

# Competitive Analysis with Differentiated Products

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**ABSTRACT.** — Differentiated products are the central economic focus of competition in consumer goods products such as cereal, soda, and beer. We first estimate demand models which do not restrict unduly the pattern of consumer preferences as does much previous research in the area of differentiated products. Using recently available transactions data we estimate own and cross price elasticities in a relatively unrestricted manner. We next turn to competitive analysis using our estimated demand system. We consider two applications in this paper. The main economic factor that we consider is that the firms which produce the differentiated products almost always tend to be multi-product firms in the given industry. Our first application is competitive analysis when two firms are allowed to merge. The other application that we consider is inference on the competitive structure in an industry. In both applications we consider the effect of a multi-product firm where its competitive decisions for one brand affects its sales and prices for other brands that it produces.

## Analyse de la concurrence avec différenciation des produits

**Résumé.** — La différenciation des produits est un aspect économique central de la concurrence sur les marchés des biens de consommations comme les céréales, les sodas ou les bières. Tout d'abord nous estimons des modèles de demande qui n'impliquent pas de restrictions excessives sur la structure des préférences du consommateur, à l'inverse de ce qui est souvent pratiqué dans les recherches portant sur des marchés de produits différenciés. L'utilisation de données d'échanges qui sont devenues récemment disponible, permet l'estimation des élasticités-prix propres et croisées d'une manière relativement peu contrainte. Ensuite nous développons une analyse de la concurrence à partir des estimations du système de demande. Nous focalisons l'étude sur deux aspects, sachant que le facteur économique principal considéré ici est la multi-production qui caractérise presque toujours les entreprises produisant des biens différenciés. La première application consiste en une analyse des conséquences des fusions d'entreprises: la seconde a pour but d'inférer la structure concurrentielle d'une industrie. Dans les deux cas nous nous intéressons à l'effet d'une entreprise multi-produit pour qui les décisions stratégiques concernant une marque affectent les ventes et les prix des autres marques qu'il produit.

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# 1 Introduction

Differentiated products are the central economic focus of competition in consumer goods products. Cereal, soda, beer, and toothpaste are all examples of products where "brand names" are important. These industries are often characterized by heavy advertising and "brand proliferation" which economics has difficulty incorporating into a useful analytical framework. Advances in economic theory over the last two decades for differentiated products industries have recognized the importance of the structure of consumer preferences in the analysis of differentiated products industries, e.g., SPENCE [1976], DIXIT and STIGLITZ [1977], HART [1985], and PERLOFF and SALOP [1985]. However the structure of consumer preferences has been somewhat special in these models, and economists have recognized that the results of the models may be highly sensitive to specific model assumptions on consumer preferences.

The two most widely used models of consumer preferences in differentiated goods analysis are either the representative consumer model or a model of different consumers with different tastes with the "symmetry property". Unfortunately, the structure of demands in these models is often in conflict with the facts since the models display various features of the independence of irrelevant alternatives (IIA) property: if a single good is eliminated from the choice set, consumers who were choosing the eliminated good will distribute themselves among the remaining goods according to the overall market shares of those goods. This property has been tested and rejected numerous times in the discrete choice literature, e.g., HAUSMAN and WISE [1978], HAUSMAN and McFADDEN [1984]. In terms of price elasticities, the IIA property implies that all cross price elasticities with respect to a given good are the same, c.f. HAUSMAN [1975]. Again this property is unlikely to hold very often and is contrary to what we would expect for most differentiated product situations.

Thus, the first aim of this paper is to estimate demand models which do not restrict unduly the pattern of consumer preferences. Over the past five years or so, weekly or monthly transactional data have become available at the individual store level for consumer products. Data are available for sales of each brand, and indeed for sales of each SKU (stock keeping unit). Since each transaction is recorded onto a computer file, the number of units and price paid per unit are available. Of course, some aggregation is needed across different size containers for each brand, but the effect of promotions when prices are lowered allows for the possibility of precise estimates of own and cross price elasticities, estimated in a relatively unrestricted manner.

However, two econometric problems arise in the use of store level data which we attempt to solve in this paper. First, it is difficult to conceive of estimating a demand equation with say 135 brands of cereal or say 40 brands of beer. The sheer number of prices included would lead to very difficult problems of estimation, if for no other reason than a lack of sufficient number of observations. We attempt to solve this problem by using the GORMAN [1971] approach to multi-level demand estimation. Here,

this approach is similar to estimating segments of a differentiated product industry which correspond to how marketing analysts consider patterns of demand. This approach implies restrictions on demand patterns which we discuss how to test. Also, the testing approach allows for consideration of different segmentation patterns since marketing analysts do not always agree on how a product space is segmented.

The other outstanding econometric problem is identification, or where do the instruments come from? At the upper levels of the demand system, e.g., the demand for beer overall, traditional cost shifting instruments can be used such as indices for different ingredients, packaging, and labor. However, at the lower levels where consumers choose among brands, the search for instruments is more difficult since excluded cost shifting variables at the brand level may be difficult to find, depending on the given situation. The approach we take here is to utilize the panel structure of the underlying data. Data are collected at the individual metropolitan (SMSA) level while, in most cases, the products are sold at wholesale at a national level with a uniform wholesale price set by the manufacturers. Our approach is to allow for both individual brand fixed effects and city fixed effects, and then to use the prices from one city as instruments for other cities applying the econometric methodology of HAUSMAN and TAYLOR [1981]. The intuition that we use is that prices in each city reflect underlying product costs and city specific factors which vary over time as supermarkets run promotions on a particular product. To the extent that the stochastic city specific factors are independent of each other, prices from one city can serve as instruments for another city.

Note that our approach to demand estimation does not imply any restrictions on patterns of competition among the differentiated products. We next turn to competitive analysis using our estimated demand system. We consider two applications in this paper. The main economic factor that we consider is that the firms which produce the differentiated products almost always tend to be multi-product firms in the given industry. Miller Beer produces Miller High Life, Miller Lite Beer, Miller Genuine Draft, Milwaukee's Best, and Meisterbrau. Cereal manufacturers such as Kellogg's produce over 30 brands of cereal. Thus, to some extent, manufacturers produce competing brands for their own products. They are well aware of this situation with great attention paid to "cannibalization" of one firm's brand by another brand produced by the same firm.

Our first application is competitive analysis when two firms are allowed to merge. To the extent that one or more brands of one of the merging parties is constraining the price of brands of the other merging party, prices may increase after the merger. When the price of a given brand increases its demand will decrease, but if sufficient amounts of the lost demand go to other brands of the merging parties the price increase may be profitable. A countervailing influence to a price increase may be decreased costs of production for one or more of the brands. We demonstrate how these possible efficiencies can be included in the analysis so that the overall impact on prices of the merger can be estimated. We then apply a welfare analysis for consumers and for the overall economy when prices change.

