Optimal Monetary Policy
with Informational Frictions *

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Abstract

This paper studies optimal policy in a business-cycle setting in which firms hold dispersed
private information about the state of the economy, or have a blurry understanding about it
due to rational inattention. The informational friction is not only the source of nominal rigidity
but also an impediment to the coordination of production. The main lesson is that the optimal
monetary policy does not target price stability; instead, it leans against the wind in the sense of
targeting a negative relation between the nominal price level and real economic activity. This
policy serves the goal of inducing the firms to utilize their information and to coordinate their
decisions in the best possible manner. An additional contribution is the adaptation of the primal
approach of the Ramsey literature to a setting with a rich form of informational friction.

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optimal policy, price stability.

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1 Introduction

In the last few years, a growing literature explores the macroeconomic implications of rational inattention (Sims, 2003), related forms of informational friction (Woodford, 2003; Mankiw and Reis, 2002), and higher-order uncertainty (Morris and Shin, 1998, 2002). In this paper we study how such frictions affect the nature of the optimal monetary policy and in particular the desirability of price stability.

To this goal, we consider a general-equilibrium macroeconomic setting in which firms fix both their prices and their production decisions on the basis of incomplete information about the state of the economy. This amounts to introducing not only a nominal rigidity that has been the focus of prior work, but also an impediment to the coordination of production.

The main lesson is a novel rationale for “leaning against the wind,” that is, for a policy that targets a negative relation between the nominal price level and real economic activity. Such a policy is optimal because it provides firms with the right incentives for how to act on their information about the state of the economy, as well as for how to collect such information in the first place.

This rationale is different from the one familiar from the textbook New Keynesian model. In that context, policies that lean against the wind are justified by assuming that the flexible-price allocations are suboptimal and by letting monetary policy substitute for missing tax instruments. Furthermore, such policies involve a trade off between minimizing relative-price distortions and stabilizing the output gap. By contrast, none of these properties apply in our context.

Understanding these subtle points and the precise nature of the optimal policy requires a revision of the efficiency benchmark relative to which the output gap and the relative-price distortions ought to be measured once the informational friction is accommodated. This brings us to the methodological contribution of our paper, which is to extend the primal approach of the Ramsey literature, and more specifically the methods of Correia, Nicolini, and Teles (2008), to a framework in which firms are informationally constrained.

Framework. Our setting features a representative household, centralized markets, and a continuum of monopolistic firms. Each such firm produces a differentiated commodity, which serves as an input into the production of a single final good, which in turn can be used for consumption and investment. A benevolent Ramsey planner sets jointly the monetary and fiscal policies, under full commitment. Lump-sum taxation is ruled out, but the tax system is otherwise rich enough to guarantee that monetary policy does not have to substitute for missing tax instruments.

These features make our framework comparable to, and indeed nest, those considered in Lucas and Stokey (1983), Chari, Christiano, and Kehoe (1994) and Correia et al. (2013). We depart from these benchmarks by letting both the firm’s price and its input choices be measurable in a noisy, private signal of the aggregate state of the economy.
In the main text, the informational friction is treated as exogenous. In Appendix C, it is recast as the product of a generalized form of rational inattention or of costly information acquisition. This illustrates the flexibility of our methods and the robustness of our insights.

**Nominal vs Real Rigidity.** The informational constraint on the firm’s pricing-setting choice represents a *nominal* rigidity. The constraint on its input choices introduces a *real* rigidity.

Although the literature has proposed the first feature as an appealing substitute to sticky prices and menu costs (Mankiw and Reis, 2002; Woodford, 2003; Mackowiak and Wiederholt, 2009), this feature does not alone upset the key normative lessons of the New Keynesian paradigm. Indeed, a corollary of our analysis is that when only prices are subject to an informational constraint, the results of Correia, Nicolini, and Teles (2008) continue to apply: price stability remains optimal insofar as monetary policy need not substitute for missing tax instruments.

The second feature is therefore crucial. Because each firm conditions its input choice on a noisy and idiosyncratic understanding of the state of the economy, production can no longer be perfectly coordinated across firms. As a result, the efficiency benchmark studied in Correia et al. (2013) and the existing literature more generally is inappropriate for gauging optimal policy in the environment under consideration. Instead, the relevant benchmark embeds the real information rigidity within the feasibility constraints of the planner. It is this imperfection which is responsible for the novel lessons delivered in our paper.

**A Primal Approach.** We start by characterizing the entire set of the allocations that can be implemented as equilibria with the available policy instruments. To shed light on the role of monetary policy, we conduct this exercise under two scenarios. The one switches off the nominal rigidity by dropping the informational constraint on the firms’ pricing decisions, the other maintains it. This adapts the concepts of flexible-price and sticky-price allocations to our context.

We next solve a *relaxed* problem in which the planner faces only three constraints: resource feasibility, the absence of lump sum taxation, and the real informational rigidity discussed above. As a result of the latter friction, the solution to this problem can display positive cross-sectional dispersion in marginal products as well as exotic business-cycle properties such as noise or sentiment-driven fluctuations.\(^1\) These deviations could be mistaken as obvious reasons for stabilization policy and yet they are inherent in the planner’s solution within this environment—thereby revising the efficiency benchmark relative to which the concept of the output gap and that of relative-price distortions should be defined.

The methodological part of our paper is completed by showing that the relaxed optimum is contained within the set of flexible-price allocations and by identifying the combination of taxes and

\(^1\)For instance, the business cycle can be driven by forces akin to “animal spirits” as formalized in Angeletos and La’O (2013) and Benhabib, Wang, and Wen (2015).
monetary policy that implement it as a sticky-price allocation. As it turns out, the optimal taxes are similar to those found in Lucas and Stokey (1983) and Chari, Christiano, and Kehoe (1994); this is despite the fact that taxes play a novel role in our setting, namely they may manipulate the decentralized use of private information. The optimal monetary policy is discussed next.

**On Price Stability.** We now turn to the main applied contribution of our paper. In the New Keynesian framework, the optimal flexible-price allocation is typically replicated with a monetary policy that targets price stability (Correia, Nicolini, and Teles, 2008). We instead show that price stability is inconsistent with replication of the optimal flexible-price allocation. Rather, optimality requires a negative relation between the nominal price level and real economic activity.

The intuition for this result is subtle, yet robust. Consider two firms with different beliefs, or different degrees of optimism, about the state of the economy. Because firms are rational, such belief differences reflect differential private information. Furthermore, it is socially optimal to let each firm condition its production on such private information (this is a key property of the efficiency benchmark identified above). As a result, it is optimal for the more optimistic firm to produce more than the less optimistic one. In short, efficiency requires that relative quantities vary with relative beliefs.

In equilibrium, this means that relative prices must also vary with relative beliefs; this is simply a consequence of downward-sloping demand. When nominal prices are flexible, the requisite co-movement between relative prices and relative beliefs is trivially implementable. But now consider a world in which firms face informational constraints on their price-setting decisions. For the more optimistic firm to produce more and charge a lower relative price than the less optimistic firm, it has to be that the nominal price set by each firm is a decreasing function of her belief: more optimistic firms ought not only to produce more but also to fix lower prices.

Consider then a shock that causes a positive mass of firms to receive favorable private information about the underlying economic fundamentals and the likely level of aggregate demand. As explained above, efficiency requires that these firms produce more and set lower nominal prices. But since there is a positive mass of them, the aggregate level of real economic activity and the nominal price level have to move in the opposite direction, which explains the result.

To sum up, the documented form of “leaning against the wind” derives from three basic properties: (i) firms have different information and different beliefs about the state of the economy; (ii) there is social value in letting relative production vary with relative beliefs; and (iii) nominal prices must move in the direction opposite to real quantities in order to implement the requisite movements in relative quantities and relative prices.

**Relation to the Literature.** The frictions we are concerned with are not only *a priori* plausible but also consistent with survey evidence (Coibion and Gorodnichenko, 2012, 2015). They also help
account for various salient features of the macroeconomic data (Angeletos and Huo, 2018; Carroll et al., 2018; Lorenzoni, 2009; Mackowiak and Wiederholt, 2015; Mankiw and Reis, 2006).

Although a few other works have also touched on the question of how such frictions affect optimal monetary policy (Ball, Mankiw, and Reis, 2005; Adam, 2007; Lorenzoni, 2010; Paciello and Wiederholt, 2014), our paper remains the first to study this question in a setting in which such frictions is the source not only of nominal rigidity but also of real rigidity (in the sense defined earlier). As already explained, this feature is responsible for the novel lessons delivered in this paper. Barring that feature, the results of Correia, Nicolini, and Teles (2008) would have applied: the relevant efficiency benchmark would not have to be modified, and price stability would have remained optimal.

Another notable aspect of our contribution is the flexibility of our primal approach. Macroeconomic models with informational frictions can be hard to analyze due to the complexity in the dynamics of higher-order beliefs. As a result, the literature typically takes one of two routes: either it imposes strong assumptions, including the absence of capital accumulation and specific signal structures, so as to solve for the equilibrium in closed form; or it resorts to numerical simulations. In contrast, our approach bypasses these obstacles and delivers sharp theoretical results despite a flexible specification of the information structure. This approach builds a bridge between the Ramsey literature and the work of Angeletos and Pavan (2007, 2009), which studies efficiency in a class of abstract incomplete-information games. Close, although not as flexible, variants of such an approach appear in Angeletos and La’O (2008) and Lorenzoni (2010).

Layout. The rest of the paper is organized as follows. Section 2 sets up our framework. Sections 3 and 4 define the appropriate concepts of sticky-price and flexible-price allocations. Section 5 defines and characterizes the optimal allocation. Section 6 presents our key results on optimal monetary policy. Section 7 contains a simple, tractable example that helps illustrate the lessons of our paper more sharply. Section 8 concludes. Appendices A and B contain the proofs. Appendix C contains an extension with endogenous information acquisition or rational inattention.

2 The Framework

In this section, we introduce our framework. We first describe the components of the environment that are invariant to the information structure. We next formalize the informational friction and its two roles (the nominal and the real).

Preliminaries. Periods are indexed by $t \in \{0, 1, 2, \ldots\}$. There is a representative household, which pools all the income in the economy and makes consumption, capital accumulation, and

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2See the discussions in Townsend (1983), Huo and Takayama (2015b) and Nimark (2017).
labor supply decisions. There is a continuum of monopolistically competitive firms, indexed by \( i \in I = [0, 1] \). These firms produce differentiated goods, which are used by a competitive retail sector as intermediate inputs into the production of a final good. The latter in turn can be used for three purposes: as consumption; as investment into capital; or as materials, that is, as intermediate input in the production of the differentiated goods. Finally, there is a government, which lacks lump-sum taxation but can levy a variety of distortionary taxes and can issue both a contingent and non-contingent debt.

**States of Nature.** In each period \( t \), Nature draws a random variable \( s_t \) from a finite set \( S_t \). This variable may contain not only innovations in the current fundamentals, namely aggregate TFP, government spending, and household preferences, but also news about future fundamentals (Beaudry and Portier, 2006; Jaimovich and Rebelo, 2009) or noise and sentiment shocks (Lorenzoni, 2009; Angeletos and La'O, 2013). The aggregate state of the economy, or the state of Nature, in period \( t \) is comprised by the history of draws of \( s_\tau \) for all \( \tau \in \{0, ..., t\} \). The state is therefore an element of \( S^t \equiv S_0 \times ... \times S_t \) and is henceforth denoted by \( s^t \equiv (s_0, ..., s_t) \). Its unconditional probability is denoted by \( \mu(s^t) \).

**Tax and debt instruments.** The government lacks access to both lump-sum taxation and firm-specific taxes. It can nonetheless impose four kinds of economy-wide taxes: a proportional tax on consumption at rate \( \tau^c_t \); a proportional tax on labor income at rate \( \tau^\ell_t \); a proportional tax on capital income, net of depreciation, at rate \( \tau^k_t \); and a 100% tax on distributed profits. In addition, the government can issue two kinds of debt instruments. The first is a one-period, non-contingent, debt instrument that costs 1 dollar in period \( t \) and pays out \( 1 + R_t \) in period \( t+1 \), where \( R_t \) denotes the nominal interest rate between \( t \) and \( t+1 \). The second is a complete set of state-contingent assets (or Arrow securities). These are indexed by \( s \in S^{t+1}_t \), they cost \( Q_{t,s} \) dollars in period \( t \), and they pay out 1 dollar in period \( t+1 \) if state \( s \) is realized and 0 otherwise. Their corresponding quantities are denoted by \( D_{t,s} \). The quantity of the non-contingent debt, on the other hand, is denoted by \( B_t \).

**The household.** Let \( K_t \) denote the capital stock accumulated by the end of period \( t \); \( L_t \) the labor supply in period \( t \); \( r_t \) and \( w_t \) the pre-tax real values of the rental rate of capital and the wage rate in period \( t \), respectively; \( C_t \) and \( X_t \) the period-\( t \) real levels of consumption and investment, respectively; and \( P_t \) the period-\( t \) price level (i.e., the nominal price of the final good). The household’s period-\( t \) budget constraint can then be expressed, in nominal terms, as follows:

\[
(1 + \tau^c_t)P_tC_t + P_tX_t + B_t + \sum_{s \in S^{t+1}_t} Q_{t,s}D_{t,s} = (1 - \tau^\ell_t)P_tw_tL_t + (1 - \tau^k_t)P_tr_{t-1}K_{t-1} + (1 + R_{t-1})B_{t-1} + D_{t-1,s}^t
\]
The law of motion of the capital stock is given by

\[ K_t = (1 - \delta)K_{t-1} + X_t, \]

where \(\delta \in [0, 1]\) is the depreciation rate of capital. Finally, the household’s preferences are given by her expectation of

\[ U = \sum_{t=0}^{\infty} \beta^t U(C_t, L_t, s^t), \]

where \(\beta \in (0, 1)\) and \(U\) is strictly increasing and strictly concave in \((C_t, -L_t)\).

The firms. Consider monopolist \(i\), that is, the firm producing variety \(i\). Its output in period \(t\) is denoted by \(y_{it}\) and is given by

\[ y_{it} = A(s^t)F(k_{it}, h_{it}, \ell_{it}), \]

where \(A(s^t)\) is an aggregate productivity shock, \(k_{it}\) is the capital input, \(h_{it}\) is the final-good input (or “materials”), \(\ell_{it}\) is the labor input, and \(F\) is a Cobb-Douglas production function.\(^3\) The firm faces a proportional revenue tax, at rate \(\tau^r_t\). Its nominal profit net of taxes is therefore given by

\[ \Pi_{it} = (1 - \tau^r_t) p_{it} y_{it} - P_t r_t k_{it-1} - P_t h_{it} - P_t w_t \ell_{it}, \]

where \(p_{it}\) denotes the nominal price of the intermediate good \(i\), \(P_t\) denotes the nominal price of the final good (also, the price level), and \(r_t\) and \(w_t\) denote, respectively, the real rental rate of capital and the real wage rate. The final good, in turn, is produced by a competitive retail sector, whose output, \(Y_t\), is a CES aggregator of all the intermediate varieties:

\[ Y_t = \left[ \int_I (y_{it})^{\frac{\rho - 1}{\rho}} \, di \right]^{\frac{\rho}{\rho-1}}, \]

where \(\rho > 1\). The profit of the retail sector is therefore given by \(P_t Y_t - \int_I p_{it} y_{it} \, di\) and its maximization yields the demand curves faced by the monopolists.

The government. The government’s period-\(t\) budget constraint, in nominal terms, is given by

\[ (1 + R_{t-1}) B_{t-1} + D_{t-1,s^t} + P_t G_t = B_t + \sum_{s \in S^{t+1}} Q_{t,s} D_{t,s} + T_t \]

where \(G_t = G(s^t)\) is the exogenous real level of government spending and \(T_t\) is the nominal level

\(^3\)The Cobb-Douglas restriction is with some, but not serious, loss of generality. For details, see Section 4 in the September 2017 version of this paper (https://economics.mit.edu/files/7025).
of tax revenue, given by

\[ T_t = \tau^r_t P_t Y_t + \tau^f_t P_t C_t + \tau^\ell_t P_t \ell_t L_t + \tau^k_t P_t k_t + \Pi_t \]

where \( \Pi_t \) are the aggregate firm profits. With some abuse of notation, we let \( D_t = (D_{t,s})_{s \in S^{t+1}} \) and \( Q_t = (Q_{t,s})_{s \in S^{t+1}} \). The planner controls the vector \( (\tau^r_t, \tau^f_t, \tau^k_t, \tau^\ell_t, B_t, D_t) \) along with \( R_t \), the nominal interest rate. Finally, to simplify the exposition and keep the analysis comparable to that of Correia, Nicolini, and Teles (2008), we abstract from the zero lower bound on the nominal interest rate.

**Market Clearing.** Market clearing in the goods market is given by

\[ C_t + H_t + X_t + G_t = Y_t, \]

where \( X_t \equiv \int x_{it} di \) is aggregate investment and \( H_t \equiv \int h_{it} di \) is the aggregate intermediate-input use of the final good. Market clearing in the labor market, on the other hand, is given by \( \int \ell_{it} di = L_t \).

**The informational friction.** The scenario most often studied in the literature allows the firm-specific variables \( (p_{it}, k_{it}, h_{it}, \ell_{it}, y_{it}) \) to be measurable in \( s^t \), for all \( i \) and all \( t \). We depart from this benchmark by requiring that each firm must act on the basis of a noisy, and idiosyncratic, signal of \( s^t \). As in the related literature, the noise can be interpreted either as the product of imperfect observability of the state or as the product of rational inattention.

More specifically, the friction takes the following form. For every \( t, s^t \), and \( i \), Nature draws a random variable \( \omega^t_i \) from a finite set \( \Omega^t \) according to a probability distribution \( \varphi \). This variable represents the entire information ("signal") that firm \( i \) has in period \( t \) about the underlying state of Nature. We denote with \( \varphi(\omega^t_i, s^t) \) the joint probability of \( (\omega^t_i, s^t) \), with \( \varphi(\omega^t_i | s^t) \) the probability of \( \omega^t_i \) conditional on \( s^t \), and with \( \varphi(s^t | \omega^t_i) \) the probability of \( s^t \) conditional on \( \omega^t_i \). Conditional on \( s^t \), the draws are i.i.d. across firms and a law of large number applies so that \( \varphi(\omega^t_i | s^t) \) is also the fraction of the population that receives the signal \( \omega^t_i \).\(^4\) Finally, we impose the following two measurability restrictions.

**Property 1.** There exists functions \( \{h_t, k_t, \ell_t, y_t\} \) such that firm-level quantities satisfy

\[
\begin{align*}
    h_{it} &= h_t(\omega^t_i), & k_{it} &= k_t(\omega^t_i), & \ell_{it} &= \ell_t(\omega^t_i, s^t), & y_{it} &= y_t(\omega^t_i, s^t),
\end{align*}
\]

for all \( i \), all \( t \), and all realizations of uncertainty.

\(^4\)See Uhlig (1996) for an applicable law of large numbers with a continuum of draws.
Property 2. There exist functions \( \{p_t\} \) such that prices satisfy
\[
p_{it} = p_t(\omega^i_t)
\]
for all \( i \), all \( t \), and all realizations of uncertainty.

These properties constitute, in effect, a definition of informationally feasibility. Property 2, which requires \( p_{it} \) to be measurable in \( \omega^i_t \) rather than \( s^i_t \), introduces the same kind of nominal rigidity as the one featured in Mankiw and Reis (2002), Woodford (2003), Mackowiak and Wiederholt (2009), and a growing literature that replaces Calvo-like sticky prices with an informational friction. Relative to this literature, the key novelty here is Property 1. This adds a real friction by requiring that \((k_{it}, h_{it})\) be also measurable in \( \omega^i_t \). Finally, letting \( \ell_{it} \) (and thereby also \( y_{it} \)) adjust to \( s^i_t \) guarantees that supply can meet demand and markets clear for all realizations of uncertainty.\(^5\)

**Interpretation and a few special cases.** Our formulation allows for great generality. For instance, because no restriction is imposed on the dynamic structure of the signals, we can accommodate arbitrary learning dynamics or even the possibility of memory loss over time. Furthermore, \( \omega^i_t \) may contain an arbitrary information, not only about the fundamentals, but also about the beliefs of other firms. This allows us to accommodate rich higher-order uncertainty.

This level of generality highlights the flexibility of our primal approach and the robustness of our lessons. It also permits us to nest a variety of specific cases found in the literature.

To start with, consider models with noisy Gaussian signals, as in Morris and Shin (2002), Woodford (2003), and Angeletos and La’O (2010). These may be nested by specifying the underlying aggregate TFP shock as a Gaussian random variable and letting each firm observe a pair of signals about it, one private and one public; see Section 7 for an example along these lines.

Alternatively, consider models with “sticky information” as in Mankiw and Reis (2002) and Chung, Herbst, and Kiley (2015). This specification is nested in our framework by letting \( \varphi \) assign probability \( \mu \) to \( \omega^i_t = (\omega^{i-1}_t, s^i_t) \) and probability \( 1 - \mu \) to \( \omega^i_t = \omega^{i-1}_t \), where \( \mu \in (0, 1) \) is the probability with which a firm updates its information set with perfect observation of the underlying state and \( 1 - \mu \) is the probability with which the firm is stuck with her old information set.

Finally, consider the different forms of rational inattention found in Sims (2003), Mackowiak and Wiederholt (2009), Myatt and Wallace (2012) and Pavan (2016), or the model of fixed observation costs found in Alvarez, Lippi, and Paciello (2011). For our purposes, these approaches boil down to

\(^5\)Although we make a specific modeling choice regarding which input choices are restricted to be contingent on \( \omega^i_t \), what is essential for our results is that some inputs are chosen on the basis of incomplete information, not the precise interpretation of these inputs. Moreover, the assumption that at least one input can adjust to the realized \( s^i \) is standard in both the New Keynesian literature and the recent literature on the informational foundations of nominal rigidity (Mankiw and Reis, 2002; Woodford, 2003; Mackowiak and Wiederholt, 2009). Without this assumption market clearing would not be possible, and some form of rationing would have to be introduced.
(i) allowing each firm to choose its own $\varphi$, the joint distribution of its signal and of the underlying state, and (ii) making different assumptions on set of feasible $\varphi$ and the associated cognitive costs. These possibilities are nested in the extension studied in Appendix C.

Relation to Ramsey and New Keynesian literatures. When both the nominal and the real rigidity are assumed away (meaning that all prices and inputs can be measurable in $s^t$), our framework reduces to a prototypical Ramsey economy such as those found in Lucas and Stokey (1983) and Chari, Christiano, and Kehoe (1994). More importantly, our framework nests the New Keynesian setting of Correia, Nicolini, and Teles (2008) by dropping the measurability constraint on $(h_{it}, k_{it}, \ell_{it}, y_{it})$, maintaining the measurability constraint on $p_{it}$, and letting $\omega_{it}^t = s^{t-1}$ with probability $\lambda$ and $\omega_{it}^t = s^t$ with probability $1 - \lambda$, which means that a fraction $\lambda$ of the firms must set their prices one period in advance while the rest can adjust their prices freely.

This nesting permits us to clarify three elementary points. First, the earlier results of Correia, Nicolini, and Teles (2008) directly extend to the alternative, information-based forms of nominal rigidity considered in Mankiw and Reis (2002), Woodford (2003), Mackowiak and Wiederholt (2009), and Alvarez, Lippi, and Paciello (2011). Second, these earlier results do not directly apply to our setting because, and only because, of the real rigidity formalized in Property 1 above and of the associated imperfection in the coordination of production. And third, this imperfection is the sole source of our result regarding the optimality of monetary policies that lean against the wind. All these points will be made clear in the sequel.

3 Sticky vs Flexible Prices: Definitions

The dual role of the informational friction as both a source of nominal and real rigidity is a defining feature of our framework. Accordingly, we are ultimately interested in the scenario in which both rigidities are present. To understand the role of monetary policy in this scenario, it is nevertheless instrumental to study the alternative scenario in which the nominal rigidity is artificially shut down by letting all prices be measurable in $s^t$. Borrowing, and paraphrasing, the terminology of the New Keynesian literature, we henceforth refer to the scenario that embeds the nominal rigidity as “sticky prices” and to the one that assumes it away as “flexible prices.” In this section, we define the sets of allocations, prices, and policies that can be part of an equilibrium under each scenario.

To start with, we introduce some useful notation. We henceforth represent an allocation by a sequence $\xi \equiv \{\xi_t(.)\}_{t=0}^\infty$, where

$$\xi_t(.) \equiv \{k_t(.), h_t(.), \ell_t(.), y_t(.); K_t(.), H_t(.), L_t(.), Y_t(.), C_t(.)\}$$

is a vector of functions that map the realizations of uncertainty to the quantities chosen by the
typical firm (for the first four components of $\xi_t$) and the aggregate quantities (for the remaining five components). We similarly represent a price system by a sequence $\varrho \equiv \{\varrho_t(\cdot)\}_{t=0}^T$, where

$$\varrho_t(\cdot) \equiv \{p_t(\cdot), P_t(\cdot), r_t(\cdot), w_t(\cdot), Q_t(\cdot)\}$$

is a vector of functions that map the realizations of uncertainty to the price set by the typical firm, the aggregate price level, the real wage rate, the real rental rate of capital, and the prices of the Arrow securities. We finally represent a policy with a sequence $\theta \equiv \{\theta_t(\cdot)\}_{t=0}^T$, where

$$\theta_t(\cdot) \equiv \{\tau^r_t(\cdot), \tau^k_t(\cdot), \tau^e_t(\cdot), B_t(\cdot), D_t(\cdot), R_t(\cdot)\}$$

is a vector of functions that map the realizations of uncertainty to the various policy instruments.

