Addendum to
"Speculative Attacks and Risk Management"

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This addendum contains unreported calculations for the main text.

Section 7.1: Attracting Foreign Investment

Consider a model where the third party are foreign lenders and there are domestic firms. The model has two periods: $t = 1, 2$ and four types of agents: domestic entrepreneurs/firms, domestic consumers, foreign investors/foreign speculators, and the government.

Let "*" denote tradable good. The utility function of foreign investors or speculators is given by:

$$U^{\text{foreigners}} = c_1^* + c_2^*,$$

while that of domestic residents and entrepreneurs is

$$U^{\text{domestic}} = c_1 + c_2 + u(c_2^*),$$

where (unless otherwise stated) $u(\cdot)$ is strictly concave and $u(c_2^*) = -\infty$ for $c_2^* \leq 0$.

Timing:

The game starts with a pre-determined peg e and level of foreign-owned liabilities (in pesos) D due at date 1.

Date 1:

(i) Government privately learns foreign reserves $\mathcal{R}$ which are either $\mathcal{R}_s$ with probability ρ or $\mathcal{R}_w$ with probability $1 - \rho$, where $\mathcal{R}_s > \mathcal{R}_w$.

(ii) D is repaid to foreign investors.
The foreign speculators take $\mathcal{S}$ out of the country. $\mathcal{S}$ is the size of the attack, which we allow to be either unbounded (unlimited short sales) or bounded by D (no short sales).
The government observes $\mathcal{S}$ and maintains the peg (converts $\mathcal{S}$ at rate e) or lets the currency float by refusing to convert.

(iii) The foreign exchange market updates its beliefs to ρ', which depends on whether the government maintained the peg. The forward exchange rate is denoted f. Domestic entrepreneurs finance equipment in tradables by borrowing abroad at rate f.

Date 2:

(iv) Entrepreneurs produce non-tradables, the foreign exchange market clears (the remaining reserves are equal to the net demand for tradables), and consumption takes place.

As in the first application, the existence of an initial debt D in pesos establishes a lower bound on the volume of funds that can be mobilized for a speculative attack: when reimbursed at date 1, foreign investors can roll over or take the money out of the country. D could also denote the amount of money that foreigners could obtain by selling their equity portfolio at date 1 or the peso-denominated collateral (e.g. real estate) seized by holders of dollar debt in defaulting firms.[1] In order to abstract away from balance-sheet effects, assume that this debt is paid by parties who are private and are different from the entrepreneurs who will need to raise funds at substage (iii).

At stage (iii), the market puts posterior probability ρ' on fundamentals being strong. With unlimited short sales or when the short sales constraint is not binding, the forward rate is f where

$$\frac{1}{f} = E_{\rho'}\left[\frac{1}{e_2}\right]. \tag{1}$$

Domestic entrepreneurs borrow $I(f)$ in tradables (with $I' < 0$) from foreigners. The proceeds of this investment at date 2 will be in non-tradables, of which $rI(f)$ will be returned back to foreign investors. Here the entrepreneur borrows in domestic currency, and the foreign lenders will need to convert into dollars at date 2. Equivalently, the foreign lenders can hedge the exchange rate risk at date 1 or the entrepreneur could borrow in dollars and hedge or not the exchange rate risk. Because we are not interested in default, all arrangements are equivalent.

[1] Dollar debt that is repaid to investors can be directly counted in $\mathcal{R}$ and so we do not need to consider it.

	Date 1 (present)		**Date 2** (future)
(i)	(ii)	(iii)	(iv)
Government privately learns foreign reserves. $\mathcal{R} = \begin{cases} \mathcal{R}_s & (\rho) \\ \mathcal{R}_w & (1-\rho) \end{cases}$ where $\mathcal{R}_s > \mathcal{R}_w$.	• Speculators non-cooperatively mount speculative attack $\mathcal{S}$. • Government observes $\mathcal{S}$ and maintains peg (converts $\mathcal{S}$ at rate e) or lets currency float (refuses to convert).	• The market updates its beliefs to ρ'. • Domestic entrepreneurs finance projects using tradables by borrowing $I(f)$.	• Entrepreneurs produce non-tradables, and pay back $rI(f)$ to foreign investors. • Foreign exchange market clears at exchange rate e_2. • Consumption takes place.

Figure A2.– Timing.

The market-clearing equation for type $i \in \{w, s\}$ at date 2 is:

$$\mathcal{R}_i - \frac{\mathcal{S}y_i}{e} - \frac{D - \mathcal{S}y_i}{e_i} = c_2^*(e_i) + \frac{r}{e_i}I(f), \tag{2}$$

where $y_i = 1$ if type i defends the peg and $y_i = 0$ if it lets the currency float.

Finally, the government's objective function is assumed to take the following form:

$$W = \pi(I, \alpha) + v(e_2), \tag{3}$$

where α is a parameter indexing the weights attached to investment,

$$\frac{\partial \pi}{\partial I} > 0 \quad \text{and} \quad \frac{\partial^2 \pi}{\partial \alpha \partial I} > 0.$$

The government's welfare improves when there is greater investment and the size of the improvement is increasing in the weight the government places on domestic entrepreneurs. The size of α governs the tension between entrepreneurs and residents in the government's objective function. It is this competition between political stakeholders that we seek to exploit in this model.

Corporate finance microfoundations

Let us show how the government's objective function (3) results from a simple weighting of corporate and non-corporate interests with a standard corporate finance determination of investment. (The specific functional form analyzed in this example will be used only in Propositions $A6$ and $A8$ and so the reader may skip this example).

