# Endogenous Groups and Dynamic Selection in Mechanism Design*

Gabriel A. Madeira Robert M. Townsend

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**Abstract**

We create a dynamic theory of endogenous risk sharing groups, with good internal information, and their coexistence with relative perfomance, individualistic regimes, which are informationaly more opaque. Inequality and organizational form are determined simultaneously. Numerical techniques and succinct re-formulations of mechanism design problems with suitable choice of promised utilities allow the computation of a stochastic steady state and its transitions. Regions of low inequality and moderate to high wealth (utility promises) produce the relative performance regime, while regions of high inequality and low wealth produce the risk sharing group regime. If there is a cost to prevent coalitions, risk sharing groups emerge at high wealth levels also. Transition from the relative performance regime to the group regime tend to occur when rewards to observed outputs exacerbate inequality, while transitions from the group regime to the relative performance regime tend to come with a decrease in utility promises. Some regions of inequality and wealth deliver long term persistence of organization form and inequality, while other re-

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gions deliver high levels of volatility. JEL Classification Numbers: D23,D71,D85,O17.
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# 1 Introduction

Imagine a landowner with two distinct plots of land and two possible tenants. One possible arrangement is for the two tenants to form a cropping group and farm all the land of the owner as a cooperative association, deciding on inputs and sharing outputs after paying the land rental. Another possible arrangement is for the landowner to rent to each of the two tenants individually, using a comparison across the plot harvests to determine the net rental payment, awarding an apparently diligent agent with a larger ex post share of the harvest. Both of these are a stylized version of what we see in tenancy data in villages in Maharastra, India, as described in more detail in Townsend and Mueller [29] and Mueller, Prescott and Sumner [19]. Some principals have their land farmed by single cropping groups while others divide up the land, farmed by several individual tenants. This gives us the first empirical observation: there is cross sectional diversity in organization arrangements, with some individuals acting cooperatively as in risk sharing groups, and others competitively, as in a relative performance arrangement.

A second empirical observation is that organization form changes over time. At the end of the season, some groups disband (or at least their composition changes). Likewise, tenants previously acting as individuals now agree to form a group (or at least enter into an existing group). Though the sample size and number of time periods were too limited to know for sure, the retrospective data in the Townsend and Mueller study are not inconsistent with the possibility that a given land owner might switch from one type of regime to another even with the same set of tenants, the latter as a risk sharing group in one period and competing under relative performance in another. Certainly flux in groups is typical of the longer village history.

More generally, sociologists with an interest in social networks have typically found them to be unstable. There is low correlation in the composition of networks over time and occasional abandonment: to cite just one example Barkey and van Rossem [3] analyzes 17th-century Ottoman villages in western Anatolia, using court records to reconstruct formal and informal networks. Under conditions of state and market expansion, those villages in intermediate positions in the regional structure tend to experience the vagaries of these changes more than central or isolated villages. Ironically, cooperative village organization in these intermediate villages is found to promote contention. Sociological theorists Leik and Chalkey [17] discuss a list of possible causes of instability documented in this and many other studies: unreliable measurement, external change, inherent instability, and systematic change from endogenous forces.

Development economists with an interest in networks seem, on the other hand, to fix apriori the likely network candidate and then test for risk sharing. Often the village is taken as the obvious group, as in Townsend [28] study villages in india. But several of the cropping groups mentioned above at the outset of this introduction exist in Aureapple, one of these villages. This raises the question of whether the correct candidate for a risk sharing network is a village or one of these groups . Others study the nuclear family in Ethiopia (Dercon and Krishnan [7]), tribes in Cote D'Ivoire as in Grimard [11], and family and friends in the Phillipines (Fafchamps and Lund [9]). Some find networks and subgroups that share risk well, but other reject the selected apriori candidate. Chiappori, *et al* [6] shows in particular that some family-related networks within Thai villages, and some villages, pass rigorous tests for risk sharing while other networks and village perform poorly. It seems the search for a stable apriori structure may be misguided.

Rather, economists and sociologists need theories to deal empirically with cross sectional variety and temporal change. Of course different disciplines, and different theories within a discipline, are likely to suggest different candidate variables as crucial. In this regard we are much motivated by one salient piece of evidence for a theory of joint liability credit groups. Ahlin and Townsend [1] test the group-versus-relative performance model of Prescott and Townsend [24], Itoh [13] and Holmstrom and Milgrom [12], finding that the higher the level of inequality in a village, and the lower the overall wealth of the village, the more likely it is that two (or more) households will borrow in a group. Conversely, relative equality is a force for individual loans. Consistent with this, but not tested, is the notion that inequality and organization structure are co-determined and that both can change over time. After all, the relative performance regime has the natural implication that some agents would be punished and other rewarded, so that starting from equality and relative performance, there would be switches to inequality and groups. In the other direction, starting from inequality and groups, successful outcomes can increase wealth and cause a switch to a more individualistic regime.

What we did not anticipate, from the data or our initial thinking, is that depending on the staring point, one could see long periods with a stable regime, either risk sharing networks or individualist structures, then periods of volatility with a switch back and forth across organization structure many times in relatively few periods. We show how in principle this model would be tested, with cross section data and a sufficiently long panel.

Various other authors have taken up the challenge of producing a theory of groups or networks, that is, making groups or group size endogenous. Kranton and Bramoullé [16] model networks as the costly formation of bilateral links. Their setup allows heterogenous treatment of otherwise identical individuals. In a dynamic version with exogenous pairwise meetings probabilities, the size of a dynamic network grows and shrinks over time with agents at the fringe at risk of being cut-off entirely. Related is Jackson and Watts [14] who focus on self-improving and myopic paths. In contrast, Genicot and Ray [10] consider a static model with a core notion of outcomes, that is, the possibility that subgroups of individuals may destabilize insurance arrangements among the larger group. Self-enforcing risk-sharing agreements are robust not only to single-person deviations but also to potential deviations by subgroups. However, such deviations must be credible, in the sense that the subgroup must pass exactly the same test that is applied to the entire group. Stable groups have bounded size and the degree of risk sharing is non-monotonic in risk.

Also delivering clusters but with varying internal risk sharing is the paper of Murgai, Winters, Sadoulet and deJanvry [20]. High association costs combined with low extraction costs lead to clusters with full insurance, while low association costs with high extraction costs will lead to community-level partial insurance. Empirical results from data on water exchanges among households along irrigation canals in Pakistan are used to support this proposition.

Here, in our paper, we take a dynamic, foresighted, mechanism design approach with information as an explicit impediment to trade. This connects us to a mechanism design literature with its focus on information and theories of inside and outside monies. Cavalcanti and Wallace [5] study implementable allocations in a random matching model in which some people have publicly known histories (banker) and others have private histories (non-bankers). Bankers can issue bank notes, to be compared with outside money. Though outside money dominates, Mills [18] shows that information lags for bankers can deliver the combined use of both kinds of monies. For us, groups have good internal information though the outsider does not, while individuals in relative performance have limited information as with the principal. Our results show that diverse information structures and organization forms can coexist and evolve in interchangeable forms.

We characterize the solution to the multiagent moral hazard model problem as a linear program. At this point, a curse of dimensionality emerges - a common problem in the linear programing solutions of contract problems, exacerbated here by the presence of more than one agent. We develop a formulation that makes computational implementation less demanding. The basic idea of this reformulation is to solve the moral hazard problem using a variable that summarizes utilities from consumption and promises to the future, instead of using consumption and promises directly. This formulation can be useful for multiagent dynamic moral hazard problems in general and facilitates empirical work.

The paper is organized as follows. In section 2 we present the model, that is basically a dynamic extension of the formulation of Prescott and Townsend [23]. In section 3 we prove some propositions about the dynamics of the model. In particular, we show that incentives for effort make the dynamics depend on output. The presence of outside options for agents and the principal can make the feasible set compact and rule out degenerate steady states. Section 4 presents and discusses numerical results obtained. The numerical procedure adopted, including some computationally convenient reformulations of the problem, is discussed in details in the appendix.

# 2 The Model

## 2.1 Environment:

A local economy has three individuals, two agents with preferences as discounted expected utilities over consumption and effort, and a principal, whose objective function is the present value of the surplus of production over the agents' consumption. The utility of agent $i$ at period $t$ is $w_i^t \equiv E\{\sum_{s=t}^{T} \beta^{s-t}[U(c_i^s) + V(e_i^s)]\}$, where $c_i^t$ and $e_i^t$ are, respectively, the consumption and the effort of agent $i$ at period $t$, and $\beta$ is a subjective discount factor. Function $U$ is strictly concave and strictly increasing. The function $V(e)$ is strictly decreasing, meaning that agents prefer to make low effort. The production technology is characterized by $p(q_1^t, q_2^t \mid e_1^t, e_2^t) > 0$, a probability distribution of outputs at $t$ that depends on the effort levels of both individuals within the period. This allows correlated outputs. The present value of the principal's surplus flow at $t$ is given by $S^t \equiv \sum_{s=t}^{T} \left(\frac{1}{1+r}\right)^s [q_1^s + q_2^s - c_1^s - c_2^s]$, where $r$ is an exogenous interest rate, as if this were a small open economy. We assume that $\frac{1}{1+r} = \beta$. For expositional convenience we denote the vectors $(c_1^t, c_2^t)$, $(q_1^t, q_2^t)$ and $(e_1^t, e_2^t)$ by $c^t$, $q^t$ and $e^t$, respectively. The sets of possible consumption, effort and output for each individual in each period are denoted, respectively, by $C$, $Q$ and $E$. Thus consumption, effort and output pairs are, respectively, in $C^2$, $Q^2$ and $E^2$.

We study allocations that are efficient conditional on the following moral hazard problem: the principal does not observe the efforts of the agents, although it is possible to allow one agent to observe the effort of the other. This implies that the decision of how much effort is done is ultimately taken by the agents. The decision process depends on how the agents are organized. Two regimes are available: a group regime in which individuals are allowed to communicate and thus to collude and jointly define levels of effort and share risks, and a relative performance regime in which individuals are not allowed to communicate and collude. Inside a group, each agent can enforce actions defined in an agreement (or contingent plan). But the members of a group can collude against the principal and take actions and consumption levels that are not recommended in the initial plan, if this can benefit the agents of the group. Therefore, the choices inside a group maximize a sum of utilities weighted by underlying internal Pareto weights. While in the relative performance regime each individual decides how much effort to do, in the group regime this decision is taken jointly, according to the Pareto weights inside the group. We assume that the implementation of the relative performance regime has a fixed cost of $k$ that represents the cost of avoiding collusion, communication and side payments among individuals.

The type of collective organization at a given date is characterized by the regime (relative performance or groups), and in the case of groups, by the inside-group utility weights. We define $O$ as the set of types of organization available. The elements of $O$ characterize the regime, and for the groups regime, the Pareto weight inside groups. We denote the set of possible utility weights inside groups by $M$.

## 2.2 The Mechanism Design Program

Our goal is to find allocations that are efficient given the environment and the organizational forms available. We formulate the overall Pareto problem as a maximization of the expected surplus of an outsider contractor, that we call the principal, conditional on initial promised utility levels for the two agents. This formulation may make it appear that the principal is the one making decisions about the choice of regime, but this is just a way to allow us to find the set of information-constrained Pareto optimal allocations, varying with the initial promise vector. The Mechanism Design problem thus determines the optimal state contingent historical path of the type of organization to which agents are assigned, their vectors of consumption, effort and output, and the surplus of the principal. The constraints on the problem include a set of incentive and technological constraints.

We follow Phelan and Townsend [21] and solve the problem sequentially. At each period the distribution of consumption, effort, organization and future expected utilities are defined by a principal-multiagent problem conditional on initial promises. This recursive formulation of the problem is not restrictive, since expected utilities from the future and their corresponding surplus levels are sufficient to characterize all incentives that come from future arrangements. Finally, for simplicity[1] and realism, we require that arrangements be renegotiation proof *i.e.*, they do not allow gains for all individuals resulting from renegotiation between the principal and the agents, as could be the case if the surplus function were non decreasing in one or both utility promises.

More formally, at any period, $t$, a principal multi-agent problem defines a probability distribution over the elements of $\Gamma_t \equiv C^2 \times Q^2 \times E^2 \times O \times W^2$, which are vectors expressing consumption, output and effort vectors, the type of organization that individuals are part of and the pair of expected future utilities. The distribution of consumption, output, effort and the type of organization in future periods is implied by the choices of promised future utilities $w_1^{t+1}$ and $w_2^{t+1}$. Throughout the remainder of the paper, we call $(w_1^t, w_2^t)$ the ex-ante utility pair at $t$, and $(w_1^{t+1}, w_2^{t+1})$ the set of promised (future) utilities assigned at $t$. For notational convenience, we sometimes omit the index $t$ on variables and refer to the initial pair of expected utilities as $w$ and the pair of promised utilities as $w'$. Again, the surplus of the principal at $t$ is $S^t(w)$ and at $t+1$, $S^{t+1}(w')$ where the superscript $t$ can also be deleted as in an infinite period problem.

The sequence of events in period $t$ is presented in Figure 1. First, agents are assigned to a regime: groups or the relative performance. When they are assigned to the group regime, utility weights inside the groups are defined. Then, the agents or the group decide their amount of effort $(e_1^t, e_2^t)$. Following the employment of effort, an output pair $(q_1^t, q_2^t)$ is obtained and

[1] Although the non-renegotiation requirement imposes additional constraints to the problem, it is helpful in obtaining numerical solutions since it guarantees that the surplus maximization problem is equivalent to utility maximization conditional on the surplus level.

observed by the principal. Finally, conditional on the output, a consumption pair $(c_1^t, c_2^t)$ and a pair of promised expected utilities $(w_1^{t+1}, w_2^{t+1})$ is assigned.

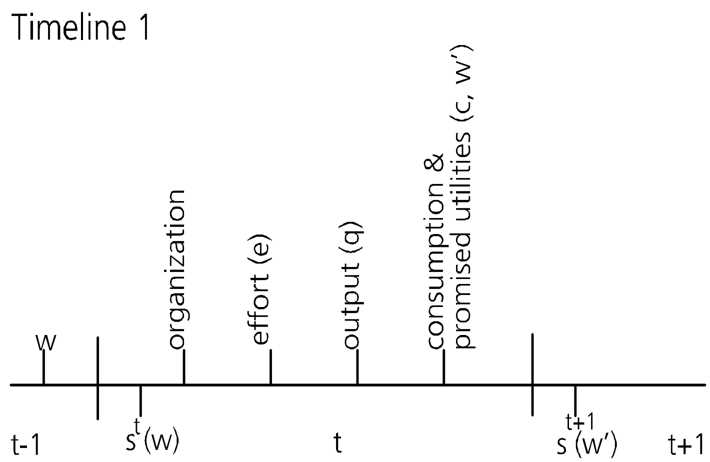


Figure 1

Whenever individuals join an organization $o$, an ex-ante organization-specific utility pair, denoted $w_o \in W^2$ is implied. The arrangement given $o$ and $w_o$ must be renegotiation proof, in the sense that there is no possibility of utility improvement to the agents or the group without a lower surplus to the principal. We can assume, without loss of generality, given the choice of organization $o$ and a corresponding organization specific utility pair $w_o$, the level of effort $e$, and the output of each agent, that the choices of consumption and promises for each agent are deterministic [2]. Therefore, given the vector of initial promises $w$, the vectors of consumption $c$ and promises for the future $w'$ can be written as functions of output $q$, effort $e$, organization $o$,and a utility pair generated under this organization $w_o$. Here again, $o \in O$ represents the

[2]Notice that consumption is separable from efforts in utility. From the fact that the utility function is concave and promises can be redefined as the expected values of some initial randomization, given any arrangement that has randomization of $c$ and $w'$, identical utility with no smaller surplus can be obtained without randomization of $c$ and $w'$, without affecting incentive constraints. If the initial arrangement is renegotiation proof, the second one is also renegotiation-proof. In any event, we can generate randoness in $w'$ by varying $w'_o$. For a more explicit formulation which allows general randomization in promises and consumption see the Appendix.

organizational form, and we use the notation $o = \mu$ when the regime is groups with internal utility weights $\mu \in M$, and $o = r$, when the regime is relative performance. Randomization between different arrangements under the same regime is allowed[3], so the decision variable includes not only a lottery over the choice of organization, but also randomization of utility pairs $w_o$ for each organization $o$. We assume that $Q$, $E$ and $O$ are finite sets. For efforts and outputs, a finite grid has a natural interpretation: $E$ has two elements $e_h$ and $e_l$, denoting high and low effort, respectively, and $Q$ has two elements $q_h$ and $q_l$, denoting success and failure, respectively. Since the Pareto weights for each individual inside a group, in principle, could have any value between zero and one, the set of possible organizations is more naturally a continuum, that is approximated here with a fine grid.

