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GENERAL COMPETITIVE ANALYSIS IN AN ECONOMY WITH PRIVATE INFORMATION*

BY EDWARD C. PRESCOTT AND ROBERT M. TOWNSEND[1]

1. INTRODUCTION

The last decade has witnessed a virtual explosion in the economics of private information and moral hazard. Models using private information constructs have now gained prominence in many of the substantive areas of economics, including monetary economics, industrial organization, finance, and labor economics. Yet despite these advances, or indeed because of them, we believe there is a need for an alternative, complementary approach — the extension of modern general equilibrium theory to such environments. In this paper, then, we extend the theory of general economic equilibrium of Arrow, Debreu, and McKenzie, among others, to a prototype class of environments with private information and examine again the role of securities in the optimal allocation of risk-bearing. We consider in particular pure exchange economies with the usual multiple commodity (intratemporal), borrowing-lending (intertemporal), and insurance (uncertainty) motives for trade but assume that households experience privately observed, period-by-period shocks to preferences (see Section 2). For that class of economies, we establish the existence of Pareto optimal allocations (in Section 3) and the existence of competitive equilibria in markets for securities of a certain kind (in Section 4). We also establish (in Section 5) the first fundamental welfare theorem, that competitive equilibrium allocations are Pareto optimal. The second fundamental welfare theorem — that the optima can be supported as competitive equilibria — does not hold, suggesting difficulties

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[1] This paper is a revised version of "On Competitive Theory with Private Information" presented at the University of Chicago, Columbia University, Cornell University, Northwestern University, Yale University, the summer meetings of the Econometric Society in Montreal, and the NBER Conference-Seminar on the Theory of General Equilibrium at Berkeley in February 1980. Helpful comments from Truman Bewley, Charles Wilson, the participants of these seminars, and anonymous referees are gratefully acknowledged. We also thank the National Science Foundation and the Federal Reserve Bank of Minneapolis for financial support and accept full responsibility for any errors as well as for the views expressed herein.

of price decentralization in economies with ex ante private information.

The class of economies we consider in this paper is large. That is, we consider economies in which the distribution of (privately observed) shocks in the population is the same as the probability distribution of shocks for each individual household. We also require that households with the same shocks be treated ex ante in the same way.[2] That is, following the Arrow [1953] and Debreu [1959] treatment of uncertainty, we index a household's allocation by that household's shock (type). In this way, there is no aggregate uncertainty, and the general equilibrium feasibility constraint is a simple vector of linear inequalities. But since shocks are privately observed, not all shock-contingent allocations that satisfy the feasibility constraint are achievable. In addition, the allocations must be such that it is not in the interest of households to misrepresent their types. This is accomplished by the imposition of additional conditions which, following Hurwicz [1972], we term incentive-compatibility constraints.[3] Still, the space of allocations that specify achievable consumptions contingent upon privately observed shocks proves to be an inappropriate commodity space for general competitive analysis. Even though the underlying utility functions are concave, the space of shock-contingent consumption allocations restricted by the incentive-compatibility constraints, in general, is not convex, and there can be gains from introducing lotteries. Consequently, the linear commodity space employed in the analysis here is the space of shock-contingent signed measures, a linear space which contains the needed shock-contingent lotteries.

Lotteries have been used in game theory to make spaces convex, following the seminal contribution of von Neumann and Morgenstern [1947]. They have been used extensively in the social choice field for similar reasons. And they have been used in various economic models to discriminate among agents with private information. But lotteries have not been used in classic, general equilibrium, competitive analysis, to the best of our knowledge. This is surely because the natural spaces are already convex, and there is no need for them.[4]

Though abstract, we think that this exercise may prove useful in the positive economics of private information. The highly abstract, Arrow-Debreu, state-contingent analysis has proven to be an invaluable tool in the study of economies with publicly observed shocks. It has proven to be particularly useful in determining whether a highly limited set of security and spot markets are sufficient

[2] The reasons why we did not introduce names were that it would be notationally cumbersome and analytically difficult and would have no econometric implications. With randomness in the allocation ex ante, there can be different outcomes for ex ante indentical agents, identical, that is, from the point of view of the econometrician.

[3] The works of Myerson [1979] and Harris and Townsend [1977] [1981] provide the justification for these additional constraints.

[4] The use of signed measures in general competitive analysis is not new. Mas-Colell [1975], and subsequently others, have exploited them in the study of economies with a continuum of differentiated products.

to exploit all the gains from trade. When this is the case, one is certain that the results do not hinge upon arbitrary exclusions of security markets but rather only upon assumptions concerning preferences, endowments, technologies, and the information structure. In any event, one gains a better understanding of existing arrangements. In an analogous way, the constructs of the paper may prove useful in verifying for a particular set of contractual or institutional arrangements and economic environments that there are no potential gains from instituting other arrangements. We also hope these constructs might help us to better interpret reality.

As noted, our general analysis allows for more than one underlying consumption good and more than one consumption date and thus allows the usual intratemporal and intertemporal motives for trade. We recognize, though, this level of generality may make it difficult to interpret the constructs we have developed in the paper.[5] Thus, in Section 6 we present a simple example economy with one consumption good and one consumption date and focus entirely on uncertainty and the insurance motive for trade. For this economy, the competitive equilibria are characterized by insurance contracts with options, the exercise of which is private information dependent. In addition, the equilibrium contracts incorporate contrived randomness, even though all agents have convex preferences. We argue that such contrived randomness is not unusual, being consistent with casual observations on security markets and the state-contingent analysis of Arrow and Debreu. We also show that a simple institutional arrangement with random allocation of "excess demand" achieves the competitive equilibrium allocation, suggesting that at least some apparent disequilibrium phenomena can be interpreted as institutional or contractual arrangements that support equilibrium allocations. (Section 6 is virtually self-contained and may be read before the more general analysis of the paper.)

2. THE GENERAL SECURITIES MODEL

Imagine an economy with a continuum of agents and l commodities. Each of the agents has an endowment vector $e_t \geq 0$ in each period t, $t=0, 1, \ldots, T$. Let c_t denote the nonnegative consumption vector in period t; then each agent has preferences over consumption sequences as described by the utility function

$$E \sum_{t=0}^{T} U(c_t, \theta_t).$$

[5] Despite the apparent generality, the analysis is limited in two ways. First, we do not allow for random, privately observed shocks to endowments, though we suspect our analysis can be extended in that direction. Second, we do not allow for statistical dependence in the preference shocks, so that agents would have private information on the probability distribution of future shocks at the time of initial trading. Current efforts in Prescott and Townsend [1982] indicate that extensions of standard competitive analysis to such environments are not straightforward.

