Skewed Bidding in Pay Per Action Auctions for Online Advertising

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Online search as well as keyword-based contextual advertising on third-party publishers is primarily priced using pay-per-click (PPC): advertisers pay only when a consumer clicks on the advertisement. Slots for advertisements are auctioned, and per-click bids are weighted by the probability of a click given that the advertisement is displayed (the "click-through rate") in addition to other factors. The PPC method allows the advertising platform (e.g. Google) to bundle together otherwise heterogeneous items (impressions on different positions on a search page, on different search phrases sharing common "keywords," and on different publishers) into more homogeneous units, simplifying the advertiser's bidding problem. However, PPC pricing has some drawbacks. First, all clicks are not created equal: clicks on a Paris, France hotel website that is displayed on a search for Paris Hilton may result in lower profit conditional on the click. Second, for infrequently searched phrases on search engines or small content providers, it is difficult for the advertiser to accurately estimate conversion rates, increasing the risk and monitoring costs for the advertiser and diminishing their incentives to advertise broadly (indeed, on contextual networks, the advertising platform may not even provide the advertiser with sufficient accounting data about where the advertisements were displayed to allow the advertiser to distinguish sources of clicks, and the publisher mix may change on an ongoing basis.) Third, the problem of *click fraud* is fairly pervasive: when publishers receive a share of advertising revenue, advertisers place a single bid applying to many publishers, and revenue is derived through clicks, a small publisher may be tempted to click on ads on its pages anonymously in order to inflate its payments.

One possible form of advertising lauded as a solution to both heterogeneity in the quality of ad impressions and click fraud is the *Pay Per Action* advertising system (Miguel Helft, 2007). In PPA, advertisers pay only when consumers complete pre-defined actions on their website (Google, 2007). Marrissa Mayer, Google's Vice President of Search Product and User Experience, deemed developing the PPA advertising system "the Holy Grail" (Stephan Spencer, 2007). The appeal of a PPA system arises from the fact that advertisers pay the platform only when valuable events occur, reducing the need for costly monitoring of conversion rates from different sources of clicks, and providing insurance (as compared to PPC) to advertisers against variable quality of impressions or dishonest publishers.[1]

At one level, it may seem that PPA and PPC are just variants of the same system: it should not matter whether the consumer action that triggers a payment is a click or an action. However, in this paper, we point out a number of potential problems with PPA pricing systems that do not arise in PPC systems. First, to maximize the value of the system to advertisers, a PPA system would allow advertisers to specify more than one action, since most advertisers sell products of varying value. Second, the probability that an action is recorded can be controlled by an advertiser in more complex ways. These two features create incentives for strategic behavior on the part of advertisers that undermines the efficiency and the objective of risk reallocation of a PPA auction. In particular, advertisers have the incentive to engage in what we call "skewed bidding," and further they have the incentive

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[1] See Benjamin Edelman (2008) for some ways in which PPA systems can be defrauded by dishonest publishers through manipulation of action reporting; that article does not consider bidding incentives.

to combine skewed bidding with strategic manipulation of the probabilities of different actions through destroyed links and artificial stockouts.

The presence of multiple actions implies that the bids of advertisers on different actions must be aggregated in some way. The most natural way to do this for a particular advertiser is to sum over the actions, adding up the product of the probability of an action and the bid for that action. However, as Susan Athey and Jonathan Levin (2001), Christian Ewerhart and Karsten Fieseler (2003), and Patrick Bajari, Stephanie Houghton, and Steven Tadelis (2006) have established in other contexts, auctions with scoring rules like this create incentives for "skewed bidding." If the platform's estimates of the relative probability of each action differ from the beliefs of the advertiser, the advertiser has the incentive to "over-bid" on actions that have been underestimated by the platform. This problem is exacerbated by the fact that advertisers have control over the reporting of actions–they control their own websites and thus the ability of users to complete designated actions. An advertiser has the ability to prevent actions from occurring or being reported and then bid a large amount for the now-impossible action, thus increasing the advertiser's ranking in an auction without a corresponding increase in advertiser payment.

