

## Weak Instruments: Diagnosis and Cures in Empirical Econometrics

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What is the weak instruments (WI) problem and what causes it? Universal agreement does not exist on these questions. We define weak instruments by two features: (1) 2SLS is badly biased toward the OLS estimate and alternative “unbiased” estimators such as LIML may not solve the problem and (2) the standard (first order) asymptotic distribution does not give an accurate framework for inference.<sup>1</sup> Thus, a researcher may estimate “bad results” and not be aware of the outcome. The cause of WI is often stated to be a low  $R^2$  or F statistic of the reduced form equation, in the most commonly occurring situation of one right hand side endogenous variable. We find the situation is more complex with an additional factor, the correlation between the stochastic disturbances of the structural equation and the reduced form, that needs to be taken into account. We discuss in this paper: a specification test of Hahn-Hausman (2002a) for WI, a caution against using “no moments” estimators such as LIML in the WI situation, suggestions for different estimators, an approach to inference of Frank Kleibergen (2002) for WI, and we end with a caution of how “small biases” can become “large biases” in the WI situation.

We begin with the limited information structural model under the assumptions of e.g. Hausman (1983):

$$(1) \quad \begin{aligned} Y_1 &= Y_2\beta + Z_1\gamma + u \\ Y_2 &= Z_1\pi_1 + Z_2\pi_2 + V \end{aligned}$$

where we assume that  $Y_1$  and  $Y_2$  are each single jointly endogenous variables. Without loss of generality, we “partial out” the  $Z_1$  variables by multiplying through each equation by the complementary projection  $Q_{Z_1} = I - Z_1'(Z_1'Z_1)^{-1}Z_1' = I - P_{Z_1}$ . We write the resulting equations as:

$$(2) \quad \begin{aligned} y_1 &= \beta y_2 + \varepsilon_1 \\ y_2 &= z\pi_2 + v_2 \end{aligned}$$

where  $\dim(\pi_2) = K$  and the sample size is  $n$ . We also assume homoscedasticity:

$$(3) \quad \begin{pmatrix} \varepsilon_{1i} \\ v_{2i} \end{pmatrix} \sim N(0, \Sigma) \sim N\left(0, \begin{bmatrix} \sigma_{\varepsilon\varepsilon} & \sigma_{\varepsilon v} \\ \sigma_{\varepsilon v} & \sigma_{vv} \end{bmatrix}\right).$$

We initially assume the presence of valid instruments,  $E[z' \varepsilon / n] = 0$  and  $\pi_2 \neq 0$ .

Without loss of generality we use the normalization (rescaling of units)  $\sigma_{\varepsilon\varepsilon} = \sigma_{vv} = 1$  so that  $Var(y_2) = 1/(1 - R^2)$  and  $\sigma_{\varepsilon v} = \rho$ .<sup>2</sup>

### I. Problems Caused by Weak Instruments

Hahn and Hausman (2002a, 2002b) (and others) derive the bias of 2SLS up to second order to be

$$(4) \quad E[b_{2SLS}] - \beta \approx \frac{K\sigma_{\varepsilon v}}{n\Theta} \approx \frac{K\sigma_{\varepsilon v}}{nR^2 \text{var}(y_2)} = \frac{K\rho(1 - R^2)}{nR^2},$$

where  $\Theta = \pi' z' z \pi / n$ , assumed to be fixed,  $R^2$  is the theoretical value from the second (reduced form) equation,  $y_2$  is normalized to have mean zero, and the last expression follows from our normalization. We find from equation (4) that 2SLS is biased towards OLS since the OLS bias also depends on the covariance term  $\sigma_{\varepsilon v}$  or the correlation parameter  $\rho$ . Thus, Hausman (1978) specification tests may incorrectly fail to reject use of the OLS estimator because of the bias. Also, while  $R^2$  and the  $F = nR^2/K$  statistics from the reduced form give information about the bias, the correlation parameter,  $\rho$ , is an important parameter in determining the bias. Thus, using “too many” instruments, the sample size,  $R^2$ , and  $\rho$  all can lead to substantial bias in the 2SLS estimator. No statistic based on a subset of these parameters seems entirely adequate in diagnosing WI.