A fraction γ of residents are entrepreneurs and a fraction $1-\gamma$ non-entrepreneurs (called "consumers"; naturally entrepreneurs also consume). The government puts weight α on the utility of the representative entrepreneur and 1 on the utility of the representative consumer. The proceeds are distributed equally among domestic residents. Thus the government's objective function is:

$$W_G = \alpha\gamma U^{\text{entrepreneurs}} + (1-\gamma)U^{\text{other residents}}.$$

The representative entrepreneur has initial wealth a in tradables, invests a variable i tradables, and produces $R(i)$ units of non-tradables, $[R(i) - ri]$ of which are non-observable by the investors (and therefore appropriated by the entrepreneur, as in, say Bolton and Scharfstein (1990)), and ri is collateral (such as real estate, etc.) that can be seized by investors. Let $R(\cdot)$ be concave. We focus on the case in which firms are financially constrained.[2]

[2] The equilibrium investment $i = \frac{a}{1-\frac{r}{f}}$ must thus be assumed to be smaller than investment i^* that maximizes the entrepreneur's date-2 peso wealth. Given that the entrepreneur must contribute $i\left(1-\frac{r}{f}\right)$ units of tradables at date 1, i^* solves:

$$\max_i \left\{ [R(i) - ri] - i\left(1 - \frac{r}{f}\right) E(e_2) \right\},$$

where $E[e_2]$ is the stage (e) expectation of the date-2 exchange rate and $\frac{1}{f} = E\left(\frac{1}{e_2}\right)$. Therefore,

$$R'(i^*) = E(e_2) - r\left[E(e_2)E\left(\frac{1}{e_2}\right) - 1\right].$$

Recalling that f denotes the (forward) exchange rate at stage (e), the foreign investors' break-even condition is:[3]

$$\frac{ri}{f} = i - a, \quad \text{or} \quad i = \frac{a}{1 - \frac{r}{f}}.$$

Let

$$I(f) \equiv \gamma i = \frac{\gamma a}{1 - \frac{r}{f}},$$

denote per capita investment and note that the expectation of a strong currency facilitates financing.

Assuming that firms face solvency constraints is one way of generating rents for the corporate sector and therefore a political stake. Furthermore, the rents depend on the degree of access to the international capital market, which itself depend on exchange rate anticipations. Alternatively, we could have introduced inframarginal rents through a decreasing returns technology. Our corporate finance approach, however, yields simpler expressions.

The government's net date-2 reserves are $\mathcal{R}_i - \mathcal{S}y_i\left(\frac{1}{e} - \frac{1}{e_i}\right)$. From the market clearing equation (2), the peso proceeds are redistributed to domestic residents and therefore equal to $[D + rI(f)] + e_2 c_2^*(e_2)$.

The government's objective function is:

$$\begin{aligned} W_G = \alpha\gamma &\left[(R(i) - ri) + (rI + D + e_2 c_2^*) + (u(c_2^*) - e_2 c_2^*)\right] \\ &+ \left[-D + (1-\gamma)(rI + D + e_2 c_2^*) + (1-\gamma)(u(c_2^*) - e_2 c_2^*)\right], \end{aligned}$$

or

$$W = \pi(I, \alpha) + v(e_2), \tag{4}$$

where $W = \frac{W_G}{\alpha\gamma + 1 - \gamma}$, $v(e_2) = u(c_2^*(e_2))$ and $\pi(I,\alpha) = \frac{\alpha\gamma R(\frac{I}{\gamma}) - r(1-\gamma)(\alpha-1)I + \gamma(\alpha-1)D}{\alpha\gamma + 1 - \gamma}$.

Notice that $\frac{\partial \pi}{\partial I} > 0$ and $\frac{\partial^2 \pi}{\partial \alpha \partial I} > 0$.[4]

Benchmark: Symmetric information

Suppose that $\mathcal{R}$ is common knowledge. Then, rational expectations imply that the date-2 exchange rate is perfectly foreseen by the market at date 1. Let e_s^{FI} and e_w^{FI} stand for the exchange rate

[3] This analysis presumes that the NPV is positive ($R'(0) > E(e_2) - r\left[E(e_2)\,E\left(\frac{1}{e_2}\right) - 1\right]$), while the borrowing capacity is finite: $f > r$ which we will assume.

[4] To see this, recall that $i \leq i^*$, where i^* maximizes $[R(i) - ri] - i\left(1 - \frac{r}{f}\right)E[e_2]$. Therefore, for the relevant range $i \leq i^*$, $R'(i) > r$, or $\left.\frac{\partial \pi}{\partial I}\right|_{I=\gamma i} \propto \alpha R'(i) - r(1-\gamma)(\alpha - 1) > 0$. Furthermore, $\left.\frac{\partial^2 \pi}{\partial \alpha \partial I}\right|_{I=\gamma i} \propto R'(i) - r > 0$.

of the strong and weak type of government under full information. Investment is $I(e_i^{FI})$ for $i = \{w, s\}$.