We may also assume that whenever agents are assigned to some type of organization, they must have an expected utility that is not smaller than some exogenously given value, that corresponds to an outside option. For example, they could be entirely on their own, with the utility from production in autarky, or possibly a lower value, if there is some punishment or social stigma from abandoning the link with the principal and the other agent. Similarly, we may assume that the principal can walk away to some outside option, so arrangements are subject to a lower bound in the level of surplus to the principal or equivalently, an upper bound in the set of promises to the agent. These bounds are not always imposed, and we point out where they are utilized in the exposition below.

The set of choice variables of the principal-agent problem consists of four objects. First, the probabilities of each organizational form and corresponding utility pair generated under this organizaional form, $\Pr(o, w_o \mid w)$. Second, the probabilities of effort levels for each choice of organization and organization-specific utilities, $\Pr(e \mid o, w_o, w)$. Finally, functions $c(q, e, o, w_o \mid w)$ and $w'(q, e, o, w_o \mid w)$, that determine the values of consumption and promises conditional

[3] Randomization given the same organization may be useful because given Pareto weights $\mu$, it is possible that $w_\mu^*$ and $w_\mu^{**}$ are feasible and renegotiation proof given $\mu$, but some utility level $w_\mu^l$ that is a convex combination of $w_\mu^*$ and $w_\mu^{**}$ is not renegotiation proof given $\mu$. Randomization across the two points, each of which is renegotiation proof can generate the utility pair $w_\mu^l$.

on outputs, effort levels, organizational form, and utility under organization.

The probability functions $\Pr(o, w_o \mid w)$ and $\Pr(e \mid o, w_o, w)$ are summarized by the joint probability distribution:

$$\Pr(q, e, o, w_o \mid w) \equiv p(q \mid e) \cdot \Pr(e \mid o, w_o, w) \cdot \Pr(o, w_o \mid w), \tag{1}$$

where $p(q \mid e)$ is determined by the technology. Indeed, for expositional convenience, we characterize the problem in terms of the joint distribution $\Pr(q, e, o, w_o \mid w)$, but for this to be consistent with the technology, the choice object $\Pr(e, q, o, w_o \mid w)$ must be constrained by:

$$\Pr(\widehat{q}, \widehat{e}, \widehat{o}, \widehat{w}_o \mid w) = p(\widehat{q} \mid \widehat{e}) \sum_q \Pr(q, \widehat{e}, \widehat{o}, \widehat{w}_o \mid w) \tag{2}$$

$\forall(\widehat{q}, \widehat{e}, \widehat{o}, \widehat{w}_o) \in Q^2 \times E^2 \times O \times W^2 \equiv X$. The choice variable is thus$\{\Pr(q, e, o, w_o \mid w), c(q, e, o, w_o \mid w), w'(q, e, o, w_o \mid w) : q \in Q^2, e \in E^2, o \in O, w_o \in W^2\} \in Y$, where $Y$ is the set of functions from $X$ to $[0,1] \times C^2 \times W^2$.

Any choice $y(w) \in Y$ determines a distribution of effort, output, consumption and promises conditional on each type of organization and corresponding organization-specific utlities. We denote these distribution of variables conditional on organization $\overline{o}$ with sutplus $\overline{w}_o$ by $y(\overline{o}, \overline{w}_o \mid w) = \{\Pr(q, e \mid \overline{o}, \overline{w}_o, w), c(q, e, \overline{o}, \overline{w}_o \mid w), w'(q, e, \overline{o}, \overline{w}_o \mid w)\}$, where $\Pr(q, e \mid \overline{o}, \overline{w}_o, w) \equiv \Pr(q, e, \overline{o}, \overline{w}_o, \mid w) / \sum_{e,q} \Pr(q, e, \overline{o}, \overline{w}_o, \mid w)$. This is a function from $Q^2 \times E^2$ to $[0,1] \times C^2 \times W^2$ which is completely characterized by the function $y(w)$ with the domain restricted to $o = \overline{o}$ and $w_o = \overline{w}_o$. Notice that :

$$\sum_{q,e} \Pr(q, e \mid \overline{o}, \overline{w}_o, w) = 1 \tag{3}$$

Further and critical, some of the constraints of the problem are specific of the organization adopted and the corresponding organization specific utility level.

**Group Constraints** Inside a group, agents can enforce any agreement (or contingent plan), but they can also collude and take decisions that are not recommended in the initial plan if this can benefit one agent without hurting the other. This is anticipated in the original plan, which is defined so that agents have no possibility of such internal Pareto improvements given assigned utility weights $\mu$. Therefore, the plan must be such that the actions taken maximize the sum of the $\mu$-weighted agent utilities. Similarly, the plan must be such that there is no gain from renegotiation between the group of agents and the principal. This implies that the plan chosen must be a surplus maximizer among all feasible plans that produce at least the same sum of $\mu$-weighted utilities.

The incentive constraints, which determine that it will be optimal for a group with Pareto weight $\mu$ (hence $o = \mu$) and utility pair $w_\mu$ to do the recommended level of effort is:

$$\begin{aligned} & \sum_q \Pr(q, e \mid \mu, w_\mu, w) \sum_i \mu_i \left[U(c_i(q, e, \mu, w_\mu \mid w) + V(e_i) + \beta w_i'(q, e, \mu, w_\mu \mid w)\right] \\ \geq \; & \sum_q \Pr(q, e \mid \mu, w_\mu, w) \frac{p(q \mid \widehat{e})}{p(q \mid e)} \sum_i \mu_i \left[U(c_i(q, e, \mu, w_\mu \mid w)) + V(\widehat{e}_i) + \beta w_i'(q, e, \mu, w_\mu \mid w)\right], \end{aligned} \tag{4}$$

$\forall e, \widehat{e}$.

The set of feasible arrangements $\widehat{y}$ that are *preferred* by a group with internal Pareto weights $\mu$ to the choices implied by $y(\mu, w_\mu \mid w)$ is:

$\Psi_{\mu,w_\mu} \equiv \{\widehat{y} : \widehat{y} = [\widehat{\Pr}(q, e, \mu, \widehat{w}_\mu \mid w), \widehat{c}(q, e, \mu, \widehat{w}_\mu \mid w), \widehat{w}'(q, e, \mu, \widehat{w}_\mu \mid w)]$ satisfying (2), (3) and (4) for any $e$ given $\mu$ and $\widehat{w}_\mu = \sum_{q,e} \widehat{\Pr}(q, e, \mu, \widehat{w}_\mu \mid w) \sum_i \mu_i [U(\widehat{c}_i(q, e, \mu, \widehat{w}_\mu \mid w)) + V(e_i) + \beta \widehat{w}_i'(q, e, \mu, \widehat{w}_\mu \mid w)] > \mu_1 w_{\mu 1} + \mu_2 w_{\mu 2}\}$,
where $w_{\mu i}$ is the $i$th element of $w_\mu$.

The conditions determining that there is no possibility of gains from renegotiation between groups and the principal are that for any $\mu$ and $w_\mu$, if $\widehat{y} \in \Psi_{\mu,w_\mu}$, the surplus is smaller than in $y(\mu, w_\mu \mid w)$ :

$$\begin{aligned} & \sum_{q,e} \widehat{\Pr}(q, e, w_\mu \mid \mu, w)[q_1 + q_2 - \widehat{c}_1(q, e, \mu, w_\mu \mid w) - \widehat{c}_2(q, e, \mu, w_\mu \mid w) \\ & + \beta S^{t+1}(\widehat{w}'(q, e, \mu, w_\mu \mid w))] < \sum_{q,e} \Pr(q, e \mid \mu, w_\mu, w)[q_1 + q_2 - c_1(q, e, \mu, w_\mu \mid w) \\ & - c_2(q, e, \mu, w_\mu \mid w) + \beta S^{t+1}(w'(q, e, \mu, w_\mu \mid w))] \end{aligned} \tag{5}$$

Thus an increase in the $\mu$-weighted sum of utilities would require some loss to the principal. Notice that this implies that the surplus function is strictly decreasing: higher utility for the group must produce lower surplus. Condition (5) determines that whenever some internal Pareto weight $\mu$ is assigned to a group, the choices conditional on this organizational form maximize the $\mu$ weighted sum of utilities conditional on the surplus they produce. But notice that (5) also implies that inside a group there is full risk sharing[4]. The consumption of individuals depend only on the aggregate consumption and on the Pareto weights inside the group. Specifically, agent levels of consumption at any period $t$ must satisfy $c_i^t = c_i(\mu, c_a)$, where the function $c_i(\mu, c_a)$ is defined by the following subproblem generating the groups risk sharing rule:

$$(c_1(\mu, c_a), c_2(\mu, c_a)) = \arg \max_{(c_1, c_2) \in C^2} \mu_1 U(c_1) + \mu_2 U(c_2),$$

s.t.

$$c_1 + c_2 = c_a,$$

where, again, $c_a$ is the aggregate consumption (with values on $C + C = 2C$). This property gives us the interpretation of a group as a risk sharing network.

**Relative Performance Constraints** In the relative performance regime, agents are prevented from communicating to coordinate efforts and make side payments that could possibly mitigate incentives. Since outputs are correlated, differences in output among agents can be used as an indication of different levels of effort. Comparative performance can be used as an incentive tool, and it is necessary for such incentive schemes that individuals consume what they are assigned and are prevented from coordinating actions.

The incentive constraints for the relative performance regime (where $o = r$) with corresponding utilities of $w_r$ are:

[4] If the consumption allocations determined in $y(\mu, w_o \mid w)$ do not solve the risk sharing problem, it is possible to find a strictly preferred allocation (that therefore belongs to $\Psi_{\mu, w_o}$) that has surplus equal to $y(\mu, w_o \mid w)$, which violates (5). Notice that this risk sharing problem does not depend on effort given the separability of effort and consumption in the utility functions.

$$\sum_{q,e_2} \Pr((q,e_1,e_2 \mid r,w_r,w)\left[U(c_1(q,e_1,e_2,r,w_r \mid w)+V(e_1)+\beta w_1'(q,e_1,e_2,r,w_r \mid w)\right] \tag{6}$$

$$\geq \sum_{q,e_2} \Pr((q,e_1,e_2 \mid r,w_r,w)\frac{p(q \mid \widehat{e}_1,e_2)}{p(q \mid e_1,e_2)}\left[U(c_1(q,e_1,e_2,r,w_r \mid w))+V(\widehat{e}_1)+\beta w_1'(q,e_1,e_2,r,w_r \mid w)\right],$$

$\forall e_1, \widehat{e}_1 \in E$, and

$$\sum_{q,e_1} \Pr((q,e_1,e_2 \mid r,w_r,w)\left[U(c_2(q,e_1,e_2,r,w_r \mid w)+V(e_2)+\beta w_2'(q,e_1,e_2,r,w_r \mid w)\right] \tag{7}$$

$$\geq \sum_{q,e_1} \Pr((q,e_1,e_2 \mid r,w_r,w)\frac{p(q \mid \widehat{e}_1,e_2)}{p(q \mid e_1,e_2)}\left[U(c_2(q,e_1,e_2,r,w_r \mid w))+V(\widehat{e}_2)+\beta w_2'(q,e_1,e_2,r,w_r \mid w)\right],$$

$\forall e_2, \widehat{e}_2 \in E$

As in the group regime, we impose the constraint that there be no possibility of improvements for the agents without a lower surplus to the principal. Given the distributions conditional on the relative performance regime $y(r, w_r \mid w)$, the set of feasible relative performance arrangements that are preferred by both agents to the choices implied by $y(r, w_r \mid w)$ is:

$\Psi_{r,w_r} \equiv \{\widehat{y} : \widehat{y} = [\widehat{\Pr}(q,e,r \mid \widehat{w}_r, w), \widehat{c}(q,e,r,\widehat{w}_r \mid w), \widehat{w}'(q,e,r,\widehat{w}_r \mid w)]$ , satisfies (2), (3), (6) and (7) for any $e$, and $\widehat{w}_r = \sum_{q,e} \widehat{\Pr}(q,e \mid r,\widehat{w}_r,w)[U(\widehat{c}_i(q,e,r,\widehat{w}_r \mid w)) + V(e_i) + \beta\widehat{w}_i'(q,e,\widehat{w}_r \mid r,w)] > w_{ri}$ for i=1 and i=2$\}$,

where $w_{ri}$ is the i$th$ element of $w_{ri}$. In order for $y$ to be such that no improvement can be obtained from renegotiation, it must be the case that the surplus of the principal when the Relative Performance regime is adopted with organization specific surplus $w_r$ is higher than any surplus in $\Psi_{r,w_r}$ :

$$\begin{aligned}\sum_{q,e} \widehat{\Pr}(q,e \quad &| \quad r,w_r,w)[q_1+q_2-\widehat{c}_1(q,e,r,w_r \mid w)-\widehat{c}_2(q,e,r,w_r \mid w)+ \\ \beta S^{t+1}(\widehat{w}'(q,e,r,w_r \quad &| \quad w))] < \sum_{q,e} \Pr(q,e \mid r,w_r,w)[q_1+q_2-c_1(q,e,r,w_r \mid w) \\ -c_2(q,e,r,w_r \quad &| \quad w)+\beta S^{t+1}(w'(q,e,r,w_r \mid w))]\end{aligned} \tag{8}$$

for any $\widehat{y}$ in $\Psi_{r,w_r}$.

**The Program** The principal-agent problem at $t$ is:

**Program 1**[5]

$$S^t(w) \equiv \max_{\{\Pr,c,w'\}\in Y} \sum_{q,e,w_r} \Pr(q,e,r,w_r \mid w)[q_1+q_2-k-c_1(q,e,r,w_r \mid w)- \tag{9}$$

$$c_2(q,e,r,w_r \mid w)+\beta S^{t+1}(w'(q,e,r,w_r \mid w))]+\sum_{q,e,\mu,w_\mu} \Pr(q,e,\mu,w_\mu \mid w)[q_1+q_2-$$

$$c_1(q,e,\mu,w_\mu \mid w)-c_2(q,e,\mu,w_\mu \mid w)+\beta S^{t+1}(w'(q,e,\mu,w_\mu \mid w))]$$

subject to the promise keeping constraints

$$\sum_{q,e,o,w_o} \Pr(q,e,o,w_o \mid w)w_o = w \tag{10}$$

and for any $\widehat{o}$ in $O$ and $\widehat{w}_o$ in $W^2$,

$$\sum_{q,e} \Pr(q,e,\widehat{o},\widehat{w}_o \mid w)\left[U(c_i(q,e,\widehat{o},\widehat{w}_o \mid w))+V(e_i)+\beta w_i'(q,e,\widehat{o},\widehat{w}_o \mid w)\right]=\widehat{w}_o \tag{11}$$

$$\sum_{q,e,o,w_o} \Pr(q,e,o,w_o \mid w)=1, \text{ and } \Pr(q,e,o,w_o \mid w)\geq 0 \tag{12}$$

and (2),(4),(5),(6),(7) and (8). Notice from the objective function of Program 1 that an amount $k$, corresponding to the cost to prevent collusion, is subtracted from the surplus whenever the regime is relative performance. Program 1 defines the surplus function at $t$, $S^t$, given the surplus function at $t+1$, $S^{t+1}$. This characterizes a functional equation $S^t = F(S^{t+1})$. As in Phelan and Townsend [21], a fixed point in this functional equation, $S = F(S)$ is a solution for an infinite period problem.