The consequence of skewing is inefficiency in the allocation of sponsored links: bidders whose action probabilities have been mis-estimated most severely by the ad platform will be favored, because those bidders perceive the largest gap between their bid as calculated by the ad platform and their payment, and thus can afford to place bids that are perceived to be advantageous by the ad platform. A second consequence is that firms that are willing to actively game the system can outbid those who do not. This leads to allocative inefficiencies. Further, the potential gain from risk-reallocation is diminished, as advertisers' optimal strategies do not accurately report actions. We argue that it is difficult to resolve the inefficiencies without losing some or all of the benefits of PPA pricing; in practice, we may expect to see advertising platforms restrict PPA systems to a single action.

I. Model

For simplicity, we assume that there is only one sponsored link that will be awarded to the highest bidder. We use a stylized model of the advertising auction that preserves the main incentives of a real-world PPA auction, while simplifying the analysis. We assume that the ad platform uses a second-price auction where firms compete in aggregate (estimated) per-impression bids (if there were more sponsored links, a generalized second-price auction in aggregate bids could be considered, similar to existing practice for search advertising; first-price auctions have more complex bidding strategies, but incentives to skew are similar, as shown in Athey and Levin (2001)). Throughout, let k_1 denote the identity of the firm with the highest aggregate bid, and let k_2 be the firm with the second-highest aggregate bid. After bids are received, the highest bidder is informed of the second-highest aggregate bid (B^{k_2}) and the estimated action rates, and must report per-action bids that each exceed a per-action reserve price r and aggregate to B^{k_2}. The winning bidder pays only for actions that are reported, where the price per action is the per-action bid for that action. In real-world implementations, firms are not typically informed of their estimated action rates, and they bid per-action bids that are aggregated by the ad platform. However, they can modify their bids in real time and in principle can learn the aggregate bid required to obtain a position for their advertisements, which creates incentives similar to our stylized model.

Denote the firms $1, ..., I$. Assume that each firm i has only two actions 1 and 2. Let p^i_j denote the true probability of firm i's action j occurring given an impression. Let V^i be the firm's aggregate expected value per impression, which may incorporate some value from having the ad displayed in addition to the expected values from the actions. Let the ad platform's estimate of p^i_j be denoted e^i_j. In the PPA pricing scheme, payments are assessed to a winning bidder only when an action is reported to the ad platform, so the ad platform receives $\sum_j p^{k_1}_j b^{k_1}_j$ in expectation, which is different than $\sum_j e^{k_1}_j b^{k_1}_j = B^{k_2}$, the aggregate bid that the winning bidder was required to achieve using the ad platform's estimated probabilities. We consider Nash equilibrium in a one-shot game unless explicitly noted otherwise.

II. Skewed bidding with exogenous action probabilities

In order to implement a PPA pricing scheme, the ad platform must estimate action probabilities. Although an ad platform may not reveal its action rate estimates (search ad platforms currently do not reveal estimated click through rates), firms can track their own past action rates, and with some guesses about the ad platform's algorithm, they can form beliefs about the ad platform estimates. Firms may be better informed than the ad platform about future action rates. For example, a firm may know that a holiday next week will lead consumers to prefer actions in different proportions than usual. Or they may have planned a sale that affects different actions differentially.

In this section, we consider a stylized assumption that firms know their own true action probabilities, which differ from the ad platform estimates. With risk neutral firms (the case we consider first), the assumption of advertiser knowledge of their own probabilities is not important, but in an extension to risk averse firms, it matters for the calculations, though not the qualitative insights.