Here E is the expectation operator with respect to the random variables c_t and θ_t (the latter random variables will be described momentarily). Each single-period utility function $U(\cdot,\ \theta_t)$ is continuous, concave, and increasing and is defined for $c_t \geq 0$. The parameter θ_t is interpreted as a shock to individual preferences at the beginning of period t, observed only by the individual agent. For simplicity, parameter θ_t is assumed to take on only a finite number of values; that is, for each t, $\theta_t \in \Theta$ where Θ has n elements. Fraction $\prod_{t=0}^{T} \lambda(\theta_t)$ of the population have shock realization $(\theta_0, \theta_1, \ldots, \theta_T)$. The individual agents at the beginning of time 0 know their own θ_0 but have no basis for forecasting their future θ_t, except that they know the fractions of the population that will realize each shock sequence. Consequently, by symmetry, the predictive probability distributions of all given agents for their future preference shocks are identically and independently distributed, with $\lambda(\theta)$ for $\theta \in \Theta$ being the probability that $\theta_t = \theta$.[6] We note that the class of economies under study is quite similar to those studied by Gale [1980] and Lucas [1980].

What is the appropriate commodity space for a given economy? One approach is to follow Arrow [1953] and Debreu [1959] and index consumption c_t for each individual by $(\theta_0, \ldots, \theta_t)$, the individual's specific history. There is a problem with this approach, however. There may be incentives (gains) for agents to enter into lotteries even though they are all risk averse. In the example of Section 6, lotteries are needed for optimal and equilibrium allocations. This arises because the space of shock-contingent consumption allocations restricted by the incentive constraints is not convex. The following simple version of the model demonstrates this nonconvexity.

Suppose $T=1$, $e_0=0$, and $l=1$, so there is consumption of the single good only in period $t=1$. Suppose also that the set $\Theta=\{1, 2\}$. For the shock-contingent indexing approach, let $c(\theta)$ for $\theta=1, 2$ denote consumption in period one of a θ-type agent. The expected utility of allocation $(c(1), c(2))$ in period zero is then

$$\lambda(1)U(c(1), 1) + \lambda(2)U(c(2), 2).$$

There will be truthful revelation of shock (types) only if

$$U(c(1), 1) \geq U(c(2), 1) \tag{2.1}$$

$$U(c(2), 2) \geq U(c(1), 2). \tag{2.2}$$

These are the appropriate incentive-compatibility constraints; they insure a type-one agent weakly prefers $c(1)$ to $c(2)$ and a type-two agent weakly prefers $c(2)$ to $c(1)$. To see that (2.1) and (2.2) do not define a convex set, consider allocations $(c(1), c(2)')$ and $(c(1), c(2)'')$ that both satisfy constraint (2.1) with equal-

[6] In assuming the agents know only what the distribution of the parameters in the population will be, we avoid measurability problems. There are problems in going in the other direction, from independently and identically distributed random variables on the continuum to measurable sample spaces, which necessitate a redefinition of the integral (see Bewley and Radner [1980]).

ity, that satisfy constraint (2.2), and that have $c(2)' \neq c(2)''$. Given the strict concavity of $U(\cdot, 1)$, any convex combination of these two allocations violates constraint (2.1).

With consumption lotteries contingent on θ, the nonconvexity is overcome. Suppose for simplicity that the underlying commodity space is finite; that is, c can be one of a finite number of possible bundles in C. Then let the vector $x(\theta)$ be a *random* assignment to each agent of type θ, where $x(c, \theta)$ is the probability of bundle c. Then a shock-contingent random allocation $(x(1), x(2))$ can be achieved in a direct-revelation mechanism with truth-telling only if

$$\sum_{c \in C} U(c, 1)x(c, 1) \geq \sum_{c \in C} U(c, 1)x(c, 2)$$

$$\sum_{c \in C} U(c, 2)x(c, 2) \geq \sum_{c \in C} U(c, 2)x(c, 1).$$

These conditions are the random analogues of (2.1) and (2.2). These conditions are linear in the $x(c, \theta)$ and therefore constitute convex constraints. Finally, the expected utility of the shock-contingent lotteries $x = (x(1), x(2))$ is

$$W(x, \theta) = \lambda(1) \sum_{c \in C} U(c, 1)x(c, 1) + \lambda(2) \sum_{c \in C} U(c, 2)x(c, 2).$$

It is linear in x so the utility function is concave in that argument. This, incidentally, is true whether the underlying utility functions $U(\cdot, \theta)$ are or are not concave.

With classical general equilibrium analysis (in finite dimensional spaces), there is no need for lotteries, for the constraint sets are convex and the utility functions concave. Relaxing either of these assumptions results in the possibility of gains from lotteries.

We now return to the more general model and prepare to establish the existence of Pareto optimal and competitive equilibrium allocations and the optimality of competitive equilibria using a commodity space that contains consumption lotteries. To simplify the notation, however, we assume $T = 2$; this is the smallest T that fully illustrates the nature of the analysis. Also, for technical reasons, we assume that $e_t \gg 0$, that consumption is bounded, $0 \leq c_t \leq b$, and that the $U(\cdot, \theta_t)$ are strictly increasing. Finally, for notational convenience, let $\Theta = \{1, 2, \ldots, n\}$ and denote θ_0 by i. Now we may refer to agents of type i, $i = 1, \ldots, n$, classified by their initial shock.

There are obvious generalizations to the model we analyze. There can be statistical dependence in the θ_t, $t \geq 1$, as long as there is independence from the initial parameter θ_0. There can be nontime-additive-separable utility functions, discounting, observable heterogeneous characteristics, and nontrivial production technologies. We did not seek generality in order to focus on private information and how general competitive analysis can be extended to include it.

To begin the formal analysis, denote the underlying consumption possibilities set by $C = \{c \in R^l \colon 0 \leq c \leq b\}$. Let the commodity space L be the space of $(1 + n + n^2)$-tuples of finite, real-valued, countable-additive set functions on the Borel subsets of C. For element $z=[z_0, \{z_1(\theta_1)\}_{\theta_1\in\Theta}, \{z_2(\theta_1, \theta_2)\}_{\theta_1,\theta_2\in\Theta}]$, z_0 is the measure on the period zero consumption good vector, the $z_1(\theta_1)$, of which there are n, are measures on the period one consumption vector conditioned upon θ_1, and $z_2(\theta_1, \theta_2)$, of which there are n^2, are measures on the period two consumption vector conditioned upon both θ_1 and θ_2. The space L is linear, a property which is needed for standard general competitive analysis. Further, the space L contains the space P of $(1+n+n^2)$-tuples of probability measures or lotteries on Borel subsets of C, which are needed for the reasons noted above.