Throughout our analysis, we let the domain of $K_t(\cdot), H_t(\cdot), L_t(\cdot), Y_t(\cdot), C_t(\cdot), P_t(\cdot), r_t(\cdot), w_t(\cdot), Q_t(\cdot), \tau^r_t(\cdot), \tau^k_t(\cdot), \tau^e_t(\cdot), B_t(\cdot), D_t(\cdot),$ and $R_t(\cdot)$ be $S^t$. This reflects the fact that our analysis abstracts from informational frictions on the side of either the representative household or the government. In contrast, the informational friction of the firms is embedded in Properties 1 and 2.

We finally express the aggregate level of output and the aggregate price level as follows:

$$Y(s^t) = \left[\sum_{\omega \in \Omega^t} (y(\omega, s^t))^{\frac{\rho - 1}{\rho}} \varphi(\omega | s^t)\right]^{\frac{1}{\rho - 1}} \quad \text{and} \quad P(s^t) = \left[\sum_{\omega \in \Omega^t} (p(\omega))^{\rho - 1} \varphi(\omega | s^t)\right]^{\frac{1}{\rho - 1}}. \quad (1)$$

We can then define our notion of sticky-price equilibria as follows.\footnote{The only essentially novelty in the definition is the pair of measurability constraints imposed on the firm’s problem. The precise formulation of this problem, as well as that of the household’s problem, can be found in Appendix A.}

**Definition 1.** A **sticky-price equilibrium** is a triplet $(\xi, \varrho, \theta)$ of allocations, prices, and policies that satisfy Properties 1 and 2 and are such that:

(i) $\{C(\cdot), L(\cdot), K(\cdot), B(\cdot), D(\cdot)\}$ solves the household’s problem;

(ii) $\{p(\cdot), k(\cdot), h(\cdot), \ell(\cdot), y(\cdot)\}$ solves the firm’s problem;

(iii) the quantity and the price of the final good are given by (1);

(iv) the government’s budget constraint is satisfied;

(v) all markets clear.

We next define our notion of flexible-price equilibria by dropping the measurability constraint on prices. Formally, we replace Property 2 with the following property:

**Property 2’.** The prices satisfy

$$p_t = p_t(\omega_t^t, s^t)$$

for all $i$, all $t$, and all realizations of uncertainty.
Accordingly, we adjust the formula for the price level in condition (1) as follows:

\[
P(s^t) = \left[ \sum_{\omega_i^t} \left( p(\omega_i^t, s^t) \right)^{\rho-1} \varphi \left( \omega_i^t \mid s^t \right) \right]^{\frac{1}{\rho-1}}
\]

We can then state the relevant definition as follows.

**Definition 2.** A *flexible-price equilibrium* is a triplet \((\xi, \varrho, \theta)\) of allocations, prices, and policies that satisfy the same conditions as those stated in Definition 1, except that Property 2 is replaced by Property 2’ and, accordingly, the price level is given by condition (2).

We let \(X^f\) and \(X^s\) denote the sets of the allocations that are part of a flexible-price and a sticky-price equilibrium, respectively. We also let \(X\) denote the (super)set of all feasible allocations, by which we mean allocations that satisfy the economy’s resource constraints along with Property 1.

### 4 Sticky vs Flexible Prices: Characterization and Replication

In this section we characterize, and compare, the sets of the allocations that can be part of either a flexible-price or a sticky-price equilibrium.

**Flexible-Price Allocations.** Consider any flexible-price equilibrium. The characterization of the household’s problem is standard. The characterization of the monopolist’s problem is slightly more exotic due to the heterogeneity in the signal \(\omega_i^t\) upon which the input choices are based. To conserve on notation, we henceforth let, for any \(z \in \{\ell, h, k\}\),

\[
MP_z (\omega_i^t, s^t) \equiv \left( \frac{y(\omega_i^t, s^t)}{Y(s^t)} \right)^{-\frac{1}{\rho}} A(s^t) \frac{\partial}{\partial z} F (k (\omega_i^t), h (\omega_i^t), \ell (\omega_i^t, s^t)).
\]

In the eyes of the planner, \(MP_z\) represents the marginal product of input \(z\) in firm \(i\), expressed in terms of the final good; in the eyes of the firm, it captures the corresponding marginal revenue product once it is multiplied by \(\chi^* \equiv \frac{\rho - 1}{\rho}\), the reciprocal of one plus the monopoly markup. As shown in Appendix A, we can express the first-order conditions of the firm as follows:

\[
(1 - \tau^r (s^t)) \chi^* MP_\ell (\omega_i^t, s^t) - w(s^t) = 0 \quad \forall t, \omega_i^t, s^t
\]

\[
\mathbb{E} \left[ \mathcal{M}(s^t) \left\{ (1 - \tau^r (s^t)) \chi^* MP_h (\omega_i^t, s^t) - 1 \right\} \mid \omega_i^t \right] = 0 \quad \forall t, \omega_i^t
\]

\[
\mathbb{E} \left[ \mathcal{M}(s^t) \left\{ (1 - \tau^r (s^t)) \chi^* MP_k (\omega_i^t, s^t) - r(s^t) \right\} \mid \omega_i^t \right] = 0 \quad \forall t, \omega_i^t.
\]

where \(\mathcal{M}(s^t) \equiv \frac{U_c(s^t)}{1 + \tau^r (s^t)}\) and \(U_c(s^t)\) is a short-cut for the marginal utility of consumption.
These conditions have a simple interpretation. The firm seeks to equate the cost of each input with its after-tax marginal revenue product. The only difference among the three conditions is the extent to which this goal is achieved. Because labor is contingent on the realized state \( s^t \), its marginal revenue product is equated with the real wage state-by-state. By contrast, the other two conditions hold only “on average,” that is, in expectation conditional on the firm’s signal.

Combining the optimality conditions of the firm with those of the household, imposing market clearing, and solving out for the prices and the policy instruments, we reach the following result.

**Proposition 1.** A feasible allocation, \( \xi \in \mathcal{X} \), is part of a flexible-price equilibrium if and only if the following two properties hold.

1. The allocation satisfies

\[
\sum_{t,s^t} \beta^t \mu (s^t) \left[ U_c (s^t) C (s^t) + U_\ell (s^t) L (s^t) \right] = 0. \tag{6}
\]

2. For every \( t \), there exist functions \( \psi^r, \psi^\ell, \psi^c, \psi^k : S^t \to \mathbb{R}_+ \) such that

\[
\psi^r (s^t) \chi^* MP_\ell (\omega^t_i, s^t) - \psi^\ell (s^t) = 0 \quad \forall \omega^t_i, s^t \tag{7}
\]

\[
\mathbb{E} \left[ \psi^r (s^t) \chi^* MP_h (\omega^t_i, s^t) - \psi^c (s^t) \mid \omega^t_i \right] = 0 \quad \forall \omega^t_i \tag{8}
\]

\[
\mathbb{E} \left[ \psi^r (s^t) \chi^* MP_k (\omega^t_i, s^t) - \psi^k (s^t) \mid \omega^t_i \right] = 0 \quad \forall \omega^t_i \tag{9}
\]

Condition (6) is familiar from the Ramsey literature. It encapsulates the absence of lump-sum taxation and follows directly from the intertemporal budget constraint of the government, after replacing the equilibrium prices and the policy instruments in terms of the allocation.

Consider next conditions (7), (8) and (9). Were the informational friction absent, each firm would know \( s^t \) and these conditions would reduce to, respectively,

\[
MP_\ell (\omega^t_i, s^t) = \frac{\psi^\ell (s^t)}{\psi^r (s^t)}, \quad MP_h (\omega^t_i, s^t) = \frac{\psi^c (s^t)}{\psi^r (s^t)}, \quad \text{and} \quad MP_k (\omega^t_i, s^t) = \frac{\psi^k (s^t)}{\psi^r (s^t)}, \quad \forall t, \omega^t_i, s^t.
\]

Since the \( \psi \)'s are free variables, these conditions would require that the marginal product of each input is equated across all firms, for all \( t \) and \( s^t \). This defines what we call “perfect coordination” in production. It also means that the sole role of the tax instruments in that benchmark is to control the wedges between the common MRTs of the firms and the corresponding MRSs of the household.

When instead the informational friction is present, each firm conditions her choices on an idiosyncratic signal of \( s^t \). As a result, marginal products are typically not equated across firms. This manifests as an aggregate TFP loss like that quantified in David, Hopenhayn, and Venkateswaran (2016). It also means that the available tax instruments may play a new role: their contingency
on $s^t$ influences how firms utilize their private information. This enables the planner to control not only the response of aggregate output to aggregate TFP and other shocks, but also the cross-sectional dispersion in produced quantities and marginal products. It is this new role of the taxes that is encoded into conditions (7)-(9). We illustrate this point with an example in Section 7.

**Sticky-Price Allocations.** We now add back the nominal rigidity (Property 2). As in the New Keynesian model, this allows the realized monopoly markup to fluctuate around the ideal one. Formally, there now exists a random variable $\chi(\omega^i_t, s^t)$, representing the reciprocal of the realized markup, such that the following properties are true. First, the optimality conditions (3)-(5) are modified by replacing $\chi^*$ with $\chi(\omega^i_t, s^t)$. And second, the following optimality condition is added:

$$\mathbb{E} \left[ M(s^t) Y(s^t)^{1/\rho} y(s^t)^{1-1/\rho} (1 - \tau^r(s^t)) \left\{ \chi(\omega^i_t, s^t) - \chi^* \right\} \bigg| \omega^i_t \right] = 0 \ \forall \ \omega^i_t$$  \hspace{1cm} (10)

This condition captures the optimal price-setting behavior of the firm. It requires, in essence, that the risk-adjusted expectation of the realized markup coincides with the ideal one.

Adapting Proposition 1 to these modifications, we reach the following result.

**Proposition 2.** A feasible allocation, $\xi \in \mathcal{X}$, is part of a sticky-price equilibrium if and only if the following three properties hold.

(i) The allocation satisfies (6).

(ii) For every $t$, there exists functions $\psi^r, \psi^k, \psi^c : \mathcal{S}^t \to \mathbb{R}_+$ and $\chi : \Omega^t \times \mathcal{S}^t \to \mathbb{R}_+$ such that the following conditions hold:

$$\chi(\omega^i_t, s^t) \psi^r(s^t) \left( MP_r(\omega^i_t, s^t) - \psi^r(s^t) \right) = 0 \ \forall \ \omega^i_t, s^t$$  \hspace{1cm} (11)

$$\mathbb{E} \left[ \chi(\omega^i_t, s^t) \psi^r(s^t) \left( MP_r(\omega^i_t, s^t) - \psi^r(s^t) \right) \bigg| \omega^i_t \right] = 0 \ \forall \ \omega^i_t$$  \hspace{1cm} (12)

$$\mathbb{E} \left[ \chi(\omega^i_t, s^t) \psi^r(s^t) \left( MP_k(\omega^i_t, s^t) - \psi^k(s^t) \right) \bigg| \omega^i_t \right] = 0 \ \forall \ \omega^i_t$$  \hspace{1cm} (13)

(iii) The function $\chi : \Omega^t \times \mathcal{S}^t \to \mathbb{R}_+$ is log-separable in the sense that there exist positive-valued functions $\chi^\omega$ and $\chi^s$ such that

$$\log \chi(\omega^i_t, s^t) = \log \chi^\omega(\omega^i_t) + \log \chi^s(s^t) \ \forall \ \omega^i_t, s^t.$$  \hspace{1cm} (15)

Clearly, the only differences from Proposition 1 are the emergence of the wedge $\chi(\omega^i_t, s^t)$ in conditions (11)-(13) and the addition of conditions (14) and (15). As already explained, condition (14) follows from the optimal price-setting behavior of the firm. Condition (15), on the other hand, follows from the iso-elastic demand structure; see the Appendix for details.
Replication. Through the lens of Proposition 2, $\chi(\omega_t^i, s^t)$ represents an additional control variable for the planner, one that encapsulates the power of monetary policy over real allocations. This power is non-trivial, but it is also restrained by conditions (14) and (15). Since both conditions are automatically satisfied by letting $\chi(\omega_t^i, s^t) = \chi^\ast$, the following is immediate.

Corollary 1. Every flexible-price allocation can be replicated as a sticky-price allocation: $X^f \subset X^s$.

This proves that an appropriate monetary policy can undo the nominal rigidity, but does not tell us whether such a policy is optimal or how it looks like. We address these questions next.

5 The Ramsey Optimum

In this section we define and characterize the efficiency benchmark that is relevant for our purposes. This leads to our main results regarding the optimal monetary policy.

An appropriate efficiency benchmark. Our ultimate goal is to solve the problem of a Ramsey planner who maximizes welfare over $X^s$, the set of sticky-price allocations. To this goal, we first characterize the allocation $\xi^\ast$ that maximizes welfare over an enlarged set, denoted by $X^R$ and consisting of all technologically and informationally feasible allocations that satisfy only condition (6). That is, from the six implementability constraints seen in Proposition 2, we maintain the first one, which encapsulates the absence of lump-sum taxation, but drop the remaining ones. This is akin to allowing the planner to impose a completely flexible set of input- and signal-specific taxes.

Proposition 3. There exists a constant $\Gamma \geq 0$, capturing the shadow value of government revenue, such that $\xi^\ast$, the optimal allocation over the enlarged set $X^R$, is given by the feasible allocation that satisfies the following conditions:

$$
\tilde{U}_c(s^t) MP_t (\omega_t^i, s^t) + \tilde{U}_t(s^t) = 0 \quad \forall \omega_t^i, s^t
$$

$$
E \left[ \tilde{U}_c(s^t) \left\{ MP_h (\omega_t^i, s^t) - 1 \right\} \right| \omega_t^i] = 0 \quad \forall \omega_t^i
$$

$$
E \left[ \tilde{U}_c(s^t) \left\{ MP_k (\omega_t^i, s^t) - \kappa (s^t) \right\} \right| \omega_t^i] = 0 \quad \forall \omega_t^i
$$

for some function $\kappa : S^t \to \mathbb{R}_+$ that captures that net-of-tax rental rate of capital and that satisfies

$$
\tilde{U}_c(s^t) = \beta E \left[ \tilde{U}_c(s^{t+1}) \left\{ 1 + \kappa (s^{t+1}) - \delta \right\} \right| s^t] \quad \forall s^t,
$$

where $\tilde{U}_c(s^t)$ and $\tilde{U}_t(s^t)$ are shortcuts for $\frac{\partial}{\partial C} \tilde{U} (C(s^t), L(s^t), s^t; \Gamma)$ and $\frac{\partial}{\partial L} \tilde{U} (C(s^t), L(s^t), s^t; \Gamma)$, respectively, and where

$$
\tilde{U} (C, L, s; \Gamma) \equiv U (C, L, s) + \Gamma \left[ C \frac{\partial}{\partial C} U (C, L, s) + L \frac{\partial}{\partial L} U (C, L, s) \right].
$$
To understand this result, momentarily shut down the informational friction. In this case, conditions (16)-(18) reduce to the following:

\[ MP_k(s^t) = \bar{U}_k(s^t), \quad MP_h(s^t) = 1, \quad \text{and} \quad \bar{U}_c(s^t) = \beta E \left[ \bar{U}_c(s^{t+1}) (1 - \delta + MP_k(s^{t+1})) \right] | s^t, \]

where \( MP_z(s^t) \) now denotes the common marginal product of input \( z \) in all firms. The first condition is identical to the one found in Lucas and Stokey (1983) and identifies the optimal tax on labor. The second condition implies that the tax on the intermediate input is zero, an example of the result in Diamond and Mirrlees (1971): taxes should not interfere with productive efficiency. The last condition is identical to that found in Chari, Christiano, and Kehoe (1994) and relates to the celebrated Chamley-Judd result about the optimality of zero taxes on capital income.

Now add back in the informational friction. In general, optimality requires that each firm condition her choices on her private information about the underlying state. Because such information contains idiosyncratic noise, the marginal products are no more equated across the firms. In comparison to the previous literature, this property may be misinterpreted as a symptom of productive inefficiency and relative-price distortions; but through the lens of Proposition 3, it is understood as the by-product of the socially optimal decentralized use of information. This explains how our analysis revisits the concept of relative-price distortions.

Proposition 3 also revises the concept of the output gap. Because the CES structure implies that the social value of producing an extra unit of any given good increases with the quantities of other goods, the planner finds it optimal to let the firms coordinate their input choices. This means that the third best characterized here allows a firm’s production to vary, not only with her information about the underlying fundamentals, but also with her beliefs about the beliefs of other firms. The business cycle can thus be driven by seemingly exotic sentiments, of the kind formulated in Angeletos and La’O (2013) and Benhabib, Wang, and Wen (2015) and quantified in Angeletos, Collard, and Dallas (2017) and Huo and Takayama (2015a). Under a traditional policy perspective, such fluctuations can be misinterpreted as fluctuations in the output gap; but through our analysis, they are recast as fluctuations in potential output.

To sum up, not only do the observable properties of the optimum have to be modified, but also the familiar goals of “minimizing relative-price distortions” and “stabilizing the output gap” must be revised before we may understand the role of monetary policy.

**Implementation.** We now show how the optimum characterized in Proposition 3 can be implemented with the available policy instruments.

---

7Formally, the optimal allocation can be understood as the Perfect Bayesian Equilibrium of a game of strategic complementarity, in line with the more abstract analysis in Angeletos and Pavan (2007).
Recall that $\mathcal{X}^R$ is a superset of both $\mathcal{X}^f$ and $\mathcal{X}^s$ because it allows the planner to make the production choices of each firm an arbitrary function of her private information, whereas $\mathcal{X}^f$ and $\mathcal{X}^s$ restrain that control in the manner described in part (ii) of, respectively, Propositions 1 and 2. Yet, the additional control afforded by $\mathcal{X}^R$ is immaterial for optimality. In particular, as shown in Appendix A, we have that $\xi^* \in \mathcal{X}^f$, meaning that $\xi^*$ can be implemented as a flexible-price equilibrium. By Corollary 1, we have $\mathcal{X}^f \subset \mathcal{X}^s$. It follows that $\xi^*$ can be implemented as a sticky-price allocation with a monetary policy that replicates flexible-prices. And because $\mathcal{X}^s \subset \mathcal{X}^R$, we have $\xi^*$ maximizes welfare over all sticky-price allocations.

Combining these findings, and identifying the taxes that support $\xi^*$ as an equilibrium, we reach the following result.

**Theorem 1.** The allocation $\xi^*$ obtained in Proposition 3 identifies the optimal allocation and is implemented with:

(i) a monetary policy that replicates flexible prices; and

(ii) the following set of taxes:

$$
\frac{1 - \tau^f(s^t)}{1 + \tau^c(s^t)} = \frac{U_\ell(s^t)}{U_\ell(s^t)} \frac{U_c(s^t)}{U_c(s^t)}, \quad 1 - \tau^k(s^t) = 1, \quad 1 - \tau^r(s^t) = \frac{\rho}{\rho - 1}, \quad 1 + \tau^c(s^t) = \delta \frac{U_c(s^t)}{U_c(s^t)}
$$

(20)

where $U_c, U_\ell, \bar{U}_c,$ and $\bar{U}_\ell$ are evaluated at $\xi^*$ and where $\delta > 0$ is any state-invariant scalar.

Part (i) extends the related result of Correia, Nicolini, and Teles (2008) to the class of economies under consideration. Part (ii) generalizes the optimal taxation results of Lucas and Stokey (1983) and Chari, Christiano, and Kehoe (1994). There are, however, three subtle differences.

First and foremost, replicating flexible prices is no more synonymous to targeting price stability. We expand on this point in the next section.

Second, the relevant wedges are evaluated at an allocation whose observable properties may differ from those characterized in the aforementioned works, for the reasons already explained. This opens the door to the possibility that the cyclical properties of the optimal taxes are different even though the tax formulas obtained are essentially the same.

Third, the consumption tax may play a novel role. In the existing literature, $\tau^c$ is typically restricted to be zero and this restriction is without loss of optimality insofar as public debt is state-contingent and the the zero lower bound on the nominal interest rate is non-binding. Here, instead, it is generally necessary to let $\tau^c$ vary with the state of Nature so as to make sure that the firms face the right price of risk when setting their prices.

The last two subtleties can be sidestepped by imposing the following, homothetic specification for preferences:

$$
U(C, L) = \frac{C^{1-\gamma}}{1-\gamma} - \eta \left( \frac{L^{1+\epsilon}}{1+\epsilon} \right),
$$

(21)
for $\gamma, \epsilon, \eta > 0$. In this case, the optimal allocation is implemented with a state-invariant tax on labor, a zero tax on capital, and a zero tax on consumption; See Lemma 5 and its proof in Appendix A. From an applied perspective, the most important lesson therefore seems to be the incompatibility of replicating flexible prices with targeting price stability, to which we turn next.

6 On the Optimal Cyclicality of the Price Level

Within the New Keynesian framework, the logic in favor of price stability is that it minimizes relative-price distortions (or, equivalently, maximizes productive efficiency). We now explain why this logic is upset once the informational friction is taken into consideration and the efficiency benchmark is revised along the lines we described in the previous section.

We start by noting that, along the optimal allocation, the output of each firm can be expressed as the logarithmic sum of two components: one measurable in the firm’s private information and the other measurable in the realized state.

Lemma 1. There exist positive-valued functions $\Psi^\omega$ and $\Psi^s$ such that, along the optimal allocation, the output of a firm can be expressed as

$$\log y(\omega^i_t, s^t) = \log \Psi^\omega(\omega^i_t) + \log \Psi^s(s^t) \quad \forall \omega^i_t, s^t.$$  \hfill (22)

The precise values of these components follow from the solution to the optimality conditions in Proposition 3. In Appendix A (see, in particular, the proof of Lemma 2), we show how $\Psi^\omega$ may be expressed as a function of the input choices that firms make on the basis of their imperfect observation of the state of the economy, whereas $\Psi^s$ captures the adjustment in the labor input that takes place in order for supply to meet the realized demand, and markets to clear, at the set prices.

In the example studied in the next section, all these objects can be solved in closed form as simple functions of the available signals. It then becomes evident how the optimal input choices and the aforementioned output components covary with the state of the economy. For the present purposes, however, it suffices to note the following general points.

Whereas $\Psi^s$ captures the component of the output that is common across all firms, $\Psi^\omega$ captures the component that is driven by each firm’s private information. The latter component can be thought of as a proxy of the firm’s idiosyncratic belief about the state of the economy. Along the optimal allocation, this typically means that an optimistic firm is associated with a higher $\Psi^\omega$, and produces more, than a pessimistic one.

Furthermore, $\Psi^\omega$ is the only source of variation in relative quantities and, thereby, in relative
prices. Indeed, by the relative demand for the goods produced by firms \(i\) and \(j\), we have

\[
\log p(\omega_t^i) - \log p(\omega_t^j) = -\frac{1}{\rho} \left[ \log y(s^t, \omega_t^i) - \log y(s^t, \omega_t^j) \right]
\]

Using condition (22), we then get

\[
\log p(\omega_t^i) - \log p(\omega_t^j) = -\frac{1}{\rho} \left[ \log \Psi(\omega_t^i) - \log \Psi(\omega_t^j) \right],
\]

which verifies that the relative price of any two firms is inversely related to their relative belief, as measured by the log-difference between \(\Psi(\omega_t^i)\) and \(\Psi(\omega_t^j)\). Intuitively, if optimistic firms are to produce more than pessimistic ones, they must also charge lower relative prices.

This elementary insight underlies our result regarding the suboptimality of price stability. As long as firm \(i\) does not know \(\omega_t^j\) and, symmetrically, firm \(j\) does not know \(\omega_t^i\), their relative price can be inversely related with their relative quantity only if the nominal price of firm \(i\) is itself negatively related to her belief, as captured by \(\Psi(\omega_t^i)\), and similarly for \(j\). Formally, it has to be that

\[
\log p(\omega_t^i) = z_t - \frac{1}{\rho} \log \Psi(\omega_t^i),
\]

for some variable \(z_t\) that is commonly known to the firms (meaning that the prices of all the firms can be contingent on \(z_t\)). Aggregating the above, we get that the aggregate price level must satisfy

\[
\log P(s^t) = z_t - \frac{1}{\rho} \log B(s^t),
\]

(23)

where

\[
B(s^t) \equiv \left[ \int \Psi(\omega_t^i) \frac{\rho-1}{\rho} d\mu(\omega_t^i|s^t) \right]^{\frac{\rho}{\rho-1}}.
\]

We thus reach the following result.

**Theorem 2.** Along any sticky-price equilibrium that implements the optimal allocation, the price level is negatively correlated with the average belief and real economic activity, as proxied by \(B(s)\).

This is our main result regarding the sub-optimality of price stability. Its applicability hinges on relating the object \(B(s^t)\) to a more concrete measure of economic activity. This is done in Section 7 within an example that allows for an explicit solution of the optimal allocation and the optimal price level. That example relies on assuming away capital and imposing a Gaussian information structure. But even without these restrictions, the following result can be shown.

**Lemma 2.** Along any sticky-price equilibrium that implements the optimal allocation, \(B(s^t)\) is, to a first-order log-linear approximation, a log-linear combination of the aggregate quantities of firm inputs; \(B(s^t)\) is therefore procyclical if inputs are also procyclical.
This corroborates the interpretation of $\mathcal{B}(s^t)$ as proxy for the aggregate level of economic activity and the interpretation of Theorem 2 as a case for “leaning against the wind.”

The logic for our result follows directly from our earlier discussion about the relation between relative prices and relative beliefs. Because optimality requires that the output of each firm varies with its belief about the state of the economy, and because relative prices are inversely related to relative quantities, the nominal price of a firm has to move in the opposite direction that its belief and its output. At the aggregate level, this translates to negative co-movement between the price level and real output—property that resembles nominal GDP targeting.

