Repeated Moral-Hazard with Unmonitored Wealth: A Recursive First-Order Approach^{*}

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Abstract

We present a recursive first-order approach that may be useful in studying repeated principal-agent relationships where the agent can trade intertemporally behind the principal's back. The method has the virtue of being analytical simple and computationally tractable. Relative to Spear and Srivastava's (1987) method, the recursive structure here is obtained by adding an additional state variable: the agent's marginal utility. The agent's intertemporal Euler condition is then added as a constraint in the principal's cost minimization problem.

1 Introduction

This paper presents a recursive first-order approach that may be useful for studying repeated principal-agent relationships where the agent can trade intertemporally behind the principal's back. The method has the virtue of being analytical simple and computationally tractable. This is important because conventional formulations of such problems quickly become unmanageable.

The study of these problems has remained elusive because, relative to the case where the agent cannot save nor borrow, the hidden-action space for the agent is greater and deviations are in a sense more permanent. This

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requires imposing a great number of incentive-constraints to rule out all deviations from the suggested action. Due to the large number of constraints the problem becomes very intractable.

The method proposed in this paper relies on using the agent's first-order conditions to characterize the agent's optimal decisions – the incentive compatibility constraints. The use of first-order conditions allows the problem to be posed in a recursive manner, simplifying it analysis and computation significantly.

As is well known, imposing first-order conditions may not be sufficient to ensure incentive compatibility. Thus, this method may not solve the true agency problem of interest [Mirrlees (1999), Rogerson (1985)]. Fortunately, because the generated allocation is recursive in the appropriate state variables, it is, in principal, straightforward to check whether the solution satisfies incentive compatibility.

Moreover, even if the first-order approach is invalid for solving the optimal incentive-constrained allocation it may still be useful as an upper bound on the Pareto frontier. Because the first-order constraints employed are weaker than the true incentive constraints, the utility frontier between the principal and the agent derived with this method provides an upper bound of the actual incentive-compatible utility frontier. Of course, it is hoped that in many interesting cases the upper bound actually coincides with the frontier and this indeed appears to be the case for many cases of interest that the author has implemented.

The main idea of this paper can be understood by relating the method to that of Spear and Srivastava's (1987). Their work has proven pivotal for repeated principal-agent problems where agents cannot trade intertemporally. They showed how to summarize history dependence in a single, intuitive variable: the remaining lifetime-utility promised to the agent: w. The cost to the principal of delivering w, C(w), can then be found by iterating on a Bellman operator.

Relative to this framework we add an additional state variable: the marginal utility of the agent: λ . When the agent can save this variable must be added to keep track of the agent's intertemporal Euler condition. The cost to the principal as a function of λ and w, $C(\lambda, w)$, can then be found by iterating on a related Bellman operator.

We apply our method to the optimal design of unemployment insurance in a separate paper. Using Spear and Srivastava's (1987) recursive approach the existing literature has focused on the case where agents cannot save nor borrow, or equivalently where the agent's wealth is observable to the principal, so that unemployment transfers are essentially equal to consumption. With this assumption, interesting results have been obtained. In particular, Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997) show that unemployment payments fall with the duration of unemployment. Little is known about the more realistic case where agents can save (and perhaps borrow) and the government cannot monitor their savings (or borrowings).

1.1 Related Literature

Doepke and Townsend (2000) examine a similar private-information problem where the agent has a hidden storage technology and provide alternative methods to those presented here. Using lotteries over discrete grids they first consider the large number of incentive-compatibility constraints required for all combinations of deviations. Even for small grids the numbers become completely unmanageable with this straightforward approach. They then show how to significantly reduce the number of constraints by re-writing the problem in various ingenious ways. Although their methods reduce the number of constraints to manageable levels it remains quite large. Lotteries are thus an essential part of the method since linear programs can handle relatively large number of constraints.

In contrast, the method proposed in this paper uses the agent's first-order conditions to reduce the number of incentive-constraints required to a bare minimum. As a consequence, lotteries are not necessarily part of the solution method. However, given that the first-order approach used here cannot always be justified, both methods should clearly be seen as complements.

