

Optimism, Deadline Effect, and Stochastic Deadlines¹

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Abstract

Under a firm deadline, agreement in bargaining is often delayed until the deadline. I propose a rationale for this deadline effect that naturally comes from the parties' optimism about their bargaining power. I then show that the deadline effect disappears if the deadline is stochastic and the offers are made arbitrarily frequently.

Key words: optimism, deadline effect, bargaining

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1 Introduction

When there is a firm deadline, the agreement is often delayed until the very last minute before the deadline. This “deadline effect” is commonly observed in a wide range of laboratory experiments as well as real-life negotiations (See e.g., Roth, Murnighan, and Schoumaker (1988)). For example, the unions and employers reach “eleventh-hour agreements” just before the unions’ deadline to strike. Litigants often reach a “settlement on the courthouse steps.” In these and many other real-life negotiations the parties reportedly hold overly optimistic views about their relative bargaining power (Babcock and Loewenstein (1997)). Such optimism has long been recognized as a major factor in bargaining delays (see, e.g., Hicks (1937)). In this paper, I propose a rationale for the deadline effect that naturally comes from the parties’ optimism about their relative bargaining power. I then show that the deadline effect disappears if the deadline is stochastic and the parties’ relative bargaining power may shift quickly.

The rationale for the deadline effect is as follows. Consider two parties, A and B, negotiating under a firm deadline d^* , which is the last date at which they can strike a deal. After a deadline, they can no longer negotiate, and each party receives his disagreement payoff, which is presumably low. That is, at d^* , the cost of delay is high. Hence, at d^* , a wide range of agreements are possible as outcomes of bargaining. For example, if party A has a strong bargaining position that allows him to set the terms of trade, leaving the other party no other option than that of accepting or rejecting these terms, then they reach an agreement in which A extracts almost all of the large gains from trade and leaves B almost indifferent to disagreement. Similarly,

if B is in such a strong position, then B extracts all the gains from trade and leave A indifferent to disagreement. Depending on the parties' relative bargaining power, they may strike any deal that is in between these two extreme points. Therefore, a party's share in an agreement at d^* is greatly affected by his relative bargaining power. Hence, at any earlier date in the negotiation, each party may have very high or very low expectations about his share in a possible agreement at d^* , depending on his level of optimism about his relative bargaining power at d^* . In case of excessive optimism, these expectations will be so high that, unless waiting until the deadline is too costly, there will be no agreement at the moment that could satisfy both parties' expectations. Therefore, the players wait and reach an eleventh-hour agreement just before the deadline. The cost may be very high; half of the gain from trade may be lost due to the deadline effect.

Now consider an environment in which the deadline is not fixed but rather stochastic. Formally, assume that the deadline is a random variable with a continuous cumulative distribution function. For example, consider a market that does not necessarily close sharply at 5:00pm but may close at any time in between 4:50pm and 5:10pm. For another example, consider a labor contract that does not expire at a fixed date but expires when a certain random event happens, e.g., when the inflation rate exceeds a certain threshold. For yet another example, consider a union that commits to a strike that will be triggered by such a random event, rather than automatically starting at a fixed date. Assume that the factors which determine parties' bargaining power may change quickly, so that whether a party has a strong bargaining position today does not have an impact on the probability of that party hav-

ing a strong position tomorrow. Assume also that the parties have frequent opportunities to strike a deal. In such an environment,¹ I show that the deadline effect disappears, and the parties reach an agreement immediately.

An intuition for this result is as follows. Consider a date t^* at which the parties find it very likely that there will be no more negotiations. As explained above, the perceived cost of delay is high at t^* , and the parties may be highly optimistic about their shares in an agreement at t^* . Nevertheless, when the deadline is stochastic, the ex-ante probability that the parties will face the deadline before such a date t^* must be very high.² Hence, at a sufficiently earlier date, the parties are not willing to wait until t^* to realize their expectations because that incurs a very high risk of receiving the low payoff of disagreement. On the other hand, if the parties expect to have many opportunities to strike a deal in the future and parties are not pessimistic about their bargaining power in the future, then their expectations about the future are high. Hence the perceived cost of delay is relatively small. That limits the scope of the individually rational trade. Then, the parties' payoffs in a possible agreement are not affected much by their bargaining power. Therefore, the parties' optimism about their bargaining power induces only low levels of optimism about their shares.³

¹In the formal model, I also make an assumption to rule out any anticipated deadline that is implicit in players' beliefs.

