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Economic Organization with Limited Communication

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This paper presents formal, stylized representations of communication-accounting systems: oral assignment, portable object, written message, and telecommunication systems are considered. The environments that allow this formalization are characterized by spatial separation, private information, and a need to keep track of past actions, transfers, and shocks.

In environments that have spatial separation and private information, beneficial multilateral arrangements can depend critically on agents' ability to communicate to one another values of contemporary shocks and to keep track of histories of past transfers or past-announced shocks. This paper formalizes this idea and focuses on communication-accounting systems. The theory of this paper allows a formal, stylized representation of a variety of systems and allows one to make precise the sense in which various systems are more or less limited. Oral assignment systems, portable object systems, written message systems, and telecommunication systems are considered.

In its method this paper follows the literature on contract theory and mechanism design, of Milton Harris and Robert Townsend (1981), Roger Myerson (1979), and Townsend (1982), for example, stressing private information and incentives. The idea, essentially, is to specify the agents' endowments and preferences and the production technology available to them, and to be precise about the information structure. Then, rather than imposing a fixed-contract form or fixed-resource allocation scheme, one considers a broad class of arrangements and determines the constraints implied by private information. One then goes on to determine Pareto-optimal arrangements by maximizing weighted sums of the agents' utilities, subject to the obvious resource constraints and these derived, information, incentive compatibility constraints.

This paper takes this method one step further by making explicit both the agents' locations at various times and the technology of communication available to agents over space and over time. Exogenous variations in the technology of communication thus cause endogenous variations in the derived incentive constraints and, in this way, in the context of the class of maximization problems, one can capture formally the idea that communication systems matter and that particular systems may be more or less limited. Indeed, oral assignment systems, portable object systems, written message systems, and electronic telecommunication systems can be ordered: these are successively less limited.

The communication-accounting systems considered in this paper are motivated by observations from "simpler," historical, and contemporary economies. Oral communication systems are those in which agents must literally get together with one another in order to give instructions to one another or to execute some prespecified arrangement. For example, banking at the beginning of the commercial revolution in Europe took the form of an oral assignment system, as described by Abbott Usher (1943). Both the

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buyer of a good and its seller had to appear before the banker of the seller, and instructions were given to the banker for the transfer of the buyer's account to the seller.

Communication systems with portable tokens are those in which agents receive or surrender tokens at various spatially dispersed locations as evidence of actions taken or claims made under some prespecified arrangement. In World War II crucial commodities apparently were rationed in this way, with coupons providing evidence to the seller that the consumer was "entitled" to the good, and providing evidence to the government that the seller was in compliance with limited sale provisions. As another instance of the use of tokens, it seems that taverns in England in the 1600's handed out idiosyncratic tokens as small change, entitling the customer, or anyone else, to future purchases. Credit could also be granted with the handing out of such tokens. Apparently related, workers in coal mining towns in West Virginia or in textile mills in South Carolina received company script, entitling them to foods and other purchases at the company store. And multiple-token systems may have been in widespread use in some economies at one time. Anthropologists Raymond Firth (1939), Lorraine Baric (1964), and B. Malinowski (1953) report on ceremonial exchanges in "simpler" economies involving the transfer of prespecified goods for prespecified objects such as shells and fishhooks, perhaps to encourage production and distribution of prespecified goods or to encourage location assignments for certain agents.

Communication systems with writing seem commonplace. A mundane but familiar example is the use of customer pickup orders, which the customer carries from the checkout cashier, where payment is made, to the warehouse department. Related are the papers that a patient hand carries in a hospital as orders for services in various decentralized clinics. Similar also are the papers that circulate in firms, written orders for materials from internal stocks. Checks are written messages sent to one's banker, and bills of exchange are written messages to one's trading partner. Telecommunication systems are more familiar recently, with electronic, interspatial verification of credit and bank balances.

Unfortunately, real-world examples bring in important complications, such as limited commitment and default possibilities. The theory is not ready for these complications. In fact the theoretical literature that allows us to begin to think about communication-accounting systems, even without these complications, is not large. The purpose of this paper, then, is to attempt to bring the theory up to speed by consideration of simple, prototype environments, starting with the premise of perfect costless enforcement and varying communication technologies exogenously. More limited commitment possibilities, and the interaction of these with communication systems, are then addressed toward the end of the paper. This leaves more detailed, applied work and the requisite modifications of the theory to subsequent research efforts.

The literature related to the research reported here is that on limited communication in resource allocation mechanisms, of Leonid Hurwicz (1972), Kenneth Mount and Stanley Reiter (1974), and others. But the formal approaches differ considerably. Here spatial separation is made explicit and the messages are restricted by whether agents can talk to one another and by the objects that agents carry from place to place. Other related literatures have to do with the interaction between communication-accounting systems and money. Douglas Gale (1980) makes reference to paper assets as accounting devices in a world with a continuum of agents and a limited social planner. But Gale focuses on conditions sufficient to ensure that allocations of sequential competitive equilibria with valued money are equivalent with full information (incentive-compatible) Pareto-optimal allocations. Here the focus is on private information Pareto-optimal allocations in worlds with spatial separation and explicit limited communication, in which various kinds of financial systems are associated with various kinds of communication systems. No effort is made here to decentralize these optima. Of course the idea that money reflects some decentralization in the exchange process appears frequently in the literature. One should note here in particular the work of Karl Brunner and Allan Meltzer (1971), in which money emerges in a world with an uneven distribution of information, by reducing somehow the costs of acquiring information and of constructing transaction chains. Also related is the work of Joseph Ostroy (1973) and Ostroy and Ross Starr (1974), in which trading rules are said to be decentralized to the extent that they do not depend on past histories. Again, this paper uses explicit communication technologies to determine the extent to which agents can use past histories, and contemporary announcements in distinct locations, in their trading rules.

Briefly, the paper proceeds as follows. Section I illustrates the role for communication when there is private information, expositing the contract theoretic, mechanism design approach to the determination of Pareto-optimal allocations. This is done in the context of a two-agent, one-period, pure exchange economy with privately observed shocks to preferences. Section II extends the two-agent, one-period environment to one with two periods and concentrates on the possible gain from intertemporal links. Section III extends the two-agent, two-period environment to one with four agents and two locations and delivers examples of beneficial quadrilateral arrangements, at least if there is full communication. Section IV then begins the analysis of limited communication systems, considering first oral assignment systems. A portable concealable object system is considered in Section V, and multiple-portable tokens and written message systems are considered in Section VI. Section VII then analyzes the most powerful technology, costless interspatial electronic telecommunications. Section VIII modifies the basic environment somewhat to show how portable concealable tokens and written messages can serve as location assignment devices. Section IX returns to the basic environment, but eliminates explicit randomness to show how portable tokens can serve as a device for enforcement. Section X concludes the analysis with some comments on privately observed random endowments.

I. Pareto-Optimal Arrangements with Private Information but with Perfect, Costless Enforcement

For an economy specified at the level of preferences, endowments, and technology, and satisfying a few regularity conditions, one can derive Pareto-optimal allocations as solutions to programming problems. That is, if utility functions of agents are concave, if more is preferred to less, and if consumption sets and production-storage technologies are convex, then the utilities possibilities frontier is concave with respect to the origin and contains no "flat" or "horizontal" segments. In that case, any Pareto-optimal allocation is associated with a point on the frontier and therefore with some set of weights associated with a supporting hyperplane. Thus, the problem of maximizing a particular weighted sum of agents' utilities subject to constraints implied by resources and technology yields, as at least one of its solutions, the Pareto point in question. And any solution to such a programming problem, with arbitrary weights, is necessarily Pareto optimal.

This paper envisions the set of institutions of an economy and the allocations that such institutions deliver as being Pareto optimal for the environment of the economy. Thus, it is as if all agents of the economy agreed at some initial date to solve one of the programming problems described above, somehow coming to an agreement about what weights to impose. Further, one might suppose full commitment, that is, perfect and costless enforcement, so that *ex post* the agents just carry out the socially sanctioned plan. Finally, as Kenneth Arrow (1953) and Gerard Debreu (1959) have emphasized, uncertainty, time and dynamics, and separation of agents in space will not alter this picture of the determination of optimal arrangements if we retain the perfect, costless enforcement premise (and do not simultaneously introduce some fundamental nonconvexities).

