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Author(s): Robert M. Townsend
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Currency and Credit in a Private Information Economy

Robert M. Townsend

University of Chicago

In an environment with private information, spatial separation, and limited communication, a currency-like object and more standard named credits can be distinguished. The credit objects can be used among agents in an enduring relationship, that is, among agents with known trading histories, whereas the currency-like object must be used among relative strangers. In this environment, collectively determined Pareto-optimal rules make the level of the currency-like object and the mix of currency to named credits responsive to individual needs and to economywide states. Total indebtedness is determined by the number of lenders, that is, by preference or demand shocks, and the mix of currency to credits is determined by transaction patterns among the agents.

I. Introduction

Can we find a physical environment in which currency-like objects play an essential role in implementing efficient allocations? Would these objects coexist with more ordinary, named credits? Can we do this without further ad hoc decentralization of the economy, without insisting as an extra condition not coming from the physical environ-

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ment that allocations be achieved in competitive markets? Put differently, if agents in the environment can precommit at an initial date to arbitrary tax and transfer schemes over time, over all the commodities of the environment up to the technology of storage and communication available to them, would they choose to distribute some currency-like object at the initial date, choose what in other models are exogenous initial conditions? Would this currency-like object coexist with named commitments by some agents to other agents to give up commodities now for commodities later on? And, if we looked at a cross section of otherwise identical economies that vary by the configuration of "demand" and "transaction" shocks, would the amount of the currency-like object issued vary in absolute amount and vary relative to the use of named credits?

The model of this paper answers all these questions in the affirmative by drawing sharp distinctions among various communication, record-keeping technologies. The currency-like object of the model plays a role in allowing exchanges among relative strangers, agents whose histories are otherwise not known to one another. The more standard credit objects of the model are used among agents who know their past histories and can keep track of past commitments to one another. Both classes of objects coexist. Indeed, the model of the paper allows cross-sectional variations in the magnitudes of each of these and in the magnitude of one relative to the other.

It is important to be clear also about what the model does not do, especially since the terms "currency" and "credit" mean different things to different people, dependent perhaps on observations of various economies over various dates and dependent also on prior monetary theories that are used to interpret observations. In the model of this paper, agents precommit in the planning period to all institutions and resource allocation rules to the extent that the information structure allows. Thus there is no government in the model, apart from these plans, and no distinction between private and public. So if agents in the model agree, as they do, to the use of currency to intertemporally reallocate consumption, it may well be said that currency is a form of private credit. Conversely, all debts may be said to be public. But that is not to say that the ordinary credit and the currency of the model are not different. They do differ in their communication and record-keeping aspects. Thus the hope is that the stylized model of the paper may allow one to see an aspect of reality that otherwise might not have been as apparent.

Formally, the paper proceeds as follows. Section II describes the basic physical environment. Section III sets out a programming problem for the determination of full communication, private information Pareto-optimal allocations. Section IV does the same for the environment with no cross-location communication. This motivates the introduction in Section V of tokens as a communication device. This is the currency-like object. To deliver a price system and hence measurable currency, Section VI weakens the planning problem, allowing unobserved cross-household exchange. Section VII then extends the environment somewhat and displays an example of optimal variations in currency relative to total indebtedness and relative to the indebtedness of nonmovers, that is, to named credits. Section VIII touches on the costs of the various record-keeping systems.

II. The Physical Environment

Consider an economy with one underlying consumption good, three dates, N locations, and N^2 agents (essentially N per location). At the first date, that is, at $t = 0$, all agents get together to decide on Pareto-optimal rules, to be described in this essay. Thus there is full commitment to the arrangement, that is, perfect costless enforcement of it. At the beginning of the second date, $t = 1$, agents are dispersed to locations, N agents to each location. Finally, at the beginning of date $t = 2$, a fraction λ of the population of each location stays put in its initial location assignment, and a fraction $(1 - \lambda)$ of the population of each location is shifted to new locations in such a way that each "shifter" encounters no agents he has known previously at date $t = 1$. Thus λN agents of each location stay in residence for two periods and $(1 - \lambda)N$ agents of each location are dispersed in some way to the other $N - 1$ locations.[1] For much of the analysis, N will be taken to be arbitrarily large; that is, the mathematics assumes $N = \infty$. The $N = \infty$ economy is envisioned as the limit of finite N economies as $N \rightarrow \infty$, but this limit is never taken explicitly.

After settling in the location assignment of date $t = 1$, every agent receives *one* unit of the single consumption good of the model. This good can never be transferred across locations. The consumption good may be eaten by someone at date $t = 1$, at the endowment location, or alternatively it may be stored at that location for consumption at that location at date $t = 2$. The gross return on storage is some parameter $R \geq 1$, so that one unit of the consumption good

[1] For example, let $N = 4$ and let λ take on values $j/N, j = 1, 2, \ldots, N - 1$. If $\lambda = 1/4$, let one agent remain at each location and let the three departing agents move to the other three locations, one to each of the three nearest location neighbors to the right, moving clockwise around a circle. If $\lambda = 2/4$, let two agents remain at each location and let the two departing agents move to the two nearest location neighbors to the right, one to each, and so on for $\lambda = 3/4$. Here, of course, λ is such that λN is an integer, and this is assumed to be true generally. Similar integer assumptions are made throughout the text. Of course when $N = \infty$, only fractions of the population need be specified.

stored at the end of date $t = 1$ yields R units of the consumption good at the beginning of date $t = 2$.

Whatever might be his location, each agent has preferences over units of consumption c_1 and c_2 in periods 1 and 2, respectively, as represented by a utility function $U(c_1, \tau) + V(c_2, \tau)$. Here τ is a shock to preferences, a "demand" shock, which among other things determines an agent's rate of intertemporal substitution. For simplicity, shock τ can take on one of a finite number of values, that is $\tau \in \{1, 2, \ldots, n\}$. For each τ, the function $U(\cdot, \tau)$ is strictly concave, is continuously differentiable, and satisfies the Inada conditions $U'(0, \tau) = \infty$ and $U'(\infty, \tau) = 0$; similarly for $V(\cdot, \tau)$.

It is supposed that at each location at date $t = 1$, a fraction $\omega(\tau)$ of agents in the population receive shock τ. Of course these fractions must add to unity, that is, $\Sigma_\tau \, \omega(\tau) = 1$. In the absence of any other information, each agent in the planning period $t = 0$ naturally views his own shock τ as determined in a random way, that shock τ will occur with probability $\omega(\tau)$.

At the same time at date $t = 1$ that the distribution of shocks τ is determined, date $t = 2$ location assignments are also determined and revealed (but not executed).[2] Similarly, with fraction $1 - \lambda$ of the agents of each location to be shifted at the beginning of date $t = 2$, each agent in the planning period views his probability of being shifted as determined in a random way. If we let θ denote the "location" or "transaction" shock, which takes on two values (i.e., $\theta = 1$ for "staying" and $\theta = 2$ for "moving"), each agent views $\theta = 1$ as being drawn with probability λ and $\theta = 2$ as being drawn with probability $1 - \lambda$. For simplicity of notation, let $\lambda(\theta)$ denote the fraction of agents who receive shock θ, so that here, for example, $\lambda(\theta = 1) = \lambda$ and $\lambda(\theta = 2) = 1 - \lambda$.

