Changing the Boston School Choice Mechanism: Strategy-proofness as Equal Access*

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Abstract

In July 2005 the Boston School Committee voted to replace the existing Boston school choice mechanism with a deferred acceptance mechanism that simplifies the strategic choices facing parents. This paper presents the empirical case against the previous Boston mechanism, a priority matching mechanism, and the case in favor of the change to a strategy-proof mechanism. We present evidence both of sophisticated strategic behavior among some parents, and of unsophisticated strategic behavior by others. We find evidence that some parents pay close attention to the capacity constraints of different schools, while others appear not to. In particular, we identify a certain kind of mistake that can be observed in the data without knowing the true preferences of a family. Families that make this mistake are disproportionately unassigned, and in many cases they would have been assigned but for the mistake. This interaction between sophisticated and unsophisticated players identifies a new rationale for strategy-proof mechanisms based on fairness, and was a critical argument in Boston's decision to change the mechanism.

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1 Introduction

In July 2005 the Boston School Committee voted to replace the existing school choice mechanism (henceforth the **Boston mechanism**) with an alternative mechanism that removes the incentives to "game the system" that handicapped the Boston mechanism. This followed two years of intensive discussion and analysis of the existing school choice system and the behavior it elicited, as well as a discussion of two different possible replacement school choice mechanisms.[1] As far as we know, it is the first time that "strategy-proofness," a central concept in the game theory literature on mechanism design, has been adopted as a public policy concern related to transparency, fairness, and equal access to public facilities.

We were first invited to meet with the Boston Public Schools (BPS) planning team in October, 2003, following a Boston Globe story (Cook 2003) highlighting some of the vulnerabilities of the Boston mechanism as analyzed by Abdulkadiroğlu and Sönmez (2003) and Chen and Sönmez (2006), and comparing it unfavorably with the clearinghouse used to match medical residents to hospitals (Roth and Peranson 1998). In that meeting, we

1. presented theoretical, historical, and experimental evidence about the vulnerability of the Boston mechanism to preference misrepresentation (Abdulkadiroğlu and Sönmez 2003, Chen and Sönmez 2006, Roth 1991),
2. explained how such "gaming" may harm the system, and
3. presented the outlines of two alternative mechanisms which are **strategy-proof** and hence immune to preference manipulation: a student-proposing deferred acceptance mechanism like that now used to match residents to hospitals, and a top trading cycles mechanism.

Loosely speaking, the Boston mechanism attempts to assign as many students as possible to their first choice school, and only after all such assignments have been made does it consider assignments of students to their second choices, etc. The problem with this is that if a student is not admitted to his first choice school, his second choice may already be filled with students who listed it as their first choice. That is, a student may fail to get a place in his second choice school that would have been available had he listed that school as his first choice. This has the potential both to change the preference rankings that some families submit, and to work to the disadvantage of families that fail to take account of such strategic considerations.

The Boston mechanism is one of a class of "priority mechanisms" that were tried to match medical graduates to positions in regions of the British National Health Service and eventually abandoned (Roth 1990, 1991). Versions are widely used in American school

[1] A school choice mechanism is a function that assigns students to schools for each school choice problem. Algorithms implement mechanisms. We will sometimes (ab)use the terms mechanism and algorithm as synonyms.

choice systems, for example in Cambridge, Charlotte-Mecklenberg, Denver, Miami-Dade, Rochester, Tampa-St. Petersburg, and White Plains.

Based on the evidence, BPS staff were willing to entertain the possibility that Boston families might be engaged in strategic behavior through preference manipulation.[2] However, they wanted us to demonstrate this, and examine its consequences empirically, in the Boston data. To this end, they provided us with micro-level datasets on the choices, student characteristics, and school characteristics, and pressed us to make the empirical case for changing the Boston mechanism.

This paper presents the results of this exercise and describes the arguments for changing the Boston mechanism. While we cannot know if any particular preference list reflects a given family's true preferences, we will show that some families explicitly strategize like game theorists, and the pattern of submitted preferences, particularly in connection with the most desirable schools, reflects in broad outline the patterns that would be predicted if families are taking into account the strategic incentives that the Boston mechanism gives them. We will also show that there are families that fail to adequately take into account the strategic properties of the mechanism, and that some of them are harmed by this.

Since we do not know students' true preferences, we will not be able to assess directly any inefficiency in the current allocation. But to the extent that families are not reporting their true preferences, and only imperfectly strategizing, the indirect evidence will be strong that the outcome is inefficient.

Although our discussions with BPS often focused on broad patterns of behavior, each of these in isolation can be difficult to interpret without knowing students' true preferences. For the present paper, to show unequivocally that there are both families that strategize well and those that do not, we will focus on a particular strategic mistake under the Boston mechanism (listing an overdemanded school as a second choice) explicitly identified by one organized parents' group, and show that many other parents fail to avoid this mistake. Because this particular mistake can be identified in the data without knowing true preferences (and because it is a mistake for any preferences) it will allow us to show both that there are strategically sophisticated and well informed families who do not feel free to submit their true preferences, and other families that fail to "play the game" optimally and sometimes suffer as a result.

2 The Boston Mechanism

In Boston, students are assigned seats at public schools through a centralized student assignment mechanism.[3] Each spring, students who seek a spot in Kindergarten,[4] Grades

[2]Throughout the paper, we define strategic behavior or strategizing as submitting a preference list to the assignment mechanism that differs from the true preferences.

[3]There are several special admission high schools that process applicants separately.

[4]Boston public schools have three kindergarten entering levels: K0 programs are for children who turn 3 years old by September 1st, K1 programs for children who turn 4, and K2 programs for children who turn 5.

1, 6, and 9 are asked to submit a preference ranking of no more than five schools. Students in the remaining non-transition grades continue on in their current school unless they request and receive a transfer.

For most schools, for half of the seats at a given school the students are priority ordered as follows:

1. Students who are guaranteed a space at the school by virtue of already attending that school or a feeder school (**guaranteed priority**),
2. students who have a sibling at the school and live in the walk zone of the school. (**sibling-walk priority**),
3. students who have a sibling at the school (but who do not live in the walk zone of the school) (**sibling priority**),
4. students who live in the walk zone of the school (but who do not have a sibling at the school) (**walk zone priority**), and
5. other students in the zone.

A random lottery number for each student breaks ties in each category (**random tie-breaker**).

For the other half of the seats, walk zone priorities do not apply, and students are priority-ordered based on guaranteed and sibling priority and the random tie-breaker. Students who are not in the main transition grades continue on in their current school through **guaranteed priority**. Guaranteed priority is also given to students in transition grades who attend combined elementary and middle schools (grades K2-8) or middle and high schools (grades 6-12). The district is divided into three zones, East, North and West. Most elementary and middle schools are zone schools, which admit students only from their zone. A few elementary and middle schools are **citywide schools**, to which students from all zones can apply. All high schools are citywide.[5]

Based on preferences, priorities and school capacities, student assignments are determined by the following algorithm:

Step 1: In Step 1 only the first choices of the students are considered. For each school, consider the students who have listed it as their first choice and assign seats of the school to these students one at a time following their priority order until either there are no seats left or there is no student left who has listed it as his first choice.[6]

In general,

[5]The details of the entire priority structure are in Appendix 1 of our NBER working paper, Abdulkadiroglu et al. (2006).