The consumption set and preferences are defined first. For $x\in P$, the utility functional for a type i is the expected utility

$$W(x, i)=\int U(c, i)x_0(dc) + \sum_{\theta_1} \lambda(\theta_1)\int U(c, \theta_1)x_1(dc, \theta_1) + \sum_{\theta_1} \lambda(\theta_1) \sum_{\theta_2} \lambda(\theta_2)\int U(c, \theta_2)x_2(dc, \theta_1, \theta_2). \tag{2.3}$$

Not all $x\in P$ satisfy the incentive-compatibility conditions, so these functionals are defined only upon a subset of P. At period $t=2$, an agent must weakly prefer $x_2(\theta_1, \theta_2)$ if the agent is type θ_2 and announced type θ_1 at $t=1$. Thus,

$$\int U(c, \theta_2)x_2(dc, \theta_1, \theta_2) \geq \int U(c, \theta_2)x_2(dc, \theta_1, \theta_2') \qquad \theta_1, \theta_2, \theta_2'\in\Theta \tag{2.4}$$

is a necessary condition for a point to be in the consumption possibility set. Given (2.4), the period $t=1$ incentive-compatibility requirement is

$$\int U(c, \theta_1)x_1(dc, \theta_1) + \sum_{\theta_2} \lambda(\theta_2)\int U(c, \theta_2)x_2(dc, \theta_1, \theta_2) \geq \int U(c, \theta_1)x_1(dc, \theta_1') + \sum_{\theta_2} \lambda(\theta_2)\int U(c, \theta_2)x_2(dc, \theta_1', \theta_2) \qquad \theta_1, \theta_1'\in\Theta. \tag{2.5}$$

If asked in period $t=1$ to choose a member of $\{x_1(\theta_1), \{x_2(\theta_1, \theta_2)\}\}$, an agent would weakly prefer the pair $(x_1(\theta_1), \{x_2(\theta_1, \theta_2)\})$ if that agent's shock is actually θ_1. Let $X\equiv\{x\in P\colon x$ satisfies (2.4) and (2.5)$\}$. The set $X\subset L$ is the consumption set of the representative agent. Given any $x\in X$, preferences of type i are ordered by $W(x, i)$. A point $x^0\in X$ is a satiation point in X for agent i if $W(x, i)\leq W(x^0, i)$ for all $x\in X$.

The endowment of agent i in each period t is an l-dimensional vector $e_t\gg 0$, $e_t\in C$. So let ξ be that element of P such that ξ_0 puts all mass on e_0, $\xi_1(\theta_1)$ puts all mass on e_1 for each $\theta_1\in\Theta$, and $\xi_2(\theta_1, \theta_2)$ puts all mass on e_2 for $\theta_1, \theta_2\in\Theta$.

We now have a pure exchange economy defined by the population fractions $\lambda(i)$, $i\in\Theta=\{1, 2,\dots, n\}$, the linear space L, the common consumption set $X\subset L$, the common endowment $\xi\in L$, and preferences $W(\cdot, i)$ defined on X for every agent of type i, $i\in\Theta$.

An implementable allocation for this economy is an n-tuple (x_i) with $x_i\in X$

for every i which satisfies the resource constraints in each period t, $t=0, 1, 2$,[7]

$$\sum_i \lambda(i) \int c x_{i0}(dc) \leq e_0 \tag{2.6}$$

$$\sum_i \lambda(i) \sum_{\theta_1} \lambda(\theta_1) \int c x_{i1}(dc, \theta_1) \leq e_1 \tag{2.7}$$

$$\sum_i \lambda(i) \sum_{\theta_1} \lambda(\theta_1) \sum_{\theta_2} \lambda(\theta_2) \int c x_{i2}(dc, \theta_1, \theta_2) \leq e_2 \tag{2.8}$$

and which satisfies a prior self-selection constraint

$$W(x_i, i) \geq W(x_j, i) \qquad i, j \in \Theta. \tag{2.9}$$

Thus, we assume that fraction $x_{i0}(B)$ of the agents of type i in period zero are assigned an allocation in Borel set B in period zero, and similarly for $x_{i1}(B, \theta_1)$, $x_{i2}(B, \theta_1, \theta_2)$. Here, then, all agents of type i have chosen lottery x_{i0} in period zero, and so on. The prior self-selection constraint captures the idea that an allocation (x_i) can be actually implemented only if all agents of type i reveal their true type by the choice of the bundle x_i from among the n-tuple (x_i).

An implementable allocation (x_i) is said to be a Pareto optimum if there does not exist an implementable allocation (x_i') such that

$$W(x_i', i) \geq W(x_i, i) \qquad i = 1, 2, \dots, n \tag{2.10}$$

with a strict inequality for some i.

3. EXISTENCE OF A PARETO OPTIMUM

To establish the existence of a Pareto optimum for our economy, it is enough to establish the existence of a solution to the problem of maximizing a weighted average of the utilities of the agent types; this is maximize

$$\sum_i w(i) W(x_i, i)$$

where

$$0 < w(i) < 1, \quad \sum_i w(i) = 1$$

by choice of the n-tuple (x_i), $x_i \in X$, subject to the resource constraints (2.6)–(2.8) and the prior self-selection constraint (2.9). To establish the existence of a solution to the problem, we make use of the theorem that continuous real-valued functions on nonempty, compact sets have a maximum.

To do this, we use the weak topology on the space of signed measures. Let the topology on L be the weak topology. The underlying commodity space C is a compact subset of R^l, a separable metric space, and so the set of probability

[7] In (2.6)–(2.8), the integration is coordinate-wise and the weak inequality holds for each of the l coordinates.

measures $P \supset X$ is compact with respect to this topology.[8] Since the resource constraints (2.6)–(2.8), the prior self-selection constraint (2.9), and the constraints (2.4)–(2.5) are all defined relative to integrals of bounded continuous functions, the constraint set is closed. It is, therefore, compact because it is a closed subset of the compact set P. The constraint set is nonempty because $x_i = \xi$ for all i is feasible. Since continuous real-valued functions on nonempty compact topological spaces achieve a maximum, a Pareto optimum is guaranteed to exist.

The above argument relies heavily on the compactness of C.[9] In fact, this assumption is crucial. By modifying the example of Section 6 where C is not compact, we have produced an environment in which one can get arbitrarily close to but not attain the utility of a full-information optimum; for that environment, then, a Pareto optimum does not exist.

4. EXISTENCE OF A COMPETITIVE EQUILIBRIUM

In this section, we establish that our economy can be decentralized with a price system, that is, that there exists a competitive equilibrium. We accomplish this task by introducing a firm into the analysis, with a judiciously chosen (aggregate) production set. We then follow the spirit of the proofs used by Bewley [1972] and Mas-Colell [1975] for establishing the existence of a competitive equilibrium with a continuum of commodities. Various approximate economies are considered, with a finite number of commodities. Existence of a competitive equilibrium for these economies is established with a theorem of Debreu [1962]. One then takes an appropriate limit.

Let there be one firm in our economy with production set $Y \subset L$, where $Y = \{y \in L\text{: (4.1), (4.2), and (4.3) below are satisfied}\}$:

$$\int c y_0(dc) \leq 0 \tag{4.1}$$

$$\sum_{\theta_1} \lambda(\theta_1) \int c y_1(dc, \theta_1) \leq 0 \tag{4.2}$$

$$\sum_{\theta_1} \lambda(\theta_1) \sum_{\theta_2} \lambda(\theta_2) \int c y_2(dc, \theta_1, \theta_2) \leq 0. \tag{4.3}$$

To be noted here is that the components of the $y \in Y$ are signed measures and thus each is a way of adding. A negative weight corresponds to a commitment to take in resources and a positive weight corresponds to a commitment to distribute resources. Thus, in (4.1), for example, the term $\int c_j y_0(dc)$ should be interpreted as the net trade (sale) of the jth consumption good in period zero. Inequality (4.1) states that, as a clearinghouse or intermediary, the firm cannot supply more of the consumption good than it acquires. When indexed by the parameter θ, a component of y should be interpreted as a commitment to agents who announce

[8] See Parthasarthy [1967, Theorem 6.4, Chapter 2.]

