Unilateral Effects with General Linear Demand

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The 2010 Department of Justice and Federal Trade Commission Horizontal Merger Guidelines (MG) apply a measure of "upward pricing pressure" to consider the effect of a merger in a differentiated products industry with two merging firms, each of which produces a single product.[2] The upward pricing pressure measure leads to a "gross upward pricing pressure index" (GUPPI) which is defined for good 1 as:[3]

$$GUPPI_1 = D_{12}\frac{p_2 - c_2}{p_1} \qquad (1)$$

where D_{12} is the diversion ratio from product 1 to product 2 when the price of product 1 increases, and c_2 is the marginal cost of product 2 post-merger.[4] Thus, GUPPI measures "the value of diverted sales … in proportion to the lost revenues attributable to the reduction in unit sales" when the price of product 1 increases.[5]

To calculate this index, one needs information about the diversion ratio, both prices and the marginal cost of the other good. To calculate the GUPPI for each product, one needs information about the two diversion ratios, D_{12} and D_{21}, along with prices and marginal cost of each good. However, in many cases only one diversion ratio needs to be estimated. First note that by definition $D_{jk} = -(\partial Q_k / \partial p_j)/(\partial Q_j / \partial p_j)$. If the merging goods are intermediate goods used as inputs by downstream firms, cost minimization implies that that the cross-price derivatives of the conditional factor demands are equal. Thus, the cross-price derivatives of the unconditional factor demands will be equal (i.e., $\partial Q_2 / \partial p_1 = \partial Q_1 / \partial p_2$) if the downstream firms

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[2] U.S. Dep't of Justice & Federal Trade Commission, Horizontal Merger Guidelines (2010), available at http://www.ftc.gov/os/2010/08/100819hmg.pdf.

[3] *See* Steven C. Salop & Serge Moresi, "Updating the Merger Guidelines: Comments," Horizontal Merger Guidelines Review Project, November 2009 (http://www.ftc.gov/os/comments/horizontalmergerguides/545095-00032.pdf); Joseph Farrell & Carl Shapiro, "Antitrust Evaluation of Horizontal Mergers: An Economic Alternative to Market Definition," The B.E. Journal of Theoretical Economics, Volume 10, Issue 1, Article 9, 2010 (http://faculty.haas.berkeley.edu/shapiro/alternative.pdf).

[4] Marginal cost may change post-merger if efficiencies lead to a lower marginal cost. Thus, merger efficiencies are incorporated into the analysis through marginal cost.

[5] MG p. 21.

have constant marginal costs.[6] Even without constant marginal costs, the cross-price derivatives will be approximately equal if the inputs are a small proportion of variable costs. For consumer goods, Slutsky symmetry implies that the same relationship holds apart from income effects, which are typically (but not always) small for differentiated consumer products involved in a merger analysis. In many situations, therefore, the numerators of the two diversion ratios can be assumed to be equal. Furthermore, as will be discussed below, the denominators of the two diversion ratios are known functions of quantities and margins under the assumption of profit maximization by single-product firms pre-merger. Thus, if D_{12} is known in these situations, then D_{21} is equal to $D_{12}(\partial Q_1 / \partial p_1)/(\partial Q_2 / \partial p_2)$.

It is straightforward to demonstrate that the diversion ratio from product 1 to product 2 is the ratio of the cross price elasticity of product 2 (with respect to the price of product 1) divided by the own price elasticity of product 1 multiplied by the ratio of unit sales of product 2 divided by the unit sales of product 1. Under the assumption of a single product firm, as used in the MG, the own price elasticity is equal to the negative inverse of the price cost margin, i.e., $M_1 = -1/e_{11}$ where $M_1 = (p_1 - c_1)/p_1$ is the price-cost margin and e_{11} is the own price elasticity of demand for product 1. Thus, an estimate of the diversion ratio implies an estimate of the cross price elasticity, which is the fundamental economic measure of competition between two products.