The other application that we consider is inference on the competitive structure in an industry. The Chamberlain model and other models of differentiated product industries often lead to implications about the markup of price over marginal cost. The most popular model of this type is the Nash-Bertrand model in prices where the percentage markup of price over marginal cost equals the reciprocal of the own price elasticity. Here we note again that in the multiproduct situation that when one firm raises the price on a given product, that some of the lost demand goes to other products of the same firm. We develop the analysis which takes the multiproduct nature of differentiated goods industries into account which should permit tests of industry competitive interaction if estimates of marginal cost are available.

## 2 A Multi-Level Demand System for Differentiated Products with An Application to Beer

The basic approach that we have found useful in estimating demand for differentiated products in a three stage demand system using GORMAN'S [1971] multi-stage budgeting approach. The basic idea is to have the top level correspond to overall demand for the product—here the product that we use is beer. The middle level corresponds to different segments for the products: premium beer, light (low calorie) beer, imported beer, and non-premium (popular) beer. These segments are used by marketing people in consideration of overall consumer preferences, and similar segmentation schemes are found in all differentiated product industries. However, agreement over the proper segmentation framework does not always exist so that we discuss econometric tests of segment definitions. The bottom level of the demand system corresponds to competition among brands in a given segment, e.g., Miller, Budweiser, and Coors in the premium beer segment. We estimate all three levels of the demand system and by combining the estimates from each level together, we are able to estimate the overall own and cross prices elasticities for each brand.

In terms of actual estimation we proceed to estimate the model in reverse order beginning at the lowest level and then use the theory of price indices to allow for consistent estimation at the higher (more aggregate) levels of demand. The third (or lowest) stage determines buying behavior within market segments. We use this approach because it accords with segmentation of brand purchasing behavior which marketing analysts claim arises with purchasing behavior, and because it limits the number of cross elasticities which will be estimated. Our econometric specification at the lowest level is the "almost ideal demand system" of Deaton and Muellbauer which allows for a second order flexible demand system, i.e., the price elasticities are unconstrained at the point of approximation, and also allows for a convenient

specification for non-homothetic behavior<sup>1</sup>. However, our experience is that the particular form of demand specification is not crucial here. Use of a flexible demand system allows for few restrictions on preferences while decreasing the number of unknown parameters through the use of symmetry and adding up restrictions from consumer theory. For each brand within the market segment the demand specification is:

$$(1) \quad s_{int} = \alpha_{in} + \beta_i \log(y_{Gnt}/P_{nt}) + \sum_{j=1}^J \gamma_{ij} \log p_{jnt} + \varepsilon_{int}, \\ i = 1, \dots, J, \quad n = 1, \dots, N, \quad t = 1, \dots, T$$

where  $s_{int}$  is the revenues share of total segment expenditure of the  $i$ th brand in city  $n$  in period  $t$ ,  $y_{Gnt}$  is overall segment expenditure,  $P_{nt}$  is a price index, and  $p_{jnt}$  is the price of the  $j$ th brand in city  $n$ . Note that a test of whether  $\beta_i = 0$  allows for a test of segment homotheticity, e.g., whether shares are independent of segment expenditure. The estimated  $\gamma_{ij}$  permit a free pattern of cross-price elasticities and Slutsky symmetry can be imposed, if desired, by setting  $\gamma_{ij} = \gamma_{ji}$ . This choice of the bottom level demand specification does not impose any restrictions on competition among brands within a given segment. In particular, no IIA-type assumptions restrict the within segment cross price elasticities. Since competition among differentiated products is typically "highest" among brands within a given segment, this lack of restrictions can be an important feature of the model. An important econometric consideration is the use of segment expenditure,  $y_{Gnt}$ , in the share specification of equation (1), rather than the use of overall expenditure. Use of overall expenditure is inconsistent with the economic theory of multi-stage budgeting, and it can lead to decidedly inferior econometric results.

Given the estimates from equation (1), we calculate a price index for each segment and proceed to estimate the next level of demand. For exact two-stage budgeting, the Gorman results impose the requirement of additive separability on the next level<sup>2</sup>. To specify the middle level demand system we use the log-log demand system<sup>3</sup>:

$$(2) \quad \log q_{mnt} = \beta_m \log y_{Bnt} + \sum_{k=1}^K \delta_k \log \pi_{knt} + \alpha_{mn} + \varepsilon_{mnt} \\ m = 1, \dots, M, \quad n = 1, \dots, N, \quad t = 1, \dots, T$$

1. See A. DEATON and J. MUELLBAUER, "An Almost Ideal Demand System", *American Economic Review*, 70, 1981, and *Economics and Consumer Behavior*, (Cambridge: Cambridge University Press, 1980).
2. W. GORMAN (1971), "Two Stage Budgeting", mimeo. This subject is also discussed in C. BLACKORBY, et al., [1978], *Duality, Separability, and Functional Structure* (New York: American Elsevier) and in DEATON and MUELLBAUER [1980], op. cit. Note that the almost ideal demand system is a generalized Gorman polar form (GGPF) so that Gorman's theorem on exact two stage budgeting applies. Since the additive demand specification at the top level imposes separability restrictions, we have also used a less restrictive specification at the middle level which is not necessarily consistent with exact two-stage budgeting. The results are quite similar.
3. Note that this specification is second-order flexible. However, the Slutsky restrictions have not been imposed on the specification.

where the left hand side variable  $q_{mnt}$  is log quantity of the  $m$ th segment in city  $n$  in period  $t$ , the expenditure variable  $y_{Bnt}$  is total beer expenditure, and the  $\Pi_{knt}$  are the segment price indices for city  $n$ . The segments that we use are premium which includes both premium beer and imported beer which we found should be included in the premium segment, light beers, and popular price beer. The price indices  $\Pi_{knt}$  can be estimated either by using an exact price index corresponding to equation (1), which is constructed from the expenditure function for each segment holding utility constant, or by using a weighted average price index of the Laspeyres type. Choice of the exact form of the price index does not typically have much influence on the final model estimates.