Under common knowledge about the future exchange rate, setting and defending an ambitious exchange rate (i.e., an exchange rate e below the full information, floating level) reduces welfare. Furthermore, under floating, speculative trades are irrelevant; and so, clearing in the foreign exchange market is equivalent to:

$$\mathcal{R}_i - \frac{D}{e_i^{FI}} = c_2^*(e_i^{FI}) + \frac{r}{e_i^{FI}} I(e_i^{FI}).$$

Since $\mathcal{R}_s > \mathcal{R}_w$,

$$e_w^{FI} > e_s^{FI}.$$

The government's utility under full information is:

$$W_i^{FI} = \pi(I(e_i^{FI}), \alpha) + v(e_i^{FI}).$$

Structure of Equilibrium

Our analysis for the corporate finance example rests on two technical assumptions that were not required in the voting model in the paper. We first need to assume that the government values capital inflows positively, at least over a range of parameters. Because foreign borrowing by non-tradable good sectors consumes reserves, it competes with consumption of tradables by consumers. It must therefore be the case that the government values corporate investment sufficiently.

Assumption A1: *(demand for capital inflows)*
The weak type prefers to pool rather than reveal its type when $\mathcal{S} = 0$:

$$W_w^{FI} < \pi(I(e^{hm}), \alpha) + v(e_w),$$

where $e^{hm} \in (e_s^{FI}, e_w^{FI})$*, the "harmonic mean" exchange rate, is defined by*

$$\frac{1}{e^{hm}} \equiv \frac{\rho}{e_s} + \frac{1-\rho}{e_w},$$

and e_i *for* $i = \{w, s\}$ *satisfies:*

$$\mathcal{R}_i - \frac{D}{e_i} = c_2^*(e_i) + \frac{r}{e_i} I(e^{hm}).$$

Assumption A1 rules out the possibility that the weak type wishes to separate when there is no speculative attack and amounts, as we noted, to the imposition of a lower bound on α. That is, it is satisfied if the government cares about its domestic entrepreneurs enough relative to its domestic residents (since $I(e^{hm}) > I(e_w^{FI})$).

Assumption A2: *(sorting condition)*
Fixing $(\mathcal{S}, e)$*, the strong type is relatively more eager to defend the peg. Let* I *and* $\hat{I}$ *denote arbitrary investment levels with* $I \in [I(e_w^{FI}), I(e_s^{FI})]$ *and* $\hat{I} \leq I$ *and let* $\hat{e}_i$ *be given by:*

$$\mathcal{R}_i - \frac{D}{\hat{e}_i} = c_2^*(\hat{e}_i) + \frac{r}{\hat{e}_i}\hat{I},$$

and e_i *be given by:*

$$\mathcal{R}_i - \frac{\mathcal{S}}{e} - \frac{D - \mathcal{S}}{e_i} = c_2^*(e_i) + \frac{r}{e_i}I,$$

then

$$v(\hat{e}_w) - v(e_w) \geq v(\hat{e}_s) - v(e_s).$$

This condition will ensure that the strong type does not choose to float when the weak type does not either. The exchange rate $\hat{e}_i$ is the level corresponding to when the government chooses not to defend the currency, which generates a forward rate $f \in [e_s^{FI}, e_w^{FI}]$ and therefore investment $I(f) \in [I(e_w^{FI}), I(e_s^{FI})]$.

We expect the sorting condition to hold because foreign reserves are more valuable to the weak type as there is a lower level of consumption of the tradable good. Also, the weak type wastes more money by maintaining the peg than the strong type.

Example: The sorting condition holds with log utility: $c_2^*(e_2) = \frac{1}{e_2}$ and $v(e_2) = -\log(e_2)$. With log utility, exchange-rate clearing yields the following relations:

$$\hat{e}_s = \hat{e}_w \frac{\mathcal{R}_w}{\mathcal{R}_s} \quad \text{and} \quad e_s = e_w \frac{\mathcal{R}_w - \frac{\mathcal{S}}{e}}{\mathcal{R}_s - \frac{\mathcal{S}}{e}}.$$

Because

$$\frac{\mathcal{R}_s - \frac{\mathcal{S}}{e}}{\mathcal{R}_w - \frac{\mathcal{S}}{e}} \geq \frac{\mathcal{R}_s}{\mathcal{R}_w},$$

$$v(\hat{e}_w) - v(e_w) \geq v(\hat{e}_s) - v(e_s).$$

Proposition A1: *If $e_s^{FI} < e < e_w^{FI}$, then for all $\mathcal{S} > 0$, $e_s < e_s^{FI}$ and $e_w > e_w^{FI}$, where e_s and e_w are the date-2 exchange rates of the strong and weak types when the government has defended the peg.*

Proof. The market clearing equations for $i \in \{w, s\}$

$$\mathcal{R}_i - \frac{\mathcal{S}}{e} - \frac{D - \mathcal{S}}{e_i} = c_2^*(e_i) + \frac{r}{e_i} I(f)$$

imply that $e_s < e_w$. This is obviously true if $\mathcal{S} \leq D$. More generally, consider the lowest $\mathcal{S}$ such that $e_w = e_s$; the two market clearing conditions are then inconsistent, which shows that there can be no such $\mathcal{S}$.

Next, let

$$\frac{1}{f} = \frac{\rho'}{e_s} + \frac{1 - \rho'}{e_w}.$$

Assume first that $e_w \leq e_w^{FI}$. Then $f < e_w^{FI}$; for, if $f = e_w^{FI}$ then $\rho' = 0$ and $e_w = e_w^{FI}$, which implies that $\mathcal{S} = 0$. But $\mathcal{S} = 0$ is not optimal for speculators as $e < f$.