PROPOSITION 1: *Suppose that* p_1^i, p_2^i, e_1^i, *and* e_2^i *are strictly positive*, $V^i \geq r$, *and label the actions such that* $p_1^i/p_2^i \leq e_1^i/e_2^i$. *Then, there exists an equilibrium in which each firm i places an aggregate bid of* $\left(e_1^i/p_1^i\right)\left(V^i - r\left(p_2^i - p_1^i e_2^i/e_1^i\right)\right)$, *and if firm i places the highest aggregate bid and the second-highest aggregate bid is* B^{k_2}, *it uses per-action bids* $b_1^i = (B^{k_2} - re_2^i)/e_1^i$ *and* $b_2^i = r$. *The expected payment per impression to the ad platform is* $(p_1^{k_1}/e_1^{k_1})(V^{k_2}/(p_1^{k_2}/e_1^{k_2})+r(e_1^{k_1}p_2^{k_1}/p_1^{k_1}\text{-}e_2^{k_1}\text{-}(e_1^{k_2}p_2^{k_2}/p_1^{k_2}-e_2^{k_2})))$.

Proof: Suppose that the maximum opponent bid is B^{k_2}, and consider bidder i's utility from choosing b_2^i and $b_1^i = (B^{k_2} - b_2^i e_2^i)/e_1^i$. The bidder's utility from winning is:

$$
\begin{aligned}
& V^i - b_1^i p_1^i - b_2^i p_2^i \\
= \quad & V^i - b_2^i\left(p_2^i - p_1^i e_2^i/e_1^i\right) - B^{k_2} p_1^i/e_1^i.
\end{aligned}
$$

If $p_2^i > p_1^i e_2^i/e_1^i$ (as assumed) the firm's expected utility is maximized by decreasing b_2^i until the reserve price binds. Then, it remains to determine the optimal B^i. Expected utility depends on B^i only to the extent it determines the winner. It is thus weakly dominated to do anything other than choose the B^i that yields 0 expected profits when $B^i = B^{k_2}$, which gives the total bid in the statement of the proposition.□

The optimal per-action bids are independent of the true per-action valuations. A bidder is advantaged by over-estimates, in particular by relatively low true action rates for the most over-estimated action. Note that the relative valuations of the two objects do not matter for the equilibrium ranking of firms. The somewhat surprising part of this result is that the revenue paid to the ad platform can be higher when it mis-estimates the probabilities than when it computes them correctly. The reason is that the incorrect estimates can effectively bias the auction in favor of the weaker bidder (lower V^i), which extracts more revenue from the strong bidder in the event that the strong bidder continues to win. Also, note that if the ad platform estimates the action rates correctly, the posited strategies are weakly optimal, and the firm with the highest value V^i wins and pays the second-highest value, in expectation; however, a small amount of risk aversion would lead a bidder to prefer more balanced per-action bids.

Now consider the case where $p_1^i = 0$ and $p_1^j = 0$: both firms cannot achieve action 1, e.g. they stocked out, while all estimated actions are positive. We need to introduce maximum per-action bids, denoted $\bar{b}$, as without these, bidders have the incentive to place unbounded bids. In this case:

PROPOSITION 2: *If there is a maximum per-action bid* $\bar{b}$, $p_1^{i'} = 0$, $e_1^{i'} > 0$, *and* $e_2^{i'} > 0$ *for* $i' = 1, 2$, *then there is an equilibrium where firm i's aggregate bid is* $e_2^i V^i/p_2^i + \bar{b}e_1^i$, *and if it places the highest aggregate bid and* B^{k_2} *is the second-highest aggregate bid, its optimal per-action bids are* $b_1^i = \bar{b}$ *and* $b_2^i = (B^{k_2} - e_1^i\bar{b})/e_2^i$. *The expected payment per impression to the ad platform is* $(e_2^{k_2}V^{k_2}/p_2^{k_2} + \bar{b}(e_1^{k_2} - e_1^{k_1}))p_2^{k_1}/e_2^{k_2}$.

Proof: Consider bidder i's utility from choosing b_1^i and $b_2^i = (B^{k_2} - b_1^i e_1^i)/e_2^i$, which is $V^i - p_2^i(B^{k_2} - b_1^i e_1^i)/e_2^i$. This is increasing in b_1^i, so the maximal choice is optimal. The aggregate bid stated in the proposition yields 0 expected profits when $B^i = B^{k_2}$.□

Despite the potential for a large gap between ex ante expected revenue and payments, competition still results in payments to the ad platform that can be larger or smaller than when the ad platform estimates correctly.