Chiappori, Macho, Rey and Salanié (1994) review several issues related to the repeated moral hazard problems with agent access to credit. Fudenberg, Holmstrom and Milgrom (1990) examine a repeated agency problem where the agent can save and borrow. They restrict the analysis to the case where the agent and principal have the same interest rate, equal to the reciprocal of the agent's subjective discount factor. Their emphasis is not on characterizing the contract, rather they are concerned with the renegotiation-proofness of the contract: whether the contract is ex-post efficient. They show that in general strong conditions are required for this to be the case, although they provide an interesting example that does.

The method used in this paper is related to the work of Kydland and Prescott (1980), Chang (1998) and Phelan and Stachetti (1999). They study

government tax policies in competitive economies. Kydland and Prescott (1980) study optimal dynamic tax policy under full commitment. They were the first to show that the introduction of the marginal utility as a state variable can be used to obtain recursive formulations of this problem. Chang (1998) and Phelan and Stachetti (1999) apply this idea to the study of all time-consistent linear tax policies in the absence of commitment using the set-operator methods of Abreu, Pearce and Stachetti (1990).

All these papers study different forms of linear taxation in a competive environment, so that agent's problem is convex. In contrast, here the set of instruments controled by the principal can introduce non-convexities. Indeed, our setup can be interpreted as an optimal tax problem with full committment, as Kydland and Prescott's, but with non-linear income taxation. A complication of introducing these non-linearities is that the agent's first order conditions are no longer sufficient to characterize the agent's maximization.

1.2 Organization

The paper is organized as follows. Section 2 introduces the economic environment. Section 3 states the maximization problem faced by agents for an arbitrary mechanism and derives the necessary first-order conditions. Section 4 then presents the principal's problem and some properties that serve to simplify it. In section 5 we show the recursive structure of set of all incentive-compatible contracts. Section 6 then shows how the frontier of this set, and thus the optimal contract, can be obtained by iterating on a Bellman equation.

2 Environment

Let $s_t \in S$ denote the state of nature at time t, the set S is assumed finite for simplicity. Let $S^t = S \times S \times ...S$ be the t set product of S, $s^t = (s_{-1}, s_0, s_1, ..., s_t) \in S^{t+2}$ denotes the history of states up to time t. The state and all its history are public information.

Output at time t is a function of the current period shock: $y(s_t)$. At the beginning of each period, before the current state s_t is realized, the agent exerts an effort level, $a_t(s^{t-1})$, that affects the distribution of current and future states. We assume a Markov process for s_t : the probability of a s_t conditional on s^{t-1} and a^t is given by $\pi(s_t|s_{t-1},a_t)$. Effort is private information leading to the classical moral-hazard problem.

Lifetime utility is given by,

$$E\left\{\sum_{t=0}^{\infty}\beta^{t}\left[u(c_{t}(s^{t})-v(a_{t}(s^{t-1}))\right]\right\}$$

or given our assumption on uncertainty:

$$\sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \left[u(c_t(s^t) - v(a_t(s^{t-1}))) \right] \prod \left(s^t | s_{-1}, a^t(\cdot) \right),$$

where $\Pi(s^t|s_{-1}, a^t(s^{t-1})) = \prod_{j=0}^t \pi(s_j|s_{j-1}, a_j(s_{j-1}))$ and $a^t(s^{t-1}) \equiv (a_0(s_{-1}), a_1(s_{-1}, s_0), a_2(s^1), ..., a_t(s^{t-1})).$

The principal selects a sequence of state contingent transfers, $\tau \equiv \{\tau_t(s^t)\}\$ which the agent takes as given. The agent may save and borrow at a gross interest rate R. For now we assume that "saving" and "borrowing" occurs through the use of a linear private-storage technology with rate of return R. This leads to the following budget constraints,

$$k_{t+1}(s^t) + c_t(s^t) = y(s_t) + \tau_t(s^t) + k_t(s^{t-1})R,$$

for all t and $s^t \in S^t$. We assume the initial value of capital, k_0 , to be public information and normalize it, without loss in generality to be zero. On the other hand, consumption and capital from period 1 onwards are unobservable. We make use of the compact notation: $c \equiv \{c_t(s^t)\}, k = \{k_{t+1}(s^t)\}$ and $a \equiv \{a^t(s^{t-1})\}$. Thus, a triple (c, a, k) full describes an agents allocation.