[9] Mas-Colell [1975] also assumes the underlying commodity space is compact.

they are of type θ. The production set Y, it should be noted, contains the zero element of L and also displays constant returns to scale.

Following Debreu [1954], we define a state of our economy as an $(n+1)$-tuple $[(x_i), y]$ of elements of L. A state $[(x_i), y]$ is said to be attainable if $x_i \in X$ for every $i \in \Theta$, $y \in Y$, and $\sum_{i=1}^{n} \lambda(i)x_i - y = \xi$. Now suppose a state $[(x_i), y]$ is attainable. Then, setting $y = \sum_i \lambda(i)x_i - \xi$ in (4.1)–(4.3), one obtains the resource constraints (2.6)–(2.8). Similarly, given any n-tuple (x_i), $x_i \in X$, satisfying the resource constraints (2.6)–(2.8), define y by $y = \sum_i \lambda(i)x_i - \xi$, and then $y \in Y$. Thus, there is a one-to-one correspondence between attainable states in the economy with production and allocations in the pure exchange economy satisfying the resource constraints. An attainable state $[(x_i), y]$ is said to be a Pareto optimum if the n-tuple (x_i) satisfies (2.9) and there does not exist an attainable state $[(x'_i), y']$ which satisfies (2.9) and Pareto dominates, that is, satisfies (2.10). Again there is a one-to-one correspondence between optimal states and optimal allocations.

A price system for our economy is some real-valued linear functional on L, that is, some mapping $v\colon L \to R$. More will be said about price systems v in what follows, but we may note here that v will have $(1+n+n^2)$ components, each of which is a continuous linear functional relative to the weak topology.[10] That is, given some $z \in L$,

$$v(z) = \int f_0(c)z_0(dc) + \sum_{\theta_1} \int f_1(c, \theta_1)z_1(dc, \theta_1)$$
$$+ \sum_{\theta_1} \sum_{\theta_2} \int f_2(c, \theta_1, \theta_2)z_2(dc, \theta_1, \theta_2)$$

where the functions $f_0(\cdot)$, $f_1(\cdot, \theta_1)$, $f_2(\cdot, \theta_1, \theta_2)$ are (bounded) continuous functions on C. (See Dunford and Schwartz [1957], Theorem 9, p. 421.)

We now make the following

Definition. A *competitive equilibrium* is a state $[(x_1^*), y^*]$ and a price system v^* such that

(i) for every i, x_i^* maximizes $W(x_i, i)$ subject to $x_i \in X$ and $v^*(x_i) \leq v^*(\xi)$

(ii) y^* maximizes $v^*(y)$ subject to $y \in Y$

(iii) $\sum_{i=1}^{n} \lambda(i)x_i^* - y^* = \xi$.

An outline of our proof for the existence of a competitive equilibrium for our economy is as follows. First, the underlying commodity space C is restricted to a finite number of points, the nodes of a mesh or grid on C. In this restricted economy a countably additive, real-valued set function is completely defined by an element of a Euclidean space, with dimension equal to the dimension of the restricted C. The linear space of these restricted economies is the $1+n+n^2$

[10] See also Mas-Colell [1975].

cross product of the Euclidean space. Consumption sets, preferences, endowments, and a production set may be defined on this space in the obvious way. The existence of a competitive equilibrium for the restricted economy is then established using a theorem of Debreu [1962]. Then, letting the grid get finer and finer, one can construct a sequence of competitive equilibria for economies which are less and less restricted. A subsequence of these competitive allocations and prices converges, and the limiting allocations and prices are shown to be a competitive equilibrium for the unrestricted economy. We now give a more detailed argument.

The first restricted economy may be constructed in any essentially arbitrary way by subdividing each of the l coordinate axes of the commodity space C into intervals, subject to the following restrictions. First, each endowment point e_t, $t=0, 1, 2$, must be one of the nodes of the consequent grid. Second, if we let

$$c_0^* > \max_i \left[\frac{e_0}{\lambda(i)}\right], \quad c_t^* > \max\left[\frac{e_t}{\lambda(\theta_t)}\right] \qquad t=1, 2 \tag{4.4}$$

each point c_t^*, $t=0, 1, 2$, must be one of these nodes. (We thus suppose that the upper bound b of C is such that $0<c_t^*\leq b$.) Third, the element zero must be an element of the consequent grid. The first of these restrictions will mean the endowment points lie in each of the restricted consumption sets, and the second will mean that no agent type can be satiated in the attainable consumption sets, given the resource constraints.

The second restricted economy is obtained from the first by equal subdivision of the original intervals of the l coordinate axes. The third is obtained by equal subdivision of the second, and so on. In what follows, we let the subscript k be the index number of the sequence of restricted economies.

For the kth restricted economy, let C^k be the restricted underlying commodity space and L^k be the finite dimensional subspace of L for which the support of each of the n^2+n+1 measures is C^k. That is, let $x_0(c)$, the $x_1(c, \theta_1)$, and the $x_2(c, \theta_1, \theta_2)$ for $c\in C^k$ each be the measure of $\{c\}$, the set containing the single point c. Then the space L^k is finite dimensional, and a point is characterized by the vector $\{x_0(c), x_1(c, \theta_1)$, and $x_2(c, \theta_1, \theta_2)\}$, $c\in C^k$, $\theta_1, \theta_2\in\Theta$. Note that the integral of an integrable function f: $C\to R$ with respect to a measure x on C^k is

$$\int_C f(c)x(dc) = \sum_{c\in C^k} f(c)x(c). \tag{4.5}$$

The consumption and production possibility sets for the kth restricted economy are $X^k=X\cap L^k$ and $Y^k=Y\cap L^k$, respectively. The integrals used in the definition of X, Y, and W, namely in (2.4)–(2.5), (4.1)–(4.3), and (2.3), respectively, have representations as finite sums over the elements of C^k. As e_0, e_1, and e_2 belong to C^k, the endowment for economy k is $\xi^k=\xi\in L^k$.

As our linear space for the kth restricted economy is a subset of Euclidean space, the price system is also an element of the Euclidean space. Thus, we may define a price system $p^k=\{(p_0^k(c)), (p_1^k(c, \theta_1)), (p_2^k(c, \theta_1, \theta_2))\}$, $c\in C^k$, $\theta_1, \theta_2\in\Theta$, where each component is an element of R.