It is worth noting, however, two subtleties. First, Theorem 2 allows for a certain degree of nominal indeterminacy: as evident in condition (23), the price level is indeterminate vis-a-vis any variable $z_t$ that is common knowledge to the firms. This is because firms can perfectly coordinate their price responses to any such shock, which in turn guarantees that varying the response of monetary policy to $z_t$ affects the variation in the price level without affecting the real allocations.\(^8\)

Second, Theorem 2 contains also a case for price stability: if the optimal allocation is invariant with a shock, then it is optimal to stabilize the price level vis-a-vis that shock. Consider, for example, a pure sunspot, namely a shock that is orthogonal not only to the underlying fundamentals but also to the entire hierarchy of beliefs about them. Alternatively, abstract from capital accumulation and consider a shock to beliefs of future TFP. In either case, the optimal allocation remains stable. If the monetary authority fails to stabilize the price level with respect to the shock under consideration, the production of a positive mass of firms will vary with it, contradicting optimality.

We close this section by emphasizing that our result hinges on allowing the informational friction to be a real friction in the sense of Property 1. We formalize this point below.

**Proposition 4.** Suppose we maintain Property 2 but drop Property 1; that is, we maintain the nominal role of the informational friction but drop the real one. Then, the optimal allocation is implemented by targeting price stability.

This is essentially the main result of Correia, Nicolini, and Teles (2008). Recall that the setting in that paper may be nested in our framework when the real rigidity is assumed away and the nominal rigidity is such that a fraction $\lambda$ of the firms set their prices one period in advance (in which case $\omega^t_i = s^{-1}$) while the remaining are free to adjust their prices (in which case $\omega^t_i = s^t$). Proposition 4 therefore replicates the main result of that paper and also extends it the alternative, information-based foundations of the nominal rigidity proposed by Mankiw and Reis (2002), Woodford (2003), Mackowiak and Wiederholt (2009), and others.

\(^8\)The source of this indeterminacy is similar to that in the older literature on nominal confusion (Lucas, 1972; Barro, 1976); it is clearly welfare-irrelevant in our setting; and can refined away by imposing that no shock is common knowledge. We suspect this indeterminacy disappears also if we add a Calvo friction, even a tiny one, for this helps anchor the optimal price level at all $t \geq 0$ to $P_{-1}$, the historical price level.
To sum up, what drives the particular kind of “leaning against the wind” documented in our paper, is precisely the real bite of the informational friction, captured herein by Property 1.\footnote{This also explains why Adam (2007) and Paciello and Wiederholt (2014), which abstract from the real rigidity that has been the focus of our paper, let monetary policy substitute for missing tax instruments. Ball, Mankiw, and Reis (2005) also abstract from the real rigidity, but focus on a different issue, the transition from a suboptimal to an optimal policy.}

7 An Illustration

In this section we use a tractable Gaussian example—one similar to those studied in Woodford (2003) and Angeletos and La’O (2010)—to illustrate the main lessons of our paper. In particular, we first demonstrate how the policy instruments can manipulate the decentralized use of information and can possibly insulate aggregate output from the effects of noise, sentiments, and the like. We next characterize the optimal allocation, contrast it to its complete-information counterpart, and show how the price level moves in the opposite direction than aggregate output.

Set up. We abstract from capital, let government spending be constant, specify preferences as in condition (21), and add idiosyncratic TFP shocks. The production function is thus given by

\[
y_{it} = A_{it} (h_{it}^\eta)^{1-\alpha} f_{it}^\alpha,
\]

where $\alpha \in (0,1)$ and $\eta \in (0,1)$ and where $A_{it}$, the productivity of firm $i$ in period $t$, is comprised of both an aggregate and a firm-specific component. In particular,

\[
a_{it} \equiv \log A_{it} = a_t + v_{it},
\]

where $a_t \equiv \log A_t$ is the aggregate component and $v_{it}$ is the idiosyncratic one. The processes of $a_t$ and $v_{it}$ are Gaussian, stationary, and orthogonal to one another. The idiosyncratic component $v_{it}$ is i.i.d. across firms but can be correlated over time within a firm. The aggregate component $a_t$ can also be correlated over time. We finally let each firm know its own productivity, $a_{it}$, but not the underlying aggregate component, $a_t$.

The results presented below impose no further restrictions on the process for $a_t$ and the available signals about it. This permits us to accommodate rich learning dynamics as well as rich higher-order uncertainty. For instance, by letting $a_t$ follow an AR(1) process and each firm observe a noisy private signal of $a_t$ in each period, we can accommodate the kind of inertial belief dynamics studied in Woodford (2003), Nimark (2008), Angeletos and Huo (2018), and elsewhere.

To fix ideas, however, the reader may restrict attention to the special case in which $a_t$ is i.i.d. over time and firm $i$’s information in period $t$ is given by the pair $(a_{it}, z_{it})$, where $z_{it}$ is a noisy
signal given by

\[ z_{it} = a_t + \sigma_v v_t + \sigma_\epsilon \epsilon_{it} \]  

where \( \epsilon_{it} \) and \( v_t \) are, respectively, idiosyncratic and aggregate noises, independent of one another and of \( a_t \). We let scalars \( \sigma_\epsilon > 0 \) and \( \sigma_v > 0 \) parameterize the level of the two noises, respectively. In this example, the shock \( v_t \) is a source of correlated noise in firms’ first- and higher-order beliefs. Also note that the case of a noisy public signal is nested by letting \( \sigma_\epsilon \to 0 \), whereas the case with purely private information and no aggregate noise is nested by letting \( \sigma_v \to 0 \).

**Remark.** As already noted, the example introduced above can be thought of as a hybrid of Woodford (2003) and Angeletos and La’O (2010). Woodford (2003) assumes away the real rigidity and lets monetary policy induce an exogenous Gaussian process for nominal GDP. Angeletos and La’O (2010) shuts down the nominal rigidity and abstracts from both fiscal and monetary policy. Relative to these earlier works, we not only combine the two forms of rigidity in the same example, but also work out the optimal policy.

**Manipulating the Decentralized Use of Information.** Before characterizing the optimal policy, we find it useful to illustrate how the state contingency of the policy instruments can influence the decentralized use of information and thereby the stochastic process of aggregate output. To this end, we specify the tax system such that the relevant wedges are log-linear functions of aggregate productivity and aggregate output only. In particular, we impose

\[
-\log (1 - \tau^r (A_t, Y_t)) = \hat{\tau}_0 + \hat{\tau}_A \log A_t + \hat{\tau}_Y \log Y_t
\]

for some scalars \( \hat{\tau}_0, \hat{\tau}_A, \hat{\tau}_Y \in \mathbb{R} \). We then let the remaining tax rates satisfy \( \tau^k(s^t) = \tau^c(s^t) = 0 \) and \( 1 + \tau^f(s^t) = 1 / (1 - \tau^r(s^t)) \). The scalars \( (\hat{\tau}_0, \hat{\tau}_A, \hat{\tau}_Y) \) can then be thought of as the policy coefficients. Finally, here and for the rest of this section, we consider the log-linearized approximation of the equilibrium allocations around the steady state in which \( A_t \) takes its unconditional mean value.

**Proposition 5.** Consider the economy and the taxes described above.

In any flexible-price equilibrium, GDP satisfies, up to a log-linear approximation,

\[
\log GDP (s^t) = \gamma_0 + \gamma_A \log A_t + \gamma_u u_t,
\]

where the scalars \( (\gamma_0, \gamma_A, \gamma_u) \) are pinned down by the policy coefficients \( (\hat{\tau}_0, \hat{\tau}_A, \hat{\tau}_Y) \) and where \( u_t \) is a Normally distributed random variable, orthogonal to \( \log A_t \), with mean 0 and variance 1.

Furthermore, by appropriately choosing the policy coefficients \( (\hat{\tau}_A, \hat{\tau}_Y) \), the planner can imple-
ment any pair \((\gamma_A, \gamma_u)\) inside the set \(\Upsilon\), where

\[
\Upsilon \equiv \{ (\gamma_A, \gamma_u) \in \mathbb{R}^2 : \text{either } \gamma_u > 0 \text{ and } \gamma_A > \hat{\gamma} + \gamma_u, \text{ or } \gamma_u < 0 \text{ and } \gamma_A < \hat{\gamma} + \gamma_u \} \tag{27}
\]

and where \(\hat{\gamma}\) is a constant that depends on the underlying preference, technology, and information parameters but is invariant to policy and the implemented allocation.

To understand this result, note that \(u_t\) is the (standardized) residual of regressing aggregate output on the current aggregate productivity. This residual is zero in the absence of the informational friction but not when it is present. For instance, in the aforementioned special case in which \(a_t\) is i.i.d. over time and the signals are as in condition (25), \(u_t\) coincides with \(v_t\), the aggregate noise in the available signals. More generally, \(u_t\) encapsulates all aggregate variation in the firms’ first- and higher-order beliefs that is orthogonal to the current fundamentals (TFP).

With these points in mind, Proposition 5 can be read as follows: by appropriately designing the coefficients \(\hat{\tau}_A\) and \(\hat{\tau}_Y\), the planner can affect both the covariation of aggregate output with the current fundamental and its residual variation due to noise or higher-order uncertainty. This is because these tax coefficients control how sensitive a firm’s net-of-taxes revenue is to, respectively, TFP and the actions of other firms. As a result, these coefficients indirectly control the incentives each firm has in reacting to different pieces of information about these objects. In sum, policy coefficients may be used to control the decentralized use of information.

It can be shown that a similar result applies to monetary policy with the analogues of \(\hat{\tau}_A\) and \(\hat{\tau}_Y\) being the responsiveness of the nominal interest rate to aggregate productivity and aggregate output, respectively. This illustrates our point that familiar policy instruments, whether fiscal or monetary, play novel roles once the informational friction is accommodated.

**The Ramsey Optimum.** Consider, as a reference point, the optimal allocation in the absence of the informational friction; this corresponds in effect to the Lucas-Stokey benchmark. In this case, it can be shown that aggregate output is given by

\[
\log Y_t = \gamma_0^{LS} + \gamma_A^{LS} \log A_t,
\]

for some scalars \(\gamma_0^{LS}\) and \(\gamma_A^{LS} > 0\) that depend on the preferences, technology, and level of government spending (or the tax distortion).

Consider now the case in which the informational friction is present. By Proposition 5, there exist policies such that aggregate output is given by (26) with \(\gamma_A = \gamma_A^{LS}\) and \(\gamma_u = 0\). That is, it is feasible for the planner to both induce the same covariation between aggregate output and aggregate productivity as in the frictionless benchmark and to insulate aggregate output from noise, sentiments, etc.
This is made possible by combining a $\tau_Y$ high enough so that the net-of-taxes returns are invariant to aggregate output and a $\tau_A$ low enough so that the net-of-taxes returns are sufficiently sensitive to aggregate productivity. The former property guarantees that the firms disregard information that is useful in predicting the choice of other firms but is not useful in predicting aggregate productivity; the latter ensures that the firms respond with enough strength to variation in aggregate productivity.

This may sound like a win-win situation. But it is not. When the planner induces the firms to disregard information about one another’s choices over information about the fundamentals, she exacerbates the mis-coordination of production and implements an inefficiently high level of cross-sectional dispersion in quantities. To economize on this margin, the optimal allocation allows the firms to utilize that kind of information, thus also allowing aggregate output to move with noise, sentiments, etc. That is, optimality calls for $\gamma_u > 0$.

For essentially the same reason, the optimal allocation also induces a lower covariation between aggregate output and aggregate productivity than in the frictionless benchmark: the alternative requires that the firms respond too strongly to their private information and induces too much cross-sectional dispersion in quantities. That is, optimality calls for $\gamma_A < \gamma_A^{LS}$.

These points are established formally in the next proposition, which characterizes the process of aggregate output along the optimal allocation.

**Proposition 6.** In any equilibrium that implements the optimal allocation, GDP is given by

$$\log GDP_t = \gamma_0^* + \gamma_A^* \log A_t + \gamma_u^* u_t,$$

where $u_t$ is a Normally distributed random variable, orthogonal to $\log A_t$, with mean 0 and variance 1, and where the scalars $\gamma_A^*$ and $\gamma_u^*$ are uniquely determined by the underlying preference, technology, and information parameters. Furthermore,

$$0 < \gamma_A^* < \gamma_A^{LS} \quad \text{and} \quad \gamma_u^* > 0.$$  \hfill (29)

This result illustrates how the efficiency benchmark identified in our paper differs from that found in the literature. First, GDP features a lower sensitivity to the underlying fundamental than in the Lucas-Stokey benchmark. And second, GDP varies with noise, sentiments, beliefs, etc.

**Monetary Policy.** We now turn attention to the optimal monetary policy and the associated price level. Theorems 1 and 2, of course, apply. The goal is to illustrate the particular form of “leaning against the wind” that obtains in the example under consideration.
Proposition 7. In any sticky-price equilibrium that implements the optimal allocation, the aggregate price level satisfies
\[\log P(s^t) = \delta_0^* - \delta_A^* \log A_t - \delta_u^* u_t,\]  
for some scalars \(\delta_0^*, \delta_A^*, \delta_u^*\) that are determined by the underlying preference, technology, and information parameters and satisfy \(\delta_A^* > 0\) and \(\delta_u^* > 0\).

Corollary 2. The optimal monetary policy targets a negative correlation between the price level and and GDP, both unconditionally and conditionally on the realized TFP.

This epitomizes our take-home policy lesson: the optimal policy leans against the wind both in response to innovations in the underlying fundamentals and in response to noise, sentiments, etc.

8 Conclusion

This paper studies the question of how informational frictions affect the nature of optimal monetary policy. To this goal, we analyze a setting in which firms had to make both their pricing and their production choices on the basis of an imperfect and idiosyncratic understanding of the state of the economy. This amounts to introducing dispersed private information and accommodating frictional coordination in the form of higher-order uncertainty.

In our setting, the optimal monetary policy is shown to replicate flexible-price allocations (properly defined). As in the New Keynesian paradigm, this holds as long as monetary policy does not have to substitute for missing tax instruments. Unlike that paradigm, however, the goal of replicating flexible-price allocations and minimizing relative-price distortions in our setting does not equate to a price-stability target. Instead, it implies a particular form of “leaning against the wind,” namely a negative correlation between the price level and aggregate output along the optimal path. This property we find is necessary in order to ensure that firms not discard valuable private knowledge about the underlying shocks.

To establish these lessons, we adapt the primal approach found in the Ramsey literature to the aforementioned kind of frictions. We show that this leads to a revision, not only of the concept of flexible-price allocations, but also of the efficiency benchmark relative to which the notions of the output gap and of relative-price distortions are to be defined. This benchmark differs from those studied in the literature because, and only because, the informational friction is a source of real rigidity. In particular, this benchmark may feature exotic business cycle properties which through the lens of conventional policy analysis could be misinterpreted as a call for stabilization policy but through the lens of our analysis are recast as symptoms of the efficient use of information.

Although real-world monetary policy surely reflects many concerns left outside our framework, our analysis emphasizes an aspect that has received relatively little attention but could be impor-
tant: the role of monetary policy in affecting the incentives agents face when deciding how to react to their decentralized private information, or when deciding what information to collect in the first place (see Appendix C).

The present paper and a few other recent papers have pushed the research frontier in this direction, but more work remains to be done. For instance, it would be interesting to extend our analysis to settings in which monetary policy must substitute for missing tax instruments; this would build a bridge to the work of Paciello and Wiederholt (2014), which concentrates on this possibility but abstracts from the real rigidity we have accommodated here. Alternatively, one could study the interaction between informational frictions and the zero lower bound on interest rates; Angeletos and Lian (2016) and Wiederholt (2016) have already pointed out how this could shed new light on the effectiveness of forward guidance, but they have not addressed optimality. Last but not least, the quantitative potential of our insights is another open question for future research.
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Appendix A: Proofs

In this Appendix, we first state and solve for the household and firms’ problems. We then prove two auxiliary lemmas which offer a complete characterization of the sets of sticky- and flexible-price equilibria. We then proceed with the proofs of all the results that appear in the main text. Throughout, we ease the notation by dropping the subscript $t$ from the functions $C_t(\cdot), L_t(\cdot), \text{etc},$ except for few occasions in which it is useful to make explicit the dependence on $t.$

The Household. Consider first the household. The statement of her problem is standard.

**Household’s Problem.** The household chooses \{\(C(t), L(t), K(t), B(t), D(t)\)\} so as to maximize expected utility,

\[
W = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu(s^t) \left[ U(C(s^t), L(s^t), s^t) \right],
\]

subject to her budget constraint,

\[
\left( 1 + \tau^c(s^t) \right) C(s^t) + X(s^t) + \frac{1}{P(s^t)} \left\{ B(s^t) + \sum_{s^{t+1}} Q(s^{t+1})D(s^{t+1}) \right\} = \left( 1 - \tau^k(s^t) \right) w(s^t) L(s^t) + \left( 1 - \tau^k(s^t) \right) r(s^t) K(s^{t-1}) + \frac{1}{P(s^t)} \left\{ (1 + R(s^{t-1})) B(s^{t-1}) + D(s^t) \right\} \quad \forall t, s^t,
\]

and the law of motion for capital,

\[
K(s^t) = (1 - \delta) K(s^{t-1}) + X(s^t) \quad \forall t, s^t.
\]

The Firm. Consider next the typical monopolistic firm. Her (ex ante) valuation is given by

\[
\mathcal{V} = \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t \mathcal{M}(s^t) \frac{\Pi(\omega^t, s^t)}{P(s^t)} \right] = \sum_{t=0}^{\infty} \sum_{\omega^t, s^t} \left\{ \beta^t \mathcal{M}(s^t) \frac{\Pi(\omega^t, s^t)}{P(s^t)} \varphi(\omega^t, s^t) \right\},
\]

where $\mathcal{M}(s^t) \equiv \frac{U_c(s^t)}{1 + \tau^c(s^t)}$ is the “pricing kernel,” $U_c(s^t)$ is a shortcut for $\frac{\partial}{\partial c} U(C(s^t), L(s^t), s^t), \text{and}$

\[
\Pi(\omega^t, s^t) \equiv (1 - \tau^r(s^t)) \frac{p(\omega^t)}{P(s^t)} y(\omega^t, s^t) - h(\omega^t) - w(s^t) \ell(\omega^t, s^t) - r(s^t) k(\omega^t-1)
\]

is the firm’s real profit net of the revenue tax. The demand faced by the monopolist is given by

\[
y(\omega^t, s^t) = \left( \frac{p(\omega^t)}{P(s^t)} \right)^{-\rho} Y(s^t).
\]
We may thus express the monopolist’s problem as follows.

**Monopolist’s Problem.** The typical monopolist chooses \( \{p, k, h, \ell, y\} \) so as to maximize its valuation,

\[
\sum_{t} \sum_{\omega \subseteq \Omega_t} \left\{ \beta^t \mathcal{M}(s^t) \left[ (1 - \tau^t(s^t)) \frac{p(\omega^t)}{P(s^t)} y(\omega^t, s^t) - h(\omega^t) - w(s^t) \ell(\omega^t, s^t) - r(s^t)k(\omega^t) \right] \varphi(\omega^t, s^t) \right\},
\]

subject to technology,

\[
y(\omega^t, s^t) = A(s^t) F(k_i(\omega^t), h_i(\omega^t), \ell_i(\omega^t, s^t)) \quad \forall t, s^t, \omega^t,
\]

and the demand for its product,

\[
y(\omega^t, s^t) = \left( \frac{p(\omega^t)}{P(s^t)} \right)^{-\rho} Y(s^t) \quad \forall t, s^t, \omega^t.
\]

Finally, since the cross-sectional distribution of the signal in period \( t \) and state \( s^t \) is given by \( \varphi(.|s^t) \), the following properties are self-evident: aggregate output is given by

\[
Y(s^t) = \left[ \sum_{\omega \in \Omega} \left( y(\omega, s^t) \right)^{\frac{\rho - 1}{\rho}} \varphi(\omega|s^t) \right]^{\frac{1}{\rho - 1}} \quad \forall t, s^t; \quad (32)
\]

the price level (the price of the final good) is given by

\[
P(s^t) = \left[ \sum_{\omega \in \Omega} \left( p(\omega) \right)^{\rho - 1} \varphi(\omega|s^t) \right]^{\frac{1}{\rho - 1}} \quad \forall t, s^t; \quad (33)
\]

the market for the final good clears if and only if

\[
C(s^t) + X(s^t) + G(s^t) + \sum_{\omega \in \Omega} h(\omega) \varphi(\omega|s^t) = Y(s^t) \quad \forall t, s^t; \quad (34)
\]

the market for labor clears if and only if

\[
\sum_{\omega \in \Omega} \ell(\omega) \varphi(\omega|s^t) = L(s^t) \quad \forall t, s^t; \quad (35)
\]

and the market for capital clears if and only if

\[
\sum_{\omega \in \Omega} k(\omega) \varphi(\omega|s^t) = K(s^t) \quad \forall t, s^t. \quad (36)
\]
We now state the two auxiliary lemmas, followed by their proofs.

**Lemma 3.** An allocation $\xi$, a policy $\theta$, and a price system $\varrho$ are part of a sticky-price equilibrium if and only if the following four properties hold.

(i) The following household optimality conditions are satisfied:

$$
\frac{U_c(s^t)}{(1 + \tau_c(s^t))P(s^t)} = \beta \left[ \frac{U_c(s^{t+1})}{(1 + \tau_c(s^{t+1}))P(s^{t+1})} \right] s^t
$$

$$
-U_\ell(s^t) = U_c(s^t) \frac{(1 - \tau^c(s^t))}{(1 + \tau_c(s^t))} w(s^t)
$$

$$
\frac{U_c(s^t)}{(1 + \tau_c(s^t))} = \beta \left[ \frac{U_c(s^{t+1})}{(1 + \tau_c(s^{t+1}))} \right] \left( 1 - R(s^{t+1}) - \delta \right) s^t
$$

$$
Q(s^{t+1}) = \beta \mu(s^{t+1}) \frac{U_c(s^{t+1})}{U_c(s^t)} \frac{(1 + \tau_c(s^{t+1}))P(s^t)}{(1 + \tau_c(s^{t+1}))P(s^{t+1})}
$$

where

$$\tilde{r}(s^t) = \left( 1 - \tau^k(s^t) \right) r(s^t)$$

is the net-of-taxes return on savings.

(ii) The following firm optimality conditions are satisfied:

$$
\lambda(\omega_i^t, s^t) A(s^t)f_\ell(\omega_i^t, s^t) - w(s^t) = 0
$$

$$
\mathbb{E} \left[ M(s^t) \left( \lambda(\omega_i^t, s^t) A(s^t)f_h(\omega_i^t, s^t) - 1 \right) \right] \omega_i^t = 0
$$

$$
\mathbb{E} \left[ M(s^t) \left( \lambda(\omega_i^t, s^t) A(s^t)f_k(\omega_i^t, s^t) - r(s^t) \right) \right] \omega_i^t = 0
$$

$$
\mathbb{E} \left[ M(s^t)y(\omega_i^t, s^t) \left\{ (1 - \tau^r(s^t)) \left( \frac{\rho - 1}{\rho} \right) \frac{p(\omega_i^t)}{P(s^t)} - \lambda(\omega_i^t, s^t) \right\} \right] \omega_i^t = 0
$$

with $M(s^t) \equiv \frac{U_c(s^t)}{1 + \tau_c(s^t)}$, along with the intermediate-good demand condition, namely,

$$
y(\omega_i^t, s^t) = \left( \frac{p(\omega_i^t)}{P(s^t)} \right)^{-\rho} Y(s^t)
$$

(iii) The household’s and the government’s budget constraints are satisfied.

(iv) All markets clear, namely, conditions (34), (35), and (36) are satisfied.

**Lemma 4.** An allocation $\xi$, a policy $\theta$, and a price system $\varrho$, are part of a flexible-price equilibrium if and only if the following four properties hold.

(i) The household optimality conditions in (37)-(40) are all satisfied;
Proof of Lemma 3. We first derive the household’s optimality conditions. Following this we derive the firm’s optimality conditions.

Household. Consider the Household’s problem stated above. Let $\Lambda(s^t)$ be the Lagrange multiplier on the Household’s budget constraint in history $s^t$. The Lagrangian for the household’s problem is given by

$$L = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu(s^t) \left[ U(C(s^t), L(s^t), s^t) \right]$$

where $\Lambda(s^t) = \frac{U_c(s^t)}{1 + \tau^c(s^t)}$, along with the intermediate-good demand condition (46).

(iii) The household’s and the government’s budget constraints are satisfied.