We work with two alternative assumptions on the feasible values for $k_t(s^{t-1})$. In the first case we do not impose a constraint, so that $k_t(s^{t-1}) \in R$. In the second case, we impose a non-negativity constraint, so that $k_t(s^{t-1}) \in R_+$. This latter case can be interpreted as

The principal's utility over transfer sequences, τ , is the negative of the expected present value of transfers using

$$E\left\{-\sum_{t=0}^{\infty}\bar{R}^{-t}\tau_t\left(s^t\right)\right\}$$

or simply:

$$-\sum_{t=0}^{\infty}\sum_{s^t\in S^t}\bar{R}^{-t}\tau_t\left(s^t\right)\Pi\left(s^t|s_{-1},a^t(\cdot)\right),$$

One interpretation for this specification of the principal's preferences, is that she has access to a linear storage technology with rate of return \bar{R} and insures an infinite amount of agents with independent shocks, thus pooling risk perfectly. We allow $R \neq \bar{R}$ to be able to examine how the possibility of insurance depends on the relative returns of the agent and principal.

3 Agent Optimization

Before setting up the optimal contract problem in terms of allocations we need to define the agent's problem given a contract. This will define the incentive compatibility constraints on the allocations.

Given initial condition s_{-1} and the contract τ , the agent solves,

$$\max_{c,a,k} \sum_{t=0}^{\infty} \sum_{s^t \in S^{t+2}} \beta^t \left\{ u(c_t(s^t) - v(a_t(s^{t-1}))) \right\} \Pi(s^t | s_{-1}, a(\cdot)),$$

subject to,

$$k_{t+1}(s^t) + c_t(s^t) = y(s_t) + \tau_t(s^t) + k_t(s^{t-1})R,$$
(1)

for all t and $s^t \in S^t$. Let the set of (c, a, k)'s that achieve the above maximization given τ and s_{-1} be denoted by $MAX(\tau, s_{-1})$

Letting $\lambda_t(s^t)$ be the multiplier on (1), then the first order conditions for this problem are the budget constraints in (1) for all t and $s^t \in S^t$ and:

$$u'(c_t(s^t)) - \lambda_t(s^t) = 0 \tag{2}$$

$$\lambda_{t-1}(s^{t-1}) - \beta R \sum_{s_t} \lambda_t(s^{t-1}, s) \pi\left(s_t | s_{t-1}, a^t(s^{t-1})\right) \ge 0$$
(3)

$$\sum_{s_t} \left\{ u(c_t(s^t) + \beta w_{t+1}(s^t)) \right\} \frac{\partial \pi}{\partial \left[a_t(s^{t-1})\right]} \left(s_t | s_{-1}, a_t(s^{t-1})\right) = v'[a_t(s^{t-1})] \quad (4)$$

where:

$$w_{t+1}(s^t) \equiv \sum_{j=1}^{\infty} \sum_{s_t^{t+j}} \beta^{j-1} \left\{ u(c_{t+j}(s^{t+j}) - v(a_{t+j}(s^{t+j-1}))) \right\} \prod \left(s_{t+1}^{t+j} | s_t, a_t^{t+j}(\cdot) \right)$$

represents the remaining expected (conditional on s^t) lifetime utility from time t + 1 on as. Condition (3) must hold with equality if borrowing constraints are not imposed or if $k_{t+1}(s^t) > 0$. Let the set of (c, a, k)'s that satisfy the above f.o.c.s for given τ and s_{-1} be denoted by $FOC(\tau, s_{-1})$. Clearly, $MAX(\tau, s_{-1}) \subseteq FOC(\tau, s_{-1})$ for all (τ, s_{-1}) . In the analysis that follows we will be using the set $FOC(\tau, s_{-1})$ instead of $MAX(\tau, s_{-1})$ this is the sense in which we are using a first-order approach to the agent's maximization problem. This is the approach widely used in static principalagent problems where sufficient conditions have been studied that ensure that it indeed characterize the problem [see Rogerson (1986)].