As we show now, this problem can be seen as a randomization between the choice of organization specific problems.

**Definition 1** *The surplus specific to organization $o$ and utilities $w_o$, $S_o(w_o)$ is the solution to Program 1 without constraint (10) and with the additional constraint that*[6] $\sum_{q,e} \Pr(q,e,o,w_o \mid$

[5] There is an abuse of notation in using a sum here, since the space of $w_o$ is defined in a continuum. In any event, the optimal solution has only a finite amount of values of $w_o$ with positive probability.

[6] Using the notation of equation (1) $\Pr(\overline{o},e \mid w) \equiv \Pr(\overline{o} \mid w)\cdot \Pr(e \mid \overline{o},w)$.

$w) = 1$. $\widetilde{W}^o$ *is the set of all values of* $w_o$ *for which this last program is defined. The policy graph for organization* $o$ *is* $\Delta_o(\widetilde{W}^o) \equiv \{[(\widetilde{\Pr}(q,e,o,w_o \mid w), \widetilde{c}(q,e,o,w_o \mid w), \widetilde{w}'(q,e,o,w_o \mid w), w_o]$ : $\widetilde{\Pr}(q,e,o,w_o \mid w), \widetilde{c}(q,e,o,w_o \mid w), \widetilde{w}'(q,e,o,w_o \mid w)$ *solves the organization specific problem given* $w_o$ *in* $\widetilde{W}^o\}$.

**Proposition 1** $S(w)$ *can always be obtained as a randomization of organization specific surpluses. More specifically:* $S(w) = \sum_{o,w_o} \Pr(o, w_o \mid w) S_o(w_o)$, *and* $w = \sum_{o,w_o} \Pr(o, w_o \mid w) w_o$.

**Proof.** Let $y = \{\Pr(q,e,o,w_o \mid w), c(q,e,o,w_o \mid w), w'(q,e,o,w_o \mid w)\}$ be a solution to program 1. Suppose organization $\overline{o}$ with utilities $\overline{w}_o$ is adopted with positive probability. Then, $y_{\overline{o},\overline{w}_o} = \{\Pr(q,e,\overline{o},\overline{w}_o \mid w)/\sum_{q,e}\Pr(q,e,\overline{o},\overline{w}_o \mid w), c(q,e,\overline{o},\overline{w}_o \mid w), w'(q,e,\overline{o},\overline{w}_o \mid w)\}$ must be a solution to the $\overline{o}$ and $\overline{w}_o$specific surplus maximization problem. Constraints (2) to (8) will remain valid. Constraint (12) and (11)will also be valid. The choice $y_{\overline{o},\overline{w}_o}$ must maximize (9) conditional on these constraints and on $\sum_{q,e}\Pr(q,e,\overline{o} \mid w) = 1$. If this were not true, another object would maximize (9) conditional on these constraint and an improvement in the objective of Program 1 would be possible by replacing the solution conditional on $\overline{o}$ and $\overline{w}_o$, $y(\overline{o}, \overline{w}_o \mid w)$, by this object, which would imply that $y(w)$ is not optimal. The optimized value of (9) is $S_{\overline{o}}(\overline{w}_o)$, and its contribution to the overall surplus of Program 1 is $\Pr(\overline{o}, \overline{w}_o \mid w) \cdot S_{\overline{o}}(w_{\overline{o}})$. So if we consider all organizational forms and corresponding utilities with positive probability, we have $S(w) = \sum_{o,w_o} \Pr(o, w_o \mid w) \cdot S_o(w_o)$. Note that the solution of the organization specific problem incorporates randomization of recommended efforts given the organization. All incentive constraints are conditional on the choice of organization, organization-specific utilities and recommended effort. Any further randomization of effort is incorporated in the randomization between organizations and organization-specific utilities. So, joint randomization of effort with organization is allowed in this formulation ■

Notice from proposition 1 that for any $w$, the optimal arrangement can be obtained through only a finite number of organizations and corresponding surplus levels.

We can impose if we wish, as an additional constraint to Program 1, that for any choice of organization, the organization specific surplus $w_o$be not smaller than a given outside option of

$\underline{w}$: agents will only agree to join an organization that will give them an utility level that is not lower than the one given by this outside option. Similarly, we can also impose that the surplus of the principal obtained from agreements under any type of organization be not smaller than an outside option $\underline{S}$.

## 2.3 The Space of Promises and the Dynamics of the Model

The space on which the surplus function is defined is denoted by $\widetilde{W} \subseteq W^2$. This space will depend on outside options of the agents and the principal and also on incentive compatibility. Some utility pairs are not achievable by any regime. Suppose, for instance, that the utility function has a lower bound or, without loss of generality, $U(0) = 0$. Then, the minimal level of utility possible for each agent is that produced by zero consumption and high effort in all periods. Even this level of utility cannot be sustained for both agents simultaneously. Incentives for high effort require that rewards be given to at least one agent when high output is obtained. At zero consumption to both agents, there is no incentive to make the high level of effort. The space on which $S$ is defined also incorporates potential lower bounds on the utility of each agent. But if $\lim_{c \to 0} U(c) = -\infty$ and agents have no outside option, the set $\widetilde{W}$ is unbounded from below. Also, if $\lim_{c \to \infty} U(c) = \infty$ and there is no outside option for the principal, the set $\widetilde{W}$ is unbounded from above.

Further, some regions of the utility map are reachable only by one regime. Under relative performance, each individual needs incentives for high effort. Suppose $U(0) = 0$ and consumption is always zero to some agent. His effort would then always be the least possible, $e_l$, and his overall utility would be $V(e_l)/(1-\beta)$. Any relative performance contracts must offer at least this utility level. However, the group regimes is capable of sustaining arrangements with utility as low as $V(e_h)/(1-\beta)$ for (at most) one of the agents. That is, in the groups regime, it is possible to impose high effort and zero consumption, for example when $\mu_i = 0$, which provides utility equal to $V(e_h)/(1-\beta) < V(e_l)/(1-\beta)$.

The solution to Program 1 determines the dynamic behavior for utility promises, and thus of output, consumption and effort levels. The following results show that whenever high effort is implemented by one of the agents, there is a positive probability that utility changes from one period to the next. This is an extension of the result presented by Rogerson [25], that in dynamic moral hazard problems, both current consumption and future utilities depend on outputs. We extend Rogerson 's proof to our model, in which the state of promises may be bounded.

An important ingredient for this time variability of utility promises is that a Moral Hazard problem actually exists, so that rewards depend on the realization of outputs. Again, however, in a two agent setting where groups are allowed, high effort by one agent does not necessarily imply the existence of a moral hazard problem. It is possible that the agent that is expected to make high effort has zero Pareto weight inside a group. Lemma 1 below establishes that a sufficiently high lower bound on promises rules out groups with zero Pareto weights, so incentives for high effort always require some dependency of utilities on outputs.

**Lemma 1** *Suppose that a lower bound $\underline{w}$ is imposed for each organization o, so that $w_o > \underline{w}$. In particular, suppose $\underline{w} > V(e_h)/(1-\beta)$. Then, whenever at least one agent makes high effort, either consumption or future promises or both depend on outputs.*

**Proof.** In the relative performance regime, agents have separate incentive constraints, so if any agent makes high effort as in the premise, then that agent will require consumption and/or promises to depend on output, otherwise his incentive constraint would not be valid. Let us now consider the group regime. If $\mu_i = 0$ for some agent $i$, the solution for a group will have zero consumption, high effort and promises equal to $\underline{w}$ for $i$. Therefore, the current utility of $i$ will be $V(e_h) + \beta\underline{w}$. But by the premise of the Lemma, $V(e_h) + \beta\underline{w} < \underline{w}$ and this violates the lower bound. So, if the group regime is chosen, $\mu_i > 0$ for both agents. This implies that, even if groups are chosen and one of the agents is expected to do low effort, consumption and (or) promises must be dependent on outputs, otherwise the incentive constraint (4) would not be satisfied.. ■

The following Lemma is a version of the envelope theorem and presents some properties relating the surplus function with the surplus obtained in each period. We use the following notation: $Z$ is the set of values of $(q, e, o, w_o)$ that are reached with positive probabilities in the solution of Program 1, $\pi_z \equiv \Pr(z)$ is the probability of state $z \in Z$. When $z = (q, e, o, w_o)$, the notation $c_z$ denotes $c(q, e, o, w_o \mid w)$, $w'_z$ denotes $w'(q, e, o, w_o \mid w)$ and $e_z$ denotes $e$. The notation $u_z \equiv U(c_z) + V(e_z)$ denotes the vector of current utilities (from current consumption and effort), and $s(u, e, q) = q_1 + q_2 - U^{-1}(u_1 - V(e_1)) - U^{-1}(u_2 - V(e_2))$ is the current surplus given current utility vector $u$, effort level $e$ and output $q$.

**Lemma 2** *The following statements are valid for the solution of Program 1:*
*(a). If $w$ is in the interior of $\widetilde{W}$, and for any $z$ in $Z$ consumption is positive, $S(w)$ is differentiable at $w$, and its gradient is given by:*

$$S'(w) = E[s_u(u_z, e_z, q_z)], \tag{13}$$

*where $s_u(u_z, e_z, q_z)$ is the gradient of $s(u, e, q)$ with respect to $u$ in state $z$.*
*(b). If $w$ has some agent $i$ at a lower bound of $\widetilde{W}$, or with consumption equal to zero in some state $z$ in $Z$.*

$$S_{i+}(w) \geq E[s_u(u_z, e_z, q_z)],$$

*where $S_{i+}(w)$ is the right derivative of the surplus function with respect to the utility of agent $i$.*
*(c). If $w$ has some agent $i$ at an upper bound of $\widetilde{W}$ with consumption positive for any $z$ in $Z$,*

$$S'_{i-}(w) \leq E[s_u(u_z, e_z, q_z)],$$

*where $S_{i-}(w)$ is the left derivative of the surplus function with respect to the utility of agent $i$.*

**Proof.** We follow the Benveniste and Scheinkman theorem, as presented by Stokey and Lucas [26]. The infinite periods surplus function $S(w)$ is

$$S(w) = \sum_{z \in Z} \pi_z [s(u_z, e_z, q_z) + \beta S(w'_z)]$$

Since randomization is allowed between points with different utilities in all organizations, $S$ is concave.

(a). Adding an amount $\varepsilon$ (a vector with two elements, that can be positive or negative) of utility in every state through consumption would not change incentives for effort by agents and would change instant utility by an amount $\varepsilon$. But from optimization and from the non-renegotiation constraint, it must be the case that the surplus obtained in this way (increasing current utilities by $\varepsilon$) is not higher than the surplus from the solution of Program 1 with initial promises $w+\varepsilon$. Therefore:

$$\sum_{z\in Z} \pi_z[s(u_z+\varepsilon, e_z, q_z) + \beta S(w'_z)] \leq S(w+\varepsilon) \tag{14}$$

Since $S(w)$ is concave and not lower than the expression on the left hand side of (14), it must also be differentiable in the interior $w$, with its gradient equal to the gradient of the left-hand side with respect to $\varepsilon$ at $\varepsilon = 0$. Therefore:

$$S'(w) = E[s_u(u_z, e_z, q_z)].$$

(b). In this case, it may not be possible to decrease utility of agent $i$ by an equal amount in any state using only consumption (either because this is a lower bound or because consumption is zero in some state and cannot be decreased). But it is possible to increase the utility of agent $i$ only through consumption in any state, thus not affecting incentives. The resulting change in surplus is given by the derivative of the left-hand side of (14) with respect to $\varepsilon_i$. From (14), the change in optimal surplus of such an increase in utility cannot be smaller than this.

(c). In this case it is not possible to increase utility of both agents (since this is an upper bound). But it is possible to decrease the utility of agent $i$ by the same amount in every state only through consumption, thus without affecting incentives. The resulting change in surplus is the negative of the derivative of the left-hand side of (14)with respect to $\varepsilon_i$. From (14), the change in optimal surplus of an decrease in utility cannot be smaller than this. ■

Note that statement $(b)$ is relevant only for the case where $\widetilde{W}$ has a lower bound (either there is an outside option for the agents or $U(0)$ has a finite value). Similarly, statement $(c)$ matters only for the case in where $\widetilde{W}$ has an upper bound. These conditions are important to prove Proposition 2.

**Proposition 2** *Suppose the conditions of Lemma 1 are valid. Suppose also that whenever one agent is at an upper bound of* $\widetilde{W}$, *consumption is always positive*[7]. *Then whenever $w$ is such that there is always at least one individual making high effort in the current period, the probability that $w' \neq w$ is positive.*

**Proof.** We start from the case with the initial promise, $w$, in the interior of $\widetilde{W}$ and consumption is positive in every state. Again, following Rogerson [25], incentives for efforts will not change if, at any realization of uncertainty, $z$, consumption is subtracted in the current period so that $u_z$ is replaced by $u_z - y$ and $w'_z$ is replaced by $w'_z + y/\beta$,where $y$ is also a vector with 2 elements. The optimal choices for any $z$ must thus solve at $y = 0$:

$$\max_{y} s(u_z - y, e_z, q_z) + \beta S(w'_z + y/\beta). \tag{15}$$

If $w_z$ is in the interior of $\widetilde{W}$ for any $z$ in $Z$

$$s_u(u_z, e_z, q_z) = S'(w'_z). \tag{16}$$

From Lemma 1, different levels of $q$ imply different levels of either $u$ (and thus consumption) and or $w'$. But different levels of $u$ imply different values for $u_z$ and thus, from (16) different levels for $w'_z$. This implies that, no matter the type of organization, for some level of output (with probability not smaller than $\min\limits_{e,q} p(q \mid e)$) $w'$ will be different from $w$.

Let us now consider the case where consumption is equal to zero in some contingency or the vector of initial promises, $w$ has some agent $i$ in a lower bound of $\widetilde{W}$. Suppose also that incentives to $i$ are given only through consumption, so $w'_{zi} = w_{zi}$ for all $z$. From Lemma 2,

[7] Positive consumption in all contingencies is an endogenous result, a solution to the model, not a primitive assumption. This is trivially true when $\lim_{c \to 0} U(c) = -\infty$, but not necessarily true for the specifications we used. However, that consumption is always positive may be verified in numerical solutions.

$S_{i+}(w) \geq \sum_{z \in Z} \pi_z s_{ui}(u_z, e_z, q_z) > s_{ui}(u_{\underline{z}}, e_{\underline{z}}, q_{\underline{z}})$, where $\underline{z}$ is the state with maximal consumption for $i$, or minimal value of $s_{ui}$ (that happens with a probability not smaller than $\min_{e,q} p(q \mid e)$). Then, decreasing $u_{i\underline{z}}$ by $\varepsilon$ and increasing $w'_{i\underline{z}}$ from $w$ to $w + \frac{\varepsilon}{\beta}$ would not change the optimal choice of efforts and utilities for agents and would increase surplus. So it must be the case that $w'_{i\underline{z}} \neq w_i$. An analogous argument can be applied for the case with one agent in an upper bound of $\widetilde{W}$. ■

Proposition 2 states that when incentives are needed, there is a positive probability that promises move from one period to the next. The nature of incentives depends on the regime and, as it will be shown by the numerical results, the regime adopted impacts on the dynamics of utility promises.

