

Commodity Taxation and Social Insurance

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Abstract

We investigate optimal commodity taxation in a social insurance framework based on Varian (1980). We show that the tax prescriptions in this moral hazard framework are notably similar to those derived from models based on Mirrlees' (1971) self-selection framework. In particular, Atkinson and Stiglitz's (1976) results on uniform commodity taxation are valid in this setup. We incorporate pre-committed goods – those whose consumption must be decided before the resolution of uncertainty – and show that tax prescriptions are also analogous to the existing literature. *Keywords:* Efficiency, Optimal Taxation, Asymmetric and Private Information. *JEL Classification Numbers:* H21, D82.

1 Introduction

There are two conceptually distinct models of optimal income taxation. The first one was introduced by Mirrlees [1971] and extended in Mirrlees [1976], Atkinson and Stiglitz [1976] and others. It emphasizes the trade-off between redistribution and efficiency in a heterogeneous population. The second one, introduced by Varian [1980] and Eaton and Rosen [1980a, 1980b, 1980c], emphasizes the trade-off between insurance and incentives which may be present even with ex-ante identical agents. At the heart of both models lies an informational asymmetry. However, these are of different nature: Mirrlees's model is one of self-selection (we shall refer to this model as SS), while Varian's is one of moral-hazard (we shall refer to this model as MH).

Even though the SS model has become the paradigm, both models capture aspects of the problem which may motivate a society's desire for taxation. Thus, they should both be relevant for thinking about the normative questions of taxation.

Arguably, the most fundamental results obtained from SS models are those related to the supplementary role of commodity taxation. In particular, using an SS model, Atkinson and Stiglitz [1976] showed that the role of commodity taxation is greatly reduced when a non-linear income tax is available. Unfortunately, not much is known about the commodity tax prescriptions implied by an MH model.

Arnott and Stiglitz [1986] address the problem of commodity taxation in the presence of moral hazard. However, their main point is to show that commodity taxes may improve welfare. They do not fully explore conditions for uniform taxation as the literature using SS models does. In this paper we address this issue exploring the commodity tax implications of an MH model.

We show that the commodity tax prescriptions of both models are essentially identical. In particular, Atkinson-Stiglitz's result on uniform taxation holds in the MH model. Following recent developments in the SS literature, we incorporate pre-committed goods in the analysis – goods whose quantity must be decided before the resolution of uncertainty. Given the lifetime interpretation of our model these goods may be a significant part of consumption. Once again the results are analogous, although more general, to those currently available in the SS literature.

In the process of interpreting our results, we are able to re-examine the intuition for certain results in SS models. In particular, Cremer and Gahvari [1995b] offer an intuitive interpretation for their result on the optimal subsidization of pre-committed goods based on precautionary behavior. We argue that their interpretation is misleading and offer an alternative one based on the general principle that whenever deviations from prescribed behavior induces changes in the consumption pattern, differential commodity taxation is a useful supplement to the income tax schedule. We show that, even with separability assumptions, under normality this is exactly the case for pre-committed goods: an agent who shirks shifts consumption away from pre-committed goods.

These interpretations also help understand the related results in dynamic principal agent problems in the contract theory literature. In these models a common characteristic is that the agents intertemporal marginal rate of

substitution is not equalized to that of the principal

The rest of the paper is organized as follows. Section 2 lays out the basic model, generalizing Varian's [1980] moral hazard model. Section 3 derives the implications for optimal commodity taxation of this model. Section 4 extends the model to incorporate pre-committed goods, analyzed in Section 5. Interpretation of the results and concluding remarks are contained in Section 6.

2 The Basic Model

The model is an extension of the standard principal-agent setup to allow multiple goods and non-separability of effort and consumption. Agents maximize expected utility with preferences, in each state of the world, defined over effort, e , and n consumption goods, $x' = (x^1, x^2, \dots, x^n)$, by the strictly quasi-concave Bernoulli utility function $u(x, e)$. Technology is linear with measurement units chosen so that all producer prices equal one.

