria will exhibit non-unique normal use values for some assets independent of the existence of any speculative role the asset may have.

This discussion shows that any interesting notion of fundamental value cannot be exogenous, but rather must be defined in the context of a particular equilibrium. But given a particular equilibrium, precisely what does it mean for the price of an asset to reflect its fundamental value? A sufficient condition would be that, in each period, every agent holding or buying that asset would willingly do so even if he were to be forced to maintain his holdings of the asset forever. Clearly, if each agent were willing to do so, the use value the agent derives cannot be less than the price. But it is clear that there are cases in which agents are willing to pay more for an asset than they would if they were to be forced to hold the asset forever. The simplest example illustrating this is an overlapping generations model with finite-lived agents and two goods, a non-storable consumption good and an infinitely lived productive asset. For an equilibrium of such a model, each agent will be willing to pay a price for the asset equal to the discounted sum of the returns he will receive while he is alive plus the present value of the sale price of the asset in the period in which he plans to sell it. The current price would exceed the owner's personal use value by the present value of the anticipated revenue from the ultimate sale of the asset. This notion of speculation, which Harrison and Kreps [10] attribute to Keynes [12], motivates what we later will call an "expected bubble." But a broader notion of fundamental notion of fundamental value should include the use value of the asset that accrues to future holders of the asset.

To summarize, we are arguing that the fundamental value of an asset is the present value of the stream of the market value of dividends or services generated by that asset. The price of an asset in any given period may be above this notion of its fundamental value, the simplest example being a monetary equilibrium in an overlapping generations model. Here, an asset that pays no dividends—and hence has fundamental value zero—has a positive price. This can only occur with infinitely lived agents if there is a constraint on short sales, as pointed out by Kocherlakota [16].

In finite horizon models with no uncertainty bubbles of this type cannot exist even with short sales constraints; the constraint that all agents close short positions by the last period is sufficient to rule out prices above fundamental value. If uncertainty is introduced, we have to extend what we mean by fundamental value since the market value of an asset's output may take on different values in different states of the world. If the uncertainty is symmetric—that is, all agents have identical probability beliefs at all nodes—we might be able to extend the notion of fundamental value by taking the expected value of the stream of output. We say "might" for several reasons. First, if agents are risk averse, there will typically be a deviation between expected value and the price agents are willing to pay and second, in the absence of complete markets, there is no guarantee that all agents will have the same marginal rates of substitution, even for equilibria in which each agent consumes on the interior of his consumption set. The consequence of the second difficulty is that there may not be a notion of the value of the output of an asset in a particular period that is identical for all agents.

Even if there were a notion of fundamental value that overcame these difficulties, more serious difficulties arise when we move from symmetric to asymmetric uncertainty. Suppose we have a model with asymmetric information and an asset is being sold by one agent to a second in a particular period and, further, it will never be resold again. If we want to determine whether the price at which the asset is sold is higher than that warranted by the value of the output stream generated by the asset to the second agent, should we take expectations with respect to the seller's probability beliefs or with respect to the buyer's? Or should we perhaps use the join of their information? Or should we take into account somehow the information held by other agents?

It is our position that even if we restrict attention to a specific equilibrium, there is simply no compelling, unambiguous way of always assigning a fundamental value to an asset that reflects the consumption value of the stream of output generated by the asset. This does not mean that it is impossible to sometimes identify equilibria in which the price of some asset is higher than warranted by the value of the stream of output it generates, however. Even if we are unable to present a completely general definition of the fundamental value of an asset, there may be instances in which an asset's price is above that which would be consistent with *any* acceptable notion of fundamental value. Consider, for example, an asymmetric information model in which there is an equilibrium with fiat money which has positive value. Even without a general definition of the fundamental value of an asset, we are comfortable with a constraint that any plausible notion of fundamental value must assign value zero to an asset that with probability one generates no output valued by any agent.

As stated in the beginning of this section, the term bubble is commonly used to designate a situation in which, for some equilibrium, there is some asset and some period such that the price of the asset is above the fundamental value of the asset. However, our aim in this paper is to show that rational expectations equilibria in finite horizon, asymmetric information economies may exist in which prices are higher than those warranted by fundamentals. As argued above, there is no compelling definition of fundamental value for this case, so we restrict attention to a restricted class of economies in which there is a clear upper bound on any reasonable fundamental value one might assign to the assets. We analyze an economy with three periods, a single consumption good, and an asset that pays a single liquidating dividend in the final period. There is a finite number of agents each of whom consumes only in the final period. We will say a "strong bubble" exists if there is a state of the world such that, in that state, every agent knows (assigns probability 1 to the event) that the price of the asset is strictly above the liquidating dividend. Thus, when a strong bubble exists, every agent holding the asset for which there is a bubble does so with the (rational) expectation that there is positive probability that he will be able to sell that asset to a second agent at a price the first agent knows to be above the second agent's valuation of the asset. It is clear that even though there may be difficulty in unambiguously defining the fundamental value of the asset, any plausible notion would set it below the price in such a case.

We close this section by pointing out that the examples we consider do not depend on the particular simple structure of the assets that we assume. It will be clear that similar examples can be constructed with arbitrary numbers of periods and with assets that pay dividends in arbitrary periods. The examples use simple assets that pay a single liquidating dividend because it is especially clear in this case when the price for the asset is above any plausible fundamental value.