Lastly, the top level equation, which we use to estimate the overall price elasticity of beer, is specified as:

$$(3) \quad \log u_t = \beta_0 + \beta_1 \log y_t + \beta_2 \log \Pi_t + Z_t \delta + \varepsilon_t$$

where  $u_t$  is overall consumption of beer,  $y_t$  is deflated disposable income,  $\Pi_t$  is the deflated price index for beer, and  $Z_t$  are variables which account for changes in demographics, monthly (seasonal) factors, and minimum age for purchasing beer. To estimate equation (3) we use national (BLS) monthly data over a sixteen year period with instrumental variables. We have found that a longer time period than may be available from store level data is often useful to estimate the top level demand elasticity. The instruments we use in estimation of equation (3), are factors which shift costs such as different ingredients, packaging, and labor. We estimate the overall price elasticity of beer to be  $-1.36$  with an estimated standard error of 0.21.

We now consider the question of identification and consistent estimation of the middle level and bottom level equations. The problem is most easily seen in equation (1), the brand level equation, although an analogous problem arises in equation (2) the segment level demand equation. Equation (1) for each brand will have a number of prices included for each brand in the segment, e.g., we include 5 brands in the premium segment in the subsequent estimation. The usual strategy of estimating demand equations where the cost function includes factor input prices, e.g., material prices, which are excluded from the demand equations to allow for identification and the application of instrumental variables may be difficult to implement. An insufficient number of input prices may exist or they may not be reported with high enough frequency to allow for instrumental variable estimation. To help solve this problem, we exploit the panel structure of our data. For instance, suppose  $N = 2$  so that weekly or monthly data from two cities is available. Note that we have included brand (or segment) and city fixed effects in the specification of equations (1) and (2). Now suppose we can model the price for a brand  $i$  in city  $n$  in period  $t$  as

$$(4) \quad \log p_{jnt} = \delta_j \log c_{jt} + \alpha_{jn} + w_{jnt}$$

where  $p_{jnt}$  is the price for brand  $j$  in city  $n$  in period  $t$ . The determinants of the brand price for brand  $j$  are  $c_{jt}$ , the cost which is assumed not to

have a city specific time shifting component which is consistent with the national shipments and advertising of most differentiated products,  $\alpha_{jn}$ , which is a city specific brand differential which accounts for transportation costs or local wage differentials, and  $w_{jnt}$ , which is a mean zero stochastic disturbance which accounts of sales promotion run for brand  $j$  in city  $n$  in time period  $t$ . The specific identifying assumption that we make is that the  $w_{jnt}$  are independent across cities. Using fixed effects the city specific components are eliminated, and we are basically applying the HAUSMAN-TAYLOR [1981] technique for instrument variables in panel data models<sup>4</sup>. The idea is that prices in one city (after elimination of city and brand specific effects) are driven by underlying costs,  $c_{jt}$ , which provide instrumental variables which are correlated with prices but are uncorrelated with stochastic disturbances in the demand equations, e.g.,  $w_{jnt}$  from equation (4) is uncorrelated with  $\varepsilon_{it}$  from equation (1) when the cities are different,  $n \neq 1$ . Thus, the availability of panel data is a crucial factor which allows for estimation of the all the own price and cross price brand elasticities.

However, another interpretation can be given to equation (4) and the question of whether  $w_{jnt}$  from equation (4) is uncorrelated with  $\varepsilon_{int}$  from equation (1). To the extent that supermarkets set their prices  $p_{jnt}$  under a constant marginal cost assumption (in the short run) and do not alter their prices to equilibrate supply and demand in a given week, prices  $p_{jnt}$  may be considered predetermined with respect to equation (1). If prices can be treated as predetermined, then IV methods would not be necessarily needed. IV methods might still be required for the segment expenditure variable  $y_{Gnt}$  in equation (1), however. The need for instruments under these hypotheses can be tested in a standard procedure using specification tests for instruments, e.g. HAUSMAN [1978].

We now discuss the middle level results, the segment price elasticities, although the estimation is done in the reverse order. The results of the estimation are given in Table 1. Because a log-log specification is used, the estimated coefficients on the log prices are elasticity estimates. We use three market segments corresponding to light beers, popular priced beers, and premium beers. We note that from equation (2) that the results are estimated conditional elasticities since  $y_{Bnt}$ , expenditure on beer, is held constant. To find the unconditional elasticities for each segment, the overall price elasticity for beer, estimated at the top level, must be included. Note that the conditional estimated own-price elasticities are quite high given that they are segment, not brand, elasticities, e.g., the premium price conditional own price elasticity is estimated to be  $-2.7$ , the popular price beer elasticities is about the same while the light beer own price elasticity is  $-2.4$ . The cross price elasticities between expenditure on light beer and premium brands and between popular price beer and premium brands both demonstrate a high degree of substitutability with cross price elasticities of 0.4 and higher.

4. See also BREUSCH, MIZON, and SCHMIDT [1989]. With more than two cities, tests of the instrumental variable assumptions can be done along the lines discussed in HAUSMAN and TAYLOR [1981].

TABLE 1

Beer Segment Conditional Demand Equations.

	Premium	Popular	Light
Constant	0.501 (0.283)	-4.021 (0.560)	-1.183 (0.377)
log (Beer Exp)	0.978 (0.011)	0.943 (0.022)	1.067 (0.015)
log (PREMIUM)	-2.671 (0.123)	2.704 (0.244)	0.424 (0.166)
log (POPULAR)	0.510 (0.097)	-2.707 (0.193)	0.747 (0.127)
log (LIGHT)	0.701 (0.070)	0.518 (0.140)	-2.424 (0.092)
Time	-0.001 (0.000)	-0.000 (0.001)	0.002 (0.000)
log (# of Stores)	-0.035 (0.016)	0.253 (0.034)	-0.176 (0.023)

Number of Observations = 101.

Indeed, these estimated segment price elasticities allow us to consider how much brands in different segments compete with each other. Claims are sometimes made that competition among brands in different segments is quite low. Economists have sometimes claimed that different segments, e.g., the light beer segment and popular beer segment, should be treated as separate (antitrust) markets. Our estimates contradict such claims. For instance, if the price index for premium beers rises by 10%, demand for the light beers (whose price is assumed to remain constant) will increase by about 4.2%. In terms of prices, both popular and light beers significantly constrain the overall prices for premium beers. For instance, the conditional own price elasticity for premium beers is -2.7 which demonstrates that a hypothetical attempt to raise the prices of premium beers would not be profitable if the prices of light and popular beers did not change. Thus, competition among segments demonstrates a sufficient proportion of marginal customers who will alter their buying patterns in response to price changes so that prices in one segment are affected by prices in other segments.