We use the following notational conventions. Prices and quantities are represented by row and column vectors, respectively. We also use 1_n to denote the n -dimensional row vector of ones.

Agents exert effort to influence the distribution of the random income, w . Thus, income is distributed according to the density function $f(w; e)$. Once income is realized agents choose their consumption basket, x .

We assume a continuum of agents and that w is i.i.d. across agents. All risk is then idiosyncratic, and if all agents choose the same effort level realized and ex-ante distributions coincide. These assumptions allow us to avoid dealing with a stochastic government budget constraint.

The government's problem is restricted only by the environment's informational structure. The individual income realizations w are publicly observed but effort levels e are not. Thus, government can set a non-linear income tax schedule $t(w)$. It may also linearly tax commodities affecting their price. The restriction to linear commodity taxes is implicitly justified by assuming that only total transactions between the consumption and production sectors are observed, and not transaction between consumers [see Guesnerie [1995]].

After uncertainty is resolved, an agent with income realization w pays income taxes according to the schedule $t(w) = y(w) - w$, and uses the remaining income, $y(w)$, to choose a bundle x that maximizes his utility.

The consumer's choice of x conditional on his choice of e and its expenditure on consumption goods, y , yields the indirect utility function:

$$\begin{aligned} V(q, y, e) &\equiv \max_x u(x, e) \\ \text{s.t.} &\quad qx \leq y \end{aligned}$$

where q denotes consumer prices. To make insurance desirable we assume that $V_{yy} \leq 0$, so that the agent is risk averse.

Before the resolution of uncertainty each agent solves:

$$\max_e \int V(q, y(w), e) f(w, e) dw \quad (1)$$

A necessary condition for (1) is:

$$\int \left(V_e + V \frac{f_e}{f} \right) f dw = 0 \quad (2)$$

To keep the analysis manageable, in setting up the government's problem below we shall replace (1) with (2) as the incentive constraint – this is known as the first-order approach. It is well known that this substitution may not be valid in some cases¹. It remains an open question whether our results are sensitive to this strategy; the economic interpretation we offer towards the end of the paper suggest these are robust.

The government can be seen as directly choosing $y(w)$ – implicitly defining the tax schedule $t(w) = w - y(w)$ – as well as consumer prices, q , to maximize the agent's expected utility:

$$\max_{q, y(\cdot), e} \int V(q, y(w), e) f(w, e) dw$$

subject to the resource constraint,

$$\int [w - 1_m x(q, y(w), e)] f(w, e) dw = 0,$$

and the incentive compatibility constraint (2).

¹See Mirrlees (1999) and Rogerson (1985).

3 Optimal Taxation

The related Lagrangian to the government problem is given by,

$$\mathcal{L} = \int \left\{ V + \mu \left[V_e + V \frac{f_e}{f} \right] + \lambda [w - 1_n x] \right\} f dw$$

The first order conditions with respect to $y(w)$, and q are,

$$V_y \left(1 + \mu \frac{f_e}{f} \right) + \mu V_{ey} - \lambda 1_n x_y = 0, \quad (3)$$

$$\int \left\{ V_q \left(1 + \mu \frac{f_e}{f} \right) + \mu V_{eq} - \lambda 1_n x_q \right\} f dw = 0, \quad (4)$$

Our results do not require the government maximization of e , thus we do not display the first order conditions for e .

From Roy's identity we have that $V_q = -x'V_y$ where x' is the transpose of vector x . This allows us to write the cross derivative term in (4) as:

$$V_{eq} = V_{qe} = -V_{ye}x' - V_yx'_e \quad (5)$$

Substituting (5) into (4), yields, after transposing the vector equation:

$$\int \left\{ -x \left[V_y \left(1 + \mu \frac{f_e}{f} \right) + \mu V_{ye} - \lambda (1_n x_y) \right] - \lambda x_q^c 1'_n - \mu V_y x_e \right\} f dw = 0$$

where $x^c(q, V, e)$, is the Hicksian demand for x conditional on e .