Given the estimates of the cross price elasticities and the own price elasticities, predicted price changes follow under a Bertrand-Nash assumption and an assumed shape of the demand curves. Indeed, it is easy to demonstrate that with linear demand and constant marginal costs in the "symmetric case" of equal diversion ratios (i.e., $D_{12} = D_{21} = D$) and equal marginal costs, prices and shares, the profit maximizing price increase post-merger is $0.5 * GUPPI/(1-D)$.[7] However, the assumptions of the symmetric case are often unrealistic for the differentiated products situation. We demonstrate how to calculate the price increases for a linear demand

[6] This result follows from the fact that with constant marginal cost the cost function takes the form C(w,q) = h(w)q. The result can be extended to increasing marginal costs if the underlying production function is homothetic so that C(w,q) = h(w)g(q).

[7] *See* the Hearing Statement of Professor Steven C. Salop and Dr. Serge Moresi, Horizontal Merger Guidelines Review Project, Washington DC, December 3, 2009 (http://crai.com/uploadedFiles/Publications/Updating-the-Merger-Guidelines-Hearings-Statement-Salop-Moresi.pdf).

system using the same information required to calculate GUPPI for the two products being analyzed.[8]

In the pre-merger equilibrium, each firm i in the industry chooses its price p_i to maximize

$$(p_i - c_i)Q_i(p_i, P_{-i}) \tag{2}$$

where c_i is the constant marginal cost of production and Q_i is the quantity demanded of firm i's product as a function of all prices. We assume that demand is linear, so that the derivative of the demand function with respect to each of its arguments is constant.

The first order condition characterizing the pre-merger equilibrium is

$$\frac{\partial Q_i(p_i^0, P_{-i}^0)}{\partial p_i} = -\frac{Q_i(p_i^0, P_{-i}^0)}{p_i^0 - c_i} \equiv -\frac{Q_i^0}{p_i^0 - c_i} \tag{3}$$

We now consider a merger between two of the firms (i = 1,2) in the industry. We solve for the price changes of the two merging firms assuming that the non-merging firms do not adjust their prices or reposition their products. This assumption has offsetting effects on the calculated price changes. The assumption of no price response will tend to understate the price changes of the merging firms, but the assumption of no product repositioning will tend to overstate the price changes of the merging firms. These assumptions are the assumptions used in the MG to calculate GUPPI.

The merged firm sets p_1 and p_2 to maximize:

$$(p_1 - c_1)Q_1 + (p_2 - c_2)Q_2 \tag{4}$$

The first order condition with respect to p_1 is

$$(p_1 - c_1)\frac{\partial Q_1}{\partial p_1} + (p_2 - c_2)\frac{\partial Q_2}{\partial p_1} = -Q_1 \tag{5}$$

[8] If the cross-price derivatives of the demand functions are not equal, then our formula also requires information on the merging firms' quantities or revenues.

Using the definition of the diversion ratio, the FOC can be rewritten

$$p_1 - c_1 - (p_2 - c_2)D_{12} = -\frac{Q_1}{\partial Q_1 / \partial p_1} \tag{6}$$

Because demand is assumed to be linear, $\partial Q_1 / \partial p_1$ and D_{12} are constants that do not depend on price. Thus, from the pre-merger FOC (equation (3)) we can replace $\partial Q_1 / \partial p_1$ with $-\frac{Q_1^0}{p_1^0 - c_1}$. Making this substitution and decomposing p_i into $p_i^0 + \Delta p_i$, the FOC can be rewritten

$$p_1^0 - c_1 + \Delta p_1 - (p_2^0 - c_2 + \Delta p_2)D_{12} = \frac{(p_1^0 - c_1)Q_1}{Q_1^0} \tag{7}$$

Dividing both sides by p_1^0 and defining $m_i^0 \equiv (p_i^0 - c_i) / p_i^0$ leads to

$$\frac{\Delta p_1}{p_1^0} - \left(m_2^0 + \frac{\Delta p_2}{p_2^0}\right)\frac{p_2^0}{p_1^0}D_{12} = \frac{m_1^0}{Q_1^0}(Q_1 - Q_1^0) \tag{8}$$

Due to linearity, the bracketed term on the right hand side is equal to

$$Q_1 - Q_1^0 = \frac{\partial Q_1}{\partial p_1}\Delta p_1 + \frac{\partial Q_1}{\partial p_2}\Delta p_2 \tag{9}$$

Using the pre-merger FOC and the definition of the diversion ratio we obtain

$$Q_1 - Q_1^0 = -\frac{Q_1^0}{m_1^0}\frac{\Delta p_1}{p_1^0} + \frac{Q_2^0 D_{21}}{m_2^0}\frac{\Delta p_2}{p_2^0} \tag{10}$$

