

THEORIES OF INTERMEDIATED STRUCTURES

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INTRODUCTION

This paper is motivated by a series of positive and normative questions concerning intermediation in the economy. More specifically, on the positive side:

1. Why are there mutual funds which intermediate assets and supermarkets which intermediate commodities? More generally, why are there brokers or middlemen?
2. From where do prices come within markets? How are mutual funds priced? What determines bid-ask spreads?
3. What determines the "extent of the market"? Why are some markets said to be "thin"?
4. How do we explain valued assets such as fiat money and circulating private debt – intermediary assets which facilitate exchange? What is the relationship between such assets and highly integrated financial sectors?

On the normative side:

1. Should the government allow unfettered competition among financial intermediaries?
2. Should the government attempt to control the quantity of inside money or near-monies?
3. Should the government regulate securities markets?

Of course this paper does not pretend to offer a definitive answer to any one of these questions. The questions do motivate the analysis, and some preliminary answers are provided. In fact, it is hoped that in the end the paper is suggestive. But the overall intent of the paper is to explore methods, conceptual frameworks, and equilibrium constructs to see what might prove useful for subsequent applications. Thus the paper proceeds at a high level of abstraction; the models are highly stylized.

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Given that the models of this paper are highly stylized while the topic of the paper is the subject of an extensive literature, some further motivation would seem to be in order.¹ On the positive side, this paper searches for an explanation of intermediaries, an explanation of market structure, and an explanation of money. Obviously, to constitute an explanation, the phenomena in question cannot be assumed *a priori*. The idea then is to specify the economy at a primitive level—the preferences of households, their endowments, and the technology of production, communication, and exchange—and hope that the phenomena will emerge endogenously. Yet, despite the basic nature of the above-mentioned phenomena, research efforts are at a relatively early stage. We are still in search of a deeper understanding of intermediaries (firms) in market economies and the nature of money, to mention just two examples.

In the search for an explanation of phenomena, some structure must be imposed. In effect, this paper is an open exploration of where one might fruitfully impose the structure, that is, of what might be a useful abstraction. As it turns out, the predictions of the theory are sensitive to where the structure is imposed and, in particular, to how transactions costs and frictions are modeled. Of course, this inevitably indicates directions for future research.

A second, closely-related reason for specifying the models of this paper at the level of preferences, endowments and technology—in effect, a general equilibrium specification—is that it allows one to do explicit welfare analysis. In particular, one can look at the welfare of households directly, without specifying an *ad hoc* criterion either for intermediaries or for the government. That is, one can analyze the effect of alternative exchange structures and the effect of proposed regulations by using the standard, Pareto criterion.

As it turns out, the welfare implications of the theory, and therefore the answers to the above-listed policy questions, are sensitive also to where the structure of the model is imposed and to the nature of transactions costs and frictions. Again, this serves to indicate directions for future research.

The paper proceeds as follows. In the next section the paper is outlined in considerable detail. In fact, virtually all the models and substantive results are described there, with references to the appropriate sections. The sections are in turn more formal, self-contained treatments. This arrangement allows the reader to explore various aspects of the paper in more detail, as preferences dictate.

Briefly, section III describes a pure-exchange, Edgeworth economy without trading frictions and its Walrasian (competitive) and cooperative (core)

¹No attempt is made here to survey the literature on intermediation (or money), although no doubt such a survey would serve a useful purpose. The interested reader is urged to consult Bhattacharya [1982], Benston and Smith [1976], and Wood [1981] who offer critical review much in the spirit of the present paper before presenting their own work.

allocations. Section IV introduces a noncooperative game for the exchange economy and a sequential Nash equilibrium notion for that game, thereby modeling unfettered competition among intermediaries and explaining in one way the determination of market prices. Section V describes the relationship among Walrasian, core, and noncooperative equilibrium allocations when arbitrage by customers across potential intermediaries is allowed. With a large number of households, all these types of allocations are equivalent. Section VI describes that relationship when intermediaries alone can compete with one another. Again, with a large number of households, there is an equivalence. Section VII presents a mutual-fund model with explicit trading frictions (costly bilateral exchange). In that model, the core and noncooperative equilibrium allocations are again equivalent, and that model explains in a more satisfactory way the existence and relative number of intermediaries, the "extent of the market," the degree of diversification, and the extent of fixed fees in mutual-fund pricing schemes. Section VIII describes spatial models with explicit dynamics, spatial models which explain valued currency and circulating private debt -- intermediary assets. But here the frictions of distinct, spatially-separated markets make competitive monetary and debt equilibria Pareto *nonoptimal*. This suggests a role for government in controlling the quantity of currency and inside monies. Finally, section IX describes how pieces of the earlier sections might be fit together to build a spatial model of financial intermediation. Such a model might explain the coexistence of high-intermediated sectors with "thin" markets and bilateral currency transactions, and how the development of markets goes hand-in-hand with the development of media of exchange. But again such a model would deliver Pareto nonoptimal allocations and suggests a role for government in coordinating activity across markets or across financial intermediaries. Section IX may in turn be viewed as an attempt to reveal the source of nonoptimality and offers some caveats for policy analysis. This motivates the directions for future research indicated in section X.

II. THE ESSENCE OF THE PAPER

The paper begins with the simplest possible economy which allows for trade, a pure exchange economy such as that analyzed by Edgeworth [1881], in which households have exogenously specified endowments and preferences over consumption goods and in which there are no trading frictions (see section III). Time and uncertainty are entirely incorporated by indexing commodities. In this context, then, the standard noncooperative and cooperative equilibrium notions are the Walrasian equilibrium and the core, respectively. The Walrasian

equilibrium is the standard, competitive equilibrium in which decisions are decentralized by a price system. A core allocation is one which cannot be improved upon by a subset of households, with their own resources. These equilibrium notions are also reviewed in section III. But there are some obvious questions. With regard to the competitive outcome, from where do market prices come? More generally, suppose one allowed relatively unfettered competition among intermediaries. Would the outcome be Pareto optimal, or in the core, or even Walrasian? Indeed, as might be suggested by a reading of Fama [1980], unfettered competition among intermediaries in the pure-exchange economy would be like free banking. Thus, following Fama, one can ask whether the decisions of the entire intermediation (banking) sector would be of no consequence for the determination of prices and real activity, those of the Walrasian equilibrium? That is, is there a kind of Modigliani-Miller or neutrality proposition?

These questions are pursued here by making precise one notion of unfettered competition among intermediaries and by making precise one notion of the outcome of that competition. This is done in section IV. To be noted first is that one can analyze in the pure-exchange economy the simplest form of intermediation – acting as a go-between or broker for the exchange of commodities or claims on commodities (securities). Thus, the economy is given more structure by the (exogenous) imposition of a dynamic, two-stage game with such brokers. In the first stage of the game, each household is free to announce any vector of prices under which it is willing to act as an intermediary (broker, middleman), offering to buy or sell unlimited quantities of commodities or claims on commodities (securities) at the specified prices.² Here a vector of prices is an accounting system, with a specified (potentially abstract) unit of account. In the second stage, households take these first-stage price announcements as given in choosing intermediaries with whom to trade and the amount to trade. These latter choices are formalized by a set of decision rules, where a rule is a function of the first-stage announcements. Then, consistent with notions of dynamic programming, households take these decision rules as given in making announcements in the first stage. In this context the natural equilibrium concept is the sequential Nash equilibrium in the space of first-stage announcements and second-stage decision rules.³ Again, households are treated symmetrically;

²In none of the games for the pure-exchange economy do intermediaries impose quantity restrictions (there are, of course, feasibility restrictions). In some sense this is a troublesome feature in the first game and a virtue in the second game (both are described below)—see n. 5 and n. 9. A general treatment of pricing games with quantity restrictions is beyond the scope of the present paper.

³The concept of equilibrium in the space of decision rules is related to the perfect equilibrium of Selten [1975], and intimately linked to dynamic programming considerations, as in Bellman [1957]. It has achieved results of interest in Dybvig and Spatt [1980], J. Friedman [1977], Harris and Townsend [1977], [1980], Kreps and Wilson [1980], Kydland [1975], Kydland and Prescott [1977]. Milgrom and Roberts [1979], and Prescott and Visscher [1977], for example.

anyone who wants may call out a price vector and attempt to attract customers. In effect, there is free entry into the intermediation sector.⁴

Within this general framework, two specific games are considered. In the first, intermediaries cannot restrict the set of potential customers, and customers can deal with more than one intermediary (see section V). In this game households can resell the commodities (securities) they have purchased, so that in effect unlimited arbitrage is allowed.⁵ Putting this another way, and continuing with the Fama, Modigliani-Miller analogy, in this first game there is open access to the capital market. Thus, as it turns out, with two or more active intermediaries, prices are necessarily competitive and the Walrasian allocation is achieved. Also, with a finite number of households overall, there can exist nonWalrasian equilibria, in which there is a single active intermediary.⁶ Such equilibrium allocations are not Pareto optimal. But the ability of a single intermediary to set nonWalrasian prices is limited by the extent of the market. As his resources become negligible relative to economy-wide aggregates, prices tend to become Walrasian.⁷

In the second game considered here, customers can choose at most one intermediary with whom to trade and intermediaries can restrict the set of potential customers (see section VI). In this game there is no possibility for resale, but intermediaries can compete among themselves. Putting this another way, and continuing again the Fama, Modigliani-Miller analogy, direct access to the capital markets by individuals is limited, but the capital market itself is competitive. Accordingly, with a finite number of households overall, the Walrasian outcome can be achieved, though there may well exist other, non-Walrasian equilibria. But again, in the limit economy, with an infinity of households, the outcome is necessarily Walrasian. This is one of the more fundamental results of the paper. All these results for the second game turn on some well-known relationships between core and Walrasian allocations.

⁴This type of free entry is a crucial determinant of the equilibrium allocation. The idea that free entry plays a central role in general equilibrium, competitive markets has been emphasized recently in another context by Novshek and Sonnenschein [1980].

⁵Unlimited arbitrage would not play a role in the presence of quantity restrictions; thus the propositions for the first game below depend on the absence of such restrictions.

⁶With a finite number of households, what is offered here in effect is a general equilibrium model of imperfect competition. In such a model price setters emerge endogenously in the sense that they are not specified *a priori* but instead are determined in equilibrium. Given the criticism of Roberts and Sonnenschein [1977] of the standard, general equilibrium approach to imperfect competition (described in Arrow and Hahn [1971]), such models might be taken seriously.

⁷Edgeworth [1881] noted that there could be an indeterminacy of final settlements on the contract curve of his economy if there were a finite number of households but that such an indeterminacy would dissipate as the number of households increased (that is, as the economy is replicated). He also noted that if by custom or convenience households were restricted to trade at fixed rates of exchange, then the indeterminacy of final settlements would abide, but by way of demand curves, not the contract curve. This paper establishes that in the limit such an indeterminacy also dissipates.

Abstracting from dynamic considerations, the equilibrium of the second game has the following properties. First, any household which is not an intermediary maximizes expected utility by choosing an intermediary with whom to trade and the amount to trade. Second, households in effect partition themselves into trading cooperatives.⁸ For each cooperative there is an active intermediary or broker who specifies the terms of trade. Third, neither entry by new intermediaries nor change of strategy by existing intermediaries is desired. As noted above, no household can propose an intermediation strategy which, if adopted by households in the named coalition, would make the intermediary and the households in that coalition better off. Thus the equilibrium has both cooperative and noncooperative aspects. A particularly nice feature in the limit economy, with a continuum of households, is that it delivers allocations in the core. In fact an equilibrium allocation cannot be improved upon by a coalition, where allocations of coalitions are restricted to those which can be achieved as Walrasian equilibria for the coalition.⁹

In summary, the sequential Nash equilibria of the two particular noncooperative-intermediation games considered here have nice positive and normative characteristics, at least in Edgeworth's pure-exchange economy. They deliver properties typically associated with intermediated structures and have the above-described optimality (neutrality) properties, at least if the number of households is large.

In other ways, however, the analysis thus far is unsatisfactory, in both positive and normative aspects. From the standpoint of positive economics, we may note that some characteristics associated with intermediated structures are missing. First, there need be only one active intermediary (though there may well be more than one—it is the threat of competition which matters). Second, everybody may be in the same market, that is, markets are not thin in any sense. Third, there is nothing which corresponds to fixed fees or to bid-ask spreads with minimum purchase requirements. More generally, it is not clear in what sense we have explained anything, since the games are imposed exogenously by the modeler. Intimately related is the fact that equilibrium allocations can be nonoptimal if there are a finite number of households. Though this outcome is consistent with observations on the inefficiency of monopoly, duopoly and the like, it is the case nevertheless that there exist alternative schemes which

⁸Two other papers dealing explicitly with the connectedness of traders are Myerson [1977] and Kalai, Postlewaite and Roberts [1978]. See also n. 22.