Finally, it will be supposed initially that the distribution of the population by preference shocks τ is independent of the distribution of the population by location movements θ, so that from an individual's point of view the random variables τ and θ are also independent. Economywide, the fraction of agents who receive shocks τ and $\theta = 2$ is $\omega(\tau)(1 - \lambda)$, and so on.

It is supposed in what follows that preference shocks τ received at the beginning of date $t = 1$ are privately observed by the individual but that future location assignments θ received at the beginning of date $t = 1$ are fully observed. Further, though the *population fractions*

[2] Here and below several alternative environments will suggest themselves. Here an alternative would be for agents to see preference shocks prior to seeing location assignments. Such alternatives suggest interesting paths to pursue in subsequent efforts. The effort here is to produce a simple, albeit dramatic, example economy.

ω and λ may be determined at random at the beginning of date $t = 1$ with probabilities prob(ω) and prob(λ), respectively, for each of a finite number of possible values of ω and λ, the actual draws of ω and λ are presumed to be public information as well.

As is evident, the environment under consideration in this paper is essentially the one considered by Diamond and Dybvig (1983), but here with a slightly more general preference specification and enlarged to accommodate distinct locations. The key idea is that the model determines the consumption paths of the individual agent types and hence determines the amount of the consumption-investment good that is invested at date $t = 1$ and claimed by the agent types at date $t = 2$. The latter amount is a natural measure of aggregate lending or indebtedness.

III. The Optimal Credit Arrangement with Full Cross-Location Communication

In the context of the environment described above, we may index the consumption of each agent by the individual-specific shocks τ and θ, at least with the imposition of certain incentive-compatibility constraints described below, so that announcements of preference shocks coincide with actual realizations. Consumption may be indexed by economywide fractions ω and λ as well. Thus the number of units of consumption at date t of the "representative agent" is denoted $c_t(\tau, \theta, \omega, \lambda)$. As is evident, then, no effort is made to distinguish agents by name or by their initial date $t = 1$ location assignment. Consumption at date $t = 1$ for an agent at one location is supposed to be the same as the consumption at date $t = 1$ for an agent at any other location *if* their shocks τ and θ coincide.[3] Similarly, the issue at date $t = 2$ for an individual is what announced (and actual) preference shocks were at date $t = 1$ and whether or not an agent moved. Thus the key assumption *in this section* is that announced preference shocks at date $t = 1$ are public at date $t = 2$ even in a location distinct from the location in which shocks were announced. In this sense full cross-location communication is assumed, histories are common knowledge, and so formally there are no strangers.

The objective in what follows, then, is to characterize an allocation

[3] Also, the assumption is that only the individual state (τ, θ) and the aggregate state (ω, λ) matter. For finite N economies, one ought to list the entire vector of states across individuals, but for $N = \infty$ with nameless individuals, that vector is captured by aggregate state (ω, λ). The $N = \infty$ case also ensures that an individual mover can infer nothing at date $t = 2$ about the preference shocks of others in a set of new arrivals. Otherwise, e.g., with $N = 4$, $\lambda = 2/4$, $n = 2$, $\omega(\tau = 1) = 1/2$, and $\omega(\tau = 2) = 1/2$, a newly arrived agent with $\tau = 1$ would know for sure that the only other new arrival is a $\tau = 2$.

of the consumption good that is Pareto optimal from the point of view of the representative agent in the planning period, at date $t = 0$. This is done by consideration of the following problem.

PROGRAMMING PROBLEM 1. Maximize by choice of the consumptions $c_t(\tau, \theta, \omega, \lambda)$ the objective function

$$\sum_{\omega} \text{prob}(\omega) \sum_{\lambda} \text{prob}(\lambda) \sum_{\theta} \lambda(\theta) \sum_{\tau} \omega(\tau) \cdot \{U[c_1(\tau, \theta, \omega, \lambda), \tau] + V[c_2(\tau, \theta, \omega, \lambda), \tau]\} \tag{1}$$

subject to the resource constraints, for every state (ω, λ),

$$\sum_{\tau} \sum_{\theta} \omega(\tau)\lambda(\theta)c_1(\tau, \theta, \omega, \lambda) \leq W(\omega, \lambda) \tag{2}$$

and

$$\sum_{\tau} \sum_{\theta} \omega(\tau)\lambda(\theta)c_2(\tau, \theta, \omega, \lambda) \leq [1 - W(\omega, \lambda)]R \tag{3}$$

and subject to incentive-compatibility constraints, for all $\theta = 1, 2$, for all $\tau', \tau = 1, 2, \ldots, n$, and for each (ω, λ),

$$\begin{aligned} &U[c_1(\tau, \theta, \omega, \lambda), \tau] + V[c_2(\tau, \theta, \omega, \lambda), \tau] \\ &\geq U[c_1(\tau', \theta, \omega, \lambda), \tau] + V[c_2(\tau', \theta, \omega, \lambda), \tau]. \end{aligned} \tag{4}$$

Here again the objective function is the expected utility of the representative agent from the point of view of date $t = 0$, with expectations over individual shocks τ and θ and over population fractions ω and λ. Constraint (2) is a resource constraint for date $t = 1$, applicable for all locations, where $W(\omega, \lambda)$ stands for per capita "withdrawals" of consumption from possible investment at date $t = 1$, conditioned on ω and λ. Constraint (3) is a resource constraint for date $t = 2$, again applicable for all locations. Constraints (2) and (3) are both written in per capita terms and hold for the $N = \infty$ economy. The class of constraints (4) is a class of incentive-compatibility constraints; it ensures that for given and fully observed location shock θ and population fractions ω and λ, an individual would prefer at date $t = 1$ to announce the actual observed preference shock τ and receive consumption stream $\{c_1(\tau, \theta, \omega, \lambda), c_2(\tau, \theta, \omega, \lambda)\}$ rather than announce counterfactual preference shock τ' and receive consumption stream $\{c_1(\tau', \theta, \omega, \lambda), c_2(\tau', \theta, \omega, \lambda)\}$. It is the imposition of these constraints that allows one to refer to announcements of τ values and actual τ values synonymously. That these constraints may be imposed without loss of generality is implied by the work of Dasgupta, Hammond, and Maskin (1979), Myerson (1979), Harris and Townsend (1981), and

Townsend (1982). Otherwise it is supposed that there is full commitment to the consumption allocations $c_t(\tau, \theta, \omega, \lambda)$; reneging and default are precluded from consideration as if enforcement were perfect and costless.

Solutions to program 1 sometimes can be characterized easily.[4] For example, it is clear that location movement per se should not matter in an optimum, and so index θ can be dropped from the notation. For purposes here, however, it suffices to concentrate on movers and to note that first- and second-period consumptions can be ordered by τ values in a nontrivial way. That is, there exists *some* (re)ordering of τ values such that, for fixed ω and λ, $c_1(\tau, \theta = 2, \omega, \lambda)$ is strictly monotone increasing in τ over some range. Further, if $c_1(\tau, \theta = 2, \omega, \lambda) > c_1(\tau', \theta = 2, \omega, \lambda)$, then $c_2(\tau, \theta = 2, \omega, \lambda) < c_2(\tau', \theta = 2, \omega, \lambda)$, for otherwise the incentive constraints at the realized value of τ would be violated, since naming value τ would be strictly preferred to naming value τ'.