Substituting into the post-merger FOC (equation (8)) and rearranging terms we obtain

$$2\frac{\Delta p_1}{p_1^0}+\left(-\frac{p_2^0}{p_1^0}D_{12}-\frac{m_1^0}{m_2^0}\frac{Q_2^0}{Q_1^0}D_{21}\right)\frac{\Delta p_2}{p_2^0}=\frac{p_2^0}{p_1^0}m_2^0 D_{12} \tag{11}$$

Combining this equation with the analogous FOC with respect to p_2, Cramer's rule provides the following solutions for the percentage price changes

$$\frac{\Delta p_1}{p_1^0}=\frac{2\frac{p_2^0}{p_1^0}m_2^0 D_{12}+m_1^0 D_{12}D_{21}+\frac{(m_1^0)^2}{m_2^0}\frac{Q_2^0 p_1^0}{Q_1^0 p_2^0}(D_{21})^2}{4-2D_{12}D_{21}-\frac{m_2^0 Q_1^0 p_2^0}{m_1^0 Q_2^0 p_1^0}(D_{12})^2-\frac{m_1^0 Q_2^0 p_1^0}{m_2^0 Q_1^0 p_2^0}(D_{21})^2} \tag{12}$$

Changing the subscripts gives the formula for the percentage price change for good 2.

Equation (12) holds for the general case in which the cross-price derivatives are not assumed to be equal. However, as discussed above in many situations it is reasonable to assume that the cross-price derivatives are equal or approximately equal. Under this assumption equation (11) reduces to:

$$2\frac{\Delta p_1}{p_1^0} - 2\frac{p_2^0}{p_1^0}D_{12}\frac{\Delta p_2}{p_2^0} = \frac{p_2^0}{p_1^0}D_{12}m_2^0 \tag{13}$$

Solving the two equations (equation (13) and the analogous equation for good 2), one obtains the following formula for the percentage price increase. This formula can also be expressed in terms of *GUPPI*.

$$\begin{aligned}\frac{\Delta p_1}{p_1^0} &= \frac{D_{12}m_2^0(p_2^0/p_1^0) + D_{12}D_{21}m_1^0}{2(1 - D_{12}D_{21})}\\ &= \frac{GUPPI_1}{2}\times\frac{1 + GUPPI_2/m_2^0}{1 - D_{12}D_{21}}\end{aligned} \tag{14}$$

Note that in the symmetric case (i.e., the two products have equal pre-merger prices, quantities, margins, and diversion ratios) equations (12) and (14) both reduce to[9]

$$\frac{\Delta p}{p^0} = \frac{Dm^0}{2(1-D)} \qquad (15)$$

In a recent article, Professors Joe Farrell and Carl Shapiro have reported a formula for the price change in the asymmetric case.[10] Their formula is (implicitly) based on the assumption that the own-price derivatives of the demand functions are equal (i.e., $\partial Q_1 / \partial P_1 = \partial Q_2 / \partial P_2$) and thus does not apply in the general asymmetric case.[11] Further, this condition is unlikely to hold, even approximately, in the differentiated products situation.

Thus, we have derived the formula for price changes with linear demand and two products in the general asymmetric situation. These price changes use the same information which is required to calculate the GUPPI measure of the new MG. However, the price changes are more informative since they measure the variables which are at the core of merger analysis, potential price changes, as well as the variables required to estimate the effect on consumer welfare and economic efficiency that arise from the merger.

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[9] This formula for the symmetric case was reported in Carl Shapiro, "Mergers with Differentiated Products," Antitrust, 10, 23-30, 1996.

[10] *See* Joseph Farrell and Carl Shapiro, "Upward Pricing Pressure and Critical Loss analysis: Response", The CPI Antitrust Journal, February 2010, p. 4.

[11] *See* Carl Shapiro, "Unilateral Effects Calculations," September 2007, revised December 2009 (available at http://faculty.haas.berkeley.edu/shapiro/unilateral.pdf). In principle, one could redefine units of measurement so that the assumption of equal own-price derivatives would be satisfied. However, that would generally require to express the data and the estimated diversion ratios using very unconventional units, and thus would likely lead to measurement and interpretation errors.