[6]For schools with walk zone priority for half of the seats, the school is treated as two identical schools, each half the size of the original school, only one of which gives priority to students from the walk zone. Students are assumed to prefer the half that gives walk zone priority, and both halves are adjacent in each student's preferences.

Step k: In Step k only the k^{th} choices of the students not previously assigned are considered. For each school with still available seats, assign the remaining seats to the students who have listed it as their k^{th} choice, one at a time following their priority order, until either there are no seats left or there is no student left who has listed it as her k^{th} choice.

The procedure terminates after any step k when every student is assigned a seat at a school, or if the only students who remain unassigned listed no more than k choices.

The Boston mechanism is not **strategy-proof**; that is, students (or their parents) may improve their assignments by misrepresenting their preferences. Since a student who ranks a school as her second choice loses her priority to students who rank it as their first choices, it is very risky for the student to "waste" her first choice at a highly sought after school if she has relatively low priority. And excellent information is available from past years about which schools are highly sought after.[7] Hence the Boston mechanism gives students and their parents a strong incentive to misrepresent their preferences by improving the ranking of schools for which they have high priority.[8]

There are many signs that both the school district and families are aware that students may not always want to rank schools truthfully. The BPS school guide [2004, p3] explicitly advises parents to strategize when submitting their preferences (quotes in original)[9]:

> For a better chance of your "first choice" school ... consider choosing less popular schools. Ask Family Resource Center staff for information on "underchosen" schools.

Although it may be difficult to identify an *optimal* strategy, there are many strategies that might produce better results than truthful revelation of preferences for a family whose top choices are overdemanded schools. Ranking an underdemanded school as first choice, as suggested by BPS, is one such possibility. Another is ranking as first choice a desirable overdemanded school for which the student has sufficient priority to have a chance of admission, and ranking a less preferred underdemanded school second.

In Boston, the West Zone Parent Group (WZPG) recommends both kinds of strategies. Their introductory meeting minutes on 10/27/03 state:

[7] Aside from being available from other parents, this information is available at Family Resource Centers, and on BPS websites. For example, in 2006 the website reports, for each school, the number of seats available in 2005, and the number of applicants who listed that school as their first, second, and third choices (see the school choice data documents at http://boston.k12.ma.us/register/documents.asp)

[8] While students are exogenously priority ordered at each school, the "effective priorities" are endogenous under the Boston mechanism in the sense that each student who ranks a school as her k^{th} choice is considered before each student who ranks it $(k+1)^{st}$. The exogenous priorities are only utilized to tie-break among students who have ranked a school at the same rank order. It is this ability of students to influence the effective priorities that makes the Boston mechanism vulnerable to preference manipulation. (And the fact that Boston students were not allowed to list more than 5 schools introduces another way in which they might not be able to report their true preferences.)

[9] Other school districts that employ variants of the Boston mechanism make similar suggestions.

> One school choice strategy is to find a school you like that is undersubscribed and put it as a top choice, OR, find a school that you like that is popular and put it as a first choice and find a school that is less popular for a "safe" second choice.

Whether either type of manipulation makes sense for a particular student depends on many factors including how popular her top choices are, her priorities at schools, and her attitude towards risk. Among experimental subjects, both rules-of-thumb are fairly common (Chen and Sönmez 2006).

Figure 1 contains illustrative quotes from parents (drawn from the WZPG's online email archive) about strategic decision-making in the submission of preferences. These quotes make clear that, in formulating their rank order lists, some parents are factoring in not only their preferences, but also the allocation procedure and which schools are highly demanded.

Figure 1 about here.

We can formalize the observations about strategy made by the West Zone Parents Group, as follows.

Definition: A school is *overdemanded* if the number of students who rank that school as their **first** choice is greater than the number of seats at the school.

Proposition: No one who lists an overdemanded school as a *second* choice will be assigned to it by the Boston mechanism, and listing an overdemanded school as a second choice can only reduce the probability of receiving schools ranked lower.

Note that it certainly need not be a mistake to submit a preference list that has an overdemanded school as the first choice: this may be a sensible gamble. But, as the West Zone Parents Group note, it *is* a mistake to submit a preference list that has *two* overdemanded schools as the top two choices, since the second choice is wasted, and since wasting the second choice reduces the chance of getting the third or subsequent choices. We will see in the data that, although many families in the West Zone and elsewhere appear to have incorporated these considerations into their preference lists, many other families make the mistake identified by the West Zone Parents Group, and some of them suffer *ex post* as a result. (We need not be concerned here whether such a mistake arises from a lack of appreciation of the strategic properties of the Boston mechanism, or from lack of information about which schools are overdemanded, because a strategy-proof mechanism will protect families against both kinds of errors.)

By concentrating on families that have either identified and avoided this strategic mistake, or failed to avoid it, we remove many of the difficulties involved in assessing empirically the extent of strategic play or its absence when the true preferences are unknown, since this particular mistake can be observed in the preference list data without knowing the true preferences of a family. To foreshadow our detailed empirical results, of the 15,135 students on whom we concentrate our analysis, 19% (2910) listed two overdemanded schools as their top two choices, and about 27% (782) of these ended up unassigned. Of these unassigned students, about 8% (63) could have gotten one of their ranked choices if they had done nothing more than omitted the second overdemanded school from their list. Thus this empirical strategy allows us to show that some families strategize well while others do not. Since there are many other ways in which families can strategize well or badly, in ways that depend on their true preferences, we do not attempt to estimate the percentage of families in either set.

3 Two Strategy-Proof Mechanisms

Before looking at the Boston data, we review the two alternative mechanisms we proposed, the deferred acceptance and top trading cycles mechanisms. In both mechanisms, students may submit preference lists containing as many schools as they wish.

The deferred acceptance algorithm was first studied by Gale and Shapley (1962) in the context of *two-sided matching markets*, i.e. markets in which there are two kinds of agents needing to be matched to one another. It produces *stable* matchings, i.e. matchings with the property that there do not exist two agents, not matched to one another, who would both prefer to be matched to one another. Its principal incentive properties were established for simple, one-to-one matching by Dubins and Freedman (1981) and Roth (1982), and for many to one matching problems of the kind we study here (in which each school admits many students, each of whom is admitted only to one school) by Roth (1985) and Roth and Sotomayor (1990).[10] For many-to-one matching problems, there do not exist any stable matching mechanisms that are strategy-proof for the schools, but the student-proposing deferred acceptance mechanism is strategy-proof for the students.