3. The Model

We consider a market with I risk averse or risk neutral agents: $i = 1, ..., I$. There are a finite number of states of the world represented by Ω. There are two assets, a riskless, divisible asset (money) and a risky asset. There exists a finite number of shares of the risky asset each of which will pay a dividend which depends on the state of the world. We represent by $d(\omega)$ the dividend per share to be paid in state $\omega \in \Omega$. Prior to the payment of the dividend, there is a finite number of periods in which the agents can exchange claims on the asset at a price p which depends upon the true state and the period. We will denote by $x_{it}(\omega)$ agent i's net trade in this asset in period t when ω is the realized state. Short sales of the risky asset (but not the riskless asset) are prohibited. Agent i's final consumption in state ω is $y_i(\omega)$ and he has expected utility function $\sum_{\omega \in \Omega} \pi_i(\omega)\, u_i[y_i(\omega)]$, each u_i concave, where π_i represents agent i's subjective beliefs; we assume each π_i strictly positive.[1] Agent i's information about the state of the world at the beginning of period $t = 1, ..., T$ is represented by a partition of the space Ω, S_{it}. We denote by $s_{it}(\omega)$ the event in S_{it} containing the state ω. This represents the exogenous information agent i has at time t when the state $\omega \in \Omega$ has occurred. We assume that the discount rate is zero, although it will be clear that this does not alter any conclusions.

A price function P associates with each state $\omega \in \Omega$ and each period $t = 1, ..., T$ a price $p = P(\omega, t)$. Agents can learn from prices: let $s_{it}^P(\omega) = s_{it}(\omega) \cap \{\omega' \mid P(\omega', t') = P(\omega, t') \text{ for all } t' \leqslant t\}$; $s_{it}^P(\omega)$ is the set of states i thinks possible at time t after observing prices in the first t periods.

We write x_i for agent i's net trades, $x_i = (x_{i1}, ..., x_{iT})$, each $x_{it}\colon \Omega \to \mathbb{R}$. Now if agent i has an initial endowment m_i of money and e_i of the risky asset, then final consumption is given by

$$y_i(\omega, P, x_i) = m_i + e_i P(\omega, T) + \sum_{t=1}^{T} x_{it}(\omega)[P(\omega, t+1) - P(\omega, t)].$$

[1] Although we restrict attention to the case in which each agent's preferences can be represented by an expected utility function, it is straightforward to generalize to arbitrary utility functions over final period consumption.

Agent i's net trades, x_i, are *information feasible* if i's net trade in each period t is measurable with respect to his price-refined information in that period, i.e., $s_{it}^P(\omega) \subset \{\omega' \mid x_{it}(\omega') = x_{it}(\omega)\}$. Agent i's net trades, x_i, satisfy *no short sales*, if agent i's period t holdings of the risky asset are non-negative in every period and state, i.e.,

$$e_i + \sum_{s=0}^{t} x_{is}(\omega) \geqslant 0, \qquad \text{for all} \quad \omega \in \Omega,\ t = 1,\ T.$$

Now write x for all agents' net trades, $x = (x_1, ..., x_I)$.

DEFINITION. (P, x) is a *rational expectations equilibrium* if

(a) Each agent i's net sales x_i are information feasible and satisfy no short sales.

(b) There do not exist net sales x_i', for agent i, satisfying information feasibility, no short sales, and

$$\sum_{\omega \in \Omega} \pi_i(\omega)\, u_i[y_i(\omega, P, x_i')] > \sum_{\omega \in \Omega} \pi_i(\omega)\, u_i[y_i(\omega, P, x_i)].$$

(c) The market clears $\sum_i x_{it}(\omega) = 0$.

(d) $P(\cdot, t)$ is measurable with respect to the join of the individuals' partitions at time t.

It is worthwhile to say a few things about these requirements before going on. First, the definition is quite standard. Part (a) simply says that for each agent and for each period, the trades are utility maximizing using the individual's private information and any information that may be contained in the price. Part (c) requires price and trades to be feasible given the totality of information available. Part (d) is the requirement that an agent must always take the same action when he has the same information set; alternatively stated, agents can learn from their own net trade.[2]

As discussed in Section 2, it is unclear what the natural notion of the fundamental value of an asset is in a world of risk averse agents. It is useful to consider two notions in this model.

We say that for a given equilibrium there is an *expected* price bubble in an asset if there is a state of the world such that when that state of the world is realized, the price of the asset is higher than every agent's marginal valuation of the asset. Formally, let

$$\alpha_i(\omega) = \frac{\pi_i(\omega)(du_i[y_i(\omega)]/dy_i)}{\sum_{\omega' \in \Omega} \pi_i(\omega')(du_i[y_i(\omega')]/dy_i)}.$$

[2] See Kreps [18] and Tirole [27] for a discussion of this assumption.

$\alpha_i(\omega)$ thus represents the marginal valuation of a unit of consumption to agent i in state ω, measured in terms of the riskless asset. Write $E_i[x \mid S] = \sum_{\omega \in S} \alpha_i(\omega)\, x(\omega) / \sum_{\omega \in S} \alpha_i(\omega)$, for any random variable x and subset $S \subset \Omega$. Now there is an *expected bubble* in state ω at time t if $\forall i$, $E_i[d \mid s_{it}^P(\omega)] < P(\omega, t)$. If agents are risk neutral, each $\alpha_i = \pi$ and there is an expected bubble if the price is strictly higher than each agent's expected value of the asset.

We say that, for a given equilibrium, there is a *strong* price bubble in an asset if there is a state of the world such that when that state of the world is realized, all agents know that the price of the asset is higher than its dividend. This represents a very conservative view of what constitutes a bubble by demanding that each agent know that with probability 1 the dividends the asset pays will be less than the current price. More formally, we say that the price function P exhibits a *strong bubble* in state ω at time t if $\forall i$, $\forall \omega' \in s_{it}^P(\omega)$, $d(\omega') < P(\omega', t)$.

Clearly, if there is a strong bubble, there is also an expected bubble.