(iv) All markets clear, namely, conditions (34), (35), and (36) are satisfied.

$$\left(1 - \tau^r(s^t)\right) \frac{p^{-1}p(\omega^t)}{P(s^t)} A(s^t) f_L(\omega^t, s^t) - w(s^t) = 0 \quad (47)$$

$$\mathbb{E} \left[ \mathcal{M}(s^t) \left(1 - \tau^r(s^t)\right) \frac{p^{-1}p(\omega^t)}{P(s^t)} A(s^t) f_h(\omega^t, s^t) - 1 \right] \omega^t = 0 \quad (48)$$

$$\mathbb{E} \left[ \mathcal{M}(s^t) \left(1 - \tau^r(s^t)\right) \frac{p^{-1}p(\omega^t)}{P(s^t)} A(s^t) f_k(\omega^t, s^t) - r(s^t) \right] \omega^t = 0 \quad (49)$$

The household’s first order conditions for consumption, labor, bonds, and state-contingent securities are given by

$$\beta^t \mu(s^t) U_c(s^t) - \Lambda(s^t) \left(1 + \tau^c(s^t)\right) P(s^t) = 0, \text{ for all } s^t \quad (50)$$

$$\beta^t \mu(s^t) U_c(s^t) + \Lambda(s^t) \left(1 - \tau^c(s^t)\right) P(s^t) w(s^t) = 0, \text{ for all } s^t \quad (51)$$

$$-\Lambda(s^t) + \sum_{s^{t+1}|s^t} \Lambda(s^{t+1}) \left(1 + R(s^t)\right) = 0, \text{ for all } s^t \quad (52)$$

$$-Q(s^{t+1}) \Lambda(s^t) + \Lambda(s^{t+1}) = 0, \text{ for all } s^{t+1} \quad (53)$$
By combining (50) and (52) we derive the household’s Euler equation,

\[
\frac{U_c(s^t)}{(1 + \tau^c(s^t)) P(s^t)} = \beta \sum_{s^{t+1}|s^t} \mu(s^{t+1}|s^t) \frac{U_c(s^{t+1})}{(1 + \tau^c(s^{t+1})) P(s^{t+1})} (1 + R(s^t)) .
\]  

(54)

And by combining (50) and (51) we derive the household’s intratemporal condition,

\[-U_c(s^t) = U_c(s^t) \frac{1 - \tau^\ell(s^t)}{1 + \tau^c(s^t)} w(s^t) .
\]

Thus we obtain optimality conditions for the household stated in (37) and (38). From (53), we have that the state-contingent price satisfies:

\[Q(s^{t+1}) = \Lambda(s^{t+1}) = \beta \mu(s^{t+1}) U_c(s^{t+1}) (1 + \tau^c(s^t)) P(s^t) .
\]

Next, the household’s optimality condition for capital is given by

\[-\Lambda(s^t) P(s^t) + \sum_{s^{t+1}|s^t} \left[ \Lambda(s^{t+1}) \left(1 - \tau^k(s^{t+1})\right) P(s^{t+1}) r(s^{t+1}) + \Lambda(s^{t+1}) P(s^{t+1}) (1 - \delta) \right] = 0 .
\]

which may be rewritten as

\[\Lambda(s^t) P(s^t) = \sum_{s^{t+1}|s^t} \Lambda(s^{t+1}) P(s^{t+1}) \left[1 + \left(1 - \tau^k(s^{t+1})\right) r(s^{t+1}) - \delta\right] .
\]

(55)

Using (50) to replace \(\Lambda(s^t) P(s^t)\) and \(\Lambda(s^{t+1}) P(s^{t+1})\) in the above equation, we get

\[\frac{U_c(s^t)}{(1 + \tau^c(s^t)) P(s^t)} = \beta \sum_{s^{t+1}|s^t} \mu(s^{t+1}|s^t) \frac{U_c(s^{t+1})}{(1 + \tau^c(s^{t+1}))} [1 + \left(1 - \tau^k(s^{t+1})\right) r(s^{t+1}) - \delta] .
\]

Thus we obtain the household optimality condition stated in (39).

**Firms.** Turning attention now to the firms, we first consider the final-good retail sector. Its optimal input choices satisfy

\[y(\omega^t, s^t) = \left(\frac{p(\omega^t)}{P(s^t)}\right)^{-\rho} Y(s^t) .
\]

(56)

This gives the demand function faced by the typical intermediate-good monopolistic firm.

Consider now the monopolist’s problem stated above. The demand function (56) implies that
we may write monopolistic firm’s real revenue as
\[
p \frac{(\omega_i^t)}{P(s^t)} y (\omega_i^t, s^t) = \left( \frac{p(\omega_i^t)}{P(s^t)} \right)^{1 - \rho} Y (s^t).
\]

We can thus state the monopolistic firm’s pricing and production problem as follows:
\[
\max E \left[ \sum_{t=0}^{\infty} \beta^t M(s^t) \left\{ (1 - \tau^r(s^t)) \left( \frac{p(\omega_i^t)}{P(s^t)} \right)^{1 - \rho} Y (s^t) - h (\omega_i^t) - w (s^t) \ell (\omega_i^t, s^t) - r(s^t) k(\omega_i^t - 1) \right\} \right] | \omega_i^t \]
subject to
\[
A (s^t) F (k_i (\omega_i^t), h_i (\omega_i^t), \ell_i (\omega_i^t, s^t)) = \left( \frac{p(\omega_i^t)}{P(s^t)} \right)^{-\rho} Y (s^t) \quad \forall \omega_i^t, s^t
\]

The first constraint is simply the law of motion for capital. The second constraint, which follows from combining condition (56) with the production function, dictates how labor adjusts so as to meet the realized demand, whatever that might be.

Let \( \beta^t M(s^t) \lambda(\omega_i^t, s^t) \) be the Lagrange multiplier on the second constraint. The first order conditions with respect to labor, intermediate inputs, and investment are given by the following:
\[
\lambda (\omega_i^t, s^t) A (s^t) f_k (\omega_i^t, s^t) - w (s^t) = 0 \quad (57)
\]
\[
E \left[ M(s^t) (\lambda(\omega_i^t, s^t) A(s^t) f_k (\omega_i^t, s^t) - 1) \right] | \omega_i^t \] = 0 \quad (58)
\[
E \left[ M(s^t) (\lambda(\omega_i^t, s^t) A(s^t) f_k (\omega_i^t, s^t) - r(s^t)) \right] | \omega_i^t \] = 0 \quad (59)

The first-order condition with respect to the price \( p(\omega_i^t) \), on the other hand, can be stated as follows:
\[
E \left[ M(s^t) y (\omega_i^t, s^t) \left\{ (1 - \tau^r(s^t)) \left( \frac{p(\omega_i^t)}{P(s^t)} \right)^{\frac{\rho - 1}{\rho}} - \lambda (\omega_i^t, s^t) \right\} \right] | \omega_i^t = 0 \quad (60)
\]

Thus we obtain optimality conditions for the firm stated in (42)-(44) and (45). QED.

**Proof of Lemma 4.** The household’s problem is the same as in the sticky price equilibrium, and hence follows the proof of Lemma 3. On the firm’s side, the demand for intermediate goods from the final-good retail sector continues to satisfy (46).

Thus, the only difference between the sticky-price and flexible-price equilibria are the intermediate good firms’ problem. We may state the monopolistic firm’s production problem as follows:
\[
\max E \left[ \sum_{t=0}^{\infty} \beta^t M(s^t) \left\{ (1 - \tau^r(s^t)) y (\omega_i^t, s^t) \left( \frac{p(\omega_i^t)}{P(s^t)} \right)^{\frac{1}{\rho}} Y (s^t) - h (\omega_i^t) - w (s^t) \ell (\omega_i^t, s^t) - r(s^t) k(\omega_i^t - 1) \right\} \right] | \omega_i^t \]
subject to the production function

\[ y(\omega^t_i, s^t) = A(s^t) F(k_i(\omega^t_i), h_i(\omega^t_i), \ell_i(\omega^t_i, s^t)) \]

The FOCs of this problem are given by

\[ \left(1 - \tau^r(s^t)\right) \frac{\rho-1}{\rho} \left(\frac{y(\omega^t_i, s^t)}{Y(s^t)}\right) - \frac{1}{\rho} A(s^t)f_\ell(\omega^t_i, s^t) - w(s^t) = 0 \quad (61) \]

\[ \mathbb{E} \left[ \mathcal{M}(s^t) \left(1 - \tau^r(s^t)\right) \frac{\rho-1}{\rho} \left(\frac{y(\omega^t_i, s^t)}{Y(s^t)}\right) - \frac{1}{\rho} A(s^t)f_h(\omega^t_i, s^t) - 1 \right] \omega^t_i = 0 \quad (62) \]

\[ \mathbb{E} \left[ \mathcal{M}(s^t) \left(1 - \tau^r(s^t)\right) \frac{\rho-1}{\rho} \left(\frac{y(\omega^t_i, s^t)}{Y(s^t)}\right) - \frac{1}{\rho} A(s^t)f_k(\omega^t_i, s^t) - \tau(s^t) \right] \omega^t_i = 0 \quad (63) \]

Combining these with the intermediate good demand in (46) yields equations (47)-(49). QED.

Equipped with the previous auxiliary results, in the remainder of this appendix we offer the proofs of the results that appear in the main text.

**Proof of Proposition 1.** *Necessity.* We first prove necessity. First, take equation (61). This may be rewritten as

\[ \left(1 - \tau^r(s^t)\right) \frac{\rho-1}{\rho} \left(\frac{y(\omega^t_i, s^t)}{Y(s^t)}\right) - \frac{1}{\rho} A(s^t)f_\ell(\omega^t_i, s^t) - w(s^t) = 0 \quad \forall t, \omega^t_i, s^t \]

Combining this with the household’s intratemporal condition (38), we obtain

\[ \frac{U_c(s^t)}{(1 + \tau^c(s^t))} \left(1 - \tau^r(s^t)\right) \frac{\rho-1}{\rho} MP_\ell(\omega^t_i, s^t) - \frac{-U_\ell(s^t)}{(1 - \tau^\ell(s^t))} = 0 \]

thereby proving necessity of (7) with

\[ \psi^r(s^t) \equiv \frac{U_c(s^t)(1 - \tau^r(s^t))}{1 + \tau^c(s^t)}, \quad \chi^* \equiv \frac{\rho-1}{\rho} \quad \text{and} \quad \psi^\ell(s^t) \equiv \frac{-U_\ell(s^t)}{(1 - \tau^\ell(s^t))} \tag{64} \]

Next, take equation (62). This may be rewritten as

\[ \mathbb{E} \left[ \frac{U_c(s^t)}{1 + \tau^c(s^t)} \left(1 - \tau^r(s^t)\right) \frac{\rho-1}{\rho} MP_h(\omega^t_i, s^t) - 1 \right] \omega^t_i = 0 \quad \forall t, \omega^t_i \]
We thereby prove necessity of (8) with

\[ \psi^c (s^t) = \frac{U_c (s^t)}{1 + \tau^c (s^t)} \]  

(65)

Next, take equation (63). This may be rewritten as follows

\[ \mathbb{E} \left[ \frac{U_c (s^t)}{1 + \tau^c (s^t)} \left( (1 - \tau^r (s^t)) \frac{\rho - 1}{\rho} MP_k (\omega^t_i, s^t) - r(s^t) \right) \right| \omega^t_i] = 0 \quad \forall t, \omega^t_i \]

We thereby prove necessity of (9) with

\[ \psi^k (s^t) = \frac{U_c (s^t)}{1 + \tau^c (s^t)} r(s^t) = \frac{U_c (s^t)}{1 + \tau^c (s^t)} \tilde{r} (s^t), \]  

(66)

So far we have established the necessity of conditions (7)-(9). The necessity of the resource constraint follows from the combination of budgets and market clearing. What remains is to prove the necessity of the implementability condition (6).

To obtain this condition, we multiply the household’s budget constraint at \( s^t \) by \( \Lambda (s^t) \) and then sum over \( s^t \) and \( t \). This gives us the following

\[
\sum_{t,s^t} \Lambda (s^t) \left[ (1 + \tau^c (s^t)) C(s^t) + B(s^t) + \sum_{s^{t+1}} Q(s^{t+1}) D(s^{t+1}) + (K(s^t) - (1 - \delta) K(s^{t-1})) \right]
\]

\[
= \sum_{t,s^t} \Lambda (s^t) \left[ (1 - \tau^f (s^t)) w(s^t) L(s^t) + (1 - \tau^k (s^t)) r(s^t) K(s^{t-1}) + (1 + R(s^{t-1})) B(s^{t-1}) + D(s^t) \right]
\]

Substituting in the FOCs for debt (52) and state contingent bonds (53) we get that

\[
\sum_{t,s^t} \Lambda (s^t) [(1 + \tau^c (s^t)) C(s^t) + K(s^t)] = \sum_{t,s^t} \Lambda (s^t) \left[ (1 - \tau^f (s^t)) w(s^t) L(s^t) \right]
\]

\[
+ \sum_{t,s^t} \Lambda (s^t) (1 + (1 - \tau^k (s^t)) r(s^t) - \delta) K(s^{t-1})
\]

where we have used \( B_0 = D_0 = 0 \). Next, substituting in the FOC for capital (55), we get

\[
\sum_{t,s^t} \Lambda (s^t) (1 + \tau^c (s^t)) C(s^t) = \sum_{t,s^t} \Lambda (s^t) \left( 1 - \tau^f (s^t) \right) w(s^t) L(s^t)
\]

Now, using the household’s FOCs for consumption and employment, (50) and (51), to substitute
out all prices, we obtain

\[ \sum_{t,s}^{t} \beta_{t} \mu (s^{t}) U_{c} (s^{t}) C (s^{t}) = - \sum_{t,s}^{t} \beta_{t} \mu (s^{t}) U_{\ell} (s^{t}) L (s^{t}) \]

which we may re-write as follows

\[ \sum_{t,s}^{t} \beta_{t} \mu (s^{t}) [U_{c} (s^{t}) C (s^{t}) + U_{\ell} (s^{t}) L (s^{t})] = 0 \]

We thus obtain condition in (6) and complete the proof of necessity.

**Sufficiency.** Consider now sufficiency. Take any allocation \( \xi_{t} \) that satisfies (6)-(9). We now prove that there exists a set of tax rates

\[ \{\tau^{c} (s^{t}), \tau^{\ell} (s^{t}), \tau^{k} (s^{t}), \tau^{r} (s^{t})\}, \]

a real wage \( w (s^{t}) \), relative prices \( p (\omega^{t}_{i}, s^{t}) \), a real rental rate \( r (s^{t}) \), an interest rate function \( R (s^{t}) \) and a path for nominal debt holdings \( B (s^{t}) \) that implement this allocation as an equilibrium. We construct the equilibrium prices and policies as follows.

First, relative prices satisfy

\[ p^{(\omega^{t}_{i}, s^{t})} = p^{(\omega^{t}_{i}, s^{t})} = \left( \frac{y^{(\omega^{t}_{i}, s^{t})} Y^{(s^{t})}}{\psi^{c} (s^{t})\psi^{c} (s^{t})} \right)^{-\frac{1}{\beta}} \]

where we normalize the aggregate price level to one: \( P (s^{t}) = 1 \). With these prices we satisfy the equilibrium conditions (46) for intermediate good demand.

Let us propose the following tax rates \( \tau^{\ell}, \tau^{c}, \) and \( \tau^{r} \):

\[ 1 + \tau^{c} (s^{t}) = \frac{U_{c} (s^{t})}{\psi^{c} (s^{t})}, \quad 1 - \tau^{\ell} (s^{t}) = \frac{-U_{\ell} (s^{t})}{\psi^{\ell} (s^{t})}, \quad \text{and} \quad 1 - \tau^{r} (s^{t}) = \frac{\psi^{r} (s^{t})}{\psi^{c} (s^{t})} \]  \hspace{1cm} (67)

We then satisfy the household’s necessary optimality condition for labor (38) with the following real wage:

\[ w (s^{t}) = \frac{\psi^{\ell} (s^{t})}{\psi^{c} (s^{t})} = \frac{-U_{\ell} (s^{t})}{U_{c} (s^{t}) \left( \frac{1 - \tau^{r} (s^{t})}{1 + \tau^{r} (s^{t})} \right)} \]  \hspace{1cm} (68)

Next, take condition (7). We may replace this with the wage from (68) and obtain

\[ \chi^{*} \psi^{r} (s^{t}) MP_{\ell} (\omega^{t}_{i}, s^{t}) - \psi^{c} (s^{t}) w (s^{t}) = 0 \]
Substituting in for \( \psi^r \) and \( \psi^c \) from (67) gives us:

\[
\left(1 - \tau^r(s^t)\right) \frac{\rho - 1}{\rho} MP_t\left(\omega_i^l, s^t\right) - w\left(s^t\right) = 0
\]

This satisfies the firm’s optimality condition for labor in (61).

Next, take implementability condition (8). Again substituting in for \( \psi^r \) and \( \psi^c \) from (67) gives us the following:

\[
\mathbb{E}\left[\frac{U_c(s^t)}{1 + \tau^c(s^t)} \left(1 - \tau^r(s^t)\right) \frac{\rho - 1}{\rho} MP_h\left(\omega_i^l, s^t\right) - 1\right] \omega_i^l = 0
\]

This satisfies the firm’s optimality condition for the intermediate good (62).

Next take implementability condition (9). Again substituting in for \( \psi^r \) from (67) gives us:

\[
\mathbb{E}\left[\frac{U_c(s^t)(1 - \tau^r(s^t))}{1 + \tau^c(s^t)} \frac{\rho - 1}{\rho} MP_k\left(\omega_i^l, s^t\right) - \psi^k\left(s^t\right)\right] \omega_i^l = 0
\]

This satisfies the firm’s optimality condition for capital (63) as long as we set the real rental rate on capital be equal to

\[
r\left(s^t\right) = \psi^k\left(s^t\right) \left(\frac{U_c(s^t)}{1 + \tau^c(s^t)}\right)^{-1}
\]

This implies that we may satisfy the household’s Euler condition (39) with the following capital-income tax rate

\[
1 - \tau^k\left(s^t\right) = \frac{\tilde{r}\left(s^t\right)}{r\left(s^t\right)}
\]

with \( r\left(s^t\right) \) given by (69). Moreover, given the allocation, the following interest rate function

\[
1 + R\left(s^t\right) = \frac{U_c\left(s^t\right)}{1 + \tau^c\left(s^t\right)} \left\{\beta\mathbb{E}\left[\frac{U_c\left(s^{t+1}\right)}{1 + \tau^c\left(s^{t+1}\right)}\left|s^t\right]\right\}^{-1}
\]

ensures that condition (37) holds.

Finally we construct bond holdings such that the household’s Euler equation (37) holds. We multiply the budget by \( \Lambda\left(s^t\right) \) and sum over all periods and states following \( s^t \):

\[
\sum_{t=r+1}^{\infty} \sum_{s^t} \Lambda\left(s^t\right) \left[\left(1 + \tau^c\left(s^t\right)\right) C\left(s^t\right) + B\left(s^t\right) + \sum_{s^{t+1}} Q(s^{t+1}) D(s^{t+1}) \right]
\]

\[
= \sum_{t=r+1}^{\infty} \sum_{s^t} \Lambda\left(s^t\right) \left[\left(1 - \tau^r\left(s^t\right)\right) w\left(s^t\right) L\left(s^t\right) + \left(1 - \tau^k\left(s^t\right)\right) r\left(s^t\right) K\left(s^{t-1}\right)\right]
\]

\[
+ \left(1 + R\left(s^{t-1}\right)\right) B\left(s^{t-1}\right) D\left(s^t\right)\right]
\]

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Substituting in the FOCs for debt (52) and state contingent bonds (53) we get that

\[
\sum_{t=r+1}^{\infty} \sum_{s^t} \Lambda (s^t) \left[ (1 + \tau^c (s^t)) C (s^t) + K (s^t) \right]
\]

\[
= \sum_{t=r+1}^{\infty} \sum_{s^t} \Lambda (s^t) \left[ \left( 1 - \tau^l (s^t) \right) w (s^t) L (s^t) + \left( 1 + \left( 1 - \tau^k (s^t) \right) r (s^t) - \delta \right) K (s^{t-1}) \right]
\]

\[
+ \sum_{s^r+1 | s^r} \Lambda (s^{r+1}) (1 + R (s^r)) B (s^r)
\]

Next, substituting in the FOC for capital (55), we get

\[
\Lambda (s^r) B (s^r) = \sum_{t=r+1}^{\infty} \sum_{s^t} \Lambda (s^t) \left[ (1 + \tau^c (s^t)) C (s^t) - \left( 1 - \tau^l (s^t) \right) w (s^t) L (s^t) \right]
\]

Next, using the household’s FOCs for consumption and labor (50) and (51) gives us

\[
\frac{\beta^r \mu (s^r) U_c (s^r)}{1 + \tau^c (s^r)} B (s^r) = \sum_{t=r+1}^{\infty} \sum_{s^t} \beta^{t-r} \mu (s^t | s^r) \left[ U_c (s^t) C (s^t) + U_\ell (s^t) L (s^t) \right]
\]

which we may rewrite as follows

\[
\frac{U_c (s^r)}{1 + \tau^c (s^r)} B (s^r) = \sum_{t=r+1}^{\infty} \sum_{s^t} \beta^{t-r} \mu (s^t | s^r) \left[ U_c (s^t) C (s^t) + U_\ell (s^t) L (s^t) \right]
\]

Therefore real bond holdings are given by

\[
B (s^r) = \left( \frac{U_c (s^r)}{1 + \tau^c (s^r)} \right)^{-1} \sum_{t=r+1}^{\infty} \sum_{s^t} \beta^{t-r} \mu (s^t | s^r) \left[ U_c (s^t) C (s^t) + U_\ell (s^t) L (s^t) \right]
\]

for any period \( r \), state \( s^r \). QED.

**Proof of Proposition 2.** Necessity. We first prove necessity. Feasibility follows from the combination of budgets and market clearing.

Next, using the intermediate demand equation in (56), we may rewrite (45) as

\[
\mathbb{E} \left[ \frac{U_c (s^t)}{1 + \tau^c (s^t)} y (\omega^t_i, s^t) \left\{ (1 - \tau^r (s^t)) \left( \frac{\rho - 1}{\rho} \right) \left( \frac{y (\omega^t_i, s^t)}{Y (s^t)} \right)^{-\frac{1}{\rho}} - \lambda (\omega^t_i, s^t) \right\} \omega^t_i \right] = 0
\]
We re-write this condition as
\[
\mathbb{E} \left[ \frac{U_c(s^t)}{(1 + \tau^c(s^t))} y(\omega^t_i, s^t) (1 - \tau^r(s^t)) \left( \frac{y(\omega^t_i, s^t)}{Y(s^t)} \right)^{-\frac{1}{\rho}} \{\chi(\omega^t_i, s^t) - \chi^*\} \left| \omega^t_i \right. \right] = 0
\]
with \( \chi^* = \frac{\rho - 1}{\rho} \) and
\[
\chi(\omega^t_i, s^t) \equiv \frac{\lambda(\omega^t_i, s^t)}{(1 - \tau^r(s^t)) \left( \frac{y(\omega^t_i, s^t)}{Y(s^t)} \right)^{-\frac{1}{\rho}}} \quad \text{(71)}
\]
Using the definition of \( \psi^r(s^t) \) in (64) we obtain
\[
\mathbb{E} \left[ \psi^r(s^t) \left( \frac{y(\omega^t_i, s^t)}{Y(s^t)} \right)^{-\frac{1}{\rho}} y(\omega^t_i, s^t) \{\chi(\omega^t_i, s^t) - \chi^*\} \left| \omega^t_i \right. \right] = 0
\]
thereby proving necessity of (14).

Next, we combine the intratemporal optimality conditions of the household and of the firm for labor. Substituting (38) into the firm’s condition (42) to replace the real wage, we obtain:
\[
\lambda(\omega^t_i, s^t) \frac{U_c(s^t)}{(1 + \tau^c(s^t))} A(s^t) f_\ell(\omega^t_i, s^t) - \left( -\frac{U_\ell(s^t)}{1 - \tau_\ell(s^t)} \right) = 0. \quad \text{(72)}
\]
From our definition of \( \chi(\omega^t_i, s^t) \) in (71), we have that
\[
\lambda(\omega^t_i, s^t) = \chi(\omega^t_i, s^t) (1 - \tau^r(s^t)) \left( \frac{y(\omega^t_i, s^t)}{Y(s^t)} \right)^{-\frac{1}{\rho}}
\]
Substituting this into (72) we obtain
\[
\chi(\omega^t_i, s^t) (1 - \tau^r(s^t)) \frac{U_c(s^t)}{(1 + \tau^c(s^t))} \left( \frac{y(\omega^t_i, s^t)}{Y(s^t)} \right)^{-\frac{1}{\rho}} A(s^t) f_\ell(\omega^t_i, s^t) - \left( -\frac{U_\ell(s^t)}{1 - \tau_\ell(s^t)} \right) = 0.
\]
We may write this as
\[
\chi(\omega^t_i, s^t) \psi^r(s^t) MP_\ell(\omega^t_i, s^t) - \psi^t(s^t) = 0.
\]
where \( \psi^r(s^t) \) and \( \psi^t(s^t) \) are given by (64), thereby proving necessity of (11).

Next, we have the firm’s optimality condition for intermediate goods given by (43). We again
substitute for $\lambda(\omega^t_i, s^t)$ from (71) into (43) and obtain

$$E \left[ \frac{U_c(s^t)}{1 + \tau^c(s^t)} \left( \chi(\omega^t_i, s^t) \left( 1 - \tau^r(s^t) \right) \left( \frac{y(\omega^t_i, s^t)}{Y(s^t)} \right)^{-\frac{1}{\rho}} A(s^t) f_h(\omega^t_i, s^t) - 1 \right) \right]_{\omega^t_i} = 0$$

We may write this as

$$E \left[ \chi(\omega^t_i, s^t) \psi^r(s^t) MP_h(\omega^t_i, s^t) - \psi^c(s^t) \right]_{\omega^t_i} = 0$$

where $\psi^r(s^t)$ and $\psi^c(s^t)$ are given by (64) and (65), thereby proving necessity of (12).

Similarly we have the firm's optimality condition for capital investment given by (44). We again substitute for $\lambda(\omega^t_i, s^t)$ from (71) into (44) and obtain

$$E \left[ \frac{U_c(s^t)}{1 + \tau^c(s^t)} \left( \chi(\omega^t_i, s^t) \left( 1 - \tau^r(s^t) \right) \left( \frac{y(\omega^t_i, s^t)}{Y(s^t)} \right)^{-\frac{1}{\rho}} A(s^t) f_k(\omega^t_i, s^t) - r(s^t) \right) \right]_{\omega^t_i} = 0$$

We may write this as

$$E \left[ \chi(\omega^t_i, s^t) \psi^r(s^t) MP_k(\omega^t_i, s^t) - \psi^k(s^t) \right]_{\omega^t_i} = 0$$

where $\psi^r(s^t)$ and $\psi^k(s^t)$ are given by (64) and (66), thereby proving necessity of (13).