A decade before deferred acceptance algorithms were first formally studied, the market for medical residents developed a clearinghouse algorithm that is "outcome equivalent" to the hospital-proposing deferred acceptance algorithm (Roth 1984, 1995). (That is, they were using a different algorithm but the same mechanism.) More recently, versions of the deferred acceptance algorithm have become widely used in centralized clearinghouses that serve two-sided matching markets. Some examples include not only medical residents in the U.S., U.K., and Canada (Roth 1990, 1991), but a variety of more advanced medical positions (including the fellowship positions through which doctors become certified into different medical subspecialties; see e.g. Niederle and Roth 2003a,b, 2005), the markets for a number of other medical and health care professionals (see e.g. Table 1 in Roth and

[10] More recently, Abdulkadiroğlu (2005) extends the incentive results in a model with type-specific quotas, which also applies to the controlled school choice problem in which choice is restricted by racial quotas at schools. Hatfield and Milgrom (2005) obtains the incentive results in a model of matching with contracts which incorporates, as special cases, the college admissions problem, the Kelso-Crawford labor market matching model, and ascending package auctions.

Rothblum, 1999), and even newly graduating Reform Rabbis (Bodin and Panken, 1999). At the time we first spoke to BPS, New York City was in the process of replacing its decentralized system of high school admissions with a centralized clearinghouse based on the deferred acceptance algorithm (Abdulkadiroğlu, Pathak, and Roth 2005a,b). In this respect NYC schools resembled most of the other matching markets in which centralized clearinghouses have been introduced, in that a centralized clearinghouse was replacing a failing decentralized matching process, and the agents on both sides of the market, e.g. hospitals and medical graduates, or high schools and students, were active players.

The situation in Boston in 2003 was quite different. Boston already had a centralized school choice mechanism. And in Boston, only the students and their families were active players, the schools were passive, with priorities set by the central administration. That is, in Boston, school choice does not involve two kinds of agents who make choices: only students make choices, by submitting preference lists. The priorities that they have at each school are fixed in advance.

Abdulkadiroğlu and Sönmez (2003) observed that priorities students have at each school have the same form as school preferences, and hence two-sided matching mechanisms have counterparts in the context of school choice problems as in Boston. They also observed that the counterpart of stability is what they referred as *elimination of justified envy*, i.e. there should not be a student who prefers to his assignment a school that either has a vacant seat or has admitted a student with lower priority. Since only students are agents in Boston, the student-proposing deferred-acceptance mechanism is strategy-proof but not always Pareto efficient (when only student welfare is considered), although it Pareto dominates any other matching that eliminates justified envy. Abdulkadiroğlu and Sönmez (2003) also considered a Pareto efficient mechanism based on the method of top trading cycles, introduced in Shapley and Scarf (1974), and further studied by Roth and Postlewaite (1977). In the original context of "housing markets" Roth (1982b) showed that this method is strategy-proof and Abdulkadiroğlu and Sönmez (1999) showed how to extend it to more complex allocation problems. At the time we first met with BPS, discussions were underway to organize what eventually became the New England Program for Kidney Exchange, based on a proposal for a version of the top trading cycles mechanism outlined in Roth, Sönmez, and Ünver (2004).

So, at the time of our initial meeting, we were in a position to offer two different kinds of strategy-proof mechanisms for consideration in Boston school choice.

1. Student-Proposing Deferred Acceptance mechanism

As already mentioned, it is costly under the Boston mechanism to list a first choice that you do not succeed in getting because, once other students are assigned their first-choice places, they cannot be displaced even by a student with higher priority. This is avoided under the **student-proposing deferred acceptance** algorithm. For a given list of priorities, student preferences and school capacities, this mechanism determines a student assignment as follows:

Step 1: Each student "proposes" to her first choice. Each school tentatively assigns its seats to its proposers one at a time in their priority order until capacity is reached. Any remaining proposers are rejected.

In general, at

Step k: Each student who was rejected in the previous step proposes to her next choice if one remains. Each school considers the set consisting of the students it has been holding and its new proposers, and tentatively assigns its seats to these students one at a time in priority order. Any students in the set remaining after all the seats are filled are rejected.

The algorithm terminates when no student is rejected, and each student is assigned her final tentative assignment.

In contrast with the Boston algorithm, the deferred acceptance algorithm assigns seats only tentatively at each step, so students with higher priorities may be considered in subsequent steps. Consequently it is **stable** in the sense that there is no student who loses a seat to a lower priority student and receives a less-preferred assignment. Moreover all students prefer their outcome to any other stable matching (Gale and Shapley 1962) and the induced mechanism is strategy-proof (Roth 1985).

2. Top Trading Cycles mechanism.

If the intention of the school board is that priorities be "strictly enforced," the student proposing deferred acceptance mechanism is a leading candidate.[11] However, if welfare considerations apply only to students, there is tension between stability and Pareto optimality (Roth 1982, Balinski and Sönmez 1999, Abdulkadiroğlu and Sönmez 2003). If priorities are merely a device for allocating scarce spaces, it might be possible to assign students to schools they prefer by allowing them to trade their priority at one school with a student who has priority at a school they prefer. The **top trading cycles** mechanism (TTC) creates a virtual exchange for priorities. For a given list of priorities, student preferences and school capacities this mechanism determines a student assignment with the following algorithm:

Step 1: Assign counters for each school to track how many seats remain available. Each student points to her favorite school and each school points to the student with the highest priority. There must be at least one cycle. (A cycle is an ordered list of schools

[11] In Turkey, admissions to colleges (public or private) is through a similar centralized clearinghouse where priorities are "earned" based on some exams and hence should be strictly enforced. Hence the student-proposing deferred acceptance mechanism is especially appealing in this context (Balinski and Sönmez 1999).

and students (student 1 - school 1 - student 2 - ... - student k - school k) with student 1 pointing to school 1, school 1 to student 2, ..., student k to school k, and school k pointing to student 1.) Each student is part of at most one cycle. Every student in a cycle is assigned a seat at the school she points to and is removed. The counter of each school is reduced by one and if it reaches zero, the school is removed.

In general, at

Step k: Each remaining student points to her favorite school among the remaining schools and each remaining school points to the student with highest priority among the remaining students. There is at least one cycle. Every student in a cycle is assigned a seat at the school she points to and is removed. The counter of each school in a cycle is reduced by one and if it reaches zero, the school is removed.

The procedure terminates when each student is assigned a seat or all submitted choices have been considered.

This version of the TTC mechanism was introduced by Abdulkadiroğlu and Sönmez (2003) and is an extension of **Gale's top trading cycles** mechanism described in Shapley and Scarf (1974). Many properties of TTC carry over to school choice including Pareto efficiency (Shapley and Scarf 1974) and strategy-proofness (Roth 1982b).

