

# A Bayesian Semiparametric Competing Risk Model with Unobserved Heterogeneity\*

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## Abstract

This paper generalizes existing econometric models for censored competing risks by introducing a new flexible specification based on a piecewise linear baseline hazard, time-varying regressors, and unobserved individual heterogeneity distributed as an infinite mixture of Generalized Inverse Gaussian (GIG) densities, nesting the gamma kernel as a special case. A common correlated latent time effect induces dependence among risks. Our model is based on underlying latent exit decisions in continuous time while only a time interval containing the exit time is observed, as is common in economic data. We do not make the simplifying assumption of discretizing exit decisions – our competing risk model setup allows for latent exit times of different risk types to be realized within the same time period. In this setting, we derive a tractable likelihood based on scaled GIG Laplace transforms and their higher-order derivatives. We apply our approach to analyzing the determinants of unemployment duration with exits to jobs in the same industry or a different industry among unemployment insurance recipients on nationally representative individual-level survey data from the U.S. Department of Labor. Our approach allows us to conduct a counterfactual policy experiment by changing the replacement rate: we find that the impact of its change on the probability of exit from unemployment is inelastic.

*JEL:* C11, C13, C14, C41, J64

*Keywords:* competing risk model, Bayesian semiparametric model, unemployment insurance.

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## 1. Introduction

In an economic competing risk (CR) model with censoring, an individual is associated with a current state (e.g. of being unemployed) with the possibility to exit to one of several different states (e.g. finding a job in the same industry or a different industry). However, only one such exit is observed for some individuals, while other (censored) individuals are never observed to exit their current state. The state duration before exit to each potential new state or censoring time is modeled with a separate latent variable but only the shortest such duration is actually observed for each individual. A key ingredient of CR models is the survival function which captures the probability that the individual will remain the current state beyond any given time. CR analysis then typically seeks to determine the impact of observable characteristics of the individual and the various states on the survival function that can lead to policy recommendations.<sup>5</sup>

In this paper we introduce a new flexible model specification for the competing risk model, extending several strands of econometric and statistical literature on duration analysis. Our model encompasses three key features: (i) we estimate non-parametrically the density of unobserved individual heterogeneity; (ii) we model correlations between the different risk types; (iii) we allow for multiple latent exits within a time period with interval outcome data. Our model nests the single exit type (so-called duration model) as a special case. We apply our method to analyzing the determinants of unemployment duration among unemployment insurance recipients using data from the U.S. Department of Labor. We conduct a counterfactual experiment by changing the replacement rate. The counterfactual results show the impact of changing key policy variables such as the replacement rate on the survival function.

We will introduce each model feature in turn and discuss the advantages of using our model over the existing alternatives. First, our model provides a flexible approach to controlling for unobserved heterogeneity in competing risk data. A typical source of unobserved

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<sup>5</sup>Applications of CR models in economics include analyzing unemployment duration (Flinn and Heckman, 1982; Katz and Meyer, 1990; Tysse and Vaage, 1999; Alba-Ramírez, Arranz, and Muñoz-Bullón, 2007), Ph.D. completion (Booth and Satchell, 1995), teacher turnover (Dolton and van der Klaauw, 1999), studies of age at marriage or cohabitation (Berrington and Diamond; 2000), mortgage termination (Deng, Quigley, and Van Order, 2000), school dropout decisions (Jakobsen and Rosholm, 2003), and manufacturing firms' exits from the market (Esteve-Perez, Sanchis-Llopis, and Sanchis-Llopis, 2010). A comprehensive overview is given in Van den Berg (2001).

heterogeneity is the omission of important but perhaps unobservable variables from the conditioning set. As an example, more motivated individuals may exit unemployment more quickly because they put more effort into the search for a new job.

It is well established that failure to account for unobserved heterogeneity biases the estimated hazard rate and the proportional effects of explanatory variables on the population hazard (Vaupel et al 1979; Lancaster 1979, 1990). A number of semi-parametric estimators for the single risk mixed proportional hazard (MPH) model have been proposed following Elbers and Ridder (1982) proof of MPH semi-parametric identification (Heckman and Singer, 1984; Honoré 1990). Han and Hausman (1990) and Meyer (1990) propose an estimator for piecewise-constant baseline hazard and gamma distributed unobserved heterogeneity. Horowitz (1999) proposed a nonparametric estimator for both the baseline hazard and the distribution of the unobserved heterogeneity, under the assumption of constant time-invariant regressors. Hausman and Woutersen (2012) show that a nonparametric estimator of the baseline hazard with gamma heterogeneity yields inconsistent estimates for all parameters and functions if the true mixing distribution is not a gamma, stressing the importance of avoiding parametric assumptions on the unobserved heterogeneity.

As the second key feature of our model, our approach allows for correlations between the different risks in the CR model environment, even in the presence of the flexible individual heterogeneity infinite GIG mixture model component. This is important since the determinants of exit can differ depending on the risk type while being correlated across the risk types, and thus our approach provides additional information to the analyst compared to single-risk duration models.

Third, in our application, we deal with interval outcome data as is common in economics and other social sciences. Even though the underlying exit decision model is set in continuous time, only the broader period in which exit occurred is observable. Our data contain the week of exit from unemployment. Based on scaled GIG Laplace transforms and their higher-order derivatives, we provide a complete likelihood specification allowing for multiple latent exits within a single time period which is more realistic than simplifying the analysis by assuming that only one latent exit can occur in a given time period. Thus, we do not rule out by assumption an individual contemplating different job offers from

firms in the same industry or in a different industry as their pre-unemployment industry within any given week.

We show that given the analytical forms newly provided in this paper our model can be implemented in a user-friendly way via a Bayesian nonparametric approach. One of the key benefits of Bayesian Markov chain Monte Carlo (MCMC) methods that we utilize is their ability to factorize a complicated joint likelihood model into a sequence of conditional tractable models, so-called Gibbs blocks, and by sampling each in turn deliver outcomes from the joint model. We detail this approach for our proposed model.

The bulk of the literature on CR model development is concentrated in the natural sciences. A recent overview of CR modeling in biostatistics is provided by Beyersmann, Schumacher, and Allignol (2012), and in medical research by Pintilie (2006). The associated estimation methods typically rely on continuous time data for the exact point of exit. In contrast, we observe only discrete time intervals within which latent exits occur.

Competing risk models suitable for economic interval outcome data have been proposed in various forms. Han and Hausman (1990), Fallick (1991), Sueyoshi (1992), and McCall (1996) provide model specifications either without or with parametric individual heterogeneity. Butler, Anderson, and Burkhauser (1989) propose a semiparametric CR model controlling for the correlation between unobserved heterogeneity components in each state, with quadratic time dependence. Lleras-Muney and Honoré (2006) analyze identification issues in a general class of competing risk models allowing for correlation among risk types. Their environment is free of many of the functional form and distributional assumptions that we impose here and hence a number of parameters are set-identified. Bierens and Carvalho (2007) consider Weibull baseline hazards and common flexible unobserved heterogeneity. Canals-Cerdá and Gurmu (2007) approximate unobserved heterogeneity distribution with Laguerre polynomials. They find that model selection rules (BIC, HQIC, and AIC) perform worse in determining the polynomial order than a naive approach of controlling for unobserved heterogeneity using simple models with a small number of points of support or a polynomial of small degree. Van den Berg, van Lomwel, and van Ours (2008) consider a model with nonparametric unobserved heterogeneity terms that is based on discrete time counts. Although the model can be

derived as a time-aggregated version of an underlying continuous-time model, the latter is different from the continuous-time mixed proportional hazard model.

The literature on Bayesian nonparametric methods in the CR environment has been scant and, to our knowledge, has only been used in biostatistics for estimation of other objects of interest than individual heterogeneity.<sup>6</sup> Variants of Bayesian Dirichlet Process analysis have been used by Gasbarra and Karia (2000) for estimating nonparametrically the overall hazard rate and in Salinas-Torres, Pereira, and Tiwari (2002) and Polpo and Sinha (2011) for the vector of risk-specific cumulative incidence functions. De Blasi and Hjort (2007) specify a beta-process prior for the baseline hazard, with asymptotic properties analyzed in De Blasi and Hjort (2009).

Identification results under various assumptions were established by Heckman and Honoré (1989), Sueyoshi (1992), Abbring and van den Berg (2003), and Lee and Lewbel (2013). In general, there have been three different approaches to identification (Honoré and Lleras-Muney, 2006): (a) to make no additional assumptions beyond the latent competing risk structure and estimate bounds on the objects of interest; (b), assume that the risks are independent conditional on a set of observed covariates and deal with a multiple duration models environment; and (c), to specify a parametric or semi-parametric model conditional on the covariates. Here we take the last approach. In particular, we do not assume that the risks are independent conditional on the observed covariates.

The remainder of the paper is organized as follows. Section 2 establishes the assumptions and building block results for a single risk duration model. Section 3 introduces assumptions and results for the competing risk model. Section 4 details our application and the counterfactual experiment and Section 5 concludes. Proofs of all theorems and additional empirical results are provided in the Appendix. The Online Appendix (Burda, Harding and Hausman, 2013) contains further results.

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<sup>6</sup>In the single-risk, duration model case, Bayesian analysis with economics application was undertaken by Ruggiero (1994), Florens, Mouchart, and Rolin (1999), Campolieti (2001), Paserman (2004), and Li (2007).