4. Necessary Conditions for Bubbles

We discuss in this section a set of necessary conditions for the existence of bubbles in rational expectations equilibria. For expected bubbles, endowments must be ex ante inefficient (Tirole [27]) and every agent must be short sale constrained in some state at some time following the state and time where the bubble occurs (proposition 1). For a strong bubble, it is also necessary that all agents have some private information not revealed (proposition 2) and agents' actions are not common knowledge (proposition 3).

Tirole [27] proved an important result in a framework similar to that here:

Theorem (Tirole [27]). *If the initial allocation is ex ante efficient, there cannot be a bubble in a rational expectations equilibrium.*

Morris [22], following Harrison and Kreps [10], showed that the key to the existence of an expected bubble is when short sale constraints bind strictly. If in state ω at time t agent i assigns positive probability to being short sale constrained at some future time in some contingency, the price must be strictly higher than agent i's marginal valuation of the asset.

Proposition 1. *There exists an expected bubble at ω, t in a rational expectations equilibrium if and only if, for each i, there exists some $\omega' \in s_{it}^P(\omega)$, $t' \geq t$, such that agent i is strictly short sale constrained at ω', t'.*

Sketch of Proof (see Morris [10]). If $E_i[P(\cdot, t+1) \mid s_{it}^P(\omega)] > P(\omega, t)$, then i could increase utility by buying ε of the asset at ω, t and selling at $t+1$ whatever information he receives in period $t+1$. If i is not short sale constrained, then the converse argument holds if $E_i[P(\cdot, t+1) \mid s_{it}^P(\omega)] < P(\omega, t)$. So $E_i[P(\cdot, t) \mid s_{it}^P(\omega)] \leqslant P(\omega, t)$ for all ω, t, with strict inequality only if the short sale constraint holds strictly. Now, by induction, $E_i[P(\cdot, t') \mid s_{it}^*(\omega)] \leqslant P(\omega, t)$ for all ω, $t' \geqslant t$, with strict inequality for all i only if the short sale constraint holds strictly for all i, for some $\omega' \in s_{it}^P(\omega)$, $t \leqslant t'' < t'$. Now the proposition is true since $P(\omega, t) = d(\omega)$, $\forall \omega$.

COROLLARY. *There is no expected bubble if $t = T$ or $T-1$.*

Proof. No agent is short sale constrained in period T (the price is equal to the dividend). At least one agent is holding the asset, and is therefore not short sale constrained, at time $T-1$.

Proposition 1 gives a necessary condition for expected bubbles and applies to models without asymmetric information as well as to this model. Propositions 2 and 3 provide additional necessary conditions on agents' information in order to get strong bubbles.

As the discussion in the introduction made clear, strong bubbles can occur despite the fact that all agents know that the price is higher than any possible realization of the dividend if agents do not know that other agents know it. In other words, it must be the case that some private information is not revealed by prices in equilibrium.

Let m_t^P be the meet (finest common coarsening) of agents' information partitions after observing prices, $\{s_{it}^P\}_{i=1,I}$.

PROPOSITION 2. *If there is a strong bubble at ω, t in a rational expectations equilibrium, then each agent has private information at ω, t (i.e., $s_{it}^P(\omega) \neq m_t^P(\omega)$, $\forall i$).*

Proof. $P(\omega, t) = E_i[P(\cdot, t+1) \mid s_{it}^P(\omega)]$, for some i, so $P(\omega, t) \leqslant \max_{i \in I} \max_{\omega' \in s_{it}^P(\omega)} P(\omega', t+1)$. This implies, by induction, $P(\omega, t) \leqslant d(\omega')$ for some $\omega' \in m_t(\omega)$. If $s_{it}^P(\omega) = m_t(\omega)$, this implies $P(\omega, t) \leqslant d(\omega')$ for some $\omega' \in s_{it}^P(\omega)$, contradicting the existence of a strong bubble.

In a rational expectations equilibrium, agents learn from prices and also from their own net trades, but not necessarily from other agents' trades. If there are three agents, my trade tells me what the sum of the other two agents' trades is, but not each individually. But Geanakoplos [9] (Section III.4) has argued that, in general, common knowledge of actions (in this case, trades) "negates asymmetric information" in the sense that agents would have behaved in the same way without the private part of their information. But since proposition 2 implied that strong bubbles require asymmetric information, this means strong bubbles must require no common knowledge of trades as well.

Agents' trades are common knowledge if x_{it} is always measurable with respect to s_{jt}^{P}, for all i, j or equivalently if x_{it} is measurable with respect to m_t^P, for all i.

PROPOSITION 3. *If there is a strong bubble in rational expectations equilibrium, then agents' trades are not common knowledge.*

Proof. If agents' trades are common knowledge, then agents' trades at time t must be measurable with respect to the meet of their information m_t^P. But now consider the economy where each agent's initial information at each state and date, $s_{it}(\omega)$, is replaced by $m_t^P(\omega)$ (so there is no asymmetric information). Then the same prices and trades would still be an equilibrium. But now, by proposition 2, there can be no strong bubble.

COROLLARY. *If there is a strong bubble in rational expectations, there must be at least three agents.*

Proof. If agents know their own actions, there are only one or two agents, and markets clear, then their actions are common knowledge and there is no strong bubble by proposition 3.