While TTC is a Pareto efficient mechanism when only students are considered, and the student-proposing deferred acceptance mechanism is not, the former does not Pareto dominate the latter. One implication is, based on a stronger efficiency notion (such as a cardinal efficiency notion relying on the rank order of schools) the student-proposing deferred acceptance mechanism may perform better than the TTC for some problems. For example, the student-proposing deferred acceptance mechanism may assign more students to their first choices than TTC. Moreover while each Nash equilibrium outcome of the complete information preference revelation game induced by the Boston mechanism is weakly Pareto dominated by each dominant-strategy equilibrium outcome of the student-proposing deferred acceptance mechanism (Ergin and Sönmez 2006), equilibrium outcomes induced by the Boston mechanism and TTC are not Pareto ranked.

4 Data

4.1 Data construction

All data for this paper were provided by Boston Public Schools from their assignment system or the school guide for the corresponding year. The data include student choices and assignments and school priorities and capacities.

The last major change to the student assignment algorithm in Boston occurred in 1999. Prior to that, the assignment mechanism was based on a system of quotas on race and other factors. The new assignment system adopted in 2000-01 and described in

Section 2 stayed mostly the same through the assignment year 2004-05.[12] To avoid the complications with the transition from the old quota-based admissions plan, we focus our empirical analysis on the second year of the system, school year 2001-02.[13]

In the 2001-2002 school year, students in transition grades submitted their application by February for a school spot in September. We focus on students in the main transition grades: K2, 6, and 9.[14]

The assignment system has three rounds with the majority of students participating in the first. Boston Public Schools strongly encourages students to apply by the first deadline and informs families that their choices are more limited the longer they wait (see e.g. Boston School Guide, 2001, page 5).[15] As a result, we focus on students submitting preferences in round 1. In 2001-02, 83% of assignments for grade K2, 94% of assignments for grade 6, and 89% of assignments for grade 9 took place in the first round.

The final population consists of students with a valid application form in the first round for 2001-02. More details on data construction are in the appendix of our working paper, Abdulkadiroglu et al. (2006).

4.2 Summary statistics

There are almost three times as many elementary schools as middle schools, and only 12 high schools. Elementary schools average a little under 40 students per incoming class, while middle schools average almost 190 and high schools have more than 530.

At all entry points, between 60%-80% of students receive a free or reduced price lunch. The concentration of Black students is highest in the East zone, and the overall school population is over 80% non-white. The North zone has the highest concentration of Hispanic students and other students who are mainly Asian students living near Chinatown. More detailed statistics can be found in our working paper, Abdulkadiroglu et al. (2006).

Table 1 presents which stated choice students received from the Boston mechanism. In grades K2, 6, and 9, between 92%-95% of students were assigned to a school of their choice, with between 77% and 86% of students receiving their stated top choice. Commentators have have often used stated preferences to evaluate the performance of choice plans.

[12] The only changes have been minor modifications to a handful of walk-zone boundaries.

[13] Results for other school years remain largely the same. In Table 3, we pool data from 2000-01 to 2003-04.

[14] Although grades K0, K1, and 1 are also considered transition years by BPS, for simplicity we do not consider these students here. For K0 and K1, there are a very limited number of school spots and BPS does not guarantee students a spot. For instance, in 2001-02, there were 654 applicants for 141 K0 spots, and 78% of applicants were unassigned. There were 1,530 applicants for 546 K1 spots, and 64% were unassigned. We do not focus on grade 1 because the vast majority of students stay in the elementary school to which they were assigned a K2 place. In 2001-02, 83% of grade 1 spots were guaranteed to continuing students.

[15] If a school's capacity is filled after round 1, its seats are not available for new students in round 2. Moreover, if there is a vacant position at a previously filled school due, say, to students leaving the system to enter private school or moving out of the school district, students in round 1 who were not admitted to that school have higher priority than students entering in later rounds. Therefore there is no strategic reason to miss round 1.

On the surface, the high fraction of students receiving their top choice might suggest that the mechanism is performing well, and/or that matching demand and supply is easy because preferences are dispersed. Indeed, the ability to tell the public that a high proportion of students receive their top choices may be a reason for the widespread popularity of the Boston mechanism. However, given the incentives of the Boston mechanism, treating stated choices as true choices does not give an accurate depiction of the performance of the mechanism. Experimental evidence also suggests that a substantial fraction of participants might not reveal their choices truthfully under the Boston mechanism (Chen and Sönmez 2006).

Table 2 shows the priority through which students are assigned. At the elementary school level, about 16% of students are assigned to their guaranteed choice. This increases to 29% and 52% at the middle and high school level. The reason is that at the K2 entry point, the students who are guaranteed their choice are mainly those who were fortunate enough to obtain a school placement for grade K1 and are continuing on in the same elementary school. For grade 6, there are a number of K2-8 schools and students in grade 5 at the school are guaranteed a spot for grade 6. For high school, the high fraction of guaranteed priority students is accounted for by both continuing students and because students who live in East Boston are guaranteed a spot at East Boston High School.

Among the students who do not use their guaranteed priority, the majority of students are assigned either through walk zone priority or without priority. Sibling-walk or sibling priority account for more assignments at elementary school than middle and high school, but for a smaller fraction of the total priority than walk or no priority students. At high school, walk zone priority is lower because there are only 8 high schools, and applicants can apply from all over the city. Across grade levels, between 26% and 31% of students are assigned to a school without priority.

5 Strategic and Unsophisticated Behavior

We have already noted that at least some families are responding to the incentives to formulate their rank order lists strategically (recall the quotes in the Figure 1 from parents in the West Zone Parents Group concerned with choosing a safe second choice if their first choice is in high demand). Now we will see that the pattern that those manipulations predict is also found in the aggregate data. We will also observe that, in contrast, there are some families that rank two overdemanded schools as their top two choices, despite the fact that this gives them no chance of receiving their second choice. We will show that many of these families suffer from this, and are unassigned, but could have been assigned to one of their choices if they had not made this mistake.

Because we do not know families' true preferences, only their rank order lists, we confine our attention here to a small subset of clear cases. But the overall patterns strongly suggest that both the number of families engaged in sophisticated strategizing and the number who suffer from ranking schools without taking into account the strategic properties of the Boston mechanism (or the information about which schools are overdemanded), are substantially larger than the populations we focus on here (see our working paper).

5.1 Applicant strategies

In Boston, students were allowed to rank up to five schools and the Boston School Guide recommends that parents "choose at least three schools" (2001, page 4). Over 75% of students in grade K2 followed BPS's suggestion and ranked at least three schools. This fraction drops to 61% and 46% for middle and high schools. This difference can be accounted for by the greater number of seats allocated via guaranteed priority for middle and high school. This also explains why a sizable number of students ranked only one school for middle and high school, since the majority of these students ranked only their guaranteed choice first. Many of these students are opting to stay in their current K-8 or 6-12 schools.

Between 7%-20% of students ranked the maximum allowed five schools on their choice form. Given that the Boston mechanism considers first choices before any second choices, the low number of students who ranked five schools is unsurprising. As Table 1 showed, less than 1% of students were assigned to their fifth choice. (This is one aspect of the data we expect will change under the new strategy-proof mechanism, which should make it worthwhile for families to investigate and list more schools.)