None of the results above depend on the existence of a steady state.

### 2.3.1 Steady State

Several models in the literature on dynamic incentive problems do not have convergence to a nondegenerate steady state. Typically, a growing fraction of the population converge to absolute misery (as in Thomas and Worrall [27]), and possibly a vanishing fraction converge to extreme prosperity (Atkeson and Lucas [2], Phelan [22]). As shown by Thomas and Worrall, interior solutions make the gradient of the surplus function follow a non-negative martingale, as implied here by equations (13) and (16). A theorem by Doob (see Billingsley [4]) tells us that non-negative martingales almost surely converge. But the need for incentives makes it necessary that promises be history dependent, and convergence to any interior point is not optimal. So, there must be convergence either to an open boundary or divergence to infinity or negative infinity. In our case, when we impose outside options $\underline{w}$ and $\underline{S}$, the utility space becomes compact. As pointed out by Phelan, the Martingale property may not be valid with such boundary constraints.

However even with these boundaries, there is, in our model, the possibility that absorbent points exist in the feasible space of promises[8]. This may be the case if both individuals are required to make low effort, which implies that consumption and promises do not need to be output dependent. However, if the boundaries on promises are such that low effort is never optimally recommended to both individuals, Proposition 2 guarantees that there are no absorbent points.

The following results show that when high effort is implemented by at least one individual in all points of $\widetilde{W}$ there is no convergence in probability to any point of the compact set $\widetilde{W}$.

**Lemma 3** *Suppose the surplus function $S(w')$ is well defined in a compact set $\widetilde{W}$. Suppose that for some pair of real numbers $\underline{w}$ and $\underline{S}$ the additional constraints $w_o \geq \underline{w}$ for any $o$ in $O$ and $S^t(w) \geq \underline{S}$ are imposed in Program 1. Then, for any $o \in O$, if the set $\Delta_o(\widetilde{W}^o)$ of Definition 1 is not empty, it is compact.*

**Proof.** It is straightforward to see that with the outside options $\underline{w}$ and $\underline{S}$, the set $\Delta_o(\widetilde{W}^o)$, is bounded for any organization $o$. We have to show that it is also closed. Indeed take a sequence in $\Delta_o(\widetilde{W}^o)$, $\{\Pr_n(q,e,o,w_o^n \mid w), c^n(q,e,o,w_o^n \mid w), w'(q,e,o,w_o^n \mid w), w_o^n\}$ converging to a limit $\{(\widetilde{\Pr}(q,e,o,\widetilde{w}_o \mid w), \widetilde{c}(q,e,o,\widetilde{w}_o \mid w), \widetilde{w}'(q,e,o,\widetilde{w}_o \mid w), \widetilde{w}_o\}$. The weak inequality constraints (2), (4), (6) ,(7) and the equality constraint (11)are clearly valid in this limit. It remains to prove that the non-renegotiation constraint is valid for this limiting point.

Suppose first the regime is relative performance, so $o = r$ and the non renegotiation constraint is not valid in this limit. Then, there is some utility level $w_r^* > \widetilde{w}_r$ that is feasible under relative performance and has a surplus level $S_r^* \geq \widetilde{S}_r$, where $\widetilde{S}_r$ is the surplus implied by $\{\widetilde{\Pr}(q,e,r,\widetilde{w}_r \mid w), \widetilde{c}(q,e,r,\widetilde{w}_r \mid w), \widetilde{w}'(q,e,r,\widetilde{w}_r \mid w)\}$. Taking $n = \overline{n}$ high enough, $w_r^{\overline{n}} < w_r^*$, so, by non-renegotiation, it must be true that the surplus associated with this element of the sequence, $S_r^{\overline{n}}$, is such that $S_r^{\overline{n}} > S_r^* \geq \widetilde{S}_r$. There is some lottery between $w_r^{\overline{n}}$ and $w_r^*$ that would generate a utility pair $w_l > \widetilde{w}_r$ and a surplus $S_l > \widetilde{S}_r$. From the concavity of the utility

[8] In Thomas and Worrall [27], Atkeson and Lucas [2] and Phelan [22], shock dependence of promises is always needed for incentive purposes. In the current model, this may not be the case if low effort is recommended for both agents.

function and the surplus function, the utilities of these loteries can be generated by an object $y_l = [\Pr(q,e,r,w_l \mid w), \widehat{c}(q,e,r,w_l \mid w), \widehat{w}'(q,e,r,w_l \mid w)]$ that does not have randomization of consumption and promises conditional on output and effort, that has a surplus $\widehat{S}_r$ not smaller than $S_l$, and still satisfies constraints (2), (3), (6) and (7). Thus, $n$ high enough would have both $w_r^n < w_l$ (so $y_l$ is an element of $\Psi_{r,w_r^n}$)and a surplus $S_r^n$ such that $S_r^n < \widehat{S}_r$, which contradicts non-renegotiation of points in the sequence. A similar proof applies to groups with Pareto weights $\mu$, focusing on the $\mu-$weighted sum of utilities instead of the vector $w$ directly.

Notice also that no matter the type of organization, the surplus in this limiting arrangement must be optimal. If not, there is an arrangement with the same surplus and higher utility. It is then possible depart from this last arrangement and increase the utility through consumption in any contingency (such that incentives for efforts are never changed) and still have a surplus higher than the elements of the sequence for high $n$ enough. This would clearly violate non renegotiation of points of the sequence. ■

**Proposition 3** *Suppose the conditions of Proposition 2 are valid, that the set $\widetilde{W}$ is compact and that in the solution to Program 1 there is always high effort by at least one agent. Then $w$ does not converge in probability to any point in $\widetilde{W}$.*

**Proof.** From Proposition 1, the policy graph $\Delta(\widetilde{W}) = \{[\Pr(q,e,o,w_o \mid w), c(q,e,o,w_o \mid w), w'(q,e,o,w_o \mid w), w] : \Pr(q,e,o,w_o \mid w), c(q,e,o,w_o \mid w), w'(q,e,o,w_o \mid w)$ solves Program 1 given $w\}$ is given by optimal lotteries over elements of the compact sets $\Delta_o(\widetilde{W^o})$[9]. It must thus be compact. We can define a continuous function from $\Delta(\widetilde{W})$ to $\Re$ as $f([\Pr(q,e,o,w_o \mid w), c(q,e,o,w_o \mid w), w'(q,e,o,w_o \mid w), w]) = \max\limits_{(q,e,o,w_o)} \left|(w'(q,e,o,w_o \mid w) - w)\Pr(q,e,o,w_o \mid w)\right|.$

[9]Each element of $\Delta(\widetilde{W})$ is determined by choices of a lottery over organizations and the organization specific utilities *that* maximize surplus conditional on overall expected utility values. The set from which these utilities (and the corresponding policies) are taken, $\Delta_o(\widetilde{W}_o)$, are compact. So, the limit of a sequence of solutions will be feasible. The maximizer of the problem is continuous in $w$ (since randomization is alowed) so the surplus in this limit will be optimal.

Since $\Delta(\widetilde{W})$ is compact, $f$ has a minimum value $d$, that must be higher than zero from Proposition 2. So, the expected difference between $w$ and and $w'$ is, at any point, not smaller than some positive number, $d$, and the sequence does not converge. ■

Proposition 3 rules out convergence to a degenerate distribution and the existence of absorbing points. It reassures us that when numerical computations find a non-degenerate distribution of promises in the long run, the fact that this distribution is non-degenerate is not merely an artifact of the discretization used in the computation. Notice that Proposition 3 depends on at least one agent making high effort, which is a result and, not an assumption of the model. But a numerical solution in which high effort is observed for every $w$ is a good indication that the dynamics have no absorbing points. Further assurance can be obtained by a comparison between the surplus that is obtained by the numerical procedure to compute Program 1 for each $w$ and the surplus required to provide an utility pair $w$ through low effort and constant consumption over time.

# 3 Numerical Results

This section presents numerical solutions for Program 1. For computational purposes, we discretised the values of consumption and promises and used linear programing, a method that is convenient for problems with incentive constraints, where there may be randomization in the optimal solution. We used grids of $C$ with 18 elements, $M$ with 101 elements, and $W$ with 30 elements. We also broke the problem in several stages to deal with dimensionality problems. The Appendix presents in more detail the procedure used to obtain these solutions.

In the numerical solutions, the set of possible effort levels and outputs per agent are respectively $E = \{2, 6\}$ and $Q = \{2, 20\}$. We use the following baseline specification: the utility of consumption and the disutility of effort are $U(c) = c^{0.5}$ and $V(e) = -e^{0.5}$, respectively. This has only a modest degree of risk aversion. The cost of blocking collusion, $k$, is equal to 0.3. The discount rate $\beta$ is 0.7 which is relatively low, but allows fast convergence to the infinite period surplus function[10]. As in Prescott and Townsend [23], the technology $P(q_1, q_2 \mid e_1, e_2)$, is given by the following table:

Table - I

| | $q=(2,2)$ | $q=(20,2)$ | $q=(2,20)$ | $q=(20,20)$ |
|---|---|---|---|---|
| $e=(2,2)$ | 0.6979 | 0.0021 | 0.0021 | 0.2979 |
| $e=(6,2)$ | 0.2991 | 0.4009 | 0.0009 | 0.2991 |
| $e=(2,6)$ | 0.2991 | 0.0009 | 0.4009 | 0.2991 |
| $e=(6,6)$ | 0.2979 | 0.0021 | 0.0021 | 0.6979 |

Notice that this matrix implies that outputs of agents are correlated[11]: if they do the same amount of effort, it is very unlikely that their outputs will be different. We also assume a lower bound on the utility of agents in each organization type of $\underline{w} = -5.8$[12].

[10] After 15 iterations, the maximum difference between $S^t$ and $S^{t+1}$ was smaller than $10^{-5}$, and we assumed that $S^t$ was a reasonable approximation of the fixed point characterizing the infinite periods surplus function.

[11] When both agents make low effort, the correlation coefficient of outputs is 0.9683, and when they make high effort this coefficient is 0.99. The static literature on groups versus relative performance [e.g. Holmstrom and Milgron [12] and Itoh [13] predicts that high correlation in outputs favors the relative performance regime. Here we get some groups, nevertheless. A comprehensive review of the empirical literature on covariate risk by Dercon [7] shows diverging results about the correlation between outputs. Some measures of covariate risk such as rainfall, show considerably high correlation among plots inside villages, close to 1.0. However, it is not obvious how to translate these results into the parameters of our model, since efforts are endogenous and correlation of outputs depend on efforts.

[12] This is smaller than the autarky utility for each agent. A possible interpretation for this lower bound smaller than the autarky utility is that agents can move to autarky as an outside option but they have some punishment when they do it, that could come from legal enforcement or social stigma. This particular lower bound has been chosen to make the ergodic distribution of contracts more interesting. Without a lower bound on promised utilities, the ergotic distribution has only group contracts at the axis region, with one of the agents having the lower level of utilities possible. For the parametrization and the lower bound on promises used, the ergodic distribution has significant probability of both assignment to the group regime and the relative performance regime.

## 3.1 Solutions on the ex-ante utilities map

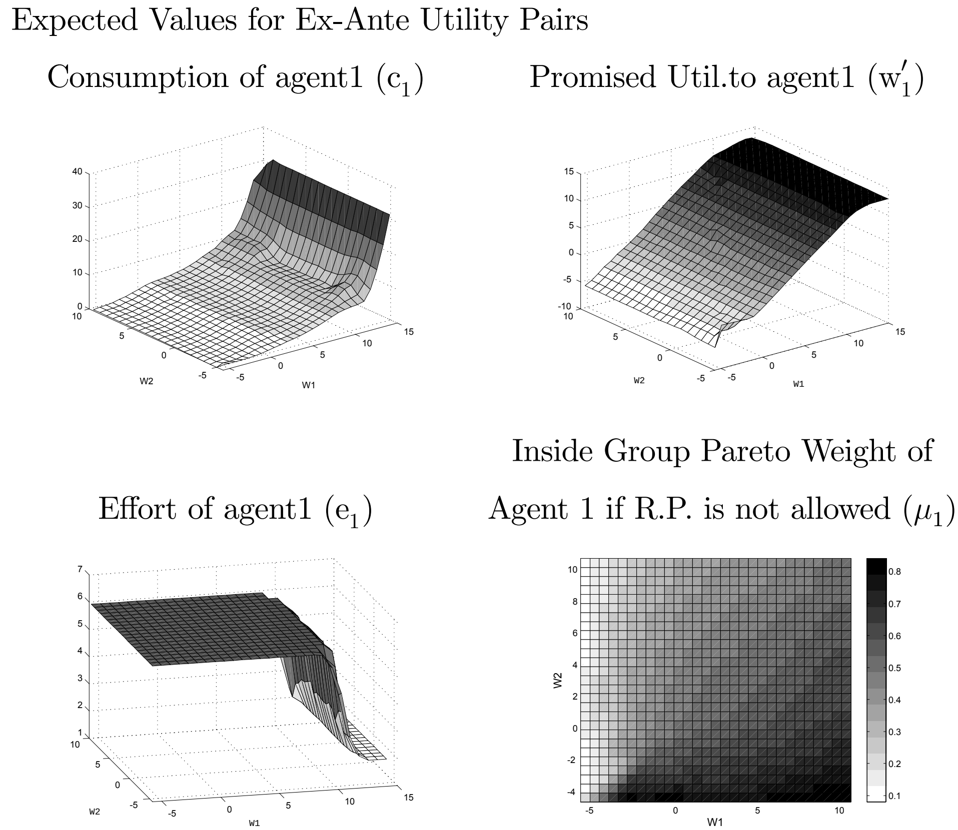


Figure 2

Figure 2 presents the expected values of effort, consumption and promised future utilities for agent 1 at each ex-ante utility pair (in the horizontal axes) for the baseline case with infinite periods. It also shows the expected value of the inside group Pareto weight of agent 1 when the problem is restricted to the group regime. Notice that agent 1 tends to make the high level of effort when his ex-ante utility is low, and the low level of effort when it is high. The expected values of consumption and promised utility of agent 1 tend to increase with his level of ex ante utility $w_1$, except in the region of transition from high effort to low effort. Note also that when the problem is restricted to groups, the Pareto weight of agent 1 tends to be higher at the southeast of the main diagonal, where the utility of agent 1 is high compared with the utility of agent 2.

Figure 3, shows the surplus that can be obtained under the relative performance regime minus the group regime surplus for each utility pair[13].

Surplus R.P. minus Surplus Groups

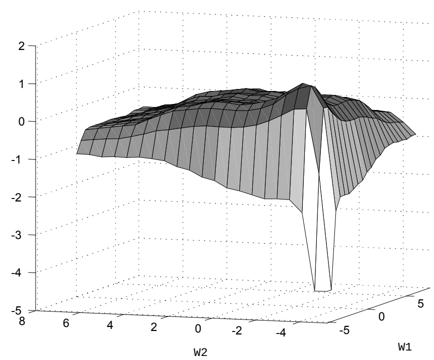


Figure 3

The horizontal axes represent ex-ante expected levels of utilities of the two agents. Notice that for very low utility levels and high inequality with respect to utility, the group regime tends to produce higher surpluses, so the net difference is negative. Note also that the fact that the set of feasible utilities under groups is bigger than the feasible set under relative performance is not the *only* potential advantage of groups: there is an area that is feasible under both regimes but generates higher surplus under the Group Regime. This is true even if the cost to avoid collusion $k$ is equal to zero. When $w_1 = -5.1767$ and $w_2 = 0.2253$, for instance, the difference between the surplus under groups and under relative performance is 1.04, bigger than the fixed cost $k = 0.3$ of the relative performance regime. For intermediate utility levels and low inequality, the relative performance regime produce higher surpluses.