5. EXAMPLE

Section 4 contains necessary conditions for bubbles. We can give a partial converse by showing the existence of bubbles in economies where these conditions are satisfied. The example may seem excessively complicated, but it contains essentially the minimal structure to get a bubble. We showed that there must be at least three agents and at least three time periods and information must not be fully revealed by prices. As Tirole showed that there cannot be even an expected bubble if there are no gains to trade, any example of a bubble must have something that generates such gains from trade. There are at least four simple ways to generate possible gains from trade: (1) let agents have heterogeneous beliefs, (2) let agents have state-dependent utility functions, (3) let agents have random endowments and identical concave utility functions, (4) let agents have identical non-random endowments and different concave utility functions. We show that examples with gains from trade arising from any of these sources can exhibit strong bubbles.[3]

For pedagogical reasons, we begin with the case of heterogeneous beliefs. While some may be skeptical of a phenomenon that stems from heterogeneous beliefs, this case demonstrates most clearly how bubbles may arise. Further, we show how our example in which agents have heterogeneous beliefs can be modified to create examples with a strong bubble in which the initial gains from trade are generated by something other than heterogeneous beliefs.

5.1. *Heterogeneous Beliefs*

Before giving the example formally, we describe it in words. There are three agents and three periods. Each agent will hold shares in an enterprise that will pay a liquidating dividend in the last period. The liquidating dividend will be either 0 or 12. In the event that the dividend will be 0, some agents may be informed that this is the case. If an agent is informed that the dividend will be 0, he will not know whether or not other agents have also been informed as well. There will be trade in the asset in each of the first two periods; in the third period, the dividend is realized and consumption takes place.

In the example, there is one state of the world where every agent knows that the price is with certainty higher than the liquidating dividend, yet each agent holds the asset. It is rational for agents to act this way because while each agent knows the asset is "overpriced," he does not know whether other agents also know. Because they do know, those holding the asset will lose money. But if only one agent had known the asset was overpriced, he would have been able to make a profit by reselling the asset at a yet higher price before the price falls. On balance, each agent is indifferent between holding the asset and selling it.

Formally, there are 11 states of the world, $\Omega = \{\omega_0, \omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8, \omega_9, \omega_{10}\}$ and three agents (A, B, and C) each initially holding four shares in an enterprise which will pay a liquidating dividend that depends on the state of the world as shown in the table below.

State	ω_0	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9	ω_{10}
Dividend	0	0	0	0	0	0	0	0	12	12	12

In keeping with the description of the example above, agents will be asymmetrically informed about the state of the world, the asymmetry

[3] Another way to generate gains from trade is if there are portfolio managers who invest on other people's behalf; the resulting agency problems can also lead to bubbles. This possibility is discussed below in Section 6.2 and in Allen and Gorton [2].

being captured by different agents having different partitions over Ω. We designate by S_{ij} the event in agent i's partition ($i = A, B, C$) in period $j = 1, 2, 3$. For our example the partitions are given by

$$S_{A1} = \{\omega_1, \omega_4\}\{\omega_0, \omega_2, \omega_3, \omega_5, \omega_6, \omega_7, \omega_8, \omega_9, \omega_{10}\}$$
$$S_{B1} = \{\omega_1, \omega_3\}\{\omega_0, \omega_2, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8, \omega_9, \omega_{10}\}$$
$$S_{C1} = \{\omega_1, \omega_2\}\{\omega_0, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8, \omega_9, \omega_{10}\}$$

$$S_{A2} = \{\omega_0\}\{\omega_1, \omega_4\}\{\omega_2, \omega_3, \omega_7, \omega_{10}\}\{\omega_5, \omega_8\}\{\omega_6, \omega_9\}$$
$$S_{B2} = \{\omega_0\}\{\omega_1, \omega_3\}\{\omega_2, \omega_4, \omega_6, \omega_9\}\{\omega_5, \omega_8\}\{\omega_7, \omega_{10}\}$$
$$S_{C2} = \{\omega_0\}\{\omega_1, \omega_2\}\{\omega_3, \omega_4, \omega_5, \omega_8\}\{\omega_6, \omega_9\}\{\omega_7, \omega_{10}\}$$

$$S_{A3} = S_{B3} = S_{C3}$$
$$= \{\omega_0\}\{\omega_1\}\{\omega_2\}\{\omega_3\}\{\omega_4\}\{\omega_5\}\{\omega_6\}\{\omega_7\}\{\omega_8\}\{\omega_9\}\{\omega_{10}\}.$$

To see where we are going, there will be a bubble in state ω_1. In this state agent A will have observed the event $\{\omega_1, \omega_4\}$; both states result in a liquidating dividend of 0. If the state is ω_1, the other two agents will also know the final dividend will be 0. However, if the state is ω_4, only agent A will know that the final dividend will be 0. Similarly, in state ω_1 agents B and C will know the final dividend will be 0 but will not know whether other agents know this as well.

All agents are risk neutral and have prior weights on the states given by (divide by 22 for probabilities)

State	ω_0	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9	ω_{10}
Agent A	10	1	1	1	1	1	1	1	1	1	3
B	10	1	1	1	1	1	1	1	1	3	1
C	10	1	1	1	1	1	1	1	3	1	1

As easily seen, the agents agree on the relative likelihood of states 0 through 7, but disagree about states 8, 9, and 10.

There are multiple equilibria for this example. The one we are interested in—the one in which there is a strong bubble—has equilibrium prices given in the following table.