Evidence of strategic behavior can be obtained by focusing on schools that are outliers. Recall that under the Boston mechanism it is a mistake to rank an overdemanded school as a second choice (since it will fill all its places with those who rank it first), which is behind the advice to rank a "safe" school second.. Two striking examples in Panel A of Table 3 illustrate this phenomenon. At the Lyndon and Quincy schools, both widely recognized as good elementary schools in their zone, and always overdemanded (in every year for which we have data), a very large number of students ranked the school first, but then there is a steep decline in the number who ranked the school second. At the Lyndon (in the West Zone), which had only 50 places, 151 students ranked it first and only 45 ranked it second; at the Quincy, which had 112 places, 187 students ranked it first and only 35 ranked it second. In both cases, the 45 students ranking the Lyndon and the 35 students ranking the Quincy second could not receive the school if they did not get their first choice because there were more applicants ranking the school first than each school's capacity. These two schools are outliers relative to the other schools in their respective zones where the distribution of students ranking schools across choices is much smoother.[16]

[16] At our first meeting with Boston Public Schools, we were met with some initial skepticism that parents would state their preferences strategically. We asked which was the most popular elementary school in the city, and were told it was the Lyndon. We asked if every nearby family ranked it first, and were told, of course not, you cannot get into the Lyndon without priority.

Panel B of Table 3 shows the relationship between the drop between first choice and second choice applicants and proxies for whether the school was overdemanded in the previous year . The regression includes all elementary schools from 2000-01 to 2003-04 and includes year and zone fixed effects, and a citywide dummy as controls. Proxies for whether the school is overdemanded are all from the previous year. The first proxy is the ratio of the number of students ranking a school first to the school's capacity in the previous year, while the second proxy is the difference in the number of students ranking a school first and the total capacity. A school is overdemanded in the previous year if the first proxy is greater than 1 or if the second proxy is positive. With either proxy, there is a statistically significant relationship between the preference discontinuity and how overdemanded the school was. If the school had 1.5 applicants for each seat in the previous year, the first regression implies that there will be over 31 more applicants ranking the school first than ranking it second in the subsequent year. The second regression implies slightly smaller magnitudes for the discontinuity, so that if a school had 20 seats and 40 first choice applicants in the previous year, there will be 15 more applicants ranking it as a first choice than as a second choice in the subsequent year.[17]

The gap between the first and second choices of overdemanded zone schools is either not present or much smaller at overdemanded *citywide* schools. This is indicated by the significance of the citywide dummy in all specifications and is apparent by looking at demand patterns at overdemanded citywide schools. For instance, at the Hernandez Elementary School which has 42 seats, there are 115 students who rank it first, 84 who rank it second, and 90 who rank it third. At the Young Achievers Elementary school which has 38 seats, there are 132 students who rank it first, 79 who rank it second, and 86 who rank it third. At Mission Hill, the other citywide elementary school, there are 19 seats, and 30 rank it first, 32 rank it second, and 40 rank it first. At the middle school level, the pattern is even more striking. The largest citywide middle school is the Timilty, which has 263 seats in 2002. At this school, there are 618 students who rank it first, 536 who rank it second, and 388 who rank it third. At each of these overdemanded citywide schools (each of which is overdemanded in every year for which we have data), the students who rank it second, third or lower have absolutely no chance of receiving an assignment there. This suggests that while most parents understand that ranking an overdemanded local school second is a mistake, fewer parents applying to citywide schools understand that ranking an overdemanded citywide school second is also a mistake. This suggests strategically unsophisticated behavior on the part of many parents interested in

[17]The gap between numbers of first and second choices could conceivably arise from geographic preferences, if e.g. one of the schools were on an island. But the schools that exhibit the greatest discontinuity are those that are consistently overdemanded in all of the years and among the most popular in the city according to our conversations with staff members at Boston Public Schools (there are 15 schools that are overdemanded for all five years, and the number of students ranking these schools as a second choice is much smaller than those ranking them as a first choice). Indeed, at the Lyndon school, there is a sizable fraction of students who rank it first who are not in close proximity to it (see Figure 4 of our working paper).

citywide schools (who do not have the support of neighborhood parent groups affiliated with the schools in which they are interested). We turn next to consider this.

5.2 Unsophisticated behavior

As noted earlier, without knowing their true preferences, it is difficult to identify students who are clearly making mistakes. However, ranking *two* overdemanded schools as a first and second choice is at least weakly a suboptimal response, since there is no chance of receiving a place at the second overdemanded school. This strategy is strictly suboptimal if the student only has random priority at her first choice so that the odds of receiving it are low: a majority of all unassigned students submitted a rank order with this property.

Table 4 looks at students who did rank two overdemanded schools as their first choices. The top panel shows that these students are disproportionately unassigned (cf. Table 1), and that some of them could have been assigned to one of their ranked schools if they had simply deleted the second overdemanded school from the preference list they submitted.[18] This is particularly true for 9th grade applicants, 23% (46/204) of whom could have gotten an assignment in this way.[19] The second panel looks at the outcome of students who rank two overdemanded schools as their first and second choice and have only random priority at their first choice. These students therefore do not have a high probability of being admitted to their first choice. At elementary school, of the 391 students ranking two overdemanded schools, 38% receive their first choice while a third receive their third or lower choice, and 29% are unassigned. At middle school, about a third receive their top choice, a third receive their third or lower choice and a third are unassigned. In high school, slightly more students receive their first choice, but still a third are unassigned. In high school, once again, 23% of these unassigned students would have been assigned if they had deleted their second choice.[20]

[18]To conduct the counterfactual exercise we must be able to reproduce the outcome of the Boston mechanism, so that we can run it with and without the change in preference lists. While we came close to exactly replicating the outcome of Boston's mechanism, there are still some differences. For grade K2, we do not match 275 assignments (8.3%); for grade 6, we do not match 415 assignments (7.2%); and for grade 9, we do not match 376 assignments (5.9%). After extensive discussions with BPS staff about potential sources of discrepancies, we believe that these fractions are as close as we can get to reproducing their match. Sources of discrepancy are related to the fact that some applicants are assigned administratively and by hand, a lack of a consistent sibling definition, priorities that are in place that are not reflected in Boston's application processing documents such as guaranteed school priority for students who transition to a different school that is in the same building, and unwritten policies related to the assignment of students to a program within a school.

[19]Omitting the second overdemanded choice from the list gives us a lower bound on how many students suffered ex post from the mistake of listing two overdemanded schools first, since this improves their chances no matter what their preferences. We can get an impression of how many more students suffered by observing, in the last panel, that in each case well over a third of the unassigned students could have been assigned to one of their ranked choices if they had chosen differently, e.g. in this case by omitting their first choice. Whether that particular decision would have been a better choice depends on their preferences and risk attitude.