Now let m be the least common denominator of the $\lambda(i)$, $i=1, 2, \ldots, n$, and consider the kth restricted finite economy containing number $\lambda(i)m$ agents of type i and production set mY^k.[11] Now restrict attention to an m-agent economy in which all agents of any given type i must be treated identically. Then, following Debreu [1962], we have the following

Definition. A *quasi-equilibrium* of the kth restricted finite economy is a state $[x_i^{k*}, y^{k*}]$ and a price system p^{k*} such that

(α) for every i, x_i^{k*} is a greatest element $\{x_i \in X^k\colon p^{k*} \cdot x_i \leq p^{k*} \cdot \xi^k\}$ under $W(\cdot, i)$ and/or $p^{k*} \cdot x_i^{k*} = p^{k*} \cdot \xi^k = \min p^{k*} \cdot X^k$

(β) $p^{k*} \cdot my^{k*} = \max p^{k*} \cdot mY^k$

(γ) $\sum_i m\lambda(i)x_i^{k*} - my^k = m\xi^k$

(δ) $p^{k*} \neq 0$.

A quasi-equilibrium is a competitive equilibrium if the first part of condition (α) holds. In what follows we shall establish the existence of a quasi-equilibrium using a theorem of Debreu [1962] and then establish directly that is is also a competitive equilibrium. It is immediate that a competitive equilibrium for the kth restricted finite economy is also a competitive equilibrium for the original kth restricted economy with a continuum of agents (m cancels out of conditions (β) and (γ)).

We make use of the following theorem, as a special case of Debreu [1962].

THEOREM (Debreu). *The kth restricted finite economy has a quasi-equilibrium if*

(a.1) $$A(mX^k) \cap (-A(mX^k)) = \{0\}$$

(a.2) X^k *is closed and convex;*

for every i,

(b.1) *for every consumption* x_i *in* $\hat{X}_i^k$, *there is a consumption in* X^k *preferred to* x_i,

(b.2) *for every* x_i' *in* X^k, *the sets*

$$\{x_i \in X^k\colon W(x_i, i) \geq W(x_i', i)\}$$

$$\{x_i \in X^k\colon W(x_i, i) \leq W(x_i', i)\} \text{ are closed in } X^k$$

(b.3) *for every* x_i' *in* X^k, *the set* $\{x_i \in X^k\colon W(x_i, i) \geq W(x_i', i)\}$ *is convex;*

(c.1) $$(\{m\xi^k\} + mY^k) \cap mX^k \neq \emptyset$$

[11] We are assuming that each $\lambda(i)$ is rational. An extension to arbitrary real $\lambda(i)$'s would entail a limiting argument.

(c.2) $(\{\xi^k\}+A(mY^k)) \cap X^k \neq \emptyset;$

(d.1) $0 \in mY^k$

(d.2) $A(mX^k) \cap A(mY^k) = \{0\};$

where $A(H)$ is the asymptotic cone of set H, $mH=\{s\colon s=mh,\ h\in H\}$, and $\hat{X}_i^k$ is the attainable consumption set for the ith *type consumer in the* kth *restricted economy.*

Each of these conditions holds for our restricted finite economy. (See Prescott and Townsend [1979] for the tedious but straightforward argument). Thus, the existence of a quasi-equilibrium is established. We now verify that the first part of condition (α) must hold. In a quasi-equilibrium, condition (β) holds, i.e., there exists a maximizing element in Y^k given p^{k*}. It follows that no component of p^{k*} can be negative. Also, from condition (δ), not all components can be zero. Therefore, at least one component of p^{k*} is positive. Maximizing $p^{k*}\cdot y$ with respect to y in Y^k, one obtains

(4.6a) $p_0^{k*}(c) - \psi_0^k\cdot c = 0 \qquad c\in C^k$

(4.6b) $p_1^{k*}(c,\theta_1) - \lambda(\theta_1)\psi_1^k\cdot c = 0 \qquad c\in C^k,\ \theta_1\in\Theta$

(4.6c) $p_2^{k*}(c,\theta_1,\theta_2) - \lambda(\theta_1)\lambda(\theta_2)\psi_2^k\cdot c = 0 \qquad c\in C^k,\ \theta_1,\ \theta_2\in\Theta$

where the ψ_t^k, $t=0$, 1, 2, are nonnegative, l-dimensional vectors of Lagrange multipliers. By virtue of the existence of a maximum and the existence of at least one positive price, one of these Lagrange multipliers is positive. Thus,

$$p^{k*}\cdot\xi^k = \psi_0^k\cdot e_0 + \psi_1^k\cdot e_1 + \psi_2^k\cdot e_2 > 0$$

since $e_t>0$, $t=0$, 1, 2. But the measure which puts mass one on the zero element of the underlying commodity space for all possible parameter draws has valuation zero under p^{k*}. Thus, $p^{k*}\cdot\xi^k > \min p^{k*}\cdot X^k$ and the second part of the condition (α) cannot hold.

Now x_i^{k*} denotes the maximizing element for the ith agent type in a competitive equilibrium of the kth restricted economy. For any i, $\{x_i^{k*}\}_{k=0}^{\infty}$ is a sequence in the space of $(1+n+n^2)$-dimensional vectors of probability measures on the underlying consumption set C. This metric space is compact, so there exists a convergent subsequence. Since there are a finite number of agent types, it is possible to construct a subsequence of the sequence allocation $\{(x_i^{k*})\}$ which converges to some allocation (x_i^{∞}). This limit, (x_i^{∞}), will constitute part of an equilibrium specification for the unrestricted economy.

For every restricted economy k, the price system is (4.6). Moreover, the price system may be normalized by dividing through by the sum of all the Lagrange multipliers so that in fact each Lagrange multiplier may be taken to be between zero and one. Thus, one may again find a further subsequence of the sequence of vectors $\{\psi_t^k\}$ which converges to some number $\{\psi_t^{\infty}\}$ with components between zero and one. Moreover, the Lagrange multipliers in $\{\psi_t^{\infty}\}$ must sum to one.

In what follows, then, we restrict attention to the subsequence of economies, indexed by h, such that for every i, $x_i^{h*w} \to x_i^\infty$ and for every t, $\psi_t^h \to \psi_t^\infty$.

For each economy h, the equilibrium price system is a linear functional v^h defined by

$$v^h(x) = \sum_{c \in C^k} p_0^{h*}(c)x_0(c) + \sum_{\theta_1} \sum_{c \in C^k} p_1^{h*}(c, \theta_1)x_1(c, \theta_1)$$
$$+ \sum_{\theta_1} \sum_{\theta_2} \sum_{c \in C^k} p_2^{h*}(c, \theta_1, \theta_2)x_2(c, \theta_1, \theta_2)$$
$$= \sum_{c \in C^k} \psi_0^h \cdot cx_0(c) + \sum_{\theta_1} \lambda(\theta_1) \sum_{c \in C^k} \psi_1^h \cdot cx_1(c, \theta_1)$$
$$+ \sum_{\theta_1} \lambda(\theta_1) \sum_{\theta_2} \lambda(\theta_2) \sum_{c \in C^k} \psi_2^h \cdot cx_2(c, \theta_1, \theta_2). \tag{4.7}$$

Taking the limit as $k \to \infty$, we see that an equilibrium price system v^∞ for the unrestricted economy will be

$$v^\infty(x) = \psi_0^\infty \cdot \int cx_0(dc) + \sum_{\theta_1} \lambda(\theta_1)\psi_1^\infty \cdot \int cx_1(dc, \theta_1)$$
$$+ \sum_{\theta_1} \lambda(\theta_1) \sum_{\theta_2} \lambda(\theta_2)\psi_2^\infty \cdot \int cx_2(dc, \theta_1, \theta_2). \tag{4.8}$$

Note that, since the sum of the Lagrange multipliers is one, a strictly positive number, $v^\infty(\xi) > 0$. Finally,

$$y^\infty = \sum_i \lambda(i)x_i^\infty - \xi \tag{4.9}$$

is an equilibrium output for the firm.