State	ω_0	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9	ω_{10}
Period 1	3	3	3	3	3	3	3	3	3	3	3
2	0	0	6	6	6	6	6	6	6	6	6
3	0	0	0	0	0	0	0	0	12	12	12

Note first that the only information revealed by prices is in period 2 where agents discover whether all agents were informed that the enterprise was bankrupt in the previous period. Period 2 information, refined by information revealed by prices, is thus

$$S^P_{A2} = \{\omega_0\}\{\omega_1\}\{\omega_4\}\{\omega_2, \omega_3, \omega_7, \omega_{10}\}\{\omega_5, \omega_8\}\{\omega_6, \omega_9\}$$

$$S^P_{B2} = \{\omega_0\}\{\omega_1\}\{\omega_3\}\{\omega_2, \omega_4, \omega_6, \omega_9\}\{\omega_5, \omega_8\}\{\omega_7, \omega_{10}\}$$

$$S^P_{C2} = \{\omega_0\}\{\omega_1\}\{\omega_2\}\{\omega_3, \omega_4, \omega_5, \omega_8\}\{\omega_6, \omega_9\}\{\omega_7, \omega_{10}\}.$$

For this equilibrium, agents do not trade in the first period; their net trades in the second period are

State	ω_0	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9	ω_{10}
Agent A	0	0	+2	+2	−4	−1	−1	+2	−1	−1	+2
B	0	0	+2	−4	+2	−1	+2	−1	−1	+2	−1
C	0	0	−4	+2	+2	+2	−1	−1	+2	−1	−1

There is no trade in the third period. Note that agents know their own actions (a requirement for REE) but actions are not common knowledge (which would make a strong bubble impossible). When A buys two units of the asset in state ω_2 in period 2, he does not know whether he is buying them both from B (i.e., he is in state ω_2), both from C (i.e., he is in state ω_3), or one from each of B and C (i.e., he is in states ω_7 or ω_{10}). Since the agents initially had four units each of the risky asset, their holdings of the risky asset going into period 3 are now

State	ω_0	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9	ω_{10}
Agent A	4	4	6	6	0	3	3	6	3	3	6
B	4	4	6	0	6	3	6	3	3	6	3
C	4	4	0	6	6	6	3	3	6	3	3

To see that these prices and asset positions constitute an equilibrium, we begin by observing that in the third period the prices are clearly the only prices consistent with the state contingent dividends of the asset and the fact that each agent has perfect information in this period. Next, note that in the second period, if agent A observed an event other than $\{\omega_1, \omega_4\}$, agent B observed an event other than $\{\omega_1, \omega_3\}$, or agent C observed an event other than $\{\omega_1, \omega_2\}$, the price conveys no information beyond that the agents already possessed. Also note that for every event other than these three, the expected price in the third period is equal to the price in the second period (the expectation being taken over each of the possible events that either agent could have observed), so agents are indifferent between holding the asset or not. For example, if the true state is ω_7, A observes $\{\omega_2, \omega_3, \omega_7, \omega_{10}\}$ in the second period, while B and C observe $\{\omega_7, \omega_{10}\}$, but all of them think there is a 1/2 chance of ω_{10}, so the price is 1/2 of 12, i.e., 6.

If any agent observes in the second period the event that contains ω_1, the price will be different for the two states contained in this event. Thus, his private information plus that revealed by the price will give him perfect information. If the state is ω_1, the price of the asset is 0, making each agent indifferent to buying, selling, or holding the asset. If he sees this event but the state is not ω_1, the price will be above the value (and hence, the price of the asset next period) and he will want to sell any of the asset he owns. The restriction on short sales means that he can sell no more than his holdings of the asset in this period. Thus, the prices in the second period are consistent with equilibrium and the prices in the third period.

It is clear that the price reveals no information in the first period since it is constant across the states. It is straightforward to verify that for every state of the world and for each individual the expected price in the second period is equal to 3, the price of the asset in the first period (the expectation being taken over an individual's observed event). Hence, the prices above constitute a rational expectations equilibrium.

There are several points to observe about the example.

1. It is possible that a bubble exists so that all agents know that the price of an asset will fall in the future but are still willing to hold the asset.

To see this, suppose that state ω_1 arises. In this state all agents observe an event which guarantees that the asset has value 0 yet each is willing to hold (or buy) the asset at price 3.

2. All agents may know that the asset will drop in price and yet it is not common knowledge.

When agent A sees the event $\{\omega_1, \omega_4\}$, he knows that the true state of the world is either ω_1 or ω_4. If the state is ω_1, agent B has observed the event $\{\omega_1, \omega_3\}$ and thus knows the value to be 0. If the state is ω_4, however, agent B has observed the event $\{\omega_0, \omega_2, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8, \omega_9, \omega_{10}\}$; his expected value of the asset, in period 1, given this information is 3. In period 2, agent B will observe the event $\{\omega_2, \omega_4, \omega_6, \omega_9\}$ and the expected value for the asset will go up to 6. A similar argument holds for agent C. Thus if the state is ω_1, all agents know that the value of the asset is 0, yet none knows whether the others know this.

3. There is a second equilibrium with an expected bubble.

Consider the prices given below.

State	ω_0	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9	ω_{10}
Period 1	4	0	0	0	0	4	4	4	4	4	4
2	0	0	0	0	0	9	9	9	9	9	9
3	0	0	0	0	0	0	0	0	12	12	12

It can easily be checked that the prices given in this table constitute a second equilibrium in which there is an expected bubble. Here, if any of the three agents receives a signal that the asset has value 0, the price immediately reflects this fact.

Finally, note that the heterogeneity of agents' beliefs was required to ensure that agent A particularly liked the shares given event $\{\omega_7, \omega_{10}\}$, B particularly liked the shares given event $\{\omega_6, \omega_9\}$, and C particularly liked the shares given event $\{\omega_5, \omega_8\}$. In the examples which follow agents have common prior beliefs, but state-dependent utility functions and differences in endowments or risk aversion can generate the same pattern.

5.2. *State-Dependent Utility Functions*

The above example relied on heterogeneous beliefs to provide expected gains from exchange. As we said prior to presenting the example, it can be modified so as to accommodate homogeneous beliefs so long as there is something else that provides expected gains from trade. It is straightforward to see how to alter the above example so that the basis for the expected gains from trade stem from state-dependent utility functions rather than from heterogeneous beliefs. Let the agents have homogeneous beliefs with relative weights as given in the table below.