⁹Thus the paper establishes that the core will not be enlarged if allocations are restricted in this way, and it thereby contributes to the literature which argues that various restrictions on coalition formation are inessential to the definition of the core—for example, Mas-Colell [1978]. Note also that if intermediaries could name prices with quantity limits, there would be fewer restrictions on the allocations achievable by coalitions, and the result would be weakened. Naming prices with quantity limits (or imposing fixed fees) may be helpful in some contexts in overcoming nonconvexities (see below).

produce Pareto superior outcomes. That is, the source of nonoptimality is the game itself, and the game is not given (here) any deep rationalization.¹⁰

There are at least two ways out of this dilemma, toward a more satisfactory positive and normative theory. These are closely related. One way, following Hurwicz [1959], [1972] is to be more formal about the mechanism which is to achieve the allocation of resources in a given environment. A mechanism is characterized by the set of possible messages which households can send (to a center) at each stage of a resource allocation process and the outcome function which maps messages into an allocation. We might ask in this context, for example, whether there is anything special about the competitive mechanism. Hurwicz [1959] and Jordan [forthcoming] have shown that there is—the competitive mechanism is the unique, efficient way to decentralize the economy. That is, in achieving Pareto optimal allocations, it requires minimal-size message space. The implication message-space considerations for the noncooperative intermediation games of sections V and VI, among others, is left as an open question.

A second, related way out of the dilemma, toward a more satisfactory positive theory, is to impose explicit trading frictions in the environment of the economy itself. Indeed, this is the route taken by Townsend [1978] for a highly stylized economy and further reported here in section VII.

Imagine in particular that each of a (countably-infinite) set of household-firms is endowed with a quantity of a capital good (the unique factor of production) and with a stochastic technology which transforms the capital good into a distribution of the unique consumption good of the model. Suppose investment project returns are independent across household-firms and that household-firms are risk averse. Finally, suppose that each bilateral link between households is *costly*, using a fixed amount of the capital good. In this economy, then, households care very much about the way in which they are linked to one another. In fact, an exchange structure in which everyone is linked to everyone else is generally inefficient—efficient structures minimize the number of bilateral connections. That is, efficient structures necessarily involve intermediation. Moreover, an efficient exchange structure may not have all households linked with one another, even indirectly. As the number of households in a trading coalition increases, there is a gain in terms of (potentially) increased risk sharing but a loss in terms of increased per capita transaction costs.

¹⁰The equivalence between competitive equilibrium allocations and the allocations of *alternative* noncooperative games is addressed by Hurwicz [1976], [1979], Pazner and Schmeidler [1978], Postlewaite and Schmeidler [1978], [1979], Schmeidler [1976], Shapley and Shubik [1976], Shubik [1973], and Wilson [1978].

To be more formal about the outcome one would predict for this economy, an equilibrium notion must be imposed. But what equilibrium notion? A Walrasian equilibrium notion which demands complete decentralization among households would seem to be inappropriate. That is, from what has been said so far, and from the analysis of sections III-VI, the equilibrium notion should take into account what can be achieved by coalitions. Thus it seems that the natural equilibrium notion here is the core. As it turns out, core allocations for this simple economy, with an infinity of virtually identical households, must treat all households symmetrically. This leads in turn to a straightforward maximization problem which has core allocations as its solutions, a problem which makes explicit the tradeoff between increased diversification and increased per capita transaction costs. The important result is that in a core allocation the number of households in a given trading coalition may well be finite. In this sense market size is limited endogenously. Finally, of course, the exchange structure *within* coalitions requires intermediation.

Again, following the lead of sections III-VI, we may ask whether unfettered competition among potential intermediaries leads to core allocations in this model *with transaction costs*. Again, an equivalence result *can* be established. That is, suppose each household-firm announces the set of household-firms with whom it is willing to deal, the yield in terms of the consumption good for one share in its portfolio, a price in terms of the capital good at which it is willing to sell an unlimited number of shares in its portfolio, a fixed fee in terms of the capital good for the purchase of shares in its portfolio, and a price in terms of the capital good at which it is willing to take in an unlimited number of shares on the project of any household-firm in its specified set of potential traders. In this context an equilibrium may be defined as a set of mutual funds, a set of consumption-investment decisions, a set of share choices, and a set of intermediation strategies such that (1) any household-firm who is not an active intermediary maximizes expected utility by choosing an intermediary with whom to trade, the number of shares in its own project to be sold to the intermediary, the number of shares in the portfolio of that intermediary to be purchased, and the amount to invest in its own project, taking as given the announced intermediation strategies of all other household-firms; (2) household-firms in effect partition themselves into mutual funds, and for each mutual fund there is one active intermediary, with a strategy and maximizing input choice which support the maximizing choices of inactive household-firms; and (3) no existing or potential intermediary can find an intermediation strategy which Pareto dominates the consumption-investment allocation. It can be established formally that the set of equilibrium allocations and set of core allocations are again equivalent. Finally, it may be supposed that the price

at which potential intermediaries are willing to buy shares is strictly less than the price at which they are willing to sell shares in their own portfolios, a kind of bid-ask spread. If in addition there is a *minimum* purchase requirement for shares in the portfolio, again core allocations can be achieved in noncooperative equilibria.

The mutual-fund model of section VII has nice positive and normative properties. It delivers the missing elements mentioned above; the relative number of active intermediaries, the extent of the market, the degree of diversification, and the relative magnitude of fixed fees can all be explained. Moreover, several of these elements come from frictions within the environment itself, the costs of bilateral exchange (the noncooperative game does impose additional structure—otherwise, who acts as an intermediary for whom is not pinned down). Finally, core allocations are Pareto optimal. There seems to be no scope for intervention.

Still, the mutual-fund model, like the frictionless Edgeworth economy, is missing observed phenomena. First, there are no active spot markets; all trade or commitments to trade take place in a market organized at the initial date. Second, there are no financial assets which play the role of money—objects which serve as media of exchange (intermediaries of another kind). To capture these phenomena, the economies must be decentralized further. This is done in sections VIII and IX.

To begin the discussion, imagine an Arrow-Debreu model which is specified at a rather general level. Each of a set of household-firms is endowed with a vector of consumption goods and a vector of factors of production such as labor and capital, but each vector may have many components which are zero. Each household-firm has preferences over consumption and resource vectors in each period of its life, as described by a well-behaved utility function; but again, not every household-firm cares about every commodity or factor of production. Finally, each household-firm is endowed with a technology for transforming factors of production into commodity outputs, but these technologies are not all alike.

Now suppose that household-firms can form bilateral links in order to trade with one another, as in the mutual-fund model, but that these links are costly. Suppose in particular that such links must be formed in any period in which a household-firm transfers a consumption good or factor of production to some other household-firm or makes a commitment to a transfer at a future date, as if communication were impossible without such links. (In the mutual-fund model a fixed cost was incurred which allowed both the making of commitments and the transfer of goods, but here a distinction is made.) It may be supposed also that the cost of such links can vary both with time and the identity of the household-firms involved.

Rather extreme but interesting versions of this trading technology suppose that trading links are either completely resource-free or infinitely costly. Such versions have the interpretation that household-firms are spatially separated, moving from location to location. Household-firms in the same location at a given date can trade with one another costlessly; and thus, with an infinity of households of each type, one may well suppose the existence of a competitive market at that location, motivated by the analysis of sections V and VI. Household-firms not in the same location at a given date can neither trade with one another nor make commitments to trade in the future. That is, there can be no communication across markets.

These spatial models are proving to be quite successful in explaining phenomena. They are reviewed in more detail in section VIII. One such model is Lucas' version of Cass-Yaari [1966], described in Townsend [1980]. Its key feature is the absence of double-coincidence of wants among groups of households who meet in bilateral pairings, a feature dating back at least to Wicksell [1935]. In this model there exists a monetary equilibrium, one in which fiat money is valued. That is, currency has a determinate price in terms of other goods. In that sense, money is explained. Also, one can introduce capital and explain rate-of-return dominance and other apparent asset price anomalies (see Townsend [1982]). Finally, one can introduce variable labor supply and explain how financial structure and real activity are intimately related (see Townsend [1982]). Another spatial model is a "turnpike" model of exchange, presented in Townsend [1980]. In it, valued fiat money facilitates intertemporal transactions, that is, it is partial substitute for borrowing and lending which is precluded by the spatial separation—the links among households are such that previous commitments cannot be honored. In fact, one may well include some within-group diversity, as Wallace [1980] and Sargent (forthcoming) have suggested, and predict fiat money transactions for households who meet only once and credit transactions for households who have more permanent relationships. Finally, one may alter the "turnpike" exchange structure so that households are more linked with one another, if only indirectly, and explain the existence of high velocity, private debt, that is, circulating IOUs (see Townsend and Wallace [1982]).

The success of these spatial models should be contrasted with their suggestive but troublesome normative features. As it turns out, (*laissez-faire*) competitive equilibria with fiat money and circulating private debt are generally not *Pareto optimal*. That is, there exists a redistribution of the consumption goods which is technically feasible and improves the welfare of everyone. These redistributions can be achieved directly with lump-sum taxes and subsidies on commodity endowments, or indirectly, in a monetary equilibrium, with lump-

sum taxes and subsidies on fiat money. Thus, the welfare analysis seems to create some scope for government policy. Indeed, one can replicate the conclusions of the money-growth literature on the optimal rate of deflation (see Friedman [1969]). In fact, Townsend [1982] seems to suggest a more activist policy in the presence of stochastic shocks, and Townsend-Wallace [1982] seems to suggest a coordination problem between markets (multiple equilibria) which might be remedied by restrictions on the issue of private debt. What is not clear is the extent to which this taxation and regulation would violate the decentralization imposed directly on the households *a priori*, namely, that reallocations be achieved by competitively determined exchanges in distinct, spatial locations and that there be no communication across such locations. Thus, the welfare analysis does not provide definitive conclusions, but it does indicate directions for future research.

At this point the paper asks the reader to imagine how pieces of the earlier sections—the noncooperative-intermediation games in sections III-VI, the mutual-fund model with costly and endogenous market formation in section VII, and the emergence of media of exchange in the spatial models of section VIII—might be fit together to build a spatial theory of intermediated financial structures. Imagine in particular that households have some control over their itinerary, that is, they can travel at some cost from exogenously specified locations at any given date. Then one might well imagine that with costly exchange, limited communication, and the absence of double-coincidence of wants for certain pairings, fiat money would be valued and would facilitate intermediation.¹¹ Thus, the development of media of exchange would go hand-in-hand with the development of markets, as Adam Smith observed. Second, and more generally, one might expect to observe the coexistence of highly intermediated sectors with thin markets and bilateral currency transactions. Third, if commitments can be made only through intermediaries, it seems likely that unfettered competition among intermediaries would *not* deliver core outcomes. In general, core allocations would seem to require some *a priori* and apparently costless cooperation across intermediaries. The paper thus concludes with several caveats for policy analysis, and, in section X, with some suggestions for future research.

¹¹The importance of this trade-enhancing rule for money has been emphasized to me by Allan Meltzer. See also Brunner and Meltzer [1971].

III. STRUCTURE OF A FRICTIONLESS, PURE-EXCHANGE, EDGEWORTH ECONOMY

Consider first a pure-exchange economy of the type considered by Edgeworth [1881]. The economy is inhabited by a finite number of households. Each of these households has an endowment of each of a finite number of commodities, and each household cares about consumption of these commodities, with preferences represented by a well-behaved utility function. There are no trading frictions. Thus the economy may well be subject to uncertainty and may well evolve over time, or even contain diverse locations. But following Arrow [1953] and Debreu [1959], these aspects are incorporated entirely by indexing commodities by realized states of nature, by time, and by location, supposing that all plans for trade are made at some initial date. In summary, then, the economy is completely determined by a specification of the endowment and utility function for each and every household. In particular, there are no firms or institutions specified *a priori*.

To be more formal, suppose there are ℓ possible commodities, so that commodities are labeled by the number 1 through ℓ with typical element j . Also, let Ω denote the set of households. Thus Ω is a set of household labels or names. A household labeled a , with $a \in \Omega$, has an endowment e_a , a finite but strictly positive ℓ -dimensional vector. Each household a has a utility function U_a , defined over the space of all nonnegative, ℓ -dimensional consumption vectors, with typical element f_a . The utility function U_a is assumed to be continuous, strictly increasing, and strictly quasi-concave. (As a technical matter, it is supposed that $U_a(\cdot)$ takes on the value $+\infty$ if any component of the consumption vector is $+\infty$.) One may let V denote the class of all such utility functions. Thus an economy E is a mapping from the set of households Ω into the space of endowment, utility functions pairs $R_{++}^{\ell} \times V$.

It may be noted that, apart from its name, each household a is completely determined by its endowment and utility function, (e_a, U_a) . Thus, households may be grouped according to types, that is, each household of type i has endowment, utility function pair (e^i, U^i) . There are at most a finite number m of household types, with type i taking on values in the index set $1, 2, \dots, m$.

A consumption allocation in this Edgeworth economy E is a specification of a consumption vector f_a for each and every household a . That is, an allocation is a function f which assigns to each household a of Ω a consumption vector f_a . A consumption allocation is said to be attainable if it can be achieved

with the endowments of the households. In other words, an allocation f for an economy E is *attainable* if $\sum_{a \in \Omega} e_a \geq \sum_{a \in \Omega} f_a$.

An especially simple and entirely familiar pure-exchange economy is the Edgeworth Box economy, it being supposed that there are two households labeled 1 and 2, and two commodities labeled 1 and 2. Essentially, the set of attainable allocations for the Edgeworth Box economy is the set of points in the Edgeworth Box.

In what follows we shall be very much concerned with how an allocation f is determined for an economy E . We shall also want to compare final allocations with those which would be achieved in perfectly competitive markets, on the one hand, and with the allocations which would be achieved if all households behaved cooperatively, on the other. Thus competitive and cooperative equilibrium notions must be formalized.