The feasibility of the limiting allocation $[(x_i^\infty)_{i \in \Theta}, y^\infty]$ follows because both constraints X and Y are closed in the weak topology. Given that $v^h(x_i^{h*}) \leq v^h(\xi)$, taking the limit as h goes to infinity yields $v^\infty(x_i^\infty) \leq v^\infty(\xi)$. All that remains is to show that (i) there is no $\hat{x}_i \in X$ which satisfies the budget constraint and for which $W(\hat{x}_i, i) > W(x_i^\infty, i)$ and (ii) there is no $\hat{y} \in Y$ for which $v^\infty(\hat{y}) > v^\infty(y^\infty)$.

The proof of (i) is by contradiction. If there is such an $\hat{x}_i$, then it is possible to select some h and $\hat{x}_i^h \in X^h$ such that $W(\hat{x}_i^h, i) > W(x_i^{h*}, i)$ and $v^h(\hat{x}_i^h) < v^h(\xi^h)$. This contradicts x_i^{h*} as maximizing in the hth restricted economy. To prove (ii), the nonnegativity of the ψ^∞ implies all points belonging to Y, that is, satisfying constraints (4.1)–(4.3), have nonnegative value with respect to the price system v^∞. Since budget constraints are binding, that is $v^\infty(x_i^\infty) - v^\infty(\xi) = 0$, from (4.9) profits at y^∞ are zero. Hence, y^∞ is profit maximizing. This completes the proof of the existence of a competitive equilibrium.

It is readily verified that for a one-period economy (with period zero only) there need be no randomness in a competitive equilibrium. Agents are risk averse, and the incentive-compatibility conditions need not be imposed explicitly. In this sense, the work developed here reduces the standard competitive analysis when the information structure is private but not sequential.

5. THE WELFARE THEOREMS

We now turn to the two fundamental theorems of contemporary welfare economics and ask whether any competitive equilibrium allocation is optimal and whether any optimum can be supported by a competitive equilibrium. The first question has an affirmative answer.

THEOREM 1. *If the allocation* $[(x_i^*), y^*]$, *together with the price system* v^*, *is a competitive equilibrium and if no* x_i^* *is a local saturation point, then* $[(x_i^*), y^*]$ *is a Pareto optimum.*

PROOF. Suppose a feasible Pareto superior allocation $((x_i), y)$ existed. Then $v^*(x_i) \geq v^*(x_i^*)$ with strict inequality for some i. Multiplying by population fractions, summing over i, and using the linearity of v^* yields $v^*(x) > v^*(x^*)$. Profit maximization implies $v^*(y^*) \geq v^*(y)$. Thus, $v^*(\xi) = v^*(x-y) > v^*(x^* - y^*) = v^*(\xi)$, which is the contradiction.

Debreu [1954] establishes that the following five assumptions are sufficient to ensure that an optimum can be supported by a quasi-competitive equilibrium.

(I) X is convex.

(II) For $x', x'' \in X$ and $i \in \theta$, $W(x', i) < W(x'', i)$ implies $W(x', i) < W(x^\alpha, i)$ where $x^\alpha = (1-\alpha)x' + \alpha x''$, $0 < \alpha < 1$.

(III) For $x, x', x'' \in X$ and $i \in \Theta$, the set $\{\alpha \in [0, 1]: W(x^\alpha, i) \leq W(x, i)\}$ is closed where $x^\alpha = (1-\alpha)x' + \alpha x''$.

(IV) Y is convex.

(V) Y has an interior point.

For property I, note that a linear combination of two probability measures is again a probability measure and that constraints (2.4) and (2.5) hold under convex combinations. Properties II and III follow immediately from the linearity of the objective function. Property IV follows from the fact that constraints (4.1)–(4.3) hold under convex combinations. For property V, pick a degenerate element of L such that (4.1)–(4.3) hold as strict inequalities.

But, with private information, these conditions, along with Debreu's argument [1954, Theorem 2], do not ensure that every Pareto optimum can be supported by a quasi-competitive equilibrium with an appropriate initial distribution of wealth. It is true that a separating hyperplane exists such that y^* maximizes value subject to the technology constraint. It does not follow, however, that x_i^* necessarily minimizes value over the set of points that yield utility to type i greater than or equal to $W(x_i^*, i)$. Rather, x_i^* minimizes value over the set $\{x \in X: W(x, i) \geq W(x_i^*, i)$ and $W(x, j) \leq W(x_j^*, j), j \neq i\}$. (For details, see Prescott and Townsend [1979].)

6. AN EXAMPLE

This section presents a simple example economy which we hope clarifies by illustration the definitions and concepts that we have developed throughout the paper. For this economy, the equilibria are characterized by insurance contracts with options, the exercise of which is private information dependent. In addition, the equilibrium contracts incorporate contrived randomness even though agents have convex preferences. For this example, all agents are alike ex ante but not ex post. This greatly simplifies the analysis, for by Theorem 1, the optima are the competitive equilibrium allocations.

Following Becker and Lancaster, we give agents a household production function mapping a time endowment, a market-produced consumption good $c \in R_+$ and a private shock $\theta \in \Theta = \{\theta_1, \theta_2\}$ into an idiosyncratic, nontradeable household consumption vector. The resulting indirect utility function is U: $R_+ \times \Theta \to R$. The function $U(c, \theta)$ is increasing, concave, and continuously differentiable in c. Further, $U'(\infty, \theta_1) = 0$, and $U(c, \theta_2) = \theta_2 c$ where $\theta_2 > 0$. Thus, households of type θ_1 are ex post risk averse and households of type θ_2 are ex post risk neutral. This somewhat extreme assumption simplifies the analysis, but is not crucial. What is needed is that there be differences in curvatures ex post.

Households know fraction $\lambda(\theta)$ of the population will experience shock θ. This is the only information they have for forecasting. We assume that agents' subjective probability beliefs are that the likelihood of being of type θ is $\lambda(\theta)$. (See footnote 6.) Finally, all agents receive endowment e of the consumption good with certainty and $U'(e, \theta_1) < \theta_2$.

Our first task is to determine a Pareto optimal allocation for this economy. This could be done formally as in Section 3 by consideration of a linear programming problem in the space of lotteries, maximizing the expected utility of the representative household subject to the incentive-compatibility and resource constraints. Here we find a solution to a simplified problem, one that takes into account the resource constraints only. We then modify that incentive-incompatible solution to obtain an allocation which is both incentive compatible and resource feasible and which yields the same expected utility as the solution to the simplified problem.