## 2. Single Risk Duration Model

Denote by  $\tau$  a continuous time variable with density  $f(\tau)$  and distribution  $F(\tau)$ . Denote a latent failure (or exit) time of individual  $i$  by  $\tau_i$ . Define the hazard rate  $\lambda_i(\tau)$  as the failure rate at time  $\tau$  conditional upon survival to time  $\tau$ ,  $\lambda_i(\tau) = \lim_{\delta \rightarrow 0} Pr(\tau < \tau_i < \tau + \delta | \tau_i \geq \tau) / \delta$  and denote the integrated hazard by:

$$(2.1) \quad \Lambda_i(\tau) = \int_0^\tau \lambda_i(u) du$$

with survivor function

$$(2.2) \quad S_i(\tau) = \exp(-\Lambda_i(\tau))$$

Denote by  $t$  a generic time period  $[\underline{\tau}, \bar{\tau})$  with end points  $\underline{\tau}$  and  $\bar{\tau}$ , and by  $t_i$  the time period  $[\underline{\tau}_i, \bar{\tau}_i) \ni \tau_i$  in which an individual  $i$  was observed to exit from a given state into another state.

**ASSUMPTION (A1).** *The data  $\{t_i\}_{i=1}^N$  consists of single spells censored at time  $T$  and drawn from a single risk process.*

**ASSUMPTION (A2).** *The hazard rate is parameterized as*

$$(2.3) \quad \lambda_i(\tau) = \lambda_0(\tau) \exp(X_i(\tau)\beta + V_i)$$

where  $\lambda_0(\tau)$  is the baseline hazard,  $X_i(\tau)$  are observed covariates that are allowed to vary over time,  $\beta$  are model parameters, and  $V_i$  is an unobserved heterogeneity component.

Hence, using (2.1) and (2.3) the integrated hazard is given by

$$(2.4) \quad \Lambda_i(\tau) = \int_0^\tau \lambda_0(u) \exp(X_i(u)\beta + V_i) du$$

**ASSUMPTION (A3).** *The baseline hazard  $\lambda_0(u)$  and the values of the covariates are constant for each time period  $t$ .*

Assumptions 1 and 2 are common in the literature. Assumption A3 is based on Han and Hausman (1990). Given Assumption A3, instead of  $\lambda_i(\tau)$  we can consider the integrated baseline hazard in the form

$$(2.5) \quad \mu_{0t} = \int_{\underline{\tau}}^{\bar{\tau}} \lambda_0(u) du,$$

where we denote the vector  $(\mu_{01}, \dots, \mu_{0T})$  by  $\mu_0$ .

For notational convenience, we will use subscripts for the time index and denote by  $\Lambda_{it}$  the quantity  $\Lambda_i(\bar{\tau})$  at the end of the time period  $t$ , and similarly for other variables. Denote the probability of the exit event in time period  $t$  by  $P(t_i = t)$ . Conditional on  $V_i$ , for outcomes that are not censored ( $t_i \leq T$ ),

$$(2.6) \quad P(t_i = t) = F_{it} - F_{i(t-1)} = (1 - S_{it}) - (1 - S_{i(t-1)}) = S_{i(t-1)} - S_{it}$$

When the duration observations are censored at the end of time period  $T$ ,

$$(2.7) \quad P(t_i > T) = 1 - F_{iT} = S_{iT}$$

The following result is familiar in the literature and we include it here for the sake of completeness as a benchmark of comparison for the new competing risk model developed in the next Section.

**RESULT 1.** *Under Assumptions A1–A3, conditional on  $V_i$ , for uncensored observations*

$$(2.8) \quad P(t_i = t|V_i) = \exp\left(-\sum_{j=1}^{t-1} \mu_{0j} \exp(X_{ij}\beta + V_i)\right) - \exp\left(-\sum_{j=1}^t \mu_{0j} \exp(X_{ij}\beta + V_i)\right)$$

*and for the censored case*

$$(2.9) \quad P(t_i > T|V_i) = \exp\left(-\sum_{j=1}^T \mu_{0j} \exp(X_{ij}\beta + V_i)\right)$$

## 2.1. Parametric Heterogeneity

**ASSUMPTION (A4).** *Let  $v_i \equiv \exp(V_i) \sim \mathcal{G}(v)$  where  $\mathcal{G}(v)$  is a generic probability measure with density  $g(v)$ .*

From (2.4), (2.5), and Assumption A3, we have

$$(2.10) \quad \tilde{\Lambda}_{it} = \sum_{j=1}^t \mu_{0j} \exp(X_{ij}\beta)$$

$$(2.11) \quad \Lambda_{it} = v_i \tilde{\Lambda}_{it}$$

If  $v$  is a random variable with probability density function  $g(v)$  then the Laplace transform of  $g(v)$  evaluated at  $s \in \mathbb{R}$  is defined as

$$(2.12) \quad \mathcal{L}(s) \equiv E_v[\exp(-vs)]$$

Using (2.2), (2.11), and (2.12), the expectation of the survival function can be linked to the Laplace transform of the integrated hazard function (Hougaard, 2000) as follows:

$$(2.13) \quad E_v [S_{it}] = \mathcal{L}(\tilde{\Lambda}_{it})$$

Using (2.10), (2.11), and (2.13) yields the unconditional exit probability of Result 1 as follows:

**RESULT 2.** *The expectation of (2.6) for the uncensored observations is*

$$(2.14) \quad E_{v_i} [P(t_i = t)] = \mathcal{L}(\tilde{\Lambda}_{i(t-1)}) - \mathcal{L}(\tilde{\Lambda}_{it})$$

*and the expectation of (2.7) for the censored observations takes the form*

$$(2.15) \quad E_{v_i} [P(t_i > T)] = \mathcal{L}(\tilde{\Lambda}_{iT})$$

Since the individual heterogeneity term  $v_i$  defined in Assumption A4 is non-negative, a suitable family of distributions  $\mathcal{G}(v)$  with support over  $[0, \infty)$  and tractable closed-form Laplace transforms is Generalized Inverse Gaussian (GIG) class of distributions, whose special case is the gamma distribution popular in duration analysis.

**ASSUMPTION (A5a).** *The unobserved heterogeneity term  $v_i$  is distributed according to the Generalized Inverse Gaussian distribution,  $\mathcal{G}(v) = \mathcal{G}^{GIG}(v; \kappa, \varphi, \theta)$ .*

The GIG has the density

$$(2.16) \quad g^{GIG}(v; \kappa, \varphi, \theta) = \frac{2^{\kappa-1}}{K_\kappa(\varphi)} \frac{\theta}{\varphi^\kappa} (\theta v)^{\kappa-1} \exp \left\{ -\theta v - \frac{\varphi^2}{4\theta v} \right\}$$

for  $\varphi, \theta > 0$ ,  $\kappa \in \mathbb{R}$ , where  $K_\kappa(\varphi)$  is the modified Bessel function of the second kind of order  $\kappa$  evaluated at  $\varphi$  (Hougaard, 2000). The GIG Laplace transform is given by

$$(2.17) \quad \mathcal{L}^{GIG}(s; \kappa, \varphi, \theta) = (1 + s/\theta)^{-\kappa/2} \frac{K_\kappa \left( \varphi (1 + s/\theta)^{1/2} \right)}{K_\kappa(\varphi)}$$

The GIG family includes as special cases the gamma distribution for  $\varphi = 0$ , the Inverse gamma distribution for  $\theta = 0$ , and the Inverse Gaussian distribution for  $\kappa = -\frac{1}{2}$ , among others.

Application of the Laplace transform of the GIG distribution (2.17) in Result 2 yields the following result that appears to not have been previously stated in the literature:



**RESULT 3.** *Under the Assumptions A1–A4, and A5a*

$$(2.18) \quad E_{v_i}^{GIG} [P(t_i = t)] = \left(1 + \frac{1}{\theta} \tilde{\Lambda}_{i(t-1)}\right)^{-\kappa/2} \frac{K_\kappa \left(\varphi \left(1 + \frac{1}{\theta} \tilde{\Lambda}_{i(t-1)}\right)^{1/2}\right)}{K_\kappa(\varphi)} - \left(1 + \frac{1}{\theta} \tilde{\Lambda}_{it}\right)^{-\kappa/2} \frac{K_\kappa \left(\varphi \left(1 + \frac{1}{\theta} \tilde{\Lambda}_{it}\right)^{1/2}\right)}{K_\kappa(\varphi)}$$

and for the censored observations

$$(2.19) \quad E_{v_i}^{GIG} [P(t_i > T)] = \left(1 + \frac{1}{\theta} \tilde{\Lambda}_{iT}\right)^{-\kappa/2} \frac{K_\kappa \left(\varphi \left(1 + \frac{1}{\theta} \tilde{\Lambda}_{iT}\right)^{1/2}\right)}{K_\kappa(\varphi)}$$

A special case of the GIG distribution is the gamma distribution, obtained from the GIG density function (2.16) when  $\varphi = 0$ . We use the gamma distribution for  $v_i$  as a benchmark model under the following alternative to Assumption 5a.