For inference Hahn-Hausman (2002c) (and others) derive the asymptotic distribution of the 2SLS estimator. Under the assumption that

$K \rightarrow \infty$  as  $n \rightarrow \infty$  such that  $K / \sqrt{n} = \mu + o(1)$  for some  $\mu \neq 0$  they derive

$$(5) \quad \sqrt{n}(b_{2SLS} - \beta) \Rightarrow N\left(\frac{\sigma_{\varepsilon v} \mu}{\Theta}, V_{2SLS}\right)$$

where  $V_{2SLS} = \sigma_{\varepsilon\varepsilon} / \Theta$ , the usual 2SLS first order asymptotic variance. Accurate estimation of the denominator is typically not difficult since it depends on the unbiased reduced form parameters estimates of  $\pi_2$ . However, the bias of the 2SLS estimator can

lead to a severe downward bias in the estimate of  $\sigma_{\varepsilon\varepsilon}$ . Hahn-Hausman (2002c) find that to second order

$$(6) \quad E[\hat{\sigma}_{2SLS}^2] \approx \sigma_{\varepsilon\varepsilon} - \frac{2(K-2)\sigma_{\varepsilon v}}{n\Theta} - \frac{1}{n}\sigma_{\varepsilon v}^2 + \frac{1}{n}\frac{\sigma_{\varepsilon\varepsilon}\sigma_{v\varepsilon}}{\Theta}.$$

Note that the leading term in the bias calculation of equation (6) can be quite large in the presence of WI. As either the number of instruments grows or the covariance between the structural and reduced term stochastic disturbances becomes large, the downward bias in the estimation of  $\sigma_{\varepsilon\varepsilon}$  will also become large. We now apply the normalization that we used above to find:

$$(7) \quad E[\hat{\sigma}_{2SLS}^2] \approx 1 - \frac{1[(2K-4)\rho^2 - 1](1-R^2)}{nR^2} - \frac{1}{n}\rho^2.$$

Equation (7) demonstrates that the downward bias can be substantial; in Monte-Carlo results Hahn-Hausman (2002c) find that for  $R^2 = .01$  and  $\rho = 0.9$  that the mean bias of the 2SLS estimate of the variance varies from  $-70\%$  to  $-80\%$  as  $K$ , the number of instruments, increases from 5 to 30.<sup>3</sup>

WI may also be an important cause of the finding that the often used test of over identifying restrictions (OID test) rejects “too often” when weak instruments are present, i.e. the actual size of the test is considerably larger than the nominal size. See Hahn-Hausman (2002a), Table III where the nominal size is 0.05 while the actual size is often greater than 0.35 and sometimes greater than 0.5. The OID test can be quite important since it tests the economic theory embodied in the model as discussed by e.g. Hausman (1983). In the weak instrument situation it may have increased importance given the substantial bias in the 2SLS estimator. From Hausman (1983, p. 433) we write the OID test as:

$$(8) \quad W = \frac{\hat{\varepsilon}' P_z \hat{\varepsilon}}{\hat{\sigma}_{\varepsilon\varepsilon}}.$$

$W$  is distributed as chi-square with  $K-1$  degrees of freedom. From equation (8) we see that a downward biased of  $\sigma_{\varepsilon\varepsilon}$  can lead to substantial over-rejection and an upward biased size of the OID test. Thus, correcting for this problem can have an important

effect on test results. This downward bias in the estimate of  $\sigma_{\varepsilon\varepsilon}$  may be especially important when WI occurs in times series model where the correlation can be quite high, e.g. recent research by Motohiro Yogo (2002) estimates the intertemporal substitution elasticity estimates the absolute value of  $\rho$  for the US to vary between 0.78-0.94 depending on the sample period used.