IV. The Optimal Credit Arrangement with No Cross-Location Communication

Now suppose that preference shock announcements of an individual agent at date $t = 1$ are *not* public information at date $t = 2$ *if* that agent shifts locations between dates 1 and 2. Thus the history of a "shifter" or "mover" would be private information, and a shifter may be said to encounter relative strangers. Otherwise, the structure of the model remains unchanged. That is, agents are still presumed to commit themselves in the planning period to some social arrangement, that is, to some economywide credit arrangement, that specifies consumptions and hence transfers to agents conditioned on the economywide state (ω, λ), on the individual-specific but publicly observed location shocks θ, and possibly on individual announcements of the individual-specific and privately observed preference shocks τ. Now, however, these latter announcements may have less content. For suppose that agents who move are to reannounce preference shocks at date $t = 2$ since there is no one present who knows the date $t = 1$ announcement. On his arrival at his new location at date $t = 2$, any mover has a choice in the family of consumptions $\{c_2(\tau, \theta = 2, \omega, \lambda)\}$, indexed by τ, and thus it is clear that any such mover would *always* name the τ value that achieves the highest level of consumption, inde-

[4] Technically, program 1 can be converted to a linear program by consideration of lotteries over consumptions, not deterministic allocations. Often, however, these lotteries do not appear in solutions, and they are ignored here altogether for simplicity of exposition and computation. For a more extended discussion of lotteries and the pitfalls of proceeding without them, see Townsend (1988*a*).

pendent of the realized τ value. In fact, with the imposition of sequential incentive compatibility, as in Townsend (1982), we must have

$$V[c_2(\tau, \theta = 2, \omega, \lambda), \tau] \geq V[c_2(\tau', \theta = 2, \omega, \lambda), \tau] \quad \text{for all } \tau, \tau' \text{ values,} \tag{5}$$

and so it is apparent that $c_2(\tau, \theta = 2, \omega, \lambda)$ must be some constant, independent of preference shocks τ, denoted $\bar{c}_2(\theta = 2, \omega, \lambda)$.

By the same logic, then, as we roll the dynamic program back to date $t = 1$, incentive-compatibility conditions (4) at $\theta = 2$ with $c_2(\tau, \theta = 2, \omega, \lambda) = \bar{c}_2(\theta = 2, \omega, \lambda)$, for all τ, imply that $c_1(\tau, \theta = 2, \omega, \lambda)$ must be some constant, independent of preference shocks τ, denoted $\bar{c}_1(\theta = 2, \omega, \lambda)$. Of course, no problem of this kind emerges for agents who do not move; with past histories fully observed by at least two agents present in each location, past histories can be made public to all agents present, as in Harris and Townsend (1981). With the assumption of full commitment, then, intertemporal tie-ins can still be used to distinguish agents by preference shocks τ in a beneficial way. That is, those who stay at a location with their cohorts can enter into more effective agreements than those who must deal in the future with relative strangers. But overall, from the point of view of the planning period, all agents are made worse off by the absence of cross-location communication. Consumption is now dependent on whether or not an agent is a mover and a mover's consumption path is independent of shocks τ.

V. Tokens as a Communication Device

Now suppose that there is some object in the environment that is intrinsically useless, that can be carried about and concealed by the agents, and that otherwise is subject to strict societal control. That is, the object can be manufactured and distributed to agents only under agreed-on rules.

Such tokens in our private information, limited communication environment can be enormously beneficial. In fact, for the environment considered in this paper, one can recover the solution to the original, full communication programming problem. Consider the following scheme. At date $t = 1$, all agents are again to announce preference shocks τ. Let those who are designated movers and who declare themselves to have a particular τ value consume the date 1, $c_1(\tau, \theta = 2, \omega, \lambda)$ solution to the original full communication programming problem. Further, let those who were to consume more in the second period under the $c_2(\tau, \theta = 2, \omega, \lambda)$ solution receive more tokens in the first period in a monotone fashion, under some allocation rule $m(\tau, \omega, \lambda)$. That is, if $c_2(\tau, \theta = 2, \omega, \lambda) < c_2(\tau', \theta = 2, \omega, \lambda)$, let

$m(\tau, \omega, \lambda) < m(\tau', \omega, \lambda)$. Thus higher levels of tokens are to entitle movers to higher levels of second-period consumption. In fact, at date $t = 2$, let movers declare one of the possible values of these privately observed token holdings, some value of $m(\tau, \omega, \lambda)$ for a possible value of τ. As more is preferred to less, agents will never declare fewer tokens than they actually hold, and the target consumption bundles $c_2(\tau, \theta = 2, \omega, \lambda)$ are achieved. As is apparent, then, movers face the same menu of consumption streams as in the solution to the original programming problem, so that solution can be implemented here in an incentive-compatible way.

Of course the key to achieving this result is the alteration, with the introduction of tokens, of the second-period incentive-compatibility constraints themselves. Conditions (5) are now replaced with the following. Let $m(\tau, \omega, \lambda)$ denote actual, beginning-of-second-period token holdings of a mover conditioned on having announced shock τ at date $t = 1$ and conditioned on the economywide state (ω, λ). Also let $f(m, \omega, \lambda)$ denote the date $t = 2$ deposit or payment of tokens conditioned on an announcement of m units of tokens, on being a mover, and on the economywide state (ω, λ). Finally, let $c_2(m, \theta = 2, \omega, \lambda)$ denote the proposed consumption bundle under these latter conditions. Now suppose that an individual enters date $t = 2$ with $m(\tau, \omega, \lambda)$ units of tokens, so that he is a mover and had announced shock τ at date $t = 1$. Then, in contemplating a counterfactual announcement of $m = m(\tau', \omega, \lambda)$, either

$$m(\tau, \omega, \lambda) < f[m(\tau', \omega, \lambda), \omega, \lambda], \tag{6}$$

so that the counterfactual announcement is not feasible, or

$$\begin{aligned} &V\{c_2[m(\tau, \omega, \lambda), \theta = 2, \omega, \lambda], \tau\} \\ &\geq V\{c_2[m(\tau', \omega, \lambda), \theta = 2, \omega, \lambda], \tau\}, \end{aligned} \tag{7}$$

so that "honest" announcements of tokens are incentive compatible. The point, of course, is that with the kind of societal control assumed in this paper, tokens are reliable records of past actions even among relative strangers. Equations (6) and (7) allow more effective indexation on unobserved first-period announcements than (5) does.

VI. Unobserved Trades and the Emergence of an Exchange Rate

Thus far tokens allow agents with different histories to be distinguished in the obvious way: those entitled to higher second-period consumptions carry a greater number of tokens. But there is no natural value for tokens per se. They serve as badges or stamps, as if a receipt for some past action or transfer. For three actions to be distinguished, say actions 1, 2, and 3, it is enough that the three actions entitle one to a different number of tokens, say low, medium, and high. Relative differences, such as between low-medium and medium-high, do not matter.

As tokens would seem in many circumstances to be more than a badge or stamp, in fact to have some value or price in terms of consumption goods, and as our interest here is in the number of tokens in the economy as a function of the state of the economy, something must be done to alter the model. The idea here is to weaken ex ante beneficial restrictions on trade by supposing additional private information, that strangers carrying tokens can make deals with one another on the side, unobserved and anonymously, both before and after they appear before the local distribution center of their new location.

It should be noted that, despite this modification, the full commitment postulate remains in place. Agents agree in the planning period to show up at the distribution center of their second-period location assignment and are restricted there to making announcements in some prespecified message space and to engaging in prespecified trades as a function of messages. This they still do. But now consumption cannot take place at the distribution center of the second period. Rather, movers are assigned, both before and after they appear at the distribution center, to a foggy location where they can make unobserved deals with one another. In fact, just prior to their arrival at the center of their new location, movers can commit in the foggy location to trades in consumption and tokens, and again, as with the $t = 0$ initial agreement, these side exchanges are honored. In short, what is weakened in the new environment is the amount of public information, not the amount of commitment.