We next discuss the lower level results which are presented in Table 2-4. It is important to note that these results are conditional on expenditure in a given category,  $y_{Gnt}$  from equation (1); the overall brand price elasticities arise from a combination of the estimates from all three levels. The estimated coefficients reported in Tables 2-4 correspond to the specification (1), and thus are not readily interpretable. However, they are used to estimate elasticities. Thus, we report in Tables 2-4 the conditional own price elasticities for each brand estimated using the reported coefficients.

For the premium beers, as shown in Table 2, the conditional own price elasticities are in the range of -3.5 to -5.0. Many of the cross-price elasticities (which are not reported in Table 2) are also quite high especially between Budweiser, Coors, and Miller. For instance, the conditional cross-price elasticities for Budweiser range between 0.4 and 1.0. Coors and Labatts have a moderately high degree of substitution between them with a conditional cross price elasticity of Coors with respect to Labatts of 1.0.

TABLE 2

Brand Share Equations: Premium.

	1	2	3	4	5
	Budweiser	Molson	Labatts	Miller	Coors
Constant	0.393 (0.062)	0.377 (0.078)	0.230 (0.056)	-0.104 (0.031)	-
Time	0.001 (0.000)	-0.000 (0.000)	0.001 (0.000)	0.000 (0.000)	-
log (Y/P)	-0.004 (0.006)	-0.011 (0.007)	-0.006 (0.005)	0.017 (0.003)	-
log (PBudweiser)	-0.936 (0.041)	0.372 (0.231)	0.243 (0.034)	0.150 (0.018)	-
log (PMolson)	0.372 (0.231)	-0.804 (0.031)	0.183 (0.022)	0.130 (0.012)	-
log (PLabatts)	0.243 (0.034)	0.183 (0.022)	-0.588 (0.044)	0.028 (0.019)	-
log (PMiller)	0.150 (0.018)	0.130 (0.012)	0.028 (0.019)	-0.377 (0.017)	-
log (# of Stores)	0.009 (0.009)	-0.010 (0.005)	-0.036 (0.008)	0.022 (0.005)	-
Conditional Own	-3.527 (0.113)	-5.049 (0.152)	-4.277 (0.245)	-4.201 (0.147)	-4.641 (0.203)
Price Elasticity	-1.436E-05 (0.000359)	-6.246E-05 (0.000109)	-2.402E-05 (0.000158)	-1.847E-05 (0.000392)	-0.000492 (0.000492)

Note: Symmetry imposed during estimation.

Similar results are found for popular price beers in Table 3 and for light beers in Table 4. We find somewhat higher own-price elasticities for the popular priced beers than for the premium or light beers which is consistent with our expectations that consumers who buy these beers are more price sensitive.

Much of the price variation that we observe arises from supermarket chains running special low price promotion on beer in a given week. A question often arises regarding household inventory behavior. The question is often stated about whether we are estimating "real" own price elasticities or whether households stock up when the price is low. With regards to economic theory if most households had strong preferences for a given brand, it would not usually make economic sense for a firm to encourage low cost stocking up when the overall quantity of the brand you would expect to sell is largely unchanged by the time series of prices charged. A possible response could be that supermarkets use this particular product as a "loss leader" - they purposely sacrifice profits on the particular brand to cause people to shop at their branches. This extra traffic could lead to higher overall profits. However, this reasoning seems inconsistent with the econometric estimates. If only inventory behavior were present, while the own price elasticity estimates could be large the cross price elasticity estimates should be very near zero. Yet an examination of the coefficient

TABLE 3

## Brand Share Equations: Popular Price.

	1	2	3	4	5
	Old Milwaukee	Genesee	Milwaukee's Best	Busch	Piels Lager
Constant	0.287 (0.062)	0.225 (0.067)	-0.019 (0.063)	0.531 (0.079)	-
Time	-0.000 (0.000)	-0.001 (0.000)	0.000 (0.000)	0.001 (0.000)	-
log (Y/P)	0.014 (0.006)	-0.018 (0.007)	0.001 (0.007)	0.004 (0.008)	-
log (P <sub>Old Milwaukee</sub> )	-0.979 (0.028)	0.235 (0.021)	0.369 (0.022)	0.257 (0.030)	-
log (P <sub>Genesee</sub> )	0.235 (0.021)	-0.698 (0.029)	0.222 (0.022)	0.205 (0.030)	-
log (P <sub>Milwaukee's Best</sub> )	0.369 (0.022)	0.222 (0.022)	-1.048 (0.036)	0.388 (0.035)	-
log (P <sub>Busch</sub> )	0.257 (0.030)	0.205 (0.030)	0.388 (0.035)	-0.892 (0.062)	-
log (# of Stores)	-0.044 (0.010)	0.122 (0.011)	-0.023 (0.010)	-0.091 (0.012)	-
Conditional Own	-4.789	-3.832	-5.813	-5.704	-3.956
Price Elasticity	(0.109)	(0.120)	(0.164)	(0.329)	(0.465)

$$\Sigma = \begin{cases} 0.000603 & -0.000123 & -0.000137 & -8.289E-05 \\ - & 0.000515 & -0.000143 & -7.136E-05 \\ - & - & 0.000758 & -0.000109 \\ - & - & - & 0.00262 \end{cases}$$

Note: Symmetry imposed during estimation.

estimates in Table 2 (which, again, are not elasticities) demonstrates that the cross price effects are relatively large in proportion to the own price coefficients. Using these coefficient estimates, we estimate the cross price elasticity of Coors with Budweiser to be 1.25, with Molson to be 0.86, and with Miller to be 0.52. Even across segments this brand switching behavior is present. Graphs 1-2 depict sales of Budweiser (premium segment) and Genesee (popular segment) in Buffalo during two different time periods. Note the high amount of negative correlation—high Bud sales correspond to lower Genny sales and vice-versa. These changes correspond to the relative price shown in the graphs which leads to the cross elasticities which our econometric models estimate. Thus, our conclusion is that estimated own and cross elasticities correspond to actual brand switching behavior and not to inventory behavior of the stocking-up type by households which would call the interpretation of the estimates into question.