Again, Figure 2 shows that for very high utility levels for agent 1, his optimal effort level is low. A symmetric picture can be observed for agent 2. Thus, in the region where both agents

[13]These graphs are ploted for a *subset* of the promises space with utilities that can be generated by both regimes. This happens for $w_1$ and $w_2$ bigger that -5.23.

have very high utilities (areas of low Surplus by the principal), no agent makes high effort. The solution in this region resembles one without a moral hazard problem, and apart from the cost to block collusion, $k$, the surplus under relative performance virtually equals the surplus under groups. That is, the surplus under groups is higher than the surplus under relative performance by an amount equal to $k$.

For Figures 2 and 3, no outside option for the principal is assumed[14]. In the following results we impose a lower bound for the surplus of the principal, $\underline{S} = -3$[15]. With the limits on surplus and utilities, one can verify that there is always one agent that makes high effort, and incentives for effort are active in every point of the space of promises.

Figure 4 shows, as an example, the maximum surplus in each regime for different levels of ex-ante expected utility for agent 1, $w_1$, when the utility of agent 2 is fixed at 0.2253[16]. Note

[14] Note however that we impose a grid of consumption with a maximum value of 40, so implicitly there is an upper bound on utility and a lower bound on surplus from the fact that consumption can never be grater than 40 in each period. Evidently this is not low enough to rule out low effort.

[15] The value of -3 for the outside option of the principal was chosen so that effort is always the maximal for at least one agent, so promises and/or consumption must be output dependent.

[16] Figure 3 is produced by the difference between the values expressed for a fixed $w_2$ in Figure 4. Notice that the graph in Figure 4 does not express the whole domain where the surplus function is well defined given $w_2$, but only a detail that makes the non convexity of the outer hull evident.

that the outer hull is not concave, so there may be randomization between regimes.

Surplus Under Both Regimes

$w_2$Fixed at 0.2253

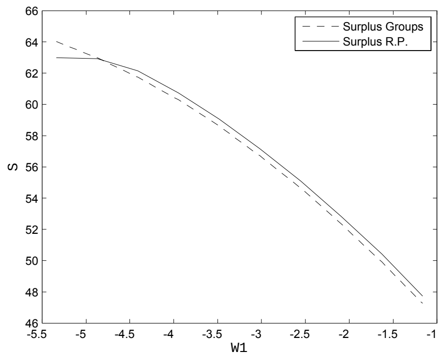


Figure 4

Regime dominance areas - Grey: No solution

Black: Prob. R.P.≥50% - White: Prob. Groups>50%

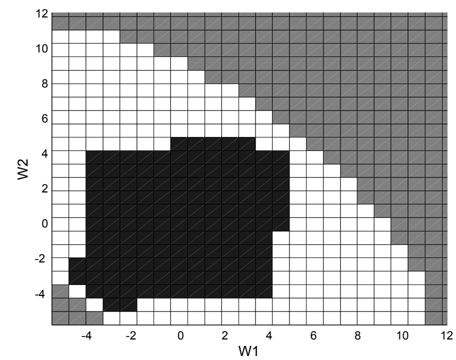


Figure 5

Figure 5 shows the areas, in the map of ex-ante utilities, $w$, where each regime dominates for the baseline case. The black area represents the region where agents are assigned to the relative performance regime with 50% or greater probability. In the white area, there is a probability greater than 50% that they are assigned to the group regime. The grey areas are outside $\widetilde{W}$,where the problem has no solution, either because surpluses are lower than $\underline{S}$ or because the non renegotiation constraints are not valid, as in the extreme southwest of the utility maps[17]. Notice again that the relative performance regime tends to dominate for intermediate levels of utility and low levels of inequality. For most rays from the origin we would see a $U$ shaped frequency of groups.

## 3.2 Transitions between regimes

We have shown that when incentives for effort are needed, there is a positive probability that $w' \neq w$. When this happens, regime change may occur. It may be the case that agents start from an ex-ante utility pair $w$ in the area of the utility map where one regime dominates, but after outputs are realized, they move to an area where the other regime dominates. Figure

[17] In this area, the surplus function would be nondecreasing, which would violate non renegotiation.

6 presents the probability of transition between regimes at each point of the ex-ante utilities map[18]. Notice that these transitions occur near the frontier between the areas in which each regime dominates.

Probabilities of Transition Between Regimes

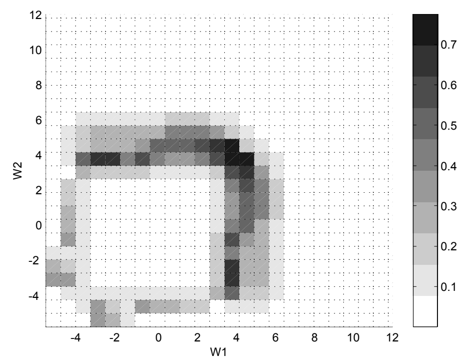


Figure 6

Tables II and III present a detailed description of the solution to the surplus maximization problem for selected points in the utility map. They show the expected values of efforts, consumption and promised future utilities conditional on output. Figures 7 and 8 show each of these points as a white spot in the regime prevalence map. Table II and Figure 7 refer to points where there is 100% of probability of Relative Performance, and Table III and Figure 8 to points where there is 100% of probability of group assignment.

Point one represents the utility pair $(w_1, w_2) = (-3.5068, -3.5068)$. In this case, both agents have 100% of probability of current assignment to the relative performance regime and the high effort level is always recommended to both agents. When their outputs are the same, there is no switching. But when the output of one agent is higher than the other, there is almost 100% probability of switching. In order to give incentives for high effort, the principal punishes agents with low output. If outputs are unequal, the rewards to agents are also unequal. But

[18] A point in the utility map may have initial randomization between regimes. In this case, we calculate the probability of transition by summing the probability of being initially in each regime multiplied by the probability of transition conditional on the initial regime.

part of the reward is achieved with promised utilities, and unequal utilities tend to be produced with a higher surplus under the group regime.

Table II – Expected Values Given Outputs for Selected Points With 100% R.P. Assignment

Point 1: w=(-3.5068,-3.5068)

Relative Performance Contracts-100% of total

| q1 | q2 | p(q1,q2) | E(e1) | E(e2) | E(c1) | E(c2) | E(w1) | E(w2) | P(switch) |
|---|---|---|---|---|---|---|---|---|---|
| 2.0000 | 2.0000 | 0.2979 | 6.0000 | 6.0000 | 1.5548 | 1.5548 | -3.4941 | -3.4941 | 0 |
| 20.0000 | 2.0000 | 0.0021 | 6.0000 | 6.0000 | 2.0000 | 0.4088 | -2.8495 | -5.7445 | 0.9933 |
| 2.0000 | 20.0000 | 0.0021 | 6.0000 | 6.0000 | 0.4088 | 2.0000 | -5.7445 | -2.8495 | 0.9933 |
| 20.0000 | 20.0000 | 0.6979 | 6.0000 | 6.0000 | 1.9318 | 1.9318 | -3.3719 | -3.3719 | 0 |

Point 2: w=(2.6085,2.6085)

Relative Performance Contracts-100% of total

| q1 | q2 | p(q1,q2) | E(e1) | E(e2) | E(c1) | E(c2) | E(w1) | E(w2) | P(switch) |
|---|---|---|---|---|---|---|---|---|---|
| 2.0000 | 2.0000 | 0.2979 | 6.0000 | 6.0000 | 9.0000 | 9.0000 | 2.6950 | 2.6950 | 0 |
| 20.0000 | 2.0000 | 0.0021 | 6.0000 | 6.0000 | 8.7352 | 6.3611 | 3.7489 | 0.3684 | 0.3849 |
| 2.0000 | 20.0000 | 0.0021 | 6.0000 | 6.0000 | 6.3611 | 8.7352 | 0.3684 | 3.7489 | 0.3849 |
| 20.0000 | 20.0000 | 0.6979 | 6.0000 | 6.0000 | 9.0000 | 9.0000 | 3.0521 | 3.0521 | 0.3061 |

Point 3: w=(2.6085,-3.5068)

Relative Performance Contracts-100% of total

| q1 | q2 | p(q1,q2) | E(e1) | E(e2) | E(c1) | E(c2) | E(w1) | E(w2) | P(switch) |
|---|---|---|---|---|---|---|---|---|---|
| 2.0000 | 2.0000 | 0.2979 | 6.0000 | 6.0000 | 9.0000 | 1.5612 | 2.4233 | -3.4867 | 0.0129 |
| 20.0000 | 2.0000 | 0.0021 | 6.0000 | 6.0000 | 9.0000 | 0.3858 | 4.0016 | -5.7959 | 0.9991 |
| 2.0000 | 20.0000 | 0.0021 | 6.0000 | 6.0000 | 6.2634 | 2.6897 | 0.1817 | -3.1137 | 0 |
| 20.0000 | 20.0000 | 0.6979 | 6.0000 | 6.0000 | 9.0000 | 2.0682 | 3.1678 | -3.4525 | 0.2917 |

Selected Starting Points With 100% of Relative Performance Assignment

Point 1 Point 2 Point 3

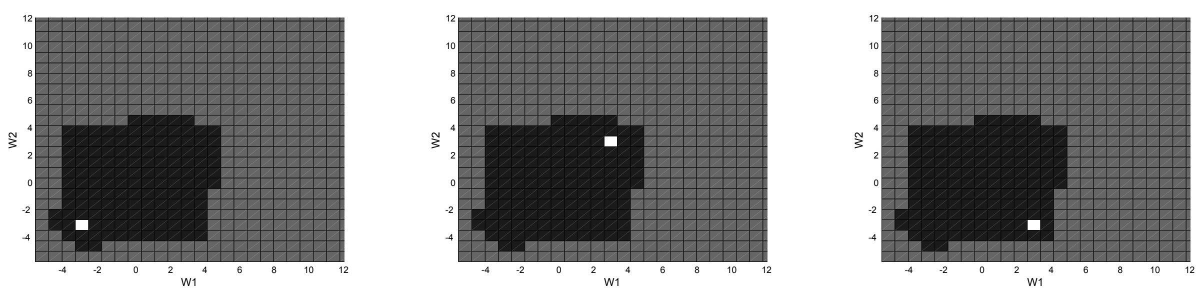


Figure 7

Table III – Expected Values Given Outputs for Selected Points With 100% Groups

Point 4: w=(-1.2136,-5.8)
Group Contracts-100% of total

| q1 | q2 | p(q1,q2) | E(e1) | E(e2) | E(c1) | E(c2) | E(w1) | E(w2) | P(switch) |
|---|---|---|---|---|---|---|---|---|---|
| 2.0000 | 2.0000 | 0.2979 | 6.0000 | 6.0000 | 1.8032 | 0.0821 | -3.1078 | -5.7899 | 0.0088 |
| 20.0000 | 2.0000 | 0.0021 | 6.0000 | 6.0000 | 3.4356 | 0.4824 | -1.1698 | -5.7899 | 0.0055 |
| 2.0000 | 20.0000 | 0.0021 | 6.0000 | 6.0000 | 2.2619 | 0.0821 | -2.5383 | -5.7899 | 0.0086 |
| 20.0000 | 20.0000 | 0.6979 | 6.0000 | 6.0000 | 5.2426 | 0.6666 | -0.2218 | - 5.5827 | 0.0855 |

Point 5: w=(7.1949,-0.44915)
Group Contracts-100% of total

| q1 | q2 | p(q1,q2) | E(e1) | E(e2) | E(c1) | E(c2) | E(w1) | E(w2) | P(switch) |
|---|---|---|---|---|---|---|---|---|---|
| 2.0000 | 2.0000 | 0.2991 | 2.0000 | 6.0000 | 13.6650 | 4.7800 | 6.0908 | -0.7910 | 0.0002 |
| 20.0000 | 2.0000 | 0.0009 | 2.0000 | 6.0000 | 13.6638 | 4.4468 | 6.1489 | -1.1000 | 0.0004 |
| 2.0000 | 20.0000 | 0.4009 | 2.0000 | 6.0000 | 15.7363 | 5.9488 | 7.4344 | -0.2345 | 0.0000 |
| 20.0000 | 20.0000 | 0.2991 | 2.0000 | 6.0000 | 13.6667 | 5.5378 | 6.8919 | -0.4852 | 0.0000 |

Point 6: w=(5.6661,2.6085)
Group Contracts-100% of total

| q1 | q2 | p(q1,q2) | E(e1) | E(e2) | E(c1) | E(c2) | E(w1) | E(w2) | P(switch) |
|---|---|---|---|---|---|---|---|---|---|
| 2.0000 | 2.0000 | 0.2991 | 2.0000 | 6.0000 | 9.4115 | 9.0000 | 4.1077 | 2.4903 | 0.5119 |
| 20.0000 | 2.0000 | 0.0009 | 2.0000 | 6.0000 | 10.6891 | 9.0000 | 3.6841 | 2.4903 | 0.6089 |
| 2.0000 | 20.0000 | 0.4009 | 2.0000 | 6.0000 | 13.6667 | 11.9464 | 5.8057 | 2.7311 | 0 |
| 20.0000 | 20.0000 | 0.2991 | 2.0000 | 6.0000 | 13.6667 | 9.5417 | 5.1694 | 2.7129 | 0 |

Selected Starting Points With 100% of Groups Assignment

Point 4 Point 5 Point 6

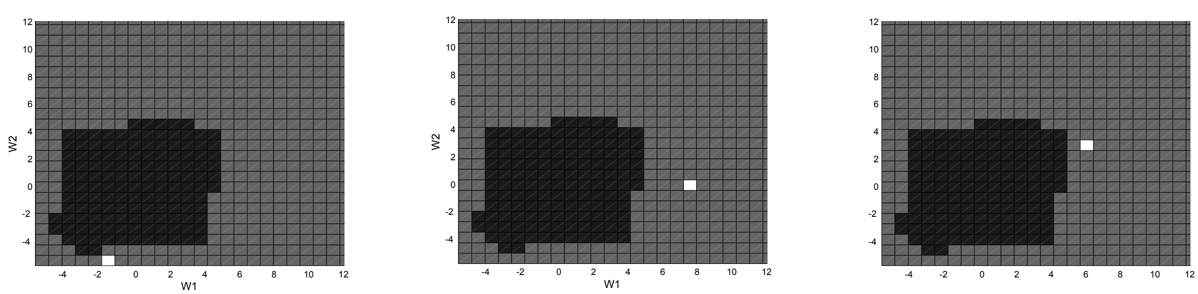


Figure 8

Point 2, utility pair $(w_1, w_2) = (2.6085, 2.6085)$, is another example of 100% of probability of assignment to the relative performance regime. As in the previous example, unequal outcomes produce an exacerbation of inequality and a significant probability of switching (close to 38%). There may also be switching when both individuals are rewarded for high outputs, as they move northeast in the utility map.

Point 3 represents utility pair $(w_1, w_2) = (2.6085, -3.5068)$. Again, there is a 100% probability of assignment to relative performance and agents make maximum effort. But in this case, agent 1 has higher ex-ante utility than agent 2, and a higher output for agent 2 than for agent 1, which makes the utility pair move northwest produces zero probability switching. However, switching happens with almost 100% probability when the output of agent 1 is higher than the output of agent 2. Notice that there is some probability of switching to groups when both outputs are high, and a small but positive probability when both outputs are low.