Because $f < e_w^{FI}$, $I(f) > I(e_w^{FI})$, and the RHS of the exchange rate clearing equation for the weak type strictly exceeds its full information level. This in turn requires that the same be true for the LHS:

$$\frac{\mathcal{S}}{e} + \frac{D - \mathcal{S}}{e_w} < \frac{D}{e_w^{FI}},$$

which in turn can hold only if $\mathcal{S} > 0$ and $e_w < e$, which itself would imply $f < e$ and so $\mathcal{S} = 0$, a contradiction. The proof that $e_s < e_s^{FI}$ is identical, by symmetry. $\diamond$

When the peg undervalues the currency in view of strong fundamentals, the exchange rate appreciates relative to the full information case for two reasons: First, the capital inflow is smaller and so the country's debt burden is alleviated. Second, the government makes a windfall profit when defending the peg ($e > e_s$), which concurs to make the currency even stronger. Conversely, when the peg overvalues the currency because fundamentals are weak, the weak type's exchange rate depreciates for the same, but opposite two reasons.

Strategic complements or substitutes?

We adapt the definition to the corporate finance example.

Definition A. *For a given peg e and an arbitrary ρ (not necessarily the prior), let $f_\rho(\mathcal{S})$ be defined by:*

$$\frac{1}{f_\rho(\mathcal{S})} \equiv \frac{\rho}{e_s} + \frac{1-\rho}{e_w},$$

and

$$\mathcal{R}_i - \frac{\mathcal{S}}{e} - \frac{D-\mathcal{S}}{e_i} = c_2^*(e_i) + \frac{r}{e_i} I(f_\rho(\mathcal{S})), \qquad \text{for} \quad i \in \{w, s\}. \tag{5}$$

Speculative activities exhibit strategic complementarity (strategic substitutability) when $f_\rho(\mathcal{S})$ is increasing (decreasing) in $\mathcal{S}$, i.e. when a large speculative attack triggers an immediate depreciation of the currency, keeping the government's strategy fixed.

Proposition $A2$ obtains sufficient conditions for SC/SS according to Definition A:

Proposition A2: *For $e_s^{FI} < e < e_w^{FI}$, there exists $\bar{\mathcal{S}} > D, \rho_1$, and ρ_2 such that $0 < \rho_1 < \rho_2 < 1$, such that speculative attacks exhibit, on $[0, \bar{\mathcal{S}}]$, SC if $\rho \in [0, \rho_1]$ and SS if $\rho \in [\rho_2, 1]$.*

Proof. Let us demonstrate this proposition for ρ small. Differentiating (5) yields:

$$\frac{de_w}{d\mathcal{S}} \left[c_2^{*'} - \frac{r}{e_w^2} I - \frac{D-\mathcal{S}}{e_w^2} \right] = \frac{1}{e_w} - \frac{1}{e} - \frac{r}{e_w} I' \frac{df_\rho}{d\mathcal{S}},$$

where $\frac{df_\rho}{d\mathcal{S}} \approx \frac{de_w}{d\mathcal{S}}$ for ρ small. Therefore,

$$\frac{de_w}{d\mathcal{S}} \left[c_2^{*'} - \frac{r}{e_w^2} I - \frac{D-\mathcal{S}}{e_w^2} + \frac{r}{e_w} I' \right] \approx \frac{1}{e_w} - \frac{1}{e}.$$

From Proposition A1, the RHS is negative and bounded away from 0. Because $I' < 0$, the coefficient of $\frac{de_w}{d\mathcal{S}}$ is negative for $\mathcal{S} \leq D$. The coefficient must actually remain negative, otherwise e_w would go to infinity for some $\mathcal{S}$. Hence,

$$\frac{de_w}{d\mathcal{S}} \geq k > 0, \qquad \text{for some } k.$$

Thus the forward rate increases with $\mathcal{S}$ for ρ small. The proof of SS for ρ large follows a similar reasoning. $\diamond$

Exchange rate defense

Unlimited Short Sales

With unlimited short sales, the forward rate when the peg is defended must be equal to the peg ($f = e$). Suppose it were higher ($f > e$); because speculators' profit is determined by what happens when the peg is defended (from an individual standpoint, a speculative trade that is not converted is equivalent to no speculative trade), the expectation of an average devaluation conditional on the peg being defended would trigger $\mathcal{S} = \infty$. Conversely, if $f < e$, foreign investors would invest an infinite amount in the country.

It is convenient to decompose the range of pegs into three regions, defined by the strong type's full-information exchange rate e_s^{FI} and the prior exchange rate, defined by the harmonic mean e^{hm}, in the absence of speculation defined above.

a) Unambitious pegs: $e \geq e^{hm}$

For $e \geq e^{hm}$, it is an equilibrium for the government to maintain the peg and for speculators not to attack. If, in the absence of speculation (or for small speculation), the government indeed defends the currency regardless of its type ($x(e) = 1$), then speculators are better off not converting pesos into dollars at the weak pegged exchange rate. Conversely, facing no or small speculation, the government prefers to defend the currency, keep market beliefs at the prior level ρ, and attract investment $I(e^{hm})$ rather than abandoning the peg, being perceived as a weak type, and attract only $I(e_w^{FI})$, as guaranteed by Assumption $A1$ for the weak type and (together with the sorting condition applied to $\hat{I} = I(e_w^{FI})$, $I = I(e^{hm})$, and $\mathcal{S} = 0$) for the strong type.

b) Ambitious pegs: $e_s^{FI} \leq e < e^{hm}$

With a more ambitious peg, a small amount of speculation is strictly profitable when the government defends the currency for sure. In this region, $\mathcal{S} > 0$.