²This is because the ex-ante probability that the deadline arrives before the date $t^* + 1$ is at least as high as the conditional probability of that event at t^* , which is assumed to be very high. But the former probability is approximately the ex-ante probability that the the deadline arrives before the date t^* , for the dates t^* and $t^* + 1$ are assumed to be very close in real-time and the cumulative distribution function is continuous.

³Although this intuition is valid also for multilateral bargaining, it may not prevent

There are arbitrarily close stochastic deadlines to any deterministic deadline. The above results describe quite different equilibrium behavior under such two similar environments. While a deterministic deadline and optimism naturally lead to delaying of agreement until the deadline, under any nearby stochastic deadline, there will be immediate agreement whenever the environment is sufficiently variable.

From a theoretical point of view, the above two results extend the critique in Yildiz (2003) of the literature that explains the bargaining delays by optimism, using informal arguments that fit to a two period model, when excessive optimism leads to a delay. In Yildiz (2003), I showed that if optimism is sufficiently persistent, then there will be an immediate agreement. The first result in this paper shows that optimism naturally leads to the deadline effect under a deterministic deadline—just as it leads to a delay in a two period model. The second result shows that, nevertheless, if there is slight uncertainty about the deadline so that it is stochastic, then there will be an immediate agreement in the continuous-time limit, the limit case that attracted most of the attention in bargaining literature.

In the continuous-time limit, the players' relative bargaining power is allowed to shift back and forth instantaneously. In practice, however, the factors that determine players' relative bargaining power do not change that suddenly and that often. Hence, optimism may lead to the deadline effect even under a stochastic deadline. Therefore, from a practical point of view, one must interpret the last result as follows: the deadline effect disappears when there is sufficient uncertainty about the deadline relative to how quickly some delay when there are many parties (Ali, 2002).

the players get a chance to strike a deal and how quickly their relative bargaining power changes.

A firm deadline is often imposed upon negotiators in order to prevent them from dragging out the negotiations indefinitely. Ironically, such deadlines themselves sometimes entice parties to delay the agreement. If this happens because of the parties' optimism, then one may be able to avoid such a deadline effect by imposing a deadline that is triggered by an event that will happen at a random time and is beyond the parties' control. The uncertainty about the timing of the deadline must be high enough so that, during the period at which such an event happens, the parties will have many opportunities to strike a deal and their relative bargaining power will potentially change many times.

Spier (1992) shows that, in a pre-trial negotiation with incomplete information, the settlement probability will be a U-shaped function of time, consistent with the deadline effect. Ma and Manove (1993) also develop a model that explains the deadline effect. In that model, the delay is costless and a party can wait as much as he wants before making an offer, and his opponent has to wait for his offer. Then, the party who is to make an offer waits until the deadline and makes a last minute take-it-or-leave-it offer. In their model, there is also a stochastic delay between the times an offer is made and it is accepted. Hence, a late offer may not go through. This suggests that the deadline effect in that model may not disappear when the deadline becomes stochastic. Indeed, Roth, Murnighan, and Schoumaker (1988) informally discuss a possible explanation based on the idea that there is no cost of delay except for a cost at the end due to a slight uncertainty

about the deadline.⁴ When the deadline becomes stochastic, the deadline effect disappears in some models while remains intact in some other. Hence one can use stochastic deadlines to test these models.

2 Model

There are two risk averse agents, $i, j \in N = \{1, 2\}$, who want to divide a dollar among themselves before a deadline; the set of all feasible expected utility pairs is $U = \{(u^1, u^2) \in [0, 1]^2 | u^1 + u^2 \leq 1\}$. The negotiation takes place on a grid $T = \{0, 1, 2, \dots\}$ of index-times t that approximates the continuum $[0, \infty)$ of real times τ . Each index t corresponds to a real time $\tau(t, k) = t/k$, where k is a large integer that measures the fineness of the grid. The players' time preferences and the deadline are given by the real time and do not depend on the grid. Each player's utility from getting x at τ is $e^{-r\tau}x$ where $r > 0$ measures the real-time impatience. The index-time discount rate is denoted by $\delta = e^{-r/k}$.

I model the deadline as a positive random variable d with cumulative distribution function F . At $\tau = d$, the negotiation automatically ends and each agent gets payoff of 0. The deadline is said to be *deterministic* if and

⁴Roth and Ockenfels (2002) consider a similar model in which the delay is motivated by the possibility that the last offer may not go through. They use this model to explain why the deadline effect is observed in e-Bay auctions but not in Amazon auctions. In e-Bay, the deadline is firm; in Amazon, auction may not end if there was a bid in the last 10 minutes. Notice that, if there are uncertainty about the entry, then the deadline in Amazon can be considered stochastic. Incidentally, the results here can also be extended to auction environments.

only if $d = d^*$ with probability 1 for some $d^* \in [0, \infty)$. The deadline is said to be *stochastic* if and only if F is continuous.

