Simplicity and Probability Weighting in Choice under Risk

By Drew Fudenberg and Indira Puri*

We discuss the work of Fudenberg and Puri (2021), which empirically implements models that combine prospect theory and cumulative prospect theory with a "complexity cost" that captures a preference for lotteries with a smaller number of outcomes, which is one measure of how "complex" the lottery is perceived to be. Fudenberg and Puri (2021) finds that this hybrid model predicts well, and conclude that both probability weighting and complexity costs have important roles to play in predicting choices among risky alternatives. Here we present a novel speculative application of the paper's estimates to field data, specifically a prize-linked saving account that was offered in South Africa in 2005-2008.

Our work builds on the *simplicity theory* of Puri (2018), which extends expected utility theory by allowing the utility of a lottery to depend on the size of its support.[1] Other work on simplicity includes Goodman and Puri (2021) and Puri (2022).[2]

I. Motivating Evidence

There is abundant evidence that people's risk-taking behavior in laboratory experiments depends nonlinearly on the stated probabilities of the various outcomes, and in particular that people overweight small probabilities and underweight large ones. This type of non-linear probability weighting is one of the two key components of prospect theory (PT, Kahneman and Tversky (1979)) and cumulative prospect theory (CPT, Tversky and Kahneman (1992)).[3] Prospect theory and cumulative prospect theory can explain many facts and observations, and have been fit to a number of lab experiments (Fehr-Duda and Epper, 2012; Starmer, 2000).

In addition, past research has shown that people tend to prefer degenerate lotteries to non-degenerate lotteries, and that violations of expected utility occur far less often when three-outcome lotteries are compared to other three-outcome lotteries, than when three-outcome lotteries are compared to lotteries with fewer outcomes.[4]

However, prominent experiments which claim that certainty is special only examine two-outcome lotteries (e.g. Tversky and Kahneman (1986), Cohen and Jaffray (1988), Andreoni and Sprenger (2011)), and although the certainty effect has been used as justification for probability weighting, it could equally well be explained by the idea that lotteries with fewer outcomes are preferred to lotteries with more.

Moreover, prior experiments found that, holding fixed the expected value, participants prefer lotteries with fewer outcomes even when these lotteries have higher variance (Moffatt, Sitzia and Zizzo, 2015; Sonsino, Benzion and Mador, 2002). Bernheim and Sprenger (2020) finds that both prospect and cumulative prospect theory fail rigorous tests which they design. Some of their results can be explained by people preferring lotter-

* MIT Economics, 50 Memorial Drive. Fudenberg: drewf@mit.edu, Puri: ipuri@mit.edu. We thank Advik Shreekumar, Charlie Sprenger, and Tavneet Suri for comments.

[1]The paper gave an axiomatic characterization for its combination of expected utility and simplicity cost. It also discussed but did not analyze combining simplicity cost with probability weighting.

[2]Puri (2022) finds experimental support for new empirical predictions made by simplicity theory that canonical decision theoretic models cannot accommodate. Goodman and Puri (2021) finds that traders are willing to pay more for simple binary options than for dominating alternatives with more possible outcomes.

[3]The other key component is reference dependence, which we do not address here.

[4]Starmer (2000) writes that "behavior on the interior of the probability triangle tends to conform more closely to the implications of Expected Utility Theory than behavior at the borders," and Fehr-Duda and Epper (2012) writes that the second stylized fact is "the reason why these models [probability-weighting] were devised in the first place."

ies with fewer outcomes. Andreoni and Sprenger (2011) reports finding evidence consistent with the idea that people have different preference functions for sure outcomes than for two-outcome lotteries.

Most work that fits PT or CPT to laboratory data uses a representative agent model, but there is no reason to believe that this is a good approximation, and allowing for heterogeneous agents provides a much richer understanding of peoples' behavior. For example, Bruhin, Epper and Fehr-Duda (2010) finds three types of CPT agents in data across two countries, with different probability weights and utility functions.[5]

II. Fudenberg and Puri (2021)

Fudenberg and Puri (2021) (FP) evaluates theories of choice under risk using a new dataset where each participant provides certainty equivalents for lotteries with two, three, four, five, and six outcomes.[6] It considers versions of prospect theory (PT), cumulative prospect theory (CPT), and simplicity theory, as well as hybrid models that combine PT or CPT with a preference for simplicity.

The CPT-Simplicity model combines probability weighting as in CPT with complexity costs as in Simplicity:

$$u(p) = \sum u(x_i) \left[\pi \left(\sum_{k=1}^{i} p_k \right) - \pi \left(\sum_{k=1}^{i-1} p_k \right) \right] - C(|\text{support}(p)|).$$

The PT-Simplicity model is similar, but without rank-dependence of the probability weighting, as in PT.[7]

To empirically implement these models, we used the functional forms used for utility and probability weighting that are standard in the literature following Kahneman and Tversky (1979): probability weighting function $\pi(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{\frac{1}{\gamma}}}$ and CRRA Bernoulli utility for u. (we consider only lotteries in the gains domain, treating 0 as the reference point). For the cost function we selected a sigmoid form so as to impose relatively few constraints:

$$C(n) = \frac{\iota}{1 + e^{\kappa(x-\rho)}} - \frac{\iota}{1 + e^{\kappa(1-\rho)}},$$

where the second term is a normalization so that $C(1) = 0$.

This functional form for simplicity may be useful in future empirical work; we find that it works reasonably for the purpose of outsample prediction. The functional form could possibly be estimated on other data following the procedure described in FP, where in addition to spelling out the econometric model, we include optimization improvements such as using validation sets to pick the number of groups.

Following Bruhin, Epper and Fehr-Duda (2010), it allows for heterogeneous preferences, and finds that a hybrid model that combines CPT with Simplicity performs the best and comes close to matching machine-learning performance; the other models all perform less well. PT performs worse than all models except expected utility; it is outperformed by both simplicity theory alone and CPT alone. Simplicity theory alone outperforms PT but does slightly less well than CPT. Thus we see that both probability weighting and simplicity have a useful role to play in predicting choice under risk.

To illustrate the potential of the hybrid model, this paper reports how the estimated model from FP performs out-of-sample on field data, specifically data on the takeup of prize-linked saving accounts in South Africa.

Our models allow for heterogeneity within one class of model, for example three CPT-Simplicity groups. For ease of estimation we do not allow two groups to be PT and one CPT, though there is no *a priori* justification for this restriction.

It finds three heterogeneous groups: One

[5]Fudenberg et al. (2022) finds that heterogeneous CPT captures most of the predictable variation in the average certainty equivalents for two-outcome lotteries.

[6]To fit a heterogeneous agent model, the same person should face lotteries with different supports, so the model can classify their behavioral type.

[7]CPT-Simplicity nests both CPT and Simplicity, so it is less restrictive than either in the sense of Fudenberg, Gao and Liang (2022); we have not quantified this.

group distorts probabilities mildly and is mildly complexity averse, and a second group heavily distorts probabilities and is more complexity averse. These groups together form a large majority. Thus, the strength of probability weighting and complexity aversion appear related, across these two groups. The third group is not complexity averse and has an intermediate level of probability weighting.

We compared the performance of these theories to machine learning performance, taking the best performance across several machine learning algorithms and found that the CPT-Simplicity model comes closest to matching ML performance. This is true both in an absolute sense and when we use 'machine learning completeness scores' which use expected value as the naive method, and modify Fudenberg et al. (2022) by using ML performance as the best performing model.

III. Prize-Linked Savings

To show how simplicity may be useful in analyzing field data, in a speculative illustration, we apply the calibrated parameters to the prize-linked savings program in South Africa. This exercise alone should not be taken as evidence of one model over another, as it is subject to strong caveats. First, we have to scale down payoffs to match those in the experiment, and since the parameter estimation is sensitive to payoff values, it is also sensitive to scaling. These are much smaller payoffs than used in PLS.[8] Second, the experimental population we use may or may not correspond to the population on which takeup was studied. Third, in our calibration, we take the model very literally; the error terms here come only from uncertainty surrounding the model parameters.

Prize-linked savings (PLS) have been introduced by government or private institutions in many countries including the UK and US, and have been studied academically (Kearney et al. (2010) for a review; later papers include Gertler et al. (2018) and Bharadwaj and Suri (2020), among others). In a PLS vehicle, rather than receiving an interest rate, the individual is entered into a drawing for a prize. The return from the drawing is typically less than the interest that would be earned on a comparable normal savings vehicle. PLS have principle guarantees, so no losses are involved.

The prize-linked savings accounts in South Africa were termed 'Million a Month', and were run by First National Bank, one of the largest retail banks in that country. The program ran from January 2005 through March 2008 before shutting down due to legal challenges from the South Africa Lottery Board.

We calibrate our model to takeup of PLS accounts, rather than to the intensive margin of savings levels, as this is a better match for the static focus of our model. To calibrate, we need a product the individual would use if they were not using PLS; we use a standard saving account.[9] Our aim is to match the takeup percentage described by Cole, Iverson and Tufano (2021), which studies the population of bank employees in the context of the South African PLS product; in addition, we compare the predicted effect of demographics to actual demographic characteristics for takeup. They write that PLS probabilities reached an equilibrium in the final year of the program, and as the participants were all bank employees they may have been well informed.

Calibration Method

We scale down the PLS lottery described above to the payoffs that would result with a \$5 principal investment (about the average payoff in FP).[10] We use the parameters calibrated by FP for the heterogeneous

[8]From Cole, Iverson and Tufano (2018), in March 2008, the monthly prize amounts were one prize of R1,000,000; four prizes of R100,000; 20 prizes of R10,000; and 200 prizes of R1,000. An investment of R100 gave a person one entry into the prize drawing.

[9]In our data, a bank launched PLS and individuals could use either a standard savings account or a PLS account, or both. In other contexts, PLS is incorporated within the standard product (Bharadwaj and Suri, 2020; Gertler et al., 2018) or the alternative option is not as clear Kearney et al. (2010).

[10]Linearizing downwards has been done in lab experiments (Conlisk, 1989; Huck and Müller, 2012).

CPT and CPT-Simplicity models, which both have three groups. We perform 1000 simulation draws; in each draw, for each heterogeneous group and for each parameter, we draw that parameter from a normal distribution with mean and standard deviation for that parameter found in FP, subject to non-negativity constraints for some parameters. In this interpretation, error arises only from uncertainty over the parameter estimates for each heterogeneous group. Given these draws, we then calculate whether the standard bank account or the PLS looks more attractive to each heterogeneous group.

Results

The heterogeneous agent CPT simulation predicts that about 84% of the time, PLS is chosen over a standard bank account. This is higher than actual takeup, which was at 63%. The reason that CPT over-predicts takeup is that, in all groups, individuals distort probabilities, so in all groups, individuals should be attracted to PLS.

CPT-Simplicity predicts that about 64% of the time, PLS is chosen over a standard bank account. This appears to be closer to the actual takeup number. CPT-Simplicity allows for probability weighting but also includes the idea that individuals may prefer the guaranteed standard bank account return over the several possible outcomes in the PLS lottery. Consequently, only the two (out of three) groups that distort probabilities more than they dislike complexity take up the product in the simulation exercise.

Cole, Iverson and Tufano (2021) reports that none of income, age, education, or employment aggregate statistics at a branch predict PLS takeup at that branch.[11] This is consistent with FP's finding that simplicity preferences are not well predicted by income, age, education, or employment.

IV. Further Remarks

There are other ways that lotteries can be complex or off-putting beyond support size. For example, for example a uniform distribution over 20 outcomes may be more appealing that a lottery over 19 outcomes that has higher mean and lower variance if the distribution of the 19-outcome lottery is very irregular. Prior papers have hypothesized that insensitivity to probabilities may be due to cognitive limitations (Viscusi, 1989; Wakker, 2010), and Enke and Graeber (2019) considers a model that, in the risk context, implies probabilities are biased towards a uniform prior. Puri (2022) finds a link between cognitive ability and simplicity preference. It would be interesting to further explore the link between cognitive ability and measures of complexity aversion.

[11] They do find an effect of race that FP's data cannot address.

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