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"COMMON AGENCY AND EXCLUSIVE DEALING"

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1. Introduction.

Common agency refers to a situation in which many principals have an interest in the actions of the same agent and therefore may try to compete for the agent's attention via various inducements or incentive schemes. As Bernheim and Whinston (1986) note, common agency is prevalent in our economy both formally (as in sales agency) and informally (through the multitude of interdependent relationships within a hierarchy, say). To the extent principals cannot enforce a cooperative design of incentives for the agent, common agency generally incurs social costs (because of externalities between principals). In this paper we will present a rather specific model in which those costs can be assessed in order to study when it would be desirable to restrict the scope of common agency by eliminating some of the principals, and in particular when exclusive dealing is a rational organizational response to the problems of common agency.

A common instance of exclusive dealing is the standard employment relationship. An employee is expected to work for his employer, not for others, at least during regular working hours. We attribute the benefits of an employment relationship at least partly to reduced common agency costs. A

single principal has better control of the agent's incentive structure. He can fashion a contract that is relatively free from distracting inducements from competing activities. The employment contract reduces the opportunity cost to the agent, and thereby incentives can be provided less expensively. Of course, the gains in incentive design must be compared with the potential losses from not being able to exploit valuable outside opportunities. It is the trade-off between productive benefits from being engaged in more activities versus the raised agency costs that such options bring, which determines the choice between exclusive dealing (eg. regular employment) and common agency (eg. self-employment).

This perspective contrasts sharply with Alchian and Demsetz's (1972) provocative view that an employment relationship is no different from arms-length market exchange: "It is common to see the firm characterized by the power to settle issues by fiat, by authority, or by disciplinary action superior to that available in the conventional market. This is delusion. The firm does not own all its inputs. It has no power of fiat, no authority, no disciplinary action any different in the slightest degree from ordinary market contracting between any two people. I can 'punish' you only by withholding future business or by seeking redress in the courts for any failure to honor our exchange agreement. That is exactly all an employer can do." This may well be true, but that does not make the two forms of exchange equivalent. The employment relationship is placed in a distinctly different legal framework than the service relationship. From our perspective, a central distinction is that the grocer is entitled to do business with whoever enters his store, while the employee is expressly restricted to serve his employer alone; the employee's fiduciary duty is such unless otherwise agreed to. Moreover, as seems apparent, the common agency costs in the

grocery setting are small relative to the gains from servicing more than one customer; customer incentives to influence the grocer in a way that imposes externalities on other patrons are relatively small.

Our perspective is much closer and complementary to Simon's (1951) (remarkably modern) view of the employment relationship. Simon identified employment with the contractual rights that an employer has in assigning the employee to a relatively broad set of tasks. The rationale for this, according to Simon, is to be found in an asymmetry of information between the employer and employee. The employer knows relatively more about the marginal returns to employee activities, while the employee's opportunity costs are more easily identified (and typically relatively insensitive to the assignment). Like Simon, we stress the advantages that an employment contract confers, because it permits the employer improved control of the employee's activities.

We will use a simple model of moral hazard (adapted from Holmstrom and Milgrom, 1987) to study the costs of common agency. We envision a risk averse agent who potentially may service several principals. The agent has to decide on how he allocates his service efforts across principals. Principals compete for the agent's services by offering him independently (ie. non-cooperatively) designed incentive schemes. The nature of a Nash equilibrium in contracts is the first object of study. Its properties will depend on what one assumes the principals can observe and hence include in a contract. In Bernheim and Whinston (1986) it was assumed that all principals observe the same performance signals. In the context of independent service relationships, that assumption is not as natural. We will also consider the opposite case in which principals only observe the agent's performance in their own service relationship. These informational assumptions have bearing

on common agency costs. Interestingly, it is not always the case that observing more signals is better overall; for instance, if the two service activities are independent (stochastically and technologically) then individualistic contracts (contracts that only depend on the agent's performance in one relationship) are efficient, while contracts that can be made contingent on performance in all activities will lead to inefficiencies.

Our chief objective is to gain an understanding of the technological conditions under which there is a strong desire to restrict the number of principals/activities of the agent. Our main result (at present) is that some activities, which would be desirable in a world of perfect information, will be dropped in a world of imperfect information. More generally, the number of activities will be reduced when agency considerations enter.

