Townsend R. "Arrow-Debreu Programs as Microfoundations for Macroeconomics" in *Advances in Economic Theory: The Fifth World Congress*, ed. Truman F. Bewley (Cambridge University Press, 1987), 379-428.

search-mediated economy (as in one with existing incomplete lagged contracts) the evolution of aggregate demand matters. This raises three questions – how aggregate demand affects the static efficiency of the economy, how government policies affect aggregate demand, and how systematic manipulation of aggregate demand affects the evolution of the economy. To pursue these questions, we need a richer set of tractable models of allocation processes that are more realistic than the Walrasian auctioneer. Staying within microeconomic rules of model construction, it has been hard to construct even special models (much less general models) encompassing the set of institutions needed to answer these questions. Nevertheless, the existing models point up the likelihood that macroeconomic policy can be used constructively, and make the case that this is potentially a very valuable research agenda, as well as one that is fun.

References

Bernanke, B. 1983. "Nonmonetary Effects of the Financial Crisis in the Propagation of the Great Depression." *American Economic Review* 73: 257–76.

Bernanke, B., and M. Gertler. 1985. "Banking in General Equilibrium." Presented at the Conference on New Approaches to Monetary Economics, Austin, Texas.

Chari, V., and R. Jagannathan. 1984. "Banking Panics, Information and Rational Expectations Equilibrium." BRC Working Paper #112, J. L. Kellogg School of Management, Northwestern University.

Cooper, R., and A. John. 1985. "Coordinating Coordination Failures in Keynesian Models." Cowles Foundation Discussion Paper No. 745, Yale University.

Diamond, D., and P. Dybvig. 1983. "Bank Runs, Deposit Insurance, and Liquidity." *Journal of Political Economy* 91: 401–19.

Diamond, P. 1984. *A Search-Equilibrium Approach to the Micro Foundations of Macroeconomics.* Cambridge, Mass.: MIT Press.

Diamond, P., and D. Fudenberg. In press. "An Example of Rational-Expectations Business Cycles in Search Equilibrium." *Journal of Political Economy.*

Grandmont, J-M. 1985. "On Endogenous Competitive Business Cycles." *Econometrica* 53: 995–1045.

Howitt, P., and R. McAfee. 1984. "Stable Low-Level Equilibrium." Research Report 8403, Department of Economics, University of Western Ontario.

Jacklin, C. 1985. "Essays in Banking." Ph.D. thesis, Stanford University.

Pagano, M. 1985. "Market Size and Asset Liquidity in Stock Exchange Economies." Ph.D. thesis, Massachusetts Institute of Technology.

Postlewaite, A., and X. Vives. 1984. "Bank Runs as an Equilibrium Phenomenon." Unpublished manuscript, University of Pennsylvania.

Woodford, M. 1984. "Indeterminacy of Equilibrium in the Overlapping Generations Model: A Survey." Unpublished manuscript, Columbia University.

CHAPTER 11

Arrow–Debreu programs as microfoundations of macroeconomics

Robert M. Townsend

1 Introduction

The class of general equilibrium models of Arrow (1964), Debreu (1959), McKenzie (1959), and others is an excellent starting point for the study of actual economies. On the positive side, this class of models can be used to address the standard macroeconomic concerns of inflation, growth, and unemployment, and also more general phenomena such as the objects and institutions of trade, the absence of insurance arrangements of some kinds, or the dispersion of consumption in a population. On the normative side, this class of models can be used to study stabilization policy and optimal monetary arrangements.

Contrary views are often expressed in professional conversations and in the literature. Indeed, the Arrow–Debreu model is often said to be operational only under such unrealistic assumptions as full information, complete markets, and no diversity. Here, however, an alternative view is argued. The Arrow–Debreu model can accommodate not only diversity in preferences and endowments but also private information, indivisibilities, spatial separation, limited communication, and limited commitment. That is, standard results on the existence of Pareto optimal allocations and on the existence and optimality of competitive equilibrium allocations can be shown to apply to a large class of environments with these elements. Further, stylized but suggestive models with these elements can be constructed and made operational so that Pareto optimal and/or competitive equilibrium allocations can be characterized. On the positive side,

Helpful comments from V. V. Chari, Peter Diamond, Gary Hansen, Lars Hansen, Robert E. Lucas, Jr., Allen Meltzer, William Rogerson – as well as research support from the National Science Foundation under grant #8406771 and the Alfred P. Sloan Foundation – are all gratefully acknowledged. The author assumes full responsibility for any errors and for the views expressed herein.

these models deliver implications for the methods of interaction of economic agents and for the outcomes from that interaction. On the normative side, the Pareto optimal or core allocations of these models allow scope for activist policy.

To sharpen the argument, a terminological point should be resolved immediately. Here an Arrow–Debreu model or (better put) an Arrow–Debreu environment is one specified at the level of the primitives of, say, Debreu (1959): households $j=1,2,\ldots,n$; linear commodity space L; consumption set X^j in space L for each household j; preferences represented by a utility function u^j on set X^j for each household j; an endowment point w^j in space L for each household j; firms $k=1,2,\ldots,m$; and production possibility set Y^k in space L for each firm k. (Here, moreover, household j will be said to have direct access to production technology Y^j, so that reference to firms as entities apart from households will be suppressed.) Thus, given an Arrow–Debreu environment, one can deliver (under specified assumptions) Pareto optimal allocations as solutions to the problems of maximizing weighted sums of the utilities of the households, subject to the constraints implied by endowments and technology. That is, maximize the objective function

$$\sum_{j=1}^{n} \lambda^j u^j(x^j)$$

subject to the constraints

$$x^j \in X^j \quad j=1,2,\ldots,n \quad \text{(feasible consumption)},$$

$$y^j \in Y^j \quad j=1,2,\ldots,n \quad \text{(feasible production)},$$

$$\sum_{j=1}^{n} x^j = \sum_{j=1}^{n} w^j + \sum_{j=1}^{n} y^j \quad \text{(resource feasibility)},$$

with values for the weights λ^j satisfying

$$0 \le \lambda^j \le 1, \quad \sum_{j=1}^{n} \lambda^j = 1.$$

Such programming problems are referred to in this chapter as Arrow–Debreu programs, and hence our title: Arrow–Debreu programs are the microfoundations for the broad positive and normative view of macroeconomics outlined above.

Of course, the outcome of an Arrow–Debreu program – a Pareto optimal allocation – may or may not be decentralizable under a price system, and in that sense the standard competitive-markets hypothesis intimately associated with the general equilibrium models of Arrow, Debreu, McKenzie, and others need not be imposed. That is, one may not necessarily

worry about whether there exist a price system p^* and allocations x^{j^*} and y^{j^*}, $j=1,2,\ldots,n$, such that:

(i) for every household j, x^{j^*} solves

$$\underset{x^j \in X^j}{\text{maximize}}\; u^j(x^j)$$

subject to $p^* x^j \le p^* w^j + p^* y^{j^*}$;

(ii) for every "firm" j, y^{j^*} solves

$$\underset{y^j \in Y^j}{\text{maximize}}\; p^* y^j;$$

(iii) $\displaystyle\sum_{j=1}^{n} x^{j^*} = \sum_{j=1}^{n} w^j + \sum_{j=1}^{n} y^{j^*}$.

On the other hand, the hypothesis that allocations are those achieved in competitive markets often serves to sharpen the predictions of the model, tightening the mapping from Arrow–Debreu environments into outcomes. And (of course), under the competitive-markets hypothesis, sense can be made empirically of observations on prices. Thus, implications under the competitive-markets hypothesis are also described in many of the environments discussed below, as a useful supplement to the discussion of Pareto optimal allocations. In fact, it is stressed in this chapter that standard theorems on the existence and optimality of competitive equilibria can be shown to apply to a large class of these environments, including environments with diversity and various impediments to trade. The point is that competitive markets need be complete only relative to the natural commodity space for a specified environment. With private information, for example, full indexation is generally neither required nor possible.

An alternative hypothesis to be coupled with Pareto optimality is the idea that allocations must be in the core. That is, an allocation x^{j^*}, y^{j^*}, $j=1,2,\ldots,n$, is in the core if it is feasible [i.e., if it satisfies the competitive equilibrium condition (iii) above] and if there does not exist any subset C of households with allocation $\bar{x}^i, \bar{y}^i$, $i \in C$, with the property that the allocation $\bar{x}^i, \bar{y}^i$ is feasible for C, that is,

$$\bar{x}^i \in X^i, \quad \bar{y}^i \in Y^i, \quad \text{and} \quad \sum_{i \in C} \bar{x}^i = \sum_{i \in C} \bar{y}^i + \sum_{i \in C} w^i;$$

and allocation $\bar{x}^i, \bar{y}^i$ improves upon the *-allocation for C, that is,

$$u^i(\bar{x}^i) \ge u^i(x^{i^*})$$

for all $i \in C$, with a strict inequality for at least one $i \in C$.

In many environments the set of core allocations is smaller than the set of Pareto optimal allocations, and so again the mapping from environments into outcomes can be tightened. In fact, core allocations and competitive-equilibrium allocations are sometimes coincident. The core notion is especially nice for an analysis of economies in which production or distribution must be done by groups of agents, as by banks or intermediaries, and should be useful also for analysis of the outcomes of political processes.

To reiterate, then: The class of general equilibrium models of Arrow, Debreu, McKenzie, and others is a useful starting point for the study of actual economies. The idea is to start with a stylized Arrow–Debreu environment; impose Pareto optimality, the competitive-markets hypothesis, or the core hypothesis; and then make predictions about the methods of interaction of economic agents or the outcomes from that interaction. This way of proceeding has theoretical content, insofar as variations in Arrow–Debreu environments do imply variations in outcomes. Indeed, these variations are especially striking when one includes diversity, uncertainty, and various impediments to trade. Furthermore, if we search broadly for observations, we discover variations in environments and in outcomes for actual economies. Thus one can contemplate matching stylized theories with observations in some way. For example, one can try first to match an environment from an actual economy having some apparent impediment to trade with the environment of a theory having a stylized version of that impediment, and then one can try to see if the observed outcome of the actual economy matches up in some way with the outcome predicted by the theory. Alternatively, one can begin with some striking arrangement in an actual economy and ask whether any theoretical environment might yield such an arrangement. Either way, one hopes to match observations with theories without making the theoretical environment too complicated or implausible.

This matching exercise is instructive, but at a relatively early stage. Thus no particular economy is studied here in great detail and no grand cross-economy comparisons are attempted. But observations from actual economies serve at least to motivate the theoretical environments studied in the sections that follow. That is, observations from the typical medieval village studied by economist McCloskey (1976), historian Bennett (1974), and sociologist Homans (1975), from the Pacific island communities studied by anthropologists Malinowski (1953) and Firth (1939), from hill tribe communities in northern Thailand studied by anthropologist van Roy (1971), from villages in southern India studied by economists Binswanger and Rosenzweig (1984), and from African hunter-gatherer tribes studied by anthropologist Lee (1979) all suggest that economywide consumption and labor sharing, the presence or absence of currency, and other forms of social interaction are interesting arrangements to study,

and also that uncertainty, private information, indivisibilities, spatial separation, limited communication, and limited commitment may be useful in helping to understand these arrangements.

In summary, then, this chapter proceeds as follows. Section 2 sets up a programming problem for a standard general equilibrium model with uncertainty and some diversity among agents. Solutions to that programming problem are then described, with an emphasis on co-movement implications for consumption. Theorems on the existence and optimality of competitive equilibria are also reviewed. Section 3 extends the analysis to an economy with multiple commodities; Section 4 deals in particular with leisure, deriving implications for leisure sharing, absence of quid pro quo, and monitoring. Section 5 extends the analysis to include capital goods. Existence and optimality theorems are reviewed for the general, extended models of these sections. Aggregation across diverse households facilitates the analysis in these sections and delivers as a special case the standard neoclassical growth model.

Section 6 introduces the possibility of private information, to cope with observations from "primitive" societies that are anomalous relative to full information theory. Section 6 also describes how to handle some nonconvexities in the underlying commodity space induced by this private information, namely by the introduction of lotteries. With these lotteries, existence and optimality results follow by more or less standard arguments. Section 7 introduces the possibility of indivisibilities in consumption and/or leisure, and asks in a preliminary way whether this might help to account for anomalous observations. Section 8 in turn introduces the possibility of spatial separation. With indivisibilities or spatial separation one must again face a nonconvexity in the underlying commodity space, and Sections 7 and 8 describe how to do this successfully. Again, lotteries are useful and allow one to recover standard optimality and existence theorems. Special cases of these models deliver the macroeconomic neoclassical models of recent literature, seeking to explain anomalous time-series observations.

Section 9 introduces the possibility of limited communication into a model with spatial separation and private information, motivated by an effort to explain the observed use of currency and financial intermediaries in some societies. Again, it is shown how to deliver a programming problem for various possible communication technologies, and an effort is made to interpret these various programming problems and their solutions relative to actual observations. As a byproduct of this discussion, one can discuss optimal currency rules and other normative issues. Finally, Section 10 introduces the possibility of limited commitment, to explain observed and more standard roles for currency and to explain the existence of markets in some societies but not others. Here a kind of sequential core notion is adopted, and it is argued that the outcomes of various competitive spatial models of currency can be interpreted as the outcomes of modified programming problems. This class of models with spatial separation and limited commitment also offers promise as a class of neoclassical macroeconomic models seeking to explain observed fluctuations.

2 Uncertainty and the standard Arrow-Debreu program with cross-household diversity

It is difficult at best to make inferences about risk sharing and co-movements of household consumption from descriptive historical or anthropological material. Still, it seems from the work of Homans (1975) and McCloskey (1976) that the dominant institutional arrangement of the medieval village economy - the so-called strip system - was designed in effect as an economywide insurance arrangement. Fields of a typical village were divided into long narrow strips, and any individual's holdings were dispersed so as to reduce the variability associated with diverse soil, topographical, and meteorological conditions. Even more obvious ex post risk sharing is apparent in the primitive island communities studied by Firth (1939) and Malinowski (1953), where relatively large portions of agricultural output were transferred to local chiefs, followed by some redistribution. In northern Thailand, hill tribe communities apparently engage in extensive intravillage and intervillage sharing of rice. The sharing of produce and equality in consumption of the hunter-gatherer tribes studied by Lee (1979) is renowned.

The theory of risk bearing and the treatment of uncertainty developed by Arrow (1964), Debreu (1959), and others predicts such economywide insurance arrangements, at least under specified assumptions and in the absence of various "frictions." In particular, imagine a stylized pure-exchange economy subject to uncertainty. There are a finite number of households indexed by j, $j=1,2,\ldots,n$, and these will be taken to be the primitive economic units. There are a finite number of consumption dates t, $t=1,2,\ldots,T$ and one planning date, $t=0$. Thus, for simplicity, identical finite lifetimes of households are assumed. Each household j has a continuous concave date-t utility function $U^j(c_t^j)$ over units of consumption c_t^j of the single underlying consumption good of the model at date t. For each household j, satiation in consumption is impossible; on the other hand, there may be a minimal subsistence level of consumption. Consumption c_t^j must lie in some a priori consumption set X_t^j, and this is assumed to be closed, convex, and bounded from below. The utility of future consumption is discounted at rate β, $0<\beta<1$, the same rate for all

households. The endowment $e_t^j(\epsilon_t)$ of each household j at date t of the single consumption good is a function of some publicly observed shock ϵ_t at date t and is in the interior of the consumption set X_t^j. The shocks ϵ_t are each presumed to take on at most a finite number of values, say from some common set S. From the point of view of the planning date $t=0$, shock sequence $(\epsilon_1,\ldots,\epsilon_t)$ is drawn with probability $\text{prob}(\epsilon_1,\ldots,\epsilon_t)$. [Extensions to more general stochastic processes are given in Hansen and Richard (in press).] There is presumed to be no storage of any kind, at least not initially. That is, the pure exchange case is studied first, for simplicity.

Following the indexation insight of Arrow (1964) and Debreu (1959), the natural commodity space in this model is the space of state-contingent consumptions, where a state at date t is a specification of the entire history of shocks through date t, namely $(\epsilon_1,\epsilon_2,\ldots,\epsilon_t)$. That is, let $c_t^j(\epsilon_1,\epsilon_2,\ldots,\epsilon_t)$ denote the proposed consumption of agent j at date t as a function of the entire history of shocks. Then a consumption point x^j to household j is the obvious vector $x^j=\{c_1^j(\epsilon_1), c_2^j(\epsilon_1,\epsilon_2),\ldots,c_T^j(\epsilon_1,\ldots,\epsilon_T)$ with indexes running over all dates t and over all histories $(\epsilon_1,\ldots,\epsilon_T)\}$. The consumption set X^j of household j is then the obvious cross-product of consumption set X_t^j and is therefore closed, convex, and bounded from below. Similarly, as a linear combination of weakly concave functions, the global utility function as of the planning date $t=0$ is

$$u^j(x^j)=E\sum_{t=1}^{T}\beta^t U^j[c_t^j(\epsilon_1,\ldots,\epsilon_t)]$$

and is obviously continuous and concave. Thus we are led to a concave programming problem for the determination of Pareto optimal allocations as follows.

Program 2.1. *Maximize by choice of the $c_t^j(\epsilon_1,\ldots,\epsilon_t)$ in the sets X_t^j the objective function*

$$\sum_{j=1}^{n}\lambda^j\left\{E\sum_{t=1}^{T}\beta^t U^j[c_t^j(\epsilon_1,\ldots,\epsilon_t)]\right\} \tag{2.1}$$

subject to the resource constraint

$$\sum_{j=1}^{n}c_t^j(\epsilon_1,\ldots,\epsilon_t)\le\sum_{j=1}^{n}e_t^j(\epsilon_t) \tag{2.2}$$

for each date t and each history $(\epsilon_1,\epsilon_2,\ldots,\epsilon_t)$, where again the weights λ^j satisfy $0\le\lambda^j\le 1$, $\sum_j\lambda^j=1$.

Here expectations are taken as of the initial date $t=0$, and all expectations are held in common. Of course, resource constraint (2.2) bounds consumptions from above.

The objective function (2.1) is continuous in the choice variables, the constraint set is closed and bounded, and autarky is feasible; therefore, a maximizing solution to Program 2.1 is guaranteed to exist. Further, it can be established that any solution to Program 2.1 is necessarily Pareto optimal. Conversely, any Pareto optimum is associated with some point on the utilities possibilities frontier, and – because the set of utility possibilities is convex – that point can be found as a solution to the program for some weights λ^j. Hereafter, then, solutions to Program 2.1 shall be taken as equivalent to the set of Pareto optimal allocations.

Supposing single-valued interior consumption solutions for all households at all dates and histories leads to first-order conditions

$$\beta^t \lambda^j \operatorname{prob}(\epsilon_1, \ldots, \epsilon_t) U^{j'}[c_t^j(\epsilon_1, \ldots, \epsilon_t)] = \mu(\epsilon_1, \epsilon_2, \ldots, \epsilon_t), \tag{2.3}$$

where $\mu(\epsilon_1, \ldots, \epsilon_t)$ is the Lagrange multiplier on the resource constraint at date t and history $(\epsilon_1, \ldots, \epsilon_t)$. It becomes apparent, then, that the aggregate endowment is to be distributed across households in such a way that weighted marginal utilities are equated across households. Furthermore, with a common discount rate and common expectations, the terms $\beta^t \operatorname{prob}(\epsilon_1, \ldots, \epsilon_t)$ factor out of these equalities, and the distribution satisfying the equalities is independent of the date and the history of shocks. Thus only the magnitude of the aggregate endowment,

$$\sum_{j=1}^{n} e_t^j(\epsilon_t) \equiv \bar{e}(\epsilon_t), \tag{2.4}$$

matters in the determination of any household's consumption at any date and any history. That is (with some abuse of notation), each c^j depends only on the aggregate endowment $\bar{e}$, and one writes $c^j(\bar{e})$. Further, this residual "static" one-period risk-allocation problem has been studied by Wilson (1968) and Diamond (1967), among others, yielding

$$\frac{\partial c^j(\bar{e})}{\partial \bar{e}} = \frac{-U^{j'}[c^j(\bar{e})]/U^{j''}[c^j(\bar{e})]}{\sum_k \{-U^{k'}[c^k(\bar{e})]/U^{k''}[c^k(\bar{e})]\}}. \tag{2.5}$$

The right-hand side of (2.5) is a number between zero and unity, and thus individual household consumption must vary positively with the aggregate endowment. Further, this derivative is shown to depend on measures of risk aversion in the population at an optimal allocation.

Sharing rules generally are not linear, as there is no reason a priori to expect the expression on the right-hand side of (2.5) to be some constant. Two special cases may be noted, however. For the first case, suppose the utility functions themselves display constant absolute risk aversion $1/\gamma_j$ for household j, with inverse γ_j; that is,

$$U^j(c) = -\gamma_j \exp[-c/\gamma_j], \quad c \geq 0. \tag{2.6}$$

Then for a two-agent economy, for example, the consumption share of the first agent is

$$c^1(\bar{e}) = \left(\sum_j \gamma_j^{-1}\right)^{-1} \log(\lambda^1/\lambda^2) + \bar{e}\frac{\gamma_1}{\gamma_1+\gamma_2}. \tag{2.7}$$

Again the λ^j are the weights from the programming problem and the coefficient on $\bar{e}$ is as predicted in (2.5). For the second case, suppose

$$U^j(c) = [(d)(1-d)]^{-1}(c-b^j)^d, \tag{2.8}$$

where $0<d<1$ with $c \geq b^j$. Then, with $a=(1-d)^{-1}$,

$$c^1(\bar{e}) = \left[\sum_j (\lambda^j)^{-a}\right]^{-1} [(\lambda^1)^{-a}b^1 - (\lambda^2)^{-a}b^2] + \bar{e}\frac{(\lambda^1)^a}{(\lambda^1)^a + (\lambda^2)^a}. \tag{2.9}$$

One can imagine "fitting" versions of (2.2) and (2.3) to actual observations. In fact, Leme (1984) has asked whether such an exercise would be reasonable for aggregate cross-country data, taking advantage of the monotonicity property (2.5) and the fact that monotonicity holds at any level of aggregation. Unfortunately, monotonicity does not hold uniformly in the postwar data. But then, the world as a whole is hardly an Arrow-Debreu economy. Smaller, self-contained economies are more obvious sources of observations.

Unfortunately, though, panel data from such sources are frequently not available; consistent observations on household consumptions are rare, as are observations on actual exchanges among households. On the other hand, there is sometimes reasonable evidence on aggregate output or aggregate consumption and reasonable evidence on prices or interest rates, say (respectively) on land or one-period loans. Thus one can try to use these observations to address the hypothesis of risk aversion – for example, to estimate the free parameter d in equation (2.8).

If one is to use available price data, one must then include as part of the model what role prices are playing. The obvious assumption analytically is that prices are such that allocations are achieved as if in an equilibrium of competitive markets. This assumption is restrictive and ad hoc – nothing in the theory predicts the existence of competitive markets. On the other hand, the competitive-markets hypothesis is clean and analytically powerful. For, in the present context, competitive equilibria can be shown to exist. Further, the usual welfare theorems apply so that competitive equilibria are necessarily Pareto optimal, and all optima can be supported as competitive equilibria.

More specifically, let the consumption set X^j of each household j with typical component x^j be defined as above. Let the endowment point w^j to household j be the vector $w^j = \{e_1^j(\epsilon_1), e_2^j(\epsilon_1, \epsilon_2), \ldots, e_T^j(\epsilon_1, \epsilon_2, \ldots, \epsilon_T)$ with indexes running over all dates t and over all histories $(\epsilon_1, \ldots, \epsilon_T)\}$. A preference ordering $\geq_j$ for household j is induced by the utility function $u^j(x^j)$ as defined above. Then certain assumptions are satisfied: (a.1) X^j is closed; (a.2) X^j is convex; (a.3) X^j has a lower bound; (a.4) there is no satiation consumption in X^j; (a.5) for every point $x' \in X^j$ the upper and lower contour sets $\{x \in X^j : x \geq_j x'\}$ and $\{x \in X^j : x \leq_j x'\}$ are closed in X^j; (a.6) if x' and x'' are two points in X^j and if t is any real number in $(0,1)$, then $x'' \geq_j x'$ implies $tx'' + (1-t)x' \geq_j x'$; (a.7) there is some x^0 in X^j such that $x^0 \ll w^j$. By Theorem 1 of Debreu (1959, §5.7), under assumptions (a.1)–(a.7) there exists a competitive equilibrium as previously defined in Section 1. In particular, there exists a price system p^* specifying $p_t^*(\epsilon_1, \dots, \epsilon_t)$ for all dates t and all histories $(\epsilon_1, \dots, \epsilon_t)$ such that, for every household j, allocation x^{j^*} maximizes $u^j(x^j)$ subject to $x^j \in X^j$ and

$$\sum_{t=1}^{T} \sum_{(\epsilon_1, \dots, \epsilon_T)} [c_t^j(\epsilon_1, \dots, \epsilon_t) - e_t^j(\epsilon_t)] p_t^*(\epsilon_1, \dots, \epsilon_t) \leq 0, \tag{2.10}$$

and the allocations x^{j^*}, $j = 1, 2, \dots, n$, satisfy market-clearing condition (2.2). Also, by Theorem 6.4 of Debreu, under assumptions (a.2), (a.5), and (a.6) for every optimum x^{j^*}, $j = 1, 2, \dots, n$, there exists a price system p^* such that x^{j^*} minimizes expenditure $p^* x^j$ on the upper contour set $\{x^j \in X^j : x^j \geq_j x^{j^*}\}$. Further, if the exceptional case

$$p^* x^{j^*} = \min_{x^j \in X^j} p^* x^j \tag{2.11}$$

does not occur, then x^{j^*} maximizes $u^j(x^j)$ subject to $x^j \in X^j$ and condition (2.10) above, with the endowment w^j replaced by x^{j^*}. Finally, by Theorem 1 of Debreu (1959, §6.3), under assumptions (a.2), (a.4), and (a.6), every competitive equilibrium is an optimum.

Again, under the competitive-equilibrium hypothesis, time series on rents or interest rates can – at least in principle – be used to check on the fit of the theory with the reality of an actual economy. At the level of generality and diversity assumed thus far, however, this still can be a challenging exercise, as it can be difficult to work backward from prices and aggregate quantities to individual demands and hence to individual preferences. More structure is needed.

An extreme but operational assumption that does allow a fitting exercise is that preferences of individual households aggregate in the sense of Gorman (1953); that is, there exists a utility function for a representative consumer which, when evaluated at aggregate quantities, can be used to deliver equilibrium prices. In fact, the asset-pricing literature of Grossman and Shiller (1981), Hall (1981), Eichenbaum, Hansen, and Richard (1987), Hansen and Singleton (1982, 1983), Mankiw, Rotemberg, and Summers (1985), Mehra and Prescott (1985), Shiller (1981), and others can be interpreted as adopting this aggregation hypothesis. It is equivalent, for the pure-exchange single-good economy here, with the supposition that preferences are of the form of (2.6) or (2.8). It must be emphasized, however, that some diversity in preferences and consumption sets as well as some competitive-equilibrium trade can be accommodated. The representative consumer construct does not require that households literally be identical.

3 Programs with multiple commodities and further aggregation possibilities

The theory thus far assumes only one consumption good, yet on the face of it this is unsatisfactory, especially if one tries to match the theory to the reality of any economy. In the medieval village, for example, various crops were grown, including wheat, oats, barley, rye, peas, and beans. And in the island community of Tikopia studied by Firth (1939), yams, fruits, and fish were all part of the regular diet.

Fortunately, formal aspects of the theory are easily modified. If there are m consumption goods, then one need only let $c_t^j(\epsilon_1, \dots, \epsilon_t)$ and $e_t^j(\epsilon_t)$ denote m-dimensional vectors and continue to make the same assumptions for the consumption sets X_t^j and utility functions $U^j(\cdot)$ as above. Program 2.1 then remains intact and its solutions still correspond with the Pareto optima. In fact, standard theorems on the existence and optimality of competitive equilibria still apply.

Aggregation possibilities in the sense of Gorman (1953) and described by Hansen (in press) are also possible. That is, under specified assumptions, a representative-consumer construct can be used to deliver competitive-equilibrium prices as marginal rates of substitution at aggregate quantities. More specifically, suppose for sake of illustration that there are two underlying commodities, so that $c_t^j = c = (c_1, c_2)$, and that preferences for each household j, $j = 1, 2, \dots, n$, over bundles c in some consumption set $X_t^j \subset R^2$ at date t are described by a family of indifference curves of the form

$$c^j = \rho^j(d) + \rho_0(d)(U^j). \tag{3.1}$$

Here, d is a 2-dimensional vector (d_1, d_2), and d_1/d_2 is the marginal rate of substitution. Thus, as d_1/d_2 is varied parametrically, one moves along an indifference curve determining bundle c^j for household j. The particular indifference curve for household j is determined by the utility index U^j, where utility number U^j can result from a monotonic transformation of some underlying utility function. The point is that diversity in preferences across households j can be accommodated, namely in the "baseline" indifference curves $\rho^j(d)$. However, the "expansion factor" $\rho_0(d)$ is common.

We may note that at a Pareto optimum with interior consumption solutions, marginal rates of substitution must be equated, so that d would be common across households. Adding up (3.1) over j for fixed d then yields

$$\bar{c} = \bar{\rho}(d) + \rho_0(d)\left[\sum_{j=1}^{n} U^j\right], \tag{3.2}$$

where the over-bar indicates aggregate numbers or functions. Of course, in any optimal allocation resources are not wasted, so $\bar{c} = \bar{e}$, the aggregate endowment. That is, from (3.2),

$$\bar{e} = \bar{\rho}(d) + \rho_0(d)\left[\sum_{j=1}^{n} U^j\right]. \tag{3.3}$$

System (3.3) has two equations in essentially two unknowns, marginal rate of substitution d_1/d_2, and aggregate utility index $\sum_{j=1}^{n} U^j$, and so under specified assumptions both these are determined by (3.3). In particular, then, the marginal rate of substitution at an optimum is pinned down by the aggregate endowment, although the distribution of utilities and hence of consumptions is not. Of course, in a competitive equilibrium this marginal rate of substitution must be the price ratio.

Important also is the fact that if there is any nontrivial choice of "endowment" e, say from some aggregate production possibilities set, then the preferences of the representative consumer with indifference curves represented by (3.2) can also be used to determine a Pareto optimal outcome. For, as in Figure 1, a move from point A to point 0 must necessarily increase the utility indexes $\sum_{j=1}^{n} U^j$. Thus, any distribution of household utilities at point A can be dominated by a suitable distribution at point 0.

The class of preferences that aggregate in the sense of Gorman include the class of utility function studied by Eichenbaum, Hansen, and Richard (1987), namely,

$$\frac{1}{\delta\sigma}\left\{\sum_{i=1}^{m} \theta_i[\delta(c_i^j - b_i^j)]^{\alpha}\right\}^{\sigma/\alpha}$$

when b_i^j is a subsistence point of household j for commodity i with $\sigma < 1$ and $\alpha < 1$. This includes as a special case $\sigma = \alpha$, so that preferences are separable over consumption goods, with each commodity utility function displaying the same index of relative risk aversion. It also includes as a special case $\theta_i = \delta = \alpha = 1$, but $\sigma < 1$ so that direct aggregation of underlying commodities and risk aversion is allowed.

Such special cases also allow Program 2.1 to deliver strong implications for co-movements of consumption as before. Essentially, separability delivers m separate marginal conditions and eliminates interaction

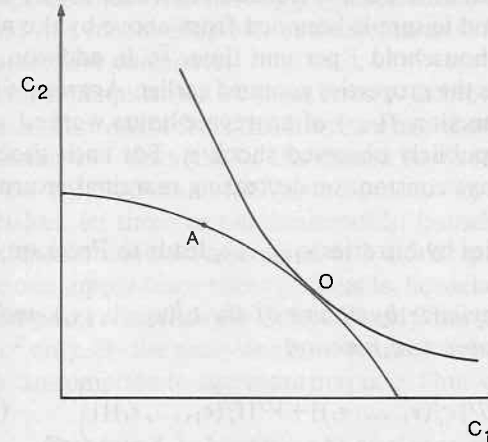

Figure 1

across commodities, something that can overturn the earlier analysis. And when direct aggregation is possible the analysis can proceed in an obvious way, namely by combining commodities directly into some aggregate. Unfortunately, strong results for the general case are not going to fall out easily, as Scheinkman (1984) has emphasized. It can be said, however, that observations have pushed the theory in an interesting direction.

4 Programs with production and with leisure as another commodity

Labor arrangements are often of interest in primitive economies, in part because such arrangements seem to differ from those to which we have become accustomed. In the medieval village, for example, virtually all residents participated in *benes*, communal works for the local lord in times of need. These and other more standard tasks were specified in great detail and were associated with an elaborate system of monitors. In the primitive island community studied by Firth (1939), there were community-wide requests for labor for household-specific projects, and this labor was often supplied without apparent compensation.

To extend the theory to incorporate leisure, let household j have preferences over units of consumption c_t^j and leisure ℓ_t^j respectively, as represented by a date-t utility function $U^j(c_t^j) + V^j(\ell_t^j)$, with each functional component continuous, weakly concave, and strictly increasing. Separability is assumed here for tractability, as motivated above. The future is discounted at common rate β, $0<\beta<1$. Consumption and leisure must be at least nonnegative, and leisure is bounded from above by the number of hours available to household j per unit time, $\bar{\ell}^j$. In addition, the consumption set X_t^j retains the properties assumed earlier. Aggregate output $\bar{q}_t$ at date t is some function $f(\cdot,\cdot)$ of aggregate hours worked, $a_t = \sum_{j=1}^n (\bar{\ell}^j - \ell_t^j)$, and some publicly observed shock ϵ_t. For each shock ϵ_t the function $f(\cdot, \epsilon_t)$ displays constant or decreasing marginal returns to hours worked.

Indexing all commodities by histories $\epsilon_1, \ldots, \epsilon_t$ leads to Program 4.1.

Program 4.1. *Maximize by choice of the* $c_t^j(\epsilon_1, \ldots, \epsilon_t)$ *and the* $\ell_t^j(\epsilon_1, \ldots, \epsilon_t)$ *the objective function*

$$\sum_{j=1}^{n} \lambda^j \left\{ E_0 \sum_{t=1}^{T} \beta^t [U^j[c_t^j(\epsilon_1, \ldots, \epsilon_t)] + V^j[\ell_t^j(\epsilon_1, \ldots, \epsilon_t)]] \right\} \tag{4.1}$$

subject to $(c_t^j(\cdot), \ell_t^j(\cdot)) \in X_t^j$,

$$\sum_{j=1}^{n} c_t^j(\epsilon_1, \ldots, \epsilon_t) \le \bar{c}_t(\epsilon_1, \ldots, \epsilon_t), \tag{4.2}$$

$$\sum_{j=1}^{n} \ell_t^j(\epsilon_1, \ldots, \epsilon_t) \le \bar{\ell}_t(\epsilon_1, \ldots, \epsilon_t), \tag{4.3}$$

$$\bar{c}_t(\epsilon_1, \ldots, \epsilon_t) \le f\left[\sum_{j=1}^{n} \bar{\ell}^j - \bar{\ell}_t(\epsilon_1, \ldots, \epsilon_t), \epsilon_t \right]. \tag{4.4}$$

Proceeding as before, one obtains the implication that consumption c_t^j of household j should move monotonically with aggregate consumption $\bar{c}_t$. Here as well, leisure ℓ_t^j of household j should move monotonically with aggregate leisure $\bar{\ell}_t$, at least if boundary constraints are avoided. Both these relationships are invariant as to date and histories. Thus consumption and leisure sharing are delivered, with possibly variable shares determined by preferences and the weights λ^j. Further, the weights λ^j may also have implications for the cross-sectional distribution of consumption and leisure. For example, a household with high consumption on average might also have high leisure on average. On the other hand, there should be no direct contemporaneous link between a household's actual effort and a household's final consumption, because ℓ_t^j depends on $\bar{\ell}_t$ and c_t^j depends on $\bar{c}_t$. For example, high labor effort to avert disaster due to some shock ϵ_t might not be associated with increased communal consumption. A relationship between aggregate consumption and aggregate leisure contingent only on contemporary shock ϵ_t can be delivered, a relationship that can also depend on the weights λ^j. However, if preferences aggregate in the sense of Gorman, this is sufficient for the relationship between aggregate consumption and aggregate leisure to not depend on the weights λ^j, and this relationship can be delivered as if from the preferences of a representative consumer.

Another implication of these Arrow–Debreu economies for labor assignments is evident from the literature on the principal–agent problem of Grossman and Hart (1983), Harris and Raviv (1979), Holmström (1979), Mirrlees (1975), Shavell (1979), and others. For specializing our notation somewhat, let there be two households: household 1 associated with an agent and household 2 associated with a principal. Suppose that the agent alone can supply labor effort a_1. That is, household 1 has preferences over consumption c^1 and leisure ℓ^1; household 2 has preferences over consumption c^2 only. By the analysis above we may consider a static problem and index consumption to aggregate output $\bar{c}$. Thus we are led to Program 4.2.

Program 4.2. *Maximize by choice of* $c^1(\bar{c}), c^2(\bar{c}), \ell(\epsilon)$ *the objective function*

$$\lambda^1\{E[U^1[c^1(\bar{c})] + V^1[\ell(\epsilon)]]\} + \lambda^2\{E[U^2[c^2(\bar{c})]]\} \tag{4.5}$$

subject to

$$c^1(\bar{c}) + c^2(\bar{c}) = \bar{c}, \tag{4.6}$$

$$\bar{c} = f[\bar{\ell}^1 - \ell(\epsilon), \epsilon]. \tag{4.7}$$

Solution $c^1(\bar{c})^*$, $c^2(\bar{c})^*$, and $\ell(\epsilon)^*$ to Program 4.2 displays an incentive problem: Given the optimal consumption schedule $c^1(\bar{c})^*$, if household 1 were asked to choose its own level of leisure (effort) $\ell(\epsilon)$ then that choice would in general differ from $\ell(\epsilon)^*$. That is,

$$U^1[c^1(\bar{c})^*] + V^1[\ell(\epsilon)] > U^1[c^1(\bar{c})^*] + V^1[\ell(\epsilon)^*], \tag{4.8}$$

where again $\bar{c}$ on the left-hand side of (4.8) is determined for some other leisure choice $\ell(\epsilon)$. In fact, household 1 would generally want to work less (i.e., consume more leisure) than at the optimum. This idea generalizes to multiagent problems.

One might also ask whether these implications of Program 4.1 survive the obvious decentralization hypothesis, that allocations are achieved as if in competitive markets. To do this, one must distinguish individual choices from aggregate outcomes. Thus suppose each household j, $j = 1, 2, \ldots, n$, has a production function $q^j = f^j(a^j, \epsilon_t)$ displaying constant or diminishing returns to scale with a^j as labor input measured in negative units. That is,

$$q_t^j = f^j\left(\sum_{k=1}^{n} a_t^{kj}, \epsilon_t \right) \quad j = 1, 2, \ldots, n,$$

where a_t^{kj}, measured in negative units, is labor hours supplied by household k to household j at date t. Of course, individual leisures satisfy

$$\ell_t^j = \bar{\ell}^j + \sum_{k=1}^{n} a_t^{jk},$$

and aggregate consumption satisfies

$$\bar{c}_t \leq \sum_{j=1}^{n} q_t^j.$$

Date-t and history $(\epsilon_1, \ldots, \epsilon_t)$-contingent production sets can now be defined for household j with the second component for labor input and the first component for commodity output, namely

$$Y_t^j(\epsilon_1, \ldots, \epsilon_t) = \{y_{1t}(\epsilon_1, \ldots, \epsilon_t), y_{2t}(\epsilon_1, \ldots, \epsilon_t): y_{1t}(\epsilon_1, \ldots, \epsilon_t) \geq 0,$$
$$y_{2t}(\epsilon_1, \ldots, \epsilon_t) \leq 0, y_{1t}(\epsilon_1, \ldots, \epsilon_t) \leq f^j[y_{2t}(\epsilon_1, \ldots, \epsilon_t), \epsilon_t]\}.$$

Evidently, with set Y^j as the obvious cross-product over all possible dates t and histories $(\epsilon_1, \ldots, \epsilon_t)$ and with aggregate set Y as the sum over the Y^j, certain assumptions are satisfied: (d.1) $0 \in Y^j$; (d.2) Y is closed; (d.3) Y is convex; (d.4) $Y \cap (-Y) \subset \{0\}$ or impossibility of free production; and (d.5) $Y \supset (-\Omega)$ or free disposal.

Preferences of each household j can be defined over consumption and labor supply measured in positive and negative units respectively. That is, the utility functions $U^j(\cdot)$ and $V^j(\cdot)$ deliver function $W^j(x_{1t}, x_{2t})$ and hence discounted expected utility function $u^j(x)$ for x in consumption set X, with the latter as the obvious cross-product over the redefined X_t^j. The endowment w^j with a component of zero for labor supply is obviously defined. It facilitates the analysis here to suppose the component for consumption of w^j is strictly positive.

Proceeding as before, define – over all dates and histories – a price system $p_t(\epsilon_1, \ldots, \epsilon_t)$ and $w_t(\epsilon_1, \ldots, \epsilon_t)$ for the consumption good and for labor effort, respectively. There follows the standard definition of a competitive equilibrium with production as given in Section 1. The existence of a competitive equilibrium follows under the previous assumptions on consumers and assumptions (d.1)–(d.5) on firms. That a competitive equilibrium is an optimum follows as before under the previous assumptions. That any optimum can be supported as a valuation equilibrium follows as before with additional assumption (d.3).

The implication of leisure sharing is robust to the decentralization by competitive markets, as leisure sharing holds for all weights λ^j; the competitive hypothesis just picks out particular weights. Absence of quid pro quo of consumption for labor supply is problematical, however. Under the decentralization hypothesis, each household j perceives itself as being free to vary labor supply, by (say) reducing $\ell_t^j(\epsilon_1, \ldots, \epsilon_t)$ and receiving $w_t^j(\epsilon_1, \ldots, \epsilon_t)$ in compensation. In this sense there is quid pro quo. In fact, if the substitution effect dominates, higher wages could effect higher effort. However, this increased purchasing power could be spent on *all* consumptions $c_\tau^j(\epsilon_1, \ldots, \epsilon_\tau)$, $\tau = 1, 2, \ldots, T$ and thus absence of movement in contemporary consumption is possible in equilibrium. Observationally, matters are subtle because one must distinguish quid pro quo in accounting credits from changes in actual consumption. That is, in an economy with diversity and trade it is a tricky business to distinguish compensation intended for *final consumption* from compensation as an accounting payment. Of course, if preferences aggregate in the sense of Gorman then the relative wage can be determined as the obvious marginal rate of substitution of a representative consumer at aggregate leisure and consumption, and thus time series on wages, aggregate consumption, and aggregate labor effort can be delivered. Thus, one can ask whether there would be quid pro quo in these data.

Under the decentralization hypothesis, household j chooses a contract specifying consumptions and labor supplies under all possible contingencies subject only to the budget constraint. In this sense, then, labor is not assigned. However, once a contract is entered into, the $\ell_t^j(\epsilon_1, \ldots, \epsilon_t)^*$ plan must be carried out, and this requires that leisure and labor effort be observed. Monitoring would still be necessary for enforcement of the competitive-equilibrium outcome.

5 Programs with capital goods and storage

A study of various primitive economies reveals the existence of storage possibilities and various capital assets. The magnitude of within-period consumption risk in the medieval village, and therefore the need for insurance and the strip system, would seem to turn on the possibility of intertemporal storage. Similarly, oxen for ploughing in the medieval village and canoes in the island economy studied by Firth represent important capital assets and seem positively associated with the owner's economic position.

To begin thinking about storage and capital assets and their implications for risk and for compensation, imagine that aggregate output $\bar{q}_t$ at date t is some function of aggregate labor supply, aggregate capital or storage, and the contemporaneous shock; that is,

$$\bar{q}_t = f\left(\sum_{j=1}^{n} \bar{\ell}^j - \bar{\ell}_t, \bar{K}_t, \epsilon_t\right),$$

and consider the following program.

Program 5.1. Maximize by choice of $c_t^j(\epsilon_1,\dots,\epsilon_t)$, $\ell_t^j(\epsilon_1,\dots,\epsilon_t)$, *and* $\bar{\imath}_t(\epsilon_1,\dots,\epsilon_t)$ *the objective function*

$$\sum_{j=1}^{n}\lambda^j\left\{E_0\sum_{t=1}^{T}\beta^t[U^j[c_t^j(\epsilon_1,\dots,\epsilon_t)]+V^j[\ell_t^j(\epsilon_1,\dots,\epsilon_t)]]\right\} \tag{5.1}$$

subject to

$$\sum_{j=1}^{n}c_t^j(\epsilon_1,\dots,\epsilon_t)\le\bar{c}_t(\epsilon_1,\dots,\epsilon_t), \tag{5.2}$$

$$\sum_{j=1}^{n}\ell_t^j(\epsilon_1,\dots,\epsilon_t)\le\bar{\ell}(\epsilon_1,\dots,\epsilon_t), \tag{5.3}$$

$$\bar{c}_t(\epsilon_1,\dots,\epsilon_t)+\bar{\imath}_t(\epsilon_1,\dots,\epsilon_t) \le f\left[\sum_{j=1}^{n}\bar{\ell}^j-\bar{\ell}_t(\epsilon_1,\dots,\epsilon_t),\bar{K}_t(\epsilon_1,\dots,\epsilon_t),\epsilon_t\right] \tag{5.4}$$

$$\bar{K}_{t+1}(\epsilon_1,\dots,\epsilon_t)=(1-\delta)[\bar{K}_t(\epsilon_1,\dots,\epsilon_{t-1})+\bar{\imath}_t(\epsilon_1,\dots,\epsilon_t)], \tag{5.5}$$

with $[c_t^j(\epsilon_1,\dots,\epsilon_t),\ell_t^j(\epsilon_1,\dots,\epsilon_t)]\in X_t^j$.

Here $\bar{c}_t$, $\bar{\imath}_t$, $\bar{\ell}_t$, and $\bar{K}_t$ are aggregate consumption, investment, leisure, and beginning-of-period capital (or storage), respectively, at date t. Inequality (5.4) divides output into consumption and investment (additions to storage), and equation (5.5) is the law of motion on aggregate capital (or storage) with δ as the rate of depreciation.

The aggregate function here, $f(\cdot,\cdot,\cdot)$, is derived from individual production functions

$$q_t^j=f^j\left(\sum_{k=1}^{n}a_t^{kj},\ \sum_{k=1}^{n}K_t^{kj},\ \epsilon_t\right) \tag{5.6}$$

for each household j, where a_t^{kj} is as before (in Section 4) and K_t^{kj}, measured in negative units, is capital supplied by household k to household j at date t. Further, let i_t^j, measured in negative units, denote additions to household j's capital at the end of date t from current production. The aggregate resource constraint may now be written

$$\sum_{j=1}^{n}(-i_t^j)+\sum_{j=1}^{n}c_t^j\le\sum_{j=1}^{n}q_t^j. \tag{5.7}$$

Despite this appearance of individual ownership, capital in Program 5.1 can be viewed as common property, allocated across projects at each date, helping to deliver aggregate output. No household should receive increased compensation for the act of supplying a capital good to anyone else's project. Consumption is determined still by aggregate consumption,

and so on as before. Of course, the weights λ^j could be related to initial capital holdings, and if so this invariance result could be weakened. More generally, one might consider whether versions of this economy with capital can be decentralized and what interpretations follow.

To do this, let the generic commodity point be a 5-tuple: the first component for consumption, the second for investment, the third for beginning-of-period capital, the fourth for end-of-period capital, and the fifth for labor–leisure. Then imagine household j has two production technologies, the first for production of the consumption-investment good of the form

$$Y_t^{1j}(\epsilon_1,\dots,\epsilon_t)=\{y_t(\epsilon_1,\dots,\epsilon_t)\in\Re^5: \underset{+}{y_{1t}}(\epsilon_1,\dots,\epsilon_t)+\underset{+}{y_{2t}}(\epsilon_1,\dots,\epsilon_t) \le f^j[y_{5t}(\epsilon_1,\dots,\epsilon_t),y_{3t}(\epsilon_1,\dots,\epsilon_t),\epsilon_t] \text{ and } \underset{+}{y_{4t}}(\epsilon_1,\dots,\epsilon_t)\le-\underset{-}{y_{3t}}(\epsilon_1,\dots,\epsilon_t)\}$$

and the second for capital accumulation of the form

$$Y_t^{2j}(\epsilon_1,\dots,\epsilon_t)=\{y_t(\epsilon_1,\dots,\epsilon_t)\in\Re^5: y_{3,t+1}(\epsilon_1,\dots,\epsilon_{t+1}) \le-\{(1-\delta)[y_{4t}(\epsilon_1,\dots,\epsilon_t)+y_{2t}(\epsilon_1,\dots,\epsilon_t)]\}\}.$$

Here the ± notation below the variables indicates + for output and − for input, and indicates as well the ranges of the indicated variables; otherwise variables must be zero. As before, let profit shares in both technologies be $\theta^{kj}=0$ for $k\ne j$ and $\theta^{jj}=1$. Evidently, assumptions (d.1)–(d.5) are satisfied with Y^{1j} and Y^{2j} as the obvious cross-products over dates t and histories $(\epsilon_1,\dots,\epsilon_t)$, Y^j as the sum of Y^{1j} and Y^{2j}, and Y as the sum over j. Households' endowment vectors in this space are all zero. But note that production technology Y^{2j} is defined relative to some fixed initial assignment of capital, $y_{3,t=1}$, and this may yield some profit to household j. Household j has preferences over consumption and labor supply,

$$\underset{+}{x_{1t}}(\epsilon_1,\dots,\epsilon_t) \quad\text{and}\quad \underset{-}{x_{5t}}(\epsilon_1,\dots,\epsilon_t)$$

at date t under history $(\epsilon_1,\dots,\epsilon_t)$.

The standard definition of a competitive equilibrium follows as before with price system $p_t(\epsilon_1,\dots,\epsilon_t)\in\Re^5$ having components for consumption, investment, beginning-of-period capital, end-of-period capital, and labor. And as above, competitive equilibria can be shown to exist and to satisfy the two welfare theorems.

In a standard competitive equilibrium, household j acting under technology Y^{1j} makes a "rental" payment $p_{3t}(\epsilon_1,\dots,\epsilon_t)-p_{4t}(\epsilon_1,\dots,\epsilon_t)$ per unit capital used during period t under history $(\epsilon_1, \ldots, \epsilon_t)$, and thus it would appear to offer explicit compensation for capital supplied to it. In this sense there is quid pro quo. However, as before, recipients of these payments - say, household h - may purchase consumption goods over the entire horizon and all contingencies. Thus consumption $c_t^h(\epsilon_1, \ldots, \epsilon_t)$ need not increase with increases in capital supplied. Thus there need be no explicit quid pro quo of contemporary capital for contemporary consumption in equilibrium. On the other hand, as noted above, there may be a more subtle relationship between capital, consumption, and leisure that would be apparent from Program 5.1: A competitive equilibrium is associated with particular Pareto weights λ^j, and initial capital holdings $y_{3,t=1}$ determine the income helping to deliver the competitive equilibrium. Thus, roughly, the higher is household j's initial capital, the higher is j's weight λ^j, and the higher (on average) might be consumption and leisure.

Of course, to follow this through one must determine the competitive equilibrium relationship among consumptions, capital (inventory) holdings, and rentals across households over time and over shocks. Unfortunately, though, with the level of diversity allowed thus far, this is not a trivial endeavor. Of course, with the aggregation hypothesis one *can* generate time series on aggregates from the maximization problem of a representative consumer, for then Program 5.1 reduces to the standard neoclassical growth model.

The standard neoclassical growth model studied by Brock and Mirman (1972), Cass (1965), Danthine and Donaldson (1981), Koopmans (1965), and Sargent (1979), among others, is often maligned as being unrealistic and a poor choice for the study of fluctuations and economywide phenomena. Yet we see now how one is led to it in an attempt to understand dynamic phenomena, starting from a natural base and getting rid of some (but not all!) of the diversity across agents. In fact, recent papers of Lucas (1985, 1987), Kydland and Prescott (1982), and Altug (1984) illustrate how little we know about dynamics induced by optimizing agents, in particular how time series of aggregate variables can depend critically on assumptions about technology and preferences. As Lucas (1987) notes, for example, solutions of the Kydland–Prescott (1982) version of Program 5.1 display too little variability in hours worked and investment and too much variability in consumption, at least relative to postwar U.S. time-series data. Yet a combination of intertemporal substitution of leisure, gestation lags, and lags in the resolution of uncertainty can at least partially overcome the deficiency. Further, the work of Altug (1984) indicates there is a tradeoff between intertemporal substitution of leisure and multiple capital goods. And it is unclear whether some reasonable combination of costly physical storage, productive planting of seed, variable weather, and risk aversion can be reconciled with the observations of low carryovers and periodic starvation in typical medieval villages. Again it seems that dynamic phenomena consistent with the neoclassical growth model are not sufficiently well understood, and are naturally the subject of continuing research.

6 Programs with private information

As noted earlier, primitive societies can generate observations that are anomalous relative to standard Arrow–Debreu theory. In the rice-sharing scheme of northern Thailand, for example, it is apparent that track is kept of households' rice "deficits" *over time*. In particular, it seems that a given deficit cannot be indefinitely large or sustained on an indefinite basis - at some critical point, reciprocity in terms of labor is required. And, in the absence of that, the deficit-ridden household may be kicked out of its village. Again, these kinds of intertemporal tie-ins are not consistent with solutions to standard programming problems of the type considered thus far. Of course, in the case of northern Thailand it is plausible that unobserved labor effort is the neglected element. One also suspects that anomalous consumption and leisure patterns abound in villages of southern India, where apparently there is no explicit monitoring of labor effort on sharecropped fields. Thus one might again suppose that private information is a neglected element.

Fortunately, the programming methods under discussion thus far can be modified to incorporate explicit private information, delivering yet another class of Arrow–Debreu programs. Moreover, the solutions to these modified programs can differ in substantial and systematic ways from the solutions to full-information, standard Arrow–Debreu programs. This section will elaborate on both these points.

To begin, it will prove instructive to return to the pure-exchange economy of Sections 2 and 3 and incorporate a minimal amount of private information. Thus, consider an essentially static pure-exchange economy with two agents, a and b, each endowed with an m-dimensional vector of consumption goods dependent on some shock ϵ. In fact, suppose realizations of agent a's endowment $e^a(\epsilon)$ are seen by agent a alone taking on (say) at most two values, θ' and θ'', with generic element θ, in some consumption period. And suppose for simplicity that realizations of agent b's endowment $e^b(\epsilon)$ are public, say some constant W. Were we to solve Program 2.1 for this special case, ignoring the private information, we would deduce the fact (with risk aversion) that consumptions of agents a and b should be functions of the aggregate endowment $W+\theta$ or (for simplicity) just θ, and we would write $c^a(\theta)$ and $c^b(\theta)$.

But now a potential problem emerges. For let $f^a(\theta)$ denote the effective net transfer that agent a is to receive when his endowment is θ; that is, $f^a(\theta) \equiv c^a(\theta) - \theta$. It is possible that

$$U^a[\theta' + f^a(\theta')] < U^a[\theta' + f^a(\theta'')], \tag{6.1}$$

so that if agent a's endowment were actually θ' and he were asked to name a value for it, he would choose to name θ'', and the allocation to him would be $\theta' + f^a(\theta'')$ rather than $\theta' + f^a(\theta')$.

As is apparent, this problem might be remedied by the imposition of (6.1) with the inequality reversed; that is,

$$U^a[\theta' + f^a(\theta')] \geq U^a[\theta' + f^a(\theta'')] \tag{6.2}$$

and similarly, for $\theta = \theta''$,

$$U^a[\theta'' + f^a(\theta'')] \geq U^a[\theta'' + f^a(\theta')]. \tag{6.3}$$

That is, one might be tempted to impose constraints (6.2)–(6.3) directly onto Program 2.1 before deriving a solution. In fact, it is the implication of the work of Myerson (1979) and of Harris and Townsend (1981) that such a procedure can be rigorously justified. In economies with private information there is essentially no loss of generality in imposing such constraints; such constraints capture all the additional restrictions associated with private information.

Of course, these restrictions can be substantial. Private-information optimal allocations often differ substantially from full-information optimal allocations – the point to be made in this section. With only one good for the economy just described, for example, the solution with private information would be autarkic.

Two qualifications to this discussion should be noted, however. First, when there is private information on quantities, as in the economy just described, constraints like (6.2) and (6.3) sometimes can be weakened. In effect, agent a could be asked both to name a value for his endowment *and* to display the endowment (if necessary) as evidence of his claim. This can be captured formally by allowing agent a to transfer some amount of his consumption good to some center, as a "tax," before receiving any compensation, as a "subsidy." Second, even without this modification, constraints (6.2) and (6.3) do not necessarily leave the space of allocations $[f^a(\theta'), f^a(\theta'')]$ convex. That is, problems such as Program 2.1 with (6.2) and (6.3) appended as constraints are not necessarily concave problems. But Prescott and Townsend (1984a, b), among others, have

shown that this problem can be remedied with the use of lotteries, and beneficial exchange is made possible as well. Intuitively, the risk aversion of agent a can vary across distinct deterministic allocations, and thus lotteries can help to weaken the effect of the incentive constraints, leading to a Pareto improvement. (An explicit example of this will be given momentarily.)

More formally for the environment at hand, let $T(\theta)$ denote the set of all feasible displays or "taxes" $\tau = (\tau^a, \tau^b)$ on agents a and b respectively, satisfying constraints $0 \leq \tau^a \leq \theta$ and $0 \leq \tau^b \leq W$. Similarly, let $S(\tau)$ denote the set of all second-round conditional "subsidies" $s = (s^a, s^b)$ on agents a and b respectively, satisfying the constraints $s^a \geq 0$, $s^b \geq 0$, and $s^a + s^b \leq \tau^a + \tau^b$, so that the sum of the subsidies is bounded by the sum of the taxes or displays. Next, let $\pi^\tau(\theta)$ be a lottery on taxes in the space $T(\theta)$, let $\pi^s(\theta, \tau)$ be a lottery on subsidies in the space $S(\tau)$ conditioned on tax τ, and let $\pi(\tau, s, \theta)$ be the joint lottery on taxes τ and subsidies s. Agent a is then imagined to effect lotteries $\pi^\tau(\theta)$ and $\pi^s(\theta, \tau)$ or (more directly) $\pi(\tau, s, \theta)$, by his announcement of θ. Program 2.1 is thus reduced to the following.

Program 6.1. Maximize by choice of the lotteries $\pi(\tau, s, \theta)$ the objective function

$$\lambda^a \left\{ E_\theta \iint U^a[\theta - \tau^a + s^a] \, \pi(d\tau, ds, \theta) \right\} + \lambda^b \left\{ E_\theta \iint U^b[W - \tau^b + s^b] \, \pi(d\tau, ds, \theta) \right\} \tag{6.4}$$

and given endowment $\theta = \theta'$ subject either to

$$\pi^\tau(\theta'') \text{ is not a lottery on the space } T(\theta'), \tag{6.5}$$

so that some realization of the tax lottery or display is not feasible given $\theta = \theta'$, or to

$$\iint U^a[\theta' - \tau^a + s^a] \, \pi(d\tau, ds, \theta') \geq \iint U^a[\theta' - \tau^a + s^a] \, \pi(d\tau, ds, \theta''), \tag{6.6}$$

so that agent a has no incentive to lie, announcing θ'' given the endowment is $\theta = \theta'$, and subject to

constraints analogous to (6.5) and (6.6) when the endowment is $\theta = \theta''$. (6.7)

In the absence of qualifications like (6.5), at least, Program 6.1 can be converted to a program that is linear in unconditional probabilities $\pi(\tau, s, \theta)$.

Techniques like this turn out to be surprisingly robust, that is, able to handle a wide range of private-information problems. First, of course, one can handle situations with private information on actions, as in the standard principal-agent problem; see Myerson (1982). Essentially, one need only reverse the strict inequality in (4.8) to a weak inequality in the other direction and place the resulting constraint into Program 4.2. Second, one can handle situations in which private information can be made public at some cost, as with (potentially random) audits or monitoring technologies, triggered by announcements of agents with private information; see Baiman and Demski (1980) and Townsend (1979). Third, one can handle multiperiod problems, even with period-by-period private information, as in W. Rogerson (1985) or Townsend (1982). And, finally, one can handle multilateral private information, as in Moore (1984), Myerson (1986), and Townsend (1986b), even in multiperiod contexts, provided there is sufficient commitment or technology to enforce internal randomization devices. These set-ups, as well as those in Section 9, allow for difficulties in processing and communicating information and for endogenous information systems.

This is not to say that the outcomes of arbitrary programming problems are necessarily sharp or well understand. It turns out to be difficult to characterize optimal consumption schedules in the classic principal-agent problem; without demanding assumptions, these schedules need not be even weakly monotone increasing. The problem of when to monitor or audit and acquire otherwise private information is also difficult analytically. Multiperiod private information problems deliver history dependence and intertemporal tie-ins, but the exact nature of these tie-ins and the extent of dependence remain open questions, especially for relatively long horizons with discounting of the future. Finally, little is known about the solutions to multilateral private-information problems, especially in multiperiod contexts. Generally, then, the solutions of many private-information programs are not neatly characterized and are sensitive to the nature of preferences, technology, and uncertainty.

Still, the potential of private-information programs to deliver interesting outcomes and interesting institutions cannot be ignored. The remainder of this section tries to establish this by consideration of three models from recent literature.

Private-information economy 6.1

First, imagine with Prescott and Townsend (1984b) an economy in which each of a continuum of households can supply its own labor to a household-specific production function, its own unobserved "back-yard technology," yielding output of an idiosyncratic nontraded consumption good contingent on some production shock $\theta \in \{1,2\}$. Each household can also supply labor to the public production of an observed transferable consumption good, and each household has its own linear technology for this production of the form $q = ba$, where variable a is market labor supply to its project and coefficient b is positive. Thus each household has an induced or indirect shock-contingent utility function over units of consumption of the publicly produced good c and units of its own market labor assignment a of the form $U_\theta(c,a) = U(c) - W_\theta(a)$, where (for simplicity) $0 \le c$ and $0 \le a \le \bar{l}$. Fraction ω_θ of agents in the population receive the household production shock θ, while in the planning period each household views shock θ as occurring with probability ω_θ.

Suppose now that, in the planning period, households enter into an agreement to receive the market consumption good in amount c and supply labor in amount a contingent on some announced value of the privately observed shock θ. That is, households can enter into a contract with individually effected options θ. Further, suppose for simplicity that consumption c and labor supply a can take on at most a finite number of values, and that these can be determined at random, so that whether or not a household works (and how much it works) are determined under a well-specified lottery. Thus, let $\pi_\theta(c,a)$ denote the probability of getting market consumption bundle c and supplying labor to the market in amount a. Then, the appropriate programming problem for the representative consumer in the planning period is the following.

Program 6.2. Maximize by choice of the $\pi_\theta(c,a)$ the objective function

$$\sum_\theta \omega_\theta \sum_{(c,a)} \pi_\theta(c,a) U_\theta(c,a), \tag{6.8}$$

subject to the resource constraints

$$\sum_\theta \omega_\theta \sum_{(c,a)} \pi_\theta(c,a) ba \ge \sum_\theta \omega_\theta \sum_{(c,a)} \pi_\theta(c,a) c; \tag{6.9}$$

subject to the incentive constraints

$$\sum_{(c,a)} \pi_1(c,a) U_1(c,a) \ge \sum_{(c,a)} \pi_2(c,a) U_1(c,a), \tag{6.10}$$

$$\sum_{(c,a)} \pi_2(c,a) U_2(c,a) \ge \sum_{(c,a)} \pi_1(c,a) U_2(c,a); \tag{6.11}$$

and subject to the obvious constraints on probabilities

$$\sum_{(c,a)} \pi_\theta(c,a) = 1, \quad \pi_\theta(c,a) \ge 0, \quad \theta \in \{1,2\}. \tag{6.12}$$

Note that here the objects $\pi_\theta(c,a)$ enter into the objective function (6.8) as probabilities of bundle (c,a) if θ is announced, and appear in the resource constraint (6.9) as fractions of type-θ agents obtaining bundle (c,a), so that average output is no less than average consumption.

Solutions to Program 6.2 can be striking. To see this, imagine an extreme case with $U(c)=\ln(c)$, $W_1(a)=a$, and $W_2(a)=a^2/3$, so that households of type $\theta=1$ are ex post neutral in labor supply although households of type $\theta=2$ are not. The full-information solution to the programming problem for the determination of Pareto optimal allocations – that is, the solution with constraints (6.10) and (6.11) deleted – is $c_1=c_2=b$, $a_1=1/2$, and $a_2=3/2$ if $\omega_1=\omega_2=1/2$. Of course, with private information this is not obtainable because consumptions are equated while type-2 households work harder. But randomness can be introduced into the labor supply of type 1, preventing type-2 households from claiming to be type 1, with no effect on the ex ante utility of type-1 households. In fact, set $c_1=c_2=b$ as before; set $a_1=\bar{\ell}$, the upper bound, with probability α; and set $a_1=0$, the lower bound, with probability $(1-\alpha)$, so that expected labor supply satisfies $\alpha\bar{\ell}+(1-\alpha)0=1/2$. This satisfies all incentive constraints for $\bar{\ell}$ sufficiently large. Less dramatic examples with lotteries can be constructed with all agents risk-averse, but with a loss of ex ante expected utility relative to the full-information solution.

Indeed, we are reminded – in this discussion of lotteries and diverse employment experience for otherwise identical households – of what might happen in a market economy with excess labor supply and sticky wages. For this and other reasons, it is interesting to ask if the private-information economy just described can be decentralized; that is: Is there a market that would support the optimum? The affirmative answer given in Prescott and Townsend (1985b) is summarized here.

Suppose that, prior to the resolution of uncertainty (i.e., in the planning-period market), each household as a firm or intermediary can make commitments to the market to supply any number of (c,a) pairs. More formally, let $y_\theta(c,a)$, if positive, denote the number of commitments to provide c units of output for and to hire a units of labor from the group of households who announce they are of type θ (such announcements are public). Each firm–intermediary is constrained by a production–intermediation set

$$Y=\left\{y=\{y_\theta(c,a)\}:\ \sum_\theta \omega_\theta \sum_{(c,a)} y_\theta(c,a)(ba-c)\geq 0\right\},$$

which states in effect that the firm–intermediary cannot plan to distribute more of the consumption good than it provides on average. The coefficients in set Y are taken as given. Also, because this production set is the

same over all households and displays constant returns to scale, there is no loss in simply positing set Y as the aggregate production–intermediation set.

Each firm–intermediary acts to maximize profits from competitive markets, taking prices as given. Thus, let $p_\theta(c,a)$ be the per-unit price of a $y_\theta(c,a)$ commitment in terms of some abstract unit of account. Then the firm–intermediary acts to maximize

$$\sum_\theta \sum_{(c,a)} y_\theta(c,a)\,p_\theta(c,a)$$

subject to $y\in Y$.

Households also make market commitments over output labor pairs, but with a different interpretation. Households are imagined to choose ex ante in the planning-period market a contract with options, indexed by $\theta\in\Theta$. Each option is a (possibly degenerate) lottery over consumption and labor-supply pairs, and the household can choose the terms of the lottery as well. But the options are such that a household of type θ ex post will choose ex post the option indexed by θ if indeed it suffers shock θ. Finally, of course, all the contracts are priced in a competitive market. More formally, motivated by the discussion above, let the common consumption set be

$$X=\Big\{\{x_\theta(c,a)\}:x_\theta(c,a)\geq 0,\ \sum_{c,a}x_\theta(c,a)=1,$$
$$\sum_{c,a}x_\theta(c,a)\,U_\theta(c,a)\geq \sum_{c,a}x_\phi(c,a)\,U_\theta(c,a),\ \theta,\psi\in\Theta\Big\}.$$

Let the endowment w be the element $w_\theta(c,a)=0$ unless $(c,a)=(0,0)$ for each $\theta\in\Theta$; that is, w puts all mass on the zero point of the underlying commodity space. Then the objective of the representative consumer is to maximize ex ante expected utility

$$\sum_\theta \omega_\theta \sum_{(c,a)} x_\theta(c,a)\,U_\theta(c,a)$$

by choosing $x\in X$ in the budget set

$$\sum_\theta \sum_{(c,a)} p_\theta(c,a)x_\theta(c,a)\leq \sum_\theta \sum_{(c,a)} p_\theta(c,a)\,w_\theta(c,a).$$

To reiterate, each household purchases a contract as a package and does not purchase the components separately. But the valuation of a contract is determined by the valuation of its individual components. In particular, with constant returns to scale in production, individual components in equilibrium must satisfy

$$p^*_\theta(c,a)=\omega_\theta(c-ba)\quad \forall\theta\in\Theta,\ \text{all }(c,a)\text{ pairs}.$$

To ensure consistency of actions taken by households and the firm–intermediaries, the usual market clearing condition is needed:

$$x_\theta(c,a) = y_\theta(c,a) + w_\theta(c,a).$$

This condition, when substituted into the production-intermediation set Y, yields the resource constraint (6.9).

A standard competitive equilibrium (as defined in Section 1) can be shown to exist and to be Pareto optimal. Here, in fact, the representative-consumer construct facilitates the argument. But the argument can be generalized considerably, as in Prescott and Townsend (1984b), for – as is now apparent – preferences are linear in lotteries, the consumption set and production set are defined by a finite set of linear inequality constraints, and so on. The set of sufficient assumptions is drawn from Debreu (1962), and the argument is given explicitly in Section 7 in an extension to indivisibilities. Further, the discrete commodity space can be filled into a continuum, following arguments like those of Bewley (1972) and Mas-Colell (1975) as well as Prescott and Townsend (1984b). The general class of private-information environments that allows standard existence and optimality theorems is given in Prescott and Townsend (1984a). Of course, not every private-information environment can be decentralized in the standard way, as might well be anticipated from the earlier work of Rothschild and Stiglitz (1976), Spence (1974), and Wilson (1978).

Private-information economy 6.2

As a second private-information economy, imagine with Ito (1984) that each of a continuum of households can supply one unit of labor either to an unobserved back-yard technology or to a publicly observed central technology. In particular, each agent of type j can produce $a_j + \epsilon$ units of the single consumption good of the model from one unit of labor supplied to its back-yard production technology, where a_j is private information to agent j and ϵ is a privately observed random variable uniformly distributed on some closed interval. Alternatively, each agent of type j can produce $m(g)$ or $m(b)$ units of the single transferable consumption good in the central public technology, depending on the state of nature s: either good, $s = g$; or bad, $s = b$. Public production $m(s)$ occurs with probability $p(s)$, $s = g, b$. Virtually all households are risk-averse, with a common strictly concave utility function $U(c)$ over units of consumption c. Labor is supplied inelastically. The fraction of agents of type j is ω_j, and there are n agent types, so that $j = 1, 2, \ldots, n$. Further, households are ordered in private productivities so that $a_1 < a_2 < \cdots < a_n$. The excep-

tion is one household with mass ω_0, say of type 0, who is risk-neutral and who supplies no labor to any technology.

All households are imagined to be gathered initially at the central public technology, and all decide on a common resource allocation scheme. Next, all households make declarations of their type. Next, all observe the public technology shock s. Then a decision is made in some way, perhaps in accord with prior plans, as to whether or not each agent of (declared) type j is to discover the ϵ-draw in its back-yard technology. The cost of this search to each agent is $z > 0$ units of consumption. Finally, with state s public and a_j and ϵ private, each household of type j is allocated in some way to one of the two available technologies.

To simplify the presentation of a programming problem, suppose initially that the ϵ-draws are all degenerate at zero and that this is common knowledge. Then the only issue is whether a type-j agent does or does not go to his back-yard technology in states s, since there is no point in returning to the public technology. Thus, let $w_j(s)$ denote the units of consumption wage for agent j from the public technology in state s, if indeed agent j is assigned to that technology; let $c_j(s)$ denote the compensation from the public technology (gross of search costs) in the event that agent j is assigned to his back-yard technology in state s; and let $\chi_j(s)$ denote an assignment function that is either one or zero depending on whether agent j is or is not assigned to the public technology in state s. [More generally, $\chi_j(s)$ could be interpreted as a population fraction for the resource constraint.] For simplicity in notation, let $w_0(s)$ denote the consumption of agent 0 with $\chi_0(s) \equiv 1$.

With this notation, the planning-period problem for the determination of a Pareto optimal allocation, with weights λ^j to agents of type j, is equivalent to the following.

Program 6.3. *Maximize by choice of the $w_j(s)$, $c_j(s)$, and $\chi_j(s)$ the objective function*

$$\sum_{j=0}^{n} \lambda^j \left\{ \sum_s p(s)[\chi_j(s) U[w_j(s)] + (1-\chi_j(s)) U[c_j(s) + a_j - z]] \right\} \tag{6.13}$$

subject to the resource constraints

$$\sum_{j=1}^{n} \chi_j(s) m(s) \omega_j \geq \sum_{j=1}^{n} (1-\chi_j(s)) c_j(s) \omega_j + \sum_{j=1}^{n} \chi_j(s) w_j(s) \omega_j + \omega_0 w_0(s), \qquad s = g, b \tag{6.14}$$

and subject to the incentive constraints

$$\sum_s p(s)[\chi_j(s)U[w_j(s)]+(1-\chi_j(s))U[c_j(s)+a_j-z]]$$
$$\geq \sum_s p(s)[\chi_i(s)U[w_i(s)]+(1-\chi_i(s))U[c_i(s)+a_j-z]]. \quad (6.15)$$

Ito (1984) examines a particular but intuitively appealing solution to this programming problem. In particular, suppose the risk-neutral agent is to receive zero (expected) consumption; suppose we ignore momentarily the incentive constraints; and suppose the economy is such that no one is to leave the public technology in the good state $s=g$. Then, for type-j agents who are to stay with the public technology in the bad state $s=b$, $w(g)=w(b)$ where for these agents $w(\cdot)$ is independent of j. Similarly, for those type-j agents who are to leave the public technology in the bad state $s=b$, $w_j(g)=c_j(b)+a_j-z$. Thus there is full insurance (constant consumption) over states s $(s=g,b)$ for each agent type, although there may be nontrivial variation over types j for j types who leave. Further, as one might hope with the presence of a risk-neutral agent, production assignments are efficient in the sense that agents leave the public project when they are relatively inefficient there; that is, $\chi_j(b)=0$ if $m(b)<a_j-z$. Indeed, individual productivities are supposed to be such that there exists some nontrivial critical value of the private production parameter a, say a^*, with $\chi_j(b)=0$ for all $a_j>a^*$ and $\chi_j(b)=1$ for $a_k\leq a^*$ (a^* nontrivial in that such a k exists). Finally, the incentive constraints can be satisfied by making the wage schedule in the good state, $w_j(g)$, monotone increasing in j and by making the compensation schedule in the bad state, $c_j(b)$, monotone decreasing in j for all j such that $\chi_j(b)=0$. The idea here is that for agent j to pretend to be a type i rather than type j with $i>j$, there is relatively higher consumption when $s=g$ [by the monotonicity of $w_j(g)$] but relatively lower compensation and hence lower consumption when $s=b$; that is, $c_i(b)+a_j-z<c_j(b)+a_j-z$. The consequent risk keeps agent j from pretending to be a type i, $i>j$. And a similar argument applies for type i, $i<j$. Indeed, Ito finds consumption and compensation schedules by decentralizing the economy, letting agents choose actuarily fair insurance ex ante, and letting agents make their own state-contingent decisions about whether or not to search and therefore what compensation to claim given their choice of an insurance package.

Ito goes on to consider an even more interesting case. Suppose in particular that the ϵ distribution is not degenerate. Then, upon seeing $a_j+\epsilon$ at cost z, agent j must decide whether or not to return to the public technology. Of course, with low ϵ's there may be a return and with high ϵ's not. But a returning agent cannot effectively claim a *particular* ϵ value, and nonreturning agents are similarly uninsured. Indeed, under his decentralization hypothesis, this lack of insurance delivers for Ito an allocation $w_j(g)$ on state $s=g$ greater than the compensation for returning workers of type j on state $s=b$. Thus there is no longer full insurance. More dramatically, relative to the case where ϵ-draws are fully observed by everyone, there are fewer agent types searching their back-yard technologies (the critical a^* is higher), and also those who search more are likely to return to the public technology. Indeed, this effect is reinforced if agents can pretend to search, returning to the public technology to receive a wage different from the one they would have received had they not "searched" at all. Ito terms the latter "phantom" search, in contrast to the case where the act of search is public.

It remains to be seen if solutions like these emerge generally for alternative weights λ^j in the programming problem. And some check should be made to see if the assumed decentralization of decisions is innocuous, especially when search is public and can be imposed or assigned. However, based on Ito's results thus far, it seems that private information can generate some kind of persistence.

Private-information economy 6.3

A third private-information economy helps make the point that private information can be an important determinant of institutions and economywide arrangements. Thus, consider with Boyd and Prescott (1986) an economy with a countable infinity of agents, each of whom lives two periods. In the first period, each agent has one unit of time and one investment project. In the second period, agents eat the single consumption good of the model. Time can be used in the first period in some investment project to help produce the consumption good for the second period, or time can be used in the first period to evaluate one project. Preferences are ordered in the first period by expected second-period consumption. The per-unit return r on an investment project if funded is either good (i.e., $r=g$) or bad (i.e., $r=b$), where of course $g>b$. The maximal investment per project is χ. Evaluation e of a project in the first period generates signal $e=g$ or $e=b$ in the first period, and these are (imperfectly) predictive of future return r on the project. Agents are ordered initially by types i, $i=g$ or $i=b$, and an agent's type i is also (imperfectly) predictive of the future returns r on his project. Here, then, $\pi(i,e,r)$ denotes the fraction of agents of type i who would receive signal e if their projects were evaluated and obtain return r if their projects were funded, and $\pi(i,e,r)$ also determines the conditional probability of the event (e,r) from the viewpoint of an individual agent of type i. Each agent's type is presumed to be private to the individual. On the other hand, the acts of evaluating and investing are supposed to be publicly observed, as are the returns r, evaluations e, and consumptions c. Finally, the return r on a project is supposed to be known ex post even if the project is not funded.

To write down a programming problem for the determination of Pareto optimal arrangements, one needs a little additional notation. Thus, let z_i denote the fraction of (declared) type-i projects evaluated, as well as the probability of an evaluation for an agent of (declared) type i; x_i the amount of time invested in a (declared) type-i project if not evaluated; x_{ie} the amount of time invested in a (declared) type-i project with evaluation e; c_{ir} the amount of consumption to a (declared) type-i agent when his own project is not evaluated and the return on it is r; and c_{ier} the amount of consumption to a (declared) type-i agent with evaluation e and return r on his project. Finally, let z, x, and c denote the associated vectors with components specifying these objects. Then, given z, x, and c, the expected utility of a type-i agent who reports to be of type j is

$$U_i(c,z,j) = z_j E_{e,r}\{c_{jer} \mid i\} + (1-z_j)E_r\{c_{jr} \mid i\}, \tag{6.16}$$

where expectations are over evaluations e and returns r given actual agent type i, and again the probabilities are determined by the $\pi(i,e,r)$. The programming problem with weights λ^j, $j=g,b$, can then be written as follows.

Program 6.4. Maximize by choice of z, x, and c the objective function

$$\lambda^g U_{i=g}(c,z,j=g) + \lambda^b U_{i=b}(c,z,j=b), \tag{6.17}$$

subject to the time constraint for investing and evaluating in per-capita terms,

$$E_{i,e}\{(1-z_i)x_i + z_i(x_{ie}+1)\} \le 1; \tag{6.18}$$

the resource constraint for distribution of the consumption good,

$$E_i\{z_i E_{e,r}\{c_{ier} \mid i\} + (1-z_i)E_r\{c_{ir} \mid i\}\}$$
$$\le E_i\{z_i E_{e,r}\{rx_{ie} \mid i\} + (1-z_i)E_r\{rx_i \mid i\}\}; \tag{6.19}$$

the incentive constraints,

$$U_i(c,z,j=i) \ge U_i(c,z,j\ne i) \quad \textit{all } i,j; \tag{6.20}$$

individual rationality constraints on participation,

$$U_i(c,z,j=i) \ge E_r\{r \mid i\}; \tag{6.21}$$

and the constraint that

$$0 \le z_i \le 1 \quad \textit{all } i,$$
$$0 \le x_i, x_{ie} < \chi \quad \textit{all } i,e. \tag{6.22}$$

Boyd and Prescott (1986) go on to define a core allocation as an allocation that cannot be blocked by a coalition. They also assume that the specification of parameters χ, g, b, and π are such that: (1) if agent types were fully known there would be evaluations of good projects; (2) there would not be evaluations of bad projects; (3) investing in $i=b$ projects dominates investing in $i=g$ projects with $e=b$; and (4) if all types of $i=g$ projects are evaluated and are funded or not at maximum level χ under the above rule then some time would remain, so that bad projects are the marginal projects. Under these assumptions there exists an (essentially) unique core allocation, one that is weighted in favor of the agents of type $i=g$.

Boyd and Prescott (1986) characterize this core allocation. They show, for example, that if evaluations e are uninformative but agent types i are private information then there can be nontrivial evaluations. Of course, these evaluations are dissipative in the sense that with full information no evaluations would take place. Even more impressive, though, are results characterizing the arrangements themselves. In particular, Boyd and Prescott show that – under their definitions – core allocations can be achieved by nontrivial coalitions of agents, whereas core allocations cannot be supported in a decentralized security market arrangement, at least not if there is private information on initial agent types and if evaluations are possible. In this sense, then, their paper delivers some kind of collective or centralized institutions, which Boyd and Prescott interpret as banks, adding to our understanding of such institutions and to the related papers of Diamond (1984) and Townsend (1978, 1983), for example.

Though they may seem alien at first, various kinds of collective or restrictive resource-allocation schemes have been observed in practice. In the Pacific island economies studied by anthropologist Malinowski, for example, only the chief was allowed to store many essential commodities, though storage displays were themselves public. Similarly, in the centralized societies of the Egyptian pharaohs there seemed to have been extensive restrictions on trade and retrade. The private-information theories described in this section are at least suggestive of these kinds of institutions.

Thus far, little has been directly said about how to tie this section's private-information programs to time-series observations and more standard macroeconomic concerns. The problem, again, is that multiperiod private-information programs are difficult to solve generally. However, progress is being made. Indeed, Green (1985) has solved an explicit infinite-horizon version of a multiperiod private-information model, at least with the assumptions of constant absolute risk aversion and unbounded negative consumption. Such contributions provide important links from the microeconomic contract-theoretic literature to the macroeconomic literature on growth and fluctuations, and support the broad view of macroeconomics adopted in this chapter.

7 Programs with indivisibilities

Observations from primitive economies suggest that certain indivisibilities may be crucial. The oxen of the medieval village can be utilized only in discrete units, and the canoes of the Polynesian islands come only in minimal sizes. Similarly, a migrant worker either does or does not labor at a specified location or project. Fortunately, an Arrow–Debreu environment with such indivisibilities can be analyzed along the lines of Prescott and Townsend (1984a, b) and the general equilibrium literature described above, so that standard existence and optimality theorems apply. Further, environments with indivisibilities can be shown in this way to have interesting dynamic properties.

For purposes of exposition we shall consider here an extended, generalized version of the environment considered by G. Hansen (1985). That environment is much like the one that underlies the neoclassical growth model (Program 5.1), but with the following modifications. First, there is a continuum of type-j households, fraction ω^j of the total set of households, with the latter being the set of households whose names lie on the unit interval. Names will be ignored in what follows, however, and type-j households will be treated identically ex ante, as if there were one type-j agent with weight ω^j. More relevant for the present discussion of indivisibilities, pair (c, a) of consumption c and labor supply a can take on at most a finite number of values for each and every household, and (for simplicity) consumption sets X_t^j are identical across households j. Thus, from the point of view of household j as consumer, the commodity point will be a specification of the probability $x^j(c, a)$ of the pair (c, a) over all pairs (c, a) in some set $\bar{X}$.

With these modifications, and the obvious adjustments for converting leisure to labor, Program 5.1 becomes the following.

Program 7.1. Maximize by choice of the $x_t^j(c, a, \epsilon_1, \ldots, \epsilon_t)$ *and the* $\bar{i}_t(\epsilon_1, \ldots, \epsilon_t)$ *the objective function*

$$\sum_{j=1}^{n} \lambda^j \left\{ E_0 \sum_{t=1}^{T} \beta^t \left\{ \sum_{(c,a)} x_t^j(c, a, \epsilon_1, \ldots, \epsilon_t)[U^j(c) - W^j(a)] \right\} \right\} \tag{7.1}$$

subject to

$$\sum_{j=1}^{n} \omega^j \sum_{(c,a)} c x_t^j(c, a, \epsilon_1, \ldots, \epsilon_t) \le \bar{c}_t(\epsilon_1, \ldots, \epsilon_t), \tag{7.2}$$

$$\sum_{j=1}^{n} \omega^j \sum_{(c,a)} a x_t^j(c, a, \epsilon_1, \ldots, \epsilon_t) \le \bar{a}_t(\epsilon_1, \ldots, \epsilon_t), \tag{7.3}$$

$$\bar{i}_t(\epsilon_1, \ldots, \epsilon_t) + \bar{c}_t(\epsilon_1, \ldots, \epsilon_t) \le f[\bar{a}_t(\epsilon_1, \ldots, \epsilon_t), \bar{K}_t(\epsilon_1, \ldots, \epsilon_t), \epsilon_t], \tag{7.4}$$

$$\bar{K}_{t+1}(\epsilon_1, \ldots, \epsilon_t) = (1-\delta)[\bar{K}_t(\epsilon_1, \ldots, \epsilon_{t-1}) + \bar{i}_t(\epsilon_1, \ldots, \epsilon_t)], \tag{7.5}$$

$$0 \le x_t^j(c, a, \epsilon_1, \ldots, \epsilon_t) \le 1 \quad \sum_{c,a} x_t^j(c, a, \epsilon_1, \ldots, \epsilon_t) = 1. \tag{7.6}$$

This is a concave program despite the indivisibilities. Further, it can be solved analytically. Hansen does so by assuming one type of consumer, so that the j notation can be suppressed; labor supply a taking on only two values, (say) $a = 0$ and $a = \bar{a}$; and consumption taking on a continuum of values. In this case, one is reduced to the representative-consumer problem of the standard neoclassical growth model, with the exception here that (say) $\alpha_t(\epsilon_1, \ldots, \epsilon_t)$, the fraction of agents who work $\bar{a}$ hours, replaces $\bar{\ell}_t(\epsilon_1, \ldots, \epsilon_t)$ in Program 5.1 as the obvious labor choice variable at date t under history $(\epsilon_1, \ldots, \epsilon_t)$.

We might note in passing that Hansen's model is designed to address the anomalies raised in the earlier work of Kydland and Prescott (1982) – that Program 5.1 delivers too little variability of hours worked relative to U.S. postwar time-series data. And Hansen is successful, inasmuch as his version of Program 7.1 delivers too much variability in hours. That is, Hansen does seem to have selected a key aspect of the model to modify. It might also be stressed that the variability is induced by the indivisibility, a "real rigidity," and that nominal rigidities so often believed to be necessary are, in fact, not needed (which is not to say that nominal rigidities are unimportant). Similarly, lotteries transform the underlying preferences of consumers to some mongrel objective function (7.1), and measured risk aversion for the mongrel may bear little relation to actual risk aversion for underlying preferences.

It should be stressed also that existence and optimality results follow as in Prescott and Townsend (1984b), despite the indivisibilities. The common consumption set of type-j households is the set of possible lotteries over consumption c (measured in positive units) and labor supply a (measured in negative units) in underlying consumption set $\bar{X}$ over all dates and histories; that is,

$$X=\left\{\{x_t(c,a,\epsilon_1,\dots,\epsilon_t)\}:\\ 0\le x_t(c,a,\epsilon_1,\dots,\epsilon_t)\le 1,\ \sum_{c,a} x_t(c,a,\epsilon_1,\dots,\epsilon_t)=1\right\}.$$

Underlying consumption set $\bar{X}$ is presumed to contain the point $(0,-\bar{\ell})$ of zero consumption and maximal labor supply for each household j. Set $\bar{X}$ also contains the endowment point of each household, the point $(0,0)$. Further, as is apparent from Program 7.1, one can trace out the path of maximal aggregate feasible consumption - say, $\bar{e}_t(\epsilon_1,\dots,\epsilon_t)$ at date t and contingent on history $(\epsilon_1,\dots,\epsilon_t)$ - by supposing that aggregate labor supply is at a maximum at each date, investment is arranged to support maximal carryover earlier, and so on. Next, define

$$\bar{e}_t^*(\epsilon_1,\dots,\epsilon_t)=\max_i\left[\frac{\bar{e}_t(\epsilon_1,\dots,\epsilon_t)}{\omega^i}\right]$$

and

$$\bar{e}^{**}=\max_{t,(\epsilon_1,\dots,\epsilon_t)}[\bar{e}_t^*(\epsilon_1,\dots,\epsilon_t)].$$

Then some consumption number $c^{**}>\bar{e}^{**}$ along with zero labor supply is supposed also to be in underlying consumption set $\bar{X}$. This will ensure that no household j is ever satiated in its attainable consumption set $\hat{X}$; that is, there is always a dominating element in consumption set X even though set X itself is bounded.

Preferences $u^j(x^j)$ over set X are defined in the obvious way, as is the endowment w^j, the latter putting all mass on the underlying endowment bundle $(c,a)=(0,0)$ uniformly for all dates and histories. Household j as producer makes $y_t(c,a,\epsilon_1,\dots,\epsilon_t)$ commitments if positive to deliver c units of output for consumption (measured in positive units), and to hire a units of labor (measured in negative units) at date t, contingent on history $(\epsilon_1,\dots,\epsilon_t)$, as well as deciding as before on components $y_{2t}(\epsilon_1,\dots,\epsilon_t)$, $y_{3t}(\epsilon_1,\dots,\epsilon_t)$, and $y_{4t}(\epsilon_1,\dots,\epsilon_t)$ for investment, beginning-of-period capital, and end-of-period capital. That is,

$$Y_t^{1j}(\epsilon_1,\dots,\epsilon_t)=\left\{y_t(\epsilon_1,\dots,\epsilon_t):\sum_{c,a} y(c,a,\epsilon_1,\dots,\epsilon_t)c+y_{2t}(\epsilon_1,\dots,\epsilon_t)\right.\\ \le f^j\left[\sum_{c,a} y(c,a,\epsilon_1,\dots,\epsilon_t)a,\ y_{3t}(\epsilon_1,\dots,\epsilon_t),\ \epsilon_t\right]\text{ and}\\ \left. y_{4t}(\epsilon_1,\dots,\epsilon_t)\le -y_{3t}(\epsilon_1,\dots,\epsilon_t)\right\}$$

with $Y_t^{2j}(\epsilon_1,\dots,\epsilon_t)$ exactly as before in Section 5. Finally, it may be supposed that each of the fractions ω^j is a rational number, so that some integer m may be taken to be a least common denominator. Then the continuum-agent economy is equivalent to a finite-agent economy with number $\omega^j m$ agents of type j.

One can now make use of the theorem in Debreu (1962): The finite-agent economy has a quasi-equilibrium (defined below) if (a.1) $A(mX)\cap(-A(mX))=\{0\}$; (a.2) X is closed and convex; (b.1) for every j and for every consumption x' in $\hat{X}$, there is a consumption in X preferred by j to x'; (b.2) for every $x'\in X$ the sets $\{x^j\in X: u^j(x^j)\ge u^j(x')\}$ and $\{x^j\in X: u^j(x^j)\le u^j(x')\}$ are closed in X; (b.3) for every x' in X, the set $\{x^j\in X: u^j(x^j)\ge u^j(x')\}$ is convex; (c.1) $(\{mw^j\}+mY)\cap mX\ne\varnothing$; (c.2) $(\{w^j\}+A(Y))\cap X\ne 0$; (d.1) $0\in Y^j$; and (d.2) $A(mX)\cap A(Y)=\{0\}$. Here $A(S)$ denotes the asymptotic cone of set S. Each of these conditions can be verified to hold for the economy with indivisibilities under consideration, just as in Prescott and Townsend (1984b). One key, of course, is that the consumption set is defined by a set of linear inequalities and that preferences are linear in lotteries.

Here a *quasi-equilibrium* is a state $[(x^{j^*})(y^{j^*})]$ and a price system p^* such that

(α) for every j, x^{j^*} is a greatest element in $\{x\in X: p^*\cdot x\le p^*\cdot w^j+p^*\cdot y^{j^*}\}$ under $u^j(x)$ and/or $p^*\cdot x^{j^*}=p^*\cdot w^j+p^*\cdot y^{j^*}=\text{Min}\, p^*\cdot X$;
(β) $p^*\cdot y^{j^*}=\text{Max}\, p^*\cdot Y^j$;
(γ) $\sum_j m\omega^j x^{j^*}=\sum_j m\omega^j y^{j^*}+\sum_j m\omega^j w^j$; and
(δ) $p^*\ne 0$.

Thus a quasi-equilibrium can differ from a standard competitive equilibrium in condition (α). Here, however, the configuration of equilibrium prices p^* can be delivered from profit-maximization condition (β) on the aggregate production set Y; in particular, the price $p_t^*(c,a,\epsilon_1,\dots,\epsilon_t)$ for the (c,a) component at date t under history $(\epsilon_1,\dots,\epsilon_t)$ satisfies

$$p_t^*(c,a,\epsilon_1,\dots,\epsilon_t)=\psi_t^1(\epsilon_1,\dots,\epsilon_t)\left[c-\frac{\partial f(\cdot,\cdot,\epsilon_t)}{\partial a}a\right],$$

where $\psi_t^1(\epsilon_1,\dots,\epsilon_t)$ is the Lagrange multiplier of the aggregate production constraint $Y_t^1(\epsilon_1,\dots,\epsilon_t)$ and its associated aggregate production function $f(\cdot,\cdot,\epsilon)$. If all the $\psi_t^1(\epsilon_1,\dots,\epsilon_t)$ were zero then all $p_t^*(c,a,\epsilon_1,\dots,\epsilon_t)$ would be zero, and this would imply from condition (β) that $p^*=0$, in contradiction to condition (δ). Thus at least one $\psi_t^1(\epsilon_1,\dots,\epsilon_t)$ is positive and this makes $p^*w^j+p^*\cdot y^{j^*}>\text{Min}\, p^*\cdot X$ because the point $x\in X$, putting mass unity on point $(0,-\bar{\ell})$ for all t and $(\epsilon_1,\dots,\epsilon_t)$, has negative valuation. Thus the first part of conditions (α) must prevail, and there exists a standard competitive equilibrium.

That competitive equilibria are necessarily Pareto optimal and that Pareto optima can be supported as competitive equilibria also follow directly from the arguments in Prescott and Townsend (1984b) and in Debreu (1954).

In summary, then, lotteries facilitate the analysis of economies with some underlying nonconvexity, here delivering existence and optimality theorems. And they help in this regard even when they are not needed - that is, even when lotteries are not actually used at an optimum or in equilibrium.

8 Programs with spatial separation

The Polynesian economy studied by Firth (1939) and the hill tribe communities studied by Van Roy (1971) have prominent aspects of spatial separation, with clusters of population separated by nontrivial distances. Still, these clusters are not isolated from one another. In the island economy studied by Firth (1939), agents occasionally migrate in order to engage in group canoe-building projects or to work in dispersed orchards. And in the economy studied by Van Roy (1971), there is (as noted earlier) sharing of rice across villages and occasional migration of some laborers from one village to another. For these environments, then, we might not want to look at individual consumption relative to aggregate consumption or individual leisure relative to aggregate leisure, at least not without taking into account some aspects of this spatial separation.

But how are we to do this? It would seem natural and instructive to answer this question first in the context of a simple pure-exchange economy of the type described at the beginning of previous sections, but modified here to accommodate a minimal amount of movement of agents across space.

Thus consider an economy with two locations, two dates, and four agents, as described in Table 1. Here agents a and a' reside in locations 1 and 2 (respectively) for both dates, while agents b and b' reside in locations 1 and 2 (respectively) at the first date, and for unspecified exogenous reasons switch locations at the beginning of the second date. Agents are presumed to have endowments and preferences over consumption goods in each location where they happen to be.

One could write down a concave programming problem for the determination of a Pareto optimum for this spatial economy, much like Program 2.1 except that here there would be a resource constraint not only for each date and history but also for each location. Thus one would distribute the consumption good at each particular location among all participants at that location in such a way as to equate weighted marginal utilities. Individual consumptions thus would vary positively (weakly)

Table 1. *Agent pairings in a spatial economy*

	loc 1	loc 2
$t=1$	(a, b)	(a', b')
$t=2$	(a, b')	(a', b)

with location-specific aggregates at a point in time over shock realizations, and - to the extent that a population remained unaltered over subsets of dates at any location - individual consumptions would vary positively (weakly) with location-specific aggregates over time. But the distribution of the consumption good would be sensitive, generally, to the population mix at any given location, making the implications of the theory more difficult to test. (Of course, conclusions like this would hold even if entire groups of a given population were to move about exogenously.)

A still more elaborate treatment of spatial separation (required in a serious application) would recognize that location choices are endogenous, that individuals are capable of supplying labor in any location they might choose, and that consumption may be transferable. This raises a potential nonconvexity problem, but fortunately the problem can be solved. This is illustrated well by a model of R. Rogerson (1985), and it will be described in this section.

So, imagine with Rogerson (1985) an economy with two sectors or locations. In each location i, $i=1,2$, production can take place according to the familiar neoclassical production function

$$q_{it}=f_i(a_{it}, K_i, \epsilon_t). \tag{8.1}$$

Here a_{it} is aggregate man-hours supplied in location i at date t measured in positive units; K_i is the stock of an immobile capital in location i, the same at any date t; ϵ_t is a common date-t shock; and so on. However, capital K_i plays no role in the analysis, and it is hereby suppressed from the notation. There is a continuum of workers, again with names on $[0,1]$, and each worker has an endowment of $\bar{\ell}$ units of leisure at each date. If a representative worker were to supply a_{1t} hours in location 1 at date t and a_{2t} hours in location 2 at date t, as well as to receive c_t units of consumption, then his utility (with the exception noted below) would be

$$U[c_t, a_{1t}, a_{2t}]=c_t+V[\bar{\ell}-(a_{1t}+a_{2t})]. \tag{8.2}$$

However, only one of either a_{1t} or a_{2t} can be positive, and this is the nonconvexity. Further, there is an assumed psychic cost to moving, so that if a worker in location 1_{t-1} at date $t-1$ is allocated to sector 1_t at date t with

$1_{t-1} \neq 1_t$, then a positive term m is subtracted from the utility function in (8.2). This induces a location-contingent utility function

$$\tilde{U}(c_t, a_t, 1_t \mid 1_{t-1}) = \begin{cases} c_t + V[\bar{\ell} - a_t] & \text{if } 1_t = 1_{t-1}, \\ c_t + V[\bar{\ell} - a_t] - m & \text{if } 1_t \neq 1_{t-1}, \end{cases} \tag{8.3}$$

where it is understood that consumption c_t and labor a_t take place at location 1_t at date t, that consumption is transferable across locations, but that labor supply is not transferable. Finally, at the first date $t=1$ the upper branch of the utility function applies, and the previous location conditioning element is suppressed from the notation.

Rogerson overcomes the nonconvexity problem by letting the location assignment of each individual be determined in a lottery. In particular, adopting notation different from Rogerson's, let $\pi_1(c_1, a_1, k, \epsilon_1)$ be the probability that an individual at date 1 and contingent on shock ϵ_1 is assigned location k, $k = 1, 2$, receives consumption in amount c_1, and supplies labor in amount a_1. Here, for simplicity of notation and analysis, both c_1 and a_1 are assumed to take on at most a finite number of values, though it may be noted that, in a Pareto optimum with a continuum of values, lotteries on c_1 and a_1 would be degenerate; that is, c_1 and a_1 would be deterministic up to ϵ_1. Indeed, c would be independent of location k. Of course, $\pi_1(c_1, a_1, k, \epsilon_1)$ also denotes the fraction of agents in the population who are assigned at date 1 and contingent on ϵ_1 the location k and the bundle (c_1, a_1). Also let $\pi_2(c_2, a_2, i, \epsilon_1, \epsilon_2 \mid k)$ be the probability that an individual is assigned at date 2 and contingent on history (ϵ_1, ϵ_2) the location i and the bundle (c_2, a_2), given the date-1 assignment to location k. Of course, $\pi_2(c_2, a_2, i, \epsilon_1, \epsilon_2 \mid k)$ also denotes the fraction of agents in the population who experience the latter event.

To further facilitate the analysis, the numbers $\pi_1(c_1, a_1, k, \epsilon_1)$ and $\pi_2(c_2, a_2, i, \epsilon_1, \epsilon_2 \mid k)$ are combined by multiplication; that is, let

$$\pi(c_1, a_1, k, c_2, a_2, i, \epsilon_1, \epsilon_2) = \pi_2(c_2, a_2, i, \epsilon_1, \epsilon_2 \mid k)\, \pi_1(c_1, a_1, k, \epsilon_1) \tag{8.4}$$

so that $\pi(\cdot)$ on the left-hand side of (8.4) is the obvious joint probability over the event in parentheses, indexed by (ϵ_1, ϵ_2). Further, let

$$\pi(c_1, a_1, k, \epsilon_1, \epsilon_2) = \sum_{c_2} \sum_{a_2} \sum_{i} \pi(c_1, a_1, k, c_2, a_2, i, \epsilon_2, \epsilon_2) \tag{8.5}$$

so that $\pi(\cdot)$ on the left-hand side of (8.5) is the marginal probability of the event in parentheses, indexed by (ϵ_1, ϵ_2). Finally, because ϵ_2 is not known at date 1, require

$$\pi(c_1, a_1, k, \epsilon_1, \epsilon_2) = \pi(c_1, a_1, k, \epsilon_1, \epsilon_2') \quad \forall \epsilon_2, \epsilon_2', \tag{8.6}$$

so that the argument ϵ_2 is not present. This thus yields the $\pi_1(c_1, a_1, k, \epsilon_1)$ specified initially.

With this notation, the determination of a social optimum in a two-period world reduces to the following.

Program 8.1. Maximize by choice of

$\pi(c_1, a_1, k, \epsilon_1)$ *and* $\pi(c_1, a_1, k, c_2, a_2, i, \epsilon_1, \epsilon_2)$

the objective function

$$E_0 \left\{ \sum_{c} \sum_{a_1} \sum_{k} \tilde{U}(c_1, a_1, k)\, \pi(c_1, a_1, k, \epsilon_1) + \sum_{c_1} \sum_{a_1} \sum_{k} \sum_{c_2} \sum_{a_2} \sum_{i} \tilde{U}(c_2, a_2, i \mid k) \cdot \pi(c_1, a_1, k, c_2, a_2, i, \epsilon_1, \epsilon_2) \right\} \tag{8.7}$$

subject to the resource constraints at date $t = 1$, contingent on ϵ_1,

$$\sum_{c_1} \sum_{a_1} \sum_{k} c_1 \pi(c_1, a_1, k, \epsilon_1) \leq \sum_{k} f_k \left[\sum_{c_1} \sum_{a_1} a_1 \pi(c_1, a_1, k, \epsilon_1), \epsilon_1 \right]; \tag{8.8}$$

subject to the resource constraints at date $t = 2$, contingent on (ϵ_1, ϵ_2),

$$\sum_{c_1} \sum_{a} \sum_{k} \sum_{c_2} \sum_{a_2} \sum_{i} c_2 \pi(c_1, a_1, k, c_2, a_2, i, \epsilon_1, \epsilon_2) \leq \sum_{i} f_i \left[\sum_{c_1} \sum_{a_1} \sum_{k} \sum_{c_2} \sum_{a_2} a_2 \pi(c_1, a_1, k, c_2, a_2, i, \epsilon_1, \epsilon_2), \epsilon_2 \right]; \tag{8.9}$$

and subject to constraints (8.5) *and* (8.6) *on* π*'s.*

Now one can let a common-date $t = 0$ consumption set X be determined by probabilities $x(c_1, a_1, k, \epsilon_1)$ and $x(c_1, a_1, k, c_2, a_2, i, \epsilon_1, \epsilon_2)$ between zero and one, summing to one for each shock ϵ_1 and history (ϵ_1, ϵ_2), respectively, with restrictions like (8.5) and (8.6) for the $x(\cdot)$. And one can let production set Y be determined by the elements $y(c_1, a_1, k, \epsilon_1)$ as units of commitments to hand out c_1 units of output at location k and hire a_1 units of labor at location k, both contingent on ϵ_1; and by the elements $y(c_1, a_1, k, c_2, a_2, i, \epsilon_1, \epsilon_2)$ as units of commitments to hand out c_1 units of output at location k and hire a_1 units of labor at location k, to hand out c_2 units of output at location i and hire a_2 units of labor at location i, all contingent on (ϵ_1, ϵ_2) with $y(\cdot)$ replacing $\pi(\cdot)$ in (8.5), (8.6), (8.8), and (8.9). Thus theorems on the existence and optimality of competitive equilibrium follow by standard arguments. In fact, the argument is virtually immediate because everyone is alike in the planning date $t=0$, so there is (in effect) a representative consumer.

R. Rogerson (1985) apparently solves Program 8.1 indirectly, by use of recursive methods. In fact, Rogerson has computed recursive equilibria to his model for example environments (though initial location assignments appear to be given, potentially inconsistent with the formulation of the optimum problem given here). In these examples, with some reluctance to move ($m>0$), changes in hours per worker lead changes in employment, employment displays some persistence, and hours per worker are more highly correlated with real wages than is employment. The point is that Rogerson's model seems to offer a rich variety of time series dynamics, and so again models motivated (as here) by a study of primitive economies seem useful in addressing standard macroeconomic concerns.

9 Programs with limited communication

We have argued thus far that programs with private information and spatial separation are rich in implications and at least suggestive of actual arrangements. Still, it is a striking fact that none of these programs can explain commonly observed arrangements such as the use of currency and financial instruments. One way to remedy this failure is to incorporate limited communication, as in Townsend (1987b) and the related work of Brunner and Meltzer (1971), Gale (1980), Ostroy (1973), and Ostroy and Starr (1974). (A second way – the incorporation of limited commitment – is discussed in Section 10.)

To begin the discussion, then, it is useful to merge the private information economy generating Program 6.1 with the spatial model depicted in Table 1. In particular, agents a and a' move between two locations according to the pattern in Table 1 (i.e., do not actually move at all), and have random privately observed endowments θ^a_{1t} and $\theta^{a'}_{2t}$ (respectively) observed at the beginning of date t, $t=1,2$, at locations 1 and 2 (respectively). Agents b and b' move according to the specified pattern of Table 1 and have public endowments w^b_{it} and $w^{b'}_{it}$ (respectively) at location i and date t. Each agent j has preferences over consumption bundles c at each date t and at his assigned location i as represented by the utility function $U^j(c^j_{it})$. Also, for simplicity, suppose there is only one underlying consumption good; that agents a and a' are identical in preferences and in the distribution of endowments; and that agents b and b' are identical to one another as well.

Now suppose the most primitive of communication technologies is in effect; that is, suppose there are no telecommunications, no recording devices, no portable but otherwise worthless tokens, and no storage possibilities for the consumption good. At each location and date agents can make announcements of their contemporary but privately observed endowments, and can make announcements as well of their histories of privately observed endowments, announcements, and trades. Thus, one can consider allocation rules $\pi^T_{it}(\cdot)$ and $\pi^S_{it}(\cdot)$ at location i and date t which have as arguments these announcements, and it is possible to write down a programming problem for the determination of Pareto optimal outcomes, much like Program 6.1, keeping track of the 4 agents and 2 locations. However, announcements of past histories have no force in any incentive-compatible arrangement. Given the imposed communication technology, there is no way to achieve bona fide intertemporal tie-ins, as agents will always make the best possible announcement given the contemporary state. Thus, the programming problem would reduce to four separate versions of Program 6.1. With only one commodity, then, the solution is necessarily autarkic, at least if utility functions display decreasing absolute risk aversion; see Townsend (1985b).

This dismal outcome can be avoided if the spatial itinerary of agents is altered or if the communication technology is slightly improved. Taking the first suggestion, suppose agents b and b' do not move in the above model. Then, as in W. Rogerson (1985) and Townsend (1982), intertemporal links and beneficial trade is possible. Indeed, more elaborate setups in which agents return periodically to some go-between allow beneficial trade and suggest a model of an intermediary.

Taking the second suggestion, while still precluding telecommunications and commodity storage, suppose the existence of portable concealable artificial tokens – objects that do not enter into anyone's utility function or into any production technology but that can be carried about and redistributed at any location where agents meet under the prespecified rules of a resource allocation process. Then, in the model considered above, beginning-of-second-period token holdings are an endogenously determined and privately observed endowment, an extra state variable that can be announced by the agents, triggering taxes and subsidies both of tokens and actual commodities. Indeed, with the symmetry assumptions, one can write down an (apparent) two-agent, two-period programming problem, much like Program 6.1 with the following exceptions: There are token as well as commodity taxes and subsidies at the first date, contingent on a's (or a''s) endowment announcement at the first date, say θ_1; there are token as well as commodity taxes and subsidies at the second date, contingent on a's (or a''s) endowment announcement at the second date, say θ_2, *and* on a's (or a''s) announced beginning-of-period token holdings, say m_2; and there are incentive constraints in both the first and second periods, to ensure truthful revelations. It can be shown that these portable concealable tokens allow beneficial multilateral trade.

These results can be extended in several directions. The first is by considering alternative communication technologies. For example, if one considers storage and bona fide commodity tokens, the intertemporal incentive constraints generally are more binding and the solutions generally are Pareto inferior; essentially, commodity storage confounds the use of objects as signals of past events. Alternatively, systems with multiple artificial tokens can dominate systems with single artificial tokens; that is, multiple tokens allow more intertemporal tie-ins and hence weakened or less binding incentive constraints. Written message systems generally do even better, in the sense that more history becomes a matter of reliable record and not subject to the requirements of incentive-compatible reporting. Finally, centralized electronic interspatial telecommunication systems represent an endpoint in the spectrum of communication technologies, removing limited communication as a constraint on the outcomes of programming problems.

These private-information, spatial-separation, limited-communication set-ups can be taken to observations from actual economies. For example, the role for intermediaries described above is not inconsistent with the role played by medieval bankers in twelfth-century Italy, as described by Townsend (1985). There, bankers were essentially person-specific oral assignment systems. Similarly, as is argued in Townsend (1987b), observations by anthropologists on the strange use of noninterchangeable multiple commodity currencies and ceremonial objects in various close-knit societies are not inconsistent with the use made (in the theory) of multiple portable tokens. There is also some evidence that financial instruments emerged in Europe in the fifteenth century as written messages sent among partners in long-term trading relationships; see Townsend (1985).

Another direction for this work is the study of optimal monetary policy, as in Townsend (1987a). For example, in writing down a programming problem for the determination of optimal arrangements in the token/currency set-up described above, one is naturally poised to ask questions concerning the optimal social use of currency in the face of various private and economywide shocks. One can use as a base the banking model of Diamond and Dybvig (1983), where groups of agents suffer from privately observed shocks determining their urgency to consume. The more urgent is consumption, the more goods an agent wishes to withdraw from an otherwise productive investment project. As for the model of Diamond and Dybvig, a Pareto optimal consumption allocation can be determined as some solution to a programming problem, as in Section 6. On this base, then, one can impose some spatial separation (say, two spatially separated investment projects) and suppose that groups of agents are exogenously shifted over time, much like single agents b and b' above.

Tokens then serve as concealable records of deferred consumption for patient movers, and it can be shown that both the level of tokens and the mix of tokens relative to location-specific "bank credits" should be responsive to economywide shocks determining the relative number of patient consumers and the relative number of movers. Thus Arrow–Debreu optimum problems allow considerable scope for activist monetary policy.

10 Programs with limited commitment

If the interpretation of currency given above seems somewhat unusual, perhaps that is because one does not tend to think of currency as an efficient communication device in the context of a social planning problem. Rather, one may view currency as an object that preserves value and facilitates trade in a decentralized market economy, an economy in which agents meet anonymously or infrequently and have little commitment to one another. Indeed, in Europe from the tenth to the fifteenth centuries, for example, gold coins circulated largely among international traders, facilitating interregional trade in occasional periodic meetings [see Townsend (1985)]. More generally, circulating currency emerged at roughly the same time as market exchange. This use of currency, then, has not yet been explained by the theory.

For that matter, markets themselves have not yet been explained by the theory. As emphasized, many of the programming problems described above can be decentralized with a price system, and in that sense an optimum is not inconsistent with the existence of markets. And price observations have been used to fit various of the economies described to data; see especially the work of Eichenbaum, Hansen, and Richard (1987). But the theory itself does not explain markets as an efficient institutional arrangement.

What the theory is missing, apparently, is some lack of commitment. That is, in the programming problems described above, it is as if agents agree at some initial date to allocation rules for future dates, contingencies, and locations – rules that are costlessly enforced and maintained despite possible time inconsistencies and incentives to renege. In fact, it may be difficult to enforce such rules and prevent reneging, and this can be an important determinant of actual arrangements.

A natural way to introduce limited commitment is to suppose that planning problems must be solved successively, period by period, perhaps for particular and potentially variable weights λ^j across agent types j. Thus there would be no precommitment to a social rule, and agents would do what is best for themselves at the moment looking forward to the future. Indeed, this leads logically to the notion of a *sequential core,* something akin to a notion suggested earlier by Gale (1980). In some last period (if there is one), the allocation of consumption goods must be in the core, not blocked by a coalition of agents. With the prespecified direct utility functions for consumption, this core outcome then induces indirect utility functions for all agents up to state variables such as beginning-of-period capital holdings or currency. Then, in the next-to-last period, the allocation of consumption goods and capital or currency must be in the core – given the current state, the contemporary direct utility functions for consumption, and the last-period value functions derived above. Continuing in this way, perhaps indefinitely (so as to be rid of sensitivity to initial conditions), one can generate sequential core outcomes.

An equivalence between core allocations and competitive-equilibrium allocations then helps to make the connection to models with sequential competitive markets. In the models of currency with spatially separated agents described in Townsend (1980), for example, agents move about exogenously from location to location, trade commodities against paper currency in competitive markets when they meet, and then continue on, perhaps never to meet again. Thus one can conjecture that the noninterventionist monetary equilibria of Townsend (1980) – equilibria with valued currency – are (essentially) equivalent to sequential core outcomes, and the role played by currency when commitment is limited would be explained. Again, this theory would be consistent with the observations given above on the emergence and use of currency.

Thus spatial models of currency, and some of the so-called Clower constraint models of currency [e.g., Lucas and Stokey (1984), Townsend (1987d)], appear to be within the Arrow–Debreu tradition described in this chapter: They seem to yield outcomes that are solutions to programming problems with limited commitment. Further, these currency models promise to be an interesting base for the study of more traditional macroeconomic phenomena, addressing observations that are anomalous relative to the standard neoclassical growth model. Finally, in better matching the environment of the theory with the environments of actual economies, this class of spatial limited-commitment models promises to be an interesting study in its own right.

References

Altug, S. J. 1984. "Gestation Lags and the Business Cycle: An Empirical Analysis." Ph.D. dissertation, Carnegie-Mellon University.

Arrow, K. 1964. "The Role of Securities in the Optimal Allocation of Risk Bearing." *Review of Economic Studies* 31: 91–6.

Baiman, S., and J. Demski. 1980. "Economically Optimal Performance Evaluations and Control Systems." *Journal of Accounting Research* (supplement): 184–220.

Bewley, T. F. 1972. "Existence of Equilibria in Economies with Infinitely Many Commodities." *Journal of Economic Theory* 4: 514–40.

Bennett, H. S. 1974. *Life on the English Manor.* Cambridge: Cambridge University Press.

Binswanger, Hans, and Mark R. Rosenzweig. 1984. *Contractual Arrangements, Employment, and Wages in Rural Labor Markets in Asia.* New Haven: Yale University Press.

Boyd, J., and E. Prescott. 1986. "Financial Intermediary Coalitions." *Journal of Economic Theory* 38: 211–32.

Brock, W. A., and L. Mirman. 1972. "Optimal Economic Growth and Uncertainty: The Discounted Case." *Journal of Economic Theory* 4: 479–513.

Brunner, K., and A. H. Meltzer. 1971. "The Uses of Money: Money in the Theory of an Exchange Economy." *American Economic Review* 61: 784–805.

Cass, D. 1965. "Optimal Growth in Aggregate Model of Capital Accumulation." *Review of Economic Studies* 32: 233–40.

Danthine, J. P., and J. B. Donaldson. 1981. "Stochastic Properties of Fast vs. Slow Growing Economies." *Econometrica* 49: 1007–33.

Debreu, G. 1954. "Valuation Equilibrium and Pareto Optimum." *Proceedings of the National Academy of Science* 40: 588–92.

1959. *The Theory of Value.* New York: Wiley.

1962. "New Concepts and Techniques for Equilibrium Analysis." *International Economic Review* 3: 257–73.

Diamond, Douglas. 1984. "Financial Intermediaries and Delegated Monitoring." *Review of Economic Studies* 51: 393–414.

Diamond, D., and P. Dybvig. 1983. "Bank Runs, Deposit Insurance, and Liquidity." *Journal of Political Economy* 91: 401–10.

Diamond, Peter. 1967. "The Role of Stock Markets in a General Equilibrium Model with Technological Uncertainty." *American Economic Review* 57: 759–76.

Eichenbaum, M. S., and L. P. Hansen. 1987. "Estimating Models with Intertemporal Substitution using Aggregate Time Series Data." NBER Working Paper No. 2181.

Eichenbaum, M. S., L. P. Hansen, and S. Richard. 1987. "Aggregation, Durable Goods, and Nonseparable Preferences in an Equilibrium Asset Pricing Model." Unpublished manuscript.

Firth, Raymond. 1939. *Primitive Polynesian Economy.* London: Stephen Austin and Sons.

Gale, D. 1980. "Money, Information and Equilibrium in Large Economies." *Journal of Economic Theory* 23: 28–65.

Gorman, W. M. 1953. "Community Preference Fields." *Econometrica* 21: 63–80.

Green, Edward J. 1985. "Lending and the Smoothing of Uninsurable Income." Unpublished manuscript.

Grossman, S., and O. Hart. 1983. "An Analysis of the Principal Agent Problem." *Econometrica* 51: 7–46.
Grossman, S., and R. Shiller. 1981. "The Determinants of the Variability of Stock Market Prices." *American Economic Review, Papers and Proceedings* 71: 222–7.
Hall, R. E. 1981. "Intertemporal Substitution in Consumption." Mimeo, Hoover Institution, Department of Economics, Stanford University.
Hansen, Gary. 1985. "Indivisible Labor and the Business Cycle." *Journal of Monetary Economics* 16: 309–27.
Hansen, Lars P. In press. "Calculating Asset Prices in Three Example Economies." In T. F. Bewley (ed.), *Advances in Economic Theory – Fifth World Congress,* Vol. I. Cambridge: Cambridge University Press.
Hansen, L. P., and S. Richard. In press. "The Role of Conditioning Information in Deducing Testable Restrictions Implied by Dynamic Asset Pricing Models." *Econometrica.*
Hansen, L. P., and K. J. Singleton. 1982. "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models." *Econometrica* 50: 1269–86.
1983. "Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Stock Market Returns." *Journal of Political Economy* 91: 249–65.
Harris, M., and A. Raviv. 1979. "Optimal Incentive Contracts with Imperfect Information." *Journal of Economic Theory* 20: 231–59.
Harris, M., and R. M. Townsend. 1981. "Resource Allocation under Asymmetric Information." *Econometrica* 49: 33–64.
Holmström, Bengt. 1979. "Moral Hazard and Observability." *Bell Journal of Economics* 10: 74–91.
1982. "Moral Hazard in Teams." *Bell Journal of Economics* 13: 324–40.
Homans, George C. 1975. *English Villagers of the Thirteenth Century.* New York: Norton.
Ito, Takatoshi. 1984. "Self-Selection and Moral Hazard in Implicit Contracts with Asymmetric Information on Workers' Search." Unpublished manuscript.
Koopmans, T. 1965. "On the Concept of Optimal Economic Growth." *Academic Science* 28: 225–300.
Kydland, F. E., and E. C. Prescott. 1982. "Time to Build and Aggregate Fluctuations." *Econometrica* 50: 1345–70.
Lee, Richard. 1979. *The !Kung San.* Cambridge: Cambridge University Press.
Leme, Paulo. 1984. "Integration of International Capital Markets." Unpublished manuscript, University of Chicago.
Lucas, Robert E. 1985. "On the Mechanics of Economic Development." Unpublished manuscript.
1987. *Models of Business Cycles.* YRJO Jahnsson Lectures. Oxford: Blackwell.
Lucas, R. E., and N. L. Stokey. 1984. "Optimal Fiscal and Monetary Policy in an Economy without Capital." *Journal of Monetary Economics* 12: 55–93.
McCloskey, Donald. 1976. "English Open Fields as Behavior Towards Risk." *Research in Economic History* pp. 124–71.

McKenzie, L. W. 1959. "On the Existence of General Economic Equilibrium for a Competitive Market." *Econometrica* 27: 54–71.
Malinowski, Bronislaw. 1953. *Argonauts of the Western Pacific,* Chapter 3. New York: Dutton.
Mankiw, N., G. and J. Rotemberg, and L. H. Summers. 1985. "Intertemporal Substitution in Macro-Economics." *Quarterly Journal of Economics* 100: 225–52.
Mas-Colell, Andreu. 1975. "A Model of Equilibrium with Differentiated Commodities." *Journal of Mathematical Economics* 2: 263–95.
Mehra, R., and E. C. Prescott. 1985. "The Equity Premium: A Puzzle." *Journal of Monetary Economics* 15: 145–61.
Mirrlees, J. A. 1975. "The Theory of Moral Hazard and Unobservable Behavior, Part 1." Oxford Mimeos, Nuffield College.
Moore, John. 1984. "Contracting Between Two Parties with Private Information." Unpublished manuscript.
Myerson, Roger B. 1979. "Incentive Compatibility and the Bargaining Problem." *Econometrica* 47: 61–74.
1982. "Optimal Coordination Mechanisms in Generalized Principal-Agent Problems." *Journal of Mathematical Economics* 10: 67–81.
1986. "Multistage Games with Communication." *Econometrica* 54: 323–58.
Ostroy, J. M. 1973. "The Informational Efficiency of Monetary Exchange." *American Economic Review* 63: 597–610.
Ostroy, J. M., and R. M. Starr. 1974. "Money and the Decentralization of Exchange." *Econometrica* 52: 1093–1113.
Prescott, E., and R. Townsend. 1984a. "Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard." *Econometrica* 52: 21–45.
1984b. "General Competitive Analysis in an Economy with Private Information." *International Economic Review* 25: 1–20.
Rogerson, Richard. 1985. "Nonconvexities and the Aggregate Labor Market." Working Paper.
Rogerson, William. 1985. "Repeated Moral Hazard." *Econometrica* 53: 69–76.
Rothschild, Michael, and Joseph Stiglitz. 1976. "Equilibrium in Competitive Insurance Market." *Quarterly Journal of Economics* 90: 629–49.
Sargent, T. J. 1979. "Tobin's q' and the Rate of Investment in General Equilibrium." In K. Brunner and A. Meltzer (eds.), *On the State of Macroeconomics.* Amsterdam: North-Holland.
Scheinkman, Jose A. 1984. "General Equilibrium Models of Economic Fluctuations: A Survey of Theory." Unpublished manuscript, University of Chicago.
Shavell, S. 1979. "Risk Sharing and Incentives in the Principal and Agent Relationship." *Bell Journal of Economics* 10: 55–73.
Shiller, R. J. 1981. "Do Stock Prices Move too Much to be Justified by Subsequent Changes in Dividends." *American Economic Review* 71: 421–36.
Spence, Michael. 1974. *Market Signaling.* Cambridge, Mass.: Harvard University Press.
Townsend, Robert. 1978. "Intermediation with Costly Bilateral Exchange." *Review of Economic Studies* 45: 417–25.

1979. "Optimal Contracts and Competitive Markets with Costly State Verification." *Journal of Economic Theory* 21: 265–93.
1980. "Models of Money with Spatially Separated Agents." In Neil Wallace and John Kareken (eds.), *Models of Monetary Economies.* Federal Reserve Bank of Minneapolis.
1982. "Optimal Multiperiod Contracts and the Gain from Enduring Relationships under Private Information." *Journal of Political Economy* 90: 1166–86.
1983. "Theories of Intermediated Structures." *Carnegie-Rochester Conference Series on Public Policy* 18: 221–72.
1985. "Taking Theory to History: Explaining Financial Structure and Economic Organization." Unpublished manuscript.
1987a. "An Optimal Activist Currency Rule." Unpublished manuscript.
1987b. "Economic Organization with Limited Communication." Unpublished manuscript.
1987c. "Private Information, Volatility, and Economic Welfare." Unpublished manuscript.
1987d. "Asset Return Anomalies in a Monetary Economy." *Journal of Economic Theory* 41: 219–47.
Van Roy, Edward. 1971. *Economic Systems of Northern Thailand: Structure and Change.* Ithaca, N.Y.: Cornell University Press.
Wilson, C. 1977. "A Model of Insurance Markets with Incomplete Information." *Journal of Economic Theory* 16: 167–207.
Wilson, Robert. 1968. "The Theory of Syndicates." *Econometrica* 36: 119–293.