I will analyze the following perfect-information game. At each t with $\tau(t, k) < d$, Nature recognizes a player $i \in N$; i offers an alternative $u = (u^1, u^2) \in U$; if the other player accepts the offer, then the game ends yielding a payoff vector $\delta^t u = (\delta^t u^1, \delta^t u^2)$; otherwise, the game proceeds to date $t + 1$, unless $\tau(t + 1, k) \geq d$, in which case the game ends yielding payoff vector $(0, 0)$. Given any t and any history with real time τ' before the recognition at t , each player i assigns probability $(1 - F(\tau(t, k) | d > \tau')) p_t^i$ to the event that i will be recognized at date t , where $p_t^i \in [0, 1]$ is his probability assessment about this event conditional on $\tau(t, k) < d$. Everything above is common knowledge.

This model is similar to that of Yildiz (2003). Its only difference is that it allows a stochastic deadline. When the discounting is due to the random bargaining breakdowns, the Rubinstein-Stahl framework (e.g., Binmore (1987), Merlo and Wilson (1995)) also allows stochastic deadlines. In that framework, the discount rate is usually taken as constant over time, and it is thereby assumed that the deadline has an exponential distribution. This paper does not impose any such restriction on deadlines because that would rule out the critical class of stochastic deadlines that approximate deterministic deadlines. Its main departure from the Rubinstein-Stahl framework, however, is that it allows the players to hold subjective beliefs about which player will be recognized and when, and these beliefs possibly differ from each other.

A player's bargaining power is determined by the recognition process; a

player's equilibrium payoff is the present value of the temporal monopoly rents he expects to extract when he is recognized (Yildiz, 2003, 2004). The differences in players' beliefs about the recognition process reflect their optimism or pessimism about their bargaining power. *The level of optimism for date t* is measured by

$$y_t = p_t^1 + p_t^2 - 1.$$

The players' probability assessments for date t agree when $y_t = 0$. The players are weakly optimistic (resp., pessimistic) for t when $y_t \geq 0$ (resp., $y_t \leq 0$). The main focus in this paper will be on the case that the players remain (weakly) optimistic throughout the game, although it is not assumed.

Notice also that p_t^i is a constant. This reflects the assumption that players do not update their beliefs about the future events as they observe which player is recognized at a given date. This independence assumption is made in order to rule out the delays that are due to learning (Yildiz (2004)), unrelated to the deadline effect.

3 Equilibrium

One can iteratively eliminate all conditionally dominated strategies to reach an essentially unique subgame-perfect equilibrium. The equilibrium is unique in the sense that the continuation value of any player at the beginning of any date is unique for all subgame-perfect equilibria; there may be a trivial multiplicity of equilibria in knife-edge cases. (The continuation values are computed as the present value of continuation payoffs at t , using the player's own expectations.) In this section, I will derive a difference equation that

determines these continuation values and describe the equilibrium behavior and the conditions that determines whether players reach an agreement at any given date.

Let V_t^i be the continuation value of a player i at the beginning of a date t conditional on that $\tau(t, k) < d$; the continuation values are identically zero when $\tau(t, k) \geq d$. Conditional on $\tau(t, k) < d$, the surplus or the perceived size of the pie at any date t is

$$S_t = V_t^1 + V_t^2.$$

If the players share a common set of beliefs, S_t is identically 1. Since the players' subjective beliefs are allowed to differ in this paper, S_t may be higher or lower than 1. For each t , write also

$$q_t = 1 - F(\tau(t+1, k) | d > \tau(t, k))$$

for the conditional probability that the deadline is not reached at index time $t+1$ given that the deadline is not reached at t . At date t players discount their payoffs at $t+1$ by the effective discount factor of δq_t . The discounted value $\delta q_t S_{t+1}$ of the next period surplus determines whether players reach an agreement at t in equilibrium if they have not yet agreed.