We now present in Table 5 the overall (unconditional) own-price elasticities for all 15 brands, and, in addition, the cross price elasticities within the light beer segment. Among the 15 brands, the lowest own-price elasticity is for Genesee Light (a light beer), -3.8, and the highest elasticity is -6.2 for Milwaukee's Best (a popular beer). Note that the own price elasticities are estimated quite precisely despite the use of the three levels of demand in the overall model. The cross price elasticities among the

TABLE 4

## Brand Share Equations: Light.

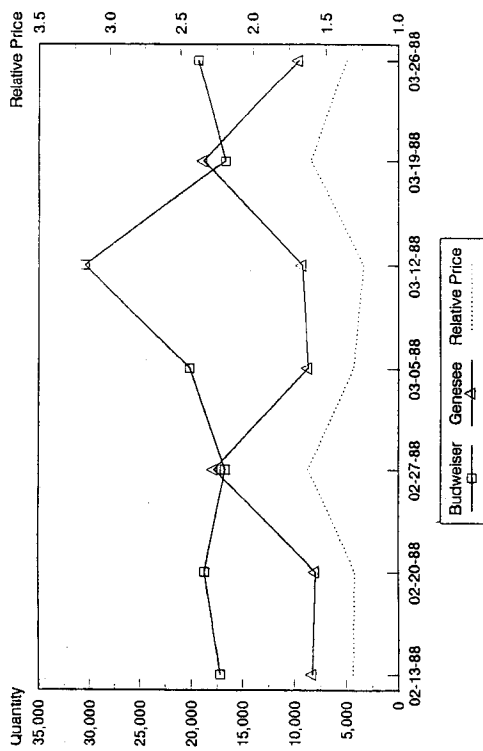
	1	2	3	4	5
	Genesee Light	Coors Light	Old Milwaukee Light	Lite	Molson Light
Constant	0.249 (0.053)	0.273 (0.054)	-0.009 (0.038)	0.194 (0.042)	-
Time	-0.000 (0.000)	0.001 (0.000)	-0.000 (0.000)	0.000 (0.000)	-
log (Y/P)	-0.014 (0.005)	0.015 (0.005)	-0.002 (0.004)	0.005 (0.004)	-
log (P <sub>Genesee Light</sub> )	-0.813 (0.023)	0.304 (0.018)	0.203 (0.013)	0.177 (0.014)	-
log (P <sub>Coors Light</sub> )	0.304 (0.023)	-0.812 (0.028)	0.154 (0.014)	0.184 (0.019)	-
log (P <sub>Old Milwaukee Light</sub> )	0.203 (0.013)	0.154 (0.013)	-0.565 (0.016)	0.124 (0.013)	-
log (P <sub>Lite</sub> )	0.177 (0.014)	0.184 (0.019)	0.124 (0.022)	-0.606 (0.022)	-
log (# of Stores)	0.032 (0.009)	-0.021 (0.010)	0.006 (0.007)	-0.009 (0.008)	-
Conditional Own	-3.225	-4.180	-5.923	-4.783	-5.682
Price Elasticity	(0.062)	(0.109)	(0.140)	(0.138)	(0.147)

$$\Sigma = \begin{cases} 0.000433 & -0.000176 & -4.636E-05 & -9.295E-05 \\ - & 0.000475 & -0.000117 & -7.556E-05 \\ - & - & 0.000226 & -7.203E-05 \\ - & - & - & 0.000305 \end{cases}$$

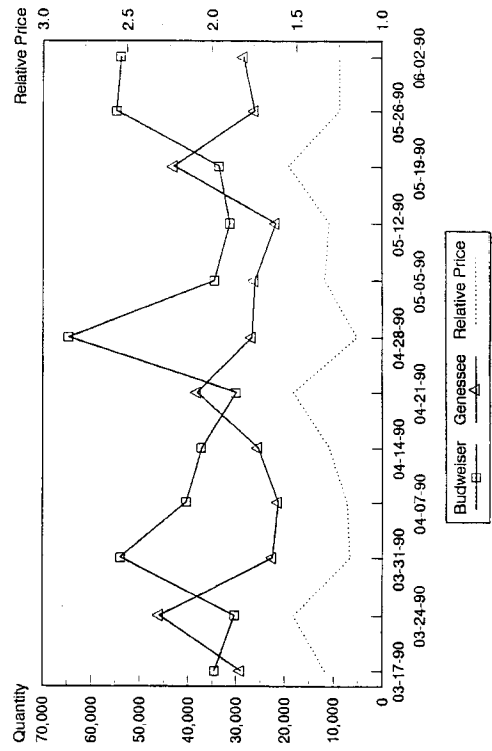
Note: Symmetry imposed during estimation.

15 brands (all of which are not reported in Table 5) also seem of reasonable size and are estimated quite precisely. The Budweiser and Miller own price elasticities are relatively similar while the Coors own price elasticity is considerable higher, which is expected given its perceived low "brand loyalty" relative to Budweiser and Miller. On the other hand, Coors Light, which is a very popular light brand, has about the same price estimate price elasticity as Miller Lite. The cross price elasticity between Coors Light and Miller Lite is 0.74—quite a high value which we expect given the closeness of competition between the brands. We also note that the popular brands, e.g., Old Milwaukee, Milwaukee's Best, Busch, and Pils, have higher own price elasticities than do the light beers or premium beers. These findings are consistent with the lower amounts of "brand loyalty" which marketing experts view these brands as having.

Lastly, we consider the question of our segment definitions. The notion of weak separability is used in multi-stage budgeting—in particular as demonstrated in equation (1) shares of a given segment depend only on segment expenditures and the prices in that segment alone. Segment expenditure, from equation (2), is determined by all brand prices through a price index, but weak separability leads to the restriction, that apart from income effects (which we estimate to be very small at the bottom level),



GRAPH 1  
Budweiser and Genesee. Quantities and Relative Price.



GRAPH 2  
Budweiser and Genesee. Quantitative and Relative Price.

a change in the price of a given premium brand, e.g., Coors, affects the price of all popular brands in the same way. Thus, the cross-price elasticity

TABLE 5  
Overall Elasticities.

	Elasticity	Standard Error
Budweiser	-4.196	0.127
Molson	-5.390	0.154
Labatts	-4.592	0.247
Miller	-4.446	0.149
Coors	-4.897	0.205
Old Milwaukee	-5.277	0.118
Genesee	-4.236	0.129
Milwaukee's Best	-6.205	0.170
Busch	-6.051	0.332
Piels	-4.117	0.469
Genesee Light	-3.763	0.072
Coors Light	-4.598	0.115
Old Milwaukee Light	-6.097	0.140
Lite	-5.039	0.141
Molson Light	-5.841	0.148

Light Segment Own and Cross Elasticities.