In the class of pure-exchange economies under consideration, a price for a commodity j specifies the number of units of credit which each household receives for each unit of commodity j supplied to the market, and the number of debits incurred for each unit of commodity j demanded from the market. Thus, a price system is a nonnegative, ℓ -dimensional vector p , with typical element p_j . Of course, the unit of account is somewhat arbitrary here; one can normalize prices by letting p_j equal unity, for example, so that commodity j is the numeraire, or by letting prices sum to unity, $\sum_j p_j = 1$, so that a particular market bundle is the numeraire. In *that* sense the price system is indeterminate. Whatever normalization is used, however, each household takes the price system p as given, and maximizes utility subject to its budget constraint, that the valuation of commodities purchased cannot exceed the valuation of commodities supplied. Of course, in equilibrium the consumption allocations which are determined in this way must clear the market. More formally, an allocation f and a price vector p is called a Walras equilibrium for an economy E if it is attainable and assigns to each household an element which is maximal in the budget hyperplane defined by p , $[x \in R_+^\ell: p \cdot e_a = p \cdot x]$. Then an allocation f is called a Walras allocation for E if there exists a price vector p such that (f, p) is a Walras equilibrium for E ; here also p may be referred to as a Walras price vector. To avoid trivialities, it is supposed that the economy is such that no Walras equilibrium can be autarkic.

The more or less standard *cooperative* equilibrium notion is the core. It is straightforward to define here. The idea behind the core is that any subset or group of households can combine and reject a proposed economy-wide allocation if that group can do better on its own, that is, with its own resources. More formally, a subset of households C is said to improve upon an allocation f

for an economy E if there exists an allocation g which is attainable for C , that is, $\sum_{a \in C} g_a \leq \sum_{a \in C} e_a$ and is such that $U_a(g_a) > U_a(f_a)$ for every household a of C . The set of attainable allocations for E which no subset of households can improve upon is called the *core* of E .

The set of core allocations for the Edgeworth Box economy is easily deduced. Obviously, an allocation f which is not on the contract curve, the locus of points where indifference curves are tangent, can be improved upon by the set of all households. Also, any allocation which puts one household on an indifference curve below the indifference curve through its endowment can be improved upon by that household itself. Thus, for the two-household, Edgeworth Box economy, core allocations lie on the portion of the contract curve which is bounded by indifference curves through the endowment. Also, as Edgeworth [1881] argued, the core shrinks to the set of Walras allocations as economy is replicated, with an increasing number n of households of type 1 and type 2, identical with households 1 and 2, respectively. This result is now known to hold for the general class of economies under consideration.

Finally, one may note again the advantage of the general equilibrium structure under consideration—one has at hand an obvious welfare criterion, namely Pareto optimality. That is, an attainable allocation f for an economy E is said to be Pareto optimal if there does not exist an alternative allocation g which is attainable and makes some households better off without making other households worse off. Under the assumptions made here, both the core and the set of Walras allocations are Pareto optimal; one may note, for example, that an allocation which is not Pareto optimal cannot lie in the core. Moreover, as the Edgeworth Box economy illustrates, the set of Walras allocations is contained in the core, and these in turn are contained in the set of Pareto optimal allocations (the entire contract curve). Thus, the core has more predictive content than the set of Pareto optimal allocations, and less predictive content than Walras allocations, except where the number of traders is large.

IV. UNFETTERED COMPETITION AMONG INTERMEDIARIES IN THE EDGEWORTH ECONOMY: A CLASS OF NONCOOPERATIVE GAMES AND A SEQUENTIAL NASH EQUILIBRIUM NOTION

This section describes a class of noncooperative games. Conceptually it will be convenient to view each of these games as consisting of two stages. In the first stage each household $t \in \Omega$ is free to announce any vector of prices p_t under which it is willing to act as an intermediary in the second stage for a specified set of households A_t' . Here it should be understood that from the

point of view of households of A_t , household t is willing to buy or sell unlimited amounts of each of the l commodities at the prices p_t . In particular there is no possibility of limiting quantities. In general A_t may be chosen from a specified set \mathcal{C} of subsets of Ω . (In particular either $\mathcal{C} = [\Omega, \phi]$ or $\mathcal{C} = 2^\Omega$, the set of all subsets of Ω .) Thus a first-stage strategy for any household t is an announcement $S_t = (p_t, A_t) \in \mathbb{R}_+^l \times \mathcal{C}$.

An announcement $S_t = (p_t, A_t)$ is said to be null if A_t equals the null set ϕ , and with some abuse of notation one writes $S_t = \phi$. Under a null announcement, household t foregoes entirely the possibility of acting as an intermediary in the second stage. Also it is understood that if S_t is nonnull, then household t names itself, that is, $t \in A_t$. Let the function $S: \Omega \rightarrow \mathbb{R}_+^l \times \mathcal{C}$ denote the announcements of all households with typical element S_t for household $t \in \Omega$. We may also write $S = [S_a, a]_{a \in \Omega}$. Also let $S_{\bar{t}}$ denote the announcements of households other than t . That is, $S_{\bar{t}} = [S_a, a]_{a \in \bar{t}}$ where $\bar{t} = \Omega - \{t\}$.

Given that the announcements S of the first stage have been determined in some way, each household in the second stage takes these announcements as given in choosing both intermediaries with whom to trade and the amount to trade, subject to the following restrictions. If for household $t \in \Omega$, $S_t = \phi$, then household t must choose intermediaries from among those whose announcements include it in a named coalition. That is, household t chooses a set of intermediaries $D_t(S)$ from the set $M_t(S) \equiv [a \in \bar{t}: S_a = (p_a, A_a), t \in A_a]$. The particular games considered below will be characterized in part by further restrictions on the choice of intermediaries from $M_t(S)$. Household t also chooses the amount to trade with each such intermediary, that is, household t chooses a trade vector $Z_t(S) \equiv [Z_{ta}(S), a \in D_t(S)]$. It is required that the trade vector $Z_{ta}(S)$ with intermediary a be a member of the budget set defined by the announced price vector p_a , $[z \in \mathbb{R}^l: p_a \cdot z = 0]$, and that the final consumption of household t be feasible, i.e., that $[\sum_{a \in D_t(S)} Z_{ta}(S)] + e_t \in X$, where $X \subseteq \mathbb{R}_+^l$ is some *a priori* feasible consumption set. Note that if $M_t(S) = \phi$, then $D_t(S) = \phi$, and so $Z_t(S)$ is undefined. Note also that thus far $D_t(S)$ and $Z_t(S)$ are introduced only as a matter of notation as choice elements; it has not yet been said how particular elements will in fact be chosen.

Now consider a household $t \in \Omega$ with a nonnull announcement $S_t = (p_t, A_t)$. Household t may be either active or inactive under its announcement. Household t is said to be active under S_t if chosen by a household in its named

coalition, that is, if $t \in D_a(S)$ for some $a \in A_t - (t)$, and is said to be inactive if chosen by no one. If household t is active, then it cannot choose another intermediary with whom to trade, that is, $D_t(S) = \phi$. If household t is inactive, it must choose an intermediary with whom to trade from $M_t(S)$, and $D_t(S)$ and $Z_t(S)$ are defined as above for the case $S_t = \phi$.¹²

In order to define the decision problem confronting each household at each stage of the game, one must specify what each household takes as given. As is implicit in the discussion above, in the second-stage each household takes as given the second-stage choices of all households other than itself, as well as the first-stage announcements S of all households, in choosing intermediaries with whom to trade and the amount to trade. Thus, given S , one may say that some specified second-stage choices $D_t^*(S), Z_t^*(S)$ are maximal for household t relative to some specified choices of others, $[D_a^*(S), Z_a^*(S), a]_{a \in \bar{t}}$, if, when there is any discretion, the choices $D_t^*(S), Z_t^*(S)$ maximize U_t . (Recall that there may be no discretion if $S_t \neq \phi$ and household t is active given $D_a^*(S), a \in \bar{t}$.) If, for a specified set of choices $[D_a^*(S), Z_a^*(S), a]_{a \in \Omega}$, the specified choices $D_t^*(S)$ and $Z_t^*(S)$ are maximal for each household t , then the set is said to be *maximal*.

It must be noted here that a *finite* maximum will not exist if $X = R_+^Q$ and there is an arbitrage possibility. In that event some components of the "maximizing" trade vector will be interpreted as being infinite in absolute value, and the utility function will be interpreted as taking on the value $+\infty$. In general no maximal choices of any kind need exist. But it *will* be part of an *equilibrium* specification (see below) that maximal choices exist.

A *decision rule* for each household t is a function which specifies the maximal choice of intermediaries, $D_t^*(S)$, and the maximal trade vector, $Z_t^*(S)$, for all possible first-stage announcements S . Thus, from the point of view of household t , its decision rule $D_t^*(\cdot), Z_t^*(\cdot)$ is determined in a maximizing way, given the specified decision rules of others, $[D_a^*(\cdot), Z_a^*(\cdot), a]_{a \in \bar{t}}$. Again, these decision rules need not be well-defined in general, but they must be well-defined in equilibrium (see below).

In the first stage of the game each household t takes as given, in

¹²Here the set of feasible choices (strategies) for any household depends on the selected choices (strategies) of the other households. In a sequential Nash equilibrium, defined below, this specification causes no difficulty, but it is not conventional in formal game-theoretic terms. In conventional game theory, feasible strategies should be independent of chosen strategies of others; if not, the game is not well-defined *out of equilibrium*. To be more formal one might imagine an extended or multi-round second stage (that is, an n -stage game) in which households are numbered exogenously and select from among potential intermediaries in order. For a similar caveat, see n. 13.

choosing its own announcement, specified first-stage announcements of all households other than itself, $[S_a^*, a]_{a \in \bar{t}}$, specified second-stage decision rules of households other than itself, $[D_a^*(\cdot), Z_a^*(\cdot), a]_{a \in \bar{t}}$, and its own maximizing decision rule $D_t^*(\cdot), Z_t^*(\cdot)$. Thus, in the view of household t , an announcement $S_t = (p_t, A_t)$ under which it is an active intermediary results in an allocation $[e_t - \sum_a Z_{at}^*(S)]$, where $S = (S_{\bar{t}}^*, S_t)$, and the summation is over all households a who choose to trade with t , $[a \in A_t: t \in D_a^*(S)]$. It is assumed here that the trades $Z_a^*(S)$ are effected. Of course if household t is inactive under S_t , its allocation will be determined by $D_t^*(S), Z_t^*(S)$ as described above. In this context a strategy $S_t = (p_t, A_t)$ may be said to be *feasible strategy* for household t relative to the specified announcements of others, $[S_a^*, a]_{a \in \bar{t}}$, and the specified decisions of others, $[D_a^*(S), Z_a^*(S), a]_{a \in \bar{t}}$, if household t is either inactive or active with the vector $[e_t - \sum_a Z_{at}^*(S)] \in X$. It is assumed that given the announcements $[S_a^*, a]_{a \in \bar{t}}$ and decision rules $[D_a^*(\cdot), Z_a^*(\cdot), a]_{a \in \Omega}$, household t varies S_t parametrically over all feasible strategies and, if possible, chooses one which is maximal under U_t .¹³ The chosen strategy is said to be maximal relative to the announcements $[S_a^*, a]_{a \in \bar{t}}$ and decision rules $[D_a^*(\cdot), Z_a^*(\cdot), a]_{a \in \Omega}$. Again maximal strategies need only exist in equilibrium (see below).

A sequential Nash equilibrium of a game is a specification of first-stage announcements and second-stage decision rules such that for each household t the specified announcement and decision rule are indeed maximal. More formally we have the following:

Definition: A *sequential Nash equilibrium* is a specification of an announcement S_t^* and decision rule $D_t^*(\cdot), Z_t^*(\cdot)$ for each household $t \in \Omega$ such that

- (1) for every S , the choices $D_t^*(S), Z_t^*(S)$ are maximal for each household t relative to the choices of others, $[D_a^*(S), Z_a^*(S), a]_{a \in \bar{t}}$; and
- (2) the announcement S_t^* is maximal for each household t relative to the announcements of others, $[S_a^*, a]_{a \in \bar{t}}$, and the decision rules of everyone, $[D_a^*(\cdot), Z_a^*(\cdot), a]_{a \in \Omega}$.

¹³Here each household t is restricted exogenously to feasible strategies. Clearly, given that a specified game is actually played, it is well-known *ex-post* whether or not a strategy was feasible, and it is in this sense that the exogenous restriction is warranted. Note also in an equilibrium, as defined below, the decision rules which household t takes as given will be those actually used, and that a specified game need not generate feasible allocations out of equilibrium. This is a common property of the noncooperative games mentioned earlier; see Postlewaite and Schmeidler [1979] for a further discussion.

Hereafter, any reference to an equilibrium of a game for the pure-exchange, Edgeworth economy will be taken to mean this sequential Nash equilibrium.

Several properties of this equilibrium concept should be noted. First, the equilibrium path is consistent in that the choices of each household are derived as the solution to a dynamic optimization problem. That is, each household t determines its optimal announcement by working backwards from the second stage, taking as given what it and other households will do given (arbitrary) first-stage announcements. In the second stage these second-stage plans will be carried out. Second, it may be noted that in specifying a particular equilibrium, it is not necessary to specify the choices $[D_a^*(S), Z_a^*(S), a]_{a \in \Omega}$ for *all* possible announcements S . From the point of view of each household t , given the announcements of others, $[S_a^*, a]_{a \in \bar{t}}$, the only announcements S which can be generated are of the form $S = (S_t, S_{\bar{t}}^*)$. In addition the rules $D_a^*(\cdot)$, $Z_a^*(\cdot)$ are often clear without elaboration. Third, an equilibrium may be said to be null if in equilibrium all households have adopted the null strategy. Since the environment is such that no Walras equilibrium is autarkic, there cannot exist a null equilibrium.¹⁴

The sequential Nash equilibria of two specific games are characterized in the next two sections.