The paper's outline is as follows. We begin with a brief description of the linear model that we employ. For further details on this model the reader should consult Holmstrom and Milgrom (1986). In section 3 we present the common agency situation and define the notion of equilibrium to be employed and prove that linear incentive schemes form an equilibrium. Section 4 is devoted to the case in which principals observe the same information, while section 5 deals with the case of independent observations. Section 6 presents the results on reduced and exclusive dealings. Section 7 discusses organizational interpretations and potential responses to common agency. We conclude with a discussion of connections to the related, but broader issue of collusion in multi-person agency.

## 2. A review of the linear model.

Applications of agency models have been limited by the paucity of simple examples that yield tractable solutions. Typically, moral hazard analyses result in complicated, non-linear incentive schemes, which make extensions to richer economic settings cumbersome. Some studies have resorted to ad hoc restrictions on incentive schemes, for instance by assuming that incentive contracts are linear. The problem with such ad hoc restrictions is that one does not know which predicted responses are induced by the restrictions and which reflect the genuine economics of the situation one is interested in.

We will apply the moral hazard model in Holmstrom and Milgrom (1987), because it leads to linear second-best incentive contracts. It may be useful to recall the setting and main results of the paper to see how they extend to common agency.

The paper analyzes a model of repeated moral hazard. An agent works for a principal for  $T$  periods, each period independent (stochastically and technologically) from the others. Periods are to be construed as short so assume the agent is only interested in what he is paid at the end of the horizon (this can be rationalized by a permanent income hypothesis, which can be shown to apply in this context). The agent's preferences over payments at the end are described by an exponential utility function:

$$(1) \quad u(s(x) - c(\mu)) = -\exp(-r[s(x) - c(\mu)]),$$

where  $r$  is the agent's constant absolute risk aversion,  $s(x)$  is what the agent receives as a function of the observables  $x$  and  $c(\mu)$  represents the costs incurred in operating the technology as a function of the agent's action  $\mu$ . As usual, the moral hazard problem enters because the agent's cost  $c(\mu)$  is not observed (nor can it be fully inferred from  $x$ ) by the principal.

The agent must be compensated indirectly for his efforts, forcing him to bear some uncertainty.

Initially, assume the technology can be described by a Bernoulli process. In each period the agent may produce a high ( $x_t = 1$ ) or a low outcome ( $x_t = 0$ ). The probability of a high outcome in period  $t$  is determined by the agent's choice  $\mu_t$ , with the vector  $\mu = (\mu_t)$  resulting in the overall cost function  $c(\mu) = \sum c(\mu_t)$  (without loss of generality one can think of  $\mu_t$  as the probability of success). Suppose further that  $x = (x_1, \dots, x_T)$ , ie. the observable vector  $x$  consists of the individual outcomes in each of the  $T$  periods. Finally, assume that the agent can observe the outcome  $x_t$  before choosing what to do in period  $t+1$ . Then it is relatively straightforward to show that the optimal incentive scheme is one which only depends on  $X_T = \sum x_t$ , ie. the number of successes, and furthermore is of the linear form:

$$(2) \quad s(x) = \alpha X_T + \beta.$$

This result follows from the more general proposition that the multi-period problem is solved by summing up the solutions to the single period problem. In a single period the agent would be paid a fixed amount for participating and a bonus if a favorable outcome occurred. In the repeated situation above, the agent is paid a fixed amount  $\beta$  and a bonus  $\alpha$  for each favorable outcome during the  $T$  periods. The bonus is the same irrespective of the period in which the favorable outcome occurred.

The result that the multiperiod problem decomposes into a set of single period problems is based on the fact that the agent's preferences for risk are independent of wealth and that the agent can observe the past before

acting further. With those two conditions, the agent's preferences over future lotteries are not dependent on history. In terms of certain equivalents, the agent and the principal face identical problems in each period, leading to a repetition of the solution for the single-period problem. In the Bernoulli case the result is (2).

The logic of decomposition extends considerably. Suppose the agent operates a general technology (the same in each period). With a finite number of outcomes, the most general technology is the multinomial one, where  $\mu_t$  can be taken to represent the probability vector over possible outcomes in period  $t$ . Again one finds that the optimal scheme aggregates over single period outcomes:

$$(3) \quad s(x) = \sum_i \alpha_i X_T(i) + \beta,$$

where  $X_T(i)$  is the number of times that outcome  $i$  occurred during the horizon. It is particularly relevant to notice that this general formulation covers both situations in which the agent observes some private information before acting in each period, as well as situations in which the agent's action for some other reason is multi-dimensional (eg. he has several activities).