Point 4, $(w_1, w_2) = (-1.236, -5.8)$, is an example of 100% of probability of assignment to groups. Notice that at this point, both agents make high effort. When both outputs are high, there is about 8.5% probability of switching to relative performance, as the utility pair moves northeast. In points 5 and 6, agent 1 makes low effort and agent 2 makes high effort. Notice that when the output of agent 2 is low, the whole group is punished for a low achievement of an agent who is supposed to do high effort. As the group makes his decision jointly and aggregate rewards are shared, the whole group is punished, not only the agent with low output. Indeed, a low output of agent 2 produces a significant decrease in reward to agent 1 even when his own output is high. In point 6, a punishment to both individuals can move them to an area where relative performance dominates. This produces a significant probability of switching.

Finally, note that Points 5 is an example where the probability of immediate switching is close to zero. This is a common property in the area where groups dominate.

In summary, numerical solutions reveal that transitions from groups to relative performance tend to occur when there is a reward or a punishment to both agents in a group. Transitions from the relative performance to the groups regime may also come when rewards to different outcomes generate an exacerbation of inequality.

## 3.3 Steady State

Steady State Distribution of Promised Utility Pairs

59.47% probability of R.P. assignment

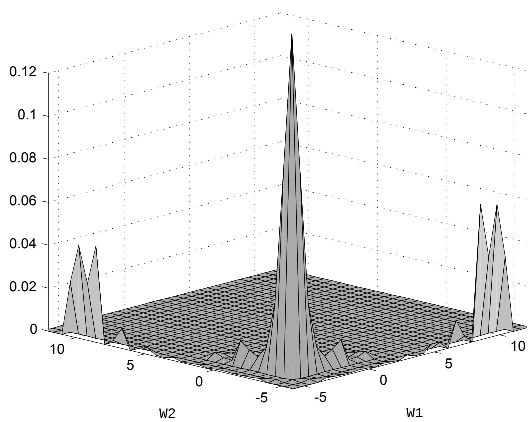


Figure 9

The steady state[19] distribution of utility pairs for the numerical solution to the baseline specification is presented in figure 9[20]. The long run distribution obtained has significant presence of both the relative performance and the group regimes (59.47% probability of R.P. assignment and 40.53% probability of Group assignment). The distribution concentrates in two separate sets: one, in the central area of the picture, where relative performance dominates, and another near the axes where group dominates. The area between these two sets has zero

[19] We verified that the choice of effort is always high for at least one agent.We also verified that the surplus obtained by the numerical procedure with discretization is always higher than the surplus with low effort and constant consumption. This reasures us that the existence of a nondegenerate long run distribution is not the result of imperfections in the numerical method.

[20] The optimal policy for problem 1 generates a transition matrix $M$ such that, when $M$ pre-multiplies a vector with the probability distribution of $w$, the result is the probability distribution of $w'$. The steady state distribution of promises is obtained by starting with some arbitrary distribution of $w$ and successively pre-multiplying it by $M$ until the result converges to some vector, an invariant measure that corresponds to the steady state distribution of promises.

or positive low mass in the steady state[21]. A possible explanation is that instead of promising utilities in this area, it is preferable to randomize between a utility pair in the area where groups dominate and another in the central area where relative performance dominates. As it can be seen in Figure 4, the function $\max\{S_r(w_1, w_2), S_g(w_1, w_2)\}$ (where $S_r(w_1, w_2)$ and $S_g(w_1, w_2)$ are the maximum surplus produced under the group and the relative performance regimes respectively), is not concave (although both $S_r(w_1, w_2)$ and $S_g(w_1, w_2)$ are each concave) so the optimal surplus may be obtained from a randomization of different points in the map.

### 3.4 Regime Volatility in the Utility Map

Figures 10 and 11 plot the probabilities of number of transitions over time[22]. More precisely, they plot the probability that a pair starting at a given point switched regimes zero, one, two, three, four or more than five times after a given amount of periods (in the horizontal axis). We plotted these distributions for 1 to 100 periods. The paths described in Figure 10 all start from the relative performance regime, but there is strong variability in organizational volatility. Point 1 has strong regime persistence, but at points 2 and 3 it is very likely that after a few periods at least one transition happens. Starting from point 2, the volatility stays high even after the first transition. Indeed, the probability of more than 5 transitions increases rapidly. This differs from the dynamics starting from point 3, that has a sharp decrease in the probability

[21] We sometimes found small positive probabilities in this area and it is possible that those result from imperfections in the numerical results. On the other hand, the computed ergodic distribution is the same regardless of the starting points and so these may play a genuine role. We conjectured, but have not proved, that the ergodic distribution with the continuum is unique.

[22] The procedure used to generate these distributions is the following: start with two state variables. A transition matrix (that comes from the solution) determines the joint probability distribution of $w_1'$ and $w_2'$ (future promises) given the joint distribution of $w_1$ and $w_2$. We added another state variable: number of transitions so far. The joint distribution of $w_1$, $w_2$ and the number of transitions until t determines its joint distribution in t+1. For dimensional reasons, only up to 5 transitions are studied, so one of the possible states is 5 transitions or more. Of course, over time, the probability that there is 5 or more transitions converge to 1 as in the steady state there is transit from one point to any other point in the support.

of no transitions in the first periods, but has relative stability after these first periods in the sense that the probabilities of 0, 1, 2, 3 and 4 transitions decrease very slowly. The probability of more than 5 transitions increases only slowly. After the first 10 periods, "survivors" that did not switch in the initial periods tend to have strong persistence in the initial regime, and economies that switch once in the early periods have relatively low volatility after this first transition.

Figure 11 describe paths starting from the groups regime. Once again, there is considerable variability in organizational volatility. In all points selected, there is a sharp decrease in the probability of zero transitions in the initial periods. For point 4, however, after 20 periods there is relatively high regime persistence both for those who switched once and for those that did not switch in the initial periods. Point 5 has considerably higher regime volatility, but after 40 periods, there is still about 10% of probability of no transition, and the "survivors" in the initial regime tend to have considerable persistence after 40 periods. Point 6 shows continuing high volatility over time, and the probability of five or more transitions grows rapidly and is higher than 80% after 50 periods.

## 3.5 Sensitivity to Alternative Parameter Specifications

This section show the response of the solution to alternative specifications of the model. We solve the model for two alternative specifications of $\beta$ and $k$. The results obtained suggest that the general characteristics of the solution are not particular to the baseline parametrization we used.

Figure 12 shows the difference between the surplus under relative performance and groups for 3 values of $\beta$. In all specifications the picture has a similar profile, with relative performance producing higher surpluses for intermediate levels of utility and low levels of inequality. At high $\beta$ one does get the impression that the peak becomes more salient. However, there may be numerical inaccuracies as $\beta$ gets higher, as the computation of the infinite period solution demands more iterations.

Figure 13, varying both $\beta$ and $k$ displays the dominance regions. Again these are qualitatively the same as in the baseline[23]. But note from Figure 13 that even when $k$, the fixed cost of the relative performance, is equal to zero, there are areas where groups dominate and areas where the relative performance regime dominates. But the area of group dominance in the northeast does disappear.

Figure 14 shows the steady state distribution of utility pairs for all these specifications of $\beta$ and $k$. Notice that in all cases presented there is a positive probability of both groups and relative performance. It follows that organization stability results will be similar qualitatively but numerically distinct. The mass on the extremes seem to increase as $\beta$ increases. The steady state probability of relative performance assignments decreases as the cost to prevent collusion $k$ increases (naturally, as $k$ is the fixed cost of relative performance assignment).

Figure 15 shows the probability of regime switching in each region of the utility map. In every specification verified, there is some probability of switching near the frontier between the areas in which regime dominate.

# 4 Conclusion

This paper presents a theory that generates coexistence of different contractual regimes, groups versus relative performance, across local economies, even when the parametric specification is fixed. We also solve the model numerically for several parametric specifications. These numerical solutions generate both cross sectional and dynamic implications for observables. The cross sectional results obtained by Prescott and Townsend [23] are maintained in our formulation: the relative performance regime dominates for intermediate levels of wealth and

[23] Note that the space in which the problem is defined change as we change $\beta$. The minimum value possible for utility when $U(0) = 0$ (as in our specifications) and the agents have no outside option is $V(e_h)/(1-\beta)$, that depends on $\beta$. We imposed an outside option for the agents roughly equal to 70% of this value in all specifications, which results in $\underline{w} = -3.5$ for $\beta = 0.5$ and $\underline{w} = -11.75$ for $\beta = 0.85$. Notice that the maximum level of utility feasible given $\underline{S}$ also depends on $\beta$.

low inequality, the group regime for extremes in wealth and inequality (see Figure 13). However, there may be randomization between regimes in optimal arrangements, so it is possible that local economies with identical histories are assigned to different regimes. The model also generates transitions between regimes. Intermediate levels of inequality are associated with a higher probability of transition, although there are also transitions with low inequality for high and low levels of wealth (see Figure 15). As it is shown in tables II and III, switches from relative performance to groups can be accompanied by big shifts in inequality, while transitions from groups to relative performance are related to changes in average wealth (promised utilities). We compute long run distributions of wealth (utility), inequality and regime for local economies, and both regimes are observed in these steady state distributions (Figure 14). The results show much variation in regime volatility (figures 10 and 11) over the wealth (utility) / inequality map. Different starting points, even in the same regime, produce highly different distributions of number of transitions over time. In some areas there is a high probability of successive transitions over time, while in others there is low probability of transition or one or a few transitions tend to be followed by relative stability.

If we do not observe or infer the initial utility promise, as a starting point, the model still has implications for observables, both the cross sectional distributions and the dynamic paths. As in Karaivanov and Townsend [16], one can estimate the unobserved distribution of utility starting points from the distributions of observables: consumption, output, and so on. But here the regime may also be observable. The distribution of starting points can be parameterized and estimated along with the parameters of technology and preferences, such as correlations in return and risk aversion. The panel data are then determined by the evolution of utility promises over time, something which comes from the model and depends on the parameters to be estimated. The criterion for estimation could be a likelihood function or a mean square error metric. This allows for measurement error and a variety of specifications of what is observable by the econometrician.

Distribution Number of Transitions - Points With 100% Relative Perf. Assignment

Point 1 Point 2 Point 3

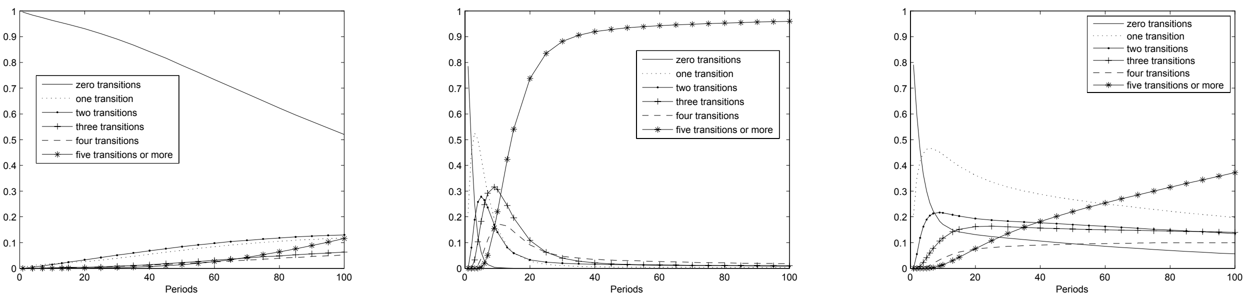


Figure 10

Distribution Number of Transitions - Points With 100% Group Assignment

Point 4 Point 5 Point 6

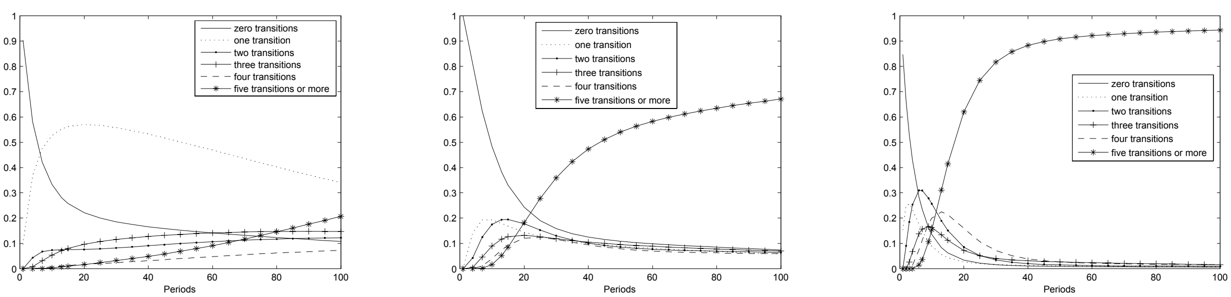


Figure 11

Surplus Under R. P.minus Surplus Under Groups

$\beta = 0.5$ $\beta = 0.7$ $\beta = 0.85$

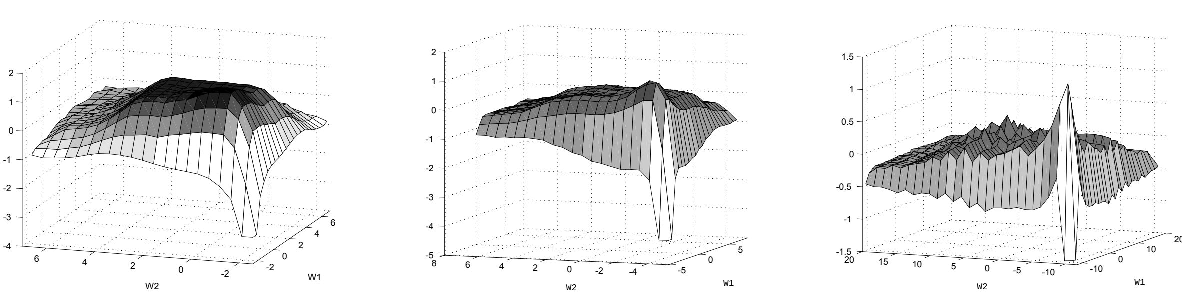


Figure 12

R.P. Dominates(Black) X Groups Dominate(White)X No solution (Grey)