A feature of this programming approach to the determination of optimal arrangements is that there is nothing for agents to communicate over time. Even the realizations of the random variables of the model need not be communicated because, by assumption, such states of the world are observed by everyone and have been incorporated into the prespecified agreement to allocate labor to production, to transfer goods, and to reallocate agents over space. To allow for communication then, that is, to allow for a discussion of communication-accounting systems, something must be done to alter the model.

The alteration discussed at length in this paper is the incorporation of private information. That is, in addition to uncertainty, realizations of some of the random variables of the model are presumed to be seen by subsets of agents only. Indeed, this is a natural way to allow for communication, because it is known from the work of Myerson (1979) and of Harris and Townsend (1981) that programs for the determination of Pareto-optimal allocations can be utilized if agents are given the opportunity to announce (communicate) values for the subset of random variables, the realizations of which they alone see. That is, one can impose without loss of generality incentive compatibility constraints which ensure that these privately observed realizations are announced truthfully.

Because this idea is at the heart of the analysis, here it is best to review it in the context of a relatively simple environment, one without any essential dynamics and without any spatial separation. Further, to avoid complications introduced by putting private information on quantities, something which will be contemplated later, it is best to begin by supposing privately observed and unverifiable shocks to preferences. The motive for trade in the planning period is insurance.

Thus imagine an economy with just two agents, named a and b, and two time periods, a planning date and a consumption date. Agent a has an endowment e^a in the consumption date, a nonnegative vector of goods. Typically, we shall consider cases in which there is only one good, or two, but the dimension of e, and of consumptions below, can be arbitrary if finite. Agent a has preferences in the consumption date over consumption vector c^a as represented by utility function $U^a(c^a, \theta^a)$. Here θ^a is a shock to a's preferences, observed by agent a alone at the beginning of the consumption date. Also, θ^a takes on values in some set Θ^a. Typically, set Θ^a contains two values, or three, and each value is at most two-dimensional, but again the number of values and the dimension can be arbitrary if finite. Agent b has endowment e^b and preferences $U^b(c^b)$ in the consumption period. Utility functions are concave, and there is no production technology. Further, this structure is common knowledge. That is, everything is known up to shocks θ^a, presumed as of the planning period to occur with probabilities $p(\theta^a)$.

In the planning period both agents commit themselves to an allocation of the consumption goods in the consumption period contingent on agent a's announcement in the consumption period of the shocks he has experienced. That is, both agents precommit to pool their endowments and redistribute the total under a prespecified plan. Further, as a technical matter, this shock contingent allocation rule can be a lottery, $\pi(c|\theta^a)$, specifying the probability of consumption bundle $c = (c^a, c^b)$ to agents a and b. This will ensure that the programming problem is concave (actually linear), despite private information, though no essential use will be made of the lotteries in the solutions reported below. Thus letting $C = \{c = (c^a, c^b); c^a + c^b \leq e^a + e^b\}$ denote the space of feasible consumptions, presumed for simplicity to be finite as if there were some indivisibility, the programming problem for the determination of Pareto-optimal allocations is

Program 1: Maximize by choice of the $\pi(c|\theta^a)$ the objective function

$$(1)\quad \lambda^a\left\{\sum_{\theta^a} p(\theta^a)\sum_c \pi(c|\theta^a)U^a(c^a, \theta^a)\right\} + \lambda^b\left\{\sum_{\theta^a} p(\theta^a)\sum_c \pi(c|\theta^a)U^b(c^b)\right\},$$

subject to the incentive compatibility constraints,

$$(2)\quad \sum_c \pi(c|\theta^a)U^a(c^a, \theta^a) \geq \sum_c \pi(c|\tilde{\theta}^a)U^a(c^a, \theta^a) \quad \forall \theta^a, \tilde{\theta}^a \in \Theta^a.$$

One can easily append onto this program *ex ante* participation constraints,

$$(3)\quad \sum_{\theta} p(\theta^a) \sum_{c} \pi(c|\theta^a) U^a(c^a, \theta^a) \geq \sum_{\theta} p(\theta) U^a(e^a, \theta^a),$$

$$(4)\quad \sum_{\theta} p(\theta^a) \sum_{c} \pi(c|\theta^a) U^b(c^b) \geq U^b(e^b).$$

In fact, weights λ^a and λ^b will be chosen implicitly in the examples below by the presumption that the expected utility of agent a is maximized subject to constraint (4) for agent b.

Solutions to program 1 can be generated as solutions to linear programs, computed numerically. Two examples provide illustrations to be carried through the subsequent analysis. The first has only one consumption good; the second, two.

For the first example, the utility function of agent a is of the form

$$U^a(c, \theta^a) = (c)^{\theta^a},$$

so that agent a is risk averse for each parameter θ^a, with $\theta^a \in (.4, .5, .6\}$, each possibility occurring with probability one-third. The utility function of agent b is of the form $U^b(c) = c$, so that b is risk neutral. Also let $e^a = e^b = 5$. Then, ignoring the discrete use of the consumption space, the full information solution would allocate the consumption good in such a way as to equate weighted marginal utilities over states. That is, agent a would receive the consumption good when he is urgent, at least at $\theta^a = .6$, and give it up when he is patient, at least at $\theta^a = .4$. On the other hand, with private information, the incentive constraints do not allow this dependence, and the solution degenerates to autarky (despite allowance for lotteries). This is natural since there is only one good and more is preferred to less. In this case, then, private information is quite damning to the social arrangement. Though trivial, the full information and private information solutions are reported in Table 1.

For the second example there are two goods, denoted generically as x and y, and θ is two-dimensional. The utility function of agent a is of the form

$$U^a(c_x, c_y, \theta^a) = (c_x)^{\theta_x^a} + (c_y)^{\theta_y^a},$$

with $(\theta_x^a, \theta_y^a) \in \{(.4, .6), (.6, .4)\}$, each possibility occurring with probability one-half. Agent b continues to be linear, now in both goods. Endowments are $e_x^a = e_x^b = e_y^a = e_y^b = 5$. The full information and private information solutions are reported in Table 2.

TABLE 1—SINGLE-PERIOD SOLUTION, ONE GOOD

Values for θ^a	c^a (Full Information)	c^a (Private Information)
.4	2.2	5
.5	4	5
.6	8.8	5

TABLE 2—SINGLE-PERIOD SOLUTION, TWO GOODS

Values for θ_x^a, θ_y^a	(Full Information) (c_x^a, c_y^a)	(Private Information) (c_x^a, c_y^a)
(.4, .6)	(2, 8)	(2, 8)
(.6, .4)	(8, 2)	(8, 2)

The full information and private information solutions are identical, because the incentive constraints (2) are not binding; that is, with two commodities, goods can be assigned optimally in such a way that agent a self-selects. (There is some *ex ante* beneficial insurance associated with the solution.)

Again, there is presumed to be perfect, costless enforcement of a selected, *ex ante* optimal arrangement. It seems best to take this as given, if only as an approximation, without spelling out the mechanics of the enforcement procedure. Otherwise, without any enforcement mechanism, one must suppose full commitment. That is, agents a and b must show up at the consumption date; agents a and b must pool their endowments then as if these had to be placed on preset conveyors, activated by weight; agent a is restricted to announce one value for θ^a in the set Θ^a, as if a computer were programmed to accept one of a prespecified set of values; consumption is reallocated on the conveyors in accordance with the announced value θ^a and the preset computer program; and finally agents eat in private, without redistribution. Such a technology may read more as science fiction than as realistic economics, but it is important to note that in principle there could exist a transfer function, message space technology that would allow one to implement a solution to a socially optimal program. That is, private information is the only impediment to trade.