The results above say nothing about the size of  $\alpha_i$ . While it is conceptually easy to solve for these coefficients, a much more attractive analysis obtains if we consider the limiting situation where the length of periods goes to zero and the problem can be represented approximately as a situation in which the agent controls the drift vector of a Brownian process. As is intuitive, the Bernoulli solution (2) converges to a one-dimensional Brownian process. The optimal incentive scheme is linear in the end of period position of the process, in analogy with (2). Corresponding to the

Bernoulli case in which the agent chooses the same action in each period irrespectively of history, the agent will choose a constant drift-rate. That in turn implies that the distribution of the end-of-period position of the Brownian process is normally distributed and that the agent is simply choosing the mean of that normal distribution. Hence, the whole problem, conditional on knowing that the optimal scheme is linear, can be solved as a static agency problem in which the principal tries to pick out the best linear scheme when the agent operates a normally distributed technology. That static problem is straightforward to solve.

To illustrate, let  $x = \mu + \epsilon$ , where  $\epsilon$  is normal with zero mean and variance  $\sigma^2$  and  $\mu$  is the mean of  $x$ , chosen by the agent. The optimal incentive scheme is of the form  $s(x) = \alpha x + \beta$ . To find the coefficients, note that the objective of the agent can be described in terms of the agent's certain equivalent:

$$(4) \quad \text{Max } \alpha\mu + \beta - (1/2)r\alpha^2\sigma^2 - c(\mu).$$

The agent will choose  $\mu$  so that:

$$(5) \quad \alpha = c'(\mu).$$

The principal's certain equivalent is,

$$(6) \quad (1 - \alpha)\mu - \beta.$$

A Pareto optimal choice of  $\alpha$  will maximize the joint surplus (sum of (4) and (6)):



$$(7) \quad \text{Max } \mu - (1/2)r\alpha^2\sigma^2 - c(\mu), \text{ subject to the constraint (5).}$$

If, for instance,  $c(\mu) = (1/2)\mu^2$ , then (5) reduces to  $\alpha = \mu$  and the solution to (7) becomes:

$$(8) \quad \alpha = (1 + r\sigma^2)^{-1}.$$

This solution reflects the moral hazard constraints in a very intuitive way. In the first-best solution (one in which the agent would not care about risk or there was no uncertainty about his action), one would set  $\alpha = 1$ , that is franchise the activity to the agent, who would just pay a constant rental fee  $\beta$  to the principal. But in the second-best situation, the agent does not want to carry all the risk. The optimal trade-off between risk and incentives is given by  $\alpha$  in (8). The agent's share is bigger the smaller is the risk and the smaller is his risk-aversion. The agency cost in this example is easily seen to be:  $1/2 - (1/2)(1 + r\sigma^2)^{-1}$ .

The case in which the agent has several activities is equally easy to solve. The multinomial process with frequent small periods is approximated well by a multi-dimensional Brownian process, with fixed covariance matrix and a drift vector controlled by the agent. The optimal scheme is linear in each component of the Brownian process, corresponding to (3). Consequently, the agent will act the same in each period and the dynamic incentive problem reduces to a static problem in which the principal has to decide on the coefficients of a linear scheme, given that the agent controls the means of a multivariate normal distribution.

To illustrate, consider the case in which the agent operates two processes:

$$(9) \quad x_i = \mu_i + \epsilon_i, \quad i = 1, 2.$$

The optimal linear scheme is of the form  $s(x_1, x_2) = \alpha_1 x_1 + \alpha_2 x_2 + \beta$ . The agent's certain equivalent is

$$(10) \quad \alpha_1 \mu_1 + \alpha_2 \mu_2 + \beta - (1/2)r[\alpha_1^2 \sigma_1^2 + \alpha_2^2 \sigma_2^2 + 2\alpha_1 \alpha_2 \sigma_{12}] - c(\mu),$$

where  $\sigma_{12} = \text{cov}(\epsilon_1, \epsilon_2)$ . The agent will choose  $\mu$  so that

$$(11) \quad \alpha_i = c_i(\mu), \quad i = 1, 2,$$

where  $c_i$  refers to the partial derivative of  $c$  with respect to  $\mu_i$ . The optimal choice of coefficients is determined again by maximizing joint surplus subject to incentive compatibility conditions in (11).

