

Walrasian Bargaining

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Abstract

Given any two-person economy, consider an alternating-offer bargaining game with complete information where the proposers offer prices, and the responders either choose the amount of trade at the offered prices or reject the offer. We provide conditions under which the outcomes of all subgame-perfect equilibria converge to the Walrasian equilibrium (the price and the allocation) as the discount rates approach 1. Therefore, price-taking behavior can be achieved with only two agents.

Key Words: Bargaining, Competitive equilibrium, Implementation

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1 Introduction

It is commonly believed that, in a sufficiently large, frictionless economy, trade results in an approximately competitive (Walrasian) allocation. In fact, the core of such an economy consists of the approximately Walrasian allocations. Moreover, in a bargaining model with a continuum of anonymous agents, Gale (1986) shows that the allocation

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is competitive in any subgame-perfect equilibrium (henceforth, SPE). In contrast, when there are only two agents, it is believed that we have a bilateral monopoly case, and the outcome is indeterminate (Edgeworth (1881)). Accordingly, in that case, the core is very large, and the SPE outcome typically differs from the competitive allocation in the usual bargaining models, in which the agents are allowed to offer any allocation.¹ In this paper, we analyze a simple two-agent bargaining model in which the agents offer price vectors (henceforth, prices) rather than allocations. We show that, under certain conditions, as the agents' discount rates approach 1, all the SPE allocations and prices converge to the competitive allocation and price, respectively. Our result suggests that the Walrasian equilibrium does not necessarily require a large economy. It simply corresponds to price-taking behavior, which can be achieved even with only two agents. Therefore, the Walrasian equilibrium may have stronger foundations than commonly thought.

Considering a pure-exchange economy with only two agents, we analyze the following alternating-offer bargaining game with complete information: Agent 1 offers a price. Agent 2 either demands a feasible trade at that price, or rejects the offer. If he demands a trade, the demanded trade is realized, and the game ends. If he rejects the offer, we proceed to the next date, when Agent 2 offers a price, and Agent 1 either demands a feasible trade or rejects the offer. This goes on until they reach an agreement. Each agent's utility function is normalized by setting it to 0 at the initial endowment. Agents cannot consume their goods until they reach an agreement, and each discounts the future at a constant discount rate.

Notice that we restrict the proposer to offering prices, and allow the other agent to choose the amount of trade at the offered price. For instance, in the case of wage bargaining, if the union sets the wage, the firm has the right to choose how much labor to hire. Likewise, if the firm sets the wage, the workers or the union have the right to choose the amount of labor they provide. In contrast, in the standard models, the proposer is allowed to offer any allocation, while the other agent can only accept or reject the offer. This is the only major difference.

In this model with price offers, price-taking behavior emerges. To see this, consider an economy with two goods. When an agent i accepts a price, the game ends, hence

¹For instance, as the common discount rate approaches 1, the SPE outcome in Rubinstein (1982) converges to the Nash (1950) bargaining solution (Binmore, Rubinstein, and Wolinsky (1987)), which is different from the Walrasian outcome, as noted by Binmore (1987). (Rubinstein's model is more general and abstract, but the set of feasible payoffs is typically taken to be the set of *all* materially feasible payoffs, allowing the agents to offer any allocation.)

he demands the optimal trade for himself at that price. Effectively, this restricts the other agent to offer a payoff vector on *the offer curve of agent i* , the payoff vectors that can be achieved when i is a price-taker. For discount rates that are close to 1, consider any stationary SPE (henceforth, SSPE) in which all the offers are accepted. Now, both agents are approximately indifferent between the allocations today and tomorrow, and the payoffs associated with these allocations are on the offer curves of two distinct agents. Therefore, the payoffs in such a SSPE must be very close to an intersection of the offer curves. Of course, each Walrasian payoff-vector is at such an intersection. Moreover, in many canonical economies with a unique Walrasian equilibrium, the Walrasian payoff-vector is the only intersection of the offer curves in the relevant region. Therefore, for such economies, the payoffs in these SSPE must converge to the Walrasian payoffs as the discount rates approach 1. Moreover, the Walrasian payoffs are obtained only at the Walrasian allocation, as we have strictly quasi-concave utility functions. By continuity, this also shows that the allocations at these SSPE converge to the Walrasian allocation. Finally, when we have a continuous inverse-demand function, we can also conclude that the prices in these SSPE converge to the Walrasian price.

We further show that in any other possible SSPE, an agent must act as a natural monopoly; he must offer his monopoly price, and it must be accepted. In order for this to be an equilibrium for large values of discount rates, the monopoly outcome must be Pareto-optimal under the constraint that one of the agents is a price-taker. We rule out such SSPE, by assuming that no monopoly is efficient in this sense. It turns out that, under these assumptions, SPE must be in between some SSPE payoffs, and hence all SPE allocations (and prices) must converge to the Walrasian allocation (and price) of the static pure-exchange economy at hand. This convergence to the Walrasian equilibrium of the static economy is independent of the way the discount rates approach 1. This further implies that this convergence does not depend on relative frequency of offers, and one can easily extend this result beyond alternating-offer bargaining.

This limiting behavior is strikingly different from that of the Rubinstein (1982) model where agents can offer any allocation. In that model, for a quasi-linear economy, how the agents share the gains from trade in the limit is solely determined by the relative frequencies the agents make offers, or the logarithmic ratio of the discount rates (as they approach 1). More generally, in such sequential bargaining models, the outcome is determined by which agent makes an offer and when. Some authors find this a weakness of the model and adhere to the traditional view that the bargaining outcome should be determined by the property rights — not by the details of the procedure (see

Nash (1950), Harsanyi (1977), Aumann (1987), Perry and Reny (1993), and Smith and Stacchetti (2001)). Some others (such as Binmore (1987)) take the position that the terms in which the bargaining is conducted may be relevant to the bargaining outcome. Our theorem has two implications on this issue. On the one hand, a change in the procedure (that the proposers are required to offer prices rather than allocations while the responders are allowed to choose the size of the trade) has a profound effect on the outcome. Therefore, *the allocation of procedural rights matters*. On the other hand, the limiting behavior under the new allocation of the procedural rights does not depend on how the discount rates approach 1—or which agent makes an offer when, provided that each agent makes offers sufficiently frequently (see footnote 5). Therefore, *such procedural details do not matter for the limiting behavior* under the new allocation of procedural rights.

This result is also relevant to the implementation literature, where there is an interest in implementing the Walrasian allocation in a Nash environment (see Hurwicz (1979), Schmeidler (1980), Hurwicz, Maskin, and Postlewaite (1995), and Bochet (2002)). Here we have a simple bargaining procedure that approximately implements the Walrasian allocation in SPE under certain conditions. Our result may appear to be weaker than the results cited above, as we use stronger concept of SPE while the previous results consider all Nash equilibria. Nevertheless, the epistemic assumptions behind a solution depend on the game. In our game, the SPE can be obtained by iteratively eliminating conditionally dominated strategies, and hence can be supported by some sequential-rationality assumptions. In contrast, it would be very hard to find any set of sensible epistemic assumptions that would support the Nash equilibrium in the above games, as these games typically involve roulette games and simultaneous declaration of types.

There is also a literature that focuses on the Walrasian payoffs in the axiomatic framework of Nash (1950) (e.g., Binmore (1987), Sertel and Yildiz (1994), and Serrano and Volij (2000)). Most notably, Binmore (1987) illustrates that the Nash bargaining-solution may yield non-Walrasian payoffs, but then characterizes the Walrasian payoffs by certain axioms on the space of allocations that are similar to the axioms of Nash (1950), which were stated in utility space. He also presents a direct mechanism and a version of the Nash Demand Game in which each player simultaneously commits to a “worst price” and a ceiling on the amount of trade. The largest possible trade under these commitments is then realized. Even though the induced game possesses abundance of equilibria (including no trade), he shows that all Pareto-efficient equilibria yield the unique Walrasian allocation.

One question remains. Is impatience or the relative frequency of offers irrelevant? We show that the standard comparative static holds: if an agent becomes more patient, he becomes better off, and his opponent becomes worse off. In fact, when the discount rates are away from 1, impatience of players has a significant effect on the SPE outcome. That is, if the agents are making offers once a year, then it does matter how patient each player is, or which player makes an offer when. But, if the agents are going to make offers every other split second, then these details do not matter.

In the next section, we lay out our model. In Section 3, we explain why the SPE outcome in the standard model is unrelated to the competitive outcome, and why our theorem is still true. In Section 4, we derive our main results and provide counterexamples. Section 5 contains extensions of the main result; Section 6 is about the role of impatience and the frequency of offers, and Section 7 concludes. Some proofs are in the Appendix.

2 Model

Let $X = \mathbb{R}_+^n$ be a commodity space with n goods, and P be the set of all price vectors $p = (p^1, \dots, p^n) \in \mathbb{R}_{++}^n$ where $p^1 = 1$, and $p^k > 0$ for all k .² (Henceforth, we will simply say prices instead of price vectors.) Consider a set $N = \{1, 2\}$ of two agents and their pure-exchange economy $e = ((u_1, \bar{x}_1), (u_2, \bar{x}_2))$, where $\bar{x}_i \in X$ and $u_i : X \rightarrow \mathbb{R}$ are the initial endowment and the utility function of agent i , respectively, for each $i \in N$. For each $i \in N$, assume that u_i is strictly quasi-concave, continuous, monotonically-increasing, and $u_i(\bar{x}_i) = 0$. Write $w = \bar{x}_1 + \bar{x}_2$.

We wish to understand the relation between the Walrasian equilibrium of e and the SPE of the following perfect-information bargaining game $G(\delta_1, \delta_2)$ with alternating offers. Let $T = \{0, 1, 2, \dots\}$ be the set of all dates. At date $t = 0$, Agent 1 offers a price $p_1 \in P$. Agent 2 either demands a consumption $x_2 \in \{x \in X | (x - \bar{x}_2) \cdot p_1 = 0, x \leq w\}$ or rejects the offer. (Note that x_2 is feasible and can be reached by reallocating the endowments at price p_1 .) If she demands x_2 , the game ends yielding the payoff vector $(\delta_1^t u_1(w - x_2), \delta_2^t u_2(x_2))$, which is associated with the allocation $(w - x_2, x_2)$, where $(\delta_1, \delta_2) \in (0, 1)^2$. If she rejects, we proceed to the next date. At $t = 1$, Agent 2 offers a price p_2 , and Agent 1 either demands $x_1 \in \{x \in X | (x - \bar{x}_1) \cdot p_2 = 0, x \leq w\}$, when the game ends yielding the payoff vector $(\delta_1^t u_1(x_1), \delta_2^t u_2(w - x_1))$, or rejects the offer,

²We write \mathbb{R} for the set of real numbers, \mathbb{R}_+^n for the non-negative orthant of a n -dimensional Euclidean space, \mathbb{R}_{++}^n for the interior of \mathbb{R}_+^n .

when we proceed to the next date, when Agent 1 offers a price again. This goes on indefinitely until they reach an agreement. If they never reach an agreement, each gets 0. We are most interested in the behavior of the SPE as (δ_1, δ_2) approaches $(1, 1)$.

Any *stationary subgame-perfect equilibrium* (SSPE) is represented by a list $(\hat{p}_1, \hat{x}_1, \hat{p}_2, \hat{x}_2)$ where, for each $i \in N$, $\hat{p}_i \in P$ is the price that the agent i offers whenever he is to make an offer, and $\hat{x}_i : P \rightarrow X \cup \{\text{Reject}\}$ is the function that determines his responses: when offered a price p_j , the agent i demands $\hat{x}_i(p_j)$ if $\hat{x}_i(p_j) \in X$, and rejects the offer if $\hat{x}_i(p_j) = \text{Reject}$. Likewise, any *subgame-perfect equilibrium* (SPE) is represented by a list $(\hat{p}_{1,2k}, \hat{x}_{1,2k+1}, \hat{p}_{2,2k+1}, \hat{x}_{2,2k})_{k=0}^{\infty}$ where the offer $\hat{p}_{i,t}$ (by i) and the response $\hat{x}_{j,t}$ at any t depends on the entire history.

Basic Definitions and Assumptions For each $i \in N$, define the (constrained) *demand function* $D_i : P \rightarrow X$ by

$$D_i(p) \in \arg \max \{u_i(x) \mid (x - \bar{x}_i) \cdot p \leq 0, x \leq w, x \in X\} \quad (\forall p \in P).$$

Since u_i is continuous and strictly quasi-concave, D_i is a well-defined function. Since u_i is continuous, by the Maximum Theorem, D_i is also continuous. A (constrained) *Walrasian equilibrium* is any pair $(p, (x_1, x_2))$ of a price $p \in P$ and an allocation $(x_1, x_2) \in X^2$ such that $x_i = D_i(p)$ for each i , and $x_1 + x_2 = w$.

Assumption 1 *There exists a Walrasian equilibrium $(p^W, (x_1^W, x_2^W))$; $(x_1^W, x_2^W) \neq (\bar{x}_1, \bar{x}_2)$, and x_i^W is in the interior of X for each $i \in N$.*

Together with strict quasi-concavity, the assumption that $(x_1^W, x_2^W) \neq (\bar{x}_1, \bar{x}_2)$ guarantees that $u_i(x_i^W) > 0$ for each i . In that case, the initial allocation is not Pareto optimal, hence there are gains from trade. The assumption that x_i^W is in the interior of X for each i is made only to make sure that $(p^W, (x_1^W, x_2^W))$ is an “unconstrained” Walrasian equilibrium.

Define the *offer curves* of agents 1 and 2 as

$$OC_1 = \{(u_1(D_1(p)), u_2(w - D_1(p))) \mid p \in P\}$$

and $OC_2 = \{(u_1(w - D_2(p)), u_2(D_2(p))) \mid p \in P\}$, respectively. OC_i is the set of all utility pairs that can be reached by offering a price to agent i , who will then maximize his payoff given the price. In general, given any payoff v_j for an agent j , there might be multiple pairs (v_1, v_2) in OC_j . In that case, if the other agent i is to choose between

these pairs by offering different prices, he will choose the pair with maximum v_i . To formulate this, for each distinct i and j , define function U_i by

$$U_i(v_j) = \max\{v_i \mid (v_1, v_2) \in OC_j\}.$$

Note that the Walrasian payoff-vector $v^W \equiv (u_1(x_1^W), u_2(x_2^W))$ is in $OC_1 \cap OC_2$, and satisfies the equations

$$v_1^W = U_1(v_2^W) \quad \text{and} \quad v_2^W = U_2(v_1^W). \quad (1)$$

That is, the graphs of U_1 and U_2 (which are typically the offer curves OC_2 and OC_1) intersect each other at the Walrasian payoff-vector. For our main result, we will assume that this is the only intersection in the relevant region (see Assumption 4 below).

Throughout the paper, we will make the following assumption, which is satisfied by many economies, such as the Cobb-Douglas economies in the Edgeworth box.

Assumption 2 *For each $i \in N$, U_i is continuous and single-peaked.*

Since U_i is single-peaked and cannot be monotonically increasing, it is maximized at some $v_j^M \in \mathbb{R}$. (Note that U_i is strictly increasing at any $v_j < v_j^M$ and strictly decreasing at any $v_j > v_j^M$.) By a *monopoly price of an agent i* , we will mean any price $p_i^M \in P$ with $u_j(D_j(p_i^M)) = v_j^M$ and $u_i(w - D_j(p_i^M)) = U_i(v_j^M)$.

3 An Example

In this section, using a canonical example, we explain our formulation, and show how the Walrasian outcome typically differs from the SPE outcome in Rubinstein's model in which the agents offer allocations. We then explain why our theorem is true.

Consider a quasi-linear economy $e = ((u_1, (0, 1)), (u_2, (M, 0)))$ with two goods where $u_1(m, y) = m + y^\alpha - 1$, $u_2(m, y) = m + y^\alpha - M$, $\alpha \in (0, 1)$, and $M > 0$. Since we have quasi-linear utility functions, take $X = \mathbb{R} \times \mathbb{R}_+$, allowing negative amounts of the first good — money.

The offer curves are plotted in Figure 1 for $\alpha = 0.5$. Notice that both U_1 and U_2 are single peaked and continuous. The offer curves OC_1 and OC_2 are simply the graphs of U_2 and U_1 , respectively. They intersect each other only at the origin and the Walrasian payoff-vector. Notice that the Walrasian payoffs are very asymmetric.

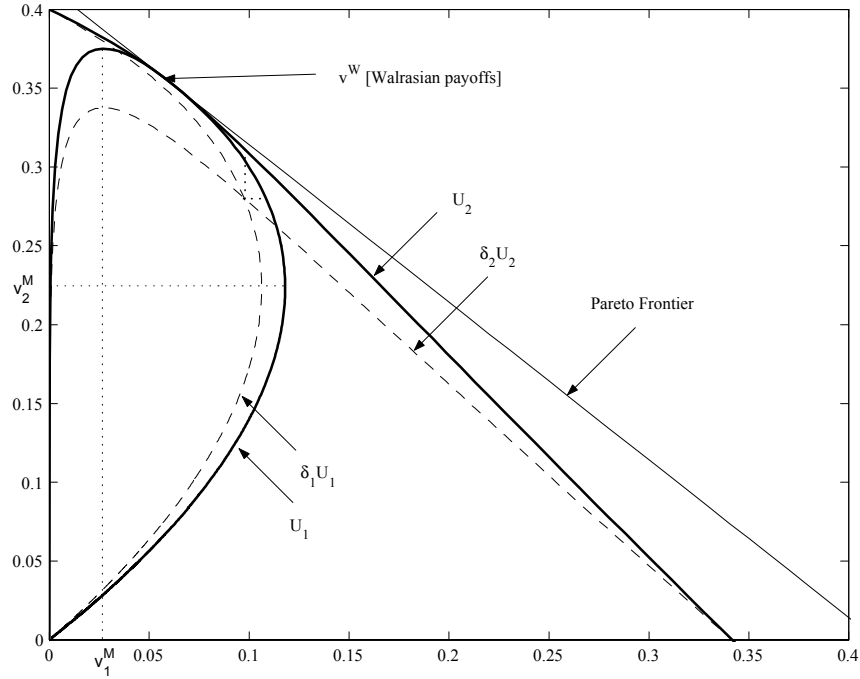


Figure 1: Offer curves for $\alpha = 1/2$, $\delta_1 = \delta_2 = 0.9$.

One can compute that the Walrasian price is $p^W = (1, \alpha 2^{1-\alpha})$, yielding the Walrasian payoff vector

$$v^W = \left(\frac{1 + \alpha}{2^\alpha} - 1, \frac{1 - \alpha}{2^\alpha} \right).$$

Hence, at the Walrasian equilibrium, the ratio of Agent 2's share to Agent 1's share is $(1 - \alpha) / (1 + \alpha - 2^\alpha)$, determined by α .

On the other hand, we are in a transferable-utility case: in any Pareto-optimal allocation, the payoffs add up to $2^{1-\alpha} - 1$. Then, the unique SPE outcome in Rubinstein's model is

$$v^R(\delta_1, \delta_2) = [2^{1-\alpha} - 1] \left(\frac{1 - \delta_2}{1 - \delta_1 \delta_2}, \frac{\delta_2 (1 - \delta_1)}{1 - \delta_1 \delta_2} \right).$$

As $(\delta_1, \delta_2) \rightarrow (1, 1)$ at the rate $r = \log(\delta_1) / \log(\delta_2)$,

$$v^R(\delta_1, \delta_2) \rightarrow [2^{1-\alpha} - 1] \left(\frac{1}{1 + r}, \frac{r}{1 + r} \right).$$

Therefore, in the limit, the ratio of Agent 2's share to Agent 1's share is r , determined solely by the discount rates, or the frequencies at which the agents make offers. This well-known fact implies that, in the limit, the SPE payoffs in Rubinstein's model cannot

possibly be related to the Walrasian payoffs, which are determined by α . In particular, when there is a common discount rate, in the limit, the SPE distributes the gains from trade equally, while the Walrasian payoffs are very asymmetric.

Now consider the bargaining procedure in $G(\delta_1, \delta_2)$. Whenever an agent i accepts a price p_j , he demands the optimal consumption $D_i(p_j)$ — for the game ends there. Thus, our bargaining procedure can be considered as the Rubinstein’s bargaining model in which each agent is restricted to offer payoffs on the other agent’s offer curve. Therefore, as in Rubinstein (1982), there is a SPE that is determined by the intersection $\hat{v}(\delta_1, \delta_2)$ of the graphs of $\delta_1 U_1$ and $\delta_2 U_2$ (see Figure 1). In this SPE, an agent i accepts an offer iff he gets at least $\hat{v}_i(\delta_1, \delta_2)$, and the other agent j offers a price that gives $\hat{v}_i(\delta_1, \delta_2)$ to i and $U_j(\hat{v}_i(\delta_1, \delta_2))$ to j . But v^W is the unique intersection of the graphs of U_1 and U_2 . Therefore, as $(\delta_1, \delta_2) \rightarrow (1, 1)$,

$$\hat{v}(\delta_1, \delta_2) \rightarrow v^W.$$

In the limit, the SPE payoff-vector itself is v^W . This convergence is independent of the rate at which the discount rates go to 1. Therefore this convergence to the Walrasian equilibrium would hold even if the agents made offers at different frequencies. (See Section 6 for a further discussion.)

Notice in Figure 1 that $(v_1^M, U_2(v_1^M))$ and $(U_1(v_2^M), v_2^M)$ are below the graphs of U_1 and U_2 , respectively. That is, if an agent offers his monopoly price, then (for large values of discount rates) the other agent can reject that offer and make a Pareto-improving counter-offer. Under this condition, for large values of discount rates, we further show that there cannot be other SPE. Therefore, when this condition and the unique intersection property of U_1 and U_2 hold, as in this example, all the SPE outcomes converge to the Walrasian outcome.

4 Theorem

In this section, we will describe the SSPE of game $G(\delta_1, \delta_2)$. We will then formally state our sufficient conditions under which the allocations and the prices at all SPE converge to the Walrasian allocation and the price, respectively, as $(\delta_1, \delta_2) \rightarrow (1, 1)$.

Our first lemma describes the basic properties of SSPE.

Lemma 1 *Given any $\delta_1, \delta_2 \in (0, 1)$, any $i \neq j \in N$, any SSPE $(\hat{p}_1, \hat{x}_1, \hat{p}_2, \hat{x}_2)$ of $G(\delta_1, \delta_2)$, under Assumption 2, the following are true:*

1. $\hat{x}_j(p_i) \in \{D_j(p_i), \text{Reject}\}$ for all $p_i \in P$;

2. if $\hat{x}_j(\hat{p}_i) \neq \text{Reject}$, then $u_i(w - D_j(\hat{p}_i)) = U_i(u_j(D_j(\hat{p}_i)))$;
3. if $\hat{x}_j(\hat{p}_i) \neq \text{Reject}$, then $u_j(D_j(\hat{p}_i)) \geq v_j^M$.

Proof. The proofs omitted in the text are in the appendix. ■

Part 1 states that, if an agent accepts a price, he demands his optimal consumption at that price, for the game ends there. Hence, in terms of equilibria, our game is equivalent to a bargaining game where each agent is restricted to offer a payoff vector in the other agent's offer curve. In that case, each agent i offers a point on the graph of U_i —hence the second part. The proof of this part uses the continuity of U_i and the availability of the prices that allow the other agent to demand consumptions better than his equilibrium demand. Part 3 simply states that each agent's offer is at least as generous as his monopoly price.

Our next lemma lists some necessary conditions for a SSPE. This is the main step towards proving our Theorem. The basic argument is the following. Under Assumption 2, if an agent j is offering a price \hat{p}_j that will be accepted and that allows the other agent i to obtain a higher payoff than his continuation value, then \hat{p}_j must be a monopoly price of j . For, otherwise, j would offer a less generous price that would be accepted and would yield a higher payoff for j . This implies that either (i) or (ii) below must hold.

Lemma 2 *Under Assumptions 1 and 2, for any SSPE $(\hat{p}_1, \hat{x}_1, \hat{p}_2, \hat{x}_2)$ of any $G(\delta_1, \delta_2)$, either (i) or (ii) is true:*

(i) **Type 1 equilibria:** for all distinct $i, j \in N$, we have $\hat{x}_i(\hat{p}_j) = D_i(\hat{p}_j)$ and

$$u_i(D_i(\hat{p}_j)) = \delta_i u_i(w - D_j(\hat{p}_i)) = \delta_i U_i(u_j(D_j(\hat{p}_i))); \quad (2)$$

(ii) **Type 2 equilibria:** there exist distinct i and j such that $\hat{x}_i(\hat{p}_j) = D_i(\hat{p}_j)$ and

$$u_i(D_i(\hat{p}_j)) = v_i^M. \quad (3)$$

Type 1 equilibria are similar to the SPE in Rubinstein (1982): each equilibrium-offer is accepted, and the equilibrium offers leave the other agents indifferent between accepting (and demanding the optimal consumption) and rejecting the offer. This indifference yields the equation system (2). At $\delta_1 = \delta_2 = 1$, this equation system is identical to (1), the system of equations satisfied by the Walrasian prices.

In a Type 2 equilibrium, there exists an agent j who always offers his monopoly price, which is accepted by the other agent. The other agent's offer is typically rejected.

This is an equilibrium iff there is no point on the offer curve of j that gives each agent at least the continuation value — the discounted value of payoffs when j is a monopoly. In that case, the other agent (i) cannot offer any price that must be accepted by the sequentially-rational agent j .³

Lemma 3 *Under Assumptions 1 and 2, for $\delta_1, \delta_2 \in (0, 1)$, $G(\delta_1, \delta_2)$ has an equilibrium of type 2 iff there does not exist any distinct i and j and any $p_i \in P$ such that*

$$(\delta_i u_i(w - D_j(p_i)), u_j(D_j(p_i))) > (v_i^M, \delta_j U_j(v_i^M)). \quad (4)$$

In order to rule out the equilibria of type 2, we will assume:

Assumption 3' *For all $i, j \in N$, there exists some $p_i \in P$ that satisfies (4).*

Assumption 3' holds iff $\delta_i U_i(\delta_j U_j(v_i^M)) > v_i^M$. That is, if j offers his monopoly price v_i^M at some t , the other agent i can gain by rejecting the offer and offering at $t + 1$ another price p_i such that j is indifferent between maximizing his payoff at p_i and enjoying his monopoly payoff at $t + 2$. This assumption not only rules out the equilibria of type 2 but also guarantees the existence of an equilibrium type 1:

Theorem 1 *Under assumptions 1 and 2, $G(\delta_1, \delta_2)$ has a SSPE. Moreover, if Assumption 3' also holds, then all SSPE of $G(\delta_1, \delta_2)$ are of type 1.*

We now assume that Assumption 3' holds at $\delta_1 = \delta_2 = 1$; it will hold for all large values of δ_1 and δ_2 by continuity.

Assumption 3 *For all $i, j \in N$, there exists some $p_i \in P$ such that*

$$(u_i(w - D_j(p_i)), u_j(D_j(p_i))) > (v_i^M, U_j(v_i^M)).$$

That is, an agent i can offer a price that is better than the monopoly price of the other agent j for both of them, provided that j maximizes his payoff given the price offered by i . In other words, any monopoly is Pareto-inefficient even under the constraint that the agents trade through prices. This assumption is the most crucial assumption in this paper, and it is used for several purposes. I do not know any set of assumptions on the preferences that imply this assumption. Under Assumptions 2 and 3, there exists some $\tilde{\delta} \in (0, 1)$ such that Assumption 3' holds for all $\delta_1, \delta_2 \in (\tilde{\delta}, 1)$. Together with Theorem 1, this yields:

³In a bargaining model where the agents bargain over trades (rather than prices), an agent can always offer the trade that will take place in the next date, guaranteeing the continuation value to each agent, but this is not true in our model.

Lemma 4 *Under Assumptions 1-3, there exists $\tilde{\delta} \in (0, 1)$ such that, for all $\delta_1, \delta_2 \in (\tilde{\delta}, 1)$, all SSPE of $G(\delta_1, \delta_2)$ are of type 1.*

We now assume that the graphs of U_1 and U_2 has a unique intersection in the relevant region:

Assumption 4 *There exists a unique $(v_1, v_2) \geq (v_1^M, v_2^M)$ such that*

$$U_1(v_2) = v_1 \text{ and } U_2(v_1) = v_2.$$

At any Walrasian equilibrium, each agent i gets at least v_i^M , and the graphs of U_1 and U_2 intersect each other. Therefore, we have the following lemma.

Lemma 5 *Under Assumptions 1 and 4, given any $p_1, p_2 \in P$, we have*

$$U_i(u_j(D_j(p_i))) = u_i(D_i(p_j)) \geq v_i^M \quad (\forall i \neq j \in N)$$

iff $p_1 = p_2 = p^W$.

This gives us our main theorem for SSPE.

Theorem 2 *Under Assumptions 1-4, let $(\hat{p}_1^\delta, \hat{x}_1^\delta, \hat{p}_2^\delta, \hat{x}_2^\delta)$ be a SSPE of games $G(\delta)$ for each $\delta = (\delta_1, \delta_2) \in (0, 1)^2$. Then,*

$$\lim_{\delta \rightarrow (1,1)} \hat{x}_j^\delta(\hat{p}_i^\delta) = x_j^W \quad (5)$$

for each distinct $i, j \in N$. Moreover, if u_1 and u_2 are continuously differentiable, then

$$\lim_{\delta \rightarrow (1,1)} \hat{p}_1^\delta = \lim_{\delta \rightarrow (1,1)} \hat{p}_2^\delta = p^W. \quad (6)$$

That is, as the discount rates approach 1, under Assumptions 1-4, the SSPE allocations converge to the Walrasian allocation. If the utility functions are continuously differentiable, this also implies that the SSPE prices converge to the Walrasian price. (We will later prove stronger results, but a direct proof of this result is given right away, as it is much more transparent than the proofs of the stronger results.)

Proof. Let $A = (D_1(P) \times D_2(P)) \cap \{(x_1, x_2) \in X^2 | \forall i \in N, u_i(x_i) \geq v_i^M\}$. Define the function $\phi : (0, 1)^2 \times A \rightarrow \mathbb{R}^2$ by

$$\phi_i(\delta_1, \delta_2, x_1, x_2) = \delta_i U_i(u_j(x_j)) - u_i(x_i) \quad (i \neq j \in N)$$

and the correspondence $\xi : [0, 1]^2 \rightarrow 2^A$ by

$$\xi(\delta_1, \delta_2) = \{(x_1, x_2) \in A \mid \phi(\delta_1, \delta_2, x_1, x_2) = 0\}.$$

By Lemma 5,

$$\xi(1, 1) = \{(x_1^W, x_2^W)\}.$$

Moreover, since ϕ is continuous, ξ has a closed graph. Since A is compact, this implies that ξ is upper semi-continuous. Hence, given any $\epsilon > 0$, there exists $\hat{\delta} \in (0, 1)$ such that, for each $\delta_1, \delta_2 > \hat{\delta}$, for each $(x_1, x_2) \in \xi(\delta_1, \delta_2)$, and for each $j \in N$, we have

$$\|x_j - x_j^W\| < \epsilon. \quad (7)$$

On the other hand, by Lemma 4, there exists a $\bar{\delta} \in (0, 1)$ such that, for each $\delta = (\delta_1, \delta_2) \in (\bar{\delta}, 1)^2$,

$$(\hat{x}_1^\delta(\hat{p}_2^\delta), \hat{x}_2^\delta(\hat{p}_1^\delta)) \in \xi(\delta_1, \delta_2). \quad (8)$$

Therefore, by (7) and (8), given any $\epsilon > 0$, there exists $\delta^* \geq \max\{\hat{\delta}, \bar{\delta}\}$ such that, for each $\delta = (\delta_1, \delta_2) \in (\delta^*, 1)^2$, we have $\|\hat{x}_j^\delta(\hat{p}_i^\delta) - x_j^W\| < \epsilon$, proving (5).

Towards proving the second part, define $\hat{X} = \{x \in X \mid 0 < x^k < w^k \ \forall k \leq n\}$, the set of consumption bundles corresponding to the interior allocations. For any distinct $i, j \in N$, if u_i is continuously differentiable, then inverse-demand function $D_i^{-1} : \hat{X} \cap D_i(P) \rightarrow P$ exists and is continuous, where $D_i(D_i^{-1}(x_i)) = x_i$ for each $x_i \in \hat{X} \cap D_i(P)$. But, since $\hat{x}_j^\delta(\hat{p}_i^\delta) \rightarrow x_j^W$, by continuity, $\hat{p}_i^\delta = D_i^{-1}(\hat{x}_j^\delta(\hat{p}_i^\delta)) \rightarrow D_i^{-1}(x_j^W) = p^W$. ■

An intuition for Theorem 2 is the following. In equilibrium, Agent 1 must be maximizing his utility at the price set by Agent 2, and Agent 2 must be maximizing his utility at the price set by Agent 1. The markets clear by definition. As the discount rates converge to 1, one would expect that the prices set by different agents will become similar, and each agent will become indifferent between today and tomorrow. That is, approximately, each agent i is indifferent between he himself maximizing his utility at the price set by the other agent j and the other agent j maximizing her payoff at approximately the same price set by i . Therefore, we must be approximately at a Walrasian equilibrium. (The logic of our proof is clearly different from this intuition, for proving convergence to a Walrasian equilibrium turns out to be more straightforward than proving that the prices converge to the same price.)

Our proof utilizes a more general fact about sequential bargaining: if (in equilibrium) we can restrict the agents to make offers from two sets whose Pareto-frontiers have a unique intersection with each other, then all SPE of the form that appears in Rubinstein

(1982) (namely the SPE of type 1) converge to the intersection as the discount rates approach 1. The limit is independent of how the discount rates approach 1. In contrast, in the original model of Rubinstein (1982) these two sets (and therefore their Pareto-frontiers) coincide, hence the limit depends on how the discount rates approach 1.

The crucial assumption behind Theorem 2 is Assumption 3 — that the monopolies are inefficient even under the constraint that one of the agents is a price-taker. Our next example illustrates that this assumption is not superfluous.

Example 1 Consider the economy $e = ((u_1, (9, 1)), (u_2, (1, 9)))$ where $u_1(x, y) = x^\alpha y^{1-\alpha} - 9^\alpha$ and $u_2(x, y) = x^{1-\beta} y^\beta - 9^\beta$. In Figure 2, we plot the offer curves for $\alpha = 0.6$, $\beta = 0.1$, and $\delta_1 = \delta_2 = 0.9$. Notice that $U_1(v_2^M) > \max\{v_1 | U_2(v_1) \geq 0\}$. Hence, for large values of (δ_1, δ_2) , we have a SSPE $(p_1^M, \hat{x}_1, p_2^M, \hat{x}_2)$ where $\hat{x}_1(p) = D_1(p)$ iff $u_1(D_1(p)) \geq \delta_1 U_1(v_2^M)$ and $\hat{x}_2(p) = D_2(p)$ iff $u_2(D_2(p)) \geq \delta_2^2 v_2^M$. (Recall that p_i^M is the monopoly price of i .) In this SSPE, Agent 1 emerges as a monopoly: he offers his monopoly price p_1^M , and it is accepted. There is no price that Agent 1 would accept and that gives positive payoff to Agent 2, so he offers the non-serious price p_2^M , which will be rejected. Clearly, this SSPE does not converge to the Walrasian equilibrium.

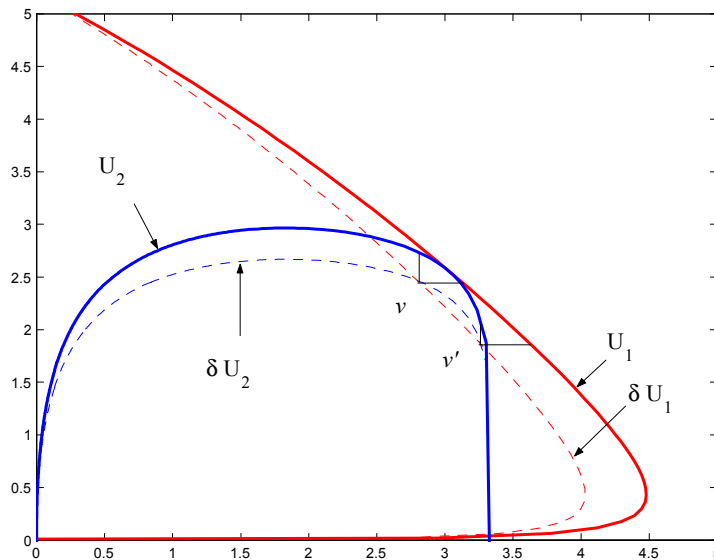


Figure 2: The offer curves for Example 1.

Observe that, for $\delta_1 = \delta_2 = 0.9$, the graphs of $\delta_1 U_1$ and $\delta_2 U_2$ intersect each other at two distinct points v and v' , yielding two more SSPE. The existence of such intersections

depends on δ_1 and δ_2 , but we can find a sequence of discount rates (δ_1, δ_2) converging to $(1, 1)$ for which there are two such SSPE. Both of these equilibria converge to the Walrasian outcome. Since we can construct non-stationary subgame-perfect equilibria using these SSPE, this reveals a weakness of Theorem 2: even if all SSPE converge to the Walrasian equilibrium, we may have non-stationary SPE, which may not converge to the Walrasian equilibrium.

It turns out that Assumptions 3 and 4 rule out this possibility. Under these assumptions, we show in the Appendix that, when U_1 and U_2 are smooth and δ_1 and δ_2 are sufficiently large, $\delta_1 U_1$ and $\delta_2 U_2$ have a unique intersection in the relevant region. Together with Assumption 3, this allows us to extend Shaked-Sutton (1984) argument for the uniqueness of SPE to our model, albeit in a more complicated form.⁴

Theorem 3 *Under Assumptions 1-4, assume that U_1 and U_2 are analytical at the Walrasian payoff vector v^W . Then, there exists some $\bar{\delta} \in (0, 1)$ such that, whenever $\delta_1, \delta_2 \in (\bar{\delta}, 1)$, the game $G(\delta_1, \delta_2)$ has a unique SPE payoff-vector, which is obtained at a SSPE.*

Since the proof of this theorem requires several technical lemmas, it is relegated to the Appendix. For sufficiently large values of discount rates, under Assumptions 1-4 and for smooth offer curves, this theorem establishes the uniqueness of SPE payoffs. These payoffs are obtained at a SSPE. Combining this with Theorem 2, we can conclude that, under the above conditions, as the discount rates approach 1, all SPE outcomes converge to the Walrasian outcome (price and allocation). It turns out that we do not need the smoothness assumptions for convergence, as the next theorem states.

Theorem 4 *Under Assumptions 1-4, for each $\delta = (\delta_1, \delta_2) \in (0, 1)^2$, let $(\hat{p}_{1,2k}^\delta, \hat{x}_{1,2k+1}^\delta, \hat{p}_{2,2k+1}^\delta, \hat{x}_{2,2k}^\delta)_{k=0}^\infty$ be a SPE of game $G(\delta)$. Then,*

$$\lim_{\delta \rightarrow (1,1)} \lim_{j,t} \hat{x}_{j,t}^\delta (\hat{p}_{i,t}^\delta) = x_j^W \quad (9)$$

for each $t \in T$ at which i makes an offer and j responds. Moreover, if u_1 and u_2 are continuously differentiable, then

$$\lim_{\delta \rightarrow (1,1)} \hat{p}_{1,t}^\delta = \lim_{\delta \rightarrow (1,1)} \hat{p}_{2,t'}^\delta = p^W \quad (10)$$

at each even t and odd t' .

⁴The arguments are more complicated in our model, because (in utility space) our agents make offers from two distinct sets, and therefore it is conceivable that an agent gets his highest SPE payoff when his offer is rejected and he is offered a more generous price in the next period.

Proof. It is an immediate corollary to Theorem 6 below. ■

It is crucial for Theorems 3 and 4 that the graphs of U_1 and U_2 have a unique intersection in the relevant region (Assumption 4). Without this assumption, there could have been multiple SSPE converging to some of these intersections. In that case, we would also have non-stationary SPE, which might not converge to any of these intersections (see the next section for extensions beyond this assumption). On the other hand, for Theorem 2, we only need that all these intersections correspond to Walrasian equilibria. The next Example illustrates that this assumption is needed even for Theorem 2, that is, if the graphs of U_1 and U_2 intersect each other at non-Walrasian payoff vectors, the SSPE can converge to non-Walrasian outcomes. This example is taken from Sertel and Yildiz (1994), who use this economy to show that there cannot be an axiomatic bargaining solution that always picks the Walrasian payoff-vectors.

Example 2 (Sertel and Yildiz (1994)) Consider the economy $e = ((u, (0, 10)), (u, (10, 0)))$ where $u(x, y) = (1/45) \min\{24x + 3y + 15, 9x + 18y, 4x + 23y + 5\} - 1$. The indifference curves and the offer curves (in utility space and in the Edgeworth box) are plotted in Figure 3. The indifference curves are tangent to each other only at the strip around the diagonal, when the common slope of the indifference curves is $-1/2$. Therefore, the unique Walrasian price is $p^W = (1, 2)$, yielding a unique Walrasian payoff-vector $v^W = (4, 1)$. In the Edgeworth box, the offer curves of Agent 1 and Agent 2 are the line segments connecting the points $[\bar{x}, a, b, b', c, d]$ and $[\bar{x}, a', b, b', g, h]$, respectively. But in the utility space, the graphs of U_1 and U_2 coincide: $U_2(v_1) = 5 - v_1$ and $U_1(v_2) = 5 - v_2$. Then, the SSPE in this economy is as in Rubinstein (1982): For any $\delta_1 = \delta_2 = \delta \in (0, 1)$, the unique SSPE payoff-vector is $(5/(1 + \delta), 5\delta/(1 + \delta))$. As $\delta \rightarrow 1$, the SSPE payoffs converge to $(5/2, 5/2)$, distinct from the Walrasian payoff-vector $v^W = (4, 1)$. The SSPE prices \hat{p}_1^δ and \hat{p}_2^δ converge to $p_1 = (1, 29/37)$ and $p_2 = (1, 7/11)$, respectively.

5 Extensions

After circulation of an earlier version of this paper that contained the results above, Davila and Eeckhout (2002) have shown that the combination of Assumptions 3 and 4 is too strong: under these two assumptions, by changing the endowments and the preferences, we can find a nearby economy in which U_1 and U_2 have a (nearby) non-Walrasian intersection. In this section, we extend our results beyond Assumption 4.

First, under Assumption 3', for any $\delta_1, \delta_2 \in (0, 1)$ and any $i \neq j \in N$, let $\underline{v}_i(\delta_1, \delta_2)$

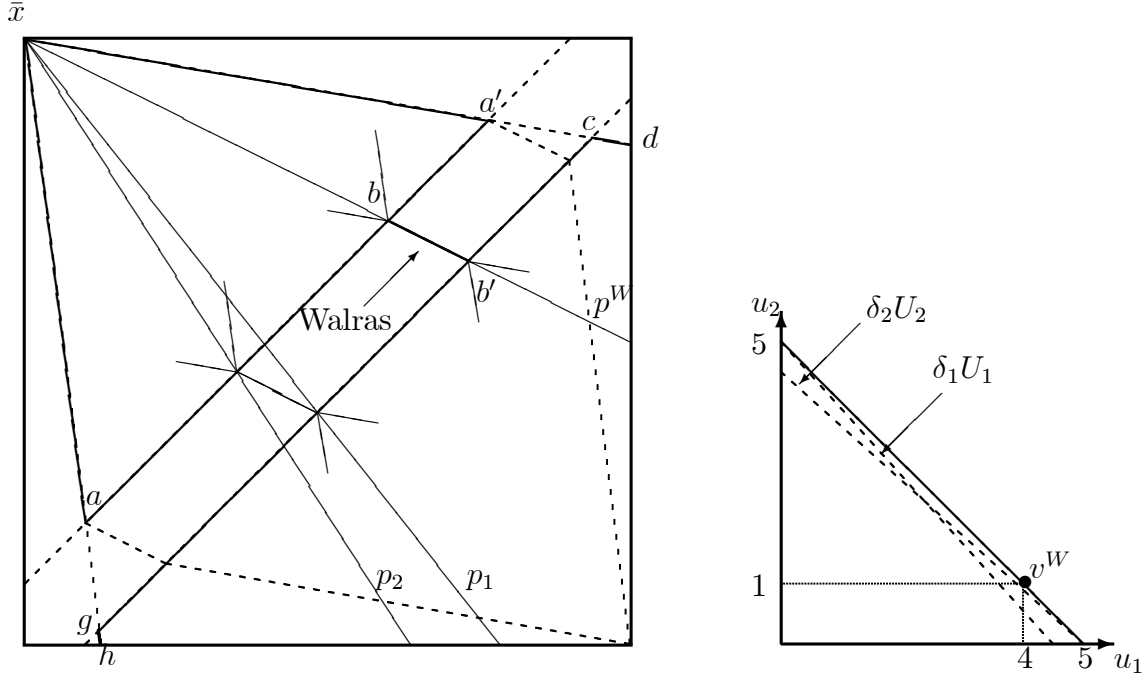


Figure 3: The offer curves for economy e of Example 2 — in the Edgeworth box and in the utility space. (The hyperplanes are indicated by the associated prices.)

and $\bar{v}_i(\delta_1, \delta_2)$ be the i th coordinates of the worst and the best intersections of the graphs of $\delta_1 U_1$ and $\delta_2 U_2$ for agent i in the relevant region, respectively. That is,

$$\underline{v}_i(\delta_1, \delta_2) = \min \{v_i \geq v_i^M \mid \exists v_j \geq v_j^M : v_i = \delta_i U_i(v_j), v_j = \delta_j U_j(v_i)\}, \quad (11)$$

$$\bar{v}_i(\delta_1, \delta_2) = \max \{v_i \geq v_i^M \mid \exists v_j \geq v_j^M : v_i = \delta_i U_i(v_j), v_j = \delta_j U_j(v_i)\}. \quad (12)$$

Note that $\underline{v}_i(\delta_1, \delta_2)$ and $\bar{v}_i(\delta_1, \delta_2)$ are well defined by Assumption 3'; $\underline{v}_i(\delta_1, \delta_2)/\delta_i$ and $\bar{v}_i(\delta_1, \delta_2)/\delta_i$ are the lowest and the highest SSPE payoffs i expects when he makes an offer, and finally $\bar{v}_i(\delta_1, \delta_2) = \delta_i U_i(\underline{v}_j(\delta_1, \delta_2))$ for $i \neq j$. Our first result states that, under Assumption 3, the extremal SPE payoffs are obtained in SSPE, and hence the expected payoffs from all SPE are contained in the rectangle generated by the SSPE.

Theorem 5 *Under Assumptions 1, 2, and 3', for any $\delta_1, \delta_2 \in (0, 1)$ and any $i \in N$, the lowest and the highest expected SPE payoffs of i at the beginning of any date at which i makes an offer are*

$$m_i \equiv \underline{v}_i(\delta_1, \delta_2)/\delta_i \text{ and } M_i \equiv \bar{v}_i(\delta_1, \delta_2)/\delta_i, \quad (13)$$

respectively.

Since the limit of SSPE payoffs must be an intersection of the offer curves, this yields bounds for the limit of the SPE payoffs.

Theorem 6 *Under Assumptions 1-3, for each distinct $i, j \in N$, let*

$$\underline{v}_i = \min \{v_i \geq v_i^M \mid \exists v_j \geq v_j^M : v_i = U_i(v_j), v_j = U_j(v_i)\}$$

and $\bar{v}_i = U_i(\underline{v}_j)$. For each $\delta \in (0, 1)^2$ and $i \in N$, let v_i^δ be a SPE payoff for i in game $G(\delta)$, such that $\lim_{\delta \rightarrow (1,1)} v_i^\delta = v_i^*$ for some v_i^* . Then,

$$v_i^* \geq \underline{v}_i. \quad (14)$$

Moreover, for any Walrasian payoff vector (v_1^W, v_2^W) and any $\epsilon > 0$, there exists $\bar{\delta} \in (0, 1)$ such that for all $\delta_1, \delta_2 \in (\bar{\delta}, 1)$, for all $i \in N$, and for all SPE payoff v_i of i in game $G(\delta_1, \delta_2)$, we have

$$|v_i - v_i^W| < \epsilon + \max \{\bar{v}_i - v_i^W, v_i^W - \underline{v}_i\}. \quad (15)$$

That is, all the limiting SPE payoffs are located in the smallest rectangle that includes all the intersections of the graphs of U_1 and U_2 . If there are intersections that are far apart, as in the Rubinstein's model, this theorem does not have much predictive power. But if all the intersections are located around a unique Walrasian payoff vector, then all SPE payoffs will be close to the Walrasian payoffs for high values of discount rates. Therefore, existence of nearby non-Walrasian intersections does not weaken our results much. Finally, under Assumption 4, by (15), all SPE converge to the Walrasian equilibrium.

6 Comparative Statics

In this section we will show that under our assumptions, if an agent gets more patient, then in equilibrium, he will be better off and the other agent will be worse off.

Theorem 7 *For any distinct i and j , and any $\delta_1, \delta_1', \delta_2, \delta_2'$ with $\delta_j = \delta_j'$, under Assumptions 1 and 2, let (v_1, v_2) and (v_1', v_2') be SSPE payoffs for $G(\delta_1, \delta_2)$ and $G(\delta_1', \delta_2')$, respectively. Assume that $G(\delta_1, \delta_2)$ has a unique SSPE and Assumption 3' is satisfied throughout. Then,*

$$\begin{aligned} v_i' &> v_i \text{ and } v_j' < v_j && \text{whenever } \delta_i' > \delta_i, \text{ and} \\ v_i' &< v_i \text{ and } v_j' > v_j && \text{whenever } \delta_i' < \delta_i. \end{aligned} \quad (16)$$

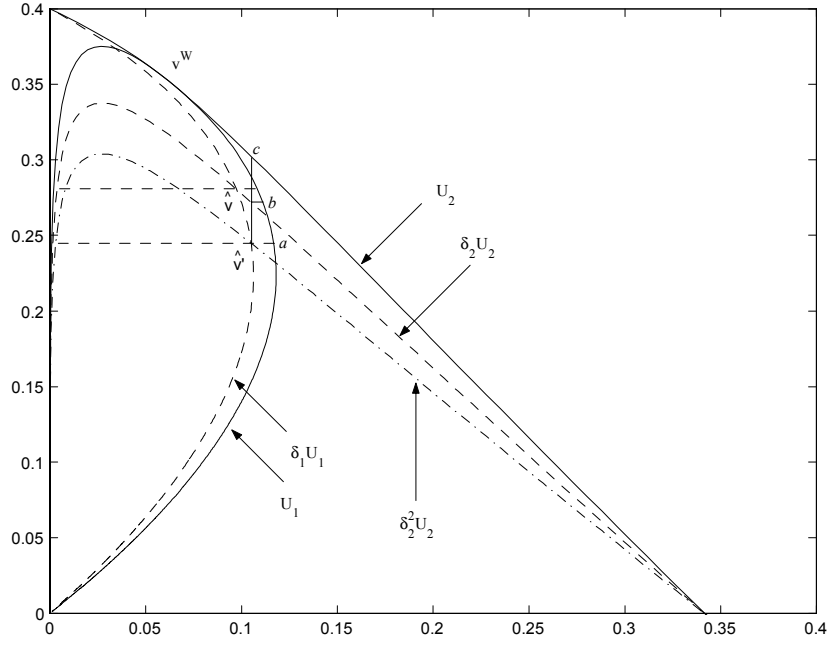


Figure 4: Agent 2 is hurt by his impatience. ($\delta_1 = \delta_2 = 0.9$.)

Theorem 7 is illustrated in Figure 4. Consider the economy in Section 3. The graphs of $\delta_1 U_1$ and $\delta_2 U_2$ have a unique intersection \hat{v} . Decrease δ_2 to δ_2^2 . In the region to the left of \hat{v} , $\delta_2 U_2$ lies below $\delta_1 U_1$. Hence, all the new intersections must be to the right of \hat{v} . Since $\delta_1 U_1$ is decreasing in this region, they must be located below \hat{v} . Then, the change hurts Agent 2 and benefits Agent 1.

In this result, Assumption 3' plays two roles. Firstly, it guarantees that the equilibria are of type 1; the allocations in type 2 equilibria do not depend on the discount rates. Second, combined with the uniqueness of SPE in the original economy, it guarantees that the graph of $\delta_2 U_2$ intersects the graph of $\delta_1 U_1$ from below—as in Figure 4. To see why we need this, consider Figure 2. If we decrease δ_2 slightly, v and v' move in opposite directions. In the equilibrium corresponding to v (where $\delta_2 U_2$ intersects $\delta_1 U_1$ from below), this change makes Agent 1 better off and Agent 2 worse off. In the SPE corresponding to v' (where $\delta_2 U_2$ intersects $\delta_1 U_1$ from above), Agent 2 gains from his impatience, which hurts his opponent.

Figure 4 also illustrates that we do have a proposer advantage in our model. To see this, modify the bargaining procedure so that Agent 1 makes two offers in a row and Agent 2 makes only one offer. (The order of proposers is 112112112. . . .) The SPE under

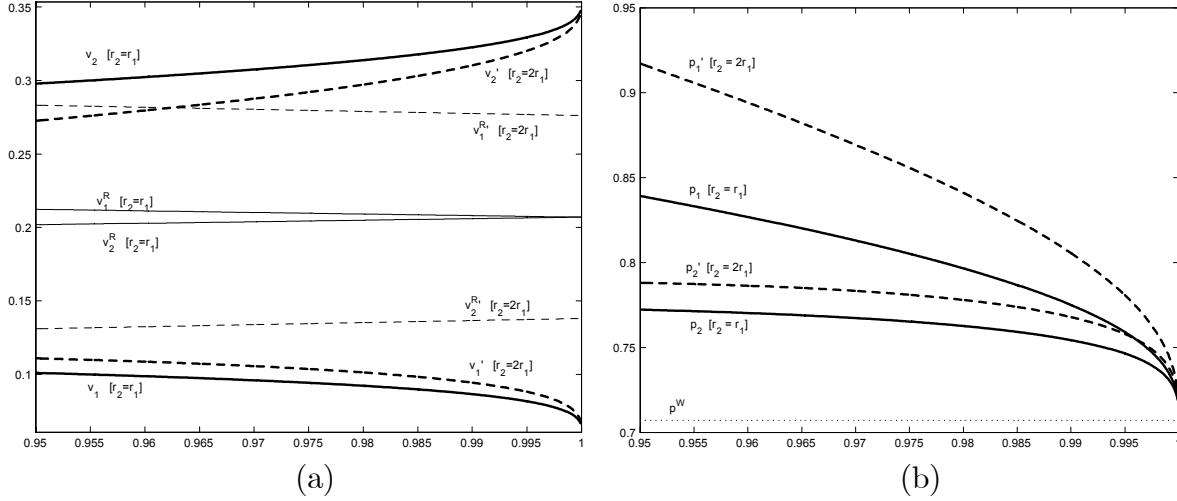


Figure 5: The SPE payoffs (a) and the prices (b) as functions of δ_1 . The superscript R and the prime signify Rubinstein's model and $r_2 = 2r_1$, respectively. (In (a), the Walrasian payoffs of the Agents 1 and 2 are located at the lower and the upper ends of the graph, respectively.)

the new bargaining procedure can be obtained from the SPE when the discount rate of Agent 2 is δ_2^2 .⁵ In the utility space, Agent 1 first offers a , and then offers b , while Agent 2 offers c ; any less generous offer is rejected. Under the new procedure, Agent 1 is better off, and Agent 2 is worse off.

In summary, the time preferences and the relative frequency of the offers do affect the bargaining outcome in the predictable way, even though all these are irrelevant to the limit as the discount rates approach 1 or as the real-time delay between the consecutive offers vanishes.

When we are away from the limit, the effect is significant: Consider the economy in Section 3, and let $\delta_1 = e^{-r_1\Delta}$ and $\delta_2 = e^{-r_2\Delta}$ with the understanding that the index time is a grid in a continuum of real time. We are interested in how the equilibrium is affected for various values of Δ when we double r_2 . Recall that doubling r_2 represents either making Agent 2 more impatient, or letting Agent 1 make two offers in a row (when Δ is doubled for Agent 2). Take $r_1 = r_2$. As exhibited in Figure 6(a), in Rubinstein's model with unrestricted offers, the agents share the surplus equally, except for a relatively small first-mover advantage that vanishes as $\Delta \rightarrow 0$. Doubling r_2 changes the outcome dramatically: now Agent 1 gets two-thirds of the surplus, in addition to the vanishing

⁵This also illustrates how our theorems can be extended beyond alternating offers.

first-mover advantage. Now consider our model. The outcome is in between the equal split and the Walrasian payoff-vector, in which Agent 2 gets a disproportionately higher share. In equilibrium the price offered by each agent for the good owned by Agent 1 is higher than the competitive price —as illustrated in Figure 6(b). Doubling r_2 benefits Agent 1 and hurts Agent 2, as the good Agent 1 owns becomes even more expensive. When Δ is away from 0, the effect is significant, even though it is not as dramatic as in the Rubinstein’s model. As $\Delta \rightarrow 0$, all these prices and the payoff vectors converge to the Walrasian price and payoff-vector, respectively. Hence, the impact of doubling r_2 monotonically decreases, and disappears in the limit.

7 Conclusion

Consistent with common sense, the SPE in usual bargaining models yield the Walrasian allocation in a large economy (Gale (1986)), while they typically yield non-Walrasian outcomes when there are only limited number of agents. In these bargaining models, agents are allowed to offer any trade among the bargaining parties. Here, we present a simple bargaining procedure where the agents are restricted to offer prices, while their trading partners optimize at these prices. As the discount rates go to 1, under certain conditions, the SPE of this mechanism yields the Walrasian equilibrium. Therefore, the Walrasian equilibrium does not necessarily require a large economy. It simply corresponds to price-taking behavior, which can be achieved even with only two agents.

A Omitted Proofs

Proof of Lemma 1. (Part 1) If $\hat{x}_j(p_i) \neq \text{Reject}$, then we are in a final decision node, hence we must have $\hat{x}_j(p_i) = D_j(p_i)$.

Claim: Assume that $\hat{x}_j(\hat{p}_i) = D_j(\hat{p}_i)$. Then, there exists $\tilde{p}_i \in P$ such that $u_j(D_j(\hat{p}_i)) < u_j(D_j(\tilde{p}_i))$.

Proof of Claim. If $u_j(D_j(\hat{p}_i)) = 0$, $\tilde{p}_i = p^W$ fits the bill. Assume that $u_j(D_j(\hat{p}_i)) > 0$. Hence, $D_j(\hat{p}_i) \neq \bar{x}_j$, and $w - D_j(\hat{p}_i) \neq \bar{x}_i$. By the Separating Hyperplane Theorem, there exists a price \tilde{p}_i such that $D_i(\tilde{p}_i) = \bar{x}_i$. For any $x \neq \bar{x}_i$ with $\tilde{p}_i \cdot (x - \bar{x}_i) \leq 0$, we have $u_i(x) < 0$. But $u_i(w - D_j(\hat{p}_i)) \geq 0$ (for i offers \hat{p}_i). Hence, this implies that $0 < \tilde{p}_i \cdot (w - D_j(\hat{p}_i) - \bar{x}_i) = \tilde{p}_i \cdot (\bar{x}_j - D_j(\hat{p}_i))$, i.e., $\tilde{p}_i \cdot (D_j(\hat{p}_i) - \bar{x}_j) < 0$. Since u_j is increasing, this yields $u_j(D_j(\hat{p}_i)) < u_j(D_j(\tilde{p}_i))$.

(Part 2) Assume that $\hat{x}_j(\hat{p}_i) \neq \text{Reject}$, i.e., $\hat{x}_j(\hat{p}_i) = D_j(\hat{p}_i)$. But, by stationarity, the continuation values do not depend on which offers are rejected, hence, for any p_i with

$u_j(D_j(p_i)) > u_j(D_j(\hat{p}_i))$, we must have $\hat{x}_j(p_i) \neq \text{Reject}$, and thus $\hat{x}_j(p_i) = D_j(p_i)$. Suppose that $u_i(w - D_j(\hat{p}_i)) < U_i(u_j(D_j(\hat{p}_i)))$. Since U_i is continuous, by Claim above, there then exists some $\epsilon > 0$ and some price p' such that $U_i(u_j(D_j(\hat{p}_i)) + \epsilon) > u_i(w - D_j(\hat{p}_i))$ and $u_j(D_j(p')) = u_j(D_j(\hat{p}_i)) + \epsilon$. Now, if i offers p' , it will be accepted, yielding higher payoff $U_i(u_j(D_j(\hat{p}_i)) + \epsilon)$, a contradiction.

(Part 3) Suppose that $\hat{x}_j(\hat{p}_i) \neq \text{Reject}$ and $u_j(D_j(\hat{p}_i)) < v_j^M \equiv u_j(D_j(p_i^M))$. Then, by stationarity, $\hat{x}_j(p_i^M) \neq \text{Reject}$, i.e., $\hat{x}_j(p_i^M) = D_j(p_i^M)$. Thus, offering p_i^M is a profitable deviation for i , a contradiction. ■

Proof of Lemma 2. There are two cases.

Case 1: Assume that $\hat{x}_1(\hat{p}_2) \neq \text{Reject} \neq \hat{x}_2(\hat{p}_1)$. Then, by Lemma 1.2, for each distinct $i, j \in N$, we have $u_i(w - D_j(\hat{p}_i)) = U_i(u_j(D_j(\hat{p}_i)))$. Since $\hat{x}_j(\hat{p}_i) \neq \text{Reject}$, it follows that the continuation value of i at the beginning of any date $t + 1$ at which he makes an offer is $U_i(u_j(D_j(\hat{p}_i)))$. This has two implications. First, since \hat{p}_j is accepted,

$$u_i(D_i(\hat{p}_j)) \geq \delta_i U_i(u_i(D_i(\hat{p}_j))). \quad (17)$$

Second, at t , for any p_j , if $u_i(D_i(p_j)) > \delta_i U_i(u_j(D_j(\hat{p}_i)))$, player i must accept the price p_j and demand $D_i(p_j)$ (see Lemma 1.1). Since j offers \hat{p}_j , it must be true that

$$U_j(u_i(D_i(\hat{p}_j))) \geq U_j(v_i) \quad (18)$$

for each $v_i > \delta_i U_i(u_j(D_j(\hat{p}_i)))$. Now, assume that (i) is not true. Then, by (17), $u_i(D_i(\hat{p}_j)) > \delta_i U_i(u_i(D_i(\hat{p}_j)))$. Hence, by (18), $U_j(u_i(D_i(\hat{p}_j))) \geq U_j(v_i)$ for each v_i with $v_i > \delta_i U_i(u_i(D_i(\hat{p}_j)))$. Since $u_i(D_i(\hat{p}_j)) > \delta_i U_i(u_i(D_i(\hat{p}_j)))$, this implies that U_j has a local maximum at $u_i(D_i(\hat{p}_j))$. Thus, by Assumption 2, $u_i(D_i(\hat{p}_j)) = v_i^M$.

Case 2: Assume that $\hat{x}_j(\hat{p}_i) = \text{Reject}$ for some $i, j \in N$. If we also had $\hat{x}_i(\hat{p}_j) = \text{Reject}$, agents would never reach an agreement, thus their continuation values would be zero. Since the Walrasian payoffs are strictly positive (by Assumption 1), this would yield a contradiction: In that case, in a stationary equilibrium each agent must accept the Walrasian price p^W , providing a profitable deviation (p^W) for the agent who makes an offer. Therefore, $\hat{x}_i(\hat{p}_j) \neq \text{Reject} = \hat{x}_j(\hat{p}_i)$. Then, $\hat{x}_i(p_j) = D_i(p_j)$ if $u_i(D_i(p_j)) > \delta_i^2 u_i(D_i(\hat{p}_j))$. Since j offers \hat{p}_j , if $u_i(D_i(\hat{p}_j)) = 0$, then $U_j(v_i) \leq U_j(0)$ for each $v_i > 0$; i.e., $v_i^M = 0 = u_i(D_i(\hat{p}_j))$. On the other hand, if $u_i(D_i(\hat{p}_j)) > 0$, then $u_i(D_i(\hat{p}_j)) > \delta_i^2 u_i(D_i(\hat{p}_j))$. As in Case 1, this yields $u_i(D_i(\hat{p}_j)) = v_i^M$. ■

Proof of Lemma 3. Assume there exists such $p_i \in P$. Then, if i offers p_i , it will be accepted, yielding a payoff higher than v_i^M/δ_i . Then, at the previous date, i should not accept \hat{p}_j , which gives him only v_i^M . ■

Proof of Theorem 1. The second statement is an immediate corollary to Lemma 3 and Assumption 3'; we prove the first statement. If Assumption 3' does not hold, we have an equilibrium of type 2 by Lemma 3. Assume Assumption 3' holds. Write $v_1 = \delta_1 U_1(\delta_2 U_2(v_1^M))$,

$v'_1 = \delta_1 U_1(v_2^M)$, and write h for the inverse of the restriction of $\delta_1 U_1$ to $[v_2^M, \infty)$. By Assumption 3', $v_1 > v_1^M$, hence $h(v_1) \equiv \delta_2 U_2(v_1^M) > \delta_2 U_2(v_1)$. Also, by Assumption 3', $h(v'_1) \equiv v_2^M < \delta_2 U_2(\delta_1 U_1(v_2^M)) = \delta_2 U_2(v'_1)$. Since h and $\delta_2 U_2$ are continuous on a convex domain, by the intermediate-value theorem, the graph of $\delta_2 U_2$ intersects the graph of h , which is a part of the graph of $\delta_1 U_1$, at some \hat{v} . That is, (2) is satisfied at some (\hat{p}_1, \hat{p}_2) . Then, $(\hat{p}_1, \hat{x}_1, \hat{p}_2, \hat{x}_2)$ is a SSPE, where $\hat{x}_i(p) = D_i(p)$ iff $u_i(D_i(p)) \geq \hat{v}_i$ for each $i \in N$. ■

In the sequel we prove Theorems 3 and 7. First a technical lemma:

Lemma 6 For any $[\underline{v}, \bar{v}] \subset \mathbb{R}$, define $f : [0, 1] \times [\underline{v}, \bar{v}] \rightarrow \mathbb{R}$ by

$$f(\delta, v) = \delta g(v) - h(v)$$

where $g : [\underline{v}, \bar{v}] \rightarrow \mathbb{R}$ and $h : [\underline{v}, \bar{v}] \rightarrow \mathbb{R}$ are any two positive, decreasing, and analytical functions with $g(\underline{v}) < h(\underline{v})$ and $g(\bar{v}) > h(\bar{v})$. Assume that $f(\delta, v) = 0$ has a unique solution $\tilde{v}(1)$ at $\delta = 1$. Then, there exists a $\hat{\delta} \in (0, 1)$ such that $f(\delta, v) = 0$ has a unique solution $\tilde{v}(\delta)$ at each $\delta > \hat{\delta}$.

Proof. Firstly, given any $v \in [\tilde{v}(1), \bar{v}]$, since $f(1, v) > 0 > f(0, v)$ and f is continuous in δ , $f(\delta, v) = 0$ has a solution $\tilde{\delta}(v)$. Since $\partial f / \partial \delta = g > 0$, $\tilde{\delta}(v)$ is unique. Second, since $\tilde{v}(1)$ is the unique solution to $f(1, v) = 0$, we must have $\tilde{\delta}(v) < 1$ at each $v > \tilde{v}(1)$. Since $\tilde{\delta}$ is analytical (by the implicit function theorem), this implies that $\tilde{\delta}$ is decreasing on $[\tilde{v}(1), \tilde{v}(1) + \epsilon]$ for some $\epsilon > 0$. Letting LM be the set of all local maxima of $\tilde{\delta}$, set

$$\hat{\delta} = \sup_{v \in LM \setminus \{\tilde{v}(1)\}} \tilde{\delta}(v).$$

Note that $LM \setminus \{\tilde{v}(1)\} \subset [\tilde{v}(1) + \epsilon, \bar{v}]$. This implies that $\hat{\delta} < 1$, for $\tilde{\delta}$ is continuous, $[\tilde{v}(1) + \epsilon, \bar{v}]$ is compact, and $\tilde{\delta}(v) < 1$ at each $v \in [\tilde{v}(1) + \epsilon, \bar{v}]$. Now, by construction, $\tilde{\delta}$ is decreasing on $\{v | \tilde{\delta}(v) > \hat{\delta}\}$, hence there exists a unique solution $\tilde{v}(\delta)$ to $f(\delta, v) = 0$ at each $\delta > \hat{\delta}$, where $\tilde{\delta}(\tilde{v}(\delta)) = \delta$. ■

Lemma 7 Under Assumptions 1-4, assume that U_1 and U_2 are analytical at the Walrasian payoff vector v^W . Then, there exists some $\bar{\delta} \in (0, 1)$ such that, whenever $\delta_1, \delta_2 \in (\bar{\delta}, 1)$, the equation system

$$v_i = \delta_i U_i(v_j) \quad (i \neq j \in N)$$

possesses a unique solution $\hat{v}(\delta_1, \delta_2)$ with $\hat{v}(\delta_1, \delta_2) > (v_1^M, v_2^M)$.

Proof. Without loss of generality, assume that U_i is analytical on some interval $[\underline{v}_i, \bar{v}_i] \subset (v_i^M, U_j(v_i^M))$ for each $i \neq j$ where $\underline{v}_i < v^W < \bar{v}_i$. We want to show that the graphs of $\delta_1 U_1$ and $\delta_2 U_2$ have a unique intersection in a neighborhood of v^W whenever (δ_1, δ_2) is in a

neighborhood of $(1, 1)$. To this end, we will apply Lemma 6 twice. First, set $g = U_1$ and $h = \left(U_{2|v_1^M, \infty} \right)^{-1}$. By Assumptions 2, 3 and 4, $g(\underline{v}_2) < h(\underline{v}_2)$ and $g(\bar{v}_2) > h(\bar{v}_2)$. Then, by Lemma 6, there exists a $\hat{\delta}_1 \in (v_1^W/\bar{v}_1, 1)$ such that the graph of $\delta_1 U_1$ intersects the graph of U_2 uniquely in $[\underline{v}_1, \bar{v}_1] \times [\underline{v}_2, \bar{v}_2]$ whenever $\delta_1 > \hat{\delta}_1$. Take any $\delta_1 > \hat{\delta}_1$. In Lemma 6, now set $g = U_2$ and $h = \left(U_{1|v_2^M, \infty} \right)^{-1}$. Take $[\underline{v}, \bar{v}] = [\underline{v}_1, \hat{\delta}_1 \bar{v}_1]$. (Together with Assumptions 2 and 3, Lemma 6 already implies that $g(\underline{v}_1) < h(\underline{v}_1)$ and $g(\delta_1 \bar{v}_1) > h(\delta_1 \bar{v}_1)$.) Then, by Lemma 6, there exists $\hat{\delta}_2(\delta_1) < 1$ such that the graph of $\delta_1 U_1$ intersects the graph of $\delta_2 U_2$ uniquely in $[\underline{v}_1, \hat{\delta}_1 \bar{v}_1] \times [\underline{v}_2, \bar{v}_2]$ whenever $\delta_2 > \hat{\delta}_2(\delta_1)$. By construction $\hat{\delta}_2(\delta_1)$ is continuous in δ_1 . Therefore, there exists a neighborhood η_1 of $(1, 1)$ such that $\delta_1 U_1$ intersects the graph of $\delta_2 U_2$ uniquely in $[\underline{v}_1, \hat{\delta}_1 \bar{v}_1] \times [\underline{v}_2, \bar{v}_2]$ whenever $(\delta_1, \delta_2) \in \eta_1$. Of course, by upper-semicontinuity and Assumption 4, there is also a neighborhood η_2 of $(1, 1)$ such that all the intersections in $(v_1^M, \infty) \times (v_2^M, \infty)$ are in $[\underline{v}_1, \hat{\delta}_1 \bar{v}_1] \times [\underline{v}_2, \bar{v}_2]$ whenever $(\delta_1, \delta_2) \in \eta_2$. Therefore, $\delta_1 U_1$ and $\delta_2 U_2$ have a unique intersection in $(v_1^M, \infty) \times (v_2^M, \infty)$ whenever $(\delta_1, \delta_2) \in \eta_1 \cap \eta_2$. ■

Lemmas 4 and 7 imply the following.

Lemma 8 *Under the assumptions and the notation of Lemma 7, given any discount rates $\delta_1, \delta_2 \in (\bar{\delta}, 1)$, there exists a unique SSPE payoff-vector, in which the proposer i gets $\hat{v}_i(\delta_1, \delta_2)/\delta_i$ and $j \neq i$ gets $\hat{v}_j(\delta_1, \delta_2)$.*

For each $i \neq j \in N$, define a function \bar{U}_i through

$$\bar{U}_i(v_j) = \begin{cases} U_i(v_j) & \text{if } v_j \geq v_j^M \\ U_i(v_j^M) & \text{otherwise.} \end{cases}$$

Recall also the definitions of $\underline{v}_i(\delta_1, \delta_2)$, $\bar{v}_i(\delta_1, \delta_2)$, \underline{v}_i , \bar{v}_i , m_i and M_i given by (11)-(13) and Theorem 6.

Lemma 9 *Under Assumptions 1, 2, and 3', for all distinct i and j , we have*

1. $m_i \geq \bar{U}_i(\delta_j M_j)$;
2. $M_i \leq \bar{U}_i(\delta_j m_j)$;
3. $m_i \geq \bar{U}_i(\delta_j \bar{U}_j(\delta_i m_i))$, and
4. $M_i \leq \bar{U}_i(\delta_j \bar{U}_j(\delta_i M_i))$.

Proof. (Part 1) In any SPE, if j rejects an offer he will get maximum $\delta_j M_j$. Hence, he will accept any price p with $u_j(D_j(p)) > \delta_j M_j$. Then, by Assumption 2, player i must be getting at least $\bar{U}_i(\delta_j M_j)$ — see the proof of Lemma 1.2.

(Part 2) Consider any SPE in which the continuation value of i is $M_i \leq v_i^M$ at some history at which i makes an offer. Since j expects to get at least m_j the next day if he rejects the offer now, he will reject any offer p with $u_j(D_j(p)) < \delta_j m_j$. Hence, i cannot get more than $\bar{U}_i(\delta_j m_j)$ when his offer is accepted. If his continuation value M_i is more than $\bar{U}_i(\delta_j m_j)$, it must be that his offer is rejected and j offers in some future period a price that gives at least M_i/δ_i to player i . But i would accept any offer that gives i more than $\delta_i M_i$, which is less than M_i/δ_i . In order this to be an equilibrium, we must have $M_i \leq v_i^M$. But this is a contradiction, because $M_i \geq \hat{v}_i/\delta_i > v_i^M$ by Theorem 1, where \hat{v}_i is defined in the proof of Theorem 1.

(Part 3) By part 2, $M_j \leq \bar{U}_j(\delta_j m_i)$. Hence, by part 1, $m_i \geq \bar{U}_i(\delta_j M_j) \geq \bar{U}_i(\delta_j \bar{U}_j(\delta_j m_i))$. The last inequality is due to the fact that \bar{U}_i is non-increasing. The proof of part 4 is similar. ■

Lemma 10 *Under Assumptions 2 and 3', for each distinct $i, j \in N$, we have*

$$\bar{U}_i(\delta_j \bar{U}_j(\delta_i v_i)) > v_i \quad \text{whenever } v_i < \underline{v}_i(\delta_1, \delta_2)/\delta_i, \text{ and} \quad (19)$$

$$\bar{U}_i(\delta_j \bar{U}_j(\delta_i v_i)) < v_i \quad \text{whenever } v_i > \bar{v}_i(\delta_1, \delta_2)/\delta_i. \quad (20)$$

Proof. Take any $v_i < \underline{v}_i(\delta_1, \delta_2)/\delta_i$. Since \bar{U}_j is non-increasing, we have $\delta_j \bar{v}_j(\delta_1, \delta_2) = \delta_j \bar{U}_j(\underline{v}_i(\delta_1, \delta_2)) \leq \delta_j \bar{U}_j(\delta_i v_i) \leq \delta_j \bar{U}_j(v_i^M)$. By Assumption 3', the domain of \bar{U}_i contains some $v_j > \delta_j \bar{U}_j(v_i^M)$. Hence, $\delta_j \bar{U}_j(\delta_i v_i)$ is in the domain of \bar{U}_i . Moreover, by Assumptions 2 and 3', the graph of $\delta_i \bar{U}_i$ is above that of $\delta_j \bar{U}_j$ in this region, i.e., $\delta_i \bar{U}_i(\delta_j \bar{U}_j(\delta_i v_i)) > \delta_i v_i$, showing the first part. The proof of the second part is similar. ■

Proof of Theorem 5. By Lemma 9.3 and (19), $m_i \geq \underline{v}_i(\delta_1, \delta_2)/\delta_i$. But, by definition of $\underline{v}_i(\delta_1, \delta_2)$, there exists a SSPE in which i gets $\underline{v}_i(\delta_1, \delta_2)/\delta_i$, yielding $\underline{v}_i(\delta_1, \delta_2)/\delta_i \geq m_i$. Therefore, $m_i = \underline{v}_i(\delta_1, \delta_2)/\delta_i$. Similarly, $M_i = \bar{v}_i(\delta_1, \delta_2)/\delta_i$. ■

Proof of Theorem 3. By Lemma 7, we have $\underline{v}_i(\delta_1, \delta_2) = \bar{v}_i(\delta_1, \delta_2)$, yielding $m_i = M_i$ by Theorem 5. ■

Proof of Theorem 6. Under Assumption 3, there exists $\tilde{\delta} \in (0, 1)$ such that Assumption 3' holds whenever $(\delta_1, \delta_2) \in (\tilde{\delta}, 1)^2$. Hence, by Theorem 5, for any $\delta \in (\tilde{\delta}, 1)^2$, $v_i^\delta \geq \underline{v}_i$, yielding (14). To prove the second statement, define the correspondence ξ by

$$\xi(\delta_1, \delta_2) = \{(v_1, v_2) \geq (v_1^M, v_2^M) \mid v_1 = \delta_1 U_1(v_2), v_2 = \delta_2 U_2(v_1)\}.$$

Write P_i for the projection operator to the i th coordinate. By Assumption 2, ξ is upper semi-continuous, and P_i is continuous. Hence, for any $\epsilon > 0$, there exists $\bar{\delta} \geq \tilde{\delta}$ such that, for all $(\delta_1, \delta_2) \in (\bar{\delta}, 1)^2$,

$$\underline{v}_i(\delta_1, \delta_2) = \min P_i(\xi(\delta_1, \delta_2)) > \min P_i(\xi(1, 1)) - \epsilon = \underline{v}_i - \epsilon$$

and

$$\bar{v}_i(\delta_1, \delta_2) = \max P_i(\xi(\delta_1, \delta_2)) < \delta_i(\max P_i(\xi(1, 1)) + \epsilon) = \delta_i(\bar{v}_i + \epsilon).$$

By Theorem 5, for any such (δ_1, δ_2) , we have $v_i \in [\underline{v}_i(\delta_1, \delta_2)/\delta_i, \bar{v}_i(\delta_1, \delta_2)/\delta_i] \subset (\underline{v}_i - \epsilon, \bar{v}_i + \epsilon)$. Since $v_i^M \in [\underline{v}_i, \bar{v}_i]$, this yields (15). ■

Proof of Theorem 7. We prove (16). By Theorem 1, $(v_1, v_2) = (U_1(\hat{v}_2(\delta_1, \delta_2)), \hat{v}_2(\delta_1, \delta_2))$ and $(v'_1, v'_2) = (U_1(\hat{v}_2(\delta'_1, \delta'_2)), \hat{v}_2(\delta'_1, \delta'_2))$ where $\hat{v}(\delta_1, \delta_2)$ (resp. $\hat{v}(\delta'_1, \delta'_2)$) is an intersection of the graphs of $\delta_1 U_1$ and $\delta_2 U_2$ (resp. $\delta'_1 U_1$ and $\delta'_2 U_2$). Let h be the inverse of the restriction of $\delta_j U_j$ to $[v_i^M, \infty)$. Since $\hat{v}(\delta_1, \delta_2)$ is the unique intersection, by Assumption 3', for each $v_j \geq \hat{v}_j(\delta_1, \delta_2)$ in the domain of h , we have $\delta_i U_i(v_j) \geq h(v_j)$. Assume that $\delta'_i > \delta_i$. Then, for each $v_j \geq \hat{v}_j(\delta_1, \delta_2)$ in the domain of h , we have $\delta'_i U_i(v_j) > \delta_i U_i(v_j) \geq h(v_j)$. Therefore, $\hat{v}_j(\delta'_1, \delta'_2) < \hat{v}_j(\delta_1, \delta_2)$. Since h is decreasing, this implies that $\hat{v}_i(\delta'_1, \delta'_2) > \hat{v}_i(\delta_1, \delta_2)$. Since U_1 is decreasing in this region, this completes the proof. ■

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