Finally, (3) allows us to cancel terms and, by using properties of homogeneity and symmetry of x_q^c we write (4) as:

$$t \int x_q^c f dw = \frac{\mu}{\lambda} \int x_e V_y f dw \quad (6)$$

This equation is the exact analogue of equation (86) in Mirrlees [1976]. Each row of this vector equation is:

$$\sum_i t_i \int x_{q_i}^{c_j} f dw = \frac{\mu}{\lambda} \int x_e^j V_y f dw$$

The term on the left hand side in the expression above is a measure of what Mirrlees calls the ‘‘encouragement’’ of good j . A good should be (weakly) encouraged whenever x_e^j is non-negative², and discouraged, otherwise.

²These goods are said to be ‘(weakly) positively related to effort’ in the terminology of Pollak (1969), who first formally analyzed those conditional demands.

Proposition 1 *If preferences are weakly separable between x and e so that*

$$u(x, e) = v(h(x), e)$$

for some function $v : \mathbb{R}^2 \rightarrow \mathbb{R}$, then uniform commodity taxation is optimal.

Proof. This is a straightforward application of (6). ■

This is the analogue of the celebrated Atkinson-Stiglitz result on uniform taxation derived in an SS model based on Mirrlees's [1971] setup. The proposition shows that their result also holds in an MH model based on Varian's [1980] setup.

4 Pre-Committed Goods

In this section, following Cremer and Gahvari [1995a, 1995b], we extend the model to introduce pre-committed goods – those whose consumption must be decided before the resolution of uncertainty. Because the uncertainty modeled of this paper represents an uncertainty of lifetime wealth, consumption in initial periods of life may be a good example of pre-committed goods.

Thus we divide the goods into two groups: m pre-committed goods, denoted by $z' = (z^1, z^2, \dots, z^m)$, and n ex-post goods, which we denote by $x' = (x^1, x^2, \dots, x^n)$. We write the Bernoulli utility function as $u(z, x, e)$. We maintain the simplifying assumption of a linear technology with producer prices normalized to unity.

We solve the consumer's choice of x conditional on z and e and expenditures for ex-post goods, y , to obtain the indirect utility function:

$$\begin{aligned} V(q, y, z, e) &\equiv \max_x u(z, x, e) \\ \text{s.t.} &\quad qx \leq y \end{aligned}$$

The agent is assumed to be risk averse so that $V_{yy} \leq 0$ for all (q, y, z, e) .

The timing of events is as follows. Agents first choose e and z , then uncertainty is realized and agents with income w pay the income tax $t(w)$ and keep the disposable income, $Y(w) \equiv w - t(w)$. Agents then pay for the pre-committed goods, at prices r , and use the remaining income $y(w) = Y(w) - rz$ to purchase x . Their ex-ante maximization problem is:

$$\max_{z, e} \int V(q, Y(w) - rz, z, e) f(w, e) dw$$

Which yields, in addition to the optimality condition for effort (2), the first order conditions for the optimal choice of z :

$$\int V_z f dw = r' \int V_y f dw \quad (7)$$

In setting up the government's problem, we can view the variables of choice as $(Y(w), r, q, e)$. However, for any $(y(w), z, q, e)$ there is a price vector r given by (7) that makes z optimal for all agents. Thus the problem can be written as choosing $(y(w), z, q, e)$ instead of $(Y(w), r, q, e)$.³

The government's problem is then,

$$\max_{z, q, y(\cdot), e} \int V(q, y(w), z, e) f(w, e) dw$$

subject to the resource constraint,

$$\int [w - 1_m z - 1_n x(q, y(w), z, e)] f(w, e) dw = 0 \quad (8)$$

and the incentive compatibility constraint (2).