Denote the function in (10) by  $CE(\alpha, \mu)$ . Then joint surplus is

$$(12) \quad S(\alpha, \mu) = CE(\alpha, \mu) + \sum (1 - \alpha_i) \mu_i.$$

Maximizing over  $\alpha$ , recognizing that  $\mu$  depends on  $\alpha$  through (11) and that the envelope theorem therefore applies, gives

$$(13) \quad \partial CE / \partial \alpha_i - \mu_i + \sum (1 - \alpha_j) \partial \mu_j / \partial \alpha_i = 0, \quad i = 1, 2.$$

This can be written out as:

$$(14) \quad \sum_j (1 - \alpha_j) \partial \mu_j / \partial \alpha_i = r[\alpha_i \sigma_i^2 + \alpha_j \sigma_{12}], \quad i = 1, 2.$$

Equation (14) expresses in a simple form the system of equations that determines the optimal coefficients when the incentives to allocate the agent's efforts across activities are provided in a coordinated fashion. This is the best one can do considering the informational constraints. We will shortly contrast this second-best solution with the situation that obtains if the agent's two activities are performed for separate principals who can influence the agent independently.

A couple of observations on the second-best solution.

1. Denote by subscripts partial derivatives of the cost function. From (11) follows

$$(15) \quad \partial \mu_i / \partial \alpha_i = c_{jj} / \Delta$$

$$\partial \mu_j / \partial \alpha_i = -c_{ij} / \Delta$$

where  $\Delta = c_{11}c_{22} - (c_{12})^2 > 0$  (by the second order condition). If  $c_{12} = 0$ , and  $\text{cov} = 0$ , ie. if the two activities are technologically and stochastically independent, then the design of  $\alpha_1$  will not interact with the design of  $\alpha_2$  and we will have from (14):

$$(16) \quad \alpha_i = (1 + r\sigma_i^2 c_{ii})^{-1}, \quad i = 1, 2.$$

It is worth noting that even if the two activities are technologically identical ( $c_{11} = c_{22}$  whenever  $\mu_1 = \mu_2$ ), the  $\alpha_i$  coefficients will not be the same unless the performance measures  $x_i$  are equally noisy. The activity

which can be measured with less noise will be relatively favored.

2. Suppose  $c_{12}$  and  $c_{ii}$  is zero whenever  $\mu_i$  is zero. Suppose further that the activities are stochastically independent. In that case the first-best and the second-best solution both have the feature that each activity is relevant in the sense that it is operated on a positive scale. (The statement also holds if the activities are negatively correlated, but it need not hold if they are positively correlated.) The reason we make this observation is that we will see that with two principals acting independently it may be desirable to close out a minor activity which is relevant both in first-best and second-best.

3. If the agent is risk neutral then  $\alpha_1 = \alpha_2 = 1$ , of course. The same is true if there is no uncertainty ( $\sigma_i=0$ ,  $i = 1,2$ ). A bit less obvious is the fact that even if  $\sigma_1=0$ , one will not have  $\alpha_1=1$  unless  $\sigma_2=0$  or  $c_{12} = 0$  as well. The point is that the principal will use  $\alpha_1$  to affect the agent's opportunity cost for choosing  $\mu_2$ , which remains a problem incentive wise because  $\sigma_2 > 0$ . In typical second-best fashion all instruments will come into play. It is not optimal to try to influence the agent's choice of  $\mu_2$  by rewards alone. Some incentives will be provided by manipulating the opportunity cost (if  $c_{12} > 0$  then  $\alpha_1 < 1$  to lower the marginal cost of  $\mu_2$ ;  $\alpha_1 > 1$  if  $c_{12} < 0$ , assuming this is feasible when all other considerations are taken into account).

4. By writing the technology in the form (9) the cost function captures all the technological features of the model. A "generalization" that can be brought immediately back into the form (9) is the following:  $x_i = f_i(\mu_i) +$

$\epsilon_i$ , where  $f_i$  is some monotone transformation. (The transformation cannot involve  $x_i$  itself, because then we would lose normality, which is essential for linearity.)

### 3. Common agency.

We consider a common agency situation in which two principals deal with the same agent independently of each other. The principals are aware of each other, but they cannot coordinate their transactions with the agent, because they are assumed unable to observe each other's contracts. For simplicity we begin with the two-activity case discussed above; extensions will be considered later.

Let  $x_i$  be as in (9);  $x_i$  defines the agent's delivered service to principal  $i$  (measured in monetary units). Let  $z_i$  denote what principal  $i$  can observe and use in a contract with the agent. We will mainly be interested in the two cases: (a)  $z_i = x_i$  and (b)  $z_i = (x_1, x_2)$ . In the first, each principal can observe the agent's performance only in the activity performed for principal  $i$ . We refer to this case as one with disjoint observations. In the second, both principals can observe the service delivered to each of them. This case, which was studied originally by Bernheim and Whinston (1986), we refer to as one with joint observations.