Assume first that the weak type is indifferent between abandoning the peg, thereby leading to a depreciation of the currency to level e_w^{FI}, and maintaining the peg:

$$W_w^{FI} = \pi(I(f), \alpha) + v(e_w). \tag{6}$$

Because $f = e$, equation (6) sets the date-2 exchange rate of the weak type when defending the currency. The market-clearing equation for the weak type then determines the level of speculative activity $\mathcal{S}$:

$$\mathcal{R}_w - \frac{\mathcal{S}}{e} - \frac{D - \mathcal{S}}{e_w} = c_2^*(e_w) + \frac{r}{e_w} I(f). \tag{7}$$

The strong type's date-2 exchange rate in turn is given by the other market-clearing condition:

$$\mathcal{R}_s - \frac{\mathcal{S}}{e} - \frac{D - \mathcal{S}}{e_s} = c_2^*(e_s) + \frac{r}{e_s} I(f). \tag{8}$$

Note that in this equilibrium, the level of investment, the posterior beliefs when the peg is defended, and the exchange rates are independent of the prior beliefs (ρ) on the strong type. By contrast, when the country's initial reputation improves (ρ increases), the probability of defending the peg ($\rho + (1 - \rho)x$) increases as well.

Last, we need to determine the equilibrium probability of pooling by the weak type. The posterior beliefs $\rho'(x)$ satisfy

$$\frac{1}{f} = \frac{\rho'(x)}{e_s} + \frac{1 - \rho'(x)}{e_w}. \tag{9}$$

There exists a unique x satisfying equation (9). If $x < 1$, then we have an equilibrium. If not, then $x = 1$ and $W_w^{FI} \leq \pi(I(f), \alpha) + v(e_w)$. The equilibrium determination is slightly different: $f = e$, (7), (8), and (9), then give equilibrium.

In either case, $x(e)$ tends to 0 as e converges from the right to e_s^{FI}. This demonstrates that, unlike in most signaling models, separation need not be costly (given costless short sales).

Under SC, if x equals 1, speculation is infinite. Equilibrium behavior therefore requires that $x < 1$. Thus the weak type must be indifferent between defending the currency and floating, which in turn requires a jump in $\mathcal{S}$ at $e = e^{hm}$. In turn, the jump in $\mathcal{S}$ implies that x jumps downward as e falls below e^{hm} in order to keep speculators indifferent.

Under SS, by contrast, the increase in $\mathcal{S}$ as e falls below e^{hm} would lower f implying $\mathcal{S} = 0$, a contradiction. So as e falls below e^{hm}, x remains equal to 1 on some interval and $\mathcal{S}$ increases continuously.

c) Unsustainable pegs: $e \leq e_s^{FI}$

In this range, the speculative attack is infinite and both types abandon the peg.

Comparative Statics

As in the paper, the equilibrium may not be unique for SC for $e > e^{hm}$; we adopt the same convention as in the paper by selecting the $\mathcal{S} = 0$ one. Similarly, for $e < e^{hm}$, we assume SS, SN or weak SC to rule out multiple equilibria. We can then examine some comparative statics, focusing on the relevant range for analysis $e \in [e_s^{FI}, e^{hm}]$.[5]

Proposition A3: *For a given peg e, an increase in the demand for foreign borrowing (an increase in α) in the government's objective function leads to*

- *an increase in speculative activity*
- *a depreciation of the currency of the weak type*
- *if speculative activities are SS (SC), an increase (decrease) in the probability of maintaining the peg.*

Proof. Recall that $f = e$ when the peg is defended. Either $W_w^{FI} < \pi(I(e), \alpha) + v(e_w)$ and then nothing changes ($x = 1$) or $W_w^{FI} = \pi(I(e), \alpha) + v(e_w)$. Because $I(e) > I_w^{FI}$ and $\frac{\partial^2 \pi}{\partial I \partial \alpha} > 0$, e_w must increase to keep the equality satisfied. Thus the weak type must lose more money on the FX market:

$$\mathcal{R}_w - \frac{\mathcal{S}}{e} - \frac{D - \mathcal{S}}{e_w} = c_2^*(e_w) + \frac{r}{e_w} I(e),$$

and so $\mathcal{S}$ increases. Last, an increase in $\mathcal{S}$ under SS (according to Definition A) raises $\frac{1}{f}$ for a given x. So x must increase in order to re-establish the speculators' zero-profit condition. ◇

Intuitively, as the government is more concerned about investment, it becomes more willing to sacrifice purchasing power of consumers to attract investment. At a given peg, this occurs through an increase in speculative activities, and, under SS, a higher probability of maintaining the peg. The same proposition also holds when the marginal productivity of investment ($\frac{\partial \pi}{\partial I}$) increases independently of α.

Adding an export sector: We can also consider a version of the model where there is a export sector with little political weight, so that the government is still in favor of a strong currency. For instance, suppose the government's objective function is:

$$\pi(I, \alpha) + \hat{\pi}(\hat{I}, \hat{\alpha}) + v(e_2),$$

[5] For $e > e^{hm}$, the outcome is the same as for $e = e^{hm}$.

where $\hat{I}$, the non-tradable investment in the export sector, is increasing in f, $\hat{\alpha}$ is increasing with the weight on the export sector and $\partial\hat{\pi}/\partial\hat{I} > 0$ and $\partial^2\hat{\pi}/\partial\hat{\alpha}\partial\hat{I} > 0$. The foreign exchange market clears as:

$$\mathcal{R}_i - \frac{\mathcal{S}}{e} - \frac{D - \mathcal{S}}{e_i} = c_2^*(e_i) + \frac{r}{e_i}I + \hat{r}\hat{I}.$$

We also have the same result as in the proposition above when we decrease the weight put on the export sector in the government's objective function.