It is important also not to weaken or, better put, to strengthen the communication technology at the same time. If agents were to carry tokens with their names on them and agents were identified by name at exchange windows of the second location, then this side exchange could be precluded. But to explore the communication and record-keeping idea of the paper, we must somehow take seriously that cross-location communication can be limited a priori, either exogenously, as here, with uniform tokens or endogenously, as in Section VIII, where it is costly to make distinctions.

To motivate the formal description of the unobserved side exchanges that now are possible, it is useful to consider first two proposed second-period allocation rules, functions that map announced (and actual) token holdings into second-period consumption. The first proposed allocation rule appears in figure 1. Under it, house-

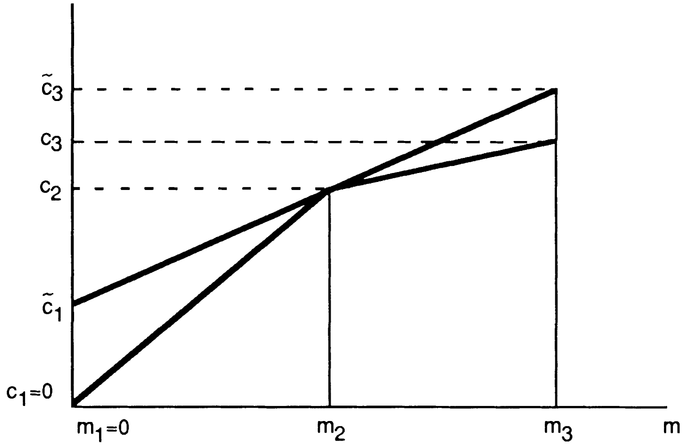

FIG. 1

holds announcing token level m_j are to get second-period consumption allocation c_j, $j = 1, 2, 3$, and it is supposed that there is a nontrivial fraction of agents who would announce each of these m's if this rule were to prevail. But now consider some linear schedule that pivots on the intermediate point (m_2, c_2) and is otherwise above the old schedule. The claim is that if agents of types m_1 and m_3 pool tokens ex ante and pool consumption ex post, then points $\tilde{c}_1$ and $\tilde{c}_3$ can be obtained for these types, respectively, so that types m_1 and m_3 are strictly better off.

Suppose that number n_3 agents are carrying m_3 units of tokens and give up $(m_3 - m_2)$ units of tokens, and number n_1 agents acquire $(m_2 - m_1)$ units of tokens. Then if

$$n_1(m_2 - m_1) = n_3(m_3 - m_2), \tag{8}$$

this redistribution of tokens is feasible, with all agents carrying m_2 units of tokens to the distribution center and acquiring c_2 there. Further, multiplying (8) by the slope s of the linear pivot schedule of figure 1 yields as a feasible redistribution $(c_2 - \tilde{c}_1)$ "taxed" from agents initially carrying m_1 and $(\tilde{c}_3 - c_2)$ given to agents initially carrying m_3. Finally, the numbers n_1 and n_2 are not critical for this analysis; only their ratio,

$$\frac{n_1}{n_3} = \frac{m_3 - m_2}{m_2 - m_1} \equiv m^*, \tag{9}$$

matters. Thus all that is required is that a group of finite size N be divided into a fraction $\rho = m^*/(1 + m^*)$ of agents initially carrying m_1 and a fraction $(1 - \rho)$ of agents carrying m_3. (Since the rational num-

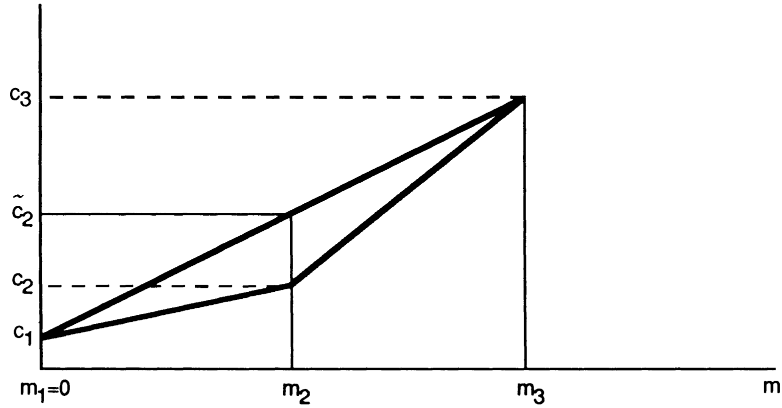

FIG. 2

bers are dense in the real line, the argument is easily modified, for ρ is not rational. On the other hand, note that the argument still holds even if values of m are restricted to integers, as if there were an indivisibility.)

The second proposed allocation rule appears in figure 2. Under it, let a group of agents of number N each initially carrying m_2 units of tokens get together; let a fraction ρ agree to acquire $(m_3 - m_2)$ units of tokens and let a fraction $(1 - \rho)$ surrender $(m_2 - m_1)$ units. If

$$\rho N(m_3 - m_2) = (1 - \rho)N(m_2 - m_1), \tag{10}$$

this redistribution of tokens is feasible. Further, multiplying (10) by the slope s of the linear schedule connecting the endpoints in figure 2 yields agents who pretend to be type m_3 surrendering $(c_3 - \tilde{c}_2)$ units of consumption and agents who pretend to be type m_1 acquiring $(\tilde{c}_2 - c_1)$ units of consumption as a feasible redistribution of the consumption good. Thus each pretender ends up with $\tilde{c}_2$. Of course, $\tilde{c}_2 > c_2$, and so each of the N agents is made better off by this scheme.

Only linear schedules escape the kind of manipulations described in figures 1 and 2. The slope of any linear schedule is the obvious price of tokens in terms of the consumption good, the inverse of the nominal price of consumption. The intercept of the linear schedule is interpreted as a guaranteed minimal consumption.

To proceed more formally, it is supposed first that some *arbitrary* schedule $c_2(m, \theta = 2, \omega, \lambda)$ is proposed for movers announcing m given state (ω, λ). Second, following Townsend (1978, 1983), a game that allows side payments yet preserves anonymity among movers is defined. Third, a symmetric outcome of the game is described, inducing some de facto consumption schedule $\tilde{c}_2(m, \theta = 2, \omega, \lambda)$. In particular, it is argued that the de facto schedule must be linear, for otherwise it could not be an equilibrium outcome, as already suggested above. Fourth, the obvious conclusion is that the initial schedule $c_2(m, \theta = 2, \omega, \lambda)$ may as well have been taken to be linear. The formal argument is somewhat tedious and is reserved for the Appendix.

VII. Optimal Variability in Tokens and Named Credits

With second-period consumption for movers a linear function of token holdings, there emerges a natural price system, and so we would appear ready to measure the amount of nominal tokens in the economy needed to support an optimal allocation for each state (ω, λ), to compute real token holdings, and to compare real measures of tokens and more standard named credits. That is, one could find solutions $c_2(\tau, \theta = 2, \omega, \lambda)$ to the full communication program, and given a price $p(\omega, \lambda)$ of consumption for tokens, holdings $m_2(\tau, \omega, \lambda)$ would be easily determined, as in figure 3, allowing a nonzero intercept. Given price $p(\omega, \lambda)$, relative differences among tokens now matter. Relative differences in consumption must be supported by relative differences in tokens. In this sense tokens are no longer just a badge or stamp.