V. ARBITRAGE RESTRICTIONS AND THE WALRASIAN OUTCOME IN THE EDGEWORTH ECONOMY

In game I no intermediary can restrict trade to a proper subset of the set of all households Ω . In particular the only nonnull strategies for any household t which are allowed are of the form $S_t = (p_t, \Omega)$, that is, $\mathcal{C} = [\phi, \Omega]$. Also, in game I any household who is not an active intermediary can choose to trade with any subset of intermediaries from the set with nonnull strategies. In this game, then, intermediaries are relatively restricted in their ability to compete with one another. Finally, attention is limited to environments with the set of all households Ω finite with $\#\Omega \geq 3$, and the consumption set $X = R_+^Q$.

It is established that any Walras allocation can be achieved in a sequential Nash equilibrium of game I. Yet the converse need not hold; it is shown by way of an example that there exist equilibria in which a single intermediary exhibits some market power. (If there are two or more active intermediaries,

¹⁴The proof is by contradiction. In a null equilibrium there is no trade. So at least one household would gain under the announcement (p, Ω) where p is a Walras price vector.

the equilibrium is necessarily competitive.) In the limit, however, as the economy is replicated, the set of equilibria of game I shrinks to the set of Walras equilibria. These results turn on the possibility of arbitrage and on the definition of feasible strategies.

First, several obvious characteristics of game I are noted. The first of these is essentially that all intermediaries must announce the same price; otherwise, there exists an arbitrage possibility for an inactive household. The second characteristic is that price-taking weakly dominates price-setting at the same price. More formally we have

Remark 1 (arbitrage): Given a set of first-stage announcements S and a set of maximal second-stage choices $[D_a^*(S), Z_a^*(S), a]_{a \in \Omega}$, suppose there exist two households, say 1 and 2, with strategies $S_1 = (p_1, \Omega)$ and $S_2 = (p_2, \Omega)$ such that S_1 is feasible, and there exists a third household, say 3, with either $S_3 = \phi$ or $3 \notin D_a^*(S), a \in \Omega - \{3\}$. Then $p_1 = p_2$.

Remark 2 (dominance of price-taking): Consider household t with a choice between trading with a set of intermediaries M with strategies $S_m = (p, \Omega)$, $m \in M$, that is, choosing from among $[z \in R^l : p \cdot z = 0]$, or being an active intermediary under a feasible strategy $S_t = (p, \Omega)$. Household t cannot prefer the latter alternative.

With these remarks it is now straightforward to establish

Proposition 1: Given any Walras equilibrium with allocation-price pair (f, p) , the allocation f can also be attained in an equilibrium of game I in which there is an active intermediary t with strategy $S_t^* = (p, \Omega)$.

For the proof of Proposition 1 (given in the Appendix), let at least two households announce the strategy (p, Ω) and let one of these households be inactive. Then if any household announces a price vector other than p , there will exist an arbitrage possibility and such a strategy cannot be feasible. Remark 2 then establishes the conclusion.

It can be established by way of an example that there exist equilibria of game I which are not competitive. For the example let there be three households, two household types, and two commodities. In particular suppose there are two households of type two, labeled 2 and 3, and one household of type one, labeled 1. For the equilibrium let $S_j^* = \phi$, $j \neq 1$. Let $S_1^* = (p_m, \Omega)$ where p_m is the monopoly price vector, the maximizing price vector set by household

1 when households 2 and 3 are price takers. This equilibrium is depicted in Figure 1. There M denotes the maximal position for household 1 relative to the offer curve of households 2 and 3, labeled $O_2 + O_3$. By construction, with $S_j^* = \phi$, $j \neq 1$, households 2 and 3 must choose household 1 as an intermediary if $S_1 \neq \phi$, that is, $[1] = D_j(S_1^*, S_1)$. Thus S_1^* is the maximal strategy for household 1. By Remark 1, households 2 and 3 are restricted to alternative strategies $S_j = (p_m, \Omega)$, $j = 2, 3$; hence by Remark 2 such strategies cannot dominate.

One's intuition is that the ability of a household to set a price other than a competitive equilibrium price should dissipate with an increase in the number of households. In order to establish this result the following lemma will prove quite useful.

Lemma 1: In any equilibrium of game I with two or more active intermediaries, each such intermediary is naming a common Walras price and the corresponding Walras allocation is achieved.

It follows from Lemma 1 that we can associate with any equilibrium of game I the price vector p which prevails. If there are two or more active intermediaries, then p is the common Walras price of their strategies; and if there is only one active intermediary, then p is the price vector of its strategy. If there are no active intermediaries, pick the price vector of one of the households with a nonnull strategy. Hence any equilibrium of game I is an allocation-price pair, as with the Walras equilibrium.

Armed with Lemma 1, the desired result can now be established, namely, the set of equilibria of game I shrinks to the set of Walras equilibria as the economy is replicated. More formally we have

Proposition 2: Consider a sequence of replica economies $E_n: \Omega_n \rightarrow V \times R_{++}^l$, $n \geq 1$, where for every E_n there are nr^i households of type i , $i = 1, 2, \dots, m$. (Each r^i is a positive integer.) For every allocation-price pair (f, p) which is not a Walras equilibrium for the E_n , there exists some N , possibly depending on f and p , such that for every $n \geq N$, (f, p) is not an equilibrium of game I.

Proposition 2 turns on the fact that the ability of a *single* active intermediary to set a nonWalras price diminishes as its endowment becomes negligible relative to the economy-wide aggregate endowment. Indeed this last result may be viewed as a formal justification of the price-taking assumption in large economies.

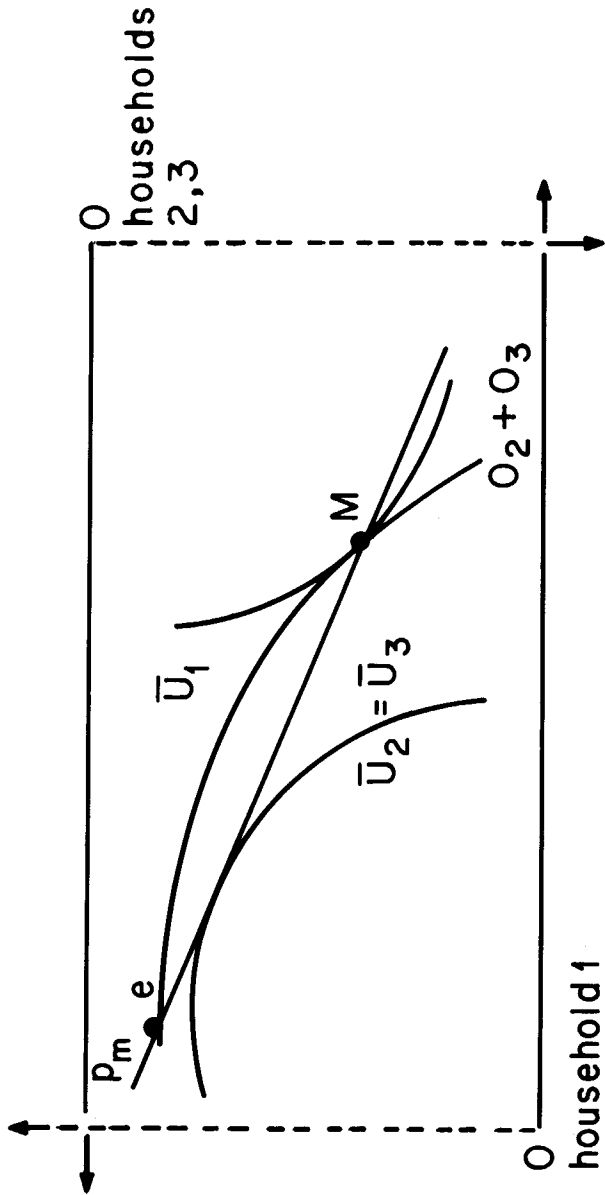


Figure 1: Monopoly Equilibrium

VI. COMPETITION FOR COALITIONS AND THE WALRASIAN OUTCOME IN THE EDGEWORTH ECONOMY

It should be reiterated that the results of game I turn on the possibility of arbitrage and the inability of a potentially active intermediary to restrict the set of households with whom it is willing to trade. That is, intermediaries are relatively restricted in their ability to compete with one another. It may be objected that in some situations an intermediary may specify the set of potential customers in order to gain market power. Moreover, in a more elaborate model in which households are spatially separated and/or exchange is costly, it may be impossible for a household to deal with more than one intermediary. It is the purpose of this section to incorporate these assumptions. In game II each household can name a (proper) subset of potential households, that is, $\mathcal{C} = 2^\Omega$, and for every a and S , $D_a(S)$ must be at most a singleton.

Despite the radically different nature of game II relative to game I, similar results are obtained. With a finite number of households ($\#\Omega \geq 3$), any Walras equilibrium can be supported as an equilibrium of game II, and in the limit, with a continuum of households, Walras allocations and game II equilibrium allocations coincide.

First, an equilibrium of game II with allocation f is characterized, in part, by the following remarks.

Remark 3: In an equilibrium of game II households are partitioned into disjoint coalitions. There may exist a set of households N_0 , each of whom is neither active nor named as a potential trader in the strategies of others. Every household of N_0 receives its endowment, that is, for every $a \in N_0$, $f_a = e_a$. The remaining set of households, $\Omega - N_0$, can be further partitioned into a set of pairwise disjoint subsets. In every set N of the partition there exists a household t with strategy $S_t^* = (p_t, A_t)$, $A_t \supset N$, who is an active intermediary for N , that is, for every $a \in N - \{t\}$, f_a is a maximal element of U_a in the set $[x \in X: p_t \cdot x = p_t \cdot e_a]$, and $\sum_{a \in N} f_a = \sum_{a \in N} e_a$.

A second characteristic of an equilibrium of game II is best stated and proved in terms of Pareto-dominating strategies and effective strategies. These are now defined. A strategy $S_b = (p_b, B)$ for household $b \in \Omega$ is said to *Pareto-dominate* an allocation f for an economy E if household b can make all households of B better off, that is, if there exists an allocation g such that $\sum_{a \in B} (g_a - e_a) = 0$, $U_a(g_a) > U_a(f_a)$ for every $a \in B$, and g_a is a maximal element under U_a in the budget set $[x \in X: p_b \cdot x = p_b \cdot e_a]$ for every household $a \in B - [b]$.

A strategy $S_b = (p_b, B)$ is said to be *effective* relative to strategies $S_{\bar{b}}$ and choices $[D_a(S), Z_a(S), a]_{a \in \bar{b}}$, $S = (S_{\bar{b}}, S_b)$, if *all* named households do choose in fact to trade with household b , that is, if $D_a(S) = \{b\}$ for every $a \in B - \{b\}$.

Now let T_ϕ denote the set of households in an equilibrium with null strategies, and as before let f denote the corresponding equilibrium allocation. Then, by the definition of an effective strategy and equilibrium condition (2), we have

Remark 4: In an equilibrium of game II there does not exist any strategy $S_b = (p_b, B)$ for any household b with $B \subset T_\phi$ which Pareto-dominates f .

The analogue of Proposition 1 for game II now follows readily from the definitions of feasible and effective strategies and the fact that any Walras allocation is necessarily a core allocation.

Proposition 3: Given any Walras equilibrium with allocation-price pair (f, p) , the allocation f can also be attained in an equilibrium of game II in which there is a single intermediary t with strategy $S_t^* = (p, \Omega)$.

For the proof (given in the Appendix) let at least three households announce the strategy (p, Ω) and let the rest announce null strategies. Then note that no alternative strategy by any household b can Pareto-dominate f since f is in the core. Thus no strategy can be effective unless household b is not better off, thus establishing that the initial strategies are indeed maximizing.

There can exist equilibria of game II which are not Walras equilibria, but again one suspects that such market power dissipates in the limit with a large number of households. Unfortunately, it seems difficult to establish this result by looking at a sequence of replica economies as in game I. The result that is established here is that no market power exists in the limit economy itself, that is, with a continuum of households.¹⁵ We first elaborate on continuum economies.

Let the set of households Ω be the unit interval $[0, 1]$. Given any Lebesgue measurable subset A of $[0, 1]$, let $\nu(A)$ denote the Lebesgue measure of A . Then $\nu(A)$ may be interpreted as the fraction of households in the population Ω who are members of A . An exchange economy associates with each household a of Ω a utility function U_a , defined on a common consumption set $X \subset R_+^k$, an element of the class V , and a strictly positive endowment e_a , an element of X . Moreover, attention is restricted to environments in which the

¹⁵The author is well aware of the asymmetric treatment of the two games.

consumption set X is bounded from above by some vector \bar{c} .¹⁶ More formally, an exchange economy E is a measurable function from the probability measure space $(\Omega, \mathcal{A}, \nu)$ into $V \times X$, where \mathcal{A} is the set of Lebesgue measurable subsets of Ω . Then, given an economy E , let A^i denote the set of all households in Ω of type i and suppose that $0 < \nu(A^i) < 1$, $i = 1, 2, \dots, m$, $2 \leq m < \infty$. (Note that there are still only a finite number of household types.)