[20]Of course, some parents may be planning to leave the school system if they are not assigned a school they

6 Changing the mechanism

6.1 Naive Comparison of the Three Mechanisms

We had two related concerns about replacing the Boston mechanism with a strategy-proof mechanism intended to help families by allowing them to state their true preferences. The first concern is that, even if, as seems to be the case, many Boston families have been manipulating their preferences in response to the strategic incentives of the Boston mechanism, it might take several years after a strategy-proof mechanism is introduced before the word-of-mouth advice about how to behave in the old mechanism is replaced with confidence that it is safe to state true preferences. In the meantime, the strategy-proof mechanism might be allocating students in its first year or years of operation with preference lists very much like those currently submitted under the Boston mechanism.[21] A second concern that would have the same outcome is that we might have misjudged the extent to which preferences are currently being manipulated. In both cases, we need to consider how the proposed new mechanisms would perform in case they are implemented and presented with preferences like those currently being submitted.[22]

We therefore compared the outcomes of the three mechanisms using the stated preferences submitted under the Boston mechanism during 2001-02. For each year and grade, the outcomes of all three mechanisms are very similar and therefore even if there is no strategic manipulation in Boston, or if it takes several years for families to learn that they no longer need to manipulate preferences, adoption of either alternative will in the meantime not substantially change the performance of the assignment system.

Since the Boston mechanism can be thought of as a version of the student-proposing deferred acceptance mechanism in which school priorities are adjusted so that after students are prioritized, those who rank a school first are elevated over those who rank it second, those who rank it second are elevated over those who rank it third, etc. it must be the case that the Boston mechanism is able to place more students in their top choice than the student-proposing deferred acceptance mechanism. Moreover, since Table 1 showed

like. But note that any student who wishes to remain unassigned rather than receive one of his lower ranked choices could have accomplished this by simply ranking fewer schools In addition, readers are referred to our working paper which observes that, even for the many unassigned students who remain in the school system, the schools to which they are eventually administratively assigned are by observable measures worse than those on their rank order lists

[21]Discussions in the theoretical mechanism design literature sometimes might lead readers to think that different mechanisms can be compared by comparing their equilibrium properties. Here we are essentially asking what will happen if adjustment to equilibrium is slow, so that in early years behavior under the new mechanism is much like recent behavior under the old mechanism.

[22]There is evidence that it takes time for participants to adjust to the use of a new matching mechanism. In the laboratory, a deferred acceptance mechanism increases the efficiency of allocation over time (Kagel and Roth 1999), and this seems to have occured in the implementation of the New York City high school match, when we compare the second year of operation with the first (Abdulkadiroğlu, Pathak and Roth, 2005b).

that the Boston mechanism is able to assign a high number of students to their top choice, stated preferences overall are such that there is not much concentration of demand for school spots. As a result, we anticipated no significant differences between the Boston mechanism and the two strategy-proof alternatives. The results confirmed this expectation. For instance, for elementary schools 77.9% of students are assigned their top choice under our replication of the Boston mechanism, while 73.7% are under student-proposing deferred acceptance and 74.1% are under TTC. The Boston mechanism assigns slightly more students to their first choice, but the number of students one of their top three stated choices is roughly the same across mechanisms. The patterns were similar for middle and high schools. (Details are in our working paper.) Thus if stated preferences change only slowly following the change to a strategy-proof mechanism, we should not anticipate large changes to the assignment in the meantime.

6.2 Strategy-proofness as public policy

The evidence summarized above convinced staff members at BPS and the Superintendent that the current student assignment algorithm should be changed. Central to their arguments to the Boston School Committee was the fact that there exist attractive strategy-proof mechanisms. In his memo to the School Committee on May 25, 2005, Superintendent Payzant wrote:[23]

> The most compelling argument for moving to a new algorithm is to enable families to list their true choices of schools without jeopardizing their chances of being assigned to any school by doing so.

Superintendent Payzant further writes:

> A strategy-proof algorithm *levels the playing field* by diminishing the harm done to parents who do not strategize or do not strategize well.

Furthermore, BPS identified the following other benefits of a change to a strategy-proof mechanism:

- A strategy-proof mechanism adds "transparency" and clarity to the assignment process, by allowing for clear and straightforward advice to parents regarding how to rank schools.[24]
- School officials will be able to use the submitted preferences as indicators of family preferences, to determine which schools are in fact the most highly regarded, and to estimate the effect of policy changes. (e.g. what would happen if some walk zone boundaries were changed?)

[23]See *Superintendent's Memorandum - May 25, 2005* at http://boston.k12.ma.us/assignment/.

[24]See *Recommendation to Implement a new BPS Assignment Algorithm - May 11, 2005* at http://boston.k12.ma.us/assignment/.

The first point especially resonated at a public hearing held shortly later, see the quotes in Figure 2.

Figure 2 around here.

6.3 Choice of strategy-proof mechanism

While the Superintendent recommended the adoption of the student proposing deferred acceptance procedure to the School Committee, there was also discussion of the alternative strategy-proof mechanism based on TTC. Indeed, the TTC mechanism was initially recommended by a taskforce of community members focused on student assignment in 2003 after over a year of community meetings with parents and families.[25]

Regarding his recommendation to adopt the student-proposing deferred acceptance mechanism, the Superintendent states:

> Another algorithm we have considered, Top Trading Cycles Mechanism, presents the opportunity for the priority for one student at a given school to be "traded" for the priority of a student at another school, assuming each student has listed the other's school as a higher choice than the one to which he/she would have been assigned. There may be advantages to this approach, particularly if two lesser choices can be "traded" for two higher choices. It may be argued, however, that certain priorities – e.g., sibling priority – apply only to students for particular schools and should not be traded away.

Central to the theoretical discussion of TTC versus student proposing deferred acceptance was the tradeoff between justified envy and efficiency. The above quotation reflects an additional, related concern: the uneasiness of BPS to allow priorities to be traded, particularly where sibling priority is involved.[26]

[25] At a public School Committee meeting on June 8th, 2005, when we were asked how the Committee and the public should think about the choice between TTC and deferred acceptance, we replied that a key question was "In case your child wants to go to the school for which my child has high priority, and my child wants to go to the school for which your child has a high priority, would anyone mind if they traded priorities?" We pointed out that this might result in a third family being excluded from a school even though a child with lower priority was admitted (but in this case the excluded child would have been unlikely to be admitted even in the absence of a trade). If this was not an objection that outweighed the benefits to the students who traded places, then we suggested TTC should be the choice, while if this were going to be a big problem, then DA might be preferred. Initially it appeared that the answer was going to be that no one minded, but as the discussion was broadened over the following weeks and months, the choice was made to go with the deferred acceptance algorithm.