II. Multiperiod Arrangements and the Gain from Intertemporal Links

The analysis above is easily extended to accommodate some nontrivial dynamics. In particular, suppose agents a and b remained paired with each other over two consumption dates t, $t=1,2$. Let preference shocks be experienced by agent a at the beginning of date t. That is, let shock θ_1^a enter into the utility function of agent a at date $t=1$ and let shock θ_2^a enter into the utility function of agent a at date $t=2$. However, some persistence in shocks is allowed. That is, $\theta_2^a = f(\theta_1^a, \delta)$, where f is a deterministic function and δ is a nondegenerate random variable, possibly dependent on θ_1^a. This allows as a special case no direct intertemporal links in shocks apart from Markov dependence in the probabilities, $p(\theta_2^a|\theta_1^a)$. Let e_t^a and e_t^b denote the vector of endowments of agents a and b at dates t, $t=1,2$, presumed for simplicity not to vary with date t. Here and below let β denote the common discount rate for time-separable utilities.[1] Finally, let $\pi_1(c|\theta_1^a)$ denote the allocation rule at date 1 and $\pi_2(c|\theta_1^a, \theta_2^a)$ denote the allocation rule at date 2, both of these specifying probabilities on set $C = \{c = (c^a, c^b), c^a + c^b \leq e^a + e^b\}$ as before, still supposing no storage. Then the program for the determination of a private information Pareto-optimal arrangement is

Program 2: Maximize by choice of $\pi_1(c|\theta_1^a)$ and $\pi_2(c|\theta_1^a, \theta_2^a)$ the objective function

$$(5)\quad \lambda^a\Big\{\sum_{\theta_1^a} p(\theta_1^a)\sum_c \pi_1(c|\theta_1^a)U^a(c^a,\theta_1^a) + \beta\sum_{\theta_1^a} p(\theta_1^a)\sum_{\theta_2^a} p(\theta_2^a|\theta_1^a) \times \sum_c \pi_2(c|\theta_1^a,\theta_2^a)U^a(c^a,\theta_2^a)\Big\} + \lambda^b\Big\{\sum_{\theta_1^a} p(\theta_1^a)\sum_c \pi_1(c|\theta_1^a)U^b(c^b) + \beta\sum_{\theta_1^a} p(\theta_1^a)\sum_{\theta_2^a} p(\theta_2^a|\theta_1^a) \times \sum_c \pi_2(c|\theta_1^a,\theta_2^a)U^b(c^b)\Big\},$$

subject to incentive constraints at date $t=2$, for all $\tilde{\theta}_1^a$ announcements in the past, for all actual contemporary values θ_2^a, and for all counterfactual announcements $\tilde{\theta}_2^a$,

$$(6)\quad \sum_c \pi_2(c|\tilde{\theta}_1^a,\theta_2^a)U^a(c^a,\theta_2^a) \geq \sum_c \pi_2(c|\tilde{\theta}_1^a,\tilde{\theta}_2^a)U^a(c^a,\theta_2^a),$$

and subject to incentive constraints at date $t=1$ for θ_1^a actuals and $\tilde{\theta}_1^a$ counterfactuals,

$$(7)\quad \sum_c U^a(c^a,\theta_1^a)\pi_1(c|\theta_1^a) + \beta\sum_{\theta_2^a} p(\theta_2^a|\theta_1^a) \times \sum_c U^a(c^a,\theta_2^a)\pi_2(c|\theta_1^a,\theta_2^a) \geq \sum_c U^a(c^a,\theta_1^a)\pi_1(c|\tilde{\theta}_1^a) + \beta\sum_{\theta_2^a} p(\theta_2^a|\theta_1^a) \times \sum_c U^a(c^a,\theta_2^a)\pi_2(c|\tilde{\theta}_1^a,\theta_2^a).$$

[1]These solutions are computed for $\beta = .95$. Also the solution is reported as deterministic, but in fact the generated solution for a grid of 101 points between 0 and 10 displayed lotteries. For example at $\theta_1^a = .6$, the computed solution is 7.2 with Prob .4872 and 7.3 with Prob .5128. As this lottery is an artifact of grid size, and would disappear with a continuum of possible consumptions, the reported solution is delivered by linear interpolation. Similar interpolations are done for the tables that follow.

TABLE 3—MULTIPERIOD PRIVATE INFORMATION SOLUTION, ONE GOOD[a]

Values for θ_1^a	c^a at Date 1	Values for θ_2^a	c^a at Date 2
		.4	6.55
.4	3.1	.5	6.55
		.6	6.55
		.4	4.87
.5	5.1	.5	4.87
		.6	4.87
		.4	3.1
.6	7.25	.5	3.1
		.6	3.1

[a]See fn. 1.

TABLE 4—MULTIPERIOD PRIVATE AND FULL INFORMATION SOLUTION, TWO GOODS

Values for $(\theta_{1x}^a,\theta_{1y}^a)$	(c_x^a,c_y^a)	Values for (δ_x,δ_y)	Values for $\theta_{2x}^a,\theta_{2y}^a$	$(c_x^a,$	$c_y^a)$
		(1,1)	(.6, .4)	8.01	2.0
(.4,.6)	(2,8)	(.5, 1.5)	(3., .6)	1.0	8.0
		(1.5, .5)	(.9, .2)	10.0	0.82
		(1,1)	(.4, .6)	2.0	8.0
(.6,.4)	(8,2)	(.5, 1.5)	(.2, .9)	0.82	10.0
		(1.5, .5)	(.6, .3)	8.0	1.0

What bears emphasis in this program is the possible dependence of allocation rule at date $t=2$ on announcement of parameter θ_1^a at date 1. Indeed, suppose there is only one good, so that beneficial trade under private information is difficult, as the no-insurance, autarkic, private information solution of Table 1 emphasizes. Suppose further, to bias the case against intertemporal tie-ins, that there is no functional persistence in the preference shocks and no Markov dependence in probabilities. That is, each shock is drawn with equal likelihood in each period. Still, as displayed in Table 3, for the environment of Table 1, there is some insurance at date $t=1$ (but not at date $t=2$) achieved by intertemporal dependence.

This dependence does not appear in the full information, full insurance solution, the full information solution of Table 1 reported twice, once for each period. In fact, this dependence would not appear in the full information solution even with nontrivial Markov probabilities, since the full information rule remains the same: allocate consumptions so as to equate weighted marginal utilities for every contemporary state. The private information solution displays dependence because intertemporal tie-ins are used to circumvent the damning effects of the incentive constraints; low consumption at date $t=1$ is tied to high consumption at date $t=2$, and conversely. Again, this allows for some insurance.

With two (or more) goods, the date $t=1$ incentive constraints are not so damning. In fact, as is evident from Table 2, it is possible to construct examples without binding incentive constraints at date 1, and if there were no persistence in shocks, intertemporal tie-ins would not be needed. On the other hand, persistence in shocks can deliver intertemporal tie-ins. For example, suppose there are two goods at each date. The utility function of agent a at date 1 is of the form

$$U^a(c_x,c_y,\theta_1^a)=(c_x)^{\theta_{1x}^a}+(c_y)_{1y}^{\theta a}$$

with $\quad(\theta_{1x}^a,\theta_{1y}^a)\in\{(.4,.6),(.6,.4)\},$

each with equal probability, and at date 2 of the form

$$U^a(c_x,c_y,\theta_2^a)=(c_x)^{(1-\theta_{1x}^a)\delta_x}+(c_y)^{(1-\theta_{1y}^a)\delta_y}$$

$$=(c_x)^{\theta_{2x}^a}+(c_y)^{\theta_{2y}^a}$$

with

$$(\delta_x,\delta_y)=\begin{cases}(1,1) & \text{with Prob .96}\\(.5,1.5) & \text{with Prob .02}\\(1.5,.5) & \text{with Prob .02}\end{cases}$$

Table 4 reports the solution.[2] To be noted is that the family of allocations available for agent a at date 2 depends on the announced (and actual) parameter draw of agent a at date 1.

Regardless of how the tie-ins are generated, the private information optimal solu-

[2]Again endowments are five uniformly and $\beta=.9$.

tion is damaged if tie-ins are not allowed, that is, if there were no record of preference shock announcements of agent a at date $t=1$ (so that at most reannouncements are viable). For the one good example above, this is obvious; with no tie-ins there is no trade, and the solution is autarky for both periods. For the two-good example, intratemporal reallocations are still viable, as in the single-period solution of Table 2. But the absence of direct tie-ins would be associated with a loss of utility.[3]

It bears repetition that perfect and costless enforcement of the private information optimal mechanism is still assumed. That is, neither agent can walk away from the agreed-upon arrangement at the end of date 1, and both are committed to comply with the multiperiod message space and transfer function requirements.

III. Optimal Multilateral Arrangements with Spatial Separation but Full Communication

With this investment in mechanism design, one can now extend the two-period model to include four agents and two locations, as a base for discussion in the sections which follow of various communication-accounting systems. This section focuses on the benefit from quadrilateral, rather than bilateral, arrangements.