In such an economy attainable allocations, core allocations, and competitive equilibria can be defined for subsets of households. Given an economy E and a subset A ($A \in \mathcal{A}$, $\nu(A) > 0$), a *subeconomy* is a specification of a utility function and an endowment for each household of A , that is, a measurable function E_A from (A, \mathcal{A}_A, ν) into $V \times X$ where $\mathcal{A}_A \equiv \{C: C = A \cap B, B \in \mathcal{A}\}$. An allocation for a subeconomy E_A is a specification of a consumption vector for every household of A , that is, an integrable function $f: A \rightarrow X$ where f_a is the allocation for household $a \in A$. An allocation f for E_A is said to be *attainable* if it can be achieved with the endowments of households of A , that is, $\int_A f d\nu = \int_A e d\nu$. A subset C ($C \in \mathcal{A}_A$, $\nu(C) > 0$) is said to improve upon an allocation f for E_A if there is an allocation which is attainable for C and Pareto dominates, that is, if there exists an attainable allocation g for E_C such that $U_a(g_a) > U_a(f_a)$ for almost every (ν) household of C . The set of attainable allocations for E_A that no subset can improve upon is called the *core* of E_A and is denoted $C(E_A)$. Notationally, for the core of the entire economy, let $C(E_\Omega) = C(E)$.

An allocation f for E_A and a price vector $p \in R_+^l$ is called a *Walras equilibrium for the subeconomy E_A* if the allocation is attainable and assigns to almost every (ν) household of A a maximal element of U_a in the budget set defined by p , $[x \in X: p \cdot x = p \cdot e_a]$. An allocation f is called a *Walras allocation* for E_A if there exists a price vector $p \in R_+^l$ such that (f, p) is a Walras equilibrium for E_A . Also, p may be called a *Walras price vector*. Let $W(E_A)$ denote the set of all Walras allocations of E_A . Notationally, for the Walras allocations of the entire economy, let $W(E_\Omega) = W(E)$. Again, it is assumed that no Walras allocation can be autarkic. It is well-known, with a continuum of households and under the assumptions made here, that the set of Walras allocations and the core coincide, i.e., $W(E_A) = C(E_A)$ for all $A \in \mathcal{A}$.

Various conventions are required of game II if it is to be played in an economy with a continuum of households. Each of these conventions is moti-

¹⁶The author is unhappy with this additional restriction. It seems to be needed for technical reasons to exclude allocations which are arbitrarily large for a set of households of a given type which is arbitrarily small in measure. See proof of Proposition 4.

vated by consideration of economies with Ω finite and the measure ν interpreted as the normalized counting measure, that is, given $A \subset \Omega$, $\nu(A) = \#A/\#\Omega$. First, a first-stage announcement $S_t = (p_t, A_t)$ for household t must be an element of $R_+^Q \times \mathcal{A}$ with either $A_t = \emptyset$ or $\nu(A_t) > 0$. Second, a nonnull strategy $S_t = (p_t, A_t)$ is said to be *feasible* for household t given $[S_a, a]_{a \in \bar{t}}$ and $[D_a(S), Z_a(S), a]_{a \in \bar{t}}$ if p_t is a Walras price vector for the subeconomy $E_{\tilde{A}_t}$ where $\tilde{A}_t = [a \in A_t: \{t\} = D_a(S)]$, and household t will be interpreted as receiving the consumption vector it would choose as a price taker under p_t (see Proposition 2 and the discussion thereafter). Similarly, in the definition of a Pareto-dominating strategy $S_b = (p_b, B)$, household b will be interpreted as receiving the consumption vector it would choose as a price taker under p_b . Finally, note that Remark 4, modified by the conventions, still applies.

Thus, with these conventions, we now have

Proposition 4: Given an economy with a continuum of households and a finite number of household types, any equilibrium allocation of game II in which the set of inactive intermediaries with nonnull strategies is of measure zero is also a Walras allocation.¹⁷

It will be helpful to sketch a heuristic proof of Proposition 4 for the special case of two commodities and two household types. If there exists an equilibrium allocation f which is not a Walras allocation, then f is not a core allocation, and there exists an attainable allocation g for a coalition C which treats all its households of type i , denoted C^i , identically and improves upon f . It can be established that $\nu(C^i) > 0$ and $U^i(g^i) \geq U^i(e^i)$ for any household of type i . As depicted in Figure 2, the allocation g for C determines a trading line PP through the endowment e with allocation g^i for household of type i . But if each household of type i were to take that price line with slope $-p$ as given and each were to choose a maximal element in the budget set, this would determine an allocation h^i such that $U^i(h^i) \geq U^i(g^i)$, $i = 1, 2$. Without loss of generality, suppose the distance $d(e, h^1)$ from e to h^1 is no greater than the distance $d(e, h^2)$ from e to h^2 and let $\rho = [d(e, h^1)]/[d(e, h^2)]$. Let $\gamma = \min[\nu(C^1), \nu(C^2)]$. As ν is Lebesgue measure, there exist sets $\hat{C}^i \subset C^i$ with $\nu(\hat{C}^i) = \gamma$, $i = 1, 2$, and there exists a set $C_*^2 \subset \hat{C}^2$ with $\nu(C_*^2) = \rho\gamma$. Let $T = \hat{C}^1 \cup C_*^2$. Then

¹⁷It is clear from Remark 3 that the set of active intermediaries is at most countable and hence of Lebesgue measure zero. Thus, under the hypothesis, the set of households with null strategies is of measure one. That the set of active intermediaries is of measure zero, so that intermediaries in effect vanish in the limit economy, is consistent with the stories about the invisible hand.

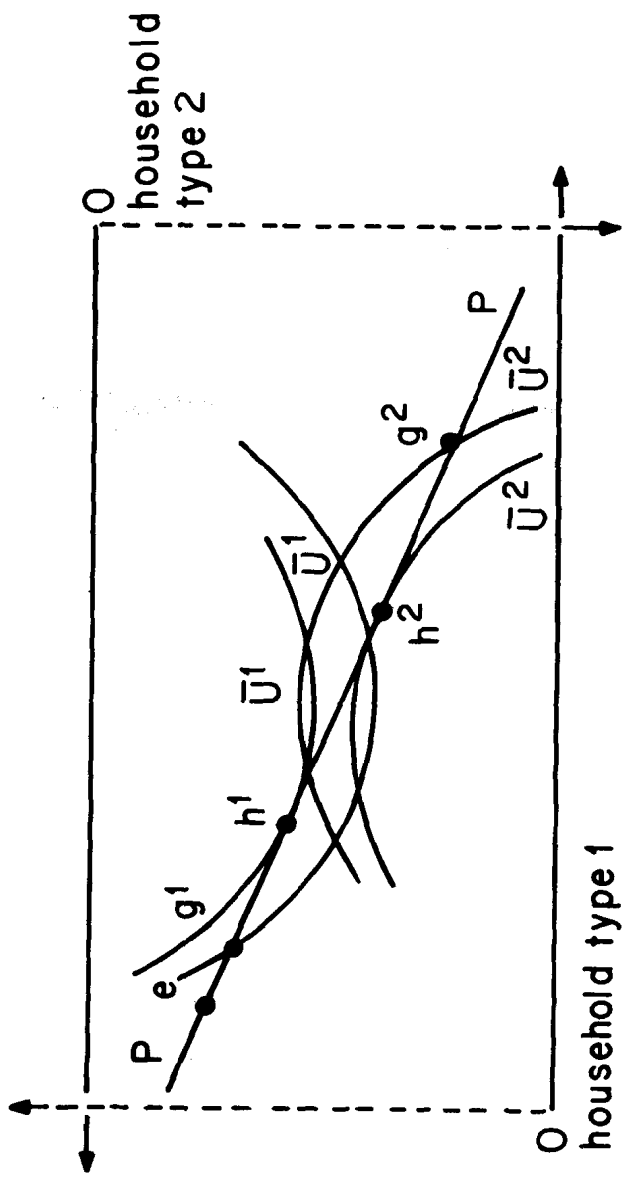


Figure 2: A Heuristic Proof of Proposition 4

allocation h is a Walras allocation for T under the implicit price vector p . Now without loss of generality it may be supposed that $T \subset T_\phi$ since by hypothesis $\nu(T_\phi) = 1$. Let household $t \in T$ adopt the strategy $S_t = (p, T)$. By construction this is a Pareto-dominating strategy relative to the equilibrium allocation f , and by Remark 4, this is the desired contradiction.

It should be noted that the allocation g for the coalition C which improves upon f need not constitute a Walras allocation for E_C , and hence cannot necessarily be supported with some price vector. But if g is not a Walras allocation for E_C , then g is not in the core of E_C and there exists an allocation h for a subset T of C which improves upon g and hence upon f , also. In the case of two commodities and two households it is obvious how to choose h and T so that h is in the core for E_T , and hence a Walras allocation for E_T , but in general an alternative approach is needed. Continuing with the process outlined above, it is clear that one can construct a sequence of Pareto-improving allocation-coalition pairs, and this suggests that perhaps a limiting allocation will have the desired property. Indeed this is the essence of the proof of Proposition 4.

It remains to note that in establishing Proposition 4 an interesting property of core allocations is also established. An allocation may be said to be in a *modified core* if it cannot be improved upon by a coalition of households where allocations of such coalitions are restricted to those which can be achieved as Walras equilibria for the coalition. Apparently, with this restriction it is more difficult for a coalition to improve upon an arbitrary allocation, so the standard core should be contained in the modified core. But the proof of Proposition 4 establishes that if an allocation is not in the core, then it is not in the modified core. Thus the modified core is contained in the standard core, and the two notions of the core are equivalent.

VII. A MODEL OF MUTUAL FUNDS WITH COSTLY BILATERAL EXCHANGE

Following Townsend [1978] imagine an economy with a set I of household-firms, where I is countably infinite. Each household-firm is endowed with K units of the unique factor of production of the model. Endowments of this capital good are identical for all household-firms and perfectly divisible. Each household-firm is also endowed with a stochastic technology which transforms the capital good into a distribution of the unique consumption good of the model. Each of these technologies or investment projects displays constant

returns to scale. Let λ_j denote the output of the consumption good per unit of the capital input y_j in project j . The $[\lambda_j]$ are assumed to be independent and identically distributed across household-firms with mean $E(\lambda_j)$. Each household-firm j has a strictly concave utility function $U(\cdot)$ over consumption c_j which displays constant relative risk aversion. Generally, consumption c_j may be regarded as a random variable, and each household-firm maximizes expected utility.

Now imagine also that exchange is costly. That is, for each bilateral deal between household-firms, a fixed cost of $2a$ units of the capital good is incurred, a per household-firm. Note that this cost is independent of the nature of the exchange and once incurred allows both prestate commitments and poststate transfers of the consumption good. Note also that this specification of costly exchange gives a real role to intermediaries. For example, suppose that there were three household-firms and each were trading with the other two directly. Then total transactions costs for the three bilateral exchanges would be $6a$. Transactions costs could be reduced to $4a$ if one household-firm were to act as a go-between or intermediary for the other two. More generally, with $\#M$ household-firms trading with one another, the most efficient number of bilateral exchanges is $\#M-1$, in contrast to an upper bound of $(\#M)(\#M-1)/2$ if each household-firm trades with everyone.

It remains to describe the set of feasible allocations given this transactions technology. Let $M \subseteq I$ denote a coalition with $\#M$ household-firms, and suppose initially that $\#M < \infty$. Let N^j denote the set of household-firms with whom household-firm j deals directly. Then a coalition C is said to constitute a market if for each $j \in M$, $N^j \subseteq M$ and if there exists no proper subset A of C such that for each $j \in A$, $N^j \subseteq A$. Thus a *market* is defined to be the smallest set of households-firms such that every household-firm of the set deals with other household-firms of the set and with no household-firm outside that set. Let $\eta(M)$ denote the number of bilateral exchanges in a market M . As already indicated $(\#M-1) \leq \eta(M) \leq (\#M)(\#M-1)/2$. Then an allocation $[c_j, y_j]_{j \in M}$ is said to be feasible for market M if $\sum_{j \in M} (K \cdot y_j) \geq (2a)\eta(M)$, and $\sum_{j \in M} \lambda_j(\omega)y_j \geq \sum_{j \in M} c_j(\omega)$ for every state of the world ω . Thus an allocation is said to be feasible for a market if it can be achieved with the resources and technologies of households of the market, taking into account the resource costs of exchange. If there are an infinity of household-firms in a market M , that is, if $\#M = \infty$, an allocation is said to be feasible if it can be achieved as the limit of allocations which are feasible for a nested, increasing sequence of finite submarkets contained in M . Finally, an allocation $[c_j, y_j]_{j \in C}$ is said to be feasible for a coalition

C if there exists a set of markets \mathcal{M} such that $\bigcup_{M \in \mathcal{M}} M = C$ and the allocation $\{c_j, y_j\}_{j \in M}$ is feasible for each market $M \in \mathcal{M}$.

The core for the economy is the set of feasible allocations for the set of all household-firms I which are not blocked by any coalition. An allocation $\{c_j, y_j\}_{j \in I}$ is said to be blocked by a coalition B and is if there exists an allocation $\{c_j^*, y_j^*\}_{j \in B}$ which is feasible for B and is such that $U(c_j^*) > U(c_j)$ for each household-firm $j \in B$. Thus, an allocation is said to be in the core if there exists no coalition which can do better with its own resources and technology.

