# The Plight of the Long Term Unemployed.

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The long term unemployed have a harder time finding jobs than the short term unemployed: their exit rate to employment is lower than that of the short term unemployed. And their plight appears to get worse in depressed labor markets (the opposite of what we might expect if the difference reflected unobserved heterogeneity. In bad times, more of the long term unemployed are there because of bad luck rather than poor characteristics.)

A natural first answer is that the long term unemployed lose skills and require training costs, making employers more reluctant to hire them. However, if firms prefer to hire the short term unemployed, the long term unemployed, who face worse labor markets, will be willing to work for less. Other things equal, this will make firms want to hire them. This paper explores the equilibrium balancing of these two factors. We find that, in tight labor markets, firms will not discriminate against the long term unemployed. But in more depressed labor markets, they will.[1]

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* Notes. This is an alternative formalization of "Ranking, unemployment duration, and training costs" with Peter Diamond, on the way to an integration of the two papers.

1. In an earlier paper, we assumed that firms could not pay different wages to workers of different

# 1 Assumptions

## 1.1 Flows

Employment is given by $N$, unemployment by $U$, and N + $U$ = **1.** The separation rate from employment is given by $\lambda$, so that the flow into unemployment is given by XN.

All unemployed are identical when they start unemployment. The short-term unemployed do not require training before beginning work. The long-term unemployed do require training, with cost equal to C. The conversion from short-term to long term unemployed is governed by a Poisson process, with parameter $\gamma$. Let $U_S$ and $U_L$ denote the pools of short and long term unemployed, $U_S + U_L = U$.

New jobs are created at rate $\theta$, and need to be filled. The number of jobs needing to be filled, vacancies, is equal to $V$. Meetings between the unemployed and vacancies are given by a constant returns meeting function $M(U, V)$. I shall often use below the special case $M = \sqrt{mUV}$. All unemployed search with the same intensity, so that firms will meet short term and long term unemployed with probabilities equal to their respective proportions in the unemployment pool.

It follows from the nature of the equilibrium characterized below that, when meeting a short term unemployed, the firm will always hire him, but, when meeting a long term unemployed, the firm will hire him with probability $\pi, \pi \leq \mathbf{1}$.

Thus, if the exit rate from short term unemployed workers is given by e $\equiv (1/U_S)(U_S/U)M = M/U$, the exit rate for the long term unemployed workers is equal to $\pi e$.[2] For future reference, note that the hiring rate $f$ is related to e, $\pi$ and the composition of the pool by $(1/V)(eU_S + \pi e U_L)$.

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durations. Here, we do not impose such an assumption.

2. In this model, if firms do not discriminate, the exit rate will be the same for both types; this

We can thus write the equations of motion for the two unemployment pools, employment and vacancies as:

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\begin{aligned}
dU_S/dt &= -eU_S + \lambda N - \gamma U_S \\
dU_L/dt &= \gamma U_S - \pi e U_L \\
dN/dt &= -\lambda N + eU_S + \pi e U_L \\
dV/dt &= \theta - eU_S - \pi e U_L
\end{aligned}
$$

In steady state (I have only looked at steady states), we can express employment and unemployment -short and long:

$$N = \theta/\lambda \qquad \theta \leq \lambda \tag{1.1}$$

$$U = 1 - e/x \tag{1.2}$$

$$U_S = \lambda/(e+\gamma) \tag{1.3}$$

$$U_L = (\gamma\lambda)/(\pi e)(e \bullet_t \gamma) \tag{1.4}$$

And the exit rate in turn is determined by the fact that N $+U_L+U_S = 1$, *so* that e satisfies:

$$(1+\frac{\lambda}{(e+\gamma)}+\frac{\gamma\lambda}{(\pi e)(e+\gamma)})=\theta/\lambda \tag{1.5}$$

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was not the case in the model of multiple applications in our previous paper, where the higher the unemployment rate, the less likely the long term unemployed were to be first in line. This may be one of the sources of differences between the results (the non monotonicity of the previous paper), as ranking may make the long term unemployed more willing to pay the upfront fee than they are in the present setup.

Again, for future reference, $f$ is given by:

$$f = (U/V)\,\frac{(e+\gamma)e\pi}{(\gamma + e\pi)}$$

For the specific meeting function above, this reduces to:

$$f = m\,\frac{(e+\gamma)\pi}{(\gamma + e\pi)e} \tag{1.6}$$

In looking at comparative steady states below, I shall vary 0. Increases in $\theta$ lead to higher employment, lower unemployment, a tighter labor market. For given $\pi$, they lead to a higher exit rate for both types of unemployed, a lower proportion of long term unemployed, and a lower hiring rate for firms.