The four-agent, two-location, two-period model is like the two-agent (one location), two-period model of Section II except that there are two agents of type a, named a and a', and two agents of type b, named b and b'. Agent a is presumed to stay at location 1 over the two periods of his lifetime, and agent a' stays at location 2. But agents b and b' switch locations between dates one and two, in accordance with Table 5.

[3]In the absence of direct tie-ins, that is, in the absence of a record of past announcements, agent a might be required to reannounce date 1 preference shocks and to announce a value for contemporary shocks. In this scheme, reannouncements of date 1 shocks may matter, and in that weak sense there are intertemporal tie-ins. But the solution is worse than if there were a record of past announcements, of $\tilde{\theta}_1^a$ in program 2.

Agent a experiences privately observed preference shocks at the beginning of each date t, $t=1,2$, and similarly for agent a', and, until otherwise specified, the distribution determining shocks for agent a is independent of the distribution determining shocks for agent a'. Agents b and b' experience no shocks. The notation for endowments is as before, and for simplicity these do not vary over agents or over time.

With $\pi_{it}(\cdot)$ as general notation for the allocation rule for the two agents present at location i and date t, $i=1,2$, $t=1,2$, let $\pi_{11}(c|\theta_1^a)$ and $\pi_{12}(c|\theta_1^a,\theta_2^a)$ denote the probabilities of consumption bundle c in common space C in location 1 at dates 1 and 2, respectively, conditioned on the announcements of agent a, and allowing intertemporal tie-ins. Also let $\pi_{21}(c|\theta_1^{a'})$ and $\pi_{22}(c|\theta_1^{a'},\theta_2^{a'})$ denote the corresponding probabilities of consumption bundle c at location 2, conditioned on the announcements of agent a'. Then the program for the determination of private information, full communication, Pareto-optimal allocations is:

Program 3: Maximize by choice of the $\pi_{it}(\cdot)$ the objective function

$$
\begin{aligned}
(8)\quad & \lambda^a\Big\{\sum_{\theta_1^a} p(\theta_1^a)\sum_c \pi_{11}(c|\theta_1^a)U^a(c^a,\theta_1^a) \\
&+\beta\sum_{\theta_1^a} p(\theta_1^a)\sum_{\theta_2^a} p(\theta_2^a|\theta_1^a)\sum_c \pi_{12}(c|\theta_1^a,\theta_2^a)U^a(c^a,\theta_2^a)\Big\} \\
&+\lambda^b\Big\{\sum_{\theta_1^a} p(\theta_1^a)\sum_c \pi_{11}(c|\theta_1^a)U^b(c^b) \\
&+\beta\sum_{\theta_1^{a'}} p(\theta_1^{a'})\sum_{\theta_2^{a'}} p(\theta_2^{a'}|\theta_1^{a'})\sum_c \pi_{22}(c|\theta_1^{a'},\theta_2^{a'})U^b(c^b)\Big\} \\
&+\lambda^{a'}\Big\{\sum_{\theta_1^{a'}} p(\theta_1^{a'})\sum_c \pi_{21}(c|\theta_1^{a'})U^{a'}(c^{a'},\theta_1^{a'}) \\
&+\beta\sum_{\theta_1^{a'}} p(\theta_1^{a'})\sum_{\theta_2^{a'}} p(\theta_2^{a'}|\theta_1^{a'})\sum_c \pi_{22}(c|\theta_1^{a'},\theta_2^{a'})U^{a'}(c^{a'},\theta_2^{a'})\Big\} \\
&+\lambda^{b'}\Big\{\sum_{\theta_1^{a'}} p(\theta_1^{a'})\sum_c \pi_{21}(c|\theta_1^{a'})U^{b'}(c^{b'}) \\
&+\beta\sum_{\theta_1^a} p(\theta_1^a)\sum_{\theta_2^a} p(\theta_2^a|\theta_1^a)\sum_c \pi_{12}(c|\theta_1^a,\theta_2^a)U^{b'}(c^{b'})\Big\},
\end{aligned}
$$

TABLE 5—AGENT PAIRINGS IN THE FOUR-AGENT, TWO-LOCATION MODEL

	Location	1	2
Date	1	(a,b)	(a',b')
	2	(a,b')	(a',b)

subject to incentive constraints for agent a at date $t=1,2$, identical to incentive constraints (6) and (7) above, except with $\pi_{1t}(\cdot)$ replacing π_t, and subject to similar incentive constraints for agent a' at date $t=1,2$.

Some results are already implicit in this program but deserve elaboration. First, the full information solution to the program would still equate weighted marginal utilities of the two agents present at each location by appropriate distribution of the total endowment of the two agents present. As only the contemporary preference shock of agent a would enter as a genuine variable into these equations for location 1, and that of agent a' for location 2, full information optimal rules depend at most on these parameters. In particular, the shock experienced by agent a' at date 1 and location 2 has no bearing on the transfer at date 2 and location 1 even though agent b' is present at both locations. That is, the history experienced by b' would not matter. This argument carries over to the private information, full communication program in question, as can be deduced by a study of first-order conditions. Similarly, contemporary announcements of agent a' at date 1 and location 2 have no bearing on the transfer at date 1 and location 1. That is, for this pure exchange environment, an interspatial telecommunication technology would not be used for *contemporary* announcements even though it is allowed. (For more on this, see Section VII below.) On the other hand, the private information, full communication program does allow the announcement of agent a at location 1 and date 1 to enter into the transfer function at location 1 and date 2. Such tie-ins are used because they weaken the damaging effect of incentive compatibility constraints at date 1 for agent a, as can be deduced also by a study of first-order conditions. A similar tie-in is allowed for announcements of agent a' at location 2 across dates 1 and 2.

These tie-ins make the optimal arrangement quadrilateral rather than bilateral. That is, apart from coordination in the choice of allocation rules over the two locations and two dates, as determined by the Pareto weights λ^j, $j=a, a', b$ and b', the solutions to the full information program, solutions to program 3 without the incentive constraints, can be implemented as a sequence of bilateral arrangements, one at each location and date. With tie-ins, however, agent a's announcment at date $t=1$ matters for both agent b at date $t=1$ and for agent b' at date $t=2$. Put crudely, agent a can "borrow" from agent b and promise to "pay" agent b', and similarly for agent a' in his dealings with agents b' and b. In fact, agent b could end up "paying" twice, if he were a "lender" to agent a at date $t=1$ and "pays" back a "loan" from agent a' at date 2. But of course these and all other possibilities are weighted optimally *ex ante* in the determination of the social optimum.

An easy way to deliver explicit examples of beneficial quadrilateral arrangements is to trick the relatively complicated program 3 into looking like program 2. To do this, suppose agents a and a' have identical utility functions, identical endowments, and suffer the same probability distribution determining preference shocks. Similarly, suppose agents b and b' have identical utility functions and identical endowments. Finally, restrict attention to Pareto weights λ^j with $\lambda^a=\lambda^{a'}$ and $\lambda^b=\lambda^{b'}$. Then, as can be deduced formally by an examination of the necessary and sufficient first-order conditions, a solution to program 2 can be viewed as a double solution to program 3, with the probabilities of consumptions to agent a identical with the probabilities of consumptions to agent a', and so on. Thus, Tables 3 and 4 are examples of beneficial quadrilateral arrangements with the understanding, though it does not appear in the notation, that agent a is dealing with agent b at date 1 but dealing with agent b' at date 2, and similarly for agent a' in his dealings with agents b' and b. This will be exploited in what follows.

Perhaps it bears repetition, however, that the perfect costless enforcement premise still underlies the analysis. That is, all agents are required to show up at the locations specified in Table 5, even though agents b and b' are traveling. Similarly, agents a and b must abide by the prespecified allocation rule at location 1 and at date 1, even though neither will deal with the other ever again. One wonders about collusion or default, even though, as before, one can tell a physical story to support the full commitment program. This subject is of sufficient interest that it is attacked head-on in Section IX below, providing an alternative rationale for communication that is ignored in the intermediate sections which follow.