State	ω_0	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9	ω_{10}
Agent A	10	1	1	1	1	1	1	1	1	1	1
B	10	1	1	1	1	1	1	1	1	1	1
C	10	1	1	1	1	1	1	1	1	1	1

These are the same as the relative beliefs in the previous section except that the agents now agree on states 8, 9, and 10.

Now let agent A have utility function $u(x) = x$ for states $i \neq 10$ and $u(x) = 3x$ in state 10; agent B have utility function $u(x) = x$ for states $i \neq 9$ and $u(x) = 3x$ in state 9; and agent C have utility function $u(x) = x$ for states $i \neq 8$ and $u(x) = 3x$ in state 8. Each agent's expected utility of any bundle will be the same as in the example above since the only change from that example is that for each agent there is one state in which we have divided the probability of the state by 3 and multiplied the utility of consumption in that state by 3. Thus, the expected utility of each bundle is as before and, consequently, the set of equilibria is unchanged.

5.3. *Concave State-Independent Utility Functions and State-Dependent Endowments*

We can modify the example in the section above that Ras state-dependent utility functions to generate an example in which the utility functions are (weakly) concave and initial endowments are state dependent. In both the initial example with heterogeneous beliefs and the modification to homogeneous beliefs and state-dependent utility functions, we ignored the initial endowment of the risk-free asset. Since the utility functions were linear, all that matters is that the initial endowments of the risk-free asset are sufficient for the agents to carry out the trades specified in the equilibria. However, once the utility functions are not linear, the demand for the risky asset may depend upon the initial endowment of the risky asset.

Suppose now that we let each agent's utility function be piecewise linear of the following form:

$$u(x) = \begin{cases} 3x - 2\bar{x} & \text{if } x \leqslant \bar{x} \\ x & \text{if } x > \bar{x}. \end{cases}$$

If agents' endowments in each state are sufficiently large (relative to $\bar{x}$), each agent's consumption will be above $\bar{x}$ with probability 1 in every state. Suppose now that agent A has constant endowment that causes this to be so in every state except 10, and in this state his endowment is sufficiently small that consumption will be below $\bar{x}$ with probability 1. Suppose analogously that agent B's endowment is sufficiently large in every state except ω_9, and low in this state, that consumption will always be above $\bar{x}$ in all states except ω_9, in which case it is certainly below $\bar{x}$; last, suppose agent C's endowment to be random so that he has small endowment only in state ω_8.

With these utility functions and random endowments, each agent's expected utility of any net trade is precisely the same as in the previous example with state-dependent utility functions. This, of course, is not surprising; one can, in general, transform examples involving state-dependent utility functions into examples in which the state dependence has been transferred from the utility functions to the endowments. In any case, since the expected utility of the relevant net trades is identical here and in the previous example, the set of rational expectations equilibria must be the same.

5.4. *State-Independent Utility Functions and Endowments*

The example in the previous section demonstrated that the prices and net trades from the original example with heterogeneous beliefs can be maintained as equilibrium prices and net trades by appropriately choosing a (common) state-independent utility function and random initial endowments. In altering the original example to accomplish this, we chose endowments and the utility function so that all the expected marginal conditions that characterize an equilibrium were identical in the two examples. We could dispense with the random initial endowments and let the three agents' utility functions differ and still accomplish the same goal; we would simply have to find for each agent a utility function that had the appropriate marginal utility at each of the finite number of relevant wealth levels to do this. There is one difficulty though; a utility function constructed in this way would not necessarily be concave. There is no reason to expect that the necessary marginal utilities would be decreasing in the level of wealth. In fact, if we were to take the initial example in Section 5.1, the necessary marginal utilities would *not* be decreasing.

In order to demonstrate that the bubble phenomenon may arise even in the case that agents have non-random initial endowments and state-independent utility functions, we construct such an example. Because of the extra constraints involved in constructing such an example, it is somewhat more complicated than the previous examples. The basic idea is simple: maintain the state space and net trades from the previous example, but alter the dividend structure, beliefs, and prices so that the necessary marginal utilities (for the net trades to be part of a rational expectations equilibrium) are decreasing.

The state space, information structure, and agents' asset trades will be exactly as in the example of Section 5.1. The share dividends and the agents' common prior are given by

State	ω_0	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9	ω_{10}
Dividend	35	35	35	35	35	32	24	0	44	72	180
Common prior	$1-10\alpha$	α	α	α	α	α	α	α	α	α	α

where $\alpha = 49/939 \approx 0.052$.

It is sufficient to identify marginal utilities at levels where consumption takes place in equilibrium. It will be seen that it is sufficient for agents to have utility functions such that marginal utilities are

Level of consumption	A	B	C
228	101		
336		104	104
372		75	
408	84		84
420			57
432	68	72	
438	54	54	54
440	51	51	51
444	49	49	49
468	34	36	
492			42
552	28		28
660		28	
876		26	26
1308	26		

Suppose agents initially have 300 units of the riskless asset. The following prices will constitute an equilibrium with the asset trades given in the example in Section 5.1:

State	ω_0	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9	ω_{10}
Period 1	35.49	35.49	35.49	35.49	35.49	35.49	35.49	35.49	35.49	35.49	35.49
2	35	35	36	36	36	36	36	36	36	36	36
3	35	35	35	35	35	32	24	0	44	72	180

Given the asset trades given in Section 5.1, this implies a final asset position of

State	ω_0	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9	ω_{10}
Agent A	440	440	438	438	444	432	408	228	468	552	1308
B	440	440	438	444	438	432	372	336	468	660	876
C	440	440	444	438	438	420	408	336	492	552	876