Now consider the case of risk aversion, and for simplicity ignore reserve prices (they will not bind when risk aversion is high enough) and assume that a firm's value from the advertisement derives entirely from actions 1 and 2, with per-action values to that firm of v_1^i and v_2^i. Further, assume that actions are mutually exclusive. We focus on the case of constant absolute risk aversion, that is, $u(x) = -e^{-\gamma x}$. Fixing B^{k_2}, the expected utility from winning is:

$$-p_1^i e^{-\gamma(v_1^i - b_1^i)} - p_2^i e^{-\gamma(v_2^i - (B^{k_2} - e_1^i b_1^i)/e_2^i)},$$

which is maximized at $b_1^i = (e_2^i/(e_1^i + e_2^i)) \cdot \left(v_1^i - v_2^i + B^{k_2}/e_2^i + (1/\gamma)\ln(e_1^i p_2^i/(e_2^i p_1^i))\right)$. Note that if a firm minimizes its risk by bidding the same profit margin on both actions, it would choose $b_1^i = (e_2^i/(e_1^i + e_2^i))\left(v_1^i - v_2^i + B^{k_2}/e_2^i\right)$. However, this is not optimal: the firm instead chooses to take on risk by bidding more for the over-estimated action by an amount that decreases with risk aversion and increases with the relative over-estimate of action 1. Note that if actions are correctly estimated, the risk-free bid becomes optimal. With mis-estimates, instead of the break-even bid being $B^{k_2} = v_1^i e_1^i + e_2^i v_2^i - (e_2^i/\gamma)\ln(e_1^i p_2^i/(e_2^i p_1^i))$, as it would be with risk-free strategies, the break-even aggregate bid will be higher to reflect the increased expected utility from optimal per-action bids.

Qualitatively, risk aversion will mitigate the incentives of firms to distort their bids. Risk-aversion also introduces some new sources of inefficiencies. All else equal, more risk averse firms will be disadvantaged in the competition. And, firms will suffer a loss of expected utility from taking on the risk of distorting their bids, relative to a scenario where they bid their values for each action, which would be optimal if action rates were correctly estimated. We focus on risk neutral firms for the remainder of this paper, but this example provides intuition about how the results would generalize.

III. Endogenous action probabilities and market design

So far, we have demonstrated that misestimates of the action rate estimation algorithm can lead firms to engage in strategic bidding. In this section, we demonstrate how firms can manipulate the PPA scheme by failing to report an action or destroying a link on their website (lowering the true probability of a given action to 0) while strategically bidding.

To begin the analysis, assume for the moment that the system is updated only periodically, and that a series of impressions occurs in a given period while bids and estimates are held fixed; in subsequent periods, the ad platform uses the action rate from the prior period to make its next-period estimates. Also for simplicity, assume that the firm can manipulate the reporting of actions by selecting the action rate per impression in each period, with a maximum action rate of one. These assumptions are stylized, but highlight the underlying incentives, incentives that would remain qualitatively in a more realistic model.

Hold fixed the maximum opponent aggregate bid B^m satisfying $r < B^m$. The following example illustrates how firm 1 can manipulate the auction. **Period 1:** Firm 1's ad is displayed and it reports action 1 at a rate of 1 per impression and action 2 at rate 0. **Period 2:** Firm 1 sets $b_1^1 = \bar{b} > B^m$ and $b_2^1 = r$ and reports action 2 at a rate of 1 per impression and action 1 at rate 0. **Period 3+:** Firm 1 continues to alternate between the period 1 and period 2 strategies. In this example, the ad platform always estimates an aggregate bid of $\bar{b}$, but firm never pays more than r per impression. The approach can also be modified to avoid such extreme behavior as destroying a link and faking reports of actions. If the firm merely alternates periods in holding "action 1 sales" and "action 2 sales," it can follow the strategy for skewed bidding from Section II, exploiting the fact that in each period, it has different beliefs than the ad platform.[2]