IV. The Limitations of Oral Assignment Systems in Spatial Settings

The next step in the consideration of communication-accounting systems is to retain the entire setup of Section III, but to limit the communication technology. The first technology to be considered is the most primitive of technologies, namely oral assignment. That is, agents a and a' can make announcements at each date and location of contemporary values for preference shocks and, where relevant, past values as well. But no other record-keeping device is available. That is, agents cannot carry commodities or tokens of any kind, cannot carry written messages, and cannot access some centralized telecommunication record-keeping system. Under these circumstances, only the contemporary state matters for the announcement of agent a or agent a', even though announcements of past histories might be permitted. And thus, by familiar, revelation principle arguments, agents a and a' may without loss of anything essential be restricted to announcing relevant contemporary values (histories can be announced only to the extent that they help to determine contemporary values).

The effect of course is to preclude direct intertemporal tie-ins of the type already analyzed. That is, program 2 reduces to two separate versions of program 1 (with identical Pareto weights in each period), and this is obviously Pareto inferior. Supposing that agents b and b' are constrained to the utility of autarky, for example, agents a and a' suffer.

In fact, though the analysis is somewhat contrived, one can see from this example how spatial organization itself can depend on communication technologies. In particular, suppose that the motive for travel of agent b to agent a' and of agent b' to agent a at the end of date 1 is that the match between a and b and the match between a' and b' deteriorate over time, mimicked by the assumption in the model that K units of the consumption good disappear from the social endowment available at each location at date 2 if agents remain paired. (Alternatively, in a more elaborate model, imagine there are gains to specialization and trade if agents move about.) Then there would be a nontrivial choice among organizations: if agents remain paired, the cost of a deteriorated match must be weighed against the gain from direct intertemporal tie-ins. In fact, agents would choose *ex ante* to remain paired for at least some nontrivial values of cost K. Yet they would not remain paired under perfect costless telecommunications, or even under some of the more restricted systems considered below.

V. Portable Concealable Objects as Record-Keeping Devices

An improved communication technology would allow agents a and a' to carry with them otherwise valueless tokens. In principle, these might be concealed at date 2, but, on the other hand, they might be displayed as a record of things past. In particular, if the allocation of tokens at date 1 is under the complete control of the predetermined allocation rules, then past announcements can be indicated, allowing some of the needed intertemporal tie-ins.

It is possible, in fact, that just one kind of token can allow recovery of the solution to the full communication, private information program, program 2 (actually program 3). In particular, for the environment generating Table 3, date 1 consumptions of agent a are ordered by values of θ_1^a, and the families of date 2 consumptions are ordered in reverse by these values. That is, the date 2, $\theta_1^a = .6$

branch is uniformly lower than the date 2, $\theta_1^a = .5$ branch, which in turn is uniformly lower than the date 2, $\theta_1^a = .4$ branch. Thus, if there were no record of first-period announcements, agent a at date 2 would prefer the $\theta_1^a = .4$ branch no matter what date 2 shock he experiences, and so on. That is, he would claim the highest branch. With tokens, claims can be limited to branches consistent with agent a's display of tokens. And, if agent a is given the least amount of tokens at date 1 for $\theta_1^a = .6$, and the most for $\theta_1^a = .4$, with $\theta_1^a = .5$ in between, then no tokens will be concealed, and the full communication solution will be effected.

Interestingly enough, the full communication solution cannot always be effected with one kind of token, as the next section illustrates.[4]

[4]On the other hand, Douglas Diamond has suggested the following ingenious scheme with the idea that even one token might not be needed for the environment of this section and that oral communication might suffice. Suppose all four agents, a, a', b, and b' are to agree a priori to implement a social optimum as if there were full communication. In particular, agents b and b' are to agree a priori, *in private*, that if agent a reports θ' at date $t=1$, then agent b is to say some prespecified password, for example, "midnight," whereas if agent a reports θ'', agent b is to say some distinct password, for example, "high noon." Upon b''s arrival to location 1, agent a is to repeat the password, out of countless thousands of possibilities. Agent b' will then know the history of actual past announcements of agent a and is to implement the specified transfer. One problem with this scheme is that the initial agreement between b and b', on passwords as a function of θ announcements, must be *private* between b and b' (if agent a knows the agreement, he will always say the password at date $t=2$, which allows him to receive the consumption good). Therefore, agents b and b' could just as easily commit themselves to a degenerate password system, always saying the same thing, and planning matters so that a is never to receive the consumption good at date $t=2$. That is, since the choice of a password function is private, agents b and b' may be supposed to make the best-unconstrained choice, and a requirement that a particular function be chosen has no force if it is not consistent with incentives. Something akin to this is assumed in the "revelation principle" literature: requirements that agents tell the truth in an announcement game have no force unless agents have an incentive to do so. This example illustrates, however, the importance of the assumption in the paper that rules be agreed to publicly and leads to an exploration of enforcement technologies and definitions of reneging. But this is left as a subject for future research.

VI. Multiple-Portable Tokens and Written Messages

The example displayed in Table 4 is one for which one kind of token is not enough. To see this, suppose agent a is given tokens at date 1 for $\theta_1^a = (.4,.6)$ and is given no tokens at date 1 for $\theta_1^a = (.6,.4)$. Then, if at date 1, $\theta_1^a = (.4,.6)$, and at date 2, $(\delta_x, \delta_y) = (.5, 1.5)$, so that $\theta_2^a = (.3,.6)$, agent a would prefer to show no tokens, would claim he was a $\theta_1^a = (.6,.4)$, and also claim $\theta_2^a = (.4,.6)$. On the other hand, if he were given tokens at date 1 under $\theta_1^a = (.6,.4)$ and not otherwise and if θ_1^a were actually (.6,.4), then he would understate tokens at $\theta_2^a = (.6,.3)$, preferring at date 2 the $\theta_1^a = (.4,.6)$ and $\theta_2^a = (.6,.4)$ outcome.

The intuition behind this result, and the contrast with Table 3, are instructive. In Table 3 agent a is either a "borrower" or a "lender" at date $t=1$, in various degrees, in the sense that the direction of the transaction is reversed at date 2. In Table 4 there are two goods, and agent a can be a "borrower" or a "lender" in either good. Still, "preference reversal" shocks at date 2 can cause agent a to want to pretend to have been a lender in the commodity he did not lend. And this can happen no matter which commodity was lent at date $t=1$. Of course, two kinds of tokens circumvent this problem, one for each commodity which can be lent. If green tokens are handed out at date 1 and $\theta_1^a = (.4,.6)$ and red at $\theta_1^a = (.6,.4)$, then a display of red tokens could be required at date 2 when $\theta_1^a = (.6,.4)$ is claimed at date 2, and this is not possible if in fact $\theta_1^a = (.4,.6)$ was claimed at date 1.

It might seem from this that the number of kinds of tokens needed to support a full communication optimum is related to the number of commodities or the number of shocks. Actually though, the current environment could be expanded to include any finite number of commodities or shocks, as Table 6 illustrates.[5] Here, if θ_1^a takes on the second value (in an ordered set) at date 1, for example, then agent a receives at date 1 two red tokens and $N-1$ green tokens. At date 2 he cannot overstate past θ_1^a values,

[5]I owe this example to Arthur Kupferman.

TABLE 6—A COMPLETE TWO-TOKEN SYSTEM

Values of θ_1^a	No. of Red Tokens	No. of Green Tokens
1	1	N
2	2	$N-1$
3	3	$N-2$
⋮	⋮	⋮
N	N	1

for example, claiming the third value, for he would be short of red tokens, and he cannot understate past θ_2^a values, for example, claiming the first, for he would be short of green tokens. Thus two kinds of tokens are enough to distinguish past histories. (We shall comment on a related issue, privately observed endowments, in Section X below.)

An interesting general issue concerns when a specified number of kinds of tokens will be enough to constitute a full language, that is, enough to achieve a full communication solution. This issue will not be pursued here, apart from noting any system with a full set of tokens would be equivalent with a system with unrestricted fully displayed written messages. That is, written message systems are the limit of concealable token systems and as such do not require a separate analysis.[6]

VII. Electronic Telecommunications—A Dominant Technology

In the explicit four-agent environment of Section III, there was no gain from electronic telecommunications over space at a point in time. But, modifications of the pure risk-sharing environment can provide a motive for such systems. For example, suppose the consumption good at date 1 can be stored in direct amount K at date 1, carried over without depreciation at the same location to date 2. Suppose also that the preference shocks of agents a and a' at date 1 are driven by some common component, with idiosyncratic noises, and that the common component persists to some extent into preference shocks at date 2. Then shock $\theta_1^{a'}$ of agent a' can be used to help forecast the marginal utility of agent a at date 2, and a high marginal utility for consumption at date 2 would motivate relatively high investment K at location one, date 1. A similar argument applies for shock θ_1^a at location two and date one.