We now prove part (iii) of Proposition 2. In any sticky-price equilibrium, prices must satisfy the intermediate good demand equation (46). Consider then the relative prices between two firms. Fix a period $t$ and a state $s^t$, and take an arbitrary pair of firms $(i, j)$, with $j \neq i$. From the consumer demand equation (46), the relative price of the two firms is pinned down by their relative output:

$$\frac{p(\omega^t_i)}{p(\omega^t_j)} = \left[ \frac{y(\omega^t_i, s^t)}{y(\omega^t_j, s^t)} \right]^{1/\rho}$$

Clearly, the above condition can hold for all realizations of $\omega^t_i$, $\omega^t_j$ and $s^t$ only if the right-hand side of this condition is independent of $s^t$ conditional on the pair $(\omega^t_i, \omega^t_j)$. This can be true if and only if there exist positive-valued functions $\Psi^\omega$ and $\Psi^s$ such that the output of a firm can be expressed as $y(\omega^t_i, s^t) = \Psi^\omega(\omega^t_i) \Psi^s(s^t)$.

Next, we may write the Cobb-Douglas production function more generally as iso-elastic in labor:

$$F(k, h, \ell) = \ell^\alpha F(k, h, 1) = \ell^\alpha g(k, h) \quad (73)$$
for all \((k, h, \ell)\) and some \(\alpha \in (0, 1)\). Output may thereby be written as

\[
y_i (\omega^t_i, s^t_i) = A (s^t_i) \ell (\omega^t_i, s^t_i)^\alpha g (k (\omega^t_i), h (\omega^t_i)) = A (s^t_i) \ell (\omega^t_i, s^t_i)^\alpha g (\omega^t_i) .
\] (74)

where, with some abuse of notation, \(g (\omega^t_i) = g (k (\omega^t_i), h (\omega^t_i))\). Thus, log-separability of output \(y (\omega^t_i, s^t_i)\) along with iso-elastic production imply log-separability of labor \(\ell (\omega^t_i, s^t_i)\).

In any sticky-price equilibrium \(\ell (\omega^t_i, s^t_i)\) is pinned down by condition (11). Given technology (74), condition (11) may be expressed as

\[
\chi (\omega^t_i, s^t_i) \frac{\psi^r (s^t_i)}{\psi^t (s^t_i)} \left( \frac{y (\omega^t_i, s^t_i)}{Y (s^t_i)} \right)^{-\frac{1}{\rho}} \frac{\alpha y (\omega^t_i, s^t_i)}{\ell (\omega^t_i, s^t_i)} = 1
\] (75)

Thus, condition (75) along with log-separability of \(y (\omega^t_i, s^t_i)\) and \(\ell (\omega^t_i, s^t_i)\) imply log-separability of \(\chi (\omega^t_i, s^t_i)\).

What remains is the implementability condition (6) in part (i) of Proposition 2. To obtain this necessary condition, we follow the exact same steps used to obtain this condition in the proof of Proposition 1.

**Sufficiency.** Consider now sufficiency. Take any allocation \(\xi^t\) that satisfies (6), (11)-(14), and is log-separable in the sense of (15). We now prove that there exists a set of tax rates\

\[
\left\{ \tau^c (s^t_i), \tau^\ell (s^t_i), \tau^k (s^t_i), \tau^r (s^t_i) \right\},
\]
a real wage \(w (s^t_i)\), nominal prices \((p (\omega^t_i))_{i \in I}, P (s^t)\), a real rental rate \(r (s^t)\), a nominal interest rate function \(R (s^t)\), and a path for nominal debt holdings \(B (s^t)\) that implement this allocation as an equilibrium. We construct the equilibrium prices and policies as follows.

First, because \(\chi (\omega^t_i, s^t)\) is log-separable, then iso-elastic technology (74) and condition (11) jointly imply that \(y (\omega^t_i, s^t)\) and \(\ell (\omega^t_i, s^t)\) are log-separable. Thereby, we have that \(y (\omega^t_i, s^t) = \Psi^\omega (\omega^t_i) \Psi^s (s^t)\) for some functions \(\Psi^\omega\) and \(\Psi^s\). Let us then propose the following nominal prices:

\[
p (\omega^t_i) = \Psi^\omega (\omega^t_i)^{-\frac{1}{\rho}},
\]

which are by construction measurable in \(\omega^t_i\). It follows that the price level satisfies

\[
P (s^t) = \left[ \sum_{\omega \in \Omega^t} p (\omega^t_i)^{1-\rho} \varphi (\omega | s^t) \right]^{\frac{1}{1-\rho}} = \left[ \sum_{\omega \in \Omega^t} \Psi^\omega (\omega^t_i)^{\frac{\rho-1}{\rho}} \varphi (\omega | s^t) \right]^{\frac{1}{1-\rho}},
\]

43
while aggregate output satisfies
\[ Y(s^t) = \Psi^s(s^t) \left[ \sum_{\omega \in \Omega} \Psi^\omega(\omega_i^t) \frac{\omega_i^t}{\rho} \varphi(\omega|s^t) \right]^{\rho \rho}, \]
and therefore relative prices satisfy
\[ \frac{p(\omega_i^t)}{P(s^t)} = \frac{\Psi^\omega(\omega_i^t)^{-\frac{1}{\rho}}}{\left[ \sum_{\omega \in \Omega} \Psi^\omega(\omega_i^t)^{\frac{1}{\rho}} \varphi(\omega|s^t) \right]^\frac{1}{1-\rho}} = \left( \frac{y(\omega_i^t, s^t)}{Y(s^t)} \right)^{-\frac{1}{\rho}}. \]

That is, we can find nominal prices that implement the right relative prices while being measurable in \( \omega_i^t \). These prices satisfy the equilibrium necessary condition (46) for intermediate good demand.

We propose tax rates \( \tau^\ell \), \( \tau^c \), and \( \tau^r \) as in (67). We then satisfy the household’s necessary optimality condition for labor (38) with the real wage proposed in (68).

Next, take implementability condition (11). We may replace this with the wage from (68) and obtain
\[ \chi(\omega, s^t) \psi^r(s^t) MP_h(\omega_i^t, s^t) - \psi^{\ell}(s^t) w(s^t) = 0. \]
Substituting in for \( \psi^r \) and \( \psi^c \) from (67) gives us:
\[ \chi(\omega, s^t) (1 - \tau^r(s^t)) MP_h(\omega_i^t, s^t) - w(s^t) = 0 \]
This satisfies the firm’s optimality condition for labor (42) as long as we let
\[ \lambda(\omega_i^t, s^t) \equiv \chi(\omega_i^t, s^t) (1 - \tau^r(s^t)) \left( \frac{y(\omega_i^t, s^t)}{Y(s^t)} \right)^{-\frac{1}{\rho}}. \quad (76) \]

Next, take implementability condition (12). Again substituting in for \( \psi^r \) and \( \psi^c \) from (67) gives us:
\[ \mathbb{E} \left[ \frac{U_c(s^t)}{1 + \tau^c(s^t)} \left( \chi(\omega, s^t)(1 - \tau^r(s^t))MP_h(\omega_i^t, s^t) - 1 \right) \bigg| \omega_i^t \right] = 0 \]
This satisfies the firm’s optimality condition for the intermediate good (43) with \( \lambda(\omega_i^t, s^t) \) given by (76).

Next take implementability condition (13). Substituting in for \( \psi^r \) from (67) gives us:
\[ \mathbb{E} \left[ \frac{U_c(s^t)(1 - \tau^r(s^t))}{1 + \tau^c(s^t)} \chi(\omega, s^t)MP_k(\omega_i^t, s^t) - \psi^k(s^t) \bigg| \omega_i^t \right] = 0 \]
This satisfies the firm’s optimality condition for capital (44) with \( \lambda(\omega_i^t, s^t) \) given by (76) and with a real rental rate on capital given by (69). This implies further that we may satisfy the household’s
Euler condition (39) with the a capital-income tax rate $\tau^k$ as in (70).

Next, take implementability condition (14). Substituting in for $\psi^r$ from (67) gives us:

$$
E \left[ Y (s^t)^{1/\rho} g (\omega^t, s^t) (1 - \tau^r(s^t)) \left\{ \chi(\omega^t, s^t) - \chi^* \right\} \mid \omega^t \right] = 0
$$

which we may rewrite as

$$
E \left[ \frac{U_c(s^t)}{1 + \tau^c(s^t)} g (\omega^t, s^t) (1 - \tau^r(s^t)) \left\{ \frac{y(\omega^t, s^t)}{Y(s^t)} \right\}^{\frac{1}{\rho}} \left\{ \chi(\omega^t, s^t) - \chi^* \right\} \mid \omega^t \right] = 0
$$

Substituting $\chi(\omega, s^t)$ from (71) gives us

$$
E \left[ \frac{U_c(s^t)}{1 + \tau^c(s^t)} g (\omega^t, s^t) \left\{ \lambda(\omega^t, s^t) - \frac{\rho - 1}{\rho} (1 - \tau^r(s^t)) \frac{p(\omega^t)}{P(s^t)} \right\} \mid \omega^t \right] = 0
$$

Using the optimality for intermediate good demand (46) we may rewrite this as

$$
E \left[ \frac{U_c(s^t)}{1 + \tau^c(s^t)} g (\omega^t, s^t) \left\{ \lambda(\omega^t, s^t) - \frac{\rho - 1}{\rho} (1 - \tau^r(s^t)) \frac{p(\omega^t)}{P(s^t)} \right\} \mid \omega^t \right] = 0
$$

and therefore the firm’s optimality condition for its nominal price (45) is satisfied.

Given the allocation and the path for the nominal price level, the following nominal interest rate ensures that condition (37) holds:

$$
1 + R(s^t) = \frac{U_c(s^t)}{1 + \tau^c(s^t)} P(s^t) \left\{ \beta \mathbb{E} \left[ \frac{U_c(s^{t+1})}{1 + \tau^c(s^{t+1})} P(s^{t+1}) \mid s^t \right] \right\}^{-1}
$$

Finally what remains is to construct bond holdings such that the household’s Euler equation (37) holds. For this we follow the exact same steps used to obtain bond holdings in the sufficiency proof of Proposition 1. Following these steps, real bond holdings are given by

$$
B(s^r) = \left( \frac{U_c(s^r)}{1 + \tau^c(s^r)} \right)^{-1} \sum_{t=r+1}^{\infty} \sum_{s^t} \beta^{t-r} \mu(s^t | s^r) \left[ U_c(s^t) C(s^t) + U_\ell(s^t) L(s^t) \right]
$$

for any period $r$, state $s^r$. **QED.**

**Proof of Corollary 1.** Follows from the main text.
Proof of Proposition 3. The Relaxed Ramsey optimal allocation solves the following problem

$$\max_{\xi_t} \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu \left( s^t \right) \left[ U \left( C \left( s^t \right) , L \left( s^t \right) , s^t \right) \right]$$

subject to the implementability condition

$$0 \leq \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu \left( s^t \right) \left[ U_c \left( s^t \right) C \left( s^t \right) + U_r \left( s^t \right) L \left( s^t \right) \right]$$ (77)

and the following resource constraints:

$$C \left( s^t \right) + K \left( s^{t+1} \right) - (1 - \delta) K \left( s^t \right) + G \left( s^t \right) + \sum_{\omega \in \Omega^t} h \left( \omega \right) \varphi \left( \omega | s^t \right)$$ (78)

where

$$L \left( s^t \right) \text{ is defined in the main text.}$$

Let $\Gamma$ be the Lagrange multiplier on the implementability constraint (77). Let $\beta^t \mu \left( s^t \right) \zeta \left( s^t \right)$ and $\beta^t \mu \left( s^t \right) \gamma \left( s^t \right)$ and $\beta^t \mu \left( s^t \right) \kappa \left( s^t \right)$ be the multipliers on the constraints (78), (79), and (80), respectively. The relaxed Ramsey problem in Lagrangian form is then given by

$$L = \sum_{t,s^t} \beta^t \mu \left( s^t \right) \tilde{U} \left( C \left( s^t \right) , L \left( s^t \right) , s^t \right)$$

$$- \sum_{t,s^t} \beta^t \mu \left( s^t \right) \zeta \left( s^t \right) \left\{ C \left( s^t \right) + K \left( s^{t+1} \right) - (1 - \delta) K \left( s^t \right) + G \left( s^t \right) + \sum_{\omega \in \Omega^t} h \left( \omega \right) \varphi \left( \omega | s^t \right) \right\}$$

$$+ \sum_{t,s^t} \beta^t \mu \left( s^t \right) \gamma \left( s^t \right) \left[ \sum_{\omega \in \Omega^t} \left( A \left( s^t \right) F \left( k \left( \omega^t_i \right) , h \left( \omega^t_i \right) , \ell \left( \omega^t_i,s^t \right) \right) \right) \frac{\rho-1}{\rho} \varphi \left( \omega | s^t \right) \right]$$

$$- \sum_{t,s^t} \beta^t \mu \left( s^t \right) \kappa \left( s^t \right) \left\{ \sum_{\omega \in \Omega^t} \ell \left( \omega \right) \varphi \left( \omega | s^t \right) - L \left( s^t \right) \right\}$$

$$- \sum_{t,s^t} \beta^t \mu \left( s^t \right) \kappa \left( s^t \right) \left\{ \sum_{\omega \in \Omega^t} k \left( \omega \right) \varphi \left( \omega | s^t \right) - K \left( s^t \right) \right\}$$

where $\tilde{U}$ is defined in the main text.
The FOCs with respect to \( C (s^t), L (s^t), \) and \( K (s^{t+1}) \) of this problem are as follows:

\[
\begin{align*}
\tilde{U}_c (s^t) - \zeta (s^t) & = 0, \\
\tilde{U}_\ell (s^t) + \zeta (s^t) \gamma (s^t) & = 0
\end{align*}
\]

\[-\beta^t \mu (s^t) \zeta (s^t) + \sum_{t,s^t} \beta^{t+1} \mu (s^{t+1}) \left[ \zeta (s^{t+1}) \kappa (s^{t+1}) + \zeta (s^{t+1}) (1 - \delta) \right] = 0.\]

The last of these conditions may be written as

\[
\zeta (s^t) = \sum_{s^{t+1}} \beta \mu (s^{t+1}|s^t) \zeta (s^{t+1}) \left[ 1 + \kappa (s^{t+1}) - \delta \right]
\]

Combining this with FOCs for \( C (s^t) \) and \( C (s^{t+1}) \), we get

\[
\tilde{U}_c (s^t) = \beta \mathbb{E} \left[ \tilde{U}_c (s^{t+1}) \{ 1 + \kappa (s^{t+1}) - \delta \} \mid s^t \right]
\]

thereby obtaining equation (19) of the proposition.

Second, the FOCs with respect to \( \ell (\omega_i, s^t) \) are given by

\[
\beta^t \mu (s^t) \varphi (\omega|s^t) \zeta (s^t) \left\{ \sum_{\omega \in \Omega^t} y (\omega_i, s^t) \frac{\rho - 1}{\rho} \varphi (\omega|s^t) \right\}^{\frac{\rho - 1}{\rho}} y (\omega_i, s^t) \frac{\rho - 1}{\rho} A (s^t) f_\ell (\omega_i, s^t) - \gamma (s^t) \right\} = 0
\]

for all \( \omega_i, s^t \), which reduces to

\[
\zeta (s^t) \left( \frac{y (\omega_i, s^t)}{s^t} \right)^{-\frac{1}{\rho}} A (s^t) f_\ell (\omega_i, s^t) - \zeta (s^t) \gamma (s^t) = 0
\]

Combining these with the FOCs for \( C (s^t) \) and \( L (s^t) \), we get

\[
\tilde{U}_c (s^t) \left( \frac{y (\omega_i, s^t)}{s^t} \right)^{-\frac{1}{\rho}} A (s^t) f_\ell (\omega_i, s^t) - \left( -\tilde{U}_\ell (s^t) \right) = 0
\]

thereby obtaining equation (16) of the proposition.

Third, the FOCs with respect to \( h (\omega_i^t) \) are given by

\[
\sum_{s^t} \zeta (s^t) \mu (s^t) \varphi (\omega|s^t) \left\{ \sum_{\omega \in \Omega^t} y (\omega_i^t, s^t) \frac{\rho - 1}{\rho} \varphi (\omega|s^t) \right\}^{\frac{\rho - 1}{\rho}} y (\omega_i^t, s^t) \frac{\rho - 1}{\rho} A (s^t) f_h (\omega_i^t, s^t) - 1 \right\} = 0
\]

for all \( \omega_i^t, s^t \). Next, by using

\[
\mu (s^t) \varphi (\omega|s^t) = \varphi (s^t|\omega_i^t) \varphi (\omega_i^t)
\]

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we have that
\[
\sum_{s'} \zeta(s') \varphi(s' | \omega_i) \varphi(\omega_i) \left\{ \left( \frac{y(\omega_i, s')}{Y(s')} \right)^{-\frac{1}{\rho}} A(s') f_h(\omega_i, s') - 1 \right\} = 0
\]
or, equivalently,
\[
\mathbb{E} \left[ \tilde{U}_c(s') \left\{ \left( \frac{y(\omega_i, s')}{Y(s')} \right)^{-\frac{1}{\rho}} A(s') f_h(\omega_i, s') - 1 \right\} | \omega_i \right] = 0
\]
We thereby obtain equation (17) of the proposition.

Fourth, the FOCs with respect to \( k(\omega_i^t) \) are given by

\[
\sum_{s'} \zeta(s') \mu(s') \varphi(\omega|s') \left\{ \left[ \sum_{\omega \in \Omega} y(\omega_i, s') \right]^{-\frac{1}{\rho}} \varphi(\omega|s') \right\} \left[ y(\omega_i, s') \right]^{\frac{1}{\rho}} A(s') f_k(\omega_i^t, s') - \kappa(s') \right\} = 0
\]
for all \( \omega_i^t, s' \). Next, by using
\[
\mu(s') \varphi(\omega|s') = \varphi(s' | \omega_i^t) \varphi(\omega_i^t)
\]
we have that
\[
\sum_{s'} \zeta(s') \varphi(s' | \omega_i^t) \varphi(\omega_i^t) \left\{ \left( \frac{y(\omega_i, s')}{Y(s')} \right)^{-\frac{1}{\rho}} A(s') f_k(\omega_i^t, s') - \kappa(s') \right\} = 0
\]
or, equivalently,
\[
\mathbb{E} \left[ \tilde{U}_c(s') \left\{ \left( \frac{y(\omega_i, s')}{Y(s')} \right)^{-\frac{1}{\rho}} A(s') f_k(\omega_i^t, s') - \kappa(s') \right\} | \omega_i^t \right] = 0
\]
We thereby obtain equation (18) of the proposition. QED.

**Proof of Theorem 1.** First, note that aside from the implementability condition (7) which holds by construction of the relaxed set \( X^R \), three conditions must be satisfied in order for an allocation to be implementable under flexible prices. These are:

\[
\chi^* \psi^* (s') MP_k(\omega_i^t, s') - \psi^c(s') = 0 \quad (81)
\]
\[
\mathbb{E} \left[ \chi^* \psi^* (s') MP_h(\omega_i^t, s') - \psi^c(s') | \omega_i^t \right] = 0 \quad (82)
\]
\[
\mathbb{E} \left[ \chi^* \psi^* (s') MP_k(\omega_i^t, s') - \psi^k(s') | \omega_i^t \right] = 0 \quad \forall \omega_i^t \quad (83)
\]
For this proof we need to show that there exists functions $\psi^c, \psi^\ell, \psi^k, \psi^r : S^t \to \mathbb{R}_+$ such that the relaxed ramsey optimal allocation $\xi^*$ satisfies these conditions.

First, consider condition (16) in the relaxed Ramsey optimal allocation,

$$\tilde{U}_c(s^t) MP_\ell (\omega^t_i, s^t) - \left( -\tilde{U}_\ell(s^t) \right) = 0$$

Let us choose $\psi^r (s^t)$ and $\psi^\ell (s^t)$ such that

$$\psi^\ell (s^t) = -\tilde{U}_\ell(s^t) \quad \text{and} \quad \chi^* \psi^r (s^t) = \tilde{U}_c(s^t) \quad (84)$$

Then (16) along with our chosen functions $\psi^r (s^t)$ and $\psi^\ell (s^t)$ in (84) ensures that the flexible-price implementability condition (81) holds.

Second, consider condition (17) in the relaxed Ramsey optimal allocation,

$$\mathbb{E} \left[ \tilde{U}_c(s^t) \{ MP_h (\omega^t_i, s^t) - 1 \} \bigg| \omega^t_i \right] = 0$$

Let us choose $\psi^c (s^t)$ such that

$$\psi^c (s^t) = \tilde{U}_c(s^t). \quad (85)$$

Then (17) along with our chosen functions $\psi^r (s^t)$ and $\psi^c (s^t)$ in (84) and (85) ensures that the flexible-price implementability condition (82) holds.

Third, consider condition (18) in the relaxed Ramsey optimal allocation,

$$\mathbb{E} \left[ \tilde{U}_c(s^t) \{ MP_k (\omega^t_i, s^t) - \kappa (s^t) \} \bigg| \omega^t_i \right] = 0$$

Let us choose $\psi^k (s^t)$ such that

$$\psi^k (s^t) = \tilde{U}_c(s^t)\kappa (s^t) \quad (86)$$

where the function $\kappa (s^t)$ satisfies (19). Then (18) along with our chosen functions $\psi^r (s^t)$ and $\psi^k (s^t)$ in (84) and (86) ensures that the flexible-price implementability condition (83) holds.

Therefore, there exists functions $\psi^c, \psi^\ell, \psi^k, \psi^r : S^t \to \mathbb{R}_+$ given specifically by

$$\psi^c (s^t) = \tilde{U}_c(s^t), \quad \psi^\ell (s^t) = -\tilde{U}_\ell(s^t), \quad \psi^r (s^t) = \tilde{U}_c(s^t)/\chi^*, \quad \text{and} \quad \psi^k (s^t) = \tilde{U}_c(s^t)\kappa (s^t) \quad (87)$$

such that the flexible price implementability conditions (81)-(83) are all satisfied at the relaxed Ramsey optimal allocation. Thus the relaxed Ramsey optimal allocation may be implemented as an equilibrium under flexible prices.

We have established that $\xi^* \in \mathcal{X}^f$. From Corollary 1 we furthermore have that $\mathcal{X}^f \subseteq \mathcal{X}^s$. Together, these imply $\xi^* \in \mathcal{X}^s$. In particular, the optimal allocation $\xi^*$ is implemented as an
equilibrium under sticky prices with functions \( \psi^c, \psi^\ell, \psi^k, \psi^r : S^t \to \mathbb{R}_+ \) given by (87), and a function \( \chi : \Omega^t \times S^t \to \mathbb{R}_+ \) that satisfies:

\[
\chi(\omega_i^t, s^t) = \chi^* \quad \forall \omega_i^t, s^t. \tag{88}
\]

**Taxes and the Interest Rate.** The functions given in (87) for \( \psi^c, \psi^\ell, \psi^k, \psi^r : S^t \to \mathbb{R}_+ \) implement the optimal allocation \( \xi^* \) as a flexible-price equilibrium. Combining these with the tax functions in (67), gives us the following tax rates consistent with this allocation:

\[
1 + \tau^c (s^t) = \delta \frac{U^c (s^t)}{\tilde{U}^c (s^t)}, \quad 1 - \tau^\ell (s^t) = \delta - \frac{U^\ell (s^t)}{\tilde{U}^\ell (s^t)}, \quad \text{and} \quad 1 - \tau^r (s^t) = (\chi^*)^{-1}
\]

where \( \delta > 0 \) is a scalar. Finally, note that the optimal \( \psi^k \) is given by

\[
\psi^k (s^t) = \tilde{U}^c (s^t) \kappa (s^t)
\]

where the function \( \kappa (s^t) \) satisfies (19). From (69), this implies an equilibrium rental rate of

\[
r (s^t) = \kappa (s^t) \tag{89}
\]

at the optimum. Recall that while \( \kappa (s^t) \) satisfies (19), the equilibrium rental rate \( r (s^t) \) must satisfy the household’s Euler condition (39). Therefore, in order for these two conditions to coincide, it must be the case that \( r (s^t) = \tilde{r} (s^t) \) which further implies

\[
1 - \tau^k (s^t) = 1
\]

as dictated by equation (70). We therefore obtain the expressions in (20) for the optimal taxes.

What remains to be shown is that there exists a nominal interest rate that replicates the flexible price allocation, i.e. one that satisfies condition (88) for \( \chi : \Omega^t \times S^t \to \mathbb{R}_+ \) and hence implements \( \xi^* \). Recall that the equilibrium nominal interest rate is pinned down by equation (37). At the optimum, consumption taxes satisfy \( 1 + \tau^c (s^t) = \delta U^c (s^t) / \tilde{U}^c (s^t) \). Substituting these taxes into (37) results in the following expression

\[
\frac{\tilde{U}^c (s^t)}{P (s^t)} = \beta \mathbb{E} \left[ \frac{\tilde{U}^c (s^{t+1})}{P (s^{t+1})} (1 + R (s^t)) \bigg| s^t \right].
\]

By Theorem 2, the price level that implements flexible price allocations is given by \( P(s^t) = e^{z_t} B(s^t)^{-\frac{1}{\gamma}} \) where \( z_t \) is commonly known. It follows that the nominal interest rate that imple-
ments the optimal allocation is given by

\[ 1 + R(s^t) = \frac{\tilde{U}_c(s^t)}{\exp(z_t - z_{t-1})B(s^t)^{-\frac{1}{\rho}}} \left\{ \beta \mathbb{E} \left[ \frac{\tilde{U}_c(s^{t+1})}{B(s^{t+1})^{-\frac{1}{\rho}}} \mid s^t \right] \right\}^{-1} \]

QED.

Proof of Lemma 1. (The argument has already been presented in the proof of Proposition 2. It is repeated here only to coincide with the order in the main text.)