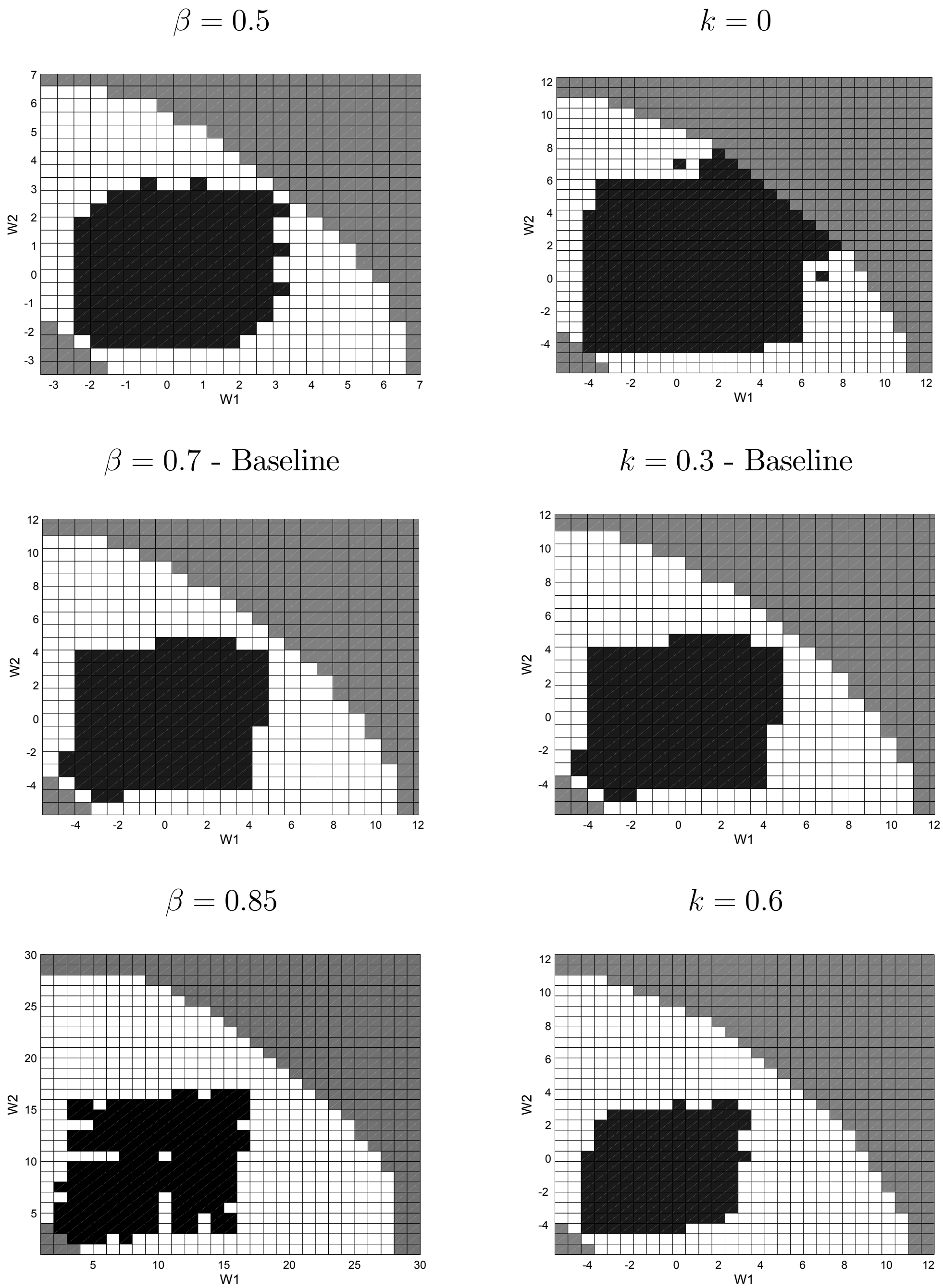


Figure 13

Steady State Distribution of Promised Utility Pairs

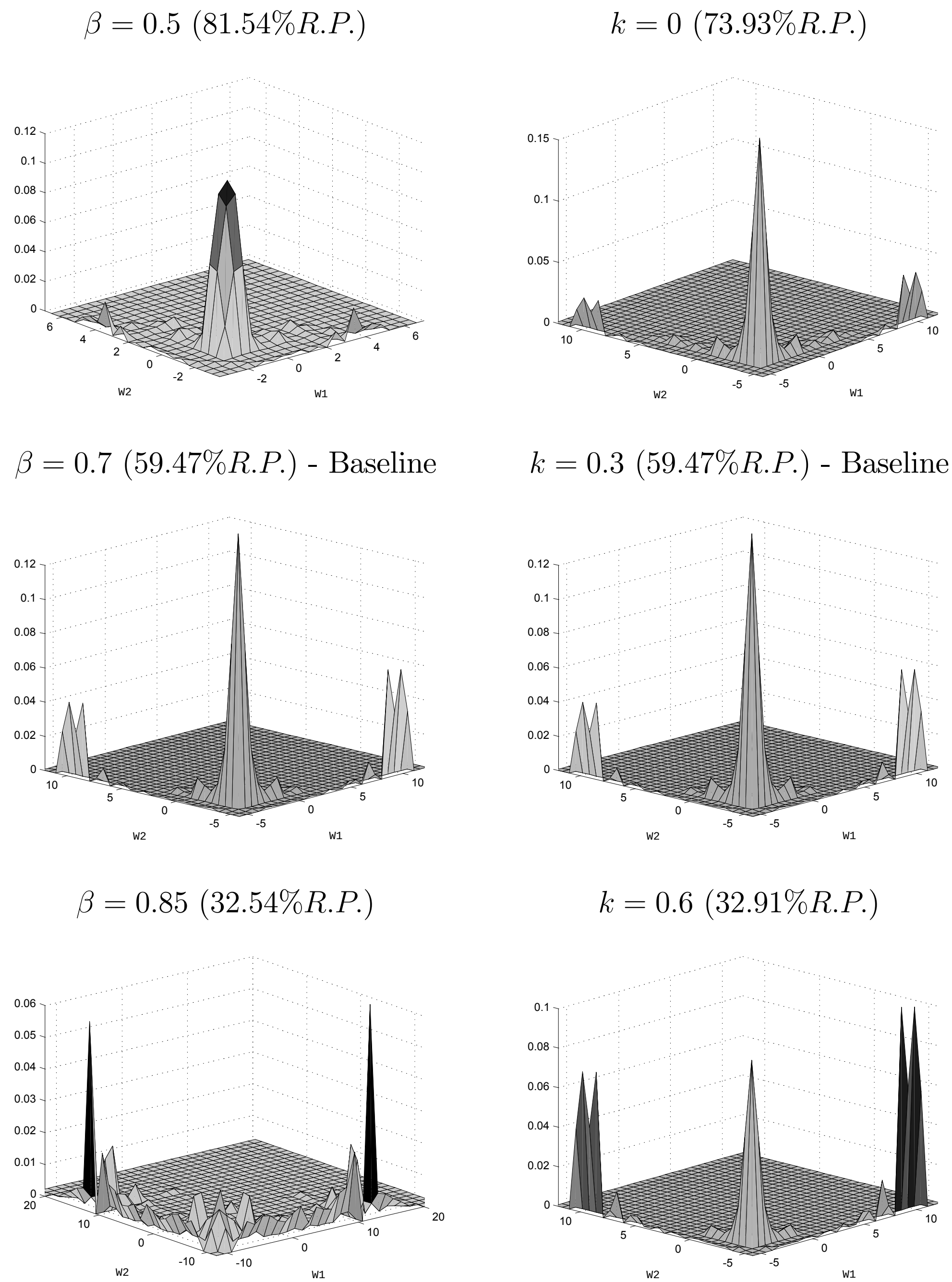


Figure 14

Probabilies of Transition

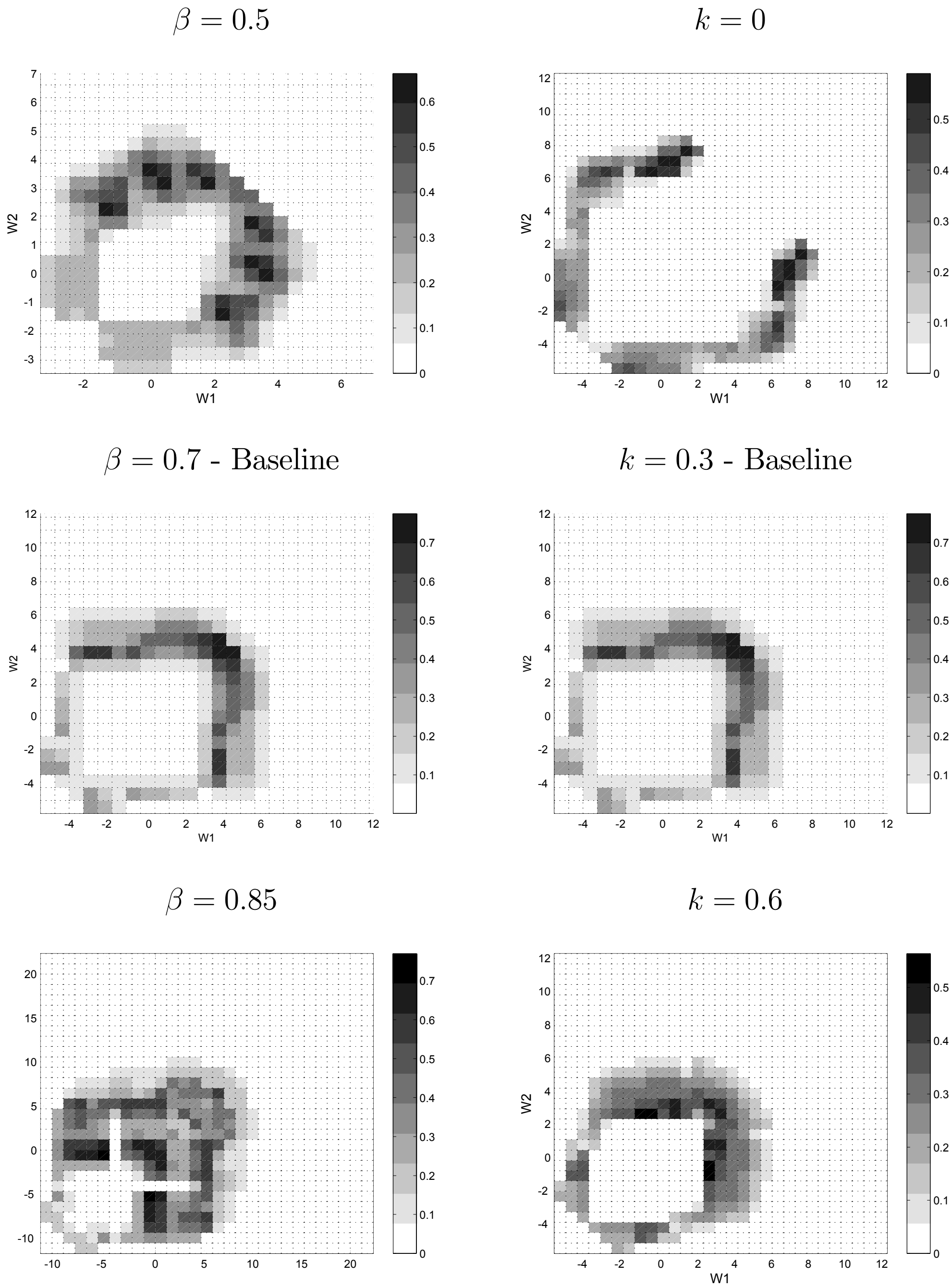


Figure 15

# 5 References


1. C. Ahlin and R. M. Townsend. "Selection into and across Credit Contracts:. Theory and Field Research." Forthcoming, Journal of Econometrics.

2. A.Atkeson and Robert E. Lucas, "On Efficient Distribution with Private Information", Review of Economic Studies, (1992), 59(3), pp. 427-453.

3. Barkey, and van Rossen, (1997), "Networks of Contention: Villages and Regional Structure in the Seventeenth Century Ottoman Empire ", American Journal of Sociology, 102:5.

4. Billingsley, P. *Probability and Measure,* Willey, New York (1986).

5. R. Cavalcanti and N.Wallace, "Inside and Outside Money as Alternative Media of Exchange", Journal of Money, Credit and Banking, 31, no. 3, (1999). 443-457.

6. P.A. Chiappori, K. Samphantharak, S. Schulhofer-Wohl, and R.Townsend "Heterogeneity and Risk Sharing in Thai villages",mimeo, (2006).

7. Dercon, S. and Krishnan, P. (2000), "In Sickness and Health: Risk Sharing within Holseholds in Rural Ethiopia", Journal of Political Economy, 108(4), 688-727.

8. S.Dercon, (2002), "Income Risk, Coping Strategies and Safety Nets", World Institute for Development Economics Research, Discussion Paper No. 2002/22.

9. M. Fafchamps and S. Lund, "Risk-sharing networks in rural Philippines," Journal of Development Economics, vol. 71(2), (2003) , 261-287

10. G.Genicot, and D. Ray, "Endogenous Group Formation in Risk-Sharing. Arrangements", Review of Economic Studies, 70, (2003), 87-11

11. Grimard, F. "Household Consumption Smoothing through Ethnicities: Evidence from Côte d'Ivoire" Journal of Development Economics, 1997. Vol. 53, 391-422.

12. B. Holmstrom and P. Milgron, "Regulating Trade Among Agents ", J. Inst. Theoret Econ. 146 (1990), 85-105.

13. H. Itoh, "Coalitions, Incentives, and Risk Sharing", Journal of Economic Theory 60 (1993), 410-427.

14. M. Jackson and A. Watts,"Social Games: Matching and the Play of Finitely Repeated Games", mimeo,(2005).

15. A. Karaivanov and Townsend, R. M. (2006), "Firm Entry and Finance: Distinguishing Information Regimes", mimeo, University of Chicago.

16. R. Kranton and Y Bramoullé, "Risk Sharing Networks," mimeo, (2005).

17. Leik, R.K. & Chalkey, M.A. (1997). On the stability of network relations under stress. Social Networks, 19 (1), 63-74

18. D. C. Mills, "A model in which outside and inside money are essential ", mimeo, (2000).

19. Mueller, R. A. E., Prescott, E. S. and Sumner, D. A.(2002), "Hired Hooves: Transactions in a South Indian Village Factor Market" . The Australian Journal of Agricultural and Resource Economics, Vol. 46, pp. 233-255.

20. Murgai, R., P. Winters, E. Sadoulet and A. de Janvry "Localized and Incomplete Mutual Insurance," Journal of Development Economics 67, (2002) 245-274.

21. C. P. Phelan and R. M. Townsend, "Computing Multi-Period, information-constrained optima", Rev. Econ. Stud. 58 (1991), 853-881.

22. C. P. Phelan, "On the Long Run Implications of Repeated Moral Hazard" Journal of Economic Theory, Volume 79, Issue 2, (1998), Pages 174-191

23. E. S. Prescott and R.A. Townsend, "Collective Organizations Versus Relative Performance Contracts: Inequality, Risk Sharing and Moral Hazard ", J. Econ Theory 103 (2002) 282-310.

24. R. T. S. Ramakrishnan and A. V. Thankor, "Cooperation Versus Competition in Agency", J. Law. Econ. Organ. 7 (1991), 248-283.

25. Rogerson, W. P., "Repeated Moral Hazard", Econometrica, Vol. 53(1), (1985), pp. 69-96.

26. N. Stokey and Lucas, R. E., *Recursive Methods in Economics,* Harvard University Press, Cambridge, MA, (1996).

27. Thomas, J and T. Worrall (1990), "Income Flutuations and Asymmetric Information: An Example of a Repeated Principal-Agent Problem.", Journal of Economic Theory, 51(2), pp. 367-390.

28. R. Townsend, "Risk and Insurance in Village India", Econometrica, 62(3), (1994), 539-591.

29. R. M. Townsend and R. A. E. Mueller, "Mechanism Design and Village Economies: Credit to Tenancy to Cropping Groups", Rev. Econ. Dynam. (1998), 119-172.

# 6 Appendix A - Reformulated discretized version and Numerical Procedure

In this appendix we present a discretized version of the mechanism design problem, which is important for the numerical methods used. This requires that we utilize a grid for consumption and promised utilities. The choice of consumption and promises are thus given by a lottery on the elements of these grids. Given a curse of dimensionality that may arise from the mere discretization of the problem as described in Program 1, we also break the problem in several stages.

One fact that is useful to reduce the dimension of the problem is that the distributions of consumption values , $c_1$and $c_2$, and promises $w'_1$ and $w'_2$ are relevant in the decisions of agents or groups only through their effects on the composite terms $U(c_1) + \beta w'_1 \equiv v_1$ and $U(c_2) + \beta w'_2 \equiv v_2$. These terms represent the utility that comes from both consumption and future contracts. Throughout the remainder of this appendix, we call $v \equiv (v_1, v_2)$ the interim utility vector. In order to diminish the size of the decision vector, we formulate the moral hazard problem making $v$ a decision variable, not $c$ and $w$ separately.

This strategy requires another maximization stage in any period $t$. For a fixed vector $(v_1, v_2)$, different distributions of $c$ and $w$ imply different expected values for the ex-post (after output is realized) surplus $-c_1 - c_2 + \frac{1}{1+r}\left[S^{t+1}(w'_1, w'_2)\right]$, denoted $\widetilde{S}^t$. In order to determine the cost (or benefit) of offering an utility pair $(v_1, v_2)$, we need to solve the problem of choosing vectors of consumption and promised future utilities that maximize the expected value of $\widetilde{S}^t$ conditional on $(v_1, v_2)$.