Still, a residual problem remains for the model under consideration: the exchange rate $p(\omega, \lambda)$ need not be pinned down until *after* state (ω, λ) is realized. To put this another way, different lines in figure 3 imply different amounts of tokens in the system, and since any line will do for each state (ω, λ), there is no legitimate way to compare nominal magnitudes across states. In fact, this is a familiar "problem" for the model as it stands: the scale or nominal magnitude on the entire stochastic process for tokens is arbitrary. Yet we shall want to compare nominal magnitudes cross-sectionally over economies that have experienced different, realized shocks.

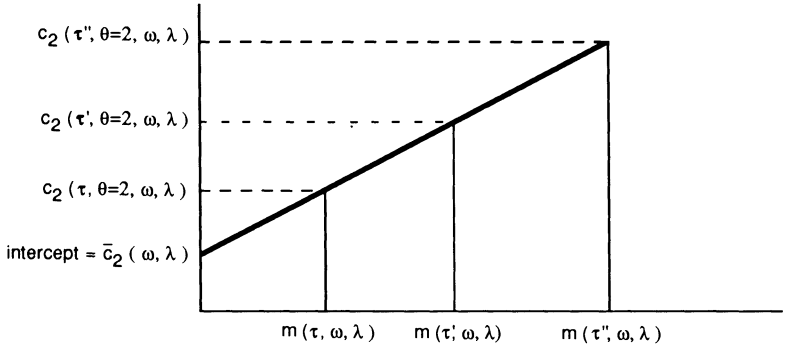

FIG. 3

Two further modifications to the model will thus be considered. The second, to be considered in the next section, allows the issue of tokens to be costly. Here, in this section, tokens are still virtually costless, but one allows some initial conditions to be predetermined and, below, constant across economies (to allow across comparisons relative to realized shocks). This is accomplished formally as follows.

Suppose that in each location at the first date there are two rounds of departures of movers, with nonmovers in the population as the residual. The fraction of first-round movers is λ_1, and a fraction $\omega_1(\tau)$ *of these movers* experience preference shock τ. For the purpose of an example, let λ_1 and $\omega_1(\tau)$ be deterministic, so that there is no uncertainty about these fractions, and notation for these fractions in consumptions can be suppressed. Consumption by first-round movers takes place in the first period prior to departure. *After* first-round movers have left, the fraction of second-round movers is determined. The fraction of second-round movers is λ_2, and a fraction $\omega_2(\tau)$ *of these movers* experience preference shock τ. The fractions λ_2 and $\omega_2(\tau)$ are drawn randomly with probabilities prob(λ_2, ω_2), and their realizations are known to everyone. However, as noted, consumptions of first-round movers are already determined, and so these cannot be indexed by the state (λ_2, ω_2). After second-round movers have departed, a fraction $\lambda_3 = 1 - \lambda_1 - \lambda_2$ of agents in the population are in the residual category, and a fraction $\omega_3(\tau)$ *of these nonmovers* experience preference shock τ. For the purpose of an example, let $\omega_3(\tau)$ be deterministic. Of course, λ_3 is determined as a residual given λ_2. Thus notation for fractions λ_3 and ω_3 is suppressed from consumptions.

Thus let $c_1(\tau, \theta = 1)$ denote consumption at date 1 for first-round movers, $\theta = 1$, as a function of announced (and actual) preference shocks τ, and let them receive tokens in the amount $m(\tau, \theta = 1)$. As will be noted, token levels $m(\tau, \theta = 1)$ must be somewhat arbitrary. Let $c_1(\tau, \theta = 2, \omega_2, \lambda_2)$ and $m(\tau, \theta = 2, \omega_2, \lambda_2)$ denote first-period consumption and token amounts to second-round movers, $\theta = 2$, as functions of announced (and actual) preference shocks τ and state (ω_2, λ_2). As will be noted, token amounts $m(\tau, \theta = 2, \omega_2, \lambda_2)$ will be determinate given the $m(\tau, \theta = 1)$ levels. Let $c_1(\tau, \theta = 3, \omega_2, \lambda_2)$ denote first-period consumption for nonmovers. Let $p(\omega_2, \lambda_2)$ denote the price of tokens in terms of consumption at date 2, the inverse of the nominal price level, and let $\bar{c}(\omega_2, \lambda_2)$ denote the intercept of the linear consumption-token schedule. Finally, let $c_2(\tau, \theta, \omega_2, \lambda_2)$ denote second-period consumption for $\theta = 1, 2, 3$ agents as a function of preference shocks τ and state (ω_2, λ_2). Note that these consumptions are endogenous for movers, $\theta = 1, 2$ agents, in the sense that they are determined by $p(\omega_2, \lambda_2)$, $\bar{c}(\omega_2, \lambda_2)$, and the specification of tokens.

That is, for each (ω_2, λ_2) state,

$$c_2(\tau, \theta = 1, \omega_2, \lambda_2) = \bar{c}(\omega_2, \lambda_2) + m(\tau, \theta = 1)p(\omega_2, \lambda_2), \quad (11)$$

$$\begin{aligned} c_2(\tau, \theta = 2, \omega_2, \lambda_2) &= \bar{c}(\omega_2, \lambda_2) \\ &+ m(\tau, \theta = 2, \omega_2, \lambda_2)p(\omega_2, \lambda_2). \end{aligned} \quad (12)$$

With this notation and with $\bar{e}$ denoting the per capita endowment of the consumption good at date 1, now possibly different from unity, the program for the determination of Pareto-optimal consumptions and tokens follows.

PROGRAMMING PROBLEM 2. Maximize by choice of $c_1(\tau, \theta = 1)$, $m(\tau, \theta = 1)$, $c_1(\tau, \theta = 2, \omega_2, \lambda_2)$, $m(\tau, \theta = 2, \omega_2, \lambda_2)$, $c_1(\tau, \theta = 3, \omega_2, \lambda_2)$, $\bar{c}(\omega_2, \lambda_2)$, $p(\omega_2, \lambda_2)$, and $c_2(\tau, \theta, \omega_2, \lambda_2)$, $\theta = 1, 2, 3$, the objective function

$$\begin{aligned} \sum_{\omega_2, \lambda_2} \text{prob}(\omega_2, \lambda_2) \Bigg(\lambda_1 \sum_{\tau} \omega_1(\tau)\{U[c_1(\tau, \theta = 1), \tau] \\ + V[c_2(\tau, \theta = 1, \omega_2, \lambda_2), \tau]\} \\ + \lambda_2 \sum_{\tau} \omega_2(\tau)\{U[c_1(\tau, \theta = 2, \omega_2, \lambda_2), \tau] \\ + V[c_2(\tau, \theta = 2, \omega_2, \lambda_2), \tau]\} \\ + \lambda_3 \sum_{\tau} \omega_3(\tau)\{U[c_1(\tau, \theta = 3, \omega_2, \lambda_2), \tau] \\ + V[c_2(\tau, \theta = 3, \omega_2, \lambda_2), \tau]\}\Bigg) \end{aligned} \quad (13)$$