**ASSUMPTION (A5b).** *The unobserved heterogeneity term  $v_i$  is distributed according to the gamma distribution,  $\mathcal{G}(v) = \mathcal{G}^G(v; \gamma, \theta)$ .*

The gamma density is parameterized as

$$(2.20) \quad g^G(v; \gamma, \theta) = \frac{\theta}{\Gamma(\gamma)} (\theta v)^{\gamma-1} \exp(-\theta v)$$

and its Laplace transform is given by

$$(2.21) \quad \mathcal{L}^G(s; \gamma, \theta) = (1 + s/\theta)^{-\gamma}$$

In the gamma density (2.20) the parameter  $\gamma > 0$  corresponds to the GIG parameter  $\kappa \in \mathbb{R}$  in (2.16) restricted to the positive part of the real line. Using the gamma distribution in place of the GIG constitutes a special case of Result 3:

**RESULT 4.** *Under the Assumptions A1–A4, and A5b,*

$$(2.22) \quad E_{v_i}^G [P(t_i = t)] = \left(1 + \frac{1}{\theta} \tilde{\Lambda}_{i(t-1)}\right)^{-\gamma} - \left(1 + \frac{1}{\theta} \tilde{\Lambda}_{it}\right)^{-\gamma}$$

and

$$(2.23) \quad E_{v_i}^G [P(t_i > t)] = \left(1 + \frac{1}{\theta} \tilde{\Lambda}_{it}\right)^{-\gamma}$$

Result 4 was obtained in Han and Hausman (1990) and Meyer (1990).

In both gamma and GIG distributions, the scale parameter  $\theta$  performs the same role. Specifically, for any  $c \in \mathbb{R}_+$ , if  $v \sim \mathcal{G}^G(v; \gamma, \theta)$  then  $cv \sim \mathcal{G}^G(v; \gamma, \theta/c)$ , and if  $v \sim \mathcal{G}^{GIG}(v; \kappa, \varphi, \theta)$  then  $cv \sim \mathcal{G}^{GIG}(v; \kappa, \varphi, \theta/c)$ . Due to this property,  $c$  and hence its inverse  $s \equiv c^{-1}$  are not separately identified from  $\theta$  in the Laplace transform expressions (2.17) and (2.21). Since all likelihood expressions are evaluated at  $s = \tilde{\Lambda}_{i(\cdot)}$  which is proportional to  $\mu_{0j}$  for all  $j$ , as specified in (2.10), any change in  $\theta$  only rescales the baseline hazard parameters  $\mu_{0j}$ , leaving the likelihood unchanged. Hence,  $\theta$  needs to be normalized to identify  $\mu_{0j}$ . In the gamma case, typically this normalization takes the form  $\theta = \gamma$  so that  $E[v] = 1$ . We use the equivalent normalization for the GIG case in order to nest the normalized gamma as a special case and to maintain the moment restriction  $E[v] = 1$ .

## 2.2. Flexible Heterogeneity

We now depart from the parametric form of the unobserved heterogeneity and consider a nonparametric infinite mixture for the distribution of  $v_i$ , as formulated in the following assumption.

**ASSUMPTION (A6).** *The prior for  $v_i|G \sim G$  takes the form of the hierarchical model  $G \sim DP(G_0, \alpha)$ ,  $\alpha \sim g^G(a_0, b_0)$ ,  $E[v_i] = 1$ .*

In Assumption A6,  $G$  is a random probability measure distributed according to a Dirichlet Process (DP) prior (Hirano, 2002; Chib and Hamilton, 2002). The DP prior is indexed by two hyperparameters: a so-called baseline distribution  $G_0$  that defines the “location” of the DP prior, and a positive scalar precision parameter  $\alpha$ . The distribution  $G_0$  may be viewed as the prior that would be used in a typical parametric analysis. The flexibility of the DP mixture model environment stems from allowing  $G$  to stochastically deviate from  $G_0$ . The precision parameter  $\alpha$  determines the concentration of the prior for  $G$  around the DP prior location  $G_0$  and thus measures the strength of belief in  $G_0$ . For large values of  $\alpha$ , a sampled  $G$  is very likely to be close to  $G_0$ , and vice versa. Assumption A6 is then completed by specifying the baseline measure  $G_0$ . We consider two cases:

**ASSUMPTION (A7a).** *In Assumption A6,  $G_0 = \mathcal{G}^{GIG}(\kappa, \varphi, \theta)$ .*

Implementation of the GIG mixture model under Assumptions A1–A3, A6, and A7a uses the probabilities (2.8), (2.9), (2.18) and (2.19). Further implementation details are given in the Online Appendix (Burda, Harding and Hausman, 2013).

**ASSUMPTION (A7b).** *In Assumption A6,  $G_0 = \mathcal{G}^G(\gamma, \theta)$ .*

Under Assumptions A6 and A7b, as a special limit case, putting all the prior probability on the baseline distribution  $G_0$  by setting  $\alpha \rightarrow \infty$  would result in forcing  $G = G_0 = \mathcal{G}^G(\gamma, \theta)$  which yields the parametric model of Han and Hausman (1990). Here we allow  $\alpha$  and hence  $G$  to vary stochastically. Furthermore, the gamma baseline in Assumption A7b results as a special case of the GIG baseline in Assumption A7a for the hyperparameter value  $\varphi = 0$ . Hence, both the gamma flexible case with  $G \sim DP(\mathcal{G}^G, \alpha)$  and the parametric benchmark Han and Hausman (1990) case with  $G = \mathcal{G}^G$  are nested within our full GIG mixture model specification.

### 3. Competing Risk Model

We will now generalize the results from the single-risk case to the competing risk (CR) environment with several different potential types of exit. Let the risk type be indexed by  $k = 1, \dots, K$ . Define the latent failure (or exit) times as  $\tau_{1i}, \dots, \tau_{Ki}$  corresponding to each risk type  $k$ , for each individual  $i$ . Define their minimum by  $\tau_i \equiv \min(\tau_{1i}, \dots, \tau_{Ki})$ . In our CR model for interval outcome data,  $\tau_i$  is not directly observed. Instead, the observed quantity is the time interval  $[\underline{\tau}_i, \bar{\tau}_i)$  labeled as  $t_i$  which contains  $\tau_i$ . This is in contrast to a large class of other types of CR models where the exact failure time  $\tau_i$  is observed, as is typical in biostatistics. Intrinsicly, the lifetimes of other risk types,  $\tau_{ji}$  for  $j \neq k$ , and their corresponding time intervals, remain unobserved.

Denote by  $f(u_1, u_2)$  the joint density of failure at time  $u = (u_1, u_2)$ . The functional form of  $f(u_1, u_2)$  is provided in the Online Appendix, (OA.2.4). For two risk types with  $K = 2$ , this yields the probability of exit in time period  $t$  of the form

$$(3.1) \quad P(t_{1i} = t, \tau_{2i} > \tau_{1i}) = \int_{\underline{\tau}_i}^{\bar{\tau}_i} \int_{u_1}^{\bar{\tau}_i} f(u_1, u_2) du_2 du_1 + \int_{\underline{\tau}_i}^{\bar{\tau}_i} \int_{\bar{\tau}_i}^{\infty} f(u_1, u_2) du_2 du_1$$

The first right-hand side term in (3.1) gives the probability that the second latent exit time occurred within the same time interval  $t$  as the first latent exit time. The second

right-hand side term in (3.1) is then the probability that the second latent exit time occurred in a later time interval than  $t$ . A key difficulty with evaluating (3.1), precluding direct factorization, is the presence of the outer integrand  $u_1$  in the lower bound in the inner integral of the first term. We deal with this issue and derive a closed-form solution for (3.1), under various assumptions on the latent model components. The joint density  $f(u_1, u_2)$  is obtained as a function of covariates and unobserved heterogeneity from the parameterization of risk-specific hazard functions, in a direct analogy to the single-risk case. Previous work using CR interval outcome data has either bypassed this link (e.g. by assuming a multivariate Gaussian density for  $f(u_1, u_2)$ ) or employed a discrete time approximation whereby only one exit type can occur per any one time period. Our model explicitly accounts for the continuous-time nature of the exit decisions. The statistical background for the stochastic environment of our CR model is given in the Online Appendix (Burda, Harding and Hausman, 2013).

For clarity of exposition, the numbering of the Assumptions and Theorems in this Section provides a direct counterpart to the Assumptions and Results of the single-risk case in the previous Section. We first treat the parametric case under the GIG and gamma distributions of unobserved heterogeneity, adding a common latent component for all risk types, and then proceed to infinite mixture modeling.

**ASSUMPTION (B1).** *The data consists of single spell data, drawn from a process with two risks  $k = 1, 2$ , and is censored at  $T_k$ .*

Assumption B1 readily generalizes to an arbitrary number of risks. Without loss of generality, suppose that the failure type is of type 1 so that  $\tau_{1i} = \min(\tau_{1i}, \tau_{2i})$ .

**ASSUMPTION (B2).** *The risk-specific hazard rate is parameterized as*

$$\lambda_{ki}(\tau) = \lambda_{0k}(\tau) \exp(X_i(\tau)\beta_k + V_{ki} + \zeta_k(\tau))$$

where  $\lambda_{0k}(\tau) > 0$  is the baseline hazard whose logarithm is independent across  $k$ ,  $X_i(\tau)$  are covariates that are allowed to vary over time with full support on all of the real line for any given  $\tau$  and one covariate common to all  $k$ ,  $\beta_k$  are model parameters,  $V_{ki}$  is an unobserved heterogeneity component, and  $\zeta_k(\tau)$  is a common time component correlated across  $k$  normalized to have mean zero in each  $k$ .