In any sticky-price equilibrium, prices must satisfy the intermediate good demand equation (46). Consider then the relative prices between two firms. Fix a period \( t \) and a state \( s^t \), and take an arbitrary pair of firms \((i, j)\), with \( j \neq i \). From the consumer demand equation (46), the relative price of the two firms is pinned down by their relative output:

\[ \frac{p(\omega^t_i)}{p(\omega^t_j)} = \left[ \frac{y(\omega^t_i, s^t)}{y(\omega^t_j, s^t)} \right]^{-1/\rho} \]

Clearly, the above condition can hold for all realizations of \( \omega^t_i, \omega^t_j \) and \( s^t \) only if the right-hand side of this condition is independent of \( s^t \) conditional on the pair \((\omega^t_i, \omega^t_j)\). This can be true if and only if \( y \) is log-separable. QED.

Proof of Theorem 2. Following the proof of Proposition 2, for any arbitrary common-knowledge process \( z_t \), nominal prices are given by

\[ p(\omega^t_i) = e^{z_t} \Psi^\omega(\omega^t_i)^{-\frac{1}{\rho}} \]

It follows that the aggregate price level satisfies

\[ P(s^t) = \left[ \sum_{\omega \in \Omega^t} p(\omega^t_i)^{1-\rho} \varphi(\omega|s^t) \right]^\frac{1}{1-\rho} = e^{z_t} \left[ \sum_{\omega \in \Omega^t} \Psi^\omega(\omega^t_i)^{\frac{1}{1-\rho}} \varphi(\omega|s^t) \right]^\frac{1}{1-\rho}, \]

We may thus express the aggregate price level in terms of \( B(s^t) \) as follows

\[ P(s^t) = e^{z_t} B(s^t)^{-\frac{1}{\rho}}, \]

thereby obtaining condition (23). QED.
Proof of Lemma 2. Take any flexible-price equilibrium. For any realization of \((\omega^f, s^f)\), output and labor must jointly satisfy the production function (74) and the optimality condition (7). Given technology (74), condition (7) may be expressed as

\[
\chi^* \frac{\psi^f (s^f)}{\psi^f (s^f)} \left( \frac{y(\omega^f, s^f)}{Y(s^f)} \right)^{\frac{1}{\rho}} \frac{y(\omega^f, s^f)}{\ell(\omega^f, s^f)} = 1 \tag{90}
\]

Note that this expression is the same as in (75) except with \(\chi^*\) replacing \(\chi(\omega^f, s^f)\).

We may solve (74 and (90) simultaneously for \(y(\omega^f, s^f)\) and \(\ell(\omega^f, s^f)\); this yields the following expression for equilibrium output:

\[
y(\omega^f, s^f) = \left[ \alpha \chi^* \frac{\psi^f (s^f)}{\psi^f (s^f)} Y(s^f)^{\frac{1}{\rho}} A(s^f)^{\frac{1}{\rho}} g(k(\omega^f), h(\omega^f))^{\frac{1}{\rho}} \right]^{1 - \frac{\alpha}{(1 - \rho)}}. \tag{91}
\]

Therefore, output \(y(\omega^f, s^f)\) and labor \(\ell(\omega^f, s^f)\) are log-separable in \(\omega^f\) and \(s^f\)

\[
y(\omega^f, s^f) = \Psi^\omega(\omega^f) \Psi^s(s^f) \tag{91}
\]

\[
\ell(\omega^f, s^f) = \Psi^\omega(\omega^f) \frac{(\Psi^s(s^f))^{\frac{1}{\rho}}}{A(s^f)^{\frac{1}{\rho}}} \tag{92}
\]

with

\[
\Psi^\omega(\omega^f) = g(k(\omega^f), h(\omega^f))^{\frac{1}{(1 - \rho)}} \tag{91}
\]

\[
\Psi^s(s^f) = \left[ Y(s^f)^{\frac{1}{\rho}} A(s^f)^{\frac{1}{\rho}} \psi^f (s^f)^{\frac{1}{\rho}} \right]^{\frac{1}{1 - \frac{\alpha}{(1 - \rho)}}} \tag{92}
\]

where we abstract from the constant scalar \((\alpha \chi^*)^{\frac{1}{1 - \frac{\alpha}{(1 - \rho)}}}\). This confirms that along any flexible-price equilibrium, \(y(\omega^f, s^f)\) is log-separable with \(\Psi^\omega(\omega^f)\) given by (91) and \(\Psi^s(s^f)\) given by (92). If technology is furthermore Cobb-Douglas, then we may write (91) as

\[
\Psi^\omega(\omega^f) = \left[ k(\omega^f)^{1 - \eta} h(\omega^f)^{\eta} \right]^{\frac{1}{1 - \frac{\alpha}{(1 - \rho)}}} \tag{93}
\]

In this case, \(B(s^f)\) is given by

\[
B(s^f) = \left[ \sum_{\omega \in \Omega} \left[ k(\omega^f)^{1 - \eta} h(\omega^f)^{\eta} \right]^{\frac{1}{1 - \frac{\alpha}{(1 - \rho)}}} \left( \frac{\rho - 1}{\rho} \right) \varphi(\omega|s^f) \right]^{\frac{\rho}{\rho - 1}} \tag{93}
\]

Next, let \(K(s^f)\) denote the aggregate capital stock and let \(H(s^f)\) denote aggregate intermediate good purchases. Then equation (93) implies that along any flexible-price equilibrium, up to a
first-order log-linear approximation $\mathcal{B}(s^t)$ may be written as

$$
\log \mathcal{B}(s^t) = \zeta_K \log K(s^t-1) + \zeta_H \log H(s^t),
$$

(94)

for some scalars $\zeta_K, \zeta_H$ given by

$$
\zeta_K \equiv (1 - \eta) \frac{1 - \alpha}{1 - \alpha \left( \frac{\rho - 1}{\rho} \right)} > 0 \quad \text{and} \quad \zeta_H \equiv \eta \frac{1 - \alpha}{1 - \alpha \left( \frac{\rho - 1}{\rho} \right)} > 0.
$$

Therefore, if the aggregate quantities of capital and intermediate goods are procyclical along the equilibrium path, then $\mathcal{B}(s^t)$ is also procyclical. QED.

Proof of Proposition 4. Suppose that $k$ and $h$ may be conditioned on $s^t$. Then by symmetry, firm optimality conditions imply

$$
y \left( \omega^t_i, s^t \right) = Y \left( s^t \right) \quad \text{for all } \omega^t_i
$$

Therefore, along any equilibrium $y(\omega^t_i, s^t)$ is log-separable with $\Psi^\omega(\omega^t_i) = 1$. This implies that along any equilibrium (including the optimal one), $\mathcal{B}(s^t) = 1$ is a constant and as a result the equilibrium allocation is implemented by targeting price stability. QED.

Finally, we conclude with an auxiliary lemma alluded to in the main text followed by its proof.

Lemma 5. Suppose preferences are given by

$$
U(C, L) = \frac{C^{1-\gamma}}{1-\gamma} - \eta \frac{L^{1+\epsilon}}{1+\epsilon}
$$

(95)

for some $\gamma, \epsilon, \eta > 0$. Then, the optimal allocation is implemented with a zero tax on capital ($\tau^k = 0$), a zero tax on consumption ($\tau^c = 0$), and a time- and state-invariant tax on labor given by

$$
1 - \tau^\ell = \frac{1 + \Gamma (1 - \gamma)}{1 + \Gamma (1 + \epsilon)},
$$

where $\Gamma$ is the Lagrange multiplier.

Proof of Lemma 5. With the homothetic preferences stated in (95),

$$
\tilde{U}(C, L) = \frac{C^{1-\gamma}}{1-\gamma} - \eta \frac{L^{1+\epsilon}}{1+\epsilon} + \Gamma \left[ (1 - \gamma) \frac{C^{1-\gamma}}{1-\gamma} - (1 + \epsilon) \eta \frac{L^{1+\epsilon}}{1+\epsilon} \right].
$$

(96)
This implies
\[ \frac{U_c(s^t)}{\tilde{U}_c(s^t)} = \frac{1}{1 + \Gamma(1 - \gamma)}, \quad \text{and} \quad \frac{U_\ell(s^t)}{\tilde{U}_\ell(s^t)} = \frac{1}{1 + \Gamma(1 + \epsilon)} \]

Consider the implementation scheme proposed; a zero tax rate on consumption implies that in order to obtain the optimal labor tax given in (20), it must satisfy
\[ 1 - \tau_\ell(s^t) = \frac{U_\ell(s^t)}{\tilde{U}_\ell(s^t)} \left( \frac{U_c(s^t)}{\tilde{U}_c(s^t)} \right)^{-1} = \frac{1 + \Gamma(1 - \gamma)}{1 + \Gamma(1 + \epsilon)} \]

The tax rate on capital follows directly from Theorem 1. \textbf{QED.}
Appendix B: Proofs for Example in Section 7

Proof of Proposition 5  Take any flexible-price equilibrium. For any realization of \((\omega^t, s^t)\), the following two equations must hold:

\[
1 = \chi^* \frac{\psi_r (s^t)}{\psi^c (s^t)} \left( \frac{y(\omega^t, s^t)}{Y(s^t)} \right) - \frac{1}{\rho} \frac{y(\omega^t, s^t)}{\ell(\omega^t, s^t)} \tag{97}
\]

\[
y(\omega^t, s^t) = A(\omega^t) h_0(\omega^t) \eta (1-\alpha) \ell(\omega^t, s^t)^{\alpha} \tag{98}
\]

Note that the main difference between these two equations and those stated previously in conditions (74) and (90) is that firm specific productivity \(A\) is now measurable in \(\omega^t\). Following the proof for Lemma 2, we can solve (97) and (98) simultaneously for \(y(\omega^t, s^t)\) and \(\ell(\omega^t, s^t)\). We thus find that in any flexible-price equilibrium, output \(y(\omega^t, s^t)\) and labor \(\ell(\omega^t, s^t)\) are log-separable in \(\omega^t\) and \(s^t\) and satisfy

\[
y(\omega^t, s^t) = \Psi^\omega(\omega^t) \Psi^s(s^t) \tag{99}
\]

\[
\ell(\omega^t, s^t) = \Psi^\omega(\omega^t)^{1-\alpha} \Psi^s(s^t)^{\frac{1}{\alpha}} \tag{100}
\]

with

\[
\Psi^\omega(\omega^t) = \left[ A(\omega^t) h_0(\omega^t)^{\eta (1-\alpha)} \right]^{1-\alpha \left( \frac{1}{\rho} - \frac{1}{\rho} \right)} \tag{101}
\]

\[
\Psi^s(s^t) = \left[ \alpha \chi^* \frac{\psi_r (s^t)}{\psi^c (s^t)} Y(s^t)^{\frac{1}{\rho}} \right]^{1-\alpha \left( \frac{1}{\rho} - \frac{1}{\rho} \right)} \tag{102}
\]

Next, consider the proposed tax policy. The revenue tax (and the associated wedges) takes the following form,

\[
\log (1 - \tau^r (A_t, Y_t)) = \tau_0 - \tau_A \log A_t - \tau_Y \log Y_t \tag{103}
\]

so that it is log-normally distributed for some scalars \(\tau_0, \tau_A, \tau_Y \in \mathbb{R}\), and the remaining tax rates satisfy \(\tau^k(s^t) = \tau^c(s^t) = 0\), and \(1 + \tau^\ell(s^t) = 1 / (1 - \tau^r(s^t))\).

Combining this last condition with the tax expressions in (67) implies

\[
\frac{\psi^r (s^t)}{\psi^c (s^t)} = \frac{\psi^\ell (s^t)}{U^\ell (s^t)} \tag{104}
\]

Rearranging and combining it with the expression for \(\psi^c(s^t)\) in (67) gives us

\[
\frac{\psi^r (s^t)}{\psi^c (s^t)} = \frac{U_c(s^t)}{-U^\ell (s^t)} \tag{105}
\]
Using the above expression to replace the wedges in (102) gives us

\[
\Psi^\omega(\omega_i^t) = \left[ A(\omega_i^t) h(\omega_i^t)^{\eta(1-\alpha)} \right]^{1-\alpha/(\rho-1)} \tag{104}
\]

\[
\Psi^s(s^t) = \left[ \frac{U_c(s^t)}{-U_l(s^t)} Y(s^t)^{1/\rho} \right]^{\alpha/(\rho-1)} \tag{105}
\]

where we abstract from the constant scalar \( (\alpha \chi^*)^{1-\alpha/(\rho-1)} \).

Aggregate output may be expressed as

\[
Y(s^t) = \left[ \sum_{\omega \in \Omega} y(\omega_i^t, s^t)^{\frac{1}{\rho-1}} \varphi(\omega|s^t) \right]^{\frac{\rho}{\rho-1}} = \Psi^s(s^t) B(s^t) \tag{106}
\]

Similarly using (100), aggregate labor may be expressed as

\[
L(s^t) = \sum_{\omega \in \Omega} \ell(\omega_i^t, s^t)^{\frac{1}{\rho}} \varphi(\omega|s^t) = \Psi^s(s^t)^{\frac{1}{\rho}} B(s^t)^{\frac{\rho-1}{\rho}}. \tag{107}
\]

Finally, the assumed specification for \( U(C, L) \) in (21) allows us to rewrite \( \Psi^s(s^t) \) in (105) as

\[
\Psi^s(s^t) = \left[ \frac{C(s^t)^{1-\gamma}}{L(s^t)^{\gamma}} Y(s^t)^{1/\rho} \right]^{\alpha/(\rho-1)} \tag{108}
\]

Taking logs of equations (106), (107), and (108) produces the following three equations.

\[
\log Y(s^t) = \log \Psi^s(s^t) + \log B(s^t) \tag{109}
\]

\[
\log L(s^t) = \frac{1}{\alpha} \log \Psi^s(s^t) + \frac{\rho-1}{\rho} \log B(s^t) \tag{110}
\]

\[
\log \Psi^s(s^t) = \alpha \zeta \left[ \frac{1}{\rho} \log Y(s^t) - \gamma \log C(s^t) - \epsilon \log L(s^t) \right] \tag{111}
\]

where \( \zeta \equiv \frac{1}{1-\alpha(\frac{\rho-1}{\rho})} \).

We combine these three equations as follows. Substituting (110) into (111) for \( L(s^t) \) yields

\[
\log \Psi^s(s^t) = \alpha \zeta \left[ \frac{1}{\rho} \log Y(s^t) - \gamma \log C(s^t) - \epsilon \left( \frac{1}{\alpha} \log \Psi^s(s^t) + \frac{\rho-1}{\rho} \log B(s^t) \right) \right].
\]

We can solve this for \( \Psi^s(s^t) \) and get

\[
\log \Psi^s(s^t) = \frac{\alpha \zeta}{(1+\epsilon \zeta)} \left[ \frac{1}{\rho} \log Y(s^t) - \gamma \log C(s^t) - \epsilon \frac{\rho-1}{\rho} \log B(s^t) \right] \tag{112}
\]
Combining this expression with equation (109) yields

\[
\log Y(s^t) = \frac{\alpha \zeta}{(1 + \epsilon \zeta)} \left[ \frac{1}{\rho} \log Y(s^t) - \gamma \log C(s^t) - \epsilon \frac{\rho - 1}{\rho} \log B(s^t) \right] + \log B(s^t)
\]

Solving the above equation for \( B(s^t) \) gives us

\[
\log B(s^t) = \frac{1}{1 - \epsilon \frac{\rho - 1}{\rho}} \left[ \left( 1 - \frac{\alpha \zeta}{1 + \epsilon \zeta} \frac{1}{\rho} \right) \log Y(s^t) + \gamma \frac{\alpha \zeta}{1 + \epsilon \zeta} \log C(s^t) \right]. \tag{113}
\]

Finally, from the definitions of \( B(s^t) \) and \( \Psi^\omega(\omega^t) \), we have the following equation.

\[
B(s^t) = \left[ \sum_{\omega \in \Omega} A(\omega^t) h(\omega^t)^{\eta(1 - \alpha)} \right]^{\frac{\rho - 1}{\rho}} \phi(\omega^t|s^t)
\]

If we log-linearize our model about the complete information equilibrium, the previous equation becomes\(^ {10} \)

\[
\log B(s^t) = \zeta \log A(s^t) + \eta(1 - \alpha) \zeta \log H(s^t). \tag{114}
\]

In summary, thus far to describe the flexible price equilibrium we have a system of two equations, (113) and (114), in four unknowns: \( Y(s^t), C(s^t), H(s^t), \) and \( B(s^t) \).

**Complete Information Case.** Our solution for the incomplete-information equilibrium will be a log-linear approximation around the complete-information Ramsey optimum. Without yet solving for the complete-information optimum, we characterize it below.

**Lemma 6.** In the complete information optimum, aggregate intermediate good purchases and aggregate consumption are log-linear in aggregate productivity:

\[
\begin{align*}
\log H^{LS}(s^t) &= \phi_A^{LS} \log A(s^t) + \text{const} \tag{115} \\
\log C^{LS}(s^t) &= \gamma_A^{LS} \log A(s^t) + \text{const} \tag{116}
\end{align*}
\]

where \( \phi_A^{LS} \) and \( \gamma_A^{LS} \) are scalar constants.

Thus, the complete information optimum is log-linear in the aggregate productivity shock, with a coefficient \( \gamma_A^{LS} \) on productivity for all aggregate variables. The scalar \( \gamma_A^{LS} \) is pinned down by preference and technology parameters along with the level of government spending (equivalently, the tightness of the government budget). For now, we take this allocation as given. We will prove Lemma 6 later when we consider the Ramsey planner’s problem in the proof of Proposition 6.

\(^ {10} \) Alternatively, we would obtain equation (114) in an exact version of our model if we assume that the information and shock structure are jointly log-Normal.
Incomplete Information Log-Linearization. We now return to characterizing the equilibrium under incomplete information. First, we log-linearize the resource constraint around the complete information equilibrium characterized in Lemma 6; this gives us

\[ \log Y(s^t) = (1 - \varsigma) \log C(s^t) + \varsigma \log H(s^t) \]  

(117)

where \( \varsigma = \eta (1 - \alpha) \) is the proportion of output that goes to intermediate good use under complete information. Substituting (117) for \( Y(s^t) \) into equation (113) produces the following expression for \( B(s^t) \):

\[ \log B(s^t) = \zeta (\Gamma_C \log C(s^t) + \Gamma_H \log H(s^t)) \]  

(118)

where

\[ \Gamma_H \equiv \frac{1 + \epsilon - \alpha}{1 + \epsilon} \varsigma \in (0, 1), \quad \text{and} \]

\[ \Gamma_C \equiv \frac{1 + \epsilon - \alpha}{1 + \epsilon} (1 - \varsigma) + \frac{\alpha \gamma}{1 + \epsilon} > 0. \]

Note that the coefficients \( \Gamma_H \) and \( \Gamma_C \) depend only on the parameters \((\alpha, \gamma, \epsilon, \eta)\) and are both strictly positive. Next, we combine (114) with (118) to obtain

\[ \Gamma_C \log C(s^t) = \log A(s^t) + (\varsigma - \Gamma_H) \log H(s^t) \]  

(119)

We thus reach an expression for aggregate GDP (consumption) in terms of \( \log A(s^t) \) and \( \log H(s^t) \).

Derivation of Beauty Contest. What remains to be characterized is the equilibrium behavior of intermediate good purchases \( H(s^t) \). We show that there exists a fixed point in \( h(\omega_{it}, s^t) \) and \( H(s^t) \) which pins down their joint solution. To do this, we use the optimality condition for intermediate good purchases given in (8). With our specification of preferences, technology, and the proposed tax scheme, this condition may be written as follows:

\[ \mathbb{E} \left[ U_c(s^t) \left( (1 - \tau^r(s^t)) \frac{\rho - 1}{\rho} \left( \frac{y(\omega^i_t, s^t)}{Y(s^t)} \right)^{-\frac{1}{\rho}} \eta (1 - \alpha) \frac{y(\omega^i_t, s^t)}{h(\omega^i_t)} \right) \omega^i_t \right] = 0 \]

where \( 1 - \tau^r(s^t) \) satisfies (103). Next, the log-separability of \( y(\omega_{it}, s^t) \) implies that this condition may be further expressed as

\[ \mathbb{E} \left[ U_c(s^t) \left( (1 - \tau^r(s^t)) \chi Y(s^t)^{\frac{1}{\rho}} \Psi_\omega(\omega^i_t)^{\frac{\epsilon - 1}{\rho}} \Psi_\delta(s^t)^{\frac{\epsilon - 1}{\rho}} - h(\omega^i_t) \right) \omega^i_t \right] = 0 \]
where $\chi \equiv \left(\frac{\rho - 1}{\rho}\right) \eta (1 - \alpha)$. Next, substituting in for $\Psi^s(\omega_i^t)$ from (104) gives us

$$\mathbb{E}\left[ U_c(s^t) \left( (1 - \tau^r(s^t)) \chi Y(s^t)^{\frac{1}{\rho}} \Psi^s(s^t)^{\frac{\rho - 1}{\rho}} \right) \right] = 0$$

Solving the above equation for $h$, we obtain the following equation characterizing the firm’s optimal choice of intermediate good purchases

$$h(\omega_i^t)^{1-\eta(1-\alpha)} \frac{\rho - 1}{1 - \alpha(\frac{\rho - 1}{\rho})} = \chi A(\omega_i^t)^{1-\alpha(\frac{\rho - 1}{\rho})} \mathbb{E}\left[ U_c(s^t) (1 - \tau^r(s^t)) Y(s^t)^{\frac{1}{\rho}} \Psi^s(s^t)^{\frac{\rho - 1}{\rho}} \right] \mathbb{E}\left[ U_c(s^t) \mid \omega_i^t \right]$$

We may re-write this in logs as follows:

$$\log h(\omega_i^t) = \frac{1}{1 - \eta(1 - \alpha)} \zeta \left(\frac{\rho - 1}{\rho}\right) \left\{ \frac{\zeta}{\rho} \log A(\omega_i^t) + \frac{1}{\rho} \mathbb{E}_i \log Y(s^t) \right\} + \frac{\rho - 1}{\rho} \mathbb{E}_i \log \Psi^s(s^t) + \mathbb{E}_i \log (1 - \tau^r(s^t))$$

where we have abstracted from the constant scalar and used $\mathbb{E}_i$ as shorthand for the conditional expectation operator: $\mathbb{E}_i x = \mathbb{E}[x \mid \omega_i^t]$. Finally, we substitute in for the tax $1 - \tau^r(s^t)$ from (103), giving us

$$\log h(\omega_i^t) = \frac{1}{1 - \eta(1 - \alpha)} \zeta \left(\frac{\rho - 1}{\rho}\right) \left\{ \frac{\zeta}{\rho} \log A(\omega_i^t) - \hat{r}_A \mathbb{E}_i \log A(s^t) \right\} + \left(\frac{1}{\rho} - \hat{r}_Y\right) \mathbb{E}_i \log Y(s^t) + \frac{1}{\rho - 1} \mathbb{E}_i \log \Psi^s(s^t)$$

(120)

Next, using the fact that $\Psi^s(s^t)$ and $B(s^t)$ simultaneously satisfy equations (112) and (118), we combine these to obtain

$$\log \Psi^s(s^t) = \frac{\alpha \zeta}{(1 + \epsilon)} \left[ \frac{1}{\rho} \log Y(s^t) - \gamma \log C(s^t) - \epsilon \frac{\rho - 1}{\rho} \zeta \left(\Gamma_C \log C(s^t) + \Gamma_H \log H(s^t)\right) \right].$$

(121)

Replacing $\Psi^s(s^t)$ in (120) with (121) gives us the following representation

$$\log h(\omega_i^t) = G_1 \left( \log A(\omega_i^t), \mathbb{E}_i \log A(s^t), \mathbb{E}_i \log Y(s^t), \mathbb{E}_i \log C(s^t), \mathbb{E}_i \log H(s^t) \right)$$

(122)

where $G_1$ is a linear function of five variables. Next, using the log-linearized resource constraint (117) to replace $Y(s^t)$, equation (122) may be reduced to

$$\log h(\omega_i^t) = G_2 \left( \log A(\omega_i^t), \mathbb{E}_i \log A(s^t), \mathbb{E}_i \log C(s^t), \mathbb{E}_i \log H(s^t) \right)$$

(123)

where $G_2$ is a linear function of four variables. Note that from (119) we may write aggregate
consumption as follows:

$$\log C(s^t) = \Gamma_C^{-1} \log A(s^t) + \Gamma_C^{-1} (\varsigma - \Gamma_H) \log H(s^t). \quad (124)$$

Using this expression to replace \(C(s^t)\) in (123) gives us the following result.