subject to resource constraints, for each (ω_2, λ_2) configuration,

$$\begin{aligned} \lambda_1 \sum_{\tau} \omega_1(\tau)c_1(\tau, \theta = 1) + \lambda_2 \sum_{\tau} \omega_2(\tau)c_1(\tau, \theta = 2, \omega_2, \lambda_2) \\ + \lambda_3 \sum_{\tau} \omega_3(\tau)c_1(\tau, \theta = 3, \omega_2, \lambda_2) = W(\omega_2, \lambda_2), \end{aligned} \quad (14)$$

$$\begin{aligned} \lambda_1 \sum_{\tau} \omega_1(\tau)c_2(\tau, \theta = 1, \omega_2, \lambda_2) + \lambda_2 \sum_{\tau} \omega_2(\tau)c_2(\tau, \theta = 2, \omega_2, \lambda_2) \\ + \lambda_3 \sum_{\tau} \omega_3(\tau)c_2(\tau, \theta = 3, \omega_2, \lambda_2) = [\bar{e} - W(\omega_2, \lambda_2)]R \end{aligned} \quad (15)$$

subject to the incentive constraints, for $\theta = 1$ movers and possible values of τ and τ',

$$U[c_1(\tau, \theta = 1), \tau] + \sum_{\omega_2, \lambda_2} \text{prob}(\omega_2, \lambda_2) V[c_2(\tau, \theta = 1, \omega_2, \lambda_2), \tau]$$
$$\geq U[c_1(\tau', \theta = 1), \tau] + \sum_{\omega_2, \lambda_2} \text{prob}(\omega_2, \lambda_2) V[c_2(\tau', \theta = 1, \omega_2, \lambda_2), \tau], \tag{16}$$

for $\theta = 2, 3$, for each (ω_2, λ_2) configuration, and for all τ and τ' values,

$$U[c_1(\tau, \theta, \omega_2, \lambda_2), \tau] + V[c_2(\tau, \theta, \omega_2, \lambda_2), \tau]$$
$$\geq U[c_1(\tau', \theta, \omega_2, \lambda_2), \tau] + V[c_2(\tau', \theta, \omega_2, \lambda_2), \tau], \tag{17}$$

and subject to constraints (11) and (12) above.

For the purpose of a numerical example, let first- and second-period utility functions be

$$U(c, \tau) = \frac{c^{\tau} - 1}{\tau}, \quad V(c, \tau) = \frac{c^{(1-\tau)} - 1}{1 - \tau},$$

with $0 < \tau < 1$, so that high τ values make a household relatively urgent to consume in the first period. For first-round movers, let fraction $\lambda_1 = 1/4$ for sure and possible τ values be .35 and .6, with fractions $\omega_1(\tau = .35) = 1/2$ and $\omega_1(\tau = .6) = 1/2$ for sure. For second-round movers, let possible values for λ_2 be $1/4$ and $1/2$, each drawn with probability $1/2$, and let the possible τ values be .35 and .6 again, with fractions $\omega_2 = [\omega_2(\tau = .35), \omega_2(\tau = .6)]$ either $[2/3, 1/3]$ or $[1/3, 2/3]$, each possibility drawn with probability $1/2$, independent of the λ_2 draw. For nonmovers, $\lambda_3 = 1 - \lambda_1 - \lambda_2$, and the possible τ value is degenerate at $\tau = .5$ so that $\omega_3(\tau = .5) = 1$ for sure. Also let the storage return $R = 1$ and let the per capita endowment $\bar{e} = 10$. Finally, for first-round movers, let $m(\tau = .35, \theta = 1) = 10$ and let $m(\tau = .6, \theta = 1) = 0$ as a somewhat arbitrary initial condition. Naturally, though, patient first-round movers, that is, low-τ agents, receive more tokens than first-round movers who are urgent to consume.

A solution to program 2 with this specification of the environment has been computed by a numerical maximum procedure.[5] Some properties can be noted.[6] First the obvious: urgent consumers eat

[5] Technically, the search for solutions is facilitated by the observation that consumptions can be found directly as a solution to a concave programming problem, and values for tokens, prices, and intercepts can be filled afterward as solutions to eqq. (11) and (12). Also, values of τ were deliberately chosen in such a way as to make the incentive constraints nonbinding. In practice this was done by computing solutions for given τ values, ignoring the incentive constraints, and then checking to see if the incentive constraints were satisfied. In principle, solutions with binding incentive constraints can be found, but the search procedure would be more time intensive.

[6] A complete tabulation appears in table 1 of my working paper (Townsend 1988*b*).

more in the first period than in the second period, uniformly over states (ω_2, λ_2). Of course, first-round movers must eat the same in the first period over all states since the state is not yet revealed, whereas second-round $\theta = 2$ movers do not. Nevertheless, there is no need to distinguish from one another the second-period consumptions of first- and second-round movers, and the optimal solution does not do so. As a consequence, the computed token holdings of second-round movers mimic the preset (arbitrary) token holdings of first-round movers and are either 10 or zero depending on whether the mover is patient or urgent. This in turn makes the per capita token balances in the population a strictly monotone increasing function of the final number of patient movers. In this sense, nominal token balances vary with the state of the economy and do so in the obvious way. Curiously, the price level term $p(\omega_2, \lambda_2)$ and intercept term $\bar{c}(\omega_2, \lambda_2)$ move over states (ω_2, λ_2) in an effort to support the configuration of consumptions of patient and urgent movers, and this configuration does change over states. These can move over states in which the number of patient movers is the same because the distribution of τ shocks in the population changes over such states. The associated price movement over these states makes real token holdings different from nominal token holdings, and thus there is more state dependence in real token holdings than in nominal holdings.

One might take as a measure of real named debt in this economy the amount of second-period consumption claimed by nonmovers, namely $\lambda_3 c_2(\tau = .5, \theta = 3, \omega, \lambda)$ since this is achieved by cohort-specific accounts with the identity of first-period agents intact. This measure of debt also moves with the state. So also does total second-period consumption, as a measure of total indebtedness. Finally, the ratio of real tokens to total indebtedness also moves with the state (and this is true as well when intercepts are added to the purchasing power of tokens), as does the ratio of real tokens to named debts.[7]

VIII. On the Costs of Tokens and Named Credit Systems

Part, but not all, of the analysis thus far hinges on some implicit assumptions about the costs of various technologies that should now be brought out more fully. First, throughout the analysis, the cost of setting up, maintaining, and using any within-location accounting system is presumed to be zero. Second, from the analysis of Section IV

[7] There are no indications that this example is exceptional in any way. Various parameter configurations have also yielded similar qualitative properties over states. Tokens and named credits are distinguished and vary optimally with the state.

onward, the cost of direct communication across locations is presumed to be infinity (whereas in Sec. III it is zero). Third, in order to remove indeterminacy as between the use of tokens and within-location accounts, the cost of tokens is nonzero but essentially negligible. This last assumption requires some elaboration.

A problem that emerges if tokens are completely costless is that tokens can be used by nonmovers. That is, within-location accounts would not be needed, and one could get by with just one asset. In this case, measures of tokens would be pinned down and would still move around, at least on the assumption that initial movers must carry some tokens with them. But one could not compare tokens to credit in such a model.

The obvious remedy is to make the issue of tokens costly. Thus suppose that there is some minimal size token, say one unit, and each unit issued at date 1 after the revelation of shocks costs ψ units of the consumption-investment good at date 1. With this cost it is clear that nonmovers should always use within-location accounts. Movers, on the other hand, do not have fruitful access to within-location accounts and should use either a minimal size configuration of tokens or nothing. The implicit assumption of the analysis is that for every draw of state (ω_2, λ_2), tokens dominate in this choice since they allow consumptions to be indexed by preference shocks τ, even though their issue is costly. This can be delivered formally for *sufficiently low cost* ψ by a continuity argument.