## II. Diagnosis

WI can cause substantial bias in the 2SLS estimator, see e.g. Hahn-Hausman (2002), Table III. However,  $n$ ,  $R^2$ ,  $K$  and  $\rho$  all affect the bias. Thus, a test that includes the effects of these factors may be useful. Hahn-Hausman (2002a) propose a specification test that includes all of these factors. They consider the “reverse” 2SLS estimator

$$(9) \quad c_{2SLS} \equiv \frac{\sum_i \hat{y}_{1i} \hat{y}_{2i}}{\sum_i \hat{y}_{1i}^2}$$

and use the fact that under conventional (first order) asymptotics that the inverse of  $c_{2SLS}$  should have correlation one with the “forward” 2SLS estimator  $b_{2SLS}$ . To construct a test they adopt the second order asymptotic approach of Paul Bekker (1994) and derive the result that the difference between the estimators takes the form

$$(10) \quad \sqrt{n} \left( b_{2SLS} - \frac{1}{c_{2SLS}} - \hat{B} \right) \rightarrow N(0, V)$$

where  $\hat{B}$  is an estimate of the probability limit of the difference between the two possible estimators of  $\beta$  and  $V$  is the variance of the “bias corrected” difference. Two primary reasons can lead to a rejection. First, the orthogonality assumptions of the instruments may be false. The traditional Sargan test of overidentifying restrictions also tests this assumption, but it is well known to have poor size properties sometimes as we discussed above. Alternatively, a rejection may occur because the finite sample properties of the first order asymptotic approximation are not sufficiently accurate (weak instruments) in the current situation to be used. Stock et. al. (2002) recommend a test based on the F statistic of the reduced form of equation (2) that does not take account of the effect of

$\sigma_{\varepsilon}$ , which is responsible for both the bias in the OLS estimator and the bias in the 2SLS estimator as well as the bias in estimating the variance of the structural equation,  $\sigma_{\varepsilon\varepsilon}$ .

### III. Possible Cures

The first approach to the 2SLS bias problem would be to use a (second-order) unbiased estimator, such as the Nagar estimator or the maximum likelihood (LIML) estimator which e.g. Hausman (1983) discusses.<sup>4</sup> However, these estimators sometimes perform well and sometimes poorly in the WI situation. The problem arises because the Nagar and LIML estimator do not have finite sample moments. While long known since Robert Mariano and Takamitsu Sawa (1972) and Sawa (1972), it had not been generally recognized that the lack of moments could cause problems in actual empirical situations. However, the empirical example and Monte-Carlo results of Hahn-Hausman (2002a) and Hahn-Hausman-Kuersteiner (HHK, 2002) demonstrate that the “moments problem” can create problems in the WI situation. HHK (2002), Tables 1 and 2, find that the inter-quartile range (IQR) of the LIML and Nagar estimators often far exceed the IQR of the 2SLS estimator when WI are present.<sup>5</sup> Thus, we recommend extreme caution in using “no moments” estimators (e.g. LIML, Nagar, and JIVE) in the presence of WI.

HHK (2002) recommend two alternative approaches, instead of using either Nagar or LIML. First, they consider the Jackknife 2SLS (JN2SLS) estimator. The JN2SLS estimator omits the  $j$ th observation in calculating the reduced form estimate of  $\pi_2$  in equation (2) when estimating the instrument for the  $j$ th observation of the structural model. Thus, it uses  $n-1$  observations to estimate  $\pi_2$ , rather than using all  $n$  observations. The JK2SLS estimator eliminates the (second-order) finite sample bias of 2SLS. Since the jackknife estimator,  $b_j$ , is a linear combination of  $n$  2SLS-type estimators it will have finite sample moments up to the degree of overidentification. Since the WI usually occurs when many instruments are used, this result solves the “moments problem.” Under an approximation where  $K$  becomes large HHK (2002) demonstrate that to a second order asymptotic approximation, the JK2SLS estimator has the same MSE as the Nagar estimator. Thus, the MSE of  $\sqrt{n}(b_j - \beta)$  is

$$(11) \quad \frac{\sigma_{\varepsilon\varepsilon}}{\Theta} + \frac{K}{n} \frac{\sigma_{\varepsilon\varepsilon} \sigma_{vv} + \sigma_{\varepsilon v}}{\Theta^2} = \frac{(1-R^2)}{R^2} + \frac{K(1+\rho^2)}{n} \left( \frac{1-R^2}{R^2} \right)^2$$

where the first term is the usual first-order asymptotic variance of the 2SLS, LIML, and Nagar estimators, and the term following the equal sign arises from our normalization.