Following the formulation described in Proposition 1, we first solve the problem assuming that only the relative performance regime is available, and then solve it using only the group regime. From these programs we construct a grid of surplus maximizing contracts for each regime. The general program in which both regimes are allowed is then solved by randomization of the elements of these grids. This procedure of solving the problem separately for each regime makes computation easier (we solve smaller programs) and allows us to compare the performance of the regimes in each area of the utility pairs map.

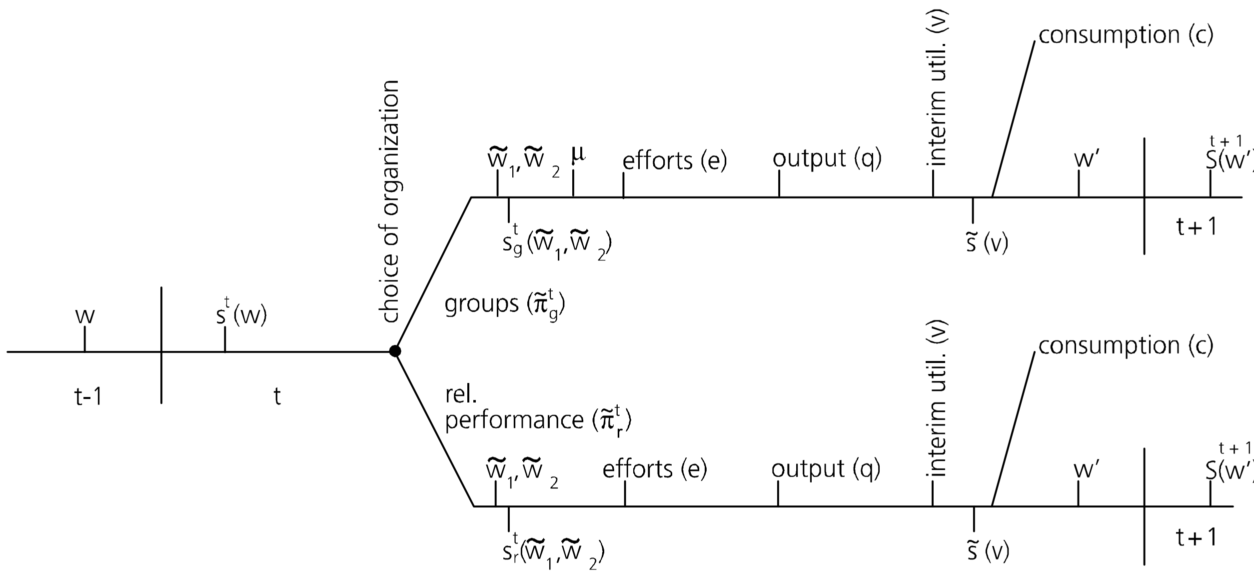


Figure A

The structure of the reformulated problem is illustrated in figure A. At the beginning of any period, a level of promises $w$ is taken as given . In order to deliver these promises, an optimal arrangement defines a joint distribution of the regime assignment and utility pairs specific to the regime $(\widetilde{w}_1, \widetilde{w}_2)$ . These utility pairs are delivered by an assignment of, first internal Pareto weights $\mu$ if the regime is groups, then levels of effort $(e)$, and finally interim utilities$(v)$ conditional on outputs $(q)$. These interim utilities are delivered by current consumption $(c)$ and promises for future periods $(w')$.

The optimal policy by the principal involves 3 maximization stages, that can be solved backward. In a first stage (the last one chronologically), the principal choses the distribution of consumption $c$ and promises $w'$ that produce each interim utility pair with maximal surplus, thus defining $\widetilde{S}^t(v)$. Notice that consumption $c$ and promises $w'$ are defined from a randomization of elements in a grid. In another stage, that depends on the regime adopted, the principal takes the interim surplus function as given, and chooses the joint distribution of internal Pareto weights (if the regime is group), efforts, output and interim promises that maximizes surplus subject to the $(\widetilde{w}_1, \widetilde{w}_2)$ promises, and the incentive and technological constraints. This stage defines regime specific surplus functions for groups $(S_g^t(\widetilde{w}_1, \widetilde{w}_2))$ and relative performance $(S_r(\widetilde{w}_1, \widetilde{w}_2))$. Finally, taking $S_g^t$ and $S_r^t$ as given, the principal chooses a lottery determining the probabilities that the two agents are assigned to the groups regime with expected utilities $(\widetilde{w}_1, \widetilde{w}_2)$, denoted by $\widetilde{\pi}_g(\widetilde{w}_1, \widetilde{w}_2)$, and that they are assigned to the relative performance regime with expected utilities $(\widetilde{w}_1, \widetilde{w}_2)$, denoted by $\widetilde{\pi}_r(\widetilde{w}_1, \widetilde{w}_2)$. This last lottery maximizes surplus conditional on the initial promises. This defines the overall surplus function, $S^t(w)$.

The maximization program that determines the optimal distribution of consumption and promised future utilities that generate interim utilities $(v_1, v_2)$ is:

**Program 2**

$$\widetilde{S}^t(v_1, v_2) \equiv \max_{\widetilde{\pi}} \sum_{w_1', w_2', c_1, c_2} \widetilde{\pi}(w_1', w_2', c_1, c_2)(-c_1 - c_2 + \beta S^{t+1}(w_1', w_2')) \tag{A1}$$

st.

$$\sum_{w_1', w_2', c_1, c_2} \widetilde{\pi}(w_1', w_2', c_1, c_2)(u(c_i) + \beta w_i') = v_i, i = 1, 2, \tag{A2}$$

$$\sum_{w_1', w_2', c_1, c_2} \widetilde{\pi}(w_1', w_2', c_1, c_2) = 1.$$

where $S^{t+1}(w_1, w_2)$ is assumed to be known from before.

The maximal surplus that can be obtained over the group regime, conditional on the utility pair $(\widetilde{w}_1, \widetilde{w}_2)$, is defined by[24]:

**Program 3-g**

$$S_g^t(\widetilde{w}_1, \widetilde{w}_2) \equiv \max_{\pi_g} \sum_{q,e,v,\mu} \pi_g(q, e, v, \mu)[q_1 + q_2 + \widetilde{S}^t(v_1, v_2)] \tag{A3}$$

$$\sum_{q,e,v,\mu} \pi_g(q, e, v, \mu)\mu_i(v_i + V(e_i)) = \widetilde{w}_i, i = 1, 2, \tag{A4}$$

[24] Here we use the notation :

$\pi_g(q, e, v \mid \mu) = \frac{\pi_g(q,e,v,\mu)}{\sum\limits_{q,e,v} \pi_g(q,e,v,\mu)}$.

$$\sum_{q,v}\pi_g(q,e,v\mid\mu)\sum_i\mu_i[v_i+V(e_i)]\geq\sum_{q,v}\pi_g(q,e,v\mid\mu)\frac{p(q\mid\widehat{e})}{p(q\mid e)}\sum_i\mu_i[v_i+V(\widehat{e}_i)],\forall e,\widehat{e},\mu. \quad \text{(A5)}$$

$$\sum_v\pi_g(\widehat{q},\widehat{e},v\mid\mu)=p(\widehat{q}\mid\widehat{e})\sum_{v,q}\pi_g(q,\widehat{e},v\mid\mu),\ \forall\widehat{q},\widehat{e},\mu \quad \text{(A6)}$$

$$\pi_g\geq 0 \text{ and } \sum_{q,e,v}\pi_g^t(q,e,v,\mu)=1. \quad \text{(A7)}$$

$$\begin{aligned}&\sum_{q,e,v}\pi_g(q,e,v\mid\mu)\left[q_1+q_2-\widetilde{S}^t(v_1,v_2)\right] \quad \text{(A8)}\\ \geq\ &\sum_{q,e,v}\widehat{\pi}(q,e,v\mid\mu)\left[q_1+q_2-\widetilde{S}^t(v_1,v_2)\right]\end{aligned}$$

for any $\mu$ and any $\widehat{\pi}$ in the set:

$\widetilde{\Psi}_r(\mu,\pi_g)\equiv\{\widehat{\pi}(q,e,v\mid\mu):\widehat{\pi}$ is positive, sums to 1 and satisfies (A5) and (A6) for any $e$ given $\mu$ and $\sum_{q,e,v}\widehat{\pi}(q,e,v\mid\mu)\sum_i\mu_i\left[v_i+V(e_i)\right]\geq w_\mu(\pi_g)\}$, where $w_\mu(\pi_g)=\sum_{q,v}\pi_g(q,e,v\mid\mu)\sum_i\mu_i\left[v_i+V(e_i)\right]$

The analogous program that generates the maximum surplus under the group regime is:

**program 3-r**

$$S_r^t(\widetilde{w}_1,\widetilde{w}_2)\equiv\max_{\pi_r}\sum_{q,e,v}\pi_r(q,e,v)[q_1+q_2+\widetilde{S}^t(v_1,v_2)] \quad \text{(A9)}$$

st.

$$\sum_{q,e,v}\pi_r(q,e,v)(v_i+V(e_i))=\widetilde{w}_i, i=1,2, \quad \text{(A10)}$$

$$\sum_{q,v}\pi_r(q,e,v)[v_i+V(e_i)]\geq\sum_{q,v}\pi_r(q,e,v)\frac{p(q\mid\widehat{e}_i,e_{-i})}{p(q\mid e)}[v_i+V(\widehat{e}_i)],\ \forall e_i,\widehat{e}_i,i=1,2, \quad \text{(A11)}$$

$$\sum_v\pi_r(\widehat{q},\widehat{e},v)=p(\widehat{q}\mid\widehat{e})\sum_{v,q}\pi_r(q,\widehat{e},v),\ \forall\widehat{q},\widehat{e}, \quad \text{(A12)}$$

$$\sum_{q,e,v} \pi_r(q,e,v) = 1. \tag{A13}$$

$$\begin{aligned} & \sum_{q,e,v} \pi_r(q,e,v) \left[ q_1 + q_2 - \widetilde{S}^t(v_1,v_2) \right] \\ \geq & \sum_{q,e,v} \widehat{\pi}(q,e,v) \left[ q_1 + q_2 - \widetilde{S}^t(v_1,v_2) \right] \end{aligned} \tag{A14}$$

for any $\widehat{\pi}$ in the set:

$\widetilde{\Psi}_r(\pi_r) \equiv \{\widehat{\pi}(q,e,v) : \widehat{\pi}$ is positive, sums to one and satisfies (A11) and (A12) for any $e$ and $i$, and $\sum_{q,e,v} \widehat{\pi}(q,e,v) [v_i + \beta w_i'] \geq w_{i,r}(\pi_r)$ for i=1,2$\}$, where

$$w_{i,r}(\pi_r) = \sum_{q,e_i,v} \pi_r(q,e,v) [v_i + V(e_i)]$$

With the surplus functions in for groups and relative performance, we are finally able to provide an approximation to the surplus function at $t$, $S^t(w_1,w_2)$. The surplus is approximated by:

**Program 4**

$$\begin{aligned} S^t(w_1,w_2) \;\; \simeq \;\; & \max_{\widetilde{\pi}_g,\widetilde{\pi}_r} \sum_{\widetilde{w}_1,\widetilde{w}_2} \widetilde{\pi}_g(\widetilde{w}_1,\widetilde{w}_2) S_g^t(\widetilde{w}_1,\widetilde{w}_2) \\ & + \sum_{\widetilde{w}_1,\widetilde{w}_2} \widetilde{\pi}_r(\widetilde{w}_1,\widetilde{w}_2)(S_r^t(\widetilde{w}_1,\widetilde{w}_2) - k), \end{aligned} \tag{A15}$$

st.

$$w_i = \sum_{\widetilde{w}_1,\widetilde{w}_2} \widetilde{w}_i \left( \widetilde{\pi}_g(\widetilde{w}_1,\widetilde{w}_2) + \widetilde{\pi}_r(\widetilde{w}_1,\widetilde{w}_2) \right), \tag{A16}$$

$i = 1, 2.$

$$\sum_{\widetilde{w}_1,\widetilde{w}_2} \left( \widetilde{\pi}_g(\widetilde{w}_1,\widetilde{w}_2) + \widetilde{\pi}_r(\widetilde{w}_1,\widetilde{w}_2) \right) = 1, \widetilde{\pi}_g, \widetilde{\pi}_r \geq 0, \tag{A17}$$

where $S_g(\widetilde{w}_1,\widetilde{w}_2)$ and $S_r(\widetilde{w}_1,\widetilde{w}_2)$ are defined in problems 3-r and 3-g.

With fine enough grids for $\widetilde{w}_1$, $\widetilde{w}_2$, $v_1$, and $v_2$, the solution of program 4 is a good approximation to the solution of Program 1. The proposed reformulation, with interim utilities, reduce considerably the dimension of the linear programming problems. Suppose again that $Q$, $E$, $C$, $W$ and $M$ have, respectively, 2, 2, 18, 30 and 101 elements, and that we adopt a grid of interim utilities with 26 elements. The decision vector in *Program 2* has 291,600 elements and the problem has 3 constraints, *Program 3-r* has a choice vector with 10,816 elements and has 21 constraints (excluding the non-renegotiation constraint that, as pointed out below, is not directly used in the computation). In contrast, the decision vector of the program for the relative performance regime without interim utilities with the same grids would have 4,665,600 elements and the problem would have the same 21 constraints. This last problem requires considerably higher computational capacity than the subprograms mentioned above.

In order to obtain an approximation of $S_g(\widetilde{w}_1, \widetilde{w}_2)$, we use the fact that , from (A8), any arrangement for a group conditional on any $\mu$, must maximize the sum of $\mu$-weighted utilities conditional on an incentive constraint (A5)(that depend on $\mu$), the technological constraint (A6) and the surplus level obtained under $\mu$. In our numerical procedure for groups, we first take a grid of 101 values of $\mu$ and 52 of values surplus level. Then, for each point in the grid, we use a linear program to maximize, by the choice of a lottery $\pi_{g,\mu}$, the $\mu$-weighted sum of utilities $(\sum_{q,e,v} \pi_{g,\mu}(q,e,v)(v_i + V(e_i)))$ conditional on incentive and technological constraints like (A5) and (A6) *and* the surplus level in the gridpoint. The solution for each point of this grid determines a triple with surplus level, utility for agent 1 and utility of agent 2. We depart from the triples thus generated to compute an approximation of the solution to program 3g. This is obtained from an optimal randomization (that maximizes surplus conditional on utilities) of the triples that are obtained in the grid of Pareto weights and surplus levels.

In the relative performance computations, we initially do not use the non-renegotiation constraint. We initially solve the problem with a weak inequality in the promise keeping constraint (we maximize surplus conditional on the minimal levels for utilities)[25]. We compute the problem with this formulation until the surplus function thus obtained converges. This will

[25] In our numerical experiments, the use of inequality constraints was faster and more stable than the use of equality constraints.

result in points where an increase in utility under relative performance does not imply a lower surplus, and thus improvements from renegotiation are possible. We suppress these areas from the choice space and recompute the problem. If a new area of noincreasing surplus is again observed, we remove it once more. This problem is recomputed until convergence to a constant space of choice is obtained. Similar procedure is used in the upper bound of the utility space. We start without imposing a lower bound on surplus, until the surplus function converges. Then we remove the area with surplus lower than the outside option from the space of promises and recompute the problem. If a new area with surplus lower than the lower bound emerges, we remove it form the space os promises. This process runs until convergence to a constant space of promises is reached.[26]

[26] This procedure produced fast convergence to a constant discrete grid of feasible promises, normally with less than 5 iterations.