**ASSUMPTION (B3).** For each  $k$ , the baseline hazard  $\lambda_{0k}(\tau)$ , the values of the covariates, and  $\zeta_k(\tau)$  are constant within each time period  $t$ .

Thus, the log-baseline hazard could also be stated as  $\delta_{0kt} = \log(\lambda_{0k}) + \zeta_{kt}$  where  $\delta_{0kt}$  is correlated across  $k$  due to the presence of  $\zeta_{kt}$ . Let  $\zeta_k = (\zeta_{k1}, \dots, \zeta_{kT})$ ,  $\zeta = (\zeta_1, \zeta_2)$ ,  $V_i = \{V_{ki}\}_{k=1}^K$ , and  $V = \{V_i\}_{i=1}^N$ . The probability (3.1), conditional on  $(V_i, \zeta)$  and a set of covariates, is

$$(3.2) \quad P(t_{1i} = t, \tau_{2i} > \tau_{1i} | V_i, \zeta) = \int_{\tau_i}^{\bar{\tau}_i} \int_{u_1}^{\bar{\tau}_i} f(u_1, u_2 | V_i, \zeta) du_2 du_1 + \int_{\tau_i}^{\bar{\tau}_i} \int_{\bar{\tau}_i}^{\infty} f(u_1, u_2 | V_i, \zeta) du_2 du_1$$

Let  $S_{kit} = \exp(-\Lambda_{kit})$  denote the risk-specific survivor function. We derive a closed-form solution to (3.2) in the following Theorem which extends Result 1 to our CR model environment.

**THEOREM 1.** Under Assumptions B1–B3, conditional on the latent vector  $(V_i, \zeta)$  and a set of covariates,

$$(3.3) \quad P(t_{1i} = t, \tau_{2i} > \tau_{1i} | V_i, \zeta) = S_{2i(t-1)} S_{1i(t-1)} \lambda_{1it} (\lambda_{2it} + \lambda_{1it})^{-1} [1 - \exp(-(\lambda_{2it} + \lambda_{1it}))]$$

for uncensored observations, and

$$(3.4) \quad P(t_{1i} > T, t_{2i} > T | V_i, \zeta) = (1 - F_{1iT}) (1 - F_{2iT}) = S_{1iT} S_{2iT}$$

for censored observations.

The proof is provided in the Online Appendix.

### 3.1. Parametric Heterogeneity in the CR Model

**ASSUMPTION (B4).** Let  $v_{ki} \equiv \exp(V_{ki}) \sim \mathcal{G}_k(v_k)$  where  $\mathcal{G}_k(v_k)$  is a generic probability measure with density  $g_k(v_k)$ . Let the correlation structure of  $\zeta_{kt}$  be given by

$$\begin{pmatrix} \zeta_{1t} \\ \zeta_{2t} \end{pmatrix} \sim N \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_2\sigma_1 & \sigma_2^2 \end{bmatrix} \right)$$

with parameters  $\rho, \sigma_1, \sigma_2$ .

As in the single-risk case, we consider two alternative forms of the distribution of unobserved heterogeneity  $\mathcal{G}(v)$  in Assumption B4. The first form is parametric given either by the GIG or gamma density. We provide new results regarding the model likelihood for the model B1–B4. These will be used in the nonparametric mixture model. This approach is different from Han and Hausman (1990) who considered the truncated multivariate Normal likelihood.

For the expected likelihood, we have two new expression for the expected probability of (3.3): one based on a quadrature, and another one with a series expansion without the need for a quadrature. The following Theorem extends Result 2 into the CR model environment.

**THEOREM 2.** *Under Assumptions B1–B4,*

$$(3.5) \quad E_v P(t_{1i} = t, \tau_{2i} > \tau_{1i}) = -\tilde{\lambda}_{1it} \int_0^1 \mathcal{L}_2 \left( \tilde{\Lambda}_{2i(t-1)} + \tilde{\lambda}_{2it} s_1 \right) \mathcal{L}_1^{(1)} \left( \tilde{\Lambda}_{1i(t-1)} + \tilde{\lambda}_{1it} s_1 \right) ds_1$$

or

$$(3.6) \quad E_v P(t_{1i} = t, \tau_{2i} > \tau_{1i}) = \sum_{r_2=0}^{\infty} \sum_{r_1=0}^{\infty} \frac{(-1)^{2r_1+2r_2+1}}{r_2! r_1! (r_2 + r_1 + 1)} \tilde{\lambda}_{1it}^{r_1+1} \tilde{\lambda}_{2it}^{r_2} \\ \times \mathcal{L}_1^{(r_1+1)} \left( \tilde{\Lambda}_{1i(t-1)} \right) \mathcal{L}_2^{(r_2)} \left( \tilde{\Lambda}_{2i(t-1)} \right)$$

for uncensored observations, and

$$(3.7) \quad E_v P(t_{1i} > T, t_{2i} > T) = \mathcal{L}_1 \left( \tilde{\Lambda}_{1iT} \right) \mathcal{L}_2 \left( \tilde{\Lambda}_{2iT} \right)$$

for censored observations, where  $\mathcal{L}_k^{(r)}(s)$  is the  $r$ -th derivative of the Laplace transform.

The proof is given in the Online Appendix. Theorem 2 is derived for a generic distribution of the unobserved individual heterogeneity term  $v_i$  and provides a direct extension of (2.14) and (2.15) to the competing risk model environment. Specific alternative distributional assumptions with corresponding likelihood expressions are provided next.

**ASSUMPTION (B5a).** *The unobserved heterogeneity term  $v_i$  is distributed according to the Generalized Inverse Gaussian distribution,  $\mathcal{G}(v) = \mathcal{G}^{GIG}(v; \kappa, \varphi, \theta)$ .*

**ASSUMPTION (B5b).** *The unobserved heterogeneity term  $v_i$  is distributed according to the gamma distribution,  $\mathcal{G}(v) = \mathcal{G}^G(v; \gamma, \theta)$ .*

The derivatives of the Laplace transform in (3.5) and (3.6) Theorem 2 depend on the functional form of the density kernel of  $v_i$ , given in Assumptions B5a and B5b. The formulas for the derivatives of arbitrary order of the Laplace transform for the GIG or gamma densities do not appear to be available in the literature; we derive them in the Online Appendix. Using those expressions in Theorem 2 yields the following two Corollaries, extending Result 3 and 4, respectively, from the single-risk case to the competing risk model environment.

**Corollary 1** (to Theorem 2). *Under Assumptions B1–B4 and B5a, the functional forms of Theorem 2 are given in (OA.2.43), (OA.2.44), and (OA.2.45) in the Online Appendix.*

**Corollary 2** (to Theorem 2). *Under Assumptions B1–B4 and B5b, the functional forms of Theorem 2 are given in (OA.2.47), (OA.2.48), and (OA.2.49) in the Online Appendix*

### 3.2. Flexible Heterogeneity in the CR Model

We will now proceed to an infinite mixture model for the distribution of  $v_{ki}$ .

**ASSUMPTION (B6).** *The prior for  $v_{ki}|G_k \sim G_k$  is specified as the hierarchical model  $G_k \sim DP(G_{0k}, \alpha_k)$ ,  $\alpha_k \sim g^G(a_{0k}, b_{0k})$ ,  $E[v_{ki}] = 1$ .*

The roles of the individual model components are described in Assumption A6 and generalize to the CR framework. Similarly to the single risk model environment, we consider two cases for the functional form of the baseline measure  $G_0$ :

**ASSUMPTION (B7a).** *In Assumption B6,  $G_{0k} = \mathcal{G}^{GIG}(\kappa_k, \varphi_k, \theta_k)$ .*

**ASSUMPTION (B7b).** *In Assumption B6,  $G_{0k} = \mathcal{G}^G(\gamma_k, \theta_k)$ .*

Implementation of the mixture models under Assumptions B1–B3, B6, and B7a or B7b uses the probabilities derived in Theorem 1, Corollary 1, and Corollary 2. Further implementation details are given in the next Section and in the Online Appendix.

Heckman and Honoré (1989) show how the introduction of covariates allows identification of a large class of dependent competing risks models without invoking distributional assumptions. Nonetheless, normalization assumptions are necessary for parameter identification. The normalization constraints generalize directly from the single-risk case and

we impose them for each risk type. Assumptions B2 and B4 impose explicit restrictions on the model behavior in continuous time within each time period. These restrictions allow us to invoke identification conditions of Heckman and Honoré (1989). A detailed discussion of identification is provided in the Online Appendix.

## 4. MCMC Posterior Sampling

### 4.1. Single Risk Model

All technical implementation details are provided in the Online Appendix. In this Section we summarize the main points. For the implementation of the Dirichlet Process Mixture model (Assumptions A6 and A7a,b) we used the Bayesian generalized Pólya urn scheme (Neal 2000 Algorithm 2; West, Müller, and Escobar, 1994; Bush and MacEachern, 1996). Implementation of the GIG mixture model (Assumptions A1–A3, A6, and A7a) uses the functions (2.8), (2.9), (2.18), and (2.19). The gamma mixture model (Assumptions A1–A3, A6, and A7b) uses (2.8), (2.9), (2.22) and (2.23), respectively. The remaining model parameters were sampled in standard Gibbs blocks using Hybrid Monte Carlo (Neal 2011) with diffuse priors unless stated otherwise above. The results are discussed in our application below.