This implies marginal utilities of final consumption

State	ω_0	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9	ω_{10}
Agent A	51	51	54	54	49	68	84	101	34	28	26
B	51	51	54	49	54	72	75	104	36	28	26
C	51	51	49	54	54	57	84	104	42	28	26

Now it can be verified that each agent's expected value of the next period price of the asset, evaluated using these marginal utilities as a probability measure, is equal to the current price, *except* when he is short sale constrained (at states ω_4, ω_3, and ω_2, respectively, for agents A, B, C). To check, note that

State	ω_0	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9	ω_{10}
$P(\omega, 2) - P(\omega, 1)$	−0.49	−0.49	0.51	0.51	0.51	0.51	0.51	0.51	0.51	0.51	0.51
$P(\omega, 3) - P(\omega, 2)$	0	0	−1	−1	−1	−4	−12	−36	8	36	144

6. Discussion

6.1. *Genericity*

Clearly the examples are special. But the necessary conditions for such a bubble in Section 4 make it clear what is special about them. The key is the non-revelation of information in rational expectations equilibrium, a result which in more general models depends, generically, on the number of prices relative to the number of sources of uncertainty. In any case, one should be careful about interpreting results on genericity of full revelation of information in rational expectations equilibrium. The idea behind generic full revelation in a model such as that in this paper is that small perturbations of the state-dependent dividends would result in different prices in each state. The interpretation is then that it would be "a coincidence" that the dividends were precisely such that prices failed to fully reveal. For example, in the example in Section 5.1. the dividend was assumed to be 0 in states 0 through 7. These can be altered slightly so that prices are fully revealing, hence the bubble could not endure.

One should not draw the conclusion that bubbles are therefore anomalous. It is well understood that the argument for revelation of all private information through prices in a rational expectations equilbrium relies on the relative dimensions of the commodity space and Ω, the set of states of the world. Rational expectations equilibria will not typically be revealing in models with a rich enough uncertainty structure, that is, a model with sufficiently many independent sources of uncertainty.[4] If one began with a model with such a rich set of uncertainty, one could presumably construct examples of bubbles similar to ours that were completely immune to any genericity concerns. The cost, of course, is both the complication in constructing the example and the likely loss of insight as to the source of the bubble.

[4] Ausabel [3] provides an example of such a model.

In any case there is some ambiguity about the relevant notion of genericity. The uncertainty in the example should be thought of as of two types: uncertainty about fundamentals and uncertainty about what other agents know. The example should be thought of as having two "physical" states of the world that embody all the uncertainty about fundamentals; either the dividend will be 0 or it will be 12. The expansion of the state space from 2 states to 11 is to model the uncertainty an agent has about what other agents know about the fundamental. But with this interpretation, the perturbations of the dividends that are "natural" are perturbations that leave the dividend constant on the events $\{\omega_0, \omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7\}$ and $\{\omega_8, \omega_9, \omega_{10}\}$; there is no reason to believe that the dividend might be slightly different in the case that I know that someone else knows the value and in the case that I do not know.

If one accepts the interpretation that there are only two fundamental states of the world and, hence, only two values of the dividend to be perturbed, the lack of genericity in the example in Section 5.1 disappears: one can perturb the two dividend values 0 and 12 and there will still be a bubble. In a similar manner, the modifications of that example to the case of state-dependent utility functions and random endowments (Sections 5.2 and 5.3) will also survive perturbations of the dividend values.[5]

6.2. *Ex Ante Inefficiency*

Milgrom and Stokey [21] proved a result that is sometimes called the no-trade theorem. It says that if two agents agree on an ex ante efficient allocation of goods, then after they get new information, there is no possibility of a transaction with the property that it is common knowledge that both agents are willing to carry out the transaction and at least one agent strictly prefers the transaction to no (further) trade. Our example is not in conflict with this result as the initial distribution of the asset was not efficient in the example. To the extent that there is a bubble of the sort exhibited in the example, we are compounding socially useful trade (efficient distribution of risk among the agents) with superfluous gambles. If agents are strictly risk averse, there is a social loss to that part of trade that is essentially a gamble. There would, in a sense, be a social gain to the elimination of the gambling aspect. That does not prevent speculation from occurring in equilibrium, however. Agents will still find it rational to trade in a world with bubbles so long as the gain from the beneficial redistribu-

[5] It should be pointed out, however, that the last example in Section 5.4 with neither state-dependent utility functions nor random endowments would typically not survive. In that case, the utility functions were constructed with the appropriate marginal utilities at the finite number of relevant consumption points. Perturbations of the dividends would alter those marginal utilities unless the utility functions were locally linear at those points.

tion of risk outweighs the loss from the costly gambles associated with the bubbles.

If equilibria without bubbles have efficiency advantages over those exhibiting bubbles, perhaps agents have a way to assure that only those equilibria without bubbles arise. For example, agents might simply trade to an efficient distribution of the assets immediately. Once the allocation is efficient, the no-trade theorem applies, and no further trade would occur (at least no trade that is socially detrimental). There are two responses to this. First, our models typically do not allow analysis of which of several equilibria in a model might arise. Given any equilibrium and its path of prices over time, each agent by definition is doing as well as he can.

There is a second and more important reason to be unconvinced by the suggestion that there might be trade to an efficient distribution of assets with no further trade necessary. This would be possible in an Arrow-Debreu world with complete state-contingent markets, but in a world of incomplete markets there may be no distribution of assets that is not only efficient at the present time, but will remain efficient over time with probability one. As the uncertainty in the environment unfolds, it may be necessary for there to be redistributions of the assets to regain an efficient distribution of risk. Even if markets were essentially complete through dynamic trading strategies, there would be no reason that the sort of bubbles exhibited in the example would be ruled out.