5 Optimal Taxation with Pre-Committed Goods

The Lagrangian for the above problem is,

$$\mathcal{L} = \int \left\{ V + \mu \left[V_e + V \frac{f_e}{f} \right] + \lambda [w - 1_m z - 1_n x] \right\} f dw$$

The first order conditions with respect to $y(w)$, q and z' are,⁴

$$V_y \left(1 + \mu \frac{f_e}{f} \right) + \mu V_{ye} - \lambda 1_n x_y = 0, \quad (9)$$

³In other words, the problem we shall setup next is equivalent to the problem of maximizing expected utility choosing z , r , q , $Y(\cdot)$ and e subject to the resource constraint (8), the incentive compatibility constraints (2), and the first order conditions (7).

⁴We use $\partial \mathcal{L} / \partial z'$ instead of $\partial \mathcal{L} / \partial z$ because it allows us to define $x_{z'}$ as an $n \times m$ matrix of similar nature of x_q . Each row is the gradient of an ex-post good's demand function with respect to the pre-committed goods - which, just like prices, are taken as parameters in the second period decision problem.

$$\int \left\{ V_q \left(1 + \mu \frac{f_e}{f} \right) + \mu V_{eq} - \lambda 1_n x_q \right\} f dw = 0, \quad (10)$$

$$\int \left\{ V_{z'} \left(1 + \mu \frac{f_e}{f} \right) + \mu V_{ez'} - \lambda [1_m + 1_n x_{z'}] \right\} f dw = 0 \quad (11)$$

First notice that (10) is analogous to (4) so using the same procedure as before we transform the expression into:

$$t \int x_q^c f dw = \frac{\mu}{\lambda} \int x_e V_y f dw \quad (12)$$

We also used both the symmetry and homogeneity properties of x^c , which hold conditional on z and e .

Equation (12) is identical to equation (6) where pre-committed goods were not considered. Including pre-committed goods in the analysis does not alter the essential results for taxation of the ex-post goods. In particular, as we show below, an analogue of proposition 1 on uniform commodity taxation exists in this case.

To further characterize optimal taxes we now impose certain separability restrictions on preferences. We start by analyzing the case where only weak separability between e and x is assumed. The most general form of such separability is represented by:

$$u(z, x, e) \equiv v(z, h(z, x), e)$$

In this way, no assumption is made on the separability of z with e or x .

Proposition 2 *If preferences are weakly separable between x and e so that*

$$u(z, x, e) \equiv v(z, h(z, x), e),$$

for some function $v : \mathbb{R}^3 \rightarrow \mathbb{R}$, then uniform commodity taxation of the ex-post goods is optimal.

Proof. A direct application of equation (12) ■

In the discussions and propositions that follow, whenever proposition 2 holds, we adopt, without loss of generality, the convenient normalization that $\tau = 0$. That is, to simplify statements, *uniform* taxation of ex-post goods is interpreted as *no taxation* of these goods.

Without separability between z and e , uniform taxation of z goods should not be expected to be optimal – nor will uniform taxation hold for all goods jointly. Thus, to give uniform taxation a chance we assume all goods to be weakly separable from effort. The next proposition shows that, even in this case, uniform taxation of pre-committed goods is not guaranteed.

Proposition 3 *Let preferences be represented by the utility function*

$$u(z, x, e) \equiv v(h(z, x), e)$$

and let the semi-indirect utility function H be defined as:

$$H(q, z, y) \equiv \max_x h(z, x) \quad \text{s.t.} \quad xq \leq y$$

Then at the optimum:

- a. ex-post goods should be untaxed;*
- b. the tax rates on pre-committed good j is given by:*

$$t^j = \frac{\text{cov}\left(V_y, \frac{H_{z^j}}{H_y}\right)}{E[V_y] E\left[\frac{H_{z^j}}{H_y}\right]} \quad (13)$$

Proof. Part (a) is implied by proposition 2, since the preference structure their is more general than the one considered here.