If θ were public, an optimal allocation would be for type θ_1 to consume c_1^* and type θ_2 to consume c_2^* where c_1^* and c_2^* are such that marginal utilities are equated across states and the endowment is exhausted. Essentially, this is full insurance. But this allocation is not achievable if θ is private information. Type θ_1 prefers the larger consumption c_2^* to its allocation c_1^* (see figure). An equally good, incentive-compatible allocation does exist, but it requires contrived randomness in the allocation. If rather than receiving c_2^* with certainty, type θ_2 receives c_3^* with probability $\alpha^* = c_2^*/c_3^*$ and consumption 0 with probability $1 - \alpha^*$, the expected utility of type θ_2 continues to be $\theta_2 c_2^*$ as type θ_2 is risk neutral. Thus, both allocations yield the same expected utility, as well as having the same

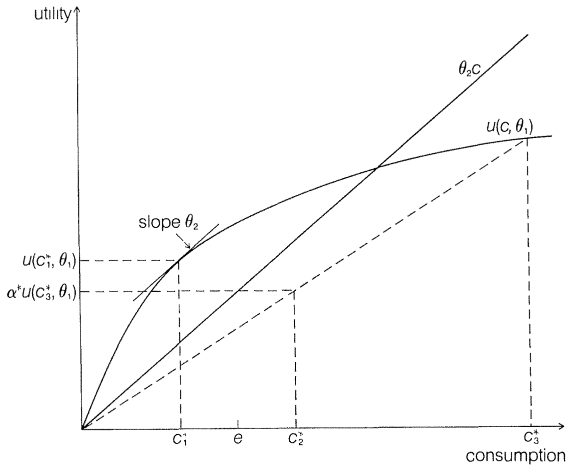

FIGURE 1

per capita consumption. Variable c_3^* can be selected sufficiently large to insure that the expected utility of this lottery for type θ_1 is less than the utility of the certainty consumption c_1^* (as in the figure). To summarize, the allocation for which type θ_1 individuals receive c_1^* with certainty and type θ_2 receive a lottery that provides c_3^* with probability α^* and 0 otherwise is an optimum. Further, no allocation without lotteries is optimal.

We shall now argue that this optimum can be achieved in a decentralized, competitive market for insurance contracts with individually effected and private information dependent options. Imagine, in particular, that households in the economy can buy and sell contracts (make commitments) in some planning period market. Clearly, with one consumption good, unconditional promises or commitments cannot be mutually beneficial. But households want commitments to be conditional on their own individual circumstances, that is, on their own shocks θ. Of course, these shocks are privately observed. Still, suppose an insurance contract has options effected entirely by the household, once its θ value is known. Then some insurance may be possible. Of course, the household will choose the option which is best given its individual circumstances, and thus we may suppose without loss of generality that options are such that the household announces its individual shocks truthfully. Finally, we allow options to affect random allocations of the consumption good.

More formally, then, a household is imagined to buy in the planning period market (say from a Walrasian auctioneer) an insurance contract $\{x(c, \theta)\}$, $c \in C$, $\theta \in \Theta$. (Here for simplicity we suppose set C is finite, though the more-general analysis of the paper allows C to contain a continuum of values.) Under this contract, the household is supposed to announce its actual shock θ in the consumption period and receive c with probability $x(c, \theta)$ (of course, $0 \leq x(c, \theta) \leq 1$ and $\sum_c x(c, \theta) = 1$).

The household can choose, in principle, apart from budget considerations, any incentive-compatible contract it wants, with the receipts varying over c in C and the probabilities varying between zero and one inclusive. A price system $\{p(c, \theta)\}$, $c \in C$, $\theta \in \Theta$ determines the cost of a contract. As for revenue, note that the household is effectively endowed with probability measures $\xi(c, \theta)$, $\theta \in \Theta$, each putting mass one on the endowment point e. These endowments are sold in the planning period market. (Alternatively, one can view $x(c, \theta) - \xi(c, \theta)$ as excess demand.) In summary, the household can choose a contract $x(c, \theta)$ to maximize

$$(6.1) \qquad \sum_{\theta} \lambda(\theta) \sum_{c} x(c, \theta) U(c, \theta)$$

subject to the budget constraint

$$(6.2) \qquad \sum_{\theta} \sum_{c} p(c, \theta) x(c, \theta) \leq \sum_{\theta} \sum_{c} p(c, \theta) \xi(c, \theta)$$

and subject to incentive-compatibility restrictions.

On the other side of the market, we suppose there are firms or intermediaries that make commitments to buy and sell the consumption good. A production-intermediation choice $y(c, \theta)$, $c \in C$, specifies the number of units of the bundle with c units of the consumption good which the firm-intermediary plans to deliver or sell to the market for use by consumers announcing they are of type θ. Thus, if $y(c, \theta)$ is negative, there is a plan to take in or buy resources. The production-intermediation set Y of each firm-intermediary is defined by

$$(6.3) \qquad Y = \{y(c, \theta),\ c \in C,\ \theta \in \Theta :\ \sum_{\theta} \lambda(\theta) \sum_{c} c y(c, \theta) \leq 0\}.$$

In effect, (6.3) requires that each firm-intermediary not deliver more of the single consumption good in the consumption period than it takes in. Note that each firm-intermediary takes the coefficients in Y, the weights on different bundles, as given, beyond its control. Note also that Y displays constant returns to scale, so we act as if there were only one firm-intermediary.

The firm-intermediary gets credit or debits for its commitments in terms of the price system $p(c, \theta)$. The firm-intermediary takes the price system as given and maximizes profits

$$(6.4) \qquad \sum_{\theta} \sum_{c} p(c, \theta) y(c, \theta).$$

It is thus clear that the constant returns to scale specification of the production set (6.3) delivers prices up to some arbitrary normalization. In fact, expressing prices in terms of the consumption good, the equilibrium price system $p^*(c, \theta)$ must satisfy

$$p^*(c, \theta) = \lambda(\theta) c.$$

This corresponds to actuarially fair insurance.

The Pareto optimal allocation to be supported in this competitive insurance market is

(6.5) $$x^*(c_1^*, \theta_1) = 1,\ x^*(c_3^*, \theta_2) = \alpha^*,\ x^*(0, \theta_2) = 1 - \alpha^*,$$
$$\text{and } x(c, \theta) = 0 \text{ otherwise.}$$

Clearly, for an equilibrium we must have $y^* = x^* - \xi$. It is easily verified under the price system p^* that x^* solves the household's problem and y^* solves the firm-intermediaries' problem.

In this analysis, we used lotteries as an allocation device. This may seem unusual, but we argue that it is not. Indeed, one can mimic exactly the effects of a lottery by indexing on the basis of a naturally occurring random variable that is unrelated to preferences and technology, provided that the random variable has a continuous density. (Kenneth Arrow pointed this out to us.) Such an arrangement would seem consistent with the existence of the usual Arrow-Debreu securities or contingent commodities.[12]

We might argue further that devices which generate lotteries or contrived risk may themselves be familiar. For returning to our competitive market setup, suppose that a group of households has entered into the contracts $\{x(c, \theta)\}$ with a broker, who acts as a firm intermediary, with terms of trade as specified in the $p^*(c, \theta)$.[13] That contract can be effected as follows.