This last point is well illustrated by the initial example in Section 5.1. In that example, agents had no incentive to trade the asset in the first period. All the gains from trade stemmed from their different values for the asset in states ω_8, ω_9, and ω_{10}. But these states cannot be distinguished with the information available to any agent in the first period. Thus, if we consider the constrained market structure in that example (that is, that the asset can be traded, but not state-contingent future contracts on the asset), the initial endowment is constrained Pareto efficient.

Another form of ex ante inefficiency which can lead to bubbles is when intermediaries allocate funds on behalf of investors; in this case agency problems mean that trading can be a positive-sum game for some market participants even though it is a zero-sum game overall. Allen and Gorton [2] consider a model of this type where there are good and bad portfolio managers who cannot be distinguished by investors; good portfolio managers can identify undervalued stocks but bad ones cannot. The bad portfolio managers pool with the good ones in equilibrium. There is limited liability and portfolio managers have no wealth of their own so the optimal compensation contract is effectively a call option. This contract form means the bad portfolio managers are prepared to buy assets with a negative expected value and, as a result, bubbles are possible. Investors earn their opportunity cost and the good portfolio managers subsidize the losses of the bad ones. This type of rationale for ex ante inefficiency could also be incorporated in an example similar to that above.

6.3. *Concluding Remarks*

Examples have been presented where the backward induction argument that stock prices reflect fundamental values fails. Bubbles can exist because agents are unaware of other agents' beliefs; beliefs are not common knowledge. Even though they all know an asset is priced above its fundamental value, they can all rationally believe that they may be able to sell it to somebody else at a higher price before its true value is publicly revealed. Although the particular example presented was rather special, it was argued that the phenomenon it illustrated can occur in a wide range of situations.

REFERENCES

1. A. ABEL AND G. MAILATH, "Financing Losers in Competitive Markets," *J. Fin. Intermediation* (1993), in press.
2. F. ALLEN AND G. GORTON, Churning bubbles, *Rev. Econ. Stud.* (1993), in press.
3. L. AUSABEL, Partially revealing rational expectations equilibrium in a competitive economy, *J. Econ. Theory* **50** (1990), 93–126
4. S. BHATTACHARYA AND B. LIPMAN, Ex ante versus interim rationality and the existence of bubbles, mimeo, Carnegie Mellon University, 1990.
5. C. CAMERER, Bubbles and fads in asset prices: A review of theory and evidence, *J. Econ. Surveys* **3** (1989), 3–41.
6. J. B. DE LONG, A. SHLEIFER, L. SUMMERS, AND R. WALDMANN, Noise trader risk in financial markets, *J. Polit. Econ.* **98** (1990), 703–738.
7. M. FLAVIN, Excess volatility in financial markets: A reassessment of the empirical evidence, *J. Polit. Econ.* **91** (1983), 929–956.
8. P. GARBER, Famous first bubbles, *J. Econ. Persp.* **4** (1990), 35–54.
9. J. GEANAKOPLOS, Common knowledge, *in* "4th Conference on Theoretical Aspects of Reasoning about Knowledge" (Y. Moses, Ed.), Morgan Kaufman, Palo Alto, 1992.
10. M. HARRISON AND D. KREPS, Speculative investor behavior in a stock market with heterogeneous expectations, *Quart. J. Econ.* **89** (19,o), 519–542.
11. M. JACKSON AND J. PECK, Speculation and price fluctuations with private, extrinsic signals, mimeo, Northwestern University, 1990.
12. J. KEYNES, "The General Theory of Employment, Interest and Money," Macmillan, London, 1936.
13. C. KINDLEBERGER, "Manias, Panics and Crashes," Basic Books, New York, 1978.
14. A. KLEIDON, Variance bounds tests and stock price valuation models, *J. Polit. Econ.* **94** (1986), 953–1001.
15. A. KLEIDON, Bias in small sample tests of stock price rationality, *J. Bus.* **59** (1986b), 237–261.
16. N. KOCHERLAKOTA, Bubbles and constraints on debt accumulation, *J. Econ. Theory*, **57** (1992), 245–256.
17. A. KRAUS AND M. SMITH, Beliefs about beliefs: The effect on equilibrium of non-payoff-relevant uncertainty, mimeo, University of British Columbia, 1990.

18. D. KREPS, A note on 'fulfilled expectations' equilibria, *J. Econ. Theory* **14** (1977), 32–44.
19. S. LEROY AND R. PORTER, The present value relation: Tests based on implied variance bounds, *Econometrica* **49** (1981), 555–574.
20. T. MARSH AND R. MERTON, Dividend variability and variance bounds tests for rationality of stock prices, *Amer. Econ. Rev.* **76** (1986), 483–498.
21. P. MILGROM AND N. STOKEY, Information, trade and common knowledge, *J. Econ. Theory* **26** (1982), 17–27.
22. S. MORRIS, Does the option to re-sell assets and durable goods raise their prices? mimeo, University of Pennsylvania, 1992.
23. P. SAMUELSON, An exact consumption-loan model of interest with or without the social contrivance of money, *J. Polit. Econ.* **66** (1958), 467–482.
24. R. SHILLER, Do stock prices move too much to be justified by subsequent changes in dividends? *Amer. Econ. Rev.* **71** (1981), 421–436.
25. R. SHILLER, "Market Volatility," MIT Press, Cambridge, MA, 1989.
26. A. SHLEIFER AND L. SUMMERS, The noise trader approach to finance, *J. Econ. Persp.* **4** (1990), 19–33.
27. J. TIROLE, On the possibility of speculation under rational expectations, *Econometrica* **50** (1982), 1163–1181.
28. K. WEST, Bubbles, fads and stock price volatility tests: A partial evaluation, *J. Finan.* (1988), 639–656.