### 4.2. Competing Risk model

Similarly to the single-risk case, for the implementation of the Dirichlet Process Mixture model in the competing risk environment (Assumptions B6 and B7a,b) we also used the Bayesian generalized Pólya urn scheme. Implementation of the GIG mixture model (Assumptions B1–B3, B6, and B7a) uses the functions (3.3) and (3.4) from Theorem 1, and Corollary 1 to Theorem 2. The gamma mixture model (Assumptions B1–B3, B6, and B7b) uses (OA.2.47) and (OA.2.49) from Corollary 2 to Theorem 2. The remaining model parameters were sampled in standard Gibbs blocks using Hybrid Monte Carlo (Neal 2011) with diffuse priors unless stated otherwise above, except the covariance matrix of Assumption B4 with parameters  $\sigma_1$ ,  $\sigma_2$ ,  $\rho$  for which we specify a proper Inverse Wishart prior with maximum possible dispersion,  $IW(K + 1, I_K)$  where  $I_K$  is the identity matrix of dimension  $K = 2$ . The results are discussed in our application below.



## 5. Application

Since its introduction in 1935 as part of Roosevelt's *Social Security Act*, unemployment insurance (UI) benefits provide partial insurance to workers who become unemployed. Most states offer unemployment insurance for up to 26 weeks. Neoclassical economic thought suggests that higher benefits also lead to reduced incentives to search for a job, thus prolonging the period of time an individual spends out of employment (Mulligan, 2012). As a result policy makers have placed increased emphasis on reforming the UI system by rewarding personal responsibility rather than bad luck. This has led to a shift away from the unqualified provision of UI benefits towards a system that is search intensive, making benefits conditional on providing evidence that the potential recipient engaged in a certain minimum amount of job search. Additionally, schemes whereby individuals are provided one-off grants that attempt to alleviate temporary hardship rather than longer term UI benefits are advocated. At the same time the recent economic crisis has forced policy makers to extend the duration of UI benefits for up to 99 weeks.<sup>7</sup>

Applied economists require econometric tools to accurately estimate the impact of unemployment insurance on the duration of unemployment, while accounting for state unemployment rates, generosity of unemployment insurance benefits and workers' observed and unobserved heterogeneity. In this section we apply our approach to analyzing the determinants of unemployment duration among unemployment insurance recipients. We will stress the importance of relaxing the parametric assumptions of the econometric models and accounting for correlations in the competing exit choices faced by workers. One of the major advantages of our approach is that it is possible to simulate counterfactual policy changes which can inform policy makers on the relative merits of various changes that may be contemplated. We will illustrate this feature by evaluating the impact of a change in the replacement rate on the duration of unemployment.

### 5.1. Data

We use data from the Needels et. al. (2001) report submitted to the U.S. Department of Labor that is based on a nationally representative sample built from individual-level surveys of unemployment insurance (UI) recipients in 25 states between 1998 and 2001.

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<sup>7</sup>The unemployment extension legislation was set to expire on January 1, 2013.

Candidates for the survey are selected on the basis of administrative records and are sampled from the pool of unemployed individuals that started collecting UI benefits at some point during the year 1998.

We are interested in analyzing the effect of unemployment insurance on the duration of unemployment. The duration of unemployment is measured in weeks. At the time of the survey and from the states that were included in the survey only two states provided UI benefits for a maximum of 30 weeks, the rest providing UI benefits for a maximum of 26 weeks. Theoretical models of the impact of UI benefits on unemployment duration, such as Mortensen (1977) and Moffitt and Nicholson (1982), predict an increasing hazard up to the point of benefit exhaustion and a flat one afterwards. We limit our study to the first 24 weeks of unemployment due to the recognized change in behavior in week 26 when UI benefits cease for a significant part of the sample (see, e.g., Han and Hausman, 1990), which would affect the econometric model.

The data contain individual-level information about labor market and other activities from the time the person entered the UI system through the time of the interview. The data include information about the individual's pre-UI job, other income or assistance received, and demographic information. We use two indicator variables, race (defined as an indicator for black) and age (defined as an indicator for over 50). We further use the replacement rate, which is the weekly benefit amount divided by the UI recipient's base period earnings. Lastly, we utilize the state unemployment rate of the state from which the individual received UI benefits during the period in which the individual filed for benefits. This variable changes over time. The Needels et. al. (2001) data shares certain similarities to the PSID dataset used in Han and Hausman (1990). The UI recipients are mostly white, young, poorly educated workers who find themselves below or very near the poverty line.<sup>8</sup>

Below we will estimate a single risk duration model for the duration to re-employment and also a competing risk duration model for the duration to re-employment using our proposed approach, which differentiates between workers who find a job in the same

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<sup>8</sup> Note that the labor market conditions captured in this dataset are substantially different than the ones experienced today. According to the Bureau of Labor Statistics (BLS), the latest figures broken down by state for September 2012 show the mean state unemployment rate is 7.5% and varies between 3% and 11.8%. In contrast, in our dataset the state unemployment rate is approximately 4.5%.

industry and workers who find a job in a different industry. For both analyses we employ subsamples from the Needels et. al. (2001) survey data, after removing outliers and observations with missing values. For the single risk model we use a subsample consisting of 15,358 individuals. Summary statistics for this sample are given in Table 2. For a subsample of 1,243 of the Needels et. al. (2001) data we also know the SIC codes of the employer before and after the unemployment spell. We denote individuals who find a job in the same industry as individuals with risk type 1, while those who find a job in a different industry as individuals with risk type 2. Summary statistics for this subsample are given in Table 2. We note a marked difference in the unemployment durations of these two groups of individuals. Figure 1 provides plots of the number of individuals who exited in each time period, shown separately for workers who find a job in the same industry and those who do not (risk type 2). Individuals who find a job in the same industry exit unemployment much faster in the first few weeks after they lose their job but conditional on not having found employment by week 8 their exit pattern resembles that of the other individuals. This is empirically interesting as it points towards the importance of industry specific human capital. We would expect variation among the industry specific human capital to vary by industry but also by individual reflecting their level of experience and motivation. It is thus to be expected that some individuals have accumulated a significant degree of human capital which makes them attractive to other employers in the same industry. Switching industries usually entails learning new skills and the incentive to do so may be affected by the duration and generosity of the unemployment insurance benefits. We would thus expect substantial behavioral differences between workers facing these two competing risks.

## 5.2. Single Risk Duration Model with Flexible Heterogeneity

Estimation results of the semiparametric duration model with a flexible form of unobserved heterogeneity under GIG mixing (Assumptions A1–A3, A6, and A7a) are presented in Table 3. In the Online Appendix we also present estimation results for two benchmark parametric models. Estimation results of a model with parametric gamma heterogeneity (Han and Hausman, 1990; Meyer 1990), as specified in Assumptions A1–A4 and A5b, are given in Table OA.1. In Table OA.2 we present estimation results for another benchmark model with parametric GIG heterogeneity (Assumptions A1–A4 and A5a).

In addition to the above mentioned censoring at  $T = 24$  weeks, we also include the benchmark cases where we censor at  $T = 6$  and  $T = 13$ . In the GIG mixture model, all of our variables (state unemployment rate, race, age, and replacement rate) are estimated to have a negative significant impact on the hazard rate of exiting unemployment. Note however, that when comparing the estimates of the  $\beta$  coefficients, the scaling changes depending on the variance of the estimated heterogeneity distribution. Thus, the ratios of the coefficients should be compared, as opposed to their absolute values. This makes the interpretation of the coefficients less transparent. We note however that the coefficient estimates obtained from the flexible model are substantively different than those obtained from the parametric model. As discussed above, the parametric restriction on the heterogeneity distribution can lead to inconsistent estimates if the true mixing distribution does not exactly correspond to the parametric specification.

We would expect to obtain similar results irrespective of the truncation point and thus the coefficients obtained for the models truncated at 6, 13, and 24 weeks to be very similar. While the coefficient ratios are not constant they tend to be relatively similar. The one exception comes from the ratios involving the replacement rate, in particular for the model with censoring at 24 periods. This is similar to the results in Hausman and Woutersen (2012) and might be explained by behavioral changes as individuals approach the date of UI exhaustion.

The estimated distribution of the unobserved individual heterogeneity is presented in Figure 2. The estimated distributions can only be very roughly approximated by the gamma distribution. While in all three cases the mode is negative, as we increase the number of periods used in the estimation the distribution acquires a more pronounced left tail. This indicates that as we observe individuals over a longer period of time the model captures to a larger extent the part of unobserved heterogeneity which prevents workers from finding employment and thus becomes indicative of the propensity for long term unemployment.

The survival function estimates along with 95% confidence bands are presented in Figure 3, featuring the anticipated downward sloping shape. The smoothing parameter  $\alpha$  of the Dirichlet Process (DP) Mixture model introduced in Assumption A6 controls the extent to which the DP draws mixture distributions that are more or less "similar" to

the baseline parametric distribution  $G_0$ . In the limiting case of  $\alpha \rightarrow \infty$  the mixture distribution becomes equivalent to  $G_0$ , while in the other extreme  $\alpha \rightarrow 0$  the mixture distribution limits to a convolution of density kernels centered at each data point without any influence of the DP prior. The posterior distribution estimates of  $\alpha$  are plotted in Figure OA.1 in the Online Appendix. The distributions are concentrated around a well-defined mode with a value of less than 1 indicating a strong influence of data relative to the baseline prior distribution thereby providing a high degree of support in favor of our nonparametric approach.