4 Pareto Problem

4.1 Statement

We wish to study the optimal incentive-compatible contracts and the resulting utility frontier between the principal and the agent. Consider the optimal contract for the principal for any given promised utility for the agent, w. **Problem 1**:

$$C(w, s_{-1}) \equiv \min_{\tau, c, a, k} \sum_{t=0}^{\infty} \sum_{s^{t} \in S^{t+2}} \bar{R}^{-t} \tau_{t}(s^{t}) \Pi\left(s^{t} | s_{-1}, a^{t}\left(s^{t-1}\right)\right)$$
$$\sum_{t=0}^{\infty} \sum_{s^{t} \in S^{t+2}} \beta^{t} \left\{ u(c_{t}(s^{t}) - v(a_{t}(s^{t-1}))) \right\} \Pi(s^{t} | s_{-1}, a\left(s^{t-1}\right)) = w$$
$$(c, a, k) \in MAX(\tau, s_{-1})$$

This problem is made quite intractable by the set $MAX(\tau, s_{-1})$ which is in general a complicated object. We thus study the related problem that replaces $MAX(\tau, s_{-1})$ with $FOC(\tau, s_{-1})$.

Problem 2:

$$C^{*}(w, s_{-1}) \equiv \min_{\tau, c, a, k} \sum_{t=0}^{\infty} \sum_{s^{t} \in S^{t+2}} \bar{R}^{-t} \tau_{t}(s^{t}) \Pi\left(s^{t} | s_{-1}, a^{t}\left(s^{t-1}\right)\right)$$
$$\sum_{t=0}^{\infty} \sum_{s^{t} \in S^{t}} \beta^{t} \left\{ u(c_{t}(s^{t}) - v(a_{t}(s^{t-1}))) \right\} \Pi(s^{t} | s_{-1}, a\left(s^{t-1}\right)) = w$$
$$(c, a, k) \in FOC(\tau, s_{-1})$$

4.2 Properties

We now derive some simple properties which help simplify the problems further. Solving the consumer's budget constraint for $\tau_t(s^t)$ yields the following present value:

$$\sum_{t=0}^{\infty} \sum_{s^t \in S^{t+2}} \bar{R}^{-t} \left[c_t(s^t) + (\bar{R} - R)k_t(s^t) \right] \prod \left(s^t | s_{-1}, a^t(s^{t-1}) \right)$$

This provides an alternative representation of the principal's expected discounted costs and will allows us to simplify Problem 1. Using this representation and forming the Lagrangian for Problem 2 the following results are immediate.

Proposition 1 At the optimum for Problem 2,

- 1. the capital accumulation constraints given by (1) are not binding; that is their related Lagrange multipliers are zero
- 2. without borrowing constraints if $R \neq \overline{R}$ the problem is not well defined
- 3. with borrowing constraints if $R < \bar{R}$ then $k_{t+1}^*(s^t) \equiv 0$
- 4. if $R = \overline{R}$ then $k_{t+1}(s^t)$ is undetermined

The intuition for these results is straightforward: the principal can always do any intertemporal transfers through the agent and vice versa, thus the only difference in returns that is allowed is $R < \bar{R}$ when the agent is constrained.