It turns out that with endogenous actions there can be a strong incumbency advantage – a firm who has a chance to be displayed has the opportunity to manipulate its action rates, which affect its future profits. To simplify the analysis, we assume that to initialize the estimated action rates, each firm at the start of the game has the opportunity to bid against (only) an aggregate per-impression reserve price R such that $r < R < \min_i V^i$. With this modification of the game, the next two results show how market design matters. First, if firms can force action rates to be zero, a cap on bids is necessary, and the auction can result in inefficient allocation and low revenue on an ongoing basis. On the other hand, if the auction design puts a lower bound on each reported per-action rates as well as on the total action rate, allocation is efficient in terms of value per impression, and revenue is not impacted over the case with exogenous, correctly estimated action rates. However, even in the latter case, firms must manipulate their reported ac-

[2] A variant of this method of gaming the system requires the firm to create new actions. As long as the system allows the firm to introduce actions that have lower true probability than the search engine's initial estimates, the firm can place bids that the search engine overvalues systematically.

tions to win the auction, and thus the benefits of PPA bidding are undermined.

PROPOSITION 3: *Suppose firms consider a time horizon of two periods in their bidding. Suppose there is a maximum per-action bid* $\bar{b}$, *tie bids are broken in favor of the most recent winner, and* $\min_i V^i > 2r$. *A subgame perfect equilibrium exists where all firms initialize their action rates to be* $(0, 1)$*; all firms select an aggregate bid of* $\bar{b}$ *in each period; a firm that wins when ad platform estimates are* $e^i_j = 1$, $e^i_k = 0$ *for* $(j, k) \in \{(1, 2), (2, 1)\}$ *uses per-action bids* $b^i_j = B^{k_2}$ *and* $b^i_k = r$ *while reporting action* j *at rate* 0 *and action* k *at rate 1; and the expected payment per impression to the ad platform is* r.

The condition $\min_i V^i > 2r$ guarantees that all firms are willing to pay up to r (reporting action at rate 1 on the action with bid r) in order to give themselves a chance of winning in the subsequent period and paying r then; a total estimated action rate of less than 1 prevents a firm from achieving an aggregate bid of $\bar{b}$ in the subsequent period, which is necessary to win the auction when opponents follow the specified strategies.

Now consider the case where the firms are *required* to report a total action rate equal to 1 in each period (note that it might be more reasonable to require the firm's action rate to be 1 conditional on a click, or to allow more than two actions, but we abstract from this). In addition, suppose that there is a minimum reporting rate for each action, $\underline{e}$. This can be interpreted as requiring firms to bid a price for each impression (more generally, it could be tied to clicks) plus a bonus payment if a desirable action occurs.

PROPOSITION 4: *Suppose that firms consider a horizon of two periods when bidding. Suppose that* $\min_i V^i > r$, *there is a minimum reporting rate for each action,* $1/2 > \underline{e} > 0$, *a requirement to report actions at a total rate equal to* 1 *per impression, and no maximum per-action bid. Then there is a subgame perfect equilibrium with the following properties: each firm i places aggregate bid* $B^i = V^i(1-\underline{e})/\underline{e} - r(1-2\underline{e})/\underline{e} > V^i$*; if a firm places the highest aggregate bid and* B^{k_2} *is the second-highest aggregate bid, and if ad platform estimates are* $e^i_j = 1-\underline{e}$, $e^i_k = \underline{e}$ *for* $(j, k) \in \{(1, 2), (2, 1)\}$*, it uses per-action bids of* $b^i_j = (B^{k_2}_t - r\underline{e})/(1-\underline{e})$ *and* $b^i_k = r$*, while reporting action* j *at the rate* $\underline{e}$ *and action* k *at the rate* $1-\underline{e}$*; the winner has the highest per-impression value; the expected payment per impression to the ad platform is* V^{k_2}*, which is the second-highest per-impression valuation.*

Conditional on winning, the bids and action rates above minimize expected payments. The winning firm's profits per period are $V^i - \underline{e}(B^{k_2}_t - r\underline{e})/(1-\underline{e}) - (1-\underline{e})r$, and so the aggregate bid in the proposition is optimal.