No Short Sales

Suppose now that speculation is bounded above by the short-term debt D in pesos. Furthermore, we assume that the government is able to verify that capital inflows are invested in physical capital, and not used for relaxing the short sales constraint. The equilibrium identified above is still an equilibrium as long as

$$\mathcal{S} \leq D.$$

In particular, when $e \geq e^{hm}$, the previous equilibrium had no speculative attack and therefore is not altered by the no-short-sales constraint. When $e < e_s^{FI}$, then $\mathcal{S} = D$: speculation is repressed by the constraint on short sales.

Focusing, last, on the interesting region, $e_s^{FI} \leq e < e^{hm}$, suppose to the contrary, that the equilibrium speculation would involve short sales. Because $\mathcal{S} = 0$ and/or $x = 0$ cannot be part of an equilibrium (by the same reasoning as in the Short Sales section), we have

$$\frac{1}{e} \geq \frac{1}{f} = \frac{\rho'}{e_s} + \frac{1 - \rho'}{e_w},$$

and

$$W_w^{FI} \leq \pi(I(f), \alpha) + v(e_w).$$

Furthermore,

$$\mathcal{R}_i - \frac{\mathcal{S}}{e} - \frac{D - \mathcal{S}}{e_i} = c_2^*(e_i) + \frac{r}{e_i}I(f) \qquad i \in \{w, s\}.$$

Consider, first, the possibility of non-repressed speculation ($f = e$). Then $\mathcal{S} \leq D$ would be an equilibrium of the short-sales situation with $x \leq 1$ and $W_w^{FI} = \pi(I(f), \alpha) + v(e_w)$ or $x = 1$ and $W_w^{FI} < \pi(I(f), \alpha) + v(e_w)$, a contradiction. Thus the equilibrium without short-sales involves *repressed speculation*:

$$f > e,$$

and so $\mathcal{S} = D$ and $I(f) < I(e)$: short-sale constraints reduce the capital inflow conditional on the peg being maintained.

The forward rate f, the date-2 exchange rates e_s and e_w, and the pooling probability are then given by the following system of equations:

$$\mathcal{R}_w - \frac{D}{e} = c_2^*(e_w) + \frac{r}{e_w} I(f),$$
$$W_w^{FI} \geq \pi(I(f), \alpha) + v(e_w), \quad \text{with equality if } x < 1,$$
$$\mathcal{R}_s - \frac{D}{e} = c_2^*(e_s) + \frac{r}{e_s} I(f),$$

and

$$\frac{1}{f} = \frac{\rho'(x)}{e_s} + \frac{1 - \rho'(x)}{e_w}.$$

Next, note that the facts that $I(f) < I(e)$ and that $\mathcal{S}$ is smaller than under short sales (short sales constraints cut the weak type's losses) imply that e_w appreciates relative to the short-sales case.

Proposition A4. *(When they are binding), short sales constraints:*

- *reduce the capital inflow when the peg is maintained*
- *appreciate the weak type date-2 exchange rate when the peg is maintained*
- *raise the probability that the weak type defends the peg when e_s is fixed.*

Choice of peg

Suppose now that the peg is chosen by the government at stage (i) instead of being determined by legacy. We assume that the peg is effectively dictated by the strong type's preferences. The welfare of the strong type as a function of e is:

$$W_s = \pi(I(f), \alpha) + v(e_s).$$

Note first that the strong type can guarantee itself W_s^{FI} by setting $e = e_s^{FI}$.

In the relevant range $e \in [e_s^{FI}, e^{hm}]$, so the equilibrium defines a feasible set $\{e, e_s(e)\}$. The strong type solves:

$$\max_e \{\pi(I(e), \alpha) + v(e_s(e))\}.$$

Setting an exchange rate e between e_s^{FI} and e^{hm} involves a trade-off: on the one hand, investment falls from $I(e_s^{FI})$ to $I(e)$, reducing corporate welfare; on the other hand, the strong type makes money on speculators, increasing future purchasing power of the consumers. Intuitively, the former effects receives more attention as α increases:

Proposition A5: *As α increases, the equilibrium peg e^* (weakly) decreases (the peg becomes more ambitious).*[6]

Proof. This follows from a simple revealed preference argument: let e_1^* be an optimal peg for α_1 and let e_2^* be an optimal peg for α_2, where $\alpha_1 > \alpha_2$:

$$\pi(I(e_1^*), \alpha_1) + v(e_s(e_1^*)) \geq \pi(I(e_2^*), \alpha_1) + v(e_s(e_2^*))$$
$$\pi(I(e_2^*), \alpha_2) + v(e_s(e_2^*)) \geq \pi(I(e_1^*), \alpha_2) + v(e_s(e_1^*))$$

Subtracting the RHS of the second equation from the LHS of the first equation and the LHS of the second equation from the RHS of the first equation and taking the limit as $\alpha_2 \to \alpha_1$ we obtain:

$$\frac{\partial \pi(I(e_1^*), \alpha)}{\partial \alpha} \geq \frac{\partial \pi(I(e_2^*), \alpha)}{\partial \alpha}$$

or $I(e_1^*) \geq I(e_2^*)$ since $\frac{\partial^2 \pi}{\partial \alpha \partial I} > 0$. Therefore, $e_2^* \geq e_1^*$, so the optimal peg becomes (weakly) more ambitious when α increases. ◇