In what follows we therefore that $R \leq \overline{R}$. In view of this proposition we may safely ignore the constraints given by (1) leaving $\{k_{t+1}(s^t)\}$ out of the maximization. Problem 2 is thus equivalent to: **Problem 3:**

$$C^{*}(w, s_{-1}) \equiv \min_{\tau, c, a} \sum_{t=0}^{\infty} \sum_{s^{t} \in S^{t+2}} \bar{R}^{-t} \tau_{t}(s^{t}) \Pi\left(s^{t} | s_{-1}, a^{t}\left(s^{t-1}\right)\right)$$
$$\sum_{t=0}^{\infty} \sum_{s^{t} \in S^{t}} \beta^{t} \left\{ u(c_{t}(s^{t}) - v(a_{t}(s^{t-1})) \right\} \Pi(s^{t} | s_{-1}, a\left(s^{t-1}\right)) = w$$
$$(c, a, 0) \in FOC(\tau, s_{-1})$$

5 Recursive Contracts

Define the set of incentive compatible allocations as:

$$IC \equiv \{ (s_{-1}, w, C, \tau, c, a) \mid (c, a, 0) \in MAX(\tau, s_{-1}) \\ C \leq \sum_{t=0}^{\infty} \sum_{s^{t} \in S^{t}} \bar{R}^{-t} \tau_{t}(s^{t}) \Pi(s^{t}|s_{-1}, a(s^{t-1})) \\ w \geq \sum_{t=0}^{\infty} \sum_{s^{t} \in S^{t}} \beta^{t} \{ u(c_{t}(s^{t}) - v(a_{t}(s^{t-1}))) \} \Pi(s^{t}|s_{-1}, a(s^{t-1})) \}$$

and analogously for those that are incentive compatible with the first-order approach:

$$IC^* \equiv \{ (s_{-1}, w, C, \tau, c, a) \mid (c, a, 0) \in FOC(\tau, s_{-1}) \\ C \leq \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \bar{R}^{-t} \tau_t(s^t) \Pi\left(s^t | s_{-1}, a\left(s^{t-1}\right)\right) \\ w \geq \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \left\{ u(c_t(s^t) - v(a_t(s^{t-1}))) \right\} \Pi(s^t | s_{-1}, a\left(s^{t-1}\right))$$

We shall exploit the fact that all contracts have a recursive structure: Φ must satisfy a self-generating operator defined by:Now define the sets:

$$\Phi \equiv \left\{ (s,\lambda,w,C) \in \mathbb{R}^4 | \exists (s_{-1},w,C,\tau,c,a) \in IC : \sum_{s_0} u'(c_0(s_0))\pi(s_0|s_{-1},a_0(s_{-1})) \le \lambda \right\}$$
$$\Phi^* \equiv \left\{ (s,\lambda,w,C) \in \mathbb{R}^4 | \exists (s_{-1},w,C,\tau,c,a) \in IC^* : \sum_{s_0} u'(c_0(s_0))\pi(s_0|s_{-1},a_0(s_{-1})) \le \lambda \right\}$$

Once again, although one is really interested in Φ , for tractability we study the simpler Φ^* .

To show the recursive structure inherent in optimal contracts we define the following set operator, which takes subsets, Q, of \mathbb{R}^4 into subsets, $\mathbb{B}(Q)$, of \mathbb{R}^4 :

$$\mathbb{B}(Q) \equiv \{(s,\lambda,w,C) \in \mathbb{R}^4 | \exists (a,\lambda'(s'),w'(s'),C'(s')) :$$

$$(s', \lambda'(s'), w'(s'), C'(s')) \in Q \ \forall s' \in S$$

$$(5)$$

$$\sum_{s'} \left[y(s') - c(s') + \bar{R}^{-1} C'(s') \right] \pi[s'|s, a] = C$$
(6)

$$\sum_{s'} \left[u(c(s')) - v(a) + \beta w'(s') \right] \pi[s'|s, a] = w$$
(7)

$$u'(c(s')) - \lambda'(s') = 0$$
 (8)

$$\lambda - \beta R \sum_{s'} \lambda'(s') \pi[s'|s, a] \ge 0 \tag{9}$$

$$\sum_{s'} \left[u(c(s')) + \beta w'(s') \right] \frac{\partial \pi[s'|s,a]}{\partial a} - v'(a) = 0 \tag{10}$$

To interpret \mathbb{B} notice that s, λ, w and C represent, respectively, the previous period's state and marginal utility of consumption and the current expected discounted utility (e.d.u.), for the agent and cost for the principal. The operator \mathbb{B} takes next period's feasible set for these variables, Q, and imposes temporary incentive compatibility^{*} constraints for the current period to find the current feasible set.