	Genesee Light	Coors Light	Old Milwaukee Light	Lite	Molson Light
Genesee Light	-3.763 (0.072)	0.464 (0.060)	0.397 (0.039)	0.254 (0.043)	0.201 (0.037)
Coors Light	0.569 (0.085)	-4.598 (0.115)	0.407 (0.058)	0.452 (0.075)	0.482 (0.061)
Old Milwaukee Light	1.233 (0.121)	0.956 (0.132)	-6.097 (0.140)	0.841 (0.112)	0.565 (0.087)
Lite	0.509 (0.095)	0.737 (0.122)	0.587 (0.079)	-5.039 (0.141)	0.577 (0.083)
Molson Light	0.683 (0.124)	1.213 (0.149)	0.611 (0.093)	0.893 (0.125)	-5.841 (0.148)

between two brands, say 1 and 2, which are in different segments G and H takes the form:

$$\delta_{12} = \lambda_{GH} \frac{\partial \log q_1}{\partial \log y_G} \frac{\partial \log q_2}{\partial \log y_H} \quad (5)$$

where  $\delta_{12}$  is the (Slutsky) compensated price elasticity and  $\lambda_{GH}$  is a constant which only depends on the segments G and H. Thus, from equation (5) we see that the compensated price elasticity depends on a segment specific constant (which does not vary across brands) and the effect of a change in expenditure in a segment which in turn depends on the price indices in equation (2). Overall, our specifications allows for completely unrestricted patterns of substitution within a given segment, e.g., premium brands, and across segments, e.g., premium and popular segments, but it does impose restrictions on price responses of brands in different segments. We now consider a test of this restriction which can be thought of as whether a

given brand which is currently excluded from a segment be included in a segment <sup>5</sup>?

As a specification test of our segment definitions using the weak separability restriction of equation (5), we re-estimated equation (2) for premium beer, the middle level equation which defines the premium beer market segment, including as additional right hand side variables the individual prices of the brands in the light beer segment. Inclusion of these individual brand prices should not affect the estimated segment elasticities if the weak separability condition holds. The results can be interpreted in either of two ways. First, none of the prices for Genessee Light, Coors Light, Old Milwaukee Light, or Molson Light were statistically significant. However, the price of Miller Lite did achieve marginal statistical significance. On the other hand, none of the estimated elasticities for the price indices estimated in Table 1 for the premium beer segment changed. Since we use these elasticities to estimate the overall own-price and cross-price elasticities for all brands, we conclude tentatively that our segment structure is adequate for estimation. However, further research on the appropriate segment framework is probably indicated.

### 3 Applications of the Demand Estimates

We have now specified the demand estimation procedure and own and cross price elasticity estimates which can be useful for competitive analysis. Note that these estimates correspond to the "primitive" notion underlying demand in the sense that we have not assumed a particular type of competition such as Bertrand, a particular type of technology such as constant returns, or a particular restriction on competitive interactions over time as in residual demand curve estimation, in making the estimates. Thus, given the demand system estimation various types of competitive responses can be either predicted or tested depending on the particular application under consideration. We now demonstrate two such applications.

5. As one keeps adding brands to a given segment, the specification becomes closer to an overall unrestricted demand system. Thus, some ordering of tests is required which uses some *a priori* knowledge about the "closeness" of brands. This knowledge is often present from perception studies and other surveys carried out by marketing analysts.

### 3.1. Price Changes Arising from Mergers

Merger analysis attempts to determine whether the increase in market power which may occur if two competitors combine their operation is sufficiently large to outweigh production efficiency benefits which may also arise from a merger <sup>6</sup>. Traditional merger analysis follows a two step procedure <sup>7</sup>. The first step involves market definition for each product and service of the merging firms. Both product market definition and geographic market definition take place in this first step of the analysis. The second step of the analysis calculates market shares of each of the products determined to be in the relevant product and geographic markets. These market shares are used to estimate the Herfindahl-Hirschman Index (HHI) of market concentration. The level of the HHI and the change in the HHI are then used as predictive measure to analyze the likely economic effects of the merger.

Here we take a more direct approach to analyzing the likely effects of a merger. We look at the price constraining effects of the merging brands and see whether removal of these constraints would lead to an increase in prices for consumers <sup>8</sup>.

We use as a basis for our analysis the Nash-Bertrand assumption under conditions where entry is expected not to occur even if prices are raised (by a relatively small amount) after a merger <sup>9</sup>. Suppose that a firm, firm 1, produces a single product in a market with  $n$  products and chooses price to maximize

$$(p_1 - mc_1) Q_1(p_1, \dots, p_n)$$

The first order condition, under a Nash assumption, is

$$Q_1(p_1, \dots, p_n) + (p_1 - mc_1) \frac{\partial Q_1}{\partial p_1} = 0$$

In equilibrium, the firm sets price according to

$$(6) \quad \frac{p_1 - mc_1}{p_1} = - \frac{1}{e_{11}}$$

where  $e_{11}$  is the firm's own price elasticity. Now consider the possible effects of a merger between firm 1 and a second firm. We assume that, before the merger, firm 1 produces its single product <sup>10</sup>, good 1, but that

6. For an introduction to the legal background of merger analysis, see the "Symposium on Horizontal Mergers and Antitrust", *Journal of Economic Perspectives*, 1, 1987.

7. Our reference to "traditional" merger analysis follows the US Department of Justice and Federal Trade Commission Horizontal Merger Guidelines (MG), dated April 2, 1992.

8. In a recent paper we demonstrate how the traditional HHI analysis can be revised when differentiated products are present, HAUSMAN-LEONARD-ZONA [1992].

9. The possibility of entry would change the post-merger analysis in a very significant manner. We are assuming that firm 1 produces a single product for simplicity of presentation. Generalization to the case where both merging firms produce a number of products is straightforward.



the combination of the merging firms produces goods  $j = 1, \dots, m$ . The combined firm will take into account that if it raises the price of good 1, that some of the lost demand will go to other goods that it produces which are substitutes for good 1. Thus, the previous price constraining effect of products now produced by the merged firm will be eliminated. When the newly merged firm sets its prices optimally, the prices solve the first order equations for each product:

$$(7) \quad \left[ \frac{p_j}{\sum_{k=1}^m p_k q_k} \right] \frac{\partial \pi}{\partial p_j} = s_j + \sum_{k=1}^m \left[ \frac{p_k - mc_k}{p_k} s_k \right] e_{kj} = 0$$

for  $j = 1, \dots, m$

where the  $e_{kj}$  denote the cross-elasticities of demand.

We now express the first order conditions of equation (7) as a system of linear equations:

$$(8) \quad s + E'w = 0$$

where  $s$  is the vector of revenues shares,  $E$  is the matrix of cross price elasticities and  $w$  is the vector of price-cost markups multiplied by the share (the term in brackets on the right hand side of equation (7)) which arise under the Nash-Bertrand assumption in equation (6). We solve for these individual terms of the markup equation by inversion of the matrix of cross elasticities:

$$(9) \quad w = -(E')^{-1} s$$

We can then use the individual elements of  $w$  to determine the change in price after the merger to the extent that marginal costs remain constant. Note that while we have derived the change in price under Nash-Bertrand assumptions, our analysis does not require this assumption. To the extent that pricing constraints will be decreased after the merger, the analysis provides a lower bound on expected price changes, absent new entry by competitors.

The main economic concern from a merger is that prices to consumers will increase. From a strict economic efficiency analysis, gains in productive efficiency may well outweigh increases in price since efficiency gains are "first-order" in size so that they will often be larger than the "second-order" efficiency losses to the economy which arise from the exercise of market power<sup>11</sup>. Nevertheless, merger analysis by the antitrust agencies in the US which judge mergers typically considers only the direct effects on consumers

11. This point was made originally by O.E. WILLIAMSON, "Economics as an Antitrust Defense: The Welfare Tradeoffs", *American Economic Review*, 58, 1968. Some economists and lawyers have argued that the presence of monopoly power will lead to first order welfare losses as firms compete for the monopoly rents, e.g., R. POSNER, *Antitrust Law: An Economic Perspective*, (Chicago: University of Chicago Press), 1976.

so we limit ourselves to this mode of analysis. We now derive an expression which allows for explicit consideration of the relaxation of the constraining effects of some of the merged goods and the effect of increased efficiencies in production to the extent that they directly affect prices to consumers. To enable us to eliminate the requirement of defining a Berkson-Samuelson social welfare function, our analysis of the change in consumer welfare derives the change in expected prices for the merging goods if the merger is allowed. The change in prices can then be applied to purchased quantities or revenues shares using social welfare weights, if desired, to do an explicit consumer welfare calculation<sup>12</sup>.

Define the price-cost markup as

$$(10) \quad \frac{p_j - mc_j}{p_j} = \theta_j$$

We use  $\theta_j^M$  to denote the markup post-merger. Now to account for merger induced efficiencies equation (10) demonstrates that decreased marginal costs can lead to lower post-merger prices if  $\theta_j^M$  does not increase too much. Using the first order conditions, we express the post merger prices as a ratio to the pre-merger prices as:

$$(11) \quad \frac{p_j^M}{p_j} = \frac{mc_j^M}{mc_j} \left( \frac{p_j}{mc_j} \right) (1 - \theta_j^M)$$

Equation (11) makes the role of post-merger induced efficiencies and the decrease in competitive pressure on the merging goods quite clear. The numerator involves the ratio of the pre-merger marginal costs. Any decreases in marginal cost will be reflected proportionately in a decrease in the post-merger prices. The denominator of equation (11) contains  $\theta_j^M$  the post-merger price-cost markups. These markups are calculated from the  $w_j$  of equation (9) after division by the revenue shares  $s_j$ . Note that as  $\theta_j^M$  decreases toward one as monopoly power increases, the post-merger prices increase compared to the pre-merger price. The overall effect on post-merger prices then trades off the price decreases which arise from efficiencies and the price increases created by the increased post-merger market power.

An alternative expression which considers the percentage increase in post-merger price may also be of interest. Note that the pre-merger price to marginal cost ratio in the denominator of equation (11) can also be written as the ratio of the own-elasticity of demand,  $(e_{jj}/(1 + e_{jj}))$ . The percentage

12. In the most straightforward case to evaluate changes in overall economic efficiency from the merger, social welfare is often taken to be the sum of the consumer and producer surpluses. For small changes, the change in social welfare is then the (unweighted) sum of the price minus the marginal cost multiplied by the change in quantity demanded of each good. However, here we consider only change in price (and quantities), not accounting for changes in producers surplus.

change in the post-merger price becomes:

$$(12) \quad \frac{p_j^M - p_j}{p_j} = \frac{mc_j^M}{mc_j} \left( \frac{e_{jj}}{1 + e_{jj}} \right) (1 - \theta_j^M) - 1$$

If we now disregard possible changes in marginal costs, we can determine the constraining influence on possible price increases by the merging products from products in other product segments, e.g., the constraining influence of Old Milwaukee on possible price increases of Coors if it is allowed to merge with Labatts. Thus, we can compute possible percentage price increases using the expression:

$$(13) \quad \alpha_j = \frac{1}{\left( \frac{e_{jj}}{1 + e_{jj}} \right) (1 - \theta_j^M)} - 1$$

where  $\alpha_j$ , the percentage price increase of each merging product, is determined by the size of  $\theta_j^M$  which is calculated from equation (8) where the other products, which would hypothetically become part of the merger (i.e., a merged firm would control the prices of all of the brands in the upper left-hand corner) are taken into account. The size of  $\theta_j^M$  will be determined by the magnitude of the estimated cross price elasticities. If these cross elasticities are zero, or near zero, then  $\theta_j^M$  in equation (13) will be approximately equal to  $\theta_j^M$  from equation (12) so that the additional products will be seen not to have placed a significant constraining influence on price increases by the merging products. On the other hand, if  $\theta_j^M$  is significantly larger than  $\theta_j^M$  because the cross price elasticities are large, then  $\alpha_j$  from equation (13) will significantly exceed the percentage price increase from equation (12). This finding will demonstrate the constraining influence on price of the additional products.

As an application of our methodology we calculate a hypothetical merger between two brands in the premium segment, Coors and Labatts, which is a popular Canadian imported beer in the US. We calculate the change in price for the hypothetically merging products, Coors and Labatts, from equation (8), both with and without any assumed efficiencies:

**Estimated Price Increases for Hypothetical Merging Brands Assumed Efficiency Gains.**

	0%	5%	10%
Coors . . . . .	4.4% (1.2)	-0.8% (1.2)	-6.1% (1.1)
Labatts . . . . .	3.3 (1.0)	-1.9 (1.0)	-7.0 (0.9)

Thus, we find that even with no efficiency gains that the estimated price increases would be quite small, because Coors price is constrained much

more by Budweiser and Miller while Labatts price is constrained more by Molsons, another Canadian import. When modest efficiency gains are included due to hypothetical decreases in marginal costs, predicted post-merger prices decrease. Now if efficiency gains of say 5% were only able to be made at Coors, then Labatts price would rise while Coors price would fall so that a (share) weighted average of the change might be the appropriate measure to consider. However, the overall constraining influence of other brands does not permit the merging brands to achieve price increases of any sizeable magnitude<sup>13</sup>.

Next we use equations (12) and (13) to analyze further the constraining influence of other segments on the potential price rise in Coors and Labatts. As we discussed above with no use of efficiency gains, the post merger increase in price for Coors is calculated to be 4.4% and 3.3% for Labatts using equation (8). If we now include all premium beers in the upper left-hand block of equation (9) the post-merger price increases of Coors and Labatts are estimated to be 23.1% and 21.0% from equation (13). Thus, the importance of other premium brands in constraining price increases in Coors and Labatts is very significant. Now we ask the question whether popular price beers constrain the post-merger price increases. If they are included in the calculation of equation (13) the post-merger price increases of Coors and Labatts would be 108.3% and 104.8%, respectively. Thus, the popular beers impose a further significant constraining influence on price increases of the merging products<sup>14</sup>. Similarly, if we include light beers in the upper left-hand block of equation (2.11) (and exclude popular price beers), the post-merger price increases of Coors and Labatts would be 29.8% and 27.6%. Thus, the light beers also impose a significant constraining influence on post-merger price increases, although not as large as popular price beers which is the expected result.

Estimating a forecasted change in price appears quite possible once demand elasticities have been estimated since the role of the price constraining influence of the merging brands can be ascertained. This direct approach yields additional useful information beyond the traditional Merger Guidelines approach of using HHIs. Indeed, the direct approach may be a preferable method to the indirect method used in the Merger Guidelines which is based on a link between changes in HHIs and changes in post-merger prices which may differ widely depending on particular economic conditions in a given industry.

**3.2. Tests of Competitive Interactions Among Firms**

Considerable attention has arisen to competitive interaction among firms in differentiated products industries, e.g., WATERSON [1984] or BRESNAHAN

13. If we consider a hypothetical merger across market segments between Labatts and Genesee, with no efficiency gains we find price increases of 2.2% (0.3) and 1.0% (0.3) respectively.

14. Another way to consider the questions of the constraining influence of brands in other segments is to note from the previous footnote that a hypothetical merger between Labatts and Genesee would allow the price of Labatts to increase by 2.2%, only slightly below the amount in the hypothetical Coors and Labatts merger.

[1989]. Various theories which involve Nash-Bertrand behavior, dominant firm behavior, or the form of reaction functions depending on the "closeness" of products all depend on own and cross-price elasticities of different products. However, these oligopoly models typically assume that each firm produces only a single differentiated product in an industry, which is not consistent with the realities of the situation. For instance, Miller produces 5 (or more) brands of beer while Kellogg's produces about 30 brands of cereal. When say Kellogg's considers changing a price of one of its brands, it presumably takes into account how much of the lost demand will go to its other products. This mutual interdependence among products can change tests of competitive interaction in a significant manner.

For instance, suppose we want to test Nash-Bertrand behavior. If we consider only a single product we would derive equation (6) where the markup of price over marginal cost depends solely on the own price elasticity. However, use of this form of the test will lead to a downward bias in the estimate of the right hand side of equation (6) because use of the own price elasticity neglects that a price increase in brand  $j$  will lead to increased demand for other products produced by the firm which is considering the price increase.

To derive the correct form of the test consider  $M$  firms which produce  $K_1, \dots, K_m$  products, respectively. Let  $E$  be a block diagonal matrix

$$(14) \quad E = \begin{bmatrix} E_1 & & & 0 \\ & \ddots & & \\ & & \ddots & \\ 0 & & & E_m \end{bmatrix}$$

with blocks  $E_i = k_i \times k_i$  of own and cross price elasticities for firm  $i$ 's products. Let  $s$  be a vector of shares of each of the products and let  $A$  be a diagonal matrix with diagonal element equal to the elements of  $s$ . Then calculate the vector  $w = -(E')^{-1}s$ . The appropriate test for Nash-Bertrand behavior for product  $j$  produced by firm  $i$  is then

$$(15) \quad \frac{P_{ij} - MC_{ij}}{P_{ij}} = \frac{W_{ij}}{S_{ij}} = \theta_{ij}$$

Thus, the test of the Nash-Bertrand markup of price over cost no longer has  $\theta_j$  depending only on the inverse of its own price elasticity as in equation (6), but instead all of the  $\theta$  are calculated from  $\theta = -(A')^{-1}(E')^{-1}s$ .

As an example consider the situation where we want to test Nash-Bertrand behavior for Miller Lite beer, the largest selling low calorie beer in the US. If we based the test on Miller Lite's estimated own price elasticity from Table 5, -5.13, the percentage price-cost markup should be 19.9% under Nash behavior. However, when we take into account that an increase in the price of Miller Lite will lead to some consumers changing to Miller High Life, Old Milwaukee Light, or Old Milwaukee, among other Miller brands, the appropriate estimate of  $\theta_{ij}$  for Miller Lite in equation (15) is 23.2%. Thus, the percentage markup under Nash-Bertrand behavior increases from 19.9% to 23.2%, or a change of about 17%. Taking into account that a

given firm produces many products can be an important determinant in models of competitive behavior with differentiated products. In the beer industry typically, firms produce only a single beer in a given segment. In other industries such as cereal or cigarettes, firms are much more likely to produce two or more brands in a given segment which is likely to increase the multi-brand effect which we have estimated. Previous attempts to test Nash behavior in a differentiated products context may have failed to find Nash-like behavior because of a failure to account for other brands produced by the same firm. This example has wider applicability than merely testing for Nash-Bertrand behavior in a differentiated goods setting. It demonstrates the potential importance of taking into account competing brands produced by the same firm, a factor which has been largely ignored in differentiated products models used in both theoretical and applied research.

## 4 Conclusions

Differentiated products industries are among the most important industries in the US economy. Firms attempt to differentiate their products from competitive products with ever increasing sophistication. In this paper we have set forth an econometric specification which allows for estimation of the underlying own and cross price elasticities for differentiated products. The specifications corresponds to market segments often used in analyzing these industries, and it allows for tests of the segmentation framework used in the analysis. We then consider two applications of the econometric framework. First, we demonstrate how to analyze the constraining effect on price that different products maintain and how these constraining effects may be attenuated by merger. However, we also consider the role of productive efficiencies and determine the predicted changes in consumer prices after a merger which seems to be the focus of US merger law. Thus, we need not depend on a purported link between changes in market share and changes in price which may vary widely across industries. Next, we demonstrate the importance of taking into account a fairly obvious economic factor—most firms in differentiated product industries produce a number of competing products. This economic factor has been largely ignored in much of the economic research on differentiated products industry, yet it can have an important effect on the analysis. Models of competition with the multi-brand effect and theories of the determinants of its relative importance across different industries seem a potentially important topic for future research. Given the estimated own and cross price elasticities, data on marginal cost over time will allow for more highly developed models of competition in differentiated product industries and tests of these models. We see this development as the next step in better understanding how firms in differentiated product industries compete with respect to their price setting behavior.

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