Special case 1: For a simple case where the optimal announced exchange rate $e^* > e_s^{FI}$ and there is a crisis ($x \in (0,1)$), consider the corporate finance example and the following parametrization of production $R(I)$ with constant returns to scale up to an investment upper capacity $\bar{I}$:

$$R(I) = \min\{RI, R\bar{I}\},$$

for some R sufficiently large that entrepreneurs want to invest as much as they can and $I(e_s^{FI}) > \bar{I} > I(e^{hm})$. Define $e^* > e_s^{FI}$ such that $\bar{I} = I(e^*)$. Intuitively, decreasing returns in the benefits of foreign borrowing dampen the strong type's incentive to separate:

Proposition A6: *In the corporate finance example, when $R(I) = \min\{RI, R\bar{I}\}$, the equilibrium*

[6] This proposition applies to the equilibrium sets if there are multiple equilibria: If e_1^* is optimal for α_1 and e_2^* is optimal for $\alpha_2 < \alpha_1$, then $e_1^* \leq e_2^*$.

peg exceeds the full information optimum for the strong type ($e^* > e_s^{FI}$).

Proof. For $e \leq e^*$, $I = \bar{I}$ and

$$\mathcal{R}_i - \frac{\mathcal{S}}{e} - \frac{D - \mathcal{S}}{e_i} = c_2^*(e_i) + \frac{r\bar{I}}{e_i}$$
$$W_w^{FI} = \pi(\bar{I}, \alpha) + v(e_w).$$

Therefore, e_w is invariant on $[e_s^{FI}, e^*]$ and hence $\mathcal{S}\left(\frac{1}{e} - \frac{1}{e_w}\right)$ is invariant. We see that

$$W_s = \pi(\bar{I}, \alpha) + v(e_s)$$

increases if e_s decreases. Using the weak type's FX clearing equation to substitute for $\mathcal{S}$, strong type FX clearing is:

$$\frac{D + r\bar{I}}{e_s} + c_2^*(e_s) = \mathcal{R}_s + \left(\frac{\frac{1}{e_s} - \frac{1}{e}}{\frac{1}{e} - \frac{1}{e_w}}\right)\left[\mathcal{R}_w - \frac{D + r\bar{I}}{e_w} - c_2^*(e_w)\right].$$

In a stable equilibrium (where "stability" refers to the Walrasian tatonnement process), e_s decreases with e. $\diamond$

Special case 2 (log utility): Suppose the utility function is:

$$u(c_2^*) = \log(c_2^*).$$

We have observed that the sorting condition holds in this case.

Proposition A7: *With log utility, the weak type's equilibrium probability of defending the exchange rate, $x(e)$, is non-decreasing in the announced exchange rate e.*

Proof. FX clearing implies

$$\frac{e_s}{e_w} = \frac{\mathcal{R}_w - \frac{\mathcal{S}}{e}}{\mathcal{R}_s - \frac{\mathcal{S}}{e}},$$

which allows us to express the strong exchange rate e_s in terms of e:

$$e_s = e\left[\rho'(x) + (1 - \rho'(x))\frac{\mathcal{R}_w - \frac{\mathcal{S}}{e}}{\mathcal{R}_s - \frac{\mathcal{S}}{e}}\right].$$

Combining this with the strong-type FX clearing equation

$$e_s = \frac{(D - \mathcal{S}) + 1 + rI}{\mathcal{R}_s - \frac{\mathcal{S}}{e}},$$

we find

$$D + 1 + rI = e[\rho'(x)\mathcal{R}_s + (1 - \rho'(x))\mathcal{R}_w].$$

Since

$$\frac{d\rho'}{de} = \frac{rI' - (\rho'(x)\mathcal{R}_s + (1 - \rho'(x))\mathcal{R}_w)}{e(\mathcal{R}_s - \mathcal{R}_w)} \leq 0,$$

we reach our desired conclusion $\frac{dx}{de} \geq 0$. ◇

For log utility, it can be easily shown that the strong type chooses to separate (chooses $e = e_s^{FI}$) if and only if $\frac{\mathcal{S}}{e}$ is maximized at $e = e_s^{FI}$ over $[e_s^{FI}, e^{hm}]$. While we have been unable to prove analytically that this condition holds, it was satisfied in numerous simulations.

Finally, we investigate the choice of peg in a structured example.

Example: Existence of a fundamental exchange rate

Let us specialize the model by assuming that, regardless of the type, the country has a fundamental exchange rate. This assumption is in line with much of the literature on speculative attacks. To obtain fundamental exchange rates, 1 and $e_w > 1$ respectively, let us assume that

$$u(c_2^*) = \begin{cases} e_w c_2^* & \text{for } c_2^* \leq c^* \\ e_w c^* + (c_2^* - c^*) & \text{for } c_2^* > c^* \end{cases}$$

and that $\mathcal{R}_s$ is large enough and $\mathcal{R}_w$ small enough that the strong and weak types' exchange rates are always 1 and e_w, respectively in the relevant range.

With fundamental exchange rates, speculative activities are weak complements or substitutes according to Definition A as $\frac{1}{f} = \rho + \frac{(1-\rho)}{e_w}$ is invariant for a given ρ. In the presence of short sales, as e increases from 1 to e^{hm}, $\mathcal{S}$ decreases continuously to 0 and x increases continuously from 0 to 1.