We seek to write the optimal contract so that s, λ and w are the natural state variables for the principal.

The sense in which \mathbb{B} reveals the recursive structure of Φ^* , and thus of contracts, will be now made apparent. The following propositions are simple applications of the ideas in Abreu, Pearce and Stacchetti (1990).

Proposition 2 The operator $\mathbb{B}(Q)$ satisfies:

- (a) (self-generation) if $Q \subseteq \mathbb{B}(Q)$ then $\mathbb{B}(Q) \subseteq \Phi$
- (b) (factorization) $\Phi = \mathbb{B}(\Phi)$

With any initial set Q_0 such that $\Phi \subseteq Q_0$ we can construct a sequence recursively $Q_n \equiv \mathbb{B}(Q_{n-1})$. If $\Phi \subseteq B(Q_0) \subseteq Q_0$, then $Q_n \subseteq Q_{n-1}$ for all $n \geq 1$. Define the limit set:

$$Q_{\infty} \equiv \lim Q_n = \bigcap_{n=0}^{\infty} Q_n \tag{11}$$

Proposition 3 $\Phi = Q_{\infty}$

6 Recursive Optimal Contracts

The set Φ is completely characterized by the frontier in terms of C, that is define the minimum cost function,

$$C^*[s,\lambda,w] = \inf \left\{ C : (s,\lambda,w,C) \in \Phi \right\},\tag{12}$$

and its domain ϕ ,

$$\phi^* = \{ (s, \lambda, w) : \exists C \text{ s.t. } (s, \lambda, w, C) \in \Phi \}.$$
(13)

Clearly, ϕ^* and its related operator, \mathbb{B} , satisfy self-generation and factorization.

The set Φ is completely characterized by ϕ and $C^* : \phi \to R$ as follows:

$$\Phi = \{ (s, \lambda, w, C) : (s, \lambda, w) \in \phi \text{ and } C \ge C^*(s, \lambda, w) \}$$
(14)

Consider now the following operator to obtain C^* and ϕ :

$$\mathbb{T}\left[\left\{\phi,C\right\}\right] \equiv \left\{\tilde{\phi},\tilde{C}\right\}$$
(15)

where, and for all $(s, \lambda, w) \in \phi$ we define:

$$\tilde{C}(s,\lambda,w) \equiv \inf\left\{\sum_{s'} \left\{ c(s') - y(s') + \bar{R}^{-1}C\left[s',\lambda'(s'),w'(s')\right] \right\} \pi[s'|s,a] \right\}$$
(16)

where the infimum is over $a, c(s'), (\lambda'(s'), w'(s')) \in \phi(s)$ subject to (7), (8), (9), and (10). Once again, the weak inequality in condition (10) must be replaced with equality if we are considering the case without borrowing constraints.

In essence the \mathbb{T} operator is the an alternative representation of the setoperator \mathbb{B} . If we iterate on \mathbb{T} starting from a pair $\{\phi, C\}$ such that $\phi^* \subseteq \phi$ and $C(s, \lambda, w) \geq C^*(s, \lambda, w)$ for all $(s, \lambda, w) \in \phi^*$, we are implementing an alternative representation of the iteration on \mathbb{B} . Thus, by proposition 3, we will converge to $\{\phi^*, C^*\}$ and characterize the set Φ . This leads us to the definition of problem 4.

Problem 4: starting from $\{\phi_0, C_0\}$ such that $\phi^* \subseteq \phi_0$ and $C_0(s, \lambda, w) \geq C^*(s, \lambda, w)$ for all $(s, \lambda, w) \in \phi^*$, iterate on \mathbb{T} for convergence.

It is also possible to iterate on the domain set ϕ till convergence and then separately iterate on the Bellman equation for $C^*(s, \lambda, w)$.

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