### 5.3. Competing Risk Model with Flexible Heterogeneity

We now present the results of our newly proposed competing risk model with a flexible form of unobserved heterogeneity using GIG mixing and correlated risks (Assumptions B1-B3, B6, and B7a). Note that in our example risk 1 corresponds to the event that a worker find a job in the same industry, while risk 2 corresponds to the event that she finds a new job in an industry that is different than the industry of her previous employer. We present the estimated coefficients in Table 4. For all three censoring times ( $T = 6, 13, 24$ ), the partial effects of race and age are not statistically significant, with a few isolated exceptions. This could be due to smaller sample size available for the competing risk case as opposed to the single-risk case, with the former consisting of less than 10% of observations of the latter. For all three censoring times, the relative influence of the replacement rate is declining from  $T = 13$  to  $T = 24$  indicating the impact of benefit exhaustion.

The estimated density<sup>9</sup> of unobserved heterogeneity  $V_{ik}$  is shown in Figure 4 for the GIG mixture model for both risk types  $k = 1, 2$ , each centered at the time average of the risk-specific latent common time effect  $\zeta_{kt}$  to reflect the overall influence of the unobserved heterogeneity component  $\zeta_{kt} + V_{ik}$ . The differences between the density of unobserved individual heterogeneity further highlight the importance of distinguishing between the different risk types in the competing risk model environment as compared to the single-risk duration case. In particular the two distributions of are distinct and well-separated

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<sup>9</sup> In Table 4 we report the estimated GIG mixture model coefficients but as these enter all mixing kernel moments their interpretation is not immediate. Hence it appears more informative to examine the resulting mixture density estimate.

indicating that conditional on observed covariates there is a significant degree of sorting between workers finding jobs in the same or in a different industry. While the mode of both distributions is negative, workers who find a job in the same industry possess latent attributes that make them more desirable than workers who are not. This may indicate the presence of unobserved industry specific human capital for these workers which make them more attractive to employers in the same industry.

The survival function estimates along with 95% confidence bands are presented in Figure 5 for both types of risks. The differences in the shapes for the first few weeks are striking and also indicative of the differences in exit rates between workers finding a job in the same industry compared who workers that find a job in a different industry. The estimated survival function for workers who find a job in the same industry is convex while that for workers who do not is concave, indicating a much slower overall re-integration into the labor market. This appears to confer a long term advantage with the overall probability of being unemployed being substantially higher for workers with no apparent industry specific human capital and which have no particular advantage in terms of finding a job in the same industry as their pre-unemployment firm.

Figure 6 shows the estimated correlation structure of the latent time variables  $\zeta_{kt}$  common to all individuals, defined in Assumption B4, for the GIG mixture model in terms of the estimated densities for the variances  $\sigma_1^2$ ,  $\sigma_2^2$ , and the correlation coefficient  $\rho$  between  $\zeta_{1t}$  and  $\zeta_{2t}$  for the two different risk types. Interestingly, most probability mass for the density of  $\rho$  is negative for  $T = 6$ , around zero for  $T = 13$  and positive for  $T = 24$ . This may correspond to a negative correlation of common shocks for jobs in the same and in different industries for the first several weeks of unemployment with a subsequent correlation reversal in later time periods, but may also be influenced by other factors, as estimates of other model elements also change. Nonetheless, such correlation pattern would explain the exit counts shown in Figure 1 for the two risks, with high ratios of jobs in the same industry to different industries for the first few weeks abetting to parity around week six.

The posterior distribution estimates of the smoothing parameter  $\alpha$  of the Dirichlet Process Mixture model (Assumption B6) are plotted in Figure OA.2 in the Online Appendix.

The distributions are concentrated around a well-defined mode of value less than five, indicating a strong influence of data relative to the baseline prior distribution.

It is informative to contrast the estimates from our preferred model with those obtained under different modeling assumptions on the unobserved heterogeneity which are detailed in the Online Appendix. Table OA.3 presents the estimated coefficients from a model that ignores the presence of individual heterogeneity, Table OA.4 corresponds to a model which assumes parametric gamma distributed heterogeneity, Table OA.5 estimates a competing risk model with parametric GIG heterogeneity, and Table OA.6 presents the estimation results from a flexible model which estimates the unobserved semi-parametrically using an infinite mixture of GIG distributions but also further imposes the assumption of independence between the different risks. In Table OA.7 we present estimation results from our single risk GIG mixture model but applied to the subsample of observations which records the outcomes for the two competing risks.

Given the large number of parameters to be considered it is helpful to compare these different models in a graphical setting. In order to facilitate the comparison between the model which pools the two risks and the models which do not we can combine the two risks into a common survival function, as discussed in the Online Appendix. Thus, in Figure 7 we compare the estimated survival function of our CR GIG mixture model (Assumptions 1–3, B6 and B7a, labeled as "CR full") with its estimates in three restricted model versions: 1) the parametric GIG case (labeled as "CR param"), 2) the independent risks case where we estimate a single-risk model separately for each risk type of data and then merge their survival functions ex-post (labeled as "CR indep"); 3) the case without individual unobserved heterogeneity under the restriction  $V_{ik} = 0$  (labeled as "CR no ihet"); and 4) the single-risk case where we do not distinguish between risk types in the competing risk data (labeled as "SR full").

Two features are particularly significant. First, we notice that if we enforce the assumption of independence of the two risk types, the resulting common survival function is severely downward biased. The magnitude of the bias dominates the other modeling choices which we make on the specification of unobserved heterogeneity. This could be due to the distributional effects of the risk correlations (Figure 7) that are absent in the independent risk model. Second, we plot the confidence bounds for our proposed model which allows for

a flexible specification of the unobserved heterogeneity but also for correlated competing risks. We notice that all other more restrictive specifications are downward biased and the differences become statistically significant as the number of time periods increases.

#### 5.4. Counterfactual Policy Evaluation

One of the advantages of our model consists in the explicit estimation of the unobserved heterogeneity components which enables us to evaluate the effectiveness of counterfactual policy experiments taking into account the distributional effects of individual heterogeneity. As discussed above, one of the main policy questions currently faced by economists is the extent to which the generosity of unemployment insurance benefits impacts the workers' incentives to find employment once they lose their job. On the one hand, more generous benefits are expensive to provide given the ongoing debt crisis and may actually prove detrimental in the long run as they may erode workers' incentives to find a job quickly. Thus they would ultimately contribute to increasing long run unemployment. On the other hand, low levels of unemployment insurance benefits can make unemployment very difficult for many low income families. Poverty can also have a negative effect on their ability to find employment since job search is costly and in the absence of unemployment insurance benefits many workers may find themselves unable to support their families while also searching for an adequate job. As a result workers may end up underemployed or leave the labor market altogether. The relative magnitude of the impact of incentives over poverty is an empirical question and a counterfactual analysis using model estimates can provide some evidence in this debate.

In the context of our model we can consider changing the replacement rate in order to investigate its impact on the probability of exit from unemployment as captured by the survival function. Note that we assume that this policy change does not impact the distribution of heterogeneity. We can perform this policy counterfactual using both the single risk and the competing risk model. For clarity, we combine the two risk types in the CR model into a common survival function as described in the Online Appendix. The counterfactual experiment consists in increasing and decreasing the replacement rate by 10%. We present counterfactual results from our preferred specification which flexibly models the unobserved heterogeneity as an infinite GIG mixture. The estimated and



counterfactual survival curves under the two scenarios are presented in Figure 8 (SR) and Figure 9 (CR).

Both Figures show that the survival function moves in the anticipated direction: for a replacement rate decrease the probability of staying unemployed is lower, and for replacement rate increase the probability of continued unemployment is higher. However, the changes are relatively small. For example, for  $T = 24$  in the final period the survival function changes by  $-1.7\%$  and  $1.6\%$  for the CR data, respectively. This suggests that while the estimated impact of a change in unemployment benefit generosity has the sign predicted by economic theory, the magnitude of the impact on the probability of unemployment exit is inelastic. Policy makers may thus wish to consider the extent to which cutting unemployment benefits may ultimately influence an unemployed worker’s welfare.

In the Online Appendix we further report on the results of two extensions of the counterfactual experiment that we briefly summarize here. First, we split individuals into two different subsamples based on their unobserved heterogeneity component – below and above the median. The results indicate that individuals with higher unobserved heterogeneity react more to replacement rate changes and have better chances exiting unemployment faster than their counterparts with lower unobserved heterogeneity. Second, we specify a time-varying replacement rate counterfactual change under two scenarios: one with sharp initial change and one with sharp late change. On average the survival function changes more under the former scenario, albeit inelastically in absolute terms.

## 6. Conclusion

We introduced a new flexible model specification for the competing risk model with piecewise linear baseline hazard, time-varying regressors, risk-specific unobserved individual heterogeneity distributed as an infinite mixture of density kernels, and a common correlated latent effect. Unobserved individual heterogeneity is assumed to be distributed according a Bayesian Dirichlet Process mixture model with a data-driven stochastic number of mixture components estimated along with other model parameters. We derive a tractable likelihood for Generalized Inverse Gaussian (GIG) mixing based on scaled GIG Laplace transforms and their higher-order derivatives. We find that mixing under

a special case of the GIG, the gamma kernel, leads to degenerate outcomes in nonparametric mixtures motivating the use of the more flexible GIG. We apply our approach to analyzing unemployment duration with exits to jobs in the same industry and to a different industry among unemployment insurance recipients on nationally representative individual-level survey data from the U.S. Department of Labor. We also conduct a counterfactual policy experiment that changes the replacement rate and find that the extent to which cuts in unemployment benefits incentivize unemployed workers is relatively very small.