**Lemma 7.** Suppose managers have Gaussian information about the aggregate state. Then the equilibrium level of intermediate good purchases satisfy the fixed point

$$\log h(\omega_i^t) = m_\omega \log A(\omega_i^t) + m_A(\tau) \mathbb{E}_t \log A(s^t) + m_H(\tau) \mathbb{E}_t \log H(s^t) \quad (125)$$

with \(H(s^t) = \sum h(\omega_i^t) \varphi(\omega|s^t)\), where \(m_\omega\) is a constant given by

$$m_\omega = \frac{\rho^{-1}}{1 - (\alpha + \eta (1 - \alpha))(\rho^{-1})} > 0 \quad (126)$$

and \(m_A(\tau)\) and \(m_H(\tau)\) are the following linear functions of the tax coefficients \(\tau = (\hat{\tau}_A, \hat{\tau}_Y)\):

$$m_A(\tau) = \delta_A + \delta_{AA} \hat{\tau}_A + \delta_{AY} \hat{\tau}_Y,$$

$$m_H(\tau) = \delta_H + \delta_{HY} \hat{\tau}_Y.$$

The coefficients \(\delta_A, \delta_H, \delta_{AA}, \delta_{AY}, \text{ and } \delta_{HY}\) are scalars that are functions only of the primitive parameters \((\alpha, \gamma, \epsilon, \eta, \rho)\).

$$\delta_A = \frac{\alpha^2 \epsilon \eta (\rho - 1) - \alpha (\gamma (1 - \rho) + \epsilon + \eta - \epsilon \rho (1 - \eta)) - (1 + \epsilon) (1 - \eta)}{(\alpha^2 \eta + \alpha (1 - \gamma - (2 + \epsilon) \eta) - (1 + \epsilon) (1 - \eta)) (\eta + (1 - \eta) (\rho - \alpha (\rho - 1)))} \quad (127)$$

$$\delta_H = \frac{(1 - \alpha) \eta (\alpha^2 (\gamma + \epsilon \eta) (\rho - 1) - \alpha (\gamma + \epsilon + \eta - \epsilon \rho (1 - \eta)) - (1 + \epsilon) (1 - \eta))}{(\alpha^2 \eta + \alpha (1 - \gamma - (2 + \epsilon) \eta) - (1 + \epsilon) (1 - \eta)) (\eta + (1 - \eta) (\rho - \alpha (\rho - 1)))} \quad (128)$$

$$\delta_{AA} = \frac{-(\rho - \alpha (\rho - 1))}{\eta + (1 - \eta) (\rho - \alpha (\rho - 1))}$$

$$\delta_{AY} = \frac{(1 + \epsilon) (1 - \eta (1 - \alpha)) (\rho - \alpha (\rho - 1))}{(\alpha^2 \eta + \alpha (1 - \gamma - (2 + \epsilon) \eta) - (1 + \epsilon) (1 - \eta)) (\eta + (1 - \eta) (\rho - \alpha (\rho - 1)))}$$

$$\delta_{HY} = \frac{\eta (1 - \alpha) ((1 + \epsilon) (1 - \eta) + \alpha (\gamma + (1 + \epsilon) \eta)) (\rho - \alpha (\rho - 1))}{(\alpha^2 \eta + \alpha (1 - \gamma - (2 + \epsilon) \eta) - (1 + \epsilon) (1 - \eta)) (\eta + (1 - \eta) (\rho - \alpha (\rho - 1)))}$$

The fixed-point representation in (125) pins down the flexible-price allocation \(h(\omega_i^t)\) and \(H(s^t)\) for any Gaussian information structure. Given the linear structure of \(m_A(\tau)\) and \(m_H(\tau)\) the following corollary is immediate.

**Corollary 3.** The tax elasticities \((\hat{\tau}_A, \hat{\tau}_Y)\) form a spanning set of \((m_A(\tau), m_H(\tau))\).
Moreover, note that one may use $\hat{\tau}_Y$ to pin down any value for $m_H$, and given this, one may use $\hat{\tau}_A$ to pin down any value for $m_A$.

**Fixed Point Solution to Beauty Contest.** We now solve the fixed point described in Lemma 7. We take the beauty contest formulation given in (125) and transform it as follows. Let us define $\tilde{h}(\omega^t_i)$ as follows

$$\log \tilde{h}(\omega^t_i) \equiv \log h(\omega^t_i) - m_\omega \log A(\omega^t_i)$$

(129)

Then combining this with (125) implies

$$\log \tilde{h}(\omega^t_i) = m_A (\hat{\tau}) \mathbb{E}_i \log A(s^t) + m_H (\hat{\tau}) \mathbb{E}_i \log H(s^t)$$

(130)

Next, aggregating over (129) gives us

$$\log H(s^t) = \log \tilde{H}(s^t) + m_\omega \log A(s^t)$$

(131)

Finally, substituting the above expression into (130) we get

$$\log \tilde{h}(\omega^t_i) = (m_A (\hat{\tau}) + m_H (\hat{\tau}) m_\omega) \mathbb{E}_i \log A(s^t) + m_H (\hat{\tau}) \mathbb{E}_i \log \tilde{H}(s^t)$$

From this formulation the following result is immediate.

**Lemma 8.** Suppose managers have Gaussian information about the aggregate state. Then the equilibrium level of intermediate good purchases satisfy the fixed point

$$\log \tilde{h}(\omega^t_i) = (1 - \tilde{\alpha}) \tilde{\chi} \mathbb{E}_i \log A(s^t) + \tilde{\alpha} \mathbb{E}_i \log \tilde{H}(s^t)$$

(132)

with $\tilde{H}(s^t) = \sum \tilde{h}(\omega^t_i) \varphi(\omega|s^t)$ and

$$\tilde{\alpha} = m_H (\hat{\tau}) \quad \text{and} \quad \tilde{\chi} \equiv \frac{m_A (\hat{\tau}) + m_H (\hat{\tau}) m_\omega}{1 - m_H (\hat{\tau})}$$

(133)

Moreover, any pair $(\tilde{\alpha}, \tilde{\chi}) \in \mathbb{R}^2$ can be attained by an appropriate choice of the pair $(\hat{\tau}_A, \hat{\tau}_Y)$.

**Proof of Lemma 8.** Equation (132) follows from the above analysis. As for the last claim in Lemma 8, the proof is straightforward. For any $(\tilde{\alpha}, \tilde{\chi}) \in \mathbb{R}^2$, choose $m_H = \tilde{\alpha}$ and $m_A = \tilde{\chi} (1 - m_H) - m_H m_\omega$. This is the pair $(m_A, m_H)$ that attains $(\tilde{\alpha}, \tilde{\chi})$ given (133). Next recall that for any pair $(m_A(\hat{\tau}), m_H(\hat{\tau})) \in \mathbb{R}^2$ there exists a pair $(\hat{\tau}_A, \hat{\tau}_Y)$ that implements these coefficients. Therefore, there exists a pair $(\hat{\tau}_A, \hat{\tau}_Y)$ that attains $(\tilde{\alpha}, \tilde{\chi})$. QED.

Although any value of $\tilde{\alpha} \in \mathbb{R}$ can be achieved with appropriate tax instruments, from now on we restrict attention to $\tilde{\alpha} \in (-\infty, 1)$ so as to ensure a unique equilibrium. Equivalently, $m_H (\hat{\tau}) < 1$. 

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With this qualification, next we note that the game in (132) is the same as in Bergemann and Morris (2013) and hence can be spanned by a private and public signal. Thus suppose the agent gets two Gaussian signals, a private and public signal, call these \((x, z)\) with mean zero and precisions \((\kappa_x, \kappa_z)\). Then the solution to this system is given by

**Lemma 9.** Suppose managers have Gaussian information about the aggregate state. Then the equilibrium level of intermediate good purchases are given by

\[
\log \tilde{h}(x, z) = \phi_0 + \phi_x x + \phi_z z
\]

where

\[
\phi_x = \frac{(1 - \tilde{\alpha}) \kappa_x}{\kappa_0 + (1 - \tilde{\alpha}) \kappa_x + \kappa_z} \tilde{\chi}
\]

\[
\phi_z = \frac{\kappa_z}{\kappa_0 + (1 - \tilde{\alpha}) \kappa_x + \kappa_z} \tilde{\chi}
\]

Let \(r_\phi \equiv \phi_z/\phi_x\) be the ratio of these coefficients, so that \(\phi_z = r_\phi \phi_x\). Any pair \((\phi_x, r_\phi) \in \mathbb{R} \times \mathbb{R}_+\) can be attained by an appropriate choice of the pair \((\hat{\tau}_A, \hat{\tau}_Y)\).

**Proof of Lemma 9.** Choose any pair \((\phi_x, r_\phi) \in \mathbb{R} \times \mathbb{R}_+\). First, note that

\[
r_\phi = \frac{\phi_z}{\phi_x} = \frac{1}{(1 - \tilde{\alpha})} \frac{\kappa_z}{\kappa_0 + (1 - \tilde{\alpha}) \kappa_x}
\]

One may choose any \(\tilde{\alpha}\) to satisfy (137). However, recall there is an upper bound on \(\tilde{\alpha} \in (-\infty, 1)\). This imposes certain bounds on the ratio \(r_\phi\) as follows.

\[
\lim_{\alpha \to -\infty} r_\phi = 0 \quad \text{and} \quad \lim_{\alpha \to 1} r_\phi = \infty
\]

Therefore the ratio \(r_\phi\) must be weakly positive. Next given the \(\tilde{\alpha}\) that satisfies (137), one need only choose the \(\tilde{\chi}\) that implements \(\phi_x\) in equation (135). Finally, recall that from Lemma 8 we know that any pair \((\tilde{\alpha}, \tilde{\chi})\) can be attained by an appropriate choice of the pair \((\hat{\tau}_A, \hat{\tau}_Y)\). This implies that for any pair \((\phi_x, r_\phi) \in \mathbb{R} \times \mathbb{R}_+\) can be attained by an appropriate choice of \((\hat{\tau}_A, \hat{\tau}_Y)\). QED.

This implies that the ratio between \(\phi_x\) and \(\phi_z\) must be weakly positive. This is intuitive: if actions are increasing in the fundamental under complete information, then also under incomplete information agents will put a positive weight on both the private and public signal; conversely if actions are decreasing in the fundamental under complete information, then under incomplete information agents will put a negative weight on both the private and public signal. Thus in either case the pair \(\phi_x, \phi_z\) are of the same sign.
Equilibrium Aggregate Intermediated Good Purchases and Consumption (GDP). Next we compute aggregate intermediate good purchases. Equation (134) in Lemma 9 implies that the aggregate intermediate good purchases satisfies

$$\log \tilde{H}(s^t) = \phi_0 + (\phi_x + \phi_z) a_t + \phi_z u_t$$

We may transform this back into the true $H(s^t)$ from (131) as follows

$$\log H(s^t) = \log \tilde{H}(s^t) + m_\omega \log A(s^t) = \phi_0 + (\phi_x + \phi_z) a_t + \phi_z u_t + m_\omega a_t$$

We thus obtain the following result.

$$\log H(s^t) = (\phi_x + \phi_z + m_\omega) \log A(s^t) + \phi_z u_t + \text{const}$$  
(138)

This is the solution to the original beauty contest game in (125). Equation (138) characterizes the equilibrium behavior of intermediate good purchases $H(s^t)$ as a function of the aggregate productivity shock and the common noise $u_t$.

Equilibrium Aggregate Consumption (GDP). Finally, we compute aggregate consumption (GDP). Using the expression in (138) to replace $H(s^t)$ in equation (124) gives us

$$\log C(s^t) = \Gamma^{-1} \log A(s^t) + \Gamma^{-1} \varsigma \left( \frac{\alpha}{1 + \epsilon} \right) ((\phi_x + \phi_z + m_\omega) \log A(s^t) + \phi_z u_t)$$

where we have used the fact that $\varsigma - \Gamma_H = \varsigma \left( \frac{\alpha}{1 + \epsilon} \right)$. Therefore

$$\log C(s^t) = \Gamma^{-1} \left( 1 + \varsigma \frac{\alpha}{1 + \epsilon} (\phi_x + \phi_z + m_\omega) \right) \log A(s^t) + \Gamma^{-1} \varsigma \frac{\alpha}{1 + \epsilon} \phi_z u_t + \text{const}$$

We thus obtain the following characterization of aggregate consumption:

$$\log C(s^t) = \log GDP(s^t) = \gamma_0 + \gamma_a \log A(s^t) + \gamma_u u_t$$

where ($\gamma_0, \gamma_A, \gamma_u$) are constants. The coefficients ($\gamma_a, \gamma_u$) satisfy

$$\gamma_a = \dot{\gamma} + v (\phi_x + \phi_z), \quad \text{and}$$  
$$\gamma_u = v \phi_z$$  
(139)

(140)
where \((\hat{\gamma}, \upsilon)\) are strictly positive scalars given by

\[
\hat{\gamma} = \Gamma^{-1}_C \left( 1 + \frac{\alpha}{1 + \epsilon} m_\omega \right) > 0 \quad \text{and} \quad \upsilon = \Gamma^{-1}_C \frac{\alpha}{1 + \epsilon} > 0.
\]  

(141)

We have thus derived equation (26) in Proposition 5. What remains to be derived are the values of \((\gamma_a, \gamma_u)\) that may be spanned with the appropriate tax instruments. To do so, we use \(\phi_z = r \phi \phi_x\) to rewrite (139) and (140) as follows

\[
\gamma_a = \hat{\gamma} + \upsilon (1 + r \phi) \phi_x \quad \text{and} \quad \gamma_u = \upsilon r \phi \phi_x
\]  

(142)

This implies that for any \(\gamma_u\), the following relation must hold

\[
\phi_x = \frac{\gamma_u}{\upsilon r \phi}
\]

where \(\upsilon r \phi > 0\). This implies that \(\gamma_u, \phi_x, \phi_z\) must all have the same sign. Plugging this into (142) gives us

\[
\gamma_a = \hat{\gamma} + \upsilon (1 + r \phi) \frac{\gamma_u}{\upsilon r \phi} = \hat{\gamma} + \left( 1 + \frac{1}{r \phi} \right) \gamma_u
\]

Recall that \(r \phi\) can take any positive number. Therefore the pair \((\gamma_A, \gamma_u)\) may take any value in the set \(\Upsilon\) defined in (27). QED.

**Proof of Proposition 6.** For any realization of \((\omega^t_i, s^t)\), at the Ramsey Optimum the following two equations must hold:

\[
-\frac{\tilde{U}_\ell(s^t)}{U_e(s^t)} = \left( \frac{y(\omega^t_i, s^t)}{Y(s^t)} \right)^{-\frac{1}{\alpha}} \frac{y(\omega^t_i, s^t)}{\ell(\omega^t_i, s^t)} \quad \text{and} \quad y(\omega^t_i, s^t) = A(\omega^t_i) h(\omega^t_i) \eta(1-\alpha) \ell(\omega^t_i, s^t)^\alpha
\]

(143)

The first is the labor-optimality condition of the Ramsey planner and the second is the production function. Note that the only main difference between (143) and the corresponding labor-optimality condition for the flexible price equilibrium, (97), is that (143) holds specifically at the Ramsey optimum. Thus in (143), \(-\tilde{U}_\ell(s^t)/\tilde{U}_e(s^t)\) is the Ramsey planner’s marginal rate of substitution between consumption and labor and there are no tax wedges.

However, recall that with homothetic preferences, the function \(\tilde{U}(s^t)\) is given by (96). We thereby replace (143) with the following equation:

\[
- \left( \frac{1 + \Gamma (1 + \epsilon)}{1 + \Gamma (1 - \gamma)} \right) \frac{U_\ell(s^t)}{U_e(s^t)} = \left( \frac{y(\omega^t_i, s^t)}{Y(s^t)} \right)^{-\frac{1}{\alpha}} \frac{y(\omega^t_i, s^t)}{\ell(\omega^t_i, s^t)}
\]

(145)
Following the proof of Proposition 5, we can solve (144) and (145) simultaneously for \( y(\omega_i^t, s^t) \) and \( \ell(\omega_i^t, s^t) \). We find that output at the Ramsey optimum must satisfy
\[
y(\omega_i^t, s^t) = \left[ \frac{U_c(s^t)}{-U_\ell(s^t)} Y(s^t)^{\frac{1}{\alpha}} \left( A(\omega_i^t, h(\omega_i^t)^{\eta(1-\alpha)}) \right) \right]^{\frac{1}{1-\alpha(\frac{\alpha}{\rho-1})}}
\]
where we have abstracted from the constant scalar \( \left( \frac{\alpha}{1+\Gamma(1-\gamma)} \right)^{\frac{1}{1-\alpha(\frac{\alpha}{\rho-1})}} \). Thus, output \( y(\omega_i^t, s^t) \) and labor \( \ell(\omega_i^t, s^t) \) are log-separable in \( \omega_i^t \) and \( s^t \) and satisfy
\[
y(\omega_i^t, s^t) = \Psi^\omega(\omega_i^t) \Psi^s(s^t) \quad (146)
\]
\[
\ell(\omega_i^t, s^t) = \Psi^\omega(\omega_i^t) \frac{1}{\rho-1} \Psi^s(s^t)^{\frac{1}{\alpha}} \quad (147)
\]
with
\[
\Psi^\omega(\omega_i^t) = \left[ A(\omega_i^t) h(\omega_i^t)^{\eta(1-\alpha)} \right]^{\frac{1}{1-\alpha(\frac{\alpha}{\rho-1})}} \quad (148)
\]
\[
\Psi^s(s^t) = \left[ \frac{U_c(s^t)}{-U_\ell(s^t)} Y(s^t)^{\frac{1}{\alpha}} \right]^{\frac{1}{1-\alpha(\frac{\alpha}{\rho-1})}} \quad (149)
\]
Comparing (148) and (149) to the corresponding equations for \( \Psi^\omega \) and \( \Psi^s \) in the flexible price allocation with the proposed tax scheme, (104) and (105), it is clear that these are identical up to a scalar multiple. This implies that we may write aggregate output as (106) and aggregate labor as in (107). Following the exact same steps as in the proof of Proposition 5, we may describe the Ramsey optimum with equations (117) for the resource constraint, (118) for aggregate sentiment, and (119) for aggregate consumption. We reproduce equation (124) here:
\[
\log C(s^t) = \Gamma_C^{-1} \log A(s^t) + \Gamma_C^{-1} (\varsigma - \Gamma_H) \log H(s^t) \quad .
\]
We thus reach the same expression for aggregate GDP (consumption) in terms of \( \log A(s^t) \) and \( \log H(s^t) \), abstracting from all constants.

*Derivation of Planner’s Beauty Contest.* What remains to be characterized is the optimal behavior of intermediate good purchases \( H(s^t) \). As in the proof for the flexible price allocation, we show that there exists a fixed point in \( h(\omega_{i,t}) \) and \( H(s^t) \) which pins down their joint solution for the Ramsey optimum. To do this, we use the optimality condition for intermediate good purchases given by (17). With our specification of preferences and technology, this optimality condition may
be written as follows:

\[
\mathbb{E} \left[ \tilde{U}_c (s^t) \left( \left( \frac{y (\omega^i t, s^t)}{Y (s^t)} \right)^{-\frac{1}{\rho}} \eta (1 - \alpha) \frac{y (\omega^i t, s^t)}{h (\omega^i t)} - 1 \right) \bigg| \omega^i t \right] = 0 \tag{151}
\]

Recall that with homothetic preferences, the function \( \tilde{U} (s^t) \) satisfies (96). We thereby rewrite equation (151) as follows:

\[
\mathbb{E} \left[ U_c (s^t) \left( \eta (1 - \alpha) Y (s^t)^{-\frac{1}{\rho}} y (\omega^i t, s^t) \frac{\rho - 1}{\rho} - h (\omega^i t) \right) \bigg| \omega^i t \right] = 0.
\]

The log-separability of \( y (\omega_i t, s^t) \) implies that this condition may be further expressed as

\[
\mathbb{E} \left[ U_c (s^t) \left( \eta (1 - \alpha) Y (s^t)^{-\frac{1}{\rho}} \Psi^\omega (\omega^i t) \frac{\rho - 1}{\rho} \Psi^s (s^t) \frac{\rho - 1}{\rho} - h (\omega^i t) \right) \bigg| \omega^i t \right] = 0.
\]

Next, plugging in the definition of \( \Psi^\omega (\omega^i t) \) from (148) gives us

\[
\mathbb{E} \left[ U_c (s^t) \left( \eta (1 - \alpha) Y (s^t)^{-\frac{1}{\rho}} \Psi^s (s^t) \frac{\rho - 1}{\rho} \left[ A (\omega^i t) h (\omega^i t)^{\eta(1-\alpha)} \right] \frac{\rho - 1}{\rho} \right) \bigg| \omega^i t \right] = 0
\]

Solving the above equation for \( h \), we obtain the following equation characterizing the firm’s optimal choice of intermediate good purchases

\[
h (\omega^i t)^{1 - \eta (1 - \alpha)} \frac{\rho - 1}{1 - \alpha (\frac{\rho - 1}{\rho})} = \eta (1 - \alpha) A (\omega^i t)^{\frac{\rho - 1}{1 - \alpha (\frac{\rho - 1}{\rho})}} \frac{\rho - 1}{\rho} \frac{\mathbb{E} \left[ U_c (s^t) Y (s^t)^{-\frac{1}{\rho}} \Psi^s (s^t)^{\frac{\rho - 1}{\rho}} \bigg| \omega^i t \right]}{\mathbb{E} \left[ U_c (s^t) \bigg| \omega^i t \right]}
\]

We may re-write this in logs as follows:

\[
\log h (\omega^i t) = \frac{1}{1 - \eta (1 - \alpha) \zeta \left( \frac{\rho - 1}{\rho} \right)} \left[ \zeta \left( \frac{\rho - 1}{\rho} \right) \log A (\omega^i t) + \frac{1}{\rho} \mathbb{E}_i \log Y (s^t) + \frac{\rho - 1}{\rho} \mathbb{E}_i \log \Psi^s (s^t) \right]
\]

where we have abstracted from the constant scalar and again used \( \mathbb{E}_i \) as shorthand for the conditional expectation operator: \( \mathbb{E}_i x = \mathbb{E} \left[ x \bigg| \omega^i t \right] \).

We use (121) to replace \( \Psi^s (s^t) \) in (153), as the former holds true also in the Ramsey optimal allocation (with different constants). This gives us the following representation

\[
\log h (\omega^i t) = G^*_1 \left( \log A (\omega^i t), \mathbb{E}_i \log Y (s^t), \mathbb{E}_i \log C (s^t), \mathbb{E}_i \log H (s^t) \right)
\]

where \( G^*_1 \) is a linear function of four variables. Next, using the log-linearized resource constraint (117) to replace \( Y (s^t) \), equation (154) may be reduced to

66
\[ \log h(\omega^t) = G^*_2(\log A(\omega^t), \mathbb{E}_t \log C(s^t), \mathbb{E}_t \log H(s^t)) \quad (155) \]

where \(G^*_2\) is a linear function of three variables. Finally, using (150) to replace \(C(s^t)\) in (155) yields the following result.

**Lemma 10.** The Ramsey optimal level of intermediate good purchases satisfy the fixed point

\[ \log h(\omega^t) = m_\omega \log A(\omega^t) + m_A^* \mathbb{E}_t \log A(s^t) + m_H^* \mathbb{E}_t \log H(s^t) \quad (156) \]

with \(H(s^t) = \sum h(\omega^t) \varphi(\omega|s^t)\), where \(m_\omega > 0\) is as defined in (126) and the coefficients \((m_A^*, m_H^*)\) are scalars given by

\[ m_A^* = m_A(0) = \delta_A, \quad \text{and} \quad m_H^* = m_H(0) = \delta_H, \]

with \(\delta_A, \delta_H\) as defined in (127) and (128).

The fixed-point representation in (156) pins down the Ramsey optimal \(h(\omega^t)\) and \(H(s^t)\) for any information structure. Note that this is the same fixed-point representation as in (125) of Lemma 7, but with the tax instruments set at \(\hat{\tau}_A = 0\) and \(\hat{\tau}_Y = 0\).

**Fixed Point Solution to Beauty Contest.** We now solve the fixed point described in Lemma 10. Following the exact same steps as in the previous derivation of Lemma 8, we may take the beauty contest formulation given in (156) and transform it as in (129). We thus reach the following result

**Lemma 11.** Suppose managers have Gaussian information about the aggregate state. Then the optimal level of intermediate good purchases satisfy the fixed point

\[ \log \tilde{h}(\omega^t) = (1 - \alpha^*) \tilde{\chi}^* \mathbb{E}_t \log A(s^t) + \alpha^* \mathbb{E}_t \log \tilde{H}(s^t) \quad (157) \]

with \(\tilde{H}(s^t) = \sum \tilde{h}(\omega^t) \varphi(\omega|s^t)\) and

\[ \alpha^* = m_H^* \quad \text{and} \quad \tilde{\chi}^* \equiv \frac{m_A^* + m_H^* m_\omega}{1 - m_H^*} \quad (158) \]

Moreover, any pair \((\tilde{\alpha}, \tilde{\chi}) \in \mathbb{R}^2\) can be attained by an appropriate choice of the pair \((\tilde{\tau}_A, \tilde{\tau}_Y)\)

Without serious loss of generality, we henceforth impose that \(\tilde{\chi}^* > 0\), which simply means that the optimal \(Ht\) comoves positively with \(A_t\) in the frictionless benchmark. Given the above characterization and the previous analysis that followed Lemma 8, it is immediate that the solution to the fixed point described in Lemma 11 is given by

\[ \log \tilde{h}^*(x, z) = \phi_0^* + \phi_x^* x + \phi_z^* z \quad (159) \]
where

\[
\phi^*_x = \frac{(1 - \alpha^*) \kappa_x \chi}{\kappa_0 + (1 - \alpha^*) \kappa_x + \kappa_z \chi} \quad (160)
\]

\[
\phi^*_z = \frac{\kappa_z \chi}{\kappa_0 + (1 - \alpha^*) \kappa_x + \kappa_z \chi} \quad (161)
\]

Equation (159) thus gives the optimal level of intermediate good purchases. Furthermore, aggregating over (159) and again transforming back into the true \( H(s^t) \) using (131), the optimal level of aggregate intermediate good purchases satisfies

\[
\log H(s^t) = (\phi^*_x + \phi^*_z + m_\omega) \log A(s^t) + \phi^*_z u_t + \text{const} \quad (162)
\]

Equation (162) characterizes the optimal behavior of intermediate good purchases \( H(s^t) \) as a function of the aggregate productivity shock and the common noise \( u_t \).

Finally, we compute optimal aggregate consumption (GDP). Using the expression in (162) to replace \( H(s^t) \) in equation (150) gives us the following characterization for aggregate consumption:

\[
\log C(s^t) = \log GDP(s^t) = \gamma^*_0 + \gamma^*_a \log A(s^t) + \gamma^*_u u_t \quad (163)
\]

where \((\gamma^*_0, \gamma^*_a, \gamma^*_u)\) are constants. The coefficients \((\gamma^*_a, \gamma^*_u)\) satisfy

\[
\gamma^*_a = \hat{\gamma} + \nu (\phi^*_x + \phi^*_z) \quad \text{and} \quad \gamma^*_u = \nu \phi^*_z
\]

where \((\hat{\gamma}, \nu)\) are strictly positive scalars as defined in (141).