Consider the case that $\delta q_t S_{t+1} > 1$. Since each player i expects a discounted payoff of $\delta q_t V_{t+1}^i$ from delaying agreement beyond t , an agreement at t must give at least $\delta q_t V_{t+1}^i$ to each player i , requiring a sum of $\delta q_t V_{t+1}^1 + \delta q_t V_{t+1}^2 = \delta q_t S_{t+1} > 1$. Hence the players cannot agree at t in equilibrium. This case is called the *disagreement regime*. The continuation value of player i at the beginning of t is

$$V_t^i = \delta q_t V_{t+1}^i,$$

and the surplus is

$$S_t = \delta q_t S_{t+1}. \quad (1)$$

Now consider the case $\delta q_t S_{t+1} \leq 1$. Now an agreement at t is not only possible but also necessary for equilibrium whenever the inequality is strict. This case is called the *agreement regime*. When a player i is recognized, the other player j is willing to agree to any division that gives j at least $\delta q_t V_{t+1}^j$; i offers j this amount and keeps the rest, $1 - \delta q_t V_{t+1}^j$, for himself, an amount that is more than $\delta q_t V_{t+1}^i$, his continuation value from delay. (The latter amount would be his share if he were not recognized.) The continuation value of player i at the beginning of t is now

$$V_t^i = p_t^i (1 - \delta q_t V_{t+1}^j) + (1 - p_t^i) \delta q_t V_{t+1}^i = p_t^i (1 - \delta q_t S_{t+1}) + \delta q_t V_{t+1}^i.$$

In that case, the surplus at the beginning of t is

$$S_t = 1 + y_t (1 - \delta q_t S_{t+1}). \quad (2)$$

4 Deadline Effect

In this section, I will show that optimism and a deterministic deadline in the near future lead players to wait until the last date to settle, but this deadline effect disappears and there is immediate agreement whenever the deadline is stochastic and the offers are made frequently (i.e., k is large).

Proposition 1 *Let there be a deterministic deadline at some $d^* > 0$, and let $t^* = \max \{t | \tau(t, k) < d^*\}$ be the last date before the deadline. Assume that $e^{-rd^*} (1 + y_{t^*}) > 1$. Then, in equilibrium, the players disagree at each $t < t^*$ and reach an agreement at t^* .*

Proof. Firstly, since $q_{t^*} = 0$, $\delta q_{t^*} S_{t^*+1} = 0$. Hence, there is an agreement regime at t^* , and $S_{t^*} = 1 + y_{t^*} (1 - \delta q_{t^*} S_{t^*+1}) = 1 + y_{t^*}$. Moreover, for each $t < t^*$, since $q_t = 1$, the equality $S_t = \delta S_{t+1} = \delta^{t^*-t} S_{t^*}$ will hold, and there will be a disagreement regime at t , so long as $\delta^{t^*-t} S_{t^*} > 1$. But $\delta^{t^*-t} S_{t^*} \geq \delta^{t^*} S_{t^*} \geq e^{-rd^*} (1 + y_{t^*}) > 1$ for each $t < t^*$ by the hypothesis. ■

Proposition 1 is a simple generalization of Example 0 of Yildiz (2003). It is based on the basic observation that the recognized player just before the deadline has a great bargaining power, as he makes a take-it-or-leave-it offer. If the deadline is not too far away and the players are sufficiently optimistic about being recognized and dictating a very favorable settlement at t^* —as it is hypothesized in Proposition 1, then there will be no feasible agreement before t^* that satisfies both players' (highly optimistic) expectations. Under this hypothesis, Proposition 1 then predicts that the players will delay the agreement until the last date before the deadline and settle at the last date. This behavior is called the deadline effect and is commonly observed in real-world negotiations and in many experiments.

The next result provides conditions under which the deadline effect disappears when the deadline becomes stochastic.

Proposition 2 *Let the deadline be stochastic (i.e., let F be continuous). Then, given any $\epsilon > 0$, there exists \bar{k} such that the players reach an agreement in equilibrium before real-time ϵ for each $k > \bar{k}$ whenever either of the following two conditions are satisfied: (i) $y_t \geq 0$ for each t , or (ii) $y_t = Y(\tau(t, k))$ for all t, k for some continuous function Y .*

That is, if the deadline is stochastic, then there will be an immediate agreement in continuous-time limit, so long as either one of the two condi-

tions above is satisfied. Such conditions are not superfluous because the players' beliefs (particularly their pessimism) may reflect an anticipated deadline, and thus considerations about a deterministic deadline may be disguised as pessimistic beliefs. To see this, note that when $y_t < 0$ (i.e., when players are pessimistic about t), the analysis remains unchanged if we instead assumed that the players commonly believed that, at t , each player i is recognized with probability p_t^i and no player is recognized with probability $-y_t$. Hence, a deterministic deadline at a date d^* can be implicitly incorporated in players' beliefs by assuming that $y_t = -1$ for each t with $\tau(t, k) > d^*$. Condition (ii) rules out such possible incorporation of deterministic deadlines in beliefs by ruling out sudden jumps in level of optimism. Consistent with this intuition, one only needs to rule out sudden drops in the level of optimism for the proof below. Condition (i), which states that the players are optimistic throughout, makes sure that all considerations about deadlines are captured by the random variable d . This condition is consistent with empirical evidence for players' persistent optimism (Babcock and Loewenstein (1997)).