[26] There is sometimes a gap in intuitions between economists and non-economists about what kinds of things should and should not be traded. As with the reservations about the trading of sibling preferences, these perceptions can play a big role in what kinds of market designs are possible. For example, the need for kidney exchange arises in part from the fact that in most countries of the world it is not permitted to buy or sell organs (Roth et al. 2004,5). In a similar way, we do not expect to see school choice mechanisms in which families bid

7 Conclusion

In Boston, strategy-proofness, a central but subtle idea from the heart of game theory, became an important policy issue.[27]

One of the challenges involved in moving from a school choice mechanism with bad incentives to one that makes it safe for families to list their true preferences is that the Boston school choice system was not broken in an obvious way. Rather, each year students were assigned to schools in an orderly manner, with a very high proportion getting their stated first choice. In this respect, Boston was like a patient with high blood pressure, a potentially deadly disease that has no easily visible symptoms.[28]

Investigating strategic play is also empirically challenging. Since the Boston mechanism is not strategy-proof, our data did not allow us to know the true preferences behind the submitted preferences. So, we cannot hope to detect the full incidence either of strategic manipulation, or of families who failed to manipulate as successfully as they could have.[29] Instead, we concentrated here on showing that there are subsets of families who have manipulated their preferences, and others who could have profited by submitting different preferences. For the first conclusion, we looked at a simple mistake identified by the West Zone Parents Group so that its members could avoid it, and showed that overdemanded schools are indeed relatively seldom listed as a second choice. For the second conclusion, we looked at the students who made the mistake of listing two overdemanded schools at the top of their lists, and showed that many of them suffered as a result.

While the available data do not permit us to assess the size of the potential welfare loss (as we can in laboratory data), what we could show was that many parents appeared to be responding to the strategic incentives to misrepresent their preferences, although many were without all the information that would be needed to do this well. This is precisely the behavior that, in the laboratory where we can measure efficiency losses, causes inefficiency in the resulting matchings. The discussions of the West Zone Parents Group further suggest that this misrepresentation of preferences was in many cases a

money for places in the most desirable schools in a direct way. Of course, economists have long recognized that there can be an implicit price to certain schools based on the costs of living in certain neighborhoods.

[27]Contrast this with the view in Rubinstein's (2005) presidential address to the Econometric Society, where he writes "I believe that as an economic theorist I have very little to say which is of relevance in the real world and I do believe that there are very few models in economic theory that could be used to provide serious advice."

[28]This is in contrast to some other recent design projects in which there was an obvious market failure to be repaired, e.g. the lack of a thick market resulting in very few kidney exchanges (Roth et al. 2004, 2005), or the unraveling that reduced mobility in the market for gastroenterologists (Niederle and Roth, 2003, 2005), or the congestion in the old New York City high school admission process (Abdulkadiroğlu et al. 2005b). To carry on the analogy, those problems were like a patient with a heart attack, where the best treatment might not be obvious, but there was little dispute that treatment was needed.

[29]The case of a family that receives its stated first choice makes both points clear. This could be their true first choice. But if it is not, then they could have profited from deviating from their true preferences, or been harmed, but this cannot be known from the data.

costly process in itself (involving e.g. the gathering of intelligence on how many siblings would enter a given school that year).

A strategy-proof mechanism has an advantage over the Boston mechanism in that, in contrast to the Boston mechanism, it would

- Allow families to list choices in order of true preferences.
- Make it easy to give parents correct advice about how to fill out their preference lists.
- Not penalize parents who are not sophisticated about the school choice process or well informed about which schools are overdemanded.
- Allow parents to spend their time visiting schools and assessing teachers, rather than researching the levels of competition for each school spot.
- Give school officials a more reliable indicator of parent preferences, which can be used not only to match students to schools, but to assess how policy changes such as a change in the borders of walk zones would influence which children would be likely to be assigned to which schools.

There are also a number of conclusions we can draw from the Boston experience that have implications for other school districts, and, more generally, for other market design problems.

First, there are quite a few other school districts with student assignment systems sharing the main features of the Boston mechanism, for example Cambridge, Charlotte-Mecklenberg, Denver, Miami-Dade, Rochester, Tampa-St. Petersburg, and White Plains.[30] Based on our analysis of the behavior of Boston families, it seems likely that in these other school districts parents are faced with solving a complex strategic problem, rather than just a problem of forming preferences over schools. A strategy-proof mechanism like top trading cycles or the deferred acceptance mechanism would lift this strategic burden from parents, and makes the school choice process more transparent. School choice is often a sensitive political issue, and transparency helps to remove some aspects of how to best assign children to schools from the political arena to the technical arena, and clarify which issues remain to be settled by the political process. There will always be such issues, since until there are enough top quality school places to satisfy all families, some aspects of school choice will be a distributive process, with only some students able to gain admission to the most desirable schools. But school assignment is far from zero sum, since different students (in different locations, and with different needs) will have different preferences.[31]

[30] See Hastings, Kane, and Staiger (2005) for a study of the effect of school choice in Charlotte-Mecklenberg on student achievement.

[31] Cullen, Jacob, and Levitt (2003), for instance, report that in an open enrollment plan in Chicago was successful at matching idiosyncratic tastes of parents and students and improving social circumstances.

Our analysis of strategic behavior in Boston also serves to emphasize the fact (lost in some theoretical discussions of mechanism design) that, in complex environments, not all players may be responding optimally to the strategic incentives of the system, i.e. the system need not elicit equilibrium behavior. The policy discussion that developed in Boston showed that one advantage of strategy-proof mechanisms is that they level the playing field between the strategically sophisticated and well informed and those who may be unsophisticated or poorly informed. For policymakers in Boston, this proved to be a major point that led to the successful adoption of the new system.

To summarize, game theory is increasingly useful for the design of markets and other allocation mechanisms. As far as we know, this is the first empirical analysis of an allocation mechanism that failed for the kind of non-obvious incentive reasons related to strategy-proofness. In the context of practical mechanisms adopted for use by a wide public, in which some participants may have the information needed to strategize well while others do not, strategy-proofness came to be seen in part as a criterion of fairness, involving equal access.

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1.) Subject: your input, 1/22/2004

For those of you considering putting the Haley as a first choice, ... you may want to put a safer school second than you had been planning to, in case the momentum builds even more than you had expected. You'll probably be fine and automatically get it as a first choice, but you may want to still play it safe.

2.) Subject: Re: Philbrick School, 1/28/2005

Have you gotten any sense if a lot of people are choosing the Philbrick as a 1st choice? We really like Philbrick ... but are not in the walk zone. We are putting Manning 1st since we're in the walk zone and Philbrick 2nd but I'm getting very nervous that Philbrick has gotten so popular that it might only be a good #1 selection. We're also looking for a good safety for 4th place, perhaps Hale or Mendell.

3.) Subject: Re: Philbrick School, 1/28/2005

I think there are probably 2-3 siblings entering K2 [at the Philbrick]. I know of 2 people who are putting it as a first choice... I don't know what to say--- according to last year's numbers, putting it second would be safe, but the year we applied, only first choice people got in. I think it would be okay if your third choice were a VERY safe bet.

4.) Subject: Re: Philbrick School, 1/30/2005

We're also having trouble with deciding on a secure 2nd choice. We are struggling with deciding between the Manning and the Haley for our 1st choice. They are both in our walk zone but for logistics and time ... the Manning works best for our family. We really loved the Haley but I'm afraid that it is not a safe 2nd choice anymore.