To determine core allocation, one may note that household-firms are faced with an interesting trade-off. On the one hand, as the number of household-firms, $\#M$, who trade with one another increases, greater portfolio diversification can be achieved. With risk aversion, this is desirable. On the other hand, even on the assumption that bilateral exchange is as efficient as possible, per capita transactions costs would be $(2a)(\#M-1)/\#M$, and this also increases with $\#M$. This intuition generates a maximization problem which makes the trade-off explicit and which determines core allocations as its solutions:

$$\text{Maximize } EU\left[\left(K - \frac{(2a)(\#M-1)}{\#M}\right) \left(\sum_{i \in M} \lambda_i / \#M\right)\right]$$

with respect to $\#M$, where E denotes the expectations operator and M denotes the set of household-firms forming a market or mutual fund.

Here, then, the solution $\#M$ is the optimal number of participants in the mutual fund. Note that with a countably infinite of household-firms, there can be an infinite number of such self-contained funds; so in principle each fund can be of any possible size, including infinite. Indeed, the solution $\#M$ may well be infinite, in which case each household-firm should receive constant consumption $c = (K-2a)E(\lambda_j)$. This allocation can be achieved (in the limit) by complete diversification. That is, each household-firm j should surrender shares in its own project, each promising to pay λ_j units of the consumption good for shares in a mutual fund, each promising to pay $E(\lambda_j)$. But it is easy to establish conditions under which there will be a *finite* solution $\#M$, so that market size is limited by transactions costs. In fact, if $K \leq 2a$, the solution must be finite; otherwise per capita transactions costs would equal or exceed the capital endowment in the limit. Also, even with $K > 2a$, if $U(c) = c^b$ with b sufficiently close to unity, then the solution must be finite; if household-firms are virtually risk neutral, there is little gain to portfolio diversification. Finally, it is easy to show that the solution need not be autarkic, that is, with $\#M = 1$. Thus transactions costs deliver a nontrivial theory of market size.

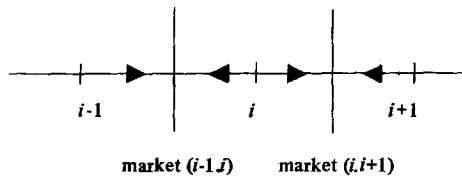
As noted in the Introduction, core allocations in this model can be achieved with intermediaries forming mutual funds in a noncooperative way, as in section VI. The reader will be spared the requisite notation. Suffice it to note that every equilibrium allocation is in the core, and all core allocations can be supported as equilibria.

VIII. SPATIAL MODELS OF MEDIA OF EXCHANGE – INTERMEDIARY ASSETS

Consider first Lucas' version of the Cass-Yaari [1966] model, described in Townsend [1980]. That is, imagine an economy with a countably infinite number of households and a countably infinite number of perishable commodities. Each (representative) household consists of a pair of agents and is imagined to be located on the real line, say, one household per integer. See Figure 3. Each household i lives forever and faces an endowment sequence of

Figure 3

Lucas' Cass-Yaari Model



commodity i (alone) which is constant. Household i cares about commodities i and $i+1$ only and discounts future over present consumption. Thus, letting $c_{it}(i)$ and $c_{i+1,t}(i)$ denote the number of units of consumption by household i at time t of commodity i and $i+1$, respectively, preferences of household i are represented by the utility function $\sum_{t=0}^{\infty} \beta^t U[c_{it}(i), c_{i+1,t}(i)]$ where $U(\cdot, \cdot)$ satisfies certain regularity properties. In each period t each member of household i is capable of moving one-half the distance to one of the two adjacent integers, $(i+1)$ and $(i-1)$. Consequently, in each period t , each household i is physically capable of carrying out transactions with households $(i-1)$ and $(i+1)$, in two spatially separated locations. As a result, the key feature of the model is the absence of double-coincidence of wants. At each time t , each household i can trade with household $i+1$, but i has no commodity $i+1$ wants.

Now suppose that households i and $i+1$ are each representative of a

large (infinite) number of households in identical situations, following identical itineraries. Suppose also that there is fiat money in the economy, pieces of paper which represent outside indebtedness (there is no redemption possibility). Then, when groups of households i and $i+1$ meet, potential intermediaries may propose the exchange of commodity $i+1$ for fiat money along the lines of the analysis of sections IV-VI. In fact, with an infinity of households, one may thus suppose the existence of a competitive market in which commodity $i+1$ is exchanged for fiat money at price $p_{i+1,t}$ in terms of fiat money. Of course, there is a market where $i-1$ and i meet, and so on. All these markets are isolated one from another.

As is discussed in Townsend [1980], there does exist a monetary equilibrium for this economy – consumption, money balance, and (positive, finite) price sequences such that the consumption and money balances sequences are maximizing for each household given the price sequences, and commodity and money markets clear. But that (fixed price) equilibrium is generally not Pareto optimal and hence not in the core. In effect, as in Clower [1967], valuation of consumption of commodity $i+1$ is limited by beginning-of-period money balances. That is, money balances must be held one period before they may be used in exchange, and this creates a distortion. As in Friedman [1969], this distortion can be remedied by lump-sum taxation of fiat money balances, inducing a deflation at the rate $1-\beta$.

The above-described model provides a rich setting which one may use to address a variety of economic phenomena. For example, one might suppose that commodity i can be stored by household i (alone), so that in effect productive capital accumulation is allowed. Then both capital and fiat money are stores of value, but fiat money alone can effect exchange for the market-produced commodity, commodity $i+1$. Hence Townsend [1982] establishes that in a monetary equilibrium the rate of return on capital can dominate the rate of return on money. In addition, one can allow shocks to technology, preferences, and the money supply (lump-sum injections of fiat money) and produce variable liquidity premia, i.e., Lagrange multipliers on the Clower constraints which move around. This in turn causes conventional intertemporal asset-pricing formulas to be overturned. Again, the monetary equilibria are generally nonoptimal. Thus, with lump-sum injections interpreted as control variables, an activist policy, one which is responsive to real shocks, would seem to be implied.

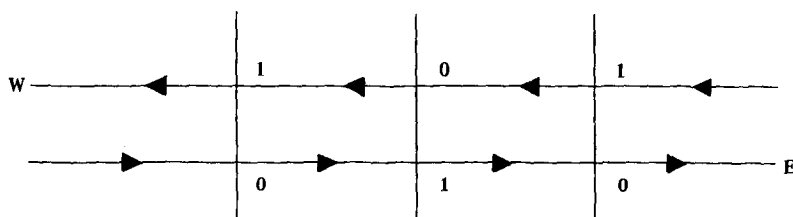
Alternatively, one may suppose that commodity i can be produced by household i each period (but not stored), that is, one may suppose the possibility of onerous labor supply. Then one may imagine that the extent to which households are linked to one another is exogenous, but variable over time. In fact, three stylized exchange-regimes may be considered: autarky, in which

households are entirely isolated one from another; a fiat money regime, as described above; and a Walrasian trade-credit regime, in which all households on the real line can trade with one another at each date, as in section III. Then, as Townsend [1982] establishes with multiple "household lines," observed comovements of economy-wide average debt and money holdings with real activity can be explained.

All the above-described models use an absence of double-coincidence of wants in intratemporal consumption. But one may imagine an absence of double-coincidence of wants for intertemporal consumption, so that the relationship of fiat money to borrowing and lending might be examined. Imagine, as in Townsend [1980], that households are traveling on a highway or turnpike, either east or west, as in Figure 4. The arrows indicate the direction of travel

Figure 4

The Turnpike Model



and the spikes indicate distinct, spatial locations where trade can occur. The numbers 0, 1 index the endowment (of the single consumption good) of a household located at the indicated position. Initially, at $t=0$, there is one (representative) household at each location, and each household moves forward one location each period. Again, these locations are isolated one from another; there can be no transactions or communication among them at any time. The absence of double-coincidence of wants is apparent here—if one indexes consumption by location and date, no two (representative) households care about the same two commodities.

Again, one may suppose each household is representative of a large (infinite) number and that intermediaries propose the exchange of the consumption good for fiat money at each location and date. Thus, one may suppose the existence of a competitive market at each location and date, a market in which the single commodity can be exchanged for fiat money at price p_t in terms of fiat money. As discussed in Townsend [1980], there does exist a (fixed price) monetary equilibrium. Again, that equilibrium is nonoptimal.

And again the inefficiency can be remedied, apparently, by lump-sum taxes on fiat money balances, but the taxes now require a distinction across household types, those who begin life with a positive endowment and those who do not.

To explain the coexistence of fiat money with borrowing and lending in this turnpike-exchange model, one can weaken the representative agent construct and suppose that, along with a group of households who begin life with zero units of the consumption good at a specified location, and thus have endowment sequence $(0, 1, 0, \dots)$ there is a smaller group following the same itinerary but with endowment sequence $(1, 0, 1, \dots)$. As Sargent [forthcoming] and Wallace [1980] have demonstrated, this within-group (within-generation) diversity explains the coexistence. The resulting model has a nice interpretation: the within-group majority uses private credit for transactions with households it meets again and uses fiat money for transactions with households it meets only once.

Finally, to explain objects like bank notes, bills of exchange, and other forms of *circulating* private debt (inside monies), imagine closing the turnpike-exchange model at both ends, forming a circle (following Cass and Yaari [1966] in a different context). Not only would representative households in such a model meet each other repeatedly, allowing the issue direct (bilateral) IOUs, there would also be chains of household-pairings which would allow the redemption of location, date specific IOUs, or securities by households other than the original acceptor. Indeed, Townsend and Wallace [1982] establish for a general class economies (of which the above is an example) the existence of competitive debt equilibria with secondary markets, third-party payment devices, and circulating high velocity private securities. However, it is also established that there can exist a multiplicity of debt equilibria, a class of equilibria unique in consumptions but not in debt issues, and that such equilibria seem to require a coordination of private debt issue across distinct, physical locations or markets at specified dates. This coordination would seem to be achieved by *a priori* restrictions on the issue of some, but certainly not all, circulating securities. And although equilibria with circulating private debt can Pareto-improve upon equilibria exogenously restricted to direct, bilateral IOUs, the equilibria with circulating private debt are generally not Pareto optimal.

IX. TOWARD A SPATIAL MODEL OF INTERMEDIATED FINANCIAL STRUCTURES

Imagine an economy inhabited by a set of H households who live for T periods (both H and T may be infinite). Each household i is endowed with

w_t^i units of the single consumption good of the model at date t . Each household i cares about sequences of units of consumption c_t^i as represented by the utility function $\sum_{t=0}^T \beta^t U^i(c_t^i)$. Here $U^i(\cdot)$ is concave and strictly increasing. Each house-

hold i has an itinerary on some spatial plane, say R^2 , which specifies its location at date t , say ϱ_t^i , in the absence of any effort (travel cost). In addition, each household can travel in the plane at each date t , with a fixed cost of a_t^i and a per-unit distance cost of γ_t^i , both in terms of the consumption good. Imagine further that households can trade with one another at date t (pass along specified units of the consumption good) if and only if they are in the same location at date t .¹⁸

Following the analysis of the mutual-fund model, an apparently natural equilibrium notion is the core. To define the core formally one needs first to define feasible allocations for arbitrary subsets of households C . Thus one must specify a consumption-location sequence for each household $i \in C$ in such a way that consumptions are feasible given the endowments of households $i \in C$, the locations visited by them, and their travel costs. The core is then a specification of a consumption-location sequence for each household $i \in H$ which is feasible for H and such that there does not exist a set of feasible consumption-

¹⁸This economy is somewhat easy to describe, but an analysis of it would not be straightforward since multiple within-period movements are allowed. To further simplify matters, one might suppose, for example, that prior to any trade at a given date, each household must specify some *ex post* location, m_t^i , possibly distinct from ϱ_t^i , that movements to these new locations occur simultaneously, and that no subsequent movements at date t are allowed. It might also be supposed that there are at most a finite number of possible locations on the plane, a finite number of households, and a finite number of dates. On the other hand, one might well have imagined that there are multiple commodities, variable factor supplies, and nontrivial production technologies, along the lines of the first class of spatial models described in section VIII.

location sequences for a subset of household C which can improve upon the original allocation.¹⁹

One might well imagine that core allocations will generally involve some intermediary or go-between activity to reduce travel costs. Some households may be centrally located, while others have relatively low travel costs. One might also imagine that there could arise dates t , due to high travel costs or low *a priori* population densities, such that there is little activity (travel or trade) on at least a portion of the spatial plane. At the same date, though, in another region, "markets" might flourish.²⁰

Again we may ask whether unfettered competition among intermediaries might achieve core allocations. Clearly this depends on how unfettered competition is modeled. Suppose, as seems natural, and following Townsend-Wallace [1982], that any household can issue at date t , in any location where it happens to be, date-location contingent securities. A typical security is a promise to pay specified units of the consumption good at a specified location at some future date s . It is assumed that such a promise must be honored. This in turn implies that a household's location at any date t may be determined (with some effort) by previously-issued securities. Now suppose in addition that a household (potential intermediary) at date t can announce the set of households with whom it is willing to deal, the set of securities it is willing to exchange, and various associated terms of trade. Here the terms of trade would