Finally, what remains to be shown is \(0 < \gamma^*_A < \gamma^*_{LS} \) and \(\gamma^*_u > 0\), where \(\gamma^*_{LS}\) is the coefficient on aggregate productivity in the complete-information Ramsey optimum. First note that

\[
\gamma^*_a = \hat{\gamma} + \nu \left( \frac{(1 - \alpha^*) \kappa_x \chi}{\kappa_0 + (1 - \alpha^*) \kappa_x + \kappa_z \chi} \right) \chi
\]

with \(\hat{\gamma}, \nu > 0\). Thus \(\chi > 0\) is sufficient for \(\gamma^*_a > 0\) and \(\gamma^*_u > 0\). Now, to compare \(\gamma^*_A\) to \(\gamma^*_{LS}\) we finally solve for the complete information optimum and offer the proof of Lemma 6 as promised previously.

**Proof of Lemma 6.** The optimal allocation under complete information is the same allocation as in (162) and (163), except with \(\kappa_x \to \infty\). In this limit,

\[
\phi^*_x + \phi^*_z \to \chi \quad \text{and} \quad \phi^*_z \to 0
\]
Therefore at the complete information optimum,

\[ \log H^{LS} (s^t) = \phi^{LS}_A \log A (s^t) + \text{const} \]
\[ \log C^{LS} (s^t) = \gamma^{LS}_A \log A (s^t) + \text{const} \]

as in (115) and (116), where \( \phi^{LS}_A \) and \( \gamma^{LS}_A \) are scalar parameters given by

\[ \phi^{LS}_A = \tilde{\chi}^* + m_\omega \quad \text{and} \quad \gamma^{LS}_A = \hat{\gamma} + v \tilde{\chi}^* \] (165)

QED.

We now take the difference between \( \gamma^{LS}_A \) and \( \gamma^*_A \); using the expressions in (164) and (165), this difference is given by

\[ \gamma^{LS}_A - \gamma^*_A = v \tilde{\chi}^* - v \left( \frac{(1 - \alpha^*) \kappa_x + \kappa_z}{\kappa_0 + (1 - \alpha^*) \kappa_x + \kappa_z} \right) \tilde{\chi}^* \]

which implies

\[ \gamma^{LS}_A - \gamma^*_A = v \left[ \frac{\kappa_0}{\kappa_0 + (1 - \alpha^*) \kappa_x + \kappa_z} \right] \tilde{\chi}^* \]

Therefore \( \tilde{\chi}^* > 0 \) is sufficient for \( \gamma^{LS}_A - \gamma^*_A > 0 \), and as a result, \( 0 < \gamma_A^* < \gamma^{LS}_A \). QED.

**Proof of Proposition 7.** Following the proof of Theorem 2, for any arbitrary common-knowledge process \( z_t \), the optimal aggregate price level is given by

\[ P(s^t) = e^{z_t} B(s^t) \]

Taking logs and combining this with expression (118) for \( B(s^t) \), we may express the optimal aggregate price level as

\[ \log P(s^t) = -\frac{1}{\rho} \log B(s^t) = -\frac{1}{\rho} \zeta \left( \Gamma_C \log C(s^t) + \Gamma_H \log H(s^t) \right) \]

where we abstract from the common-knowledge process \( z_t \). Next, by substitution of \( H(s^t) \) and \( C(s^t) \) from (162) and (163), we may express the aggregate price level as a log-linear function of \( A_t \) and \( u_t \) as follows

\[ \log P(s^t) = -\frac{1}{\rho} \zeta \left\{ (\Gamma_C \gamma^*_u + \Gamma_H (\phi^*_x + \phi^*_z + m_\omega)) \log A(s^t) + (\Gamma_C \gamma^*_u + \Gamma_H \phi^*_z) u_t \right\} \]
This yields the following expression for the aggregate price level at the Ramsey optimum:

$$\log P(s^t) = -\delta^*_A \log A (s^t) - \delta^*_u u_t + \text{const}$$

as in (30) where $\delta^*_A$ and $\delta^*_u$ are constants given by

$$\delta^*_A \equiv \frac{1}{\rho} \zeta (\Gamma_C \gamma^*_a + \Gamma_H (\phi^*_x + \phi^*_z + m_\omega)) \quad \text{and} \quad \delta^*_u \equiv \frac{1}{\rho} \zeta (\Gamma_C \gamma^*_u + \Gamma_H \phi^*_z).$$

Finally, note that

$$\frac{\delta^*_A}{\gamma^*_a} = \frac{1}{\rho} \zeta \left[ \Gamma_C + \Gamma_H \left( \frac{\phi^*_x + \phi^*_z + m_\omega}{\gamma^*_a} \right) \right] = \frac{1}{\rho} \zeta \left[ \Gamma_C + \Gamma_H \left( \frac{\phi^*_x + \phi^*_z + m_\omega}{\gamma + v (\phi^*_x + \phi^*_z)} \right) \right] > 0$$

and

$$\frac{\delta^*_u}{\gamma^*_u} = \frac{1}{\rho} \zeta \left( \Gamma_C + \Gamma_H \frac{\phi^*_z}{\gamma^*_u} \right) = \frac{1}{\rho} \zeta \left( \Gamma_C + \Gamma_H \frac{1}{v} \right) > 0$$

Therefore, the ratios $\delta^*_A/\gamma^*_a$ and $\delta^*_u/\gamma^*_u$ are strictly positive. This, along with $\gamma^*_a > 0$ and $\gamma^*_u > 0$, imply that $\delta^*_A$ and $\delta^*_u$ are strictly positive. \textbf{QED.}
Appendix C: Endogenous Information/Rational Inattention

In the preceding analysis, we have treated $\varphi$, the distribution from which a firm’s signal is drawn conditional on the underlying state of Nature, as an exogenous object. We now allow each firm to choose her $\varphi$ optimally, subject to some cost. One can think of this either as costly acquisition of information or the firm’s decision of how much attention to pay to the available data (Sims, 2003, 2010) or how much cognitive effort to put into comprehending what’s happening around her and how to best respond (Tirole, 2015). The key result of this section is that the policies that are optimal in our baseline framework remain optimal in the extended framework. This means that these policies implement not only the optimal allocation taking the stochastic process of the signals as given but also the socially optimal choice of this process itself.\footnote{The analysis in this section is most closely connected to Paciello and Wiederholt (2014). Like that paper, we endogenize the signal structure. Unlike it, we do not require that monetary policy substitute for missing tax instruments. Most crucially, we let the informational friction be the source of a real friction (in the sense of Property 1). We also allow for a more general formulation of rational inattention (namely, arbitrary $\Phi$ and arbitrary $\kappa$).}

Set up

We extend our baseline framework as follows. For any $i$, let $\varphi_i \equiv \{\varphi^t_i\}_{t=0}^\infty$, where $\varphi^t_i$ denotes the distribution from which $\omega^t_i$ is drawn conditional on $s^t$. Note that $\varphi_i$ represents a complete description of how the information or the cognitive state of firm $i$ evolves over time and over the different realizations of the underlying state of Nature. So far, $\varphi_i$ was restricted to be the same across all $i$ and was exogenously fixed. We now let each each firm choose her own $\varphi_i$, at the beginning of time, from some set $\Phi$, subject to a cost represented by a function $\kappa : \Phi \to \mathbb{R}_+$.\footnote{That is, we set $k_{it} = 1$ and $x_{it} = 0$ for all $i$, all $t$, and all realizations of uncertainty.}

To simplify the exposition, we shut down capital\footnote{This assumption guarantees that, whenever $\varphi_i = \varphi$ for all $i$ and for some $\psi$, the definition and the characterization of the sets of feasible, flexible-price, sticky-price, and optimal allocations conditional on $\psi$ remain exactly the same as in our baseline model. If, instead, we had specified the cost in terms of final good (or, say, labor), we would have to adjust appropriately all the earlier analysis: the cost would show up in firm profits and in the resource constraint.} and assume that the aforementioned cost is in terms of utility or “cognitive effort”.\footnote{This is because any asymmetric equilibrium (or optimum) can be replicated by a symmetric one that let’s each firm condition her production choices on the aforementioned extrinsic variable.} As will become evident, the arguments we develop in this section do not hinge on these simplifications. We also bypass the technical issue of the existence of an equilibrium or existence of a Ramsey optimum by requiring that all maximization and fixed-point problems defined henceforth admit a solution. We finally impose that for every $\varphi \in \Phi$, the firm learns the realization of an extrinsic random variable that is independent of $s^t$ for all $t$, is i.i.d. across firms, and is drawn from a uniform distribution over $[0, 1]$. This guarantees that it is without loss of generality to concentrate on equilibria and optima in which all firms end up choosing the same distribution and the same strategies.\footnote{This is because any asymmetric equilibrium (or optimum) can be replicated by a symmetric one that let’s each firm condition her production choices on the aforementioned extrinsic variable.}
More crucially, no restriction of economic substance is imposed on the set $\Phi$ nor on the function $\kappa$. For instance, there is no need to order the elements of $\Phi$ in terms of more or less information or to model $\kappa$ in terms of relative (Shannon) entropy or Kullback-Leibler divergence. There is also no need to take a stand on whether firms can recall their past signals effortlessly or suffer from partial amnesia, nor specify whether the cost $\kappa$ is separable across time or signals. We can thus nest, inter alia, the specifications considered in Sims (2003), Myatt and Wallace (2012), Paciello and Wiederholt (2014), and Pavan (2016). Last but not least, since the domains of $s^t$ and of $\omega^t$ are allowed to be arbitrary, the economy can be understood as a “cognitive game” in the sense of Tirole (2015). This refers to a class of two-stage games such that: in stage 2, the players play a standard game with a fixed distribution for their Harsanyi types; in stage 1, the players jointly choose the distribution of their stage-2 Harsanyi types. It follows that the choice of $\varphi$ can capture not only how much information the firms possess about the exogenous fundamentals but also how well they can grasp the endogenous behavior of one another. In short, choosing $\varphi$ is like choosing how much to know about everything that is going on in the economy.

Equilibria, Implementability, and Optimality

We now proceed to define and characterize the equilibria and the Ramsey optimum of the economy with endogenous information (or endogenous cognition). To simplify, we concentrate on the case with flexible prices; the case with sticky prices is analogous.

Consider the problem faced by an arbitrary firm $i$. This problem can be split into two subproblems: the “outer” problem of choosing a $\varphi_i$; and the “inner” problem of choosing the optimal input and output strategies for given $\varphi$. Recall that any given triplet $(\xi, \rho, \theta)$ contains a unique collection $\{Y_t(\cdot), C_t(\cdot), W(\cdot), \theta_t(\cdot)\}_t^\infty$, that is, it is associated with a unique stochastic process for aggregate output, aggregate consumption, the wage rate, and taxes. With this in mind, we can represent the firm’s inner problem as follows:

$$
\Pi(\varphi; \xi, \rho, \theta) = \max_{y, \ell, h} \sum_t \sum_{\omega, s} \beta^t \mathcal{M}(s^t) \pi(\omega^t, s^t) \varphi^t(\omega^t | s^t) \mu^t(s^t)
$$

s.t. $y(\omega^t, s) = A(s^t) F(h(\omega^t), \ell(\omega^t, s^t))$,

where $\mathcal{M}(s^t) = \frac{U_c(C_t(s^t))}{1 + \tau_c(s^t)}$ and

$$
\pi(\omega^t, s^t) \equiv (1 - \tau^r(s^t)) \left(\frac{y(\omega^t, s)}{Y(s)}\right)^{\frac{1}{\gamma}} y(\omega^t, s^t) - h(\omega^t) - W(s^t) \ell(\omega^t, s^t).
$$
We can then represent the solution to the outer problem as follows:

$$\varphi \in \Gamma (\xi, \rho; \theta) \equiv \arg \max_{\varphi} \{ \Pi (\varphi; \xi, \rho; \theta) - \kappa (\varphi) \}$$  \hspace{1cm} (167)$$

To interpret these representations, note that the first problem takes \( \varphi \) as given but lets the firm optimize her input and output choices. The second problem then describes the optimal choice of \( \varphi \).

The above determines the firm’s optimal choice of \( \varphi \) for any triplet \((\xi, \rho, \theta)\). But not every such triplet is relevant: \((\xi, \rho, \theta)\) can be part of an equilibrium of the “overall game” in which firms choose both their information structures and their input/output strategies only if it is also an equilibrium of the “continuation game” that obtains once the firms’ information structures have been fixed. We therefore define an equilibrium as follows.

**Definition 3.** In the economy with endogenous information, a flexible-price equilibrium is a collection \((\varphi, \xi, \rho, \theta)\) such that: (i) \((\xi, \rho, \theta)\) \(\in\) \(\mathcal{E}^{\text{flex}}(\varphi)\); and (ii) \(\varphi \in \Gamma (\xi, \rho; \theta)\).

An equilibrium now contains not only the triplet \((\xi, \rho, \theta)\) that describes the allocation (or the firm strategies), the price system, and the government policy, but also the information structure \(\varphi\). Part (i) requires that, taking \(\varphi\) as given, the triplet \((\xi, \rho, \theta)\) constitutes an equilibrium in the sense of Definition 2. Part (ii) on the other hand requires that \(\varphi\) is itself a solution to the optimal information/cognition problem that the typical firm faces when the rest of the economy is described by \((\xi, \rho, \theta)\). An equilibrium of the economy with endogenous information is therefore a fixed point between the mapping \(\mathcal{E}\), which was studied earlier (see especially Proposition 1), and the mapping \(\Gamma\), which is defined by condition (167) above.

Consider next the planner’s problem. By manipulating the available policy instruments, the planner can now influence not only the equilibrium allocation in the “continuation game” that obtains once \(\varphi\) is fixed but also the optimal choice of \(\varphi\) in the first place. To understand how this modifies the planner’s problem relative to the one studied earlier on, pick an arbitrary \(\hat{\varphi}\) and let \(\hat{\xi}\) be the allocation that is optimal in the sense described in Section 5 (that is, when treating \(\hat{\varphi}\) as exogenous). Relative to this benchmark, the planner’s problem has been eased by the introduction of the option to choose a \(\varphi \neq \hat{\varphi}\). However, the planner’s problem has also been worsened by the introduction of an additional implementability constraint: namely the requirement that the pair \((\varphi, \xi)\) must be consistent with the individually optimal information/cognition problem the firms.

To formalize this point, we first adapt the notion of implementability as follows.

**Definition 4.** A pair \((\varphi, \xi)\) of an information or cognition structure and an allocation is implementable (under flexible prices) if there exists a policy \(\theta\) and a price system \(\rho\) such that the collection \((\varphi, \xi, \rho, \theta)\) is an equilibrium in the sense of Definition 3.
We then state the following result, which can be proved following similar steps as in the proof of Proposition 1.

**Proposition 8.** A pair \((\varphi, \xi)\) is implementable if and only if the following properties hold.

(i) The following constraint is satisfied at the aggregate level:

\[
\sum_{t,s^t} \beta^t \mu \left( s^t \right) \left[ U_c \left( s^t \right) C \left( s^t \right) + U_\ell \left( s^t \right) L \left( s^t \right) \right] = 0;
\]

(ii) There exist wedges \(\psi = (\psi^c, \psi^\ell, \psi^r) : S^{3t} \rightarrow \mathbb{R}\) such that the following conditions hold at the firm level:

\[
\psi^r \left( s^t \right) \left[ \frac{\rho - 1}{\rho} MP_\ell \left( \omega^t, s^t \right) - \psi^\ell \left( s^t \right) \right] = 0 \quad \forall \ \omega^t, s^t \quad (169)
\]

\[
\sum_{s^t} \left\{ \psi^r \left( s^t \right) \left[ \frac{\rho - 1}{\rho} MP_h \left( \omega^t, s^t \right) - \psi^c \left( s^t \right) \right] \right\} \varphi \left( s^t \mid \omega^t \right) = 0 \quad \forall \ \omega^t 
\]

where

\[
\bar{\Pi} (\phi; \xi, \psi) \equiv \max_{\omega,t,h} \sum_{t} \sum_{\omega,s} \beta^t \left\{ \psi^r \left( s^t \right) \left[ \frac{\rho - 1}{\rho} y \left( \omega^t, s^t \right) \right] - \psi^c \left( s^t \right) h \left( \omega^t \right) + \psi^\ell \left( s^t \right) \ell \left( \omega^t, s^t \right) \right\} \phi \left( s^t \mid \omega^t \right) \mu^t \left( s^t \right)
\]

s.t. \( y \left( \omega^t, s \right) = A(s^t)F \left( h \left( \omega^t \right) , \ell \left( \omega^t, s^t \right) \right) \),

Comparing this result to Proposition 1 makes clear that the option to choose \(\varphi\) adds an extra degree of freedom to the planner’s problem, whereas condition (171) adds an extra implementability constraint. This constraint reflects the lack of a certain class of policy instruments, namely instruments that would permit the planner to manipulate the equilibrium value of \(\varphi\) while holding \(\xi\) constant. Think, in particular, of a direct Pigouvian tax or subsidy on the firm’s acquisition of information or cognition effort. If the planner had access to such an instrument, condition (171) would drop out of Proposition 8, and the planner would be free to control the equilibrium allocation \(\xi\) without having to worry how this affects the firms’ choice of \(\varphi\). Now, by contrast, the planner must take into account the feedback effect from the equilibrium value of \(\xi\) to that of \(\varphi\). In other words, the planner faces a potential trade off between influencing the *use* of information and influencing the *collection* of information.\(^{15}\)

With slight abuse of notation, let \(\mathcal{X}^{\text{flex}}\) denote the set of the pairs \((\varphi, \xi)\) that are implementable

\(^{15}\)We have qualified the trade off as a *potential* one because it remains to be seen whether this trade off is relevant for understanding the solution to the planner’s problem.
in the sense of Definition 3. The planner’s problem is to maximize welfare (defined as the ex ante utility of the representative agent, net of the cost $\kappa$) over the set $X^{\text{flex}}$.

**Definition 5.** The Ramsey optimum is given by a pair $(\varphi, \xi)$ that maximizes welfare over $X^{\text{flex}}$.

We characterize the Ramsey optimum again by adapting the methods developed in Section 5 to the endogeneity of $\varphi$. In particular, we let $X^{\text{relax}}$ denote the set of the pairs $(\varphi, \xi)$ that satisfy only condition (168) and note that, trivially, $X^{\text{flex}} \subset X^{\text{relax}}$. We then consider the following object.

**Definition 6.** The relaxed optimum is given by a pair $(\varphi^*, \xi^*)$ that maximizes welfare over $X^{\text{relax}}$.

The next lemma provides two necessary conditions for a pair $(\varphi^*, \xi^*)$ to be a relaxed optimum.

**Lemma 12.** If $(\varphi^*, \xi^*)$ is a relaxed optimum, the following two properties must hold.

(i) taking $\varphi^*$ as given, $\xi^*$ is optimal in the sense of Section 5; and

(ii) taking $\xi^*$ as given, $\varphi^*$ satisfies

\[
\varphi^* \in \arg\max_{\varphi} \{ Z(\varphi; \xi^*) - \kappa(\varphi) \}, \tag{173}
\]

where

\[
Z(\varphi; \xi) \equiv \max_{y, \ell, h} \sum_t \sum_{\omega, s} \beta^t \left[ \bar{U}_c(s^t) \left( \int_0^{y(\omega, s)} \left( \frac{s^t}{Y(s)} \right)^{-\frac{1}{\rho}} dz - h(\omega^t) \right) + \bar{U}_\ell(s^t) \ell(\omega^t, s^t) \right] \varphi(\omega|s^t) \mu(s^t).
\]

Part (i) states that $\xi^*$ is optimal whether the planner takes into account the endogeneity of $\varphi$ or treats $\varphi$ as fixed at $\varphi^*$. This is trivially true because the relaxed problem has dropped the implementability constraint (171): the aforementioned trade off between the collection and the use of information has been removed by assumption. To understand part (ii), note that, because each firm is infinitesimal, the planner can vary both a firm’s production choices and her information structure without affecting the aggregate outcomes. It follows that the contribution of any firm to social welfare is captured by $Z(\varphi; \xi)$; this measures the social surplus generated by the optimal production choices of the firm, when her information structure is fixed at $\varphi$. By the same token, the socially optimal choice of $\varphi$ maximizes the aforementioned surplus net of the information cost, which is what part (ii) states.

We next prove that any pair $(\varphi^*, \xi^*)$ that satisfies the aforementioned two properties belongs to the set $X^{\text{flex}}$. This guarantees that the solution to the relaxed problem coincides with the solution to the actual Ramsey problem, a property that mirrors the one encountered in Section 5. We furthermore prove that the same taxes that are optimal in the baseline economy in which $\varphi$ is exogenously fixed at $\varphi^*$ permit the planner to implement the pair $(\varphi^*, \xi^*)$ as an equilibrium of the (extended) economy in which $\varphi$ is endogenously chosen.
Proposition 9. Let \((\varphi^*, \xi^*)\) be a relaxed optimum. This can be implemented as part of a flexible-price equilibrium (in the sense of Definition 3) with the same taxes as in Theorem 1.

This follows directly from Lemma 12 together with the following argument. Let \(\theta^*\) be the taxes identified in Theorem 1 and let \(\rho^*\) be the associated price system. From our earlier analysis, we know that \((\xi^*, \theta^*, \rho^*)\) is an equilibrium of the (restricted) economy in which the \(\varphi\) is exogenously fixed at \(\varphi^*\). What remains to show is that, when the firm faces \((\xi^*, \theta^*, \rho^*)\), she finds it individually optimal to pick \(\varphi = \varphi^*\).

To establish that this is indeed true, consider the firm’s market valuation, as given in condition (166). At \((\xi, \theta, \rho) = (\xi^*, \theta^*, \rho^*)\), this reduces to the following:

\[
\Pi (\varphi; \xi^*) = \max_{\omega^t, s^t} \sum_t \beta^t \left[ \tilde{U}_c (s^t) \left( \frac{Y^*}{\rho Y^*} (s^t) \frac{1}{\rho} y (\omega^t, s^t) 1 - \frac{1}{\rho} h (\omega^t) \right) + \tilde{U}_\ell (s^t) \ell (\omega^t, s^t) \right] \varphi (\omega^t, s^t).
\]

Next, evaluating the innermost integral of (173), the social surplus generated by firm \(i\) can be expressed as follows:

\[
Z (\varphi; \xi^*) = \max_{\omega^t, s^t} \sum_t \beta^t \left[ \tilde{U}_c (s^t) \left( \frac{Y^*}{\rho Y^*} (s^t) \frac{1}{\rho} y (\omega^t, s^t) 1 - \frac{1}{\rho} h (\omega^t) \right) + \tilde{U}_\ell (s^t) \ell (\omega^t, s^t) \right] \varphi (\omega^t, s^t).
\]

It follows that \(\Pi (\varphi; \xi^*, \theta^*, \rho^*) = Z (\varphi; \xi^*)\) for every \(\varphi\). Combining this property with part (ii) of Lemma 12, we conclude that

\[\varphi^* \in \Gamma (\xi^*, \theta^*, \rho^*),\]

which verifies the claim that \(\varphi^*\) is optimal in the eyes of the typical firm and completes the proof of Proposition 9.

To understand this result, it is useful to build an analogy. Consider a neoclassical growth model in which a monopolist can choose her production technology (e.g., as in Romer, 1990) and ask the following question: can a uniform subsidy on firm sales induce both the efficient level of output for given technology and the efficient choice of technology? The answer to this question is positive as long as one maintains the usual Dixit-Stiglitz specification for intermediate good demand and abstract from any knowledge spillovers. These conditions suffice for the aforementioned subsidy to equate both the marginal revenue of the firm with the marginal utility of the consumer and the total profit made from any given technology with the corresponding social surplus.\(^\text{16}\) Our result can thus be understood as a variant of this observation: the choice of an information structure in our context is the analogue of the choice of technology in the growth context, the Dixit-Stiglitz

\(^{16}\)Without the aforementioned conditions, the planner may need a non-linear subsidy, as in a two-part tariff, in order to hit both goals.
specification has been maintained, and spillovers are ruled out, i.e., the cost $\kappa$ faced by each firm is independent of the choices of other firms.\footnote{The latter condition can be violated if the firms have access to, and can digest with little or no cognitive effort, the information of other firms either directly (e.g., by sharing information with one another) or indirectly (e.g., by observing for free macroeconomic statistics or the choices of other firms). These possibilities amount to introducing informational externalities; see the remark at the end of this section.} One subtlety with our result is that the appropriate notion of social surplus takes into account both the measurability constraint that precludes the firm from conditioning its choices on the true state as well as the shadow value of the government budget constraint.

It is straightforward to extend the above arguments to the more general case that allows for capital accumulation and for nominal rigidity (in the sense of Property 2). We conclude that the policy lessons provided in the earlier sections of our paper are robust to endogenous acquisition of information, rational inattention, and the like.

**Theorem 3.** Theorems 1 and 2 continue to hold in the extended framework described in this section, despite the influence that the policy instruments can exert on the information acquisition, or the cognitive effort, of the firms and thereby on the severity of the considered friction.

**Remark.** Following Sims (2003), Mackowiak and Wiederholt (2009), Paciello and Wiederholt (2014), an others, the analysis in this section has allowed the information of each firm to be endogenous to her own choices, but has abstracted from the possibility that this information is endogenous to the choices of other firms. That is, we have assumed away information externalities. In an earlier version of our paper (Angeletos and La’O, 2008) we accommodated such externalities at the expense of a more narrow specification of the information structure and proceeded to show how this is likely to reinforce the optimality of a negative relation between the price level and real economic activity.