Another estimator that is designed to solve the “moments problem” is the Wayne Fuller (1977) modification of LIML. The Fuller estimator depends on an unknown parameter that can be chosen to eliminate finite sample (second-order) bias.

Alternatively, the parameter can be chosen to yield a smaller MSE according to second order calculations, but the resulting estimator does have finite sample bias. The approximate second-order MSE of the unbiased Fuller estimator  $\sqrt{n}(b_F - \beta)$  is

$$(12) \quad \frac{\sigma_{\varepsilon\varepsilon}}{\Theta} + \frac{K}{n} \frac{\sigma_{\varepsilon\varepsilon} \sigma_{vv} - \sigma_{\varepsilon v}}{\Theta^2} = \frac{(1-R^2)}{R^2} + \frac{K(1-\rho^2)}{n} \left( \frac{1-R^2}{R^2} \right)^2.$$

Equation (12) will be smaller than equation (11) because the correlation enters with a minus sign.

HHK (2002) use the Monte-Carlo design of Hahn-Hausman (2002a) to investigate the finite sample performance of the estimators. They find that JN2SLS performs considerably better than Nagar, which is supposed to be equivalent according to the asymptotic approximations but suffers from the “no moments” problem. The Fuller estimators do well, and outperform JN2SLS in MSE when the number of instruments is not large with a reverse ordering with more instruments. When the IQR is used, LIML and Nagar are again found to have significantly larger IQR than the other estimators that have moments. Here the Fuller unbiased estimator does not do as well as the Fuller estimator with lower MSE. HHK conclude that the 2SLS, JN2SLS, and biased Fuller estimators perform best.

HHK (2002) also use their Monte-Carlo results to perform an empirical investigation to see how well the second order asymptotic approximations do in explaining the empirical results. They find that the sample size  $N$  and the ratio  $(R^2/(1-R^2))$  have approximately the effect that the asymptotic formulae predict. For the correlation coefficient,  $\rho$ , they find that the 2SLS formulae has approximately the

expected effect. However, the effect for JN2SLS and the Fuller estimator is only about  $\frac{1}{2}$  as large as the asymptotic formulae of equation (11) and (12) predict. Thus, the advantage of the Fuller estimator over JN2SLS is smaller than the asymptotic expansions predict. Lastly, they find that the effect of the number of instruments is only about 40% as large for 2SLS as predicted by the asymptotic 2SLS MSE formula. Thus, “instrument pessimism” seems overstated for 2SLS, which may be why 2SLS often performs better than expected in terms of MSE in the WI situation.

Kleibergen (2002) takes a quite different approach to a cure for the WI problem. Rather than focusing on parameter estimators, he attempts to correct the statistical inference problem in the WI situation. We discussed above that 2SLS, for instance, often lead to asymptotic distribution, which yields standard errors and confidence intervals that are “too small.” Thus, the standard statistics may be unreliable on which to base inference. Kleibergen modifies the Anderson-Rubin (AR) statistic by projecting the stochastic disturbances on only the IV estimate of the endogenous variables, rather than on all the instruments as does the AR statistic. In the model of equations (1) and (2) the Kleibergen statistic is distributed as chi-square with one degree of freedom, rather than  $K$  degrees of freedom as is the AR statistic. Thus, it usually leads to better inferential procedures because of its greater power. The Kleibergen statistic is defined for a null hypothesis of  $\beta_0$  using equation (2)

$$(13) \quad K(\beta_0) = \frac{\varepsilon_0' P_{y_2(\beta_0)} \varepsilon_0}{\frac{1}{n-K} \varepsilon_0' Q_z \varepsilon_0}$$

where  $\varepsilon_0 = y_1 - \beta_0 y_2$  and  $y_2(\beta_0) = z \pi_2(\beta_0)$  and

$$(14) \quad \pi_2(\beta_0) = (z'z)^{-1} z'[y_2 - \varepsilon_0 \sigma_{\varepsilon v}(\beta_0) / \sigma_{\varepsilon \varepsilon}(\beta_0)]$$

The estimate of the reduced form parameter is the same as the LIML estimate where the estimate is  $