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## 7. Appendix: Tables and Figures

TABLE 1. Overview of Assumptions

<i>Heterogeneity type</i>	<i>Single Risk</i>	<i>Competing Risks</i>
No heterogeneity	A1–A3	B1–B3
Parametric GIG	A1–A4, A5a	B1–B4, B5a
Parametric gamma	A1–A4, A5b	B1–B4, B5b
Flexible GIG mixture	A1–A3, A6, A7a	B1–B3, B6, B7a
Flexible gamma mixture	A1–A3, A6, A7b	B1–B3, B6, B7b

TABLE 2. Summary Statistics

Variable	Duration Data				Competing Risk Data			
	Mean	S.D.	Min	Max	Mean	S.D.	Min	Max
Duration	23.245	16.334	1	63	27.958	28.410	1	140
Race	0.117	0.321	0	1	0.107	0.310	0	1
Age	0.177	0.382	0	1	0.181	0.385	0	1
Replacement Rate	0.6558	0.3082	0.016	1.8434	0.660	0.324	0.015	2.173
State unemp rate:								
period 1	4.686	1.087	2	6.9	4.579	1.125	2	7.5
period 2	4.672	1.083	2	6.9	4.568	1.120	2	7.5
period 3	4.660	1.079	2	6.9	4.562	1.107	2	7.5
period 4	4.645	1.074	2	6.9	4.553	1.103	2	7.8
period 5	4.630	1.069	2	6.9	4.536	1.092	2	8.1
period 6	4.621	1.064	2	6.9	4.533	1.083	2	8.1
period 7	4.616	1.066	2	6.9	4.538	1.080	2	8.1
period 8	4.598	1.064	2	6.9	4.527	1.071	2	7.4
period 9	4.570	1.061	2	6.9	4.518	1.070	2	7.2
period 10	4.538	1.061	2	6.9	4.486	1.070	2	7.2
period 11	4.531	1.063	2	6.9	4.481	1.072	2	7.2
period 12	4.509	1.067	2	6.9	4.458	1.082	2	6.9
period 13	4.483	1.075	2	6.9	4.438	1.091	2	6.9
period 14	4.462	1.080	2	6.9	4.412	1.098	2	6.9
period 15	4.460	1.075	2	6.9	4.404	1.102	2	6.9
period 16	4.449	1.073	2	6.9	4.390	1.097	2	6.9
period 17	4.439	1.067	2	6.9	4.376	1.088	2	7.8
period 18	4.440	1.055	2	6.9	4.368	1.078	2	7.8
period 19	4.431	1.054	2	6.9	4.357	1.073	2	7.8
period 20	4.420	1.045	2	6.9	4.342	1.066	2	7.8
period 21	4.423	1.033	2	6.9	4.338	1.052	2	7.5
period 22	4.431	1.029	2	6.8	4.335	1.038	2	7.4
period 23	4.436	1.024	2	6.7	4.345	1.037	2	7.4
period 24	4.441	1.015	2	6.7	4.353	1.037	2	7.4
Observations		15, 358				1, 243		

TABLE 3. New Semiparametric Duration Model, GIG Mixture

		6 periods		13 periods		24 periods	
		Mean	s.e.	Mean	s.e.	Mean	s.e.
$\kappa$		-0.963	0.039	-1.226	0.034	-1.659	0.101
$\varphi$		2.284	0.110	3.022	0.097	4.240	0.285
<i>Urate</i>		-0.184	0.021	-0.214	0.014	-0.327	0.034
<i>Race</i>		-0.055	0.069	-0.126	0.042	-0.145	0.050
<i>Age</i>		-0.194	0.057	-0.178	0.035	-0.187	0.039
<i>Rrate</i>		-0.924	0.076	-0.473	0.051	-0.147	0.066
$t$	1	-2.142	0.116	-2.264	0.090	-1.971	0.181
	2	-1.716	0.109	-1.843	0.088	-1.573	0.175
	3	-2.026	0.112	-2.157	0.096	-1.878	0.182
	4	-1.733	0.112	-1.865	0.085	-1.578	0.182
	5	-2.070	0.112	-2.198	0.099	-1.928	0.185
	6	-1.833	0.102	-1.829	0.084	-1.547	0.184
	7			-2.336	0.100	-2.061	0.205
	8			-2.026	0.087	-1.743	0.189
	9			-2.347	0.097	-2.099	0.181
	10			-2.130	0.087	-1.871	0.189
	11			-2.347	0.087	-2.086	0.194
	12			-2.120	0.095	-1.856	0.196
	13			-2.277	0.074	-1.904	0.195
	14					-1.840	0.191
	15					-1.775	0.191
	16					-1.740	0.195
	17					-1.981	0.206
	18					-1.641	0.191
	19					-1.848	0.194
	20					-1.674	0.191
	21					-1.832	0.198
	22					-1.738	0.199
	23					-1.913	0.211
	24					-1.006	0.179

$N = 15,491$ , *Urate* denotes the state unemployment rate, *Rrate* denotes the replacement rate.



TABLE 4. New Semiparametric Competing Risk Model, GIG Mixture

		6 periods				13 periods				24 periods			
		Risk 1		Risk 2		Risk 1		Risk 2		Risk 1		Risk 2	
		Mean	s.e.	Mean	s.e.	Mean	s.e.	Mean	s.e.	Mean	s.e.	Mean	s.e.
$\kappa$		-1.535	0.046	-1.516	0.046	-1.482	0.057	-1.554	0.062	-1.407	0.044	-1.492	0.047
$\varphi$		3.890	0.129	3.838	0.304	3.741	0.162	3.944	0.176	3.532	0.126	3.771	0.134
<i>Urate</i>		-0.111	0.055	-0.103	0.083	-0.107	0.048	-0.117	0.063	-0.216	0.040	-0.095	0.056
<i>Race</i>		0.054	0.225	0.033	0.424	-0.044	0.183	-0.377	0.343	-0.046	0.157	0.312	0.256
<i>Age</i>		-0.008	0.176	-0.562	0.357	-0.016	0.154	-0.700	0.290	-0.036	0.130	-0.461	0.212
<i>Rrate</i>		-1.062	0.227	-0.465	0.390	-0.903	0.181	-0.411	0.288	-0.385	0.156	-0.331	0.226
$t$	1	-2.553	0.314	-4.864	0.516	-2.680	0.276	-4.741	0.503	-2.496	0.256	-4.947	0.511
	2	-1.993	0.296	-4.625	0.491	-2.113	0.255	-4.514	0.504	-1.933	0.234	-4.715	0.475
	3	-2.324	0.311	-4.937	0.565	-2.445	0.272	-4.830	0.558	-2.263	0.253	-5.027	0.554
	4	-2.022	0.302	-3.633	0.400	-2.138	0.263	-3.506	0.388	-1.962	0.243	-3.728	0.372
	5	-2.470	0.325	-3.638	0.413	-2.580	0.28	-3.516	0.391	-2.406	0.273	-3.729	0.376
	6	-3.854	0.290	-3.711	0.212	-3.044	0.322	-3.327	0.386	-2.872	0.308	-3.543	0.367
	7					-2.947	0.321	-4.361	0.518	-2.764	0.304	-4.599	0.508
	8					-2.493	0.295	-3.068	0.372	-2.318	0.277	-3.285	0.355
	9					-3.750	0.422	-3.984	0.479	-3.571	0.417	-4.203	0.460
	10					-2.620	0.305	-3.950	0.471	-2.449	0.293	-4.149	0.451
	11					-3.232	0.362	-3.925	0.480	-3.078	0.355	-4.129	0.449
	12					-2.791	0.326	-3.866	0.445	-2.625	0.305	-4.093	0.454
	13					-3.799	0.341	-3.829	0.409	-2.703	0.317	-4.057	0.453
	14									-2.738	0.329	-3.897	0.437
	15									-2.511	0.309	-3.634	0.406
	16									-3.074	0.376	-3.927	0.452
	17									-2.127	0.284	-3.872	0.453
	18									-2.564	0.335	-4.589	0.569
	19									-2.513	0.322	-3.285	0.388
	20									-2.702	0.347	-3.884	0.457
	21									-1.850	0.278	-3.569	0.424
	22									-3.795	0.540	-4.719	0.652
	23									-2.133	0.307	-3.780	0.464
	24									-3.640	0.322	-3.770	0.106

$N = 1, 243$ , *Urate* denotes the state unemployment rate, *Rrate* denotes the replacement rate.