Figure 1: Excerpts from the e-mail archive of West Zone Parents Group, a volunteer group of parents who either have children in Boston Public Schools or are considering enrolling their children (Last accessed on June 17, 2005.)

1.) Parent Comment

It [changing assignment algorithms] is a long time coming. I'm extremely pleased to see the adopting of an assignment method that does not penalize students for ranking one school over another. I was very involved with the student assignment process: I was on a taskforce back in 1985, was co-chair of a taskforce on K2s, and have been pushing for a method like for 20 years. I totally agree with a method where you don't penalize students for ranking a particular school.

2.) Parent Comment

[…] I have participated in the selection process 12 times with 4 different children. […] I find the current system of maximizing first choice to be insidious and destructive. I urge each school committee member to vote enthusiastically for this new algorithm proposal. […] My wife and I take dozens of phone calls around choice time in Dorchester. We have to tell people that it doesn't make sense to choose our children's elementary school. And that is absurd. And the people who get that advice get very angry. […] Because to get into the O'Hearn you need to be luckier than megabucks. So I have to say [to these parents], don't make your first choice your first choice.

That's enraging. It is at the bottom of the anger that you [the School Committee] get from West Roxbury.

But it is even more cruel and unusual for `non-savvy' parents. And I've never met anyone who was savvy after ten minutes who wasn't just angry. It's the Timilty problem. I've never chosen the Timilty middle school for my four children. I'm very happy with the McCormick and Rogers, but I knew it would be a bad mistake because if you choose the Timilty your risk of getting an administrative assignment goes up astronomically. That's wrong because if you don't know this, your chances of choosing your first choice and getting an administrative assignment skyrocket.

It angers the parents who figure it out because they are told not to make their first choice the first one. And it hurts those who don't figure it out because they choose a popular school and end up in the administrative assignment bin.

This new system [...] will heal the problem and quelch the anger. We will be able to give parents good advice. They will make more choices because they will not be confused. More importantly, they will not be harmed.

Figure 2: Excerpts from Parents Hearing on June 8, 2005 (Source: Audio Recording of Public Hearing obtained from School Committee Executive Secretary)

Table 1— 2001-2002 Stated Choice Received[a]

	Total Applicants	1st Choice	2nd Choice	3rd Choice	4th Choice	5th Choice	Unassigned[b]
Elementary school applicants							
Total	3,326	2,598 (78%)	301 (9%)	131 (4%)	61 (2%)	33 (1%)	202 (6%)
Middle school applicants							
Total	5,429	4,157 (77%)	415 (8%)	294 (5%)	61 (1%)	26 (0%)	476 (9%)
High school applicants							
Total	6,380	5,497 (86%)	428 (7%)	100 (2%)	42 (1%)	11 (0%)	302 (5%)

[a]Notes: Statistics tabulated using data provided by Boston Public Schools. Details on definition of student population in appendix of working paper.

[b]Unassigned students are those who are either hand assigned or receive a school that is not on their preference list.

Table 2— 2001-2002 Priority Eligibility at Assigned School[a]

	Guaranteed	Sibling-Walk	Sibling	Walk[b]	No Priority
Elementary school applicants					
Total	514 (16%)	387 (12%)	310 (9%)	888 (27%)	1,025 (31%)
Middle school applicants					
Total	1,582 (29%)	158 (3%)	225 (4%)	1,333 (24%)	1,655 (30%)
High school applicants					
Total	3,297 (52%)	105 (2%)	150 (2%)	844 (13%)	1,682 (26%)

[a]Notes: Statistics tabulated using data provided by Boston Public Schools. Details on definition of student population in appendix of working paper.

[b]Students who live in geocodes with no walk zone schools, but are assigned via non-walk priority are counted as receiving walk priority.

Table 3— Preference Discontinuities at Elementary School[a]

Panel A: Discontinuity in 2001-02[b] at Specific Schools

School	Seats	Number ranking 1st	Number ranking 2nd	Number ranking 3rd
Lyndon	50	151	45	30
Other West Zone	940	839	655	604
Quincy	112	187	35	28
Other North Zone	960	885	679	574

Panel B: Discontinuity for All Schools (from 2000-01 to 2003-04)[c]

Regression Equation:

$$\Delta_{it} = \text{Controls} + \beta \text{ f(overdemanded}_{i,t-1}) + \epsilon_{it}$$

where Δ_{it} = (Number ranking i first - Number ranking i second)$_{it}$

$f(\cdot)$ = Number ranking first / total capacity	21.14	-
	(2.48)	
$f(\cdot)$ = (Number ranking first - total capacity)	-	0.75
		(0.05)
Citywide dummy	-16.44	-24.18
	(7.79)	(6.42)
N	340	340
R^2-adj	0.24	0.42

[a]Notes: Statistics tabulated using data provided by Boston Public Schools. Details on definition of student population in appendix of working paper.

[b]We present representative numbers from 2001-02 only. In all years that we have data, there is a drop of similar magnitude as this year in the number ranking either the Lyndon or Quincy first, and ranking these programs second.

[c]Note: Table presents regressions of the difference in the number of students ranking a school first and the number of students ranking a school second on proxies for whether that school was overdemanded in the previous year, a citywide dummy variable, zone (unreported) and year dummies (unreported). Standard errors in parenthesis.

Table 4— 2001-2002 Ranking Two Overdemanded Schools[a]

Panel A: Students who Rank Two Overdemanded Schools

	Number of Students	Unassigned	Unassigned who benefit by removing 2nd choice	Unassigned who benefit by removing 1st choice
Elementary school applicants	715	174 (24%)	10 (6%)	76 (44%)
Middle school applicants	1,412	404 (29%)	7 (2%)	153 (38%)
High school applicants	783	204 (26%)	46 (23%)	79 (39%)

Panel B: Students who Rank Two Overdemanded Schools with Random Priority at First

	Number of Students	Received 1st Choice	Received Other Choice	Unassigned	Unassigned who benefit by removing 2nd choice	Unassigned who benefit by removing 1st choice
Elementary school applicants	391	147 (38%)	129 (33%)	115 (29%)	4 (3%)	51 (44%)
Middle school applicants	1,035	351 (34%)	327 (32%)	357 (34%)	5 (1%)	134 (38%)
High school applicants	555	253 (46%)	114 (21%)	188 (34%)	44 (23%)	76 (40%)

[a]Notes: Statistics tabulated using data provided by Boston Public Schools. Details on definition of student population are in our working paper. Boston Public Schools reports that in this population there are 3 students who receive their second choice in elementary school, 24 who receive their second choice in middle school, and 9 in high school. It is impossible for these students to receive their second choice under the Boston mechanism as described; this discrepancy is the result either of a miscoded assignment or an error in the priority assigned to these students. When we simulate the Boston mechanism, none of these students receive their second choice.