To check on this logic, it is useful to go through the formal exercise of writing down a programming problem for the determination of Pareto-optimal allocations with electronic, interspatial telecommunications and storage. In particular, invoking the kind of symmetry assumptions used earlier to trick the four-agent program to a two-agent program, let $\pi_1(c, K|\theta_1^a, \theta_1^{a'})$ denote the probability of consumption bundle c *and* storage K given announced (and actual) $\theta_1^a, \theta_1^{a'}$ values, where, given K, bundle c lies in the set $\{(c^a, c^b)\colon\ c^a + c^b \leq e^a + e^b - K\}$. Similarly, let $\pi_2(c, K|\theta_1^a, \theta_1^{a'}, \theta_2^a)$ denote the probability of consumption bundle c *and* storage K given the specified triple of announced (and actual) parameter values, where, for given K, the bundle c lies in $\{(c^a, c^b)\colon\ c^a + c^b \leq e^a + e^b + K\}$. Then the program for the determination of a full communication private information optimal arrangement is

Program 4: Maximize by choice of the $\pi_1(\cdot)$ and $\pi_2(\cdot)$ the objective function

$$
\begin{aligned}
(9)\quad & \lambda^a\Bigg\{\sum_{\theta_1^a}\sum_{\theta_1^{a'}} p(\theta_1^a, \theta_1^{a'}) \\
&\times \sum_K \sum_c \pi_1(c, K|\theta_1^a, \theta_1^{a'}) U^a(c^a, \theta_1^a) \\
&+ \beta \sum_{\theta_1^a}\sum_{\theta_1^{a'}}\sum_{\theta_2^a} p(\theta_2^a|\theta_1^a, \theta_1^{a'}) p(\theta_1^a, \theta_1^{a'}) \\
&\times \sum_K \sum_c \pi_2(c, K|\theta_1^a, \theta_1^{a'}, \theta_2^a) U^a(c^a, \theta_2^a)\Bigg\} \\
& \lambda^b\Bigg\{\sum_{\theta_1^a}\sum_{\theta_1^{a'}} p(\theta_1^a, \theta_1^{a'}) \sum_K \sum_c \pi_1(c, K|\theta_1^a, \theta_1^{a'}) U^b(c^b) \\
&+ \beta \sum_{\theta_1^a}\sum_{\theta_1^{a'}}\sum_{\theta_2^a} p(\theta_2^a|\theta_1^a, \theta_1^{a'}) p(\theta_1^a, \theta_1^{a'}) \\
&\times \sum_K \sum_c \pi_2(c, K|\theta_1^a, \theta_1^{a'}, \theta_2^a) U^b(c^b)\Bigg\},
\end{aligned}
$$

[6] More generally, a token system is a kind of written message system, a limited one if combinations of tokens cannot adequately convey past history. The idea that tokens are related to writing as "words" are to language may be familiar—real languages actually evolved from token accounting systems. See Schmandt-Besserat (1979).

subject to consistency in the choice of K, namely, for all $\theta_1^a, \theta_1^{a'}, \theta_2^a$

$$(10)\qquad \sum_c \pi_2(c, K|\theta_1^a, \theta_1^{a'}, \theta_2^a) \equiv \sum_c \pi_1(c, K|\theta_1^a, \theta_1^{a'}),$$

subject to incentive constraints for agent a at date $t=2$, for all past announcements $\theta_1^a, \theta_1^{a'}$, and for all actual θ_2^a and counterfactual $\tilde{\theta}_2^a$,

$$(11)\qquad \sum_c \sum_K U^a(c^a, \theta_2^a)\pi_2(c, K|\theta_1^a, \theta_1^{a'}, \theta_2^a) \geq \sum_c \sum_K U^a(c^a, \theta_2^a)\pi_2(c, K|\theta_1^a, \theta_1^{a'}, \tilde{\theta}_2^a),$$

subject to incentive constraints for agent a at date $t=1$, for all actual θ_1^a and counterfactual $\tilde{\theta}_1^a$

$$(12)\qquad \sum_{\theta_1^{a'}} p(\theta_1^{a'}|\theta_1^a)\sum_c\sum_K U^a(c^a, \theta_1^a)\pi_1(c, K|\theta_1^a, \theta_1^{a'}) + \beta\sum_{\theta_1^{a'}}\sum_{\theta_2^a} p(\theta_2^a, \theta_1^{a'}|\theta_1^a) \times \sum_c\sum_K U^a(c^a, \theta_2^a)\pi_2(c, K|\theta_1^a, \theta_1^{a'}, \theta_2^a)$$
$$\geq \sum_{\theta_1^{a'}} p(\theta_1^{a'}|\theta_1^a)\sum_c\sum_K U^a(c^a, \theta_1^a)\pi_1(c, K|\tilde{\theta}_1^a, \theta_1^{a'}) + \beta\sum_{\theta_1^{a'}}\sum_{\theta_2^a} p(\theta_2^a, \theta_1^{a'}|\theta_1^a) \times \sum_c\sum_K U^a(c^a, \theta_2^a)\pi_2(c, K|\tilde{\theta}_1^a, \theta_1^{a'}, \theta_2^a).$$

The incentive constraints are much as before except that agent a takes as given that agent a' is announcing truthfully at date 1, and agent a learns this parameter announcement *after* the allocation is effected at date 1. Constraint (10) ensures that the choice of K is the same whether viewed as determined by the allocation rule $\pi_1(\cdot)$ or the allocation rule $\pi_2(\cdot)$. Alternatively, and more naturally, one could have let the allocation rule $\pi_2(c|\theta_1^a, \theta_1^{a'}, \theta_2^a, K)$ be conditioned on K, but then it would not have been obvious that the essential program is linear.

VIII. Portable Messages as a Location Assignment Device

We have now passed through the gamut of communication technologies in the context of the same four-agent, two-location model. Each of the communication technologies considered in the model played a role as a device for keeping track, if possible, of past announcements by the agents with privately observed preference shocks, agents a and a'. No records were needed of the histories experienced by agents b and b'. In contrast, this section shows how records may be needed for travelers who experience no direct shocks *if* their assignments to locations is endogenous, part of the (optimal) social mechanism.

The model is modified by supposing there is only one traveler, agent b, who is paired initially at date $t=1$ with an agent a, experiencing preference shock θ_1^a, then is paired at date $t=2$ either with an agent d, who is to experience shocks θ_2^d, or with an agent e, who is to experience shocks θ_2^e, but is never paired to both. Further, shocks θ_1^a contain information on forthcoming θ_2^d and θ_2^e, so that an assignment at date $t=1$ matters. That is, assignment at date $t=1$ is a nontrivial function of announced (and actual) θ_1^a. For simplicity, suppose agent a cares about date $t=1$ consumption only and that agents d and e care about date $t=2$ consumption only, though agent b, the traveler, cares about consumption at both dates. Then, to ensure some mutual beneficial trade, suppose there are two goods at each date. The notation for endowments e_t^i is as before.