Using (a) and the normalization previously discussed, we have that $q = 1_n$. Adding up property of demand imply the two following conditions:

$$1_n x_y = 1 \quad \text{and} \quad 1_n x_z = 0 \quad (14)$$

The first expression in (14) can be used to rewrite (9), the first order condition for $y(\cdot)$, as:

$$H_y \left\{ v_h + \mu \left[v_{he} + v_h \frac{f_e}{f} \right] \right\} = \lambda \quad (15)$$

Substituting the second equation of (14) into (11) yields:

$$\int H_{z^j} \left\{ v_h + \mu \left[v_{he} + v_h \frac{f_e}{f} \right] \right\} f dw = \lambda 1_m \quad (16)$$

Notice that the term multiplying H_{z^j} in the integrand of (16) is by virtue of (15) equal to λ/H_y , simplifying this expression to,

$$\int \frac{H_{z^j}}{H_y} f dw = 1_m. \quad (17)$$

Using (7) and (17) and the fact that

$$V_{z^j} = v_h H_{z^j} = (v_h H_y) \frac{H_{z^j}}{H_y} = V_y \frac{H_{z^j}}{H_y},$$

we find that the price of good j equals

$$\frac{\int V_y \frac{H_{z^j}}{H_y} f dw}{\left[\int V_y f dw \right] \left[\int \frac{H_{z^j}}{H_y} f dw \right]} = r^j \quad (18)$$

We can rewrite this expression as,

$$\frac{\text{cov} \left(V_y, \frac{H_{z^j}}{H_y} \right)}{E[V_y] E \left[\frac{H_{z^j}}{H_y} \right]} = r^j - 1 = t^j$$

which concludes our proof. ■

Proposition 3 suggests that, in general, pre-committed goods should not be uniformly taxed. The following example illustrates a simple case where optimal taxation of pre-committed goods is in fact not uniform. Let the utility function be,

$$\begin{aligned} h(z^1, z^2, x^1, x^2) &\equiv h^1(z^1) + h^2(z^2 + x^1) + h^3(x^2) \\ v(h, e) &= h + l(e) \end{aligned}$$

then z^1 should be subsidized, while z^2 should be left untaxed.

To see this, notice that $v_h = 1$, $V_y = H_y$ and that $H_{z^1} = h^1'(z_1)$, the marginal utility of z^1 , is constant across realizations of w . Then equation (13) becomes:

$$\frac{\text{cov} \left(V_y, \frac{H_{z^1}}{H_y} \right)}{E[V_y] E \left[\frac{H_{z^1}}{H_y} \right]} = \frac{\text{cov} \left(H_y, \frac{1}{H_y} \right)}{E[H_y] E \left[\frac{1}{H_y} \right]} = t^1 < 0. \quad (19)$$

Because H_{z^2}/H_y is constant we have that $t^2 = 0$.

The intuition for this example is, of course, that since z^2 is a perfect substitute for x^1 , they are essentially the same good. Because preferences satisfy proposition 2, all x goods should not be taxed, and therefore, neither should z^2 . On the other hand, according to (19) good z^1 should be subsidized. The example illustrates that even with separability between goods and effort Atkinson and Stiglitz's [1976] result on uniform taxation cannot generally hold for the pre-committed goods in our setup.

Next we examine the case where utility is weakly separable not only between all consumption goods and effort, but also between pre-committed and ex-post goods. The next proposition shows that in this case pre-committed goods should generally be uniformly subsidized.

Proposition 4 *Let preferences be represented by the utility function*

$$u(z, x, e) \equiv v(h(g(z), k(x)), e)$$

then at the optimum:

- a. ex-post goods should be untaxed;*
- b. pre-committed goods should be uniformly taxed or subsidized;*
- c. if pre-committed good are jointly normal (for a fixed e), they should be subsidized, otherwise, they should be taxed*

Proof. Under these assumptions on preferences, we have that:

$$\frac{H_{z^j}}{H_y} = g_{z^j} \frac{h_g}{H_y}$$

Substituting this into (13),

$$t^j = \frac{\text{cov}\left(V_y, g_{z^j} \frac{h_g}{H_y}\right)}{E[V_y] E\left[g_{z^j} \frac{h_g}{H_y}\right]} = \frac{\text{cov}\left(V_y, \frac{h_g}{H_y}\right)}{E[V_y] E\left[\frac{h_g}{H_y}\right]} \equiv \tau. \quad (20)$$

since g_z does not depend on w .