FIGURE 1. Empirical Exit Count for Competing Risk Data

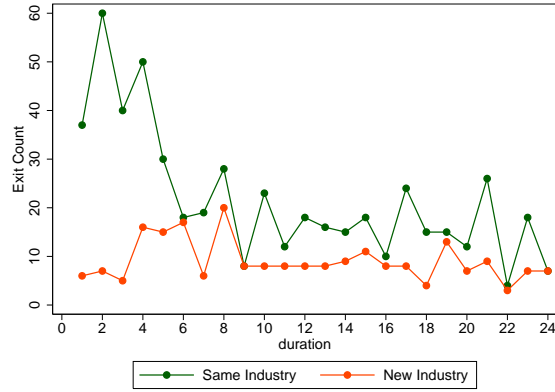


FIGURE 2. Density of individual heterogeneity component  $v_i$ , GIG mixture

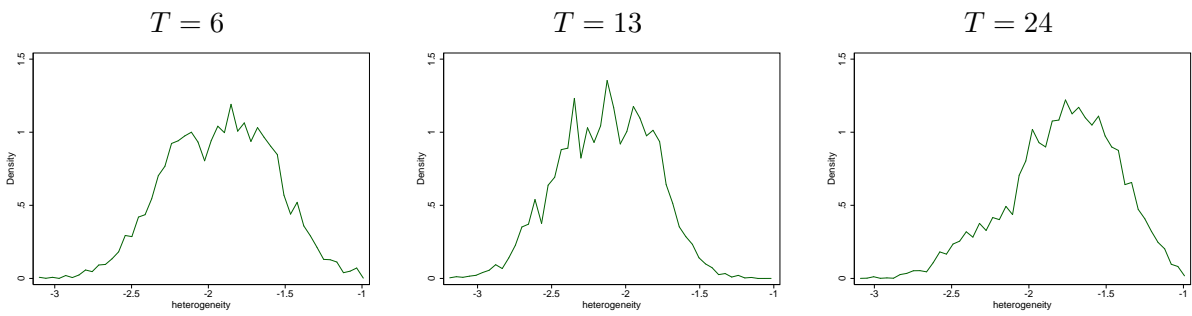


FIGURE 3. Survival function, GIG mixture

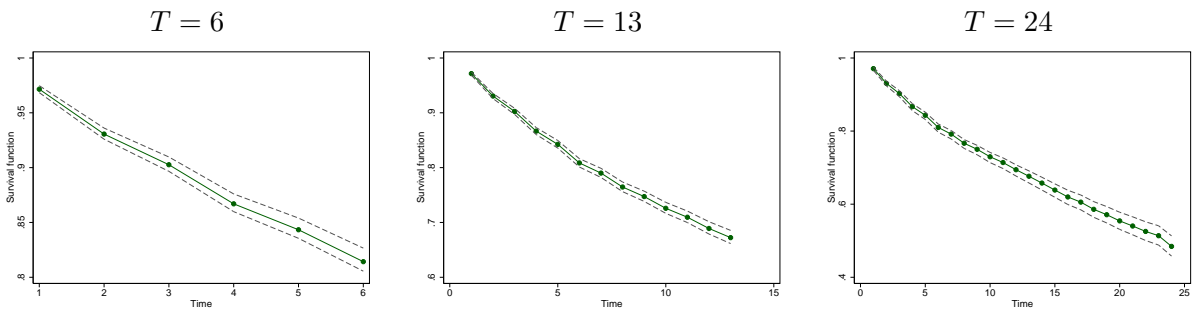


FIGURE 4. Heterogeneity density, GIG mixture

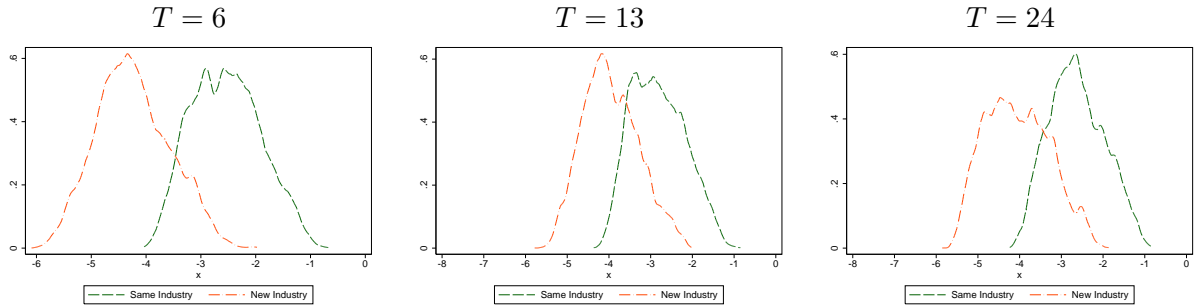


FIGURE 5. Survival function, GIG mixture

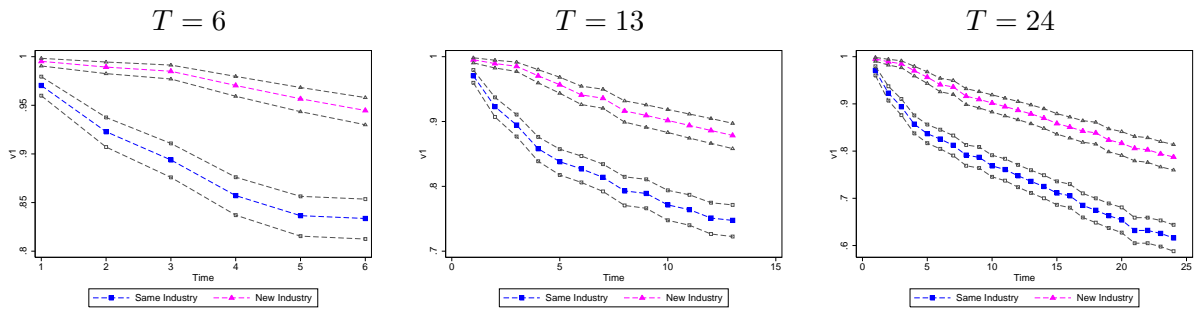


FIGURE 6. Correlation structure of  $\zeta_t$ : density of  $\sigma_1^2$ ,  $\sigma_2^2$ , and  $\rho$ , GIG mixture

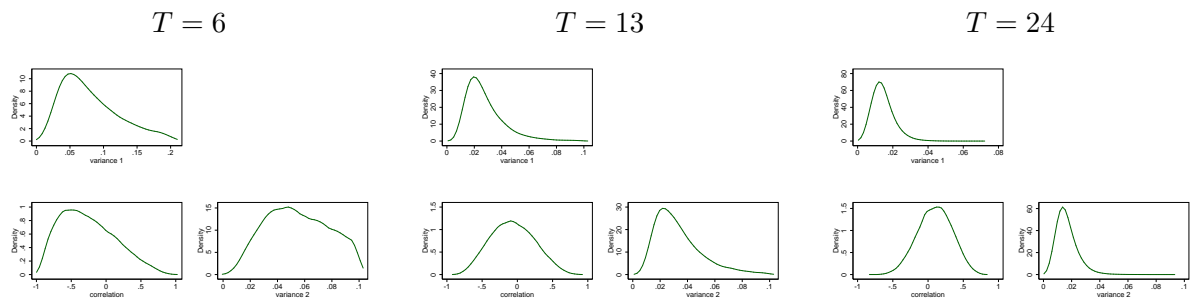


FIGURE 7. Model Comparison in Terms of Survival Functions

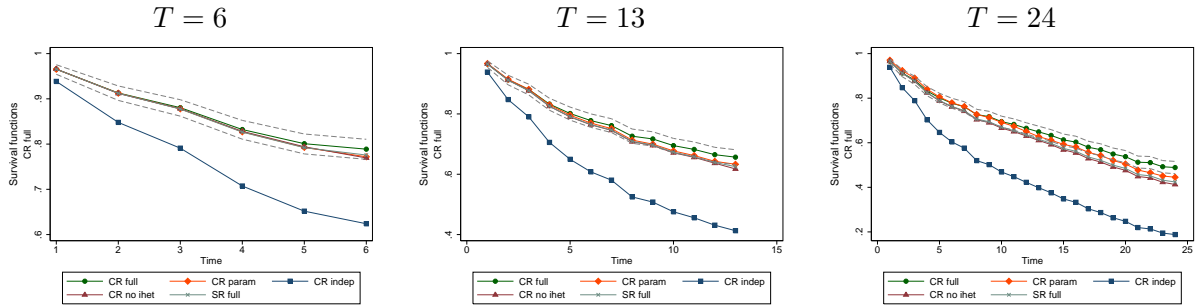


FIGURE 8. Counterfactual Experiment for the Single Risk GIG Mixture

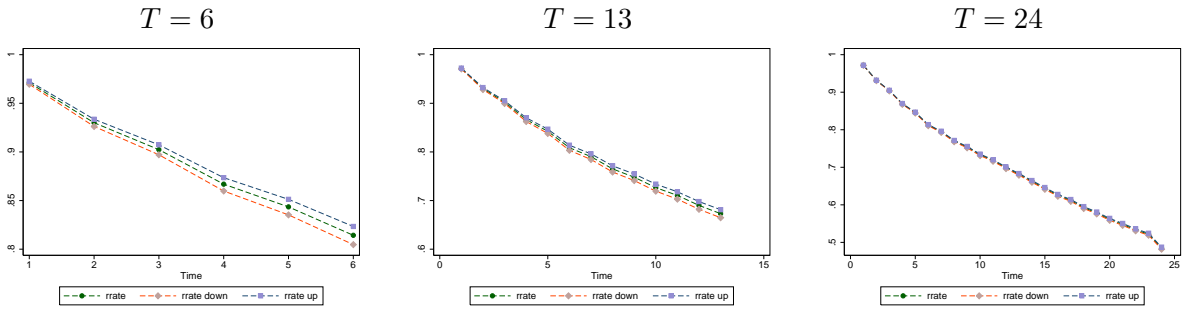


FIGURE 9. Counterfactual Experiment for the Competing Risks Model using a GIG Mixture and Combining the Risks.