## 1.2 Value equations

Let $V_S, V_L$ and $V_E$ be the values of being short term unemployed, long term unemployed, and employed respectively. All agents are risk neutral, and have the same discount rate, $r$, so that how the compensation is divided between a wage and an upfront fee is arbitrary. Let's assume that all new hires are paid the same wage, $w$, but the long term unemployed have to pay an upfront fee $F$.

In steady state, the three arbitrage equations are given by:

$$rV_S = e(V_E - V_S) + \gamma(V_L - V_S) \tag{1.7}$$

$$rV_L = \pi e(V_E - V_L - F) \tag{1.8}$$

$$rV_E = w \; t \; \lambda(V_S - V_E) \tag{1.9}$$

Let $V_F$ and $V_V$ be the value of a filled job and of a vacancy respectively. A filled job produces a flow of output equal to $y$. In steady state, the two values satisfy:

$$rV_F = y - w + \lambda(V_V - V_F) \tag{1.10}$$

$$rV_V = f(V_F - V_V - \phi(C - F)) \tag{1.11}$$

$\phi$ is the proportion of hires who are long term unemployed and thus require training. From above $\phi \equiv \pi U_L/(U_S + \pi U_L) = \gamma/(e + \gamma)$.

## 1.3 Nash bargains

Wages and the upfront fee are determined by the usual conditions that the surplus from a match is split equally between the firm and the worker.

For meetings between the short term unemployed and firms, the condition is given by:

$$(V_E - V_S) = (V_F - V_V) \tag{1.12}$$

For meetings between the long term unemployed and firms, the condition is given by:

$$(V_E - V_L) - F = (V_F - V_V) - C + F \tag{1.13}$$

One last condition must be satisfied, that firms are actually willing (or at least indifferent) to hire the long term unemployed rather than wait for a short term unemployed (otherwise the pool of long term unemployed would be infinite) :

$$(V_F - V_V) - C + F \geq 0$$

This implies that the equilibrium can take two forms (depending on the values of the parameters). Either the firms strictly gain from hiring the long term unemployed, so that

$$(V_F - V_V) - C + F \geq 0; \qquad \pi = 1 \tag{1.14}$$

Or the firms are indifferent to hiring the long term unemployed rather than waiting, and the probability that they hire them is such as to sustain the equilibrium:

$$(V_F - V_V) - C + F = 0; \qquad \pi \leq 1 \tag{1.15}$$

We want to know for what parameter values one or the other equilibrium will prevail, and how $\pi$ varies with labor market conditions.

## 2 Equilibrium with no discrimination ($\pi = 1$)

If there is no discrimination, the equilbrium has a simple recursive structure:

First, under the assumption that $\pi = 1$, the exit rate from equation (1.5) is given by:

$$e = \theta\lambda/(\lambda - \theta)$$

And $f$ is given by $f = m/e$.

Second, subtracting (1.8) from (1.7) gives:

$$V_S - V_L = e/(r + e + \gamma)F$$

This plays an important role below. As e increases from zero to infinity, the difference goes from 0 to $F$. In words, the tighter the labor market, the larger the difference between the value of being short term unemployed and of being long term unemployed. The reason is that in a tight labor market, the short term unemployed are very likely to find a job before they become long term unemployed and thus are very unlikely to have to pay an upfront fee; they perceive themselves as very different from those who are long term unemployed. For the same reasons, the larger $\gamma$ the smaller the difference.

Taking the difference between the two Nash bargaining conditions gives:

$$-(V_S - V_L) + F = C - F$$

Or equivalently:

$$F = (C + (V_S - V_L))/2$$

Thus, the larger the difference between being short and long term unemployed, the larger the upfront fee the long term unemployed will be willing to pay. Replacing $V_S - V_L$ from above gives:

$$F/C = (r + e + \gamma)/(2r + e + 2\gamma) \tag{2.1}$$

The upfront fee varies between half and the full training cost. It is increasing in the exit rate. In a tight labor market, the long term unemployed are willing to pay a larger proportion of the training costs.

Given $F$, we can solve for the wage, call it $w^*$, from the first Nash bargaining relation:

$$V_E - V_S = V_F - V_V$$

where:

$$(V_E - V_S) = \frac{1}{r + \lambda + e}(w^* + \gamma(V_S - V_L))$$

And:

$$(V_F - V_V) = \frac{1}{r + \lambda + f}(y - w^* + f\phi(C - F))$$

As the two values are linear in w and $F$ is given from above, one can solve explicitly for the wage. But the expression is not particularly revealing.