Two further points should be noted in passing. First, the determination of optimal token issue for any positive finite cost ψ would be nontrivial. With the minimal size or indivisibility assumption on tokens, integer amounts of them must be assigned to support planned consumption levels. Still, the token-consumption points must all lie on a linear schedule (from the earlier analysis). A sufficient, but not necessary, condition for this is that planned consumptions be rational numbers, so that one can find among them a least common denominator. But one can get arbitrarily close to any arbitrary consumption array *if* one is willing (and able) to issue the requisite number of tokens. Second, with costly token issue, the nominal issue of tokens would be pinned down for all economywide states even if schedules are determined ex post. The complaint at the beginning of Section VII motivating two rounds of movers is now moot. But examples seem difficult to compute. Again, the solution described earlier is an example in the limit for virtually negligible costs.

Another possibility to ensure the use of both within-location accounts and tokens would be to make tokens work less well when they are used. In particular, one could increase the number of underlying commodities and add second-period shocks to second-period preferences. Then with only one instrument to index second-period consumptions, it is possible that agents might wish to understate token holdings.[8] Since understatement cannot be precluded but is not incentive compatible, the effect would be to mitigate the ability to index second-period consumptions. Thus within-location accounts might dominate. Distinguishing tokens from internal accounts in this manner begs the question, however, of multiple-token issue. In particular, multiple-token issue would give moving agents a complete communication system and, in the absence of costs, could be complete for nonmovers also. Similarly, multiple tokens would allow the early movers in the solution described earlier to be distinguished from late movers, and so again in the absence of costs, there would arise an indeterminacy. Thus it seems that one is driven again to an explicit consideration of costly token issue and the idea that a one-token system might dominate a two-token system if there were differential setup costs. But like the earlier discussion of costs, this takes us beyond the analysis of the present paper.

Appendix

Derivation of de Facto Linear Schedules

The first step is to start with an arbitrary schedule $c_2(m, \theta = 2, \omega, \lambda)$ with the sole restrictions that it be monotone increasing in m (for incentive compatibility) and that m range over a finite number of values. However, if agents are to be allowed to collude in arbitrary ways, then this schedule must specify what is to happen for *arbitrary* fractions $f(m)$ of agents reporting m at the distribution center. That is, some feasible outcome must be specified no matter what agents report. Denote this modified schedule by $c_2[m, \theta = 2, \omega, \lambda, \mathbf{F}]$, where $\mathbf{F}$ denotes the vector of fractions $f(m)$, $m \in M$, and suppose for feasibility that

$$\lambda \sum_{\tau} \omega(\tau) c_2[\tau, \theta = 1, \lambda, \omega, \mathbf{F}] + (1 - \lambda) \sum_{m} f(m) c_2[m, \theta = 2, \lambda, \omega, \mathbf{F}]$$

$$\leq R\left[1 - \lambda \sum_{\tau} \omega(\tau) c_1(\tau, \theta = 1, \lambda, \omega) - (1 - \lambda) \sum_{\tau} \omega(\tau) c_1(\tau, \theta = 2, \lambda, \omega)\right]. \tag{A1}$$

Note that this specification would allow for group penalties for "collusion" *if* this could be detected. "Deviation" by a finite number of players carries no weight, however, that is, does not influence averages. Also note that agents are still restricted at the distribution centers to announcing values of tokens m. Crucial in this is that they do not know whom they have dealt with in the foggy location, for otherwise their collusion could be detected by a more elaborate scheme. Also, despite the possibility of agreed-on side exchanges,

[8] Townsend (1987) provides an example of limited single-token systems and beneficial multiple-token systems.

tokens m on hand are still the only relevant state variable, and so announcements can be restricted to it.

The game to be played by movers against schedule $c_2[\tau, \theta = 2, \lambda, \omega, \mathbf{F}]$ given beginning-of-second-period conditions, M, the finite set of possible values for m under date 1 handouts $m(\tau, \omega, \lambda)$, and $h(m)$ the actual fraction of movers holding balances m, $m \in M$, is described as follows. First, moving agents arriving at a particular location are assigned names in an anonymous fashion, taking numbers one at a time and unobserved by others. The set of names can be taken to be the set of positive integers, and we may refer to a generic agent named i, $i = 1, 2, \ldots$.

Second, each of these agents announces a strategy under which he is willing to be an anonymous go-between, naming a local location for trades but not revealing his identity. More specifically, let $\Delta^i_m(m, +)$ denote the number of pieces of paper (hence the m subscript) proposer i is willing to give out to anyone in the set of movers if m is named (hence the first argument in parentheses) and a plus sign (for handout) is indicated (hence the second argument in parentheses), let $\Delta^i_m(m, -)$ denote the number of pieces of paper required to be handed in to proposer i if m is named and a minus sign is indicated, and let $\Delta^i_c(m, +)$ and $\Delta^i_c(m, -)$ denote handouts and take-ins of the consumption good if m is named and a plus or minus is indicated, respectively. Here it is understood that if an agent goes to proposer i and names m and a plus for paper, he *must* choose a minus for consumption and vice versa. Also, he cannot claim both a plus and a minus for paper at the same m. Of course, an agent could choose to do nothing with a particular proposer i, effectively setting $\Delta^i_m(\cdot, \cdot)$ and $\Delta^i_c(\cdot, \cdot)$ to zero, and the proposer can specify that some of these components be zero as well. For example, motivated by figure 1, some agents of type m_3 might want proposer i to name $\Delta^i_m(m_3, -) = m_3 - m_2$ and agents of type m_1 want $\Delta^i_m(m_1, +) = m_2 - m_1$, with $\Delta^i_c(m_3, +) = s\Delta^i_m(m_3, -)$ and $\Delta^i_c(m_1, -) = s\Delta^i_m(m_1, +)$. Alternatively, motivated by figure 2, some agents of type m_2 might want proposer i to name $\Delta^i_m(m_2, +) = m_3 - m_2$ and $\Delta^i_m(m_2, -) = m_2 - m_1$, with $\Delta^i_c(m_2, -) = s\Delta^i_m(m_2, +)$ and $\Delta^i_c(m_2, +) = s\Delta^i_m(m_2, -)$. A further component of the strategy of proposer i is specification of a maximal *finite* number $\bar{g}^i(m, +)$ of households that can come to him and announce m and a plus for paper, and similarly $\bar{g}^i(m, -)$ for announcement of m and a minus. Let S_i denote the strategy of agent i.