Proof of Proposition 2. For each k , let t_k be the first date with an agreement regime. I will show that $\lim_{k \rightarrow \infty} \tau(t_k, k) = 0$. Since S_{t_k} is bounded by 2, $\tau(t_k, k) \in [0, \log(2)/r]$ for each k , and hence it suffices to show that 0 is the only limit point of the sequence $\tau(t_k, k)$. Let τ^* be any limit point of this sequence, and consider a subsequence that converges to τ^* . Consider any k with $t_k > 0$. By (1),

$$S_0 = \delta^{t_k} \left(\prod_{t=0}^{t_k-1} q_t \right) S_{t_k} = \delta^{t_k} (1 - F(\tau(t_k, k))) S_{t_k} \geq 1; \quad (3)$$

the inequality is due to the fact that there is a disagreement regime at 0.

Hence,

$$\frac{1}{A_k} \equiv \frac{1}{(1 - F(\tau(t_k, k))) S_{t_k}} \leq \delta^{t_k} < 1. \quad (4)$$

I will show that $1/A_k$ converges to 1, proving that

$$e^{-r\tau^*} = \lim_{k \rightarrow \infty} \delta^{t_k} = 1,$$

and therefore, $\tau^* = 0$. Towards this goal, note first that

$$S_{t_k} = 1 + y_{t_k} (1 - \delta q_{t_k} S_{t_k+1}) \quad (5)$$

as there is an agreement regime at t_k , which also implies that $\delta q_{t_k} S_{t_k+1} \leq 1$.

I consider the two conditions separately. First assume (i). Then, $S_{t_k+1} \geq 1$. [If $\delta q_{t_k+1} S_{t_k+2} > 1$, then $S_{t_k+1} = \delta q_{t_k+1} S_{t_k+2} > 1$. If $\delta q_{t_k+1} S_{t_k+2} \leq 1$, then $S_{t_k+1} = 1 + y_{t_k+1} (1 - \delta q_{t_k+1} S_{t_k+2}) \geq 1$, as $y_{t_k+1} \geq 0$.] Hence, by (5),

$$S_{t_k} \leq 2 - \delta q_{t_k}. \quad (6)$$

Thus,

$$\begin{aligned} A_k &\leq (1 - F(\tau(t_k, k))) (2 - \delta q_{t_k}) \\ &= 2(1 - F(\tau(t_k, k))) - \delta(1 - F(\tau(t_k + 1, k))). \end{aligned}$$

Since F is continuous and $\lim_{k \rightarrow \infty} \tau(t_k, k) = \lim_{k \rightarrow \infty} \tau(t_k + 1, k) = \tau^*$, the last expression converges to $1 - F(\tau^*) \leq 1$. (Recall that $\lim_{k \rightarrow \infty} \delta = 1$).

Since $A_k > 1$ by (4), this implies that $A_k \rightarrow 1$, and thus $1/A_k \rightarrow 1$.

Now assume (ii) without assuming (i). It is now clear that it suffices to find an upper bound for A_k that is not greater than 1 in the limit. If $y_{t_k} \leq 0$, by (5), $S_{t_k} \leq 1$ yielding $A_k \leq 1$. Assume $y_{t_k} > 0$. If we also have

$y_{t_k+1} \geq 0$, then $S_{t_k+1} \geq 1$, implying the bound in (6), which has just proven to be sufficient. Hence, assume $y_{t_k+1} < 0$. But, if this case occurs infinitely often, then $y_{t_k} = Y(\tau(t_k, k)) > 0$ implies that $Y(\tau^*) \geq 0$ by continuity of Y , and similarly $y_{t_k+1} = Y(\tau(t_k + 1, k)) < 0$ implies that $Y(\tau^*) \leq 0$, showing that $Y(\tau^*) = 0$. By (5), this yields $A_k \leq S_{t_k} \leq 1 + y_{t_k} \rightarrow 1$. All the cases are covered, and the proof is complete. ■

5 Conclusion

Parties often negotiate under a deadline. These deadlines often entice the parties to wait until the deadline to make a last minute agreement. I show that such a deadline effect may naturally arise from the parties' optimism about their bargaining power. More interestingly, I show that one may avoid the deadline effect, by making the deadline stochastic, i.e., by making the deadline contingent upon an event that will happen at a random date. In order this to succeed there must be sufficient uncertainty about the deadline so that parties have many opportunities to strike a deal and their relative bargaining power has a potential to change during the period in which they may face the deadline.

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