Ignore C and thus $F$. The effects of a tight labor market on the wage are then standard: As the exit rate goes up, and the hiring rate goes down, the wage increases.

At a given exit rate, the presence of C has two effects on wage determination. If no deal is struck, a higher value of C leads to a larger upfront fee, $F$, and thus a bigger loss if the worker becomes long term unemployed: $V_S - V_L$ increases. If no deal is struck, a higher value of C leads to a larger cost to the firm if it has to hire a long term unemployed: $f\phi(C - F)$. The effects on the wage are ambiguous.

For a given positive C, a tighter labor market has additional effects on the wage. It increases $F$, the upfront fee to be paid, but it decreases the probability that a short term unemployed will become unemployed and pay it. It decreases $C - F$, the net cost of training to the firm, and it decreases $\phi$, the probability that the firm will have to pay it. The net effect appears ambiguous, with some presumption that the effect will be to strengthen the hand of the firm, and increase the wage beyond the usual effect (the effect when $C = 0$).

Finally, given $w^*$, the condition for the equilibrium to be indeed such that $\pi = 1$ is that $V_E - V_F + F - C \geq 0$. One can write down this condition only as a function of the exogenous parameters, but again this is not particularly revealing.

### 2.1 A numerical exercise/calibration

One way of getting a sense of the solution is to solve for the maximum level of training costs, $C^*$, consistent with no discrimination. (That there is such a level follows from the fact that for $C=0$, the firm will never discriminate, and for C large enough, $C - F$ can be made arbitrarily large, and the firm will not want to hire the long term unemployed.)

I have chosen the following parameters. I think of a time period as roughly a quarter. I choose the separation rate $\lambda = .05$. I then vary $\theta$ from ,495 to .400.

This range for $\theta$ implies in turn an unemployment rate ranging from 1% to 20% and an exit rate ranging from 4.95 to 0.2. The other coefficients are as follows: $m = .45$, so that the hiring rate varies from 0.9 to 0. (For an unemployment rate equal to 10%, the exit rate is equal to .45, the hiring rate to 1.0. The expected duration of unemployment is thus about 6 months, the expected duration of a vacancy 3 months. The parameter $\gamma$, which captures how fast workers need training is set at .45 (an expected duration of two quarters to become long term unemployed). Thus, if unemployment is equal to 10%, the proportion of long term unemployed is equal to 1/2. The discount rate $r$ is equal to 5%.

The maximum training cost consistent with no discrimination is drawn as a function of the unemployment rate in Figure 1. For an unemployment rate of 20%, a very depressed labor market, training costs must be less than 2.6 times y (thus roughly six months output) for discrimination not to occur. For an unemployment rate of 10%, they can be as high as 4. For an unemployment rate of 5%, they can be as high as 5.

Put another way, for given training costs, discrimination will emerge if the labor market is sufficiently depressed. If training costs are equal to 4, then discrimination will emerge when unemployment exceeds 10%.

## 3 Equilibrium with discrimination ($\pi \leq 1$)

When $\pi$ is positive, the equilibrium loses its simple recursive structure. For given exogenous parameters, the equilibrium is characterized by values of e, $f, (V_E - V_S), (V_E - V_L), (V_F - V_V)$, w, $F, \pi$ which satisfy equations (1.5),(1.6),(1.7), (1.8),(1.10),(1.12), (1.13) and (1.15). (This can be reduced to a smaller system, as $(V_E - V_S) = (V_F - V_V) =$ C $- F$ and $(V_E - V_L) = F$. But it does not yield any particularly intuitive result. So I turn to calibration.

The results of calibration using the same assumptions as above (but now taking the training cost as exogenous) are given in Figures 2 and 3. Figure 2 gives the results for C / y = 4. For unemployment rates below 10%, firms do not discriminate, and the long term unemployed are always hired. When unemployment exceeds 10%, $\pi$ becomes less than 1. For an unemployment rate of 20%, the probability of being hired if long term unemployed is equal to 57%. Figure 3 does the same for C / y = 6. Discrimination starts at an unemployment rate of 5% and the probability drops to .3 for an unemployment rate of 20%. (In these simulations, the wage remains invariant to the state of the labor market. This is not (or at least does not appear to be) a mistake, but it is a fluke. It is not true in general, although the result that, in this case, the short term unemployed see no drop in their wage as the labor market worsens is interesting.)

## 4 Where do we go from there?

With multiple applications and ranking, we had non monotonicity of C $*/y$ as a function of labor market conditions. With limits on the upfront fee paid by the long term unemployed, this may be true as well.

Whether the efficient outcome would involve more or less discrimination is not clear to me. In the other paper, strict ranking was always efficient. There is no such simple result here.