The third event of the game is that moving agents go in turn to the locations of the proposers and choose actions $(m, \pm)$, where here and below the plus/minus notation indicates that either a plus or a minus must be filled in. Thus $(m_{ji}, \pm)$ denotes the action taken by player j with proposer i. Whatever nontrivial action $(m_{ji}, \pm)$ is taken by player j with proposer i, it is registered on proposer i's computer. Further, proposer i himself may choose to take an action with himself, and if done this is also registered on his computer. The strategy that player j adopts with proposer i is a function of balances m carried in from his initial location as well as whether or not the quotas $\bar{g}^i(\tilde{m}, \pm)$, $\tilde{m} \in M$, are filled or not by the time it is player j's turn, something that is observable at the location of proposer i (though the identity and specific moves of previous players with proposer i are concealed from player j). Thus let $g^i_j(\tilde{m}, \pm)$ denote the number of players who have come to proposer i and indicated $(\tilde{m}, \pm)$ prior to the arrival of j, and let quota *indicator* $I_j(\cdot, \cdot)$ be

$$I^i_j(\tilde{m}, \pm) = \begin{cases} 1 & \text{if } g^i_j(\tilde{m}, \pm) = \bar{g}^i(\tilde{m}, \pm) \\ \\ 0 & \text{if } g^i_j(\tilde{m}, \pm) < \bar{g}^i(\tilde{m}, \pm). \end{cases}$$

The strategy of player j with proposer i is denoted $\sigma_{ji}(m, \{I_j^k(\tilde{m}, \pm)\})$, where it is understood that k ranges over proposers $\{1, 2, \ldots\}$, and for each k, $\tilde{m}$ ranges over M. A player j can deal with more than one proposer i. But each player j is fully committed (somehow) to carry out his action with any proposer i, that is, eventually to arrive with currency balances if necessary prior to participation at the distribution center and to arrive with the consumption good if necessary afterward. We do not ask how this commitment is enforced but take it as given that some commitment among agents is necessary for the initial allocaton rule $c_2(\cdot)$ to be weakened.

Of course, each agent j knows the actual distribution of type m movers in the population. Thus given a specification of strategies $\sigma_l^*(\cdot, \cdot)$ of each of the other players l, $l = 1, 2, \ldots, l \neq i$, each player i has well-defined expectations over the eventual outcome, in particular over the induced $\mathbf{F}$. Thus in a *Nash equilibrium* of the second stage of the game, given individual balances m for player j and quota indicators $\{I_j^k(\tilde{m}, \pm)\}$ over proposers k, strategy $\sigma_{ji}^*(m, \{I_j^k(\tilde{m}, \pm)\})$, $i = 1, 2, \ldots$, for agent j solves the following problem: Maximize by choice of the $(m_{ji}, \pm)$ the objective function

$$EU\left\{c_2\left[m + \sum_{i=1}^{\infty} \Delta_m^i(m_{ji}, +) - \sum_{i=1}^{\infty} \Delta_m^i(m_{ji}, -), \theta = 2, \omega, \lambda, \mathbf{F}\right] + \sum_{i=1}^{\infty} \Delta_c^i(m_{ji}, +) - \sum_{i=1}^{\infty} \Delta_c^i(m_{ji}, -)\right\}. \tag{A2}$$

Here, of course, currency balances are adjusted before announcements are made at the distribution center and consumption levels are adjusted afterward.

Given a specification of Nash equilibrium decision rules $\sigma_l^*(\cdot, \cdot)$ at the second stage over all players l, each agent i chooses an initial intermediation strategy $S_i = S_i^*$ to be maximal among all feasible strategies given the strategies S_k^* of other players k. Feasibility means that i believes that he is able to abide by the strategy he announces for all possible realizations of random variables, that he can honor his commitments to hand out tokens and consumption.

A complete specification of a *Nash equilibrium* for a game is a specification of Nash strategies S_k^* at the first stage of the game and Nash equilibrium decision rules $\sigma_l^*(\cdot, \cdot)$ at the second.

Attention will be restricted in what follows to equilibrium outcomes of a game that are the same for any moving agent of type m, regardless of the numbered turn a type m agent is assigned. It will be supposed similarly that the numbering of players is immaterial to the outcome of a game. Then, for any proposed schedule $c_2(m, \theta = 2, \lambda, \omega, \mathbf{F})$, there emerges a de facto schedule $\tilde{c}_2(m, \theta = 2, \lambda, \omega)$ defined in the obvious, more concise notation than (A2), for any player j of type m:

$$\tilde{c}_2(m, \theta = 2, \lambda, \omega) = c_2\left[m + \sum_{i=1}^{\infty} \Delta_m^{i*}(m_{ji}, \pm), \theta = 2, \lambda, \omega, \mathbf{F}^*\right] + \sum_{i=1}^{\infty} \Delta_c^{i*}(m_{ji}, \pm), \tag{A3}$$

where here $\Delta_m^{i*}(\cdot, \cdot)$ and $\Delta_c^{i*}(\cdot, \cdot)$ denote part of the Nash equilibrium strategy S_i^* of proposer i, where the $(m_{ji}, \pm)$ are determined under the Nash equilibrium decisions $\sigma_{ji}^*(m, \cdot)$, and where $\mathbf{F}^*$ is the distribution of types at the distribution point induced by the Nash equilibrium strategies.

It should now become apparent that the effective schedule $\tilde{c}_2(m, \theta = 2, \lambda, \omega)$ must be linear in m. For suppose otherwise. In fact suppose that the de facto schedule appears as in figure 1. We may suppose that second-period strategies are maximal given a configuration of first-period proposals, consistent with the definition of equilibrium and the definition of a de facto schedule. But the specification of proposals itself could not be maximal. One agent type m_1 or m_3 could propose some preplay redistribution of m and postplay redistribution of c for some *finite* coalition of these types, as described above. All such types would voluntarily come to such a proposer making themselves strictly better off, and since they have zero mass in the aggregate, $\mathbf{F}^*$ would be unaltered by these actions. Further, even if the proposer himself changed from an earlier strategy, this cannot alter $\mathbf{F}^*$ since the proposer was able to deal only with a finite number of agents. That is, virtually all agents must have achieved the earlier supposed equilibrium outcome without going through the proposer, and so agents who had gone through the proposer originally still have the same distribution of the outcomes available to them, doing now what others had done after them originally, and so on. Thus the original specification could not have been a Nash equilibrium. Similarly, the de facto schedule depicted in figure 2 cannot be associated with a Nash equilibrium. And since this argument applies for *any* three specifications of m, regardless of the dimensionality of M, linearity must hold over all $m \in M$.

References

Dasgupta, Partha S.; Hammond, Peter J.; and Maskin, Eric S. "The Implementation of Social Choice Rules: Some General Results on Incentive Compatibility." *Rev. Econ. Studies* 46 (April 1979): 185–216.

Diamond, Douglas W., and Dybvig, Philip H. "Bank Runs, Deposit Insurance, and Liquidity." *J.P.E.* 91 (June 1983): 401–19.

Harris, Milton, and Townsend, Robert M. "Resource Allocation under Asymmetric Information." *Econometrica* 49 (January 1981): 33–64.

Myerson, Roger B. "Incentive Compatibility and the Bargaining Problem." *Econometrica* 47 (January 1979): 61–73.

Townsend, Robert M. "Intermediation with Costly Bilateral Exchange." *Rev. Econ. Studies* 45 (October 1978): 417–25.

———. "Optimal Multiperiod Contracts and the Gain from Enduring Relationships under Private Information." *J.P.E.* 90 (December 1982): 1166–86.

———. "Theories of Intermediated Structures." *Carnegie-Rochester Conf. Ser. Public Policy* 18 (Spring 1983): 221–72.

———. "Economic Organization with Limited Communication." *A.E.R.* 77 (December 1987): 954–71.

———. "Information Constrained Insurance: The Revelation Principle Extended." *J. Monetary Econ.* 21 (March/May 1988): 411–50. (*a*)

———. "Optimal Activist Currency Rules." Manuscript. Chicago: Univ. Chicago, Dept. Econ., March 1988. (*b*)