As for part (b), both V_y and h_g/H_y are functions of w only through $y(w)$. By concavity of V , V_y is decreasing in y . With normality h_g/H_y is increasing

in y – see appendix A – and the covariance is negative⁵. The case of inferiority reverses the result. ■

Before discussing this result further we consider a stronger separability assumption that delivers simple results. With additive separability, without any auxiliary assumptions (such as normality), we show that pre-committed goods should be uniformly subsidized in the following proposition.

Proposition 5 *Let preferences be represented by the utility function*

$$u(z, x, e) \equiv g(z) + k(x) + l(e)$$

then pre-committed goods are uniformly subsidized.

Proof. Under these assumptions (20) becomes:

$$\tau = \frac{\text{cov}\left(V_y, \frac{1}{V_y}\right)}{E[V_y] E\left[\frac{1}{V_y}\right]} < 0$$

■

Note that in this case, normality and concavity are not be invoked. This is because concavity (convexity) and normality (inferiority) are related under this additive specification. Cremer and Gahvari [1995b] obtain similar results in an SS model with additive separability.

We conclude this section with a loose end, showing that weaker conditions than those used in proposition 4 guarantee uniform tax rates for the two groups of goods.

Proposition 6 *Let preferences be represented by the utility function*

$$u(z, x, e) \equiv v(g(z), k(x), e)$$

then uniform tax rates are used for both groups of goods.

Proof. See appendix. ■

⁵Whether or not V_Y is decreasing and ζ is increasing in Y depends on whether $Y'(w) = y'(w) \lesseqgtr 0$. With further assumptions we could guarantee $Y'(w) \geq 0$. However, this is not required for the result.

6 Discussion

This paper investigates how commodity taxes should supplement an optimally designed non-linear income tax, when it is social insurance rather than redistribution that motivates the second-best problem. It is shown that rules very similar to the ones derived by Atkinson and Stiglitz [1976] and Mirrlees [1976] are optimal for this type of model. This should come as a relief given the unresolved issue of which model best addresses the problem of income taxation.

We also show the similarity of optimal tax results when pre-committed goods are incorporated, as defined by Cremer and Gahvari [1995]. In the rest of this section we discuss the intuition for our results with special emphasis on the subsidization result for pre-committed goods.

Non-linear income taxation is a powerful instrument that leaves few leftovers for commodity taxation. In SS and MH models redistribution and insurance, respectively, are constrained by the incentive effects they generate – summarized in the incentive constraints imposed on the problem. Consequently, differential taxation is only beneficial when it relaxes incentive constraints.

Incentive constraints are relaxed when the demand for goods is affected by deviating behavior – from truth telling in the SS model and from the prescribed effort level in the MH model – since then the tax incidence is different for deviators and abiders. Exploiting this difference allows for greater redistribution or insurance. In a sense, in these cases commodity taxation acts as a tax on dishonesty or as a subsidy to honesty. Thus separability always plays a key role in uniform taxation.

Cremer and Gahvari [1995b] offer the following intuition for their result that pre-committed goods should be subsidized in an SS model. They argue that the uncertainty of income creates a precautionary behavior leading to “low” consumption of z goods. Consequently, they argue, to encourage consumption of z it is optimal to subsidize them.

Based on the proof of our results we believe this explanation to be misleading both for the SS and MH models⁶. First, note that income uncertainty is already being taken care of by the income tax schedule; it is not clear whether any further role should be played by the subsidization of pre-

⁶Da Costa and Werning (2000) show that the exact same propositions obtained here are true in an SS framework thus generalizing Cremer and Gahvari’s (1985b) results.

committed goods.

Second, consider the case where there is only one good of each type and assume the sub-utility functions of these two goods to be equal. For a fixed level of effort this problem is equivalent to an intertemporal savings problem under uncertainty. Then, uncertain income induces agents to consume more or less of z good depending on the concavity or convexity of the marginal utility of income – the conditions required for precautionary savings. Yet, no assumptions are made regarding these third derivatives.

Finally, notice that if $\mu = 0$, then $V_y = \lambda$ and $r = I_m$, which shows that incentive compatibility constraints must play a crucial role in the story. Hence, an argument based purely on precautionary behavior towards risk appears misleading.

We offer an alternative explanation based precisely on the incentive effects differential taxation may have on effort. This interpretation applies both to the SS and the MH models and coincides with the argument used for the x goods.

Consider an agent deciding to shirk, lowering effort below the optimal level e^* . For every realization of w this agent receives the same amount of after-tax income as an agent choosing e^* . Ex-ante, however, the agent is poorer since higher realizations of w are less likely⁷. Even with separability, if pre-committed goods are normal, the deviating agent will consume relatively less pre-committed goods. Consistent with the intuition used for the taxation of x goods – that what is relevant is the change in consumption by the deviator – these goods should be subsidized.

A Appendix

A.1 Normality Condition

Consider the following problem. For a given level of total income y the agent solves the optimal allocation of income between the two groups:

$$\max_{y^0} h(G(r, y^0), K(q, y - y^0))$$

⁷First order stochastic dominance is not required for the argument. The first order condition of the agent with respect to e ensures that at the margin an increase in e increases his income in the sense that the change in distribution increases his expected utility. That is, $\int V \frac{\partial f}{\partial e} dw = - \int V_e f dw > 0$.

where G and K are the indirect sub-utility functions corresponding to $g(z)$ and $k(x)$. The f.o.c. for this problem is simply:

$$h_g G_y - h_k K_y = 0 \quad (21)$$

Where subscripts denote partial derivatives.

Applying the implicit function theorem to (21),

$$\frac{dy_0}{dy} = - \frac{h_{gk} G_y K_y - h_{kk} (K_y)^2 - h_k K_{yy}}{\Delta} \quad (22)$$

where $\Delta < 0$ is the second order condition for the problem. Thus, the sign of dy_0/dy is equal to the sign of the numerator in (22).

Consider, now, the following derivative (we hold y_0 fixed):

$$\left. \frac{d}{dy} \left(\frac{h_g}{h_k K_y} \right) \right|_{y_0} = \frac{h_{gk} K_y}{h_g K_y} - \frac{h_g h_{kk} (K_y)^2}{(h_k K_y)^2} - \frac{h_g h_k K_{yy}}{(h_k K_y)^2}$$

Using the first order conditions, one can rewrite the above expression as:

$$\left. \frac{d}{dy} \left(\frac{h_g}{h_k K_y} \right) \right|_{y_0} = \frac{1}{h_k K_y G_y} [h_{gk} G_y K_y - h_{kk} (K_y)^2 - h_k K_{yy}]$$

Since $h_k K_y G_y > 0$ the sign of dy_0/dy and $\left. \frac{d}{dy} \left(\frac{h_g}{h_k K_y} \right) \right|_{y_0}$ are the same.

A.2 Proof of Proposition 6

Proof. We can write the agent's ex-ante problem as:

$$\max_{z,e} \int v(g(z), K(y(w) - rz, q), e) f(w, e) dw$$

where K is the indirect utility function of $k(x)$. The first order conditions with respect to z are

$$g_z \int v_g f dw = r' \int v_k K_y f dw \quad (23)$$

Manipulating (18) yields,

$$g_{z'} = K_y \frac{\{v_k + \mu [v_{ek} + v_k f_e / f]\}}{\int \{v_g + \mu [v_{eg} + v_g f_e / f]\} f dw} 1_m \quad (24)$$

Finally using (23) and (24)

$$r \frac{\int K_y v_k f dw}{\int v_g f dw} = \frac{K_y \{v_k + \mu [v_{ek} + v_k f_e / f]\}}{\int \{v_g + \mu [v_{eg} + v_g f_e / f]\} f dw} 1_m$$

Hence, r is proportional to 1_m – uniform taxation of pre-committed goods is optimal. ■

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