We use the example presented in the microfoundation with the specifications $R(i) = Ri$ (linear technology) and $\alpha = 1$ (the government puts equal weight on both constituencies). Let us introduce two exchange rates, e^- and $\underline{e}$. Define e^- to satisfy:

$$I(1) - I(e_w) = \frac{D}{R - r}\left(\frac{e_w}{e^-} - 1\right)$$

(note that e^- goes to 0 as D goes to 0). It can be shown that $e^- < 1$ if the no-short sale constraint is ever binding. Let us further assume that the exchange rate cannot fall below some level $\underline{e} < 1$; for example, there is a technology that transforms $\frac{1}{\underline{e}}$ units of tradables into 1 unit of non-tradables.

For instance, US real estate, which is normally non-tradable by and large, becomes tradable if the dollar becomes too weak as foreigners buy secondary residences there.

Proposition A8. *In the corporate finance example, with* $R(i) = Ri$ *and* $\alpha = 1$*, and under a fundamental exchange rate (1 for the strong type,* $e_w > 1$ *for the weak type), and without short sales, the equilibrium peg is* $\max(e^-, \underline{e})$*. If this value is* e^-*, the strong type separates. Otherwise, there is some pooling by the weak type (and therefore financial crises).*

Proof. Without short sales, there exists some e^+ in $(1, e^{hm})$ such that the constraint $\mathcal{S} \leq D$ is binding exactly on $[1, e^+)$. We know the constraint must be binding for some $e^+ > 1$ since when $e = 1$, $\mathcal{S}$ must satisfy:

$$(R - r)[I(1) - I_w^{FI}] = \mathcal{S}(e_w - 1),$$

which implies that $\mathcal{S} > D$ for this value. We first show that the no-short-sales constraint x remains increasing in e to the left of e^+. Simple computations show that:

$$W_w = (R - r)I - D\frac{e_w}{e} + e_w \mathcal{R}_w$$

and

$$W_s = (R - r)I - \frac{D}{e} + \mathcal{R}_s,$$

where I is the investment corresponding to investment e (as we have seen $I = I(f(e)) < I(e)$). Let us look for a mixed strategy with $x < 1$ for the weak type and for $e < e^+$:

$$W_w = (R - r)I - D\frac{e_w}{e} + e_w \mathcal{R}_w = (R - r)I_w^{FI} - D + e_w \mathcal{R}_w.$$

As e increases, I must decrease to keep the weak type indifferent, and so f must increase. Because

$$\frac{1}{f} = \rho' + \frac{1 - \rho'}{e_w},$$

x must increase with e.

The weak type no longer pools ($x = 0$) when $\rho' = 1$ (i.e. $f = 1$). Thus, x increases from 0 to 1 as e goes from e^- to e^{hm}, where

$$I(1) - I_w^{FI} = \frac{D}{R - r}\left(\frac{e_w}{e} - 1\right).$$

The strong type's welfare is thus:

$$W_s = W_w + \frac{e_w - 1}{e}D + \mathcal{R}_s - e_w \mathcal{R}_w,$$

so W_s is decreasing in e. The peg is therefore set at $\max(e^-, \underline{e})$. ◇

Risk Management

The reasoning behind the next two proofs is the same as the main text.

Proposition A9: *In the extended game in which the government chooses the extent to which the domestic residents can hedge, the equilibrium obtained when residents cannot hedge is still an equilibrium. Indeed:*

- *either the government maintains the peg and then fully prohibits hedging: $V = 0$ (this happens with probability $\rho + (1-\rho)x(e)$ if the peg is e)*
- *or the government abandons the peg in which case there is no hedging either.*

Proposition A10: *The maturity structure of liabilities*

(i) is neutral under costless short sales;

(ii) is not neutral in the absence of short sales. Domestic borrowers prefer to offer short term liabilities as the exchange rate does not follow a martingale; the government cannot tilt the maturity structure of these liabilities without confessing a future depreciation.

Section 7.2: Choice of peg, log utility example

Footnote 34 in section 7.2 refers to another example we considered with log utility. Formally, we can state:

Proposition: *With log utility ($u(c_2^*) = \log(c_2^*)$), as α increases,*

(i) the weak type's equilibrium probability of defending the exchange rate, $x(e)$, is non-decreasing in the announced exchange rate e,

(ii) the equilibrium peg e^* (weakly) decreases (the peg becomes more ambitious).

Proof. (i) With log utility, FX clearing implies that

$$\frac{e_s}{e_w} = \frac{\mathcal{R}_w - \frac{\mathcal{S}}{e}}{\mathcal{R}_s - \frac{\mathcal{S}}{e}},$$

which allows us to express the strong exchange rate e_s in terms of e:

$$e_s = e\left[\rho'(x) + (1-\rho'(x))\frac{\mathcal{R}_w - \frac{\mathcal{S}}{e}}{\mathcal{R}_s - \frac{\mathcal{S}}{e}}\right].$$

Combining this with the strong-type FX clearing equation

$$e_s = \frac{D - \mathcal{S} + 1}{\mathcal{R}_s - \frac{\mathcal{S}}{e}}$$

we find

$$D + 1 = e[\rho'(x)\mathcal{R}_s + (1 - \rho'(x))\mathcal{R}_w].$$

Since

$$\frac{d\rho'}{de} = \frac{-[\rho'(x)\mathcal{R}_s + (1 - \rho'(x))\mathcal{R}_w]}{e(\mathcal{R}_s - \mathcal{R}_w)} \leq 0,$$

we reach our desired conclusion $\frac{dx}{de} \geq 0$.

(ii) This follows from the same revealed preference argument as the main text where we use the fact that x is weakly increasing in e. $\diamond$

References

Bolton, Patrick and David Scharfstein, "A Theory of Predation Based on Agency Problems in Financial Contracting," *American Economic Review*, 1990, *80*, 93–106.