Let  $s_i(z_i)$  be the contract offered to the agent by principal  $i$ . Define:

$$U(s_1, s_2, \mu) = E_\mu(u(s_1(z_1) + s_2(z_2)) - c(\mu)),$$

the agent's utility given the contracts and an action  $\mu$ ;

$$\hat{U}(s_1, s_2) = \max U(s_1, s_2, \mu) \text{ over } \mu,$$

the maximum utility the agent can achieve given the two contracts;

$$B(s_1, s_2) = \operatorname{argmax} U(s_1, s_2, \hat{\mu}) \text{ over } \mu,$$

the set of actions that are optimal for the agent given contracts;

$$R_i(s_i, \mu) = E_\mu(x_i - s_i(z_i)),$$

principal  $i$ 's expected profit, given  $\mu$ ;

$$\hat{R}_i(s_1, s_2) = \max R_i(s_i, \mu) \text{ over } \mu \in B(s_1, s_2),$$

principal  $i$ 's expected profit if the agent is offered the contract pair  $(s_1, s_2)$  and he chooses principal  $i$ 's most favored action among those that maximize his expected utility.

A missing contract will be indicated by the symbol  $\emptyset$ . The equilibrium concept we employ is the following:

Definition. Let

$$S = \{(s_1, s_2) \mid \hat{U}(s_1, s_2) \geq \max[\hat{U}(\emptyset, s_2), \hat{U}(s_1, \emptyset), \hat{U}(\emptyset, \emptyset)]\},$$

denote the set of contract pairs, such that the agent will find both contracts acceptable. The pair  $(s_1(z_1), s_2(z_2))$  constitutes a Contractual Nash Equilibrium (CNE), if:

- (a)  $(s_1, s_2) \in S$ , and
- (b) for either  $i$ , there does not exist a contract  $\tilde{s}_i$ , such that  $(\tilde{s}_i, s_j) \in S$  and  $\hat{R}_i(\tilde{s}_i, s_j) > \hat{R}_i(s_i, s_j)$ .

Note that in an equilibrium one principal may offer no contract (ie.  $s_i$  could be  $\emptyset$ ). Consequently, the definition above assures that principals earn at least zero profits. On the other hand, the definition of  $S$  assures that the agent gets at least as much utility as he could get without any contracts.

Proposition 1: If one principal uses linear contracts, it is optimal for the other principal to do likewise.

This result follows from noting that linearity in the continuous time model is the same as stationarity in the discrete version. In the discrete version, if one principal offers the agent a stationary contract, the other principal will find it optimal to offer a stationary contract as well, since each period will look contracting-wise the same by virtue of stochastic independence and exponential utility. Proposition 1 says of course nothing about the possibility of equilibria in non-linear contracts. Nor does the result say anything about existence of equilibria in linear contracts. For existence we will rely on constructing a pair of linear equilibrium contracts.

#### 4. Common agency with disjoint observations.

In this section we consider the case  $z_i = x_i$ , ie. the case in which principals only observe the service they receive and not what the other receives.

By Proposition 1 we look for equilibrium in linear sharing rules  $s_i(x_i) = \alpha_i x_i + \beta_i$ . Let  $S_i$  denote the joint surplus available to principal  $i$  and the agent. Then, given the contract offered by principal 2:

$$(4.1) \quad S_1 = \mu_1 + \alpha_2 \mu_2 - c(\mu) - (1/2)r[\alpha_1^2 \sigma_1^2 + \alpha_2^2 \sigma_2^2 + 2\alpha_1 \alpha_2 \sigma_{12}],$$

Principal 1 will choose  $\alpha_1$  to maximize  $S_1$ , recognizing that the agent's

action  $\mu$  depends on the  $\alpha$ -vector, ie.  $\mu = \mu(\alpha)$ ;  $\alpha_2$  is here to be treated as a parameter. We have:

$$(4.2) \quad \partial S_1 / \partial \alpha_1 = \partial \mu_1 / \partial \alpha_1 + \alpha_2 \partial \mu_2 / \partial \alpha_1 - c_1 \partial \mu_1 / \partial \alpha_1 - c_2 \partial \mu_2 / \partial \alpha_1 \\ - r[\alpha_1 \sigma_1^2 + \alpha_2 \sigma_{12}].$$

Recall that  $c_i$  denotes partial derivative of cost with respect to  $\mu_i$ . From the agent's first-order condition on the choice of  $\mu$ ,  $c_i = \alpha_i$ . Substituting this into (4.2) (and recognizing that the symmetric analysis applies to  $S_2$ ) gives as equilibrium conditions for the disjoint observation case:

$$(4.3) \quad (1 - \alpha_i) \partial \mu_i / \partial \alpha_i = r[\alpha_i \sigma_i^2 + \alpha_j \sigma_{12}], \quad i = 1, 2; j \neq i.$$