¹⁹More formally, for the more restricted economy described in n. 18, given a subset of households C , let $C_t^i(m)$ denote the subset of C which ends up at location m at date t . That is, $C_t^i(m) = \{i \in C: m_t^i = m\}$. Then consumption-location sequences $\{c_t^i, m_t^i\}_{t=0}^T$, $i \in C$, are said to be feasible for C if

$$\sum_{C_t^i(m)} c_t^i \leq \sum_{C_t^i(m)} w_t^i \cdot \gamma_t^i (|\varrho_t^i - m|) - \sum_{[i: \varrho_t^i \neq m]} \alpha_t^i \quad (1)$$

for all feasible locations m and for all dates t ;

$$c_t^i \geq 0; \quad (2)$$

$$\alpha_t^i + \gamma_t^i (|\varrho_t^i - m|) \leq w_t^i \quad \text{if } \varrho_t^i \neq m. \quad (3)$$

A consumption-location sequence $\{c_t^i, m_t^i\}_{t=0}^T$ for all households $i \in H$ is said to be in the *core* if there does not exist a subset of households C with feasible consumption-location sequence $\{c_t^{i*}, m_t^{i*}\}$ such that

$$\sum_{t=0}^T \beta^t u^i(c_t^{i*}) > \sum_{t=0}^T \beta^t u^i(c_t^i) \quad \text{for all } i \in C.$$

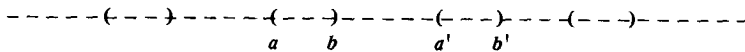
²⁰Having said this, though, it is apparent that even for the more restricted economy described in n. 18, core allocations would not be easy to determine. One might well make use of a computer for parametric specifications.

include the number of units of the date t consumption good the potential intermediary is willing to surrender for each unit of fiat money or for each security, either issued previously (by someone) and held by a named customer or issued contemporaneously by a named customer. Similarly, the terms of trade would include the number of units of the date t consumption good the potential intermediary is willing to accept for each unit of fiat money or for each security, either currently held by it (from previous encounters) or issued by it (contemporaneously). One might well imagine that a potential intermediary might be willing to exchange only a subset of potential securities and might announce a schedule of fixed fees as well.

In this setting, then, one can define a multiperiod, sequential Nash equilibrium with two stages at each date, along the lines of section IV.²¹ In such an equilibrium (assuming existence), one might expect to observe active intermediaries. That is, one might suppose that centrally-located or low-travel-cost households would act as brokers, both taking in the consumption good from "lenders," in exchange for the issue of its own securities (or previously-acquired securities), and passing along the consumption good to "borrowers," in exchange for their IOUs (or previously-acquired securities).

A multiperiod, sequential Nash equilibrium might also illustrate how money, the "extent of the market," and intermediation are all interrelated. Consider the following suggestive (but nonrigorous) scenario, in the spirit of the turnpike-exchange model of section VIII. Suppose representative households a and b , a' and b' , and so on, are in effect paired at date t as indicated in Figure 5. That is, a and b are substantially nearer to each other than to an

Figure 5



alternative trading partner. At date $t+1$ household advances a (exogenously) to the previous position of a' , and a' advances to the previous position of some a'' , and so on. Households b , b' , and so on, stay put. Suppose also that households a and a' have an endowment which is low and then high at dates t and $t+1$, respectively, and conversely for households b and b' . Thus it is supposed that households a and b would like to engage in a two-period, borrowing-lending agreement (with a borrowing from b), similarly for a' and b' , and so on. But to

²¹If T is finite, one could determine maximizing strategies by working backward from the last date. Here, following Prescott and Vischer [1977], it may be especially useful to suppose that choices at the second stage of each date are made sequentially (as if households were ordered).

effect this agreement, not only must a and b travel to some intermediate location at date t (between the points marked a and b), they must do so again at date $t+1$ (between the points marked b and a'), when they are relatively far apart.

If the gains from trade were substantial, they might well make such an arrangement. But if travel costs were sufficiently high, they might not. That is, on another real line, similarly situated but more distant households might not trade. Alternatively, suppose there is (valued) fiat money in the economy. Then when households a and b meet at date t , at some cost, let a (the potential borrower) exchange fiat money for the consumption good, and similarly for a' and b' . Then at date $t+1$, let households a and b' meet (at some lesser cost) and let b' surrender fiat money for the consumption good. Thus, one might well expect the monetary economy to Pareto dominate the nonmonetary economy, and thus money would replace privately-issued securities. Further, trade-pairings which did not occur in the nonmonetary economy might now be effected. In this sense the extent of the market might increase with the advent of fiat money. Moreover, if households a , b , and so on are each representative of a large number of households, then trade might well be effected with intermediaries, and intermediation might increase with the advent of fiat money.

We now return to the question of whether or not a multiperiod, sequential Nash equilibrium allocation would lie in the core. The answer would depend in part on the strategy spaces available to the households. Again one suspects that fixed fees would be needed, and these should vary with *a priori* customer locations. In addition, a relatively small number of households in a given location *a priori* might cause problems of imperfect competition, along the lines of those in sections V and VI—the analysis of sections V and VI indicates that this consideration would be mitigated by the extent that (now costly) arbitrage can take place across intermediaries and by the extent to which potential intermediaries compete with one another.

But even if small numbers and limited strategy spaces do not cause problems, it will generally be the case that multiperiod, sequential Nash equilibrium allocations do *not* lie in the core. To see this, note that the above-described noncooperative game essentially limits binding commitments—in the nonmonetary economy, at least, transfers of the consumption good must be achieved by security issue, redemption, or trade; and securities can be issued only when households meet (perhaps at some cost) and can travel only with households (perhaps at some cost). Again, a suggestive scenario, similar to the one given earlier, may help to illustrate this point (see also Townsend-Wallace [1982]).

Suppose there are four households (a , a' , b , and b'), two dates (t and

$t+1$), and two locations (1 and 2). Suppose at date t households a and b are paired exogenously at location 1 and households a' and b' are paired exogenously at location 2. Suppose also that at date $t+1$ households a and a' switch locations. Finally, suppose that travel costs are virtually infinite. Then under the above-described security trading rules, no security can be redeemed by the issuer at $t+1$. Consequently, no household would relinquish the consumption good for such a security at t . That is, there can be no mutually beneficial exchange. But it is easy to specify endowments in such a way that autarky can be improved upon by all four households, say by a "borrowing" from b but paying back b' , and similarly for a' and b . In fact, any symmetric bilateral arrangement between a and b and between a' and b' can be effected in this way *at no cost*. Thus, even if travel costs were sufficiently low that direct symmetric borrowing-lending takes place, such an arrangement would not be Pareto-optimal.

Of course, fiat money would help to remedy this situation, but in general one does not expect to overturn the conclusion that multiperiod, sequential Nash equilibrium allocations will not lie in the core. The general problem is that the multiperiod sequential Nash equilibrium assumes limited communication and makes it costly to enter into binding agreements, whereas the core takes into account only the cost of exchanging the consumption good itself. A core allocation is in effect a social agreement which *de facto* is "entered into" costlessly. It seems there are two ways to resolve this conflict. First, one might imagine a model of competition among intermediaries in which securities can be costlessly traded and transferred so that, in effect, there is unlimited communication. But that attack strategy would, apart from consumption-trading costs, return us to the virtually timeless analysis of Edgeworth and mutual-fund models, removing any essential dynamics. Second, one might contemplate altering the definition of the core, somehow taking into account limits on communication and the costs of commitment. But is not yet obvious how that ought to be done. One might also note in this regard that the multiperiod, sequential Nash equilibrium itself seems to assume some *a priori* communication and costless commitment, as somehow intermediaries (or customers) coordinate travel and security trades across distinct locations. That is, multiperiod sequential Nash equilibria are almost surely not unique.²²

As it stands, the divergence between the core and the multiperiod, sequential Nash equilibrium indicates there would be pressures for institutions to emerge which reduce contract costs and facilitate *a priori* coordination. Clearly, one would like a , b , a' and b' in the example just described to form a syndicate, with mutually binding (group) commitments. Alternatively, the core

²²There are some links here to multiplicities in the search and matching literature which the author is exploring in another paper. See, for example, Diamond [forthcoming] and Mortensen [1974].

might also be taken as an "ideal" in the evaluation of government policy. One can imagine default rules in the example just described in which a defaults at date $t+1$ on its date t debt to b , but is penalized at date $t+1$, that is, is forced to pay b' . But in the absence of a theory of government with costly constitutions or costly regulation, the core as specified here could be a misleading welfare criterion.

X. DIRECTIONS FOR RESEARCH

This paper should conclude with some caveats. The idea that trade links are costly, *per se*, seems to be a useful formalism, presumably capturing the cost of bookkeeping, the cost of enforcement, the cost of monitoring when there is imperfect information, the physical cost of exchange (transportation), the difficulties of communication, and so on. Similarly, Hotelling's spatial plane is a useful formalism, presumably capturing various aspects of diversity and changing circumstance. But it may well be that for some purposes, both positive and normative, we shall want to distinguish these costs and gain a better understanding of the diversity, modeling both at a somewhat deeper level. The divergence between the core and noncooperative equilibrium allocations reinforces that conclusion.

Appendix - Formal Proofs

Proof of Remark 1: The proof is by contradiction. Suppose $p_1 \neq p_2$. Then household 3 is confronted with an arbitrage possibility. Namely, there exist commodities i and j such that $(p_1^i/p_1^j) < (p_2^i/p_2^j)$. (Here the superscript i denotes the i^{th} component of the indicated price vector and so on.) Hence household 3 could exchange commodity j in return for commodity i from intermediary 1 and reverse the transaction with intermediary 2, ending with a surplus of commodity j . With the nonsatiation assumption this exchange would be carried on without limit, and hence the strategy S_1 is not feasible.

Proof of Remark 2: As an active intermediary under p , household t attains an allocation on the budget hyperplane $\{x \in R_+^L : p \cdot e_t = p \cdot x\}$, a choice which is available under the first alternative

Proof of Proposition 1: The numbering of households is arbitrary. Let $S_1^* = S_2^* = (p, \Omega)$, $S_j^* = \phi$, $j \in \Omega - \{1, 2\}$, $D_a^*(S^*) = \{1\}$ for $a \in \Omega - \{1\}$, and let Z_{a1}^* be the maximal element under U_a in $\{z \in R^L : p \cdot z = 0\}$. Consider first some alternative feasible strategy $S_j = (p_j, \Omega)$ for any household $j \in \Omega - \{1, 2\}$. If this strategy is to strictly dominate S_j^* for j , then j must be active, for if inactive his choice set is as before. Hence $j \in D_a^*(S_{\bar{j}}^* S_j)$ for some $a \in \bar{j}$. In this event there exists a household who can arbitrage between household 3 and one of the households 1 and 2. Since S_j must be feasible, Remark 1 implies that $p_j = p$, and Remark 2 implies that such a strategy cannot dominate. With $S_j^* = \phi$, $j \in \Omega - \{1, 2\}$ a similar argument establishes that neither 1 nor 2 can find alternative strategies that dominate.

Proof of Lemma 1: Note first that with two or more active intermediaries there exists a third household which can arbitrage. Hence by Remark 1 each active intermediary m has equilibrium strategy $S_m^* = (p, \Omega)$. Consider an intermediary t of type i . Intermediary t cannot be worse off than it would be if it were a price-taker under p , that is, choosing trade Z^{*i} from among $\{z \in R^L : p \cdot z = 0\}$ (given the strict quasi-concavity of U^i , Z^{*i} is unique). Otherwise t would announce the null strategy and plan to trade with one of the active intermediaries in the second stage. Yet by Remark 2, household t as an active intermediary cannot be better off than as such a price-taker. It follows that each active inter-

mediary of type i achieves the allocation $e^i + Z^{*i}$. Clearly so do all other households of type i . Of course the choice of type i was arbitrary.

It remains to establish that the allocation $[e^i + Z^{*i}, i = 1, 2, \dots, m]$ and the price vector p constitute a Walras equilibrium. Evidently, for every household a of type i , $e^i + Z^{*i}$ is a maximal element of U_a in $[x \in R_+^l : p \cdot x = p \cdot e_a]$. As for attainability, note that the net trade for any active intermediary of type i is Z^{*i} so $Z^{*i} = -\sum_{at} Z_{at}^*(S^*)$ where the summation is over $[a : t \in D_a^*(S^*)]$. Then, summing over all active intermediaries yields $\sum_{i=1}^m Z^{*i} \#(A^i) = 0$. (Note that the argument here does not require that the second intermediary be active.)

Proof of Proposition 2: Consider any allocation-price pair (f, p) which is not a Walras equilibrium for the E_n . Suppose, contrary to the conclusion of the Proposition, that (f, p) is an equilibrium of game I for infinitely many of the E_n . By Lemma 1, (f, p) must be attained under a *feasible* strategy for a single intermediary t . A contradiction can now be established. As p is not a Walras price, there exists some commodity h such that $\sum_{j=1}^m [f_h^j - e_h^j] r^j > 0$ where f_h^j denotes the maximal choice of commodity h under U^j in the budget set $[x \in R_+^l : p \cdot x = p \cdot e^j]$ and e_h^j denotes the h^{th} component of e^j . But if the intermediary t is of type i , feasibility of its strategy for $E_n, n \geq 2$, requires that

$$e_h^j \geq \sum_{j \neq i} (f_h^j - e_h^j)(nr^j) + (f_h^i - e_h^i)[(nr^i - 1)]$$

or equivalently

$$f_h^i \geq n[\sum_{j=1}^m (f_h^j - e_h^j)(r^j)] .$$

For sufficiently large n this inequality cannot be satisfied for any i , the desired contradiction.

Proof of Remark 3: That households are partitioned into disjoint coalitions follows from the specification that $D_a(S)$ is at most a singleton for all a and S and that for active intermediary t , $D_t(S) = \phi$. The other statements are obvious.

Proof of Remark 4: Suppose the contrary and note that such a strategy would be effective relative to the equilibrium strategies S^*_b and choices $[D_a^*(S), Z_a^*(S), a]_{a \in \bar{b}}$, $S = (S^*_b, S_b)$ and hence would be proposed by b . This contradicts equilibrium condition (2).

Proof of Proposition 3: Let three households (labeled 1, 2, and 3 without loss of generality) adopt the strategy $S_i^* = (p, \Omega)$, $i = 1, 2, 3$, and let all other households, if any, adopt the null strategy, that is, $S_a^* = \phi$, $a \neq 1, 2, 3$. Let $D_a(S^*) = \{1\}$, $a \neq 1$. Clearly the Walras allocation can be effected. Now without loss of generality any household b can be restricted to alternative strategies which are either feasible and effective or null. (For given a feasible strategy $S_b = (p_b, B)$ let $\tilde{B} = \{a \in B: D_a(S^*_b, S_b) = \{b\}\}$ and consider instead the strategy $S_b = (p_b, \tilde{B} \cup \{b\})$ if $\tilde{B} \neq \phi$ or $S_b = \phi$ if $\tilde{B} = \phi$.) First, consider any alternative nonnull feasible strategy $S_1 = (p_b, B)$ by household 1. As f is a Walras allocation for E , f is in the core of E ; thus no such alternative strategy can Pareto-dominate f . So let F denote the set of households in $B - \{1, 2, 3\}$ who would be no better off relative to f , let $D_a^*(S^*_1, S_1) = \{2\}$ for $a \in F$, and let $D_a^*(S^*_1, S_1) = \{1\}$ for a in the complement of F . If $F \neq \phi$, S_1 is not effective. If $F = \phi$ then some household of $\{1, 2, 3\}$ would be no better off. If $2 \in B$ would be no better off, let $D_2^*(S^*_1, S_1) = \{3\}$ (otherwise let $D_2^*(S^*_1, S_1) = \{1\}$) so that S_1 could not be effective. Similarly, if $3 \in B$ is not better off, let $D_3^*(S^*_1, S_1) = \{2\}$. Hence the only alternative effective nonnull strategies must be such that 1 would be no better off; hence S_1^* is maximal. If $S_1 = \phi$, let $D_1^*(S^*_1, S_1) = \{j\}$ for $j \in \{2, 3\}$ so that again such a strategy cannot dominate S_1^* .

Now suppose 2 considers alternative feasible effective nonnull strategies $S_2 = (p_b, B)$. Again let F denote the set of households which would be no better off in $B - \{1, 2, 3\}$ and let $D_a^*(S^*_2, S_2) = \{1\}$ if $a \in F$. If $F \neq \phi$, S_2 is not effective. If $F = \phi$, and $1 \in B$ would no better off, let $D_1^*(S^*_2, S_2) = \{3\}$. If $3 \in B$ would be no better off, let $D_3^*(S^*_2, S_2) = \{1\}$. For $S_2 = \phi$, let $D_2^*(S^*_2, S_2) = \{j\}$ for $j \in \{1, 3\}$. A similar argument applies for 3 and for $a \in \Omega - \{1, 2, 3\}$.

Proof of Proposition 4: Suppose there exists some allocation f for E such that f is an equilibrium allocation of game II but $f \notin W(E)$. As $W(E) = C(E)$, $f \notin C(E)$. Hence there exists at least one allocation-coalition pair (h, T) where h is an attainable allocation for E_T , which improves upon f . Now the following claim is established below.

Claim 1: Given any pair (h, T) which improves upon an allocation f , there exists an allocation-coalition pair (g, C) with $C \subset T$ which also improves upon f and treats all households of any given type identically.

Let $C^i = E^{-1}[U^i, e^i] \cap C, i = 1, 2, \dots, m$. Thus $C^i \in \mathcal{A}$ is the set of households of type i in C . Let M denote the set of integers $[1, 2, \dots, m]$ where m is the number of household types. Let $I(C) = \{i \in M: \nu(C^i) \neq 0\}$. Thus $I(C)$ is the set of indices of household types in C with nonzero measure. Let $\#[I(C)]$ denote the number of elements of $I(C)$. Then the pair (g, C) which improves upon f is given utility representation \tilde{u} in $R^{\#[I(C)]}$ where $\tilde{u}^i = U^i(g^i), i \in I(C)$ and g^i is the allocation under g to any household of type i . In general, given some set $N \subset M$, let $B^N \subset R^{\#(N)}$ denote the set of all such utility representations associated with (g, C) pairs which improve upon f with $I(C) = N$. Thus a particular (g, C) pair has utility representation in one of $\#(2^M)$ possible sets. (Here 2^M is the set of all subsets of M .) Also, since $f \notin C(E)$, at least one of these sets is nonempty. From the *nonempty* sets pick one of lowest dimension and denote it B^{N^*} . That is, N^* is a set $N^* \subset M$ and $\#(N^*) \leq \#(N)$ for every $N \in 2^M$ with $B^N \neq \emptyset$. For notational simplicity let $B^{N^*} \equiv B^*$.

A sequence $[b_k]_{k=1}^\infty$ of elements of B^* is said to be *monotone increasing* if for every $k, i, b_k^i \leq b_{k+1}^i$. Here the subscript k indicates the element of the sequence and the superscript i indicates the component of the vector. The following claim is established below.

Claim 2: If $[b_k]_{k=1}^\infty$ is a monotone increasing sequence in B^* with limit b , then $b \in B^*$.

Now define the set of least upper bounds of B^* , denoted $F(B^*)$ as the set of points $\hat{b} \in R^{\#(N^*)}$ so that the following two conditions hold:

- (1) There does not exist some $b \in B^*$ such that $b \geq \hat{b}$. That is, $F(B^*)$ is a set of upper bounds (here and below the notation \geq for vectors means b exceeds \hat{b} in at least one component and is no less in any component).
- (2) For every $\delta > 0$ there exists some $b \in 0(\hat{b}, \delta) \cap (\{\hat{b}\} - R^{\#(N^*)}) \cap B^*$. (Here $0(\hat{b}, \delta)$ is the sphere of radius δ centered at \hat{b} , and $\hat{b} - R^{\#(N^*)}$ is the nonpositive orthant with origin at \hat{b} .)

Now any nonempty subset of k -dimensional Euclidean space R^k which is bounded from above also has a least upper bound. The set B^* is bounded from above since for any $b \in B^*, b^i \leq U^i(\bar{c})$ for every $i \in N^*$. Therefore, $F(B^*) \neq \emptyset$.

Given some $\hat{b} \in F(B^*)$, it is possible to construct a monotone increasing sequence of elements of B^* which have \hat{b} as a limit by letting $\delta \rightarrow 0$. Hence by Claim 2, $\hat{b} \in B^*$.

Let (g, C) denote the allocation-coalition pair associated with the chosen \hat{b} . It is now established that $g \in C(E_C)$. For suppose the contrary. Then

by Claim 1 there exists an allocation-coalition pair (g', C') which improves upon g and treats all households of any given type identically. Note that $C' \subset C$. The inclusion here and below is weak. Hence (g', C') improves upon f also. Clearly $I(C') \subset I(C)$. But if $I(C')$ is a proper subset of $I(C)$, then (g', C') has utility representation in a space of lower dimension than $\#(N^*)$, a contradiction to the choice of N^* . If $I(C') = I(C)$, then there exists some $b \in B^*$ such that $b > \hat{b}$, since (g', C') improves upon g , a contradiction to the choice of \hat{b} in $F(B^*)$ (see defining property (1) above).

Since $g \in C(E_C)$, g is also a Walras allocation for E_C under a price vector p . Now without loss of generality it may be supposed that $C \subset T_\phi$, since $v(T_\phi) = 1$. Let some household $t \in C$ adopt the strategy $S_t = (p, C)$. Since g improves upon f for E_C and can be supported with the price vector p , this strategy is Pareto-dominant relative to f , and by Remark 4 this is the desired contradiction.

Proof of Claim 1: As h is attainable for E_T ,

$$\sum_{i \in I(T)} [\int_{T^i} h \, dv - e^i v(T^i)] = 0.$$

Therefore, letting

$$\rho^i = \frac{v(T^i)}{\sum_{j \in I(T)} v(T^j)},$$

$$\sum_{i \in I(T)} \rho^i \left[\frac{\int_{T^i} h \, dv}{v(T^i)} - e^i \right] = 0. \quad (1)$$

Let $Q = [q \in I(T): h_a \text{ is not equal to some constant vector for almost every } a \in T^q]$. Then if $Q = \phi$, the claim is trivial. So suppose $Q \neq \phi$. Then for every $q \in Q$ there exist some set $B^q \subset T^q$, $v(B^q) > 0$ such that, from the strict quasi-concavity of U^q ,

$$U^q(h_a) < U^q \left[\frac{\int_{T^q} h \, dv}{v(T^q)} \right] \quad \text{almost every } a \in B^q. \quad (2)$$

Clearly

$$U^i(h_a) = U^i \left\{ \frac{\int_{T^i} h^i dv}{\nu(T^i)^q} \right\} \quad \text{almost every } a \in T^i, i \in I(T), i \notin Q. \quad (3)$$

Let $\gamma_1 = \min_{q \in Q} \nu(B^q)$. Let $\gamma_2 = \min_{i \notin Q} \nu(T^i)$. Let $\gamma = \min[\gamma_1, \gamma_2]$. Then as ν is Lebesgue measure, there exist sets $B_{**}^q \subset B^q$ with $\nu(B_{**}^q) = \gamma$ and sets $B_*^q \subset B_{**}^q$ with $\nu(B_*^q) = \rho^q \gamma$ for every $q \in Q$. Similarly for $i \in I(T), i \notin Q$, there exist sets $B_*^i \subset T^i$ with $\nu(B_*^i) = \rho^i \gamma$. Let $C = \cup_{i \in I(T)} B_*^i$ and for almost every $a \in B_*^i$, let $g_a = [\int_{T^i} h^i dv] / [\nu(T^i)]$, $i \in I(T)$. Then multiplying both sides of (1) by γ it is clear that $\int_C [g - e] dv = 0$, so that g is an attainable allocation for C . From (2) and (3), g also improves upon f .

Proof of Claim 2: Let (g_k, C_k) denote the allocation-coalition pair associated with b_k which improves upon f . Then for every $i \in N^* = I(C_k)$, $0 \leq g_k^i \leq \bar{c}$. The left inequality follows from the fact that $g_k^i \in R_+^Q$, and the right inequality follows from the boundedness of the consumption set X . Define $\rho_k^i = [\nu(C_k^i)] / \sum_{j \in N^*} \nu(C_k^j)$. Then $0 \leq \rho_k^i \leq 1, i \in N^*$. Hence by compactness one can construct from the sequences $[\rho_k^i, g_k^i, i \in N^*]$ sequences $[\rho_n^i, g_n^i, i \in N^*]$ such that $\rho_n^i \rightarrow \rho^i \in [0, 1]$ and $g_n^i \rightarrow g^i \in [0, \bar{c}]$ as $n \rightarrow \infty$. By construction $\int_{C_k} g_k dv = \int_{C_k} e dv$. That is,

$$\sum_{i \in N^*} [g_k^i - e^i] \nu(C_k^i) = 0.$$

Therefore, dividing through by $\sum_{j \in N^*} \nu(C_k^j)$,

$$\sum_{i \in N^*} [g_n^i - e^i] \rho_n^i = 0$$

for every n . Taking the limit as $n \rightarrow \infty$, one obtains

$$\sum_{i \in N^*} (g^i - e^i) \rho^i = 0. \quad (4)$$

By continuity $U^i(g_n^i) \rightarrow U^i(g^i)$ as $n \rightarrow \infty$. Also, $U^i(g_n^i) = b_n^i$ and $b_n^i \rightarrow b^i$ from below. So $U^i[g^i] = b^i$, and for every n

$$U^i(g_n^i) \leq U^i(g^i) \quad i \in N^* . \quad (5)$$

Upon picking an arbitrary n , there is associated an allocation-coalition pair (g_n, C_n) which improves upon f . Define $\gamma = \min_{i \in N^*} \nu(C_n^i)$. As ν is Lebesgue measure, for every $i \in N^*$ there exists some set $C_*^i \subset C_n^i$ with $\nu(C_*^i) = \gamma$, and there exists a set $C^i \subset C_*^i$ with $\nu(C^i) = \rho^i \gamma$. Let $C = \cup_{i \in N^*} C^i \subset C_n$. But multiplying (4) through by γ one obtains

$$\sum_{i \in N^*} [g^i - e^i] \nu(C^i) = 0,$$

so g is feasible for C . As (g_n, C_n) improves upon f , it is clear from (5) that (g, C) improves upon f . Moreover $\rho^i > 0$ for every $i \in N^*$. For suppose $\rho^i = 0$ for some $i \in N^*$. Then (g, C) improves upon f and has a utility representation in a space of lower dimension than $\#(N^*)$, the desired contradiction. Therefore $b \in B^*$.

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