So far we have used a naive method of estimating action rates. However, Nicole Immorlica et al (2006) designed an algorithm such that, asymptotically, click fraud only minimally changes the advertising fees. In Theorem 1 of Mohammad Mahdian and Kerem Tomak (2007), the authors extend this approach to develop an "action-based" algorithm designed such that, asymptotically, PPA fraud only minimally affects advertising fees; this result also applies to our model when the PPA system allows only a single action. Though the framework in Mahdian and Tomak (2007) only allows for one action per sponsored link, we can slightly modify their proposal to allow for multiple actions. In this section, we show that when bidding on multiple actions is permitted in the PPA system, the moral hazard stemming from the firm's ability to destroy the "complete-action-j" link allows firms to successfully defraud the ad platform even if it uses the "fraud-resistant" algorithms described above.

Following Mahdian and Tomak (2007), if there are t_j periods since the last time action j occured for firm i, then let $e^i_j = \frac{1}{t_j}$, except when $t_j > \hat{t}$, where $e^i_j = 0$. Fix initial values of estimates $0 = e^i_1 < e^i_2$ and $B^{k_2} < e^i_2 \bar{b}$. Let t' be the maximum value of t that satisfies $\bar{b}/\hat{t} > B^{k_2}$. Firm i can strictly profit from manipulating action reports as follows. **Period 0:** Bid $B^i = \bar{b}e^i_2$, and upon winning, set $b^i_1 = r$, $b^i_2 = B^{k_2}/e^i_2$; report action 1 at rate 1 and pay r per impression. **Periods** t **=1 to t′−1:** Bid $B^i = \bar{b}$ and, upon winning, set $b^i_1 = t \cdot B^{k_2}$ but do not not report any actions. The firm stays in the top position, since $\bar{b}/\hat{t} > B^{k_2}$. **Period** t'**:** The firm sets $b^i_2 = r$ and reports action 2, paying r. **Subsequent periods:** Firm 1 then repeats the cycle exchanging the roles of the actions. The firm only generates an average of r/t' per impression.

A full analysis of equilibrium behavior in this model is beyond the scope of this paper, but the intuition from the above analysis can be applied to this variant of action rate estimation as well.

IV. Remarks

While PPA advertising appears appealing at first glance, it possess a number of vulnerabilities. First, when action rates must be (for exogenous reasons) reported honestly, but firms have better estimates of action probabilities than the ad platform, the firms have an incentive to skew their bids. Skewed bidding allows a firm with a lower valuation to outbid one with a higher valuation. Since firm strategies depend on both the estimated and true action rates, in practice firms would have an incentive to expend resources to learn them if necessary. Generally, firms that have more precise estimates of them will see higher profits. In addition, risk aversion in the presence of mis-estimates reduces the profitability of advertising. These forces will increase the fixed costs of participation, and reduce the incentive for advertisers to enter PPA advertising auctions, relative to the ideal with common knowledge of correctly estimated action rates.

Second, because firms can manipulate the action probabilities by destroying links on their websites and misreporting actions, firms have the incentive to inflate their rankings through a combination of skewed bidding and misreporting action rates. Improvements in market design can increase revenue, but they cannot restore the benefits of PPA pricing, since payments will not depend on the true action rates for the firms.

Although PPA auctions are not currently in use at the major search engines in the U.S. (Google discontinued its PPA program), they are used in some display advertising networks. This paper suggests that they will be more successful when implemented either with some restrictions on how reported action rates and/or bids can vary over time, or when only one action is permitted. In another approach, Google's search advertiser optimization tools can be used to suggest PPC bids based on estimated action rates and firms' stated values per action. These tools help minimize bidding costs, but since advertisers must honor their PPC bids, the platform is not exposed to the kind of manipulation described in this paper.

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