To get a slightly better feel for the equilibrium, let us use a Taylor approximation for the cost function. In that case:

$$(4.4) \quad (1 - \alpha_i) c_{jj} = r \Delta [\alpha_i \sigma_i^2 + \alpha_j \sigma_{12}], \quad i = 1, 2; j \neq i,$$

where, recall,  $\Delta = c_{11} c_{22} - (c_{12})^2$ . Compare this to the second-best conditions:

$$(4.5) \quad (1 - \alpha_i) c_{jj} - (1 - \alpha_j) c_{ij} = r \Delta [\alpha_i \sigma_i^2 + \alpha_j \sigma_{12}], \quad i = 1, 2; j \neq i.$$

First we notice that if the two activities are technologically independent, ie. if  $c_{12} = 0$ , then there will be no harm from common agency with disjoint observations. (Somewhat surprisingly, stochastic dependencies do not cause problems alone.) Another case without common agency costs arises



if second-best has  $\alpha_i = 1$ ,  $i = 1, 2$ . That occurs with full observability of the agent's action (in both activities; see remark 3 above) or if the agent is risk neutral ( $r = 0$ ). (The risk neutral case is treated in Bernheim and Whinston, 1986). On the other hand, if  $c_{12}$  is not zero, then we have distortions from common agency. Let  $\delta_i$  denote the difference between the second-best value of  $\alpha_i$  (as defined by (4.5)) and the value in a common agency equilibrium (as defined by (4.4)). Assume that the two activities are stochastically unrelated ( $\sigma_{12} = 0$ ). Then

$$(4.6) \quad (c_{jj} + r\sigma_i^2\Delta)\delta_i = -(1 - \alpha_j)c_{12}, \quad i = 1, 2, i \neq j, \text{ where } \alpha_j \text{ is second-best.}$$

Consequently, if  $c_{12} > 0$  ( $< 0$ ) then uncoordinated bidding for services will lead to too high (low) incentive provision. This is intuitive. When a principal raises the agent's incentive in order to attract more services, the agent will reduce his services to the other principal if  $c_{12} > 0$ . Since only a fraction of that loss is borne by the agent, there will be inadequate consideration of the loss, resulting in an excessively high commission (conversely if there is a positive externality between the principals, ie if  $c_{12} < 0$ ). We summarize our discussion in:

Proposition 2: There will be no losses from common agency with disjoint observations if the agent is risk neutral ( $r = 0$ ), if actions can be observed fully ( $\sigma_1 = \sigma_2 = 0$ ) or if there are no technological links between the two activities ( $c_{12}$ ).

If the two activities are stochastically independent ( $\sigma_{12} = 0$ ) then there will be too much (too little) incentive provision (relative to second-

best) if  $c_{12} > 0$  ( $< 0$ ).

An extreme form of excessive competition between the principals is given by the following cost function:

Example 1:  $c(\mu) = 0$  if  $\mu_1, \mu_2 \geq 0$ ,  $\mu_1 + \mu_2 \leq 1$  and  $\infty$  otherwise. With this cost structure the agent is happy to work a full unit in any combination and is unable to work any more. At the critical point, this cost function is not differentiable so the calculus above does not apply. But it is easy enough to see what the equilibrium outcome is. Assume  $\sigma_{12} = 0$  and that  $\sigma_1 < \sigma_2$ . The equilibrium will have the agent accept only one of the two offers (since the agent will at best be indifferent between the two activities). Let  $\hat{\alpha}_2$  be the lowest  $\alpha_2$  value such that the agent gets no surplus if he were to operate the second activity alone with such a share; ie. the risk  $\hat{\alpha}_2$  imposes will offset all the benefits from producing. Then the contractual equilibrium has both principals offer the agent a share equal to  $\min(\hat{\alpha}_2, 1)$  with the agent accepting the first principal's offer (since it has less risk). The second principal's offer, while rejected, prevents the first principal from reducing the commission rate  $\alpha_1$ . The bidding between principals will spoil effectively all the rents. This is all the more remarkable since the agent is perfectly content with working without any incentive at all. The example suggests that bidding for the agent's services is more detrimental the more easily the agent's choices can be affected. Parametrized versions of the cost function does support such an intuition.

5. Common agency with joint observations.

It is of interest to go through common agency under the informational hypothesis that both principals observe the same, that is,  $z_i = (x_1, x_2)$ . Bernheim and Whinston (1986) have dealt with this case in considerable generality.

Let  $\alpha_{ij}$  denote the share on  $x_j$  offered by principal  $i$ . Then the agent's incentives are determined by the aggregate share

$$(5.1) \quad \alpha_j = \alpha_{1j} + \alpha_{2j}.$$

The joint surplus of principal 1 and the agent is given in this case by:

$$(5.2) \quad S_1 = (1 + \alpha_{21})\mu_1 + \alpha_{22}\mu_2 - c(\mu) - (1/2)r[\alpha_1^2\sigma_1^2 + \alpha_2^2\sigma_2^2 + 2\alpha_1\alpha_2\sigma_{12}]$$

The choice variables here are the aggregate shares  $\alpha_i$ . Correspondingly we have the surplus expression for principal 2 and the agent:

$$(5.3) \quad S_2 = \alpha_{11}\mu_1 + (1 + \alpha_{12})\mu_2 - c(\mu) - (1/2)r[\alpha_1^2\sigma_1^2 + \alpha_2^2\sigma_2^2 + 2\alpha_1\alpha_2\sigma_{12}]$$

In equilibrium both principals wish the same aggregate share  $(\alpha_1, \alpha_2)$  for the agent. Therefore it must be the case that:

$$(5.4) \quad \alpha_{11} = 1 + \alpha_{21} \text{ and } \alpha_{22} = 1 + \alpha_{12}.$$

Combining (5.1) and (5.4) we can express the four coefficients of the incentive schemes in terms of the aggregate shares alone:

$$(5.5) \quad \alpha_{ii} = (1 + \alpha_i)/2 \quad \text{and} \quad \alpha_{ji} = (\alpha_i - 1)/2, \quad i, j = 1, 2.$$

Assuming that aggregate shares  $\alpha_i > 0$ ,  $i=1,2$ , we see that  $\alpha_{ii} > 1/2$  and  $\alpha_{ij} < 0$ . The logic is this. Principal  $i$  offers the agent more than 50% of the returns, since much of it will be passed onto principal  $j$  ( $\alpha_{ij} < 0$ ). Effectively, the principals offer each other side-payments via the agent. An alternative would be to write a contract between the principals. With secret side-payments to the agent the ultimate outcome would be the same.

Substituting expressions (5.5) back into (5.2) (or alternatively by adding (5.2) and (5.3) since both expressions have the same maximizer), we see that  $\alpha_1$  and  $\alpha_2$  must maximize:

$$(5.6) \quad (1 + \bar{\alpha}_1)\mu_1 + (1 + \bar{\alpha}_2)\mu_2 - 2c(\mu) - r[\alpha_1^2\sigma_1^2 + \alpha_2^2\sigma_2^2 + 2\alpha_1\alpha_2\sigma_{12}].$$

Here we have put a bar on the equilibrium values, which the optimal variable choices must equal. Taking first-order conditions and using the agent's optimality conditions as before, we get that the equilibrium with joint observations is characterized by:

$$(5.7) \quad \sum (1 - \alpha_i)\partial\mu_i/\partial\alpha_j = 2r[\alpha_j\sigma_j^2 + \alpha_i\sigma_{12}], \quad j = 1, 2.$$

Put in terms of a Taylor approximation of cost:

$$(5.8) \quad \begin{aligned} (1 - \alpha_1)c_{22} - (1 - \alpha_2)c_{12} &= 2r[\alpha_1\sigma_1^2 + \alpha_2\sigma_{12}], \\ - (1 - \alpha_1)c_{12} + (1 - \alpha_2)c_{11} &= 2r[\alpha_2\sigma_1^2 + \alpha_1\sigma_{12}]. \end{aligned}$$

Compared with the second-best solution, aggregate shares in the common agency case are set as if the agent were twice as risk averse (or the risk was doubled). Naturally, this leads to smaller aggregate shares:

Proposition 3: The aggregate shares offered to the agent in the case of symmetric observations are smaller than in second-best unless second-best equals first-best in which case all three have settings have identical solutions.

The last part of the proposition holds more generally as proved by Bernheim and Whinston. Thus, if we consider Example 1 in this setting, first-best will obtain, since second-best equals first-best given that the agent has nothing against working. On the other hand, it is noteworthy that second-best will not obtain in some of the situations where it does obtain given disjoint observations. Consequently, we have:

Proposition 4: Having one or the other or both principals receive more information about the agent's doings may lead to a strictly worse equilibrium outcome.

The most striking case of this occurs when  $c_{12} = 0$ . With disjoint observations one would reach second-best. However, with joint observations, we do strictly worse. The explanation is as follows: If principal 1 can observe  $x_2$  he can offer to insure the agent partly against the excess risk that principal 2 is trying to impose on the agent to provide better incentives. Such side-contracting destroys the socially optimal contract that principal 2 would otherwise have designed. Recognizing the problem, the

first principal will raise the commission rate with the second principal carrying a larger and larger share of the first principal's project. This will induce the second principal to have an interest in the first principal's project. Eventually an equilibrium is reached, but since both principals have a shared interest in the project, free riding will cause too little incentive provision in the aggregate for the agent.

#### 6. Exclusive dealing.

Given common agency costs, will it ever be better to engage in exclusive dealing by eliminating one or the other principal? More generally, will it be desirable to reduce the number of principals to fewer than what is technologically efficient? The answers to these two questions are not a priori obvious in the case activities are all relevant as defined earlier.

We will answer these questions by first considering a related one: Suppose that the agent has an outside option - a project in which he holds a fixed share  $\lambda$  (instead of the share interpretation one can see  $\lambda$  as a productivity parameter). Will it ever be desirable to remove that option, if possible, when contracting with a single principal?

Let the outside option return  $\lambda x_2$  and let  $x_1$  stand for the return from activities related to the principal. The principal cannot observe the return from the outside option. By proposition 1 a linear sharing rule is among the optimal schemes that the principal can offer the agent. The joint surplus is:

$$(6.1) \quad S = \mu_1 + \lambda\mu_2 - c(\mu) - (1/2)r[\alpha^2\sigma_1^2 + \lambda^2\sigma_2^2 + 2\alpha\lambda\sigma_{12}],$$

where the incentive scheme is  $s(x_1) = \alpha x_1 + \beta$ . The optimal choice of  $\alpha$  is determined by:

$$(6.2) \quad (1 - \alpha)\partial\mu_1/\partial\alpha = r[\alpha\sigma_j^2 + \lambda\sigma_{12}].$$

We have:

$$(6.3) \quad \partial S/\partial\lambda = (1 - \alpha)\partial\mu_1/\partial\lambda + \mu_2 - r[\lambda\sigma_2^2 + \alpha\sigma_{12}],$$

using the envelope theorem twice. If  $c_{12} > 0$  so that the two activities are substitutes in the cost function, then the first term is negative as are the last two terms provided the covariance is positive. It follows that at  $\lambda = 0$ ,  $\partial S/\partial\lambda < 0$ . Consequently, it would be better to be entirely without the outside option for small enough  $\lambda$ , even though the option is productively valuable.

The externality responsible for this outcome is that the agent ignores a portion of the lost returns that come from engaging in the outside option. Hence the agent will always invest excessively in that activity. One might think that this will induce the principal to use stronger incentives to get his share of the agent's attention as  $\lambda$  increases. That need not be true. For instance, if we consider the quadratic cost case with  $\sigma_{12} = 0$ , then (6.2) tells us that the principal will offer the same  $\alpha$  irrespective of  $\lambda$ . In this case, using (6.3), one finds that for  $\lambda < \bar{\lambda}$ ,  $\bar{\lambda} > 0$ , it is better to shut out the outside option.

We can apply the analysis rather directly to the common agency situation (with disjoint observations). Suppose there is a second principal with a

project that returns  $y = \lambda x_2$ , where  $\lambda$  is a scale parameter and  $x_2$  is as before. Since  $\alpha_2 < 1$ , we have:

Proposition 5: For sufficiently small  $\lambda$ , it is better to drop the second principal and deal exclusively with the first. For a quadratic cost function and stochastically independent activities, there is a  $\bar{\lambda} > 0$  such that for all  $\lambda < \bar{\lambda}$ , exclusive dealing will be preferred.

The results on exclusive dealing remind us that there are two means for providing incentives to an agent. One is by designing suitable financial rewards. The other is by altering the agents opportunity cost. Exclusive dealing is a method of the second kind. The agent's temptation to misallocate his efforts across various tasks is removed by reducing his options. That is cheaper in marginal cases than trying to influence his allocation via financial rewards. This may be another reason for bringing employees to a joint workplace. Even in cases where it may be technologically as good or better to have the employees work at home, the incentive costs may make such an arrangement undesirable. Correspondingly, we would expect piece rates or other incentives to be used when work is conducted at home, whereas time rates are common for office workers.

(To be continued)



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