Agents are required to surrender their endowment e to the broker and then, subsequent to the revelation of the shocks, they have a choice between two distribution centers. If they choose the first, they are guaranteed c_1^* units of the good. If they choose the second, they receive c_3^* units if it is available. Households choosing the second center are imagined to arrive in a random fashion and to receive c_3^* on a first-come, first-served basis.[14] All households know that the likelihood of receiving c_3^* if they choose center two is α^*. Agents are not permitted to recontract contingent upon whether or not they are served.[15]

Upon observing the number of unserved customers in the second center, a casual observer might find the above-described scheme somewhat unsatisfactory. Since some go away empty-handed, the "price" must be too low; that is, the potential allotment of c_3^* is too high. In fact, if the receipt were lowered to c_2^*, all could be served.[16] But, of course, the allocation achieved by the above-

[12] Cass and Shell [1983] have an example of an economy with an equilibrium characterized by allocations being indexed by an exogenous random variable that is unrelated to either preferences or technologies.

[13] In a literal sense, we would not expect to see the highly centralized Walrasian arrangement, with an auctioneer who calls out prices until all markets are clear. We believe, though, that such an arrangement might well predict the outcome of arrangements in which the market assignment process and the price determination process are explicit.

[14] Obviously, for the analysis of some queues, one wants to take starting times as choice variables. For our purposes here, we abstract away from such considerations.

[15] We thank John Bryant for pointing out this implicit restriction.

[16] Of course, this is not the only model of apparent underpricing. In a provocative article Cheung [1977] argues that apparent underpricing of better seats in theaters, so that they fill up early on, is a way of reducing the costs of monitoring seat assignments. But the theory developed

(*Continued on next page*)

described resource allocation scheme is private information optimal. The point is that the queue (rationing) is a device which induces the requisite artificial risk.

7. A CONCLUDING REMARK

The essential difference between our private information competitive analysis and the contingent claim approach of Arrow and Debreu is that options are needed and these options are exercised contingent upon private information. If we are to use competitive analysis to explain the existence of contractual arrangements with options, the exercise of which cannot be perfectly predicted given publicly available information, such a theory is needed. Given the wide use of such arrangements, we are optimistic that this formulation will prove useful in substantive economic analyses.

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REFERENCES

AKERLOF, G.A., "The Market for 'Lemons': Quality Uncertainty and the Market Mechanism," *Quarterly Journal of Economics*, 84 (August, 1970), 488–500.

ARROW, K. J., "Le rôle des valeurs boursieres pour la répartition la meilleure des risques," *Econometrie*, Paris, Centra National de la Recherche Scientifique, (1953), 41–48.

BEWLEY, T. F., "Existence of Equilibria in Economies with Infinitely Many Commodities," *Journal of Economic Theory*, 4 (Jume, 1972), 514–540.

——— AND R. RADNER, "Stationary Monetary Equilibrium with a Continuum of Independently Fluctuating Consumers," Northwestern University Working Paper (October, 1980).

CASS, D. AND K. SHELL, "Do Sunspots Matter?" *Journal of Polifical Economy*, 91 (April, 1983), 193–227.

CHEUNG, S. N. H., "Why Are Better Seats 'Underpriced'?" *Economic Inquiry*, 15 (October, 1977), 513–522.

———, "Valuation Equilibrium and Pareto Optimum," *Proceedings of the National Academy of Science*, 40 (1954), 588–592.

———, *Theory of Value: An Axiomatic Analysis of Economic Equilibrium* (New Haven: Cowles Foundation, 1959).

DEBREU, G., "New Concepts and Techniques for Equilibrium Analysis," *International Economic Review*, 3 (September, 1962), 257–273.

DUNFORD, N. AND J. SCHWARTZ, *Linear Operators, Part I: General Theory* (New York: Interscience, 1957).

(*Continued*)

here has something in common with Cheung's the: use of apparent underpricing to discriminate among potential buyers with unobserved characteristics. Such discrimination also underlies the model of credit rationing of Stiglitz and Weiss [1980], though they proceed in a different way and draw somewhat different conclusions than we do in this paper; see also Akerlof [1970], Stiglitz [1976], Weiss [1976], and Wilson [1980].

GALE, D., "Money, Information and Equilibrium in Large Economies," *Journal of Economic Theory*, 23 (August, 1980), 28–65.

HARRIS, M. AND R. M. TOWNSEND, "Allocation Mechanisms of Asymmetrically Informed Agents," Carnegie-Mellon Working Paper No. 35-76-77, (March, 1977).

———, "Resource Allocation Under Asymmetrically Information," *Econometrica*, 49 (January, 1981), 33–64.

HURWICZ, L., "On Informationally Decentralized Systems," in, R. Radner and B. McGuire, eds., *Decision and Organization* (Amsterdam: North Holland, 1971).

LUCAS, R. E., Jr., "Equilibrium in a Pure Currency Economy," in, J. Kareken and N. Wallace, eds., *Models of Monetary Economies* (Minneapolis: Federal Reserve Bank of Minneapolis, 1980).

MAS-COLELL, ANDREU, "A Model of Equilibrium with Differentiated Commodities," *Journal of Mathematical Economics*, June-September, 2 (1975), 263–295.

MCKENZIE, L. W., "On the Existence of General Equilibrium for a Competitive Market," *Econometrica*, 27 (January,1959), 54–71.

MYERSON, R. B., "Incentive Compatibility and the Bargaining Problem," *Econometrica*, 47 (January, 1979), 61–74.

NEUMANN, J. VON, and O. MORGENSTERN, *Theory of Games and Economic Behavior* (Princeton: Princeton University Press, 1947).

PARTHASARTHY, K. R., *Probability Measures on Metric Spaces* (New York: Academic Press, 1967).

PRESCOTT, E. C. AND R. M. TOWNSEND, "On Competitive Theory with Private Information," manuscript, (June, 1979).

———, "Pareto Optimal and Competitive Equilibria with Adverse Selection and Moral Hazard," Carnegie-Mellon University Working Paper (1980) forthcoming *Econometrica*.

STIGLITZ J. E., "Prices and Queues as Screening Devices in Competitive Markets," Stanford University, IMSS Technical Report, No. 212, (August, 1976).

——— AND A. WEISS, "Credit Rationing in Markets with Imperfect Information, Part II: Constraints as Incentive Devices," Research Memorandum No. 268, Princeton, (August, 1980).

WEISS, L. "The Desirability of Cheating Incentives and Randomness in the Optimal Income Tax," *Journal of Political Economy*, 84 (December, 1976), 1343–1352.

WILSON, C., "The Nature of Equilibrium in Markets with Adverse Selection," *The Bell Journal of Economics*, 11 (Spring, 1980), 108–130.