Letting variable l denote the location assignment of agent b at the end of date 1, to either agent d or e, that is either $l=d$ or $l=e$, the programming problem for the determination of a *full communication* private information optimum is

Program 5: Maximize by choice of date $t=1$ (potentially random) consumption and assignment rules, $\pi_1(c|\theta_1^a)$ and $\pi_1(l|\theta_1^a)$, respectively, and by choice of date $t=2$ consumption rules, $\pi_2(c|\theta_2^d, l=d)$ and $\pi_2(c|\theta_2^e, l=e)$, the objective function

$$(13)\quad \lambda^a\left\{\sum_{\theta_1^a} p(\theta_1^a)\sum_c \pi_1(c|\theta_1^a)U^a(c,\theta_1^a)\right\}$$

$$+\lambda^b\left\{\sum_{\theta_1^a} p(\theta_1^a)\sum_c \pi_1(c|\theta_1^a)U^b(c)+\beta\sum_{\theta_1^a} p(\theta_1^a)\right.$$

$$\times\left[\pi_1(l=d|\theta_1^a)\sum_{\theta_2^d} p(\theta_2^d|\theta_a^a)\sum_c U^b(c)\pi_2(c|\theta_2^d,l=d)\right.$$

$$\left.\left.+\pi_1(l=e|\theta_1^a)\left\{\sum_{\theta_2^e} p(\theta_2^e|\theta_1^a)\sum_c U^b(c)\pi_2(c|\theta_2^e,l=e)\right]\right\}\right.$$

$$+\lambda^d\left\{\sum_{\theta_1^a} p(\theta_1^a)\left[\pi_1(l=d|\theta_1^a)\sum_{\theta_2^d} p(\theta_2^d|\theta_1^a)\right.\right.$$

$$\times\sum_c U^d(c,\theta_2^d)\pi_2(c|\theta_2^d,l=d)$$

$$\left.\left.+\pi_1(l=e|\theta_1^a)\sum_{\theta_2^d} p(\theta_2^d|\theta_1^a)U^d(e_2^d,\theta_2^d)\right]\right\}$$

$$+\lambda^e\left\{\sum_{\theta_1^a} p(\theta_1^a)\left[\pi_1(l=d|\theta_1^a)\sum_{\theta_2^e} p(\theta_2^e|\theta_1^a)U^e(e_2^e,\theta_2^e)\right.\right.$$

$$\left.\left.+\pi_1(l=e|\theta_1^a)\sum_{\theta_2^e} p(\theta_2^e|\theta_1^a)\sum_c U^e(c,\theta_2^e)\pi_2(c|\theta_2^e,l=e)\right]\right\},$$

subject to incentive constraints for agent a at date $t=1$, and incentive constraints for agents e and d at date $t=2$. Formally, program 5 can be converted to a linear program, for example by letting

$$(14)\quad \pi_2(c,l=d|\theta_1^a,\theta_2^d)$$
$$\equiv \pi_2(c|\theta_2^d,l=d)\pi_1(l=d|\theta_1^a),$$

and similarly for $\pi_2(c,l=e|\theta_1^a,\theta_2^e)$; by imposing a linear equation to ensure that the left-hand side of (14) is independent of θ_2^d, for example; and by imposing consistency with $\pi_1(l=d|\theta_1^a)$, as in equation (10), program 4.

With limited communication, an otherwise optimal assignment of agent b to either agent d or to agent e may not be assured. Announced shock θ_1^a at date $t=1$, and, in shorthand notation, assignment $l(\theta_1^a)$ at date $t=1$, are private to agent b when b meets agent d or agent e at date $t=2$.

An example helps to make the point. In particular, suppose agent b has preferences of the form $U^b(c_x,c_y)=c_x+c_y$ at each date, with discount rate $\beta=.9$. Preferences of agent a at date 1 are of the form

$$U^a(c_x,c_y,\theta_1^a)=(c_x)^{\theta_x^a}+(c_y)^{\theta_y^a},$$

with $\theta_1^a=(\theta_x^a,\theta_y^a)$ either (.4,.6) or (.6,.4), each drawn with probability 1/2. Preferences of agents d and e at date 2 are of a similar form to agent a's with correlated parameter draws, that is,

$\theta_1^a=(.4,.6)$

$$\Rightarrow\begin{cases}\theta_2^d=(.7,.5) \text{ and } \theta_2^e=(.3,.5) & \text{with Prob .8}\\ \theta_2^d=(.3,.5) \text{ and } \theta_2^e=(.7,.5) & \text{with Prob .2}\end{cases}$$

$\theta_1^a=(.6,.4)$

$$\Rightarrow\begin{cases}\theta_2^d=(.7,.5) \text{ and } \theta_2^e=(.3,.5) & \text{with Prob .2}\\ \theta_2^d=(.3,.5) \text{ and } \theta_2^e=(.7,.5) & \text{with Prob .8}\end{cases}$$

Then the consumption allocations for agents a, d, and e and assignment rule $l(\theta)$ are displayed[7] in Table 7.

TABLE 7—OPTIMAL LOCATION ASSIGNMENT

(θ_x^a,θ_y^a)	(c_x^a,c_y^a)	$l(\theta_1^a)$	(θ_x^d,θ_y^d)	(c_x^d,c_y^d)	(θ_x^e,θ_y^e)	(c_x^e,c_y^e)
(.4,.6)	(2,6.31)	d	(.7,.5)	(10,3)		
			(.3,.5)	(2,6.24)		
(.6,.4)	(6,2)	e			(.7,.5)	(10,3)
					(.3,.5)	(2,6.24)

When $\theta_1^a=(.4,.6)$, agent a is supposed to be assigned to agent d, and when $\theta_1^a=(.6,.4)$, agent a is supposed to be assigned to agent e, each assignment consistent with the likelihood of high marginal utility for agent d or agent e, respectively. But agent b, anticipating high transfers under the full communication-assignment rule, wants to go in just the contrary direction.

[7]Endowments are five uniformly, and $\lambda^a=\lambda^d=\lambda^e$ with the utility of agent b at autarky.

However, if agent b carries a token of one kind when $l(\theta_1^a)=d$ and a token of another kind when $l(\theta_1^a)=e$; if he must be with agent a at date $t=1$, and then with agent d or agent e at date $t=2$; and if he must abide by the prespecified allocation rule when paired, then a failure to display tokens of the right kind can indicate that agent b is not abiding by the rules. Curiously, two kinds of tokens are needed. For suppose there were only one kind of token in use and agent b were given it for $\theta_1^a=(.4,.6)$ and not when $\theta_1^a=(.6,.4)$, for example. Then when $\theta_1^a=(.6,.4)$, agent b wants to go to agent d where the likelihood of high transfers is less. But he has no tokens to show, so he must go to agent e. But when $\theta_1^a=(.4,.6)$, agent b wants to go to agent e and can do so by concealing his tokens. Again, two kinds of tokens circumvent this problem.

IX. Portable Messages as a Device for Enforcement

The structure of Section VIII takes as given that agent b must show up and be paired with agent a at date $t=1$ and must choose to be paired with either agent d or agent e at date $t=2$. A slight weakening of the commitment premise would allow agent b to choose whether to participate at some dates. In fact, spatial separation alone could then imply private information in the sense that failure of one agent to participate with a second agent at a given location in a given date might not be known by a third party at a subsequent date. That is, communication can be valuable if commitment is limited, even with no underlying uncertainty.

The delicate part of schemes with limited commitment is that one can undo the structure of the problem altogether, so that *ex ante* agreements have no force at all. In that case, there would be nothing to communicate since plans have no content, are known to have no content, and so on. Thus, *some* prior commitment is needed. In this section one starts with essentially full commitment, supposing that if agents show up at a prespecified location they *must* abide by the prespecified allocation mechanism in place at that location. Further, at the very last date, agents *must* show up at prespecified locations, no matter what. Hence, the (only) aspect of limited commitment which is introduced is the idea that each agent can choose at all dates but the last whether to participate under prespecified allocation rules. The alternative to participation is to eat one's endowment.

For clarity, we return to the basic four-agent, two-location, two-period model of the paper, but eliminate shocks to preferences. Otherwise, the structure is as before. And as before it is best to begin with the full communication record-keeping technology, supposing actions at locations 1 and 2 at the first date are fully recorded and can be used in the allocation rules at date $t=2$. The objective in the planning period is to choose (deterministic) allocations c_t^i, $i=a,a',b,b'$, $t=1,2$, to maximize a weighted sum of discounted utility streams across the four agents, namely

$$\sum_i \lambda^i\left\{\sum_{t=1}^{2}\beta^{t-1}U^i\left(c_t^i\right)\right\}, \tag{15}$$

subject to the obvious resource constraints

$$c_t^i+c_t^j=e_t^i+e_t^j, \tag{16}$$

and so on for the relevant agent pairing (i,j) at each date t.

The relevant decision for a given agent at the first date is whether to participate as planned at date $t=1$; participation cannot be assumed and must be induced. But since constraints which ensure participation at date 1 generally damage the program, relative to the full commitment program, one wants such participation constraints to be as weak as possible. This is done by making the "penalty" at date 2 for failure to participate at date 1 as strong as possible. That is, the first-period participation constraint for agent i takes the form

$$U^i\left(c_1^i\right)+\beta U^i\left(c_2^i\right) \geq U^i\left(e_1^i\right)+\beta U^i(0), \tag{17}$$

where $c_2^i=0$ on the right-hand side of (17) is the obvious penalty, the lower bound on i's consumption at date 2. Again, the interpretation is that if agent i participates at date 1, receiving c_1^i as planned, then when i shows up at date 2, and he must by assumption do so, he will receive c_2^i as planned, whereas if agent i does not participate at date $t=1$, eating his endowment e_1^i, something which cannot be directly thwarted, then when agent i shows up at date $t=2$, the fact of default is known and he receives zero. In short, the program for the determination of Pareto-optimal allocations is one of maximizing objective function (15) subject to four resource constraints (16) and subject to four participation constraints (17).[8]

To implement a solution to this program *without* costless telecommunicated record keeping, but with portable concealable objects such as tokens, suppose that if agent i participates at date $t=1$ he receives c_1^i *and* a prespecified number of tokens (one will do). Tokens, by assumption, cannot be acquired elsewhere. Then a display of tokens at date 2 effects c_t^i to agent i, and a failure to display tokens effects the penalty, zero consumption. Thus tokens communicate whether or not agent i reneged at date 1, an action which is otherwise unknown to agent i's trading partner at date 2.

This argument can be extended to any T-period, finite horizon model with choice of participation at all dates t but the last. There would be a participation constraint for each agent i at each date t, $t=1,2,\dots,T-1$, and the constraint for agent i at date t would be[9]

$$\sum_{s=t}^{T} \beta^{(s-1)} U^i(c_s^i) \geq \sum_{s=t}^{T-1} \beta^{(s-1)} U^i(e_s^i) + \beta^{T-1} U^i(0). \tag{18}$$

With discount rate β less than one, participation at relative early stages is achieved not by high penalties at the last date T but by the gain from the enduring relationship, that is, from participation at intermediate dates. The number of tokens, or length of written messages, needed to implement the limited commitment solution with otherwise limited communication may get large as T goes to infinity.

X. Privately Observed Endowments

The analysis above is now easily generalized to incorporate the case of random and privately observed endowments. Indeed, as with tokens, claimed values of endowments can trigger required displays, some of which would prove infeasible for certain actual realizations of endowments.[10] In short, incentive constraints need be imposed only for claimed endowment vectors which are less than or equal to realized endowment vectors, component by component.

More formally, for the two-agent, one-consumption period model of Section II, let θ^a denote the vector of endowments of agent a, as well as a vector of shocks to the preferences of agent a as before. Also let $\tau = (\tau^a, \tau^b)$ denote a vector of transfers from agents a and b, supposing for simplicity a finite number of possible values for these transfers. Then let $\pi(\tau|\theta^a)$ denote the probability of transfer τ conditioned on announcement θ^a, so that for each θ^a, $\pi(\tau|\theta^a)$ is a lottery over τ values satisfying $\theta^a - \tau^a \geq 0$, $e^b - \tau^b \geq 0$, $\tau^a + \tau^b = 0$. The program for the determination of Pareto optimal allocations is then

Program 6: Maximize by choice of the $\pi(\tau|\theta^a)$ the objective function

$$\lambda^a\Big\{\sum_{\theta^a} p(\theta^a) \sum_{\tau} U^a[\theta^a - \tau^a, \theta^a]\pi(\tau|\theta^a)\Big\} + \lambda^b\Big\{\sum_{\theta^a} p(\theta^a) \sum_{\tau} U^b[e^b - \tau^b]\pi(\tau|\theta^a)\Big\}, \tag{19}$$

[8] More generally, we can write down a program which lets participation or assignment at date 1 be a choice variable, subject to constraint that if the assignment is not to participate, then agent i receives his endowment, and also subject to obedience constraints, that if agent i is assigned to participate, then he must prefer to do so, and if he is assigned not to participate, then he must prefer not to do so. It can be established that for any solution with nonparticipation as the assignment, there is a utility equivalent solution with participation as the assignment, hence the program given above.

[9] That this is without loss of generality can be established as in fn. 8.

[10] It might be noted that much of the literature on resource allocation mechanisms ignores privately observed endowments. Important exceptions are Andrew Postlewaite (1974), Hurwicz, Eric Maskin, and Postlewaite (1980), and also Pipat Pithyachariyakul (1982).

subject to incentive constraints, for every $\tilde{\theta}^a \leq \theta^a$,

$$(20)\quad \sum_{\tau} U^a[\theta^a - \tau^a, \theta^a]\pi(\tau|\theta^a) \geq \sum_{\tau} U^a[\theta^a - \tau^a, \theta^a]\pi(\tau|\tilde{\theta}^a).$$

Again, solutions to program 6 can require actual displays. For example, with one good, if a high endowment θ^a for agent a is associated with a high marginal utility shock θ^a for agent a, then the *ex ante* optimal insurance solution may have agent a receiving the good when his endowment is high, after it is displayed, and surrendering it otherwise, when his endowment is low.[11] Without pretransfer displays this would not be incentive compatible, for agent a would always claim to have a high endowment, in order to receive the higher net transfer.

Generally, though, pretransfer displays cannot completely overcome the incentive problems of private information, and the analysis of communication-accounting systems with privately observed endowment shocks would proceed along the lines of the paper as for the case of privately observed preference shocks. But a new and interesting case for analysis now emerges, the case of commodity storage coupled with the presence of *no* intrinsically useless (storable) tokens. In this case, displays of otherwise private stocks can serve in part as worthless tokens did to reveal past histories. But the communication-accounting system with these bonafide commodity tokens would be more limited than the system with intrinsically useless tokens.

Consider the four-agent, two-location, two-period economy, tricked into the two-agent, one-location, two-period economy by the symmetry conditions, and suppose no communication system is available other than that achieved by direct commodity storage, publicly observed at the time storage decisions are taken but not necessarily observed later. Then let $\pi(\tau, K|\theta_1^a)$ denote a lottery over transfers $\tau = (\tau^a, \tau^b)$ and storage in amount K at date 1, conditioned on announcement θ_1^a by agent a at date 1, with transfers τ and storage K satisfying $\theta_1^a - \tau^a \geq 0$, $e^b - \tau^b \geq 0$, $K = \tau^a + \tau^b$. Also, let $\pi_2(\tau|\theta_2^a + K)$ denote a lottery over transfers $\tau = (\tau^a, \tau^b)$ at date 2, conditioned on announcement $\theta_2^a + K$ by agent a at date 2, with $\theta_2^a + K - \tau^a \geq 0$, $e^b - \tau^b \geq 0$, $\tau^a + \tau^b = 0$.

The point is that the natural state variable at date 2 is $\theta_2^a + K$; only this *sum* can enter into the allocation rule at date 2. Thus if endowment θ_2^a were constant, storage K, in varying with endowment θ_1^a, might allow a complete revelation of past histories, as for the earlier examples with intrinsically useless tokens. But with endowment θ_2^a nonconstant, a display of goods may be possible, even when commodity storage itself is inadequate. That is, in an effort to achieve the former allocation, agent a may occasionally have the ability to display when formerly he did not, and this would undercut the information revelation role of tokens. On the other hand, if storage K were constant, independent of endowment θ_1^a, then allocations can be indexed by θ_2^a. But with K nonconstant, depending on θ_1^a, the θ_2^a-information role of contemporary displays is mitigated. In general, then, neither complete past histories nor contemporary states will enter into the de facto allocation rules at date 2. And further constraining the information role of commodity tokens is the fact that commodity storage K as an information variable distorts intertemporal allocations from what they would have been with costless, intrinsically useless tokens.

[11] This example may seem somewhat contrived, but it can be given a more compelling interpretation. Briefly, imagine that agents a and b each have an investment project and that returns from investment projects at date 1 must be reinvested at date 1 in order to yield an idiosyncratic, household-specific, nontraded consumption good at date 2. The investment good can be reallocated at date 1, however, and would be reallocated in an *ex ante* optimal arrangement if high investment good returns at date 1 were indicative of high consumption returns at date 2, yielding high marginal return of the *indirect* utility function for contemporary investment. For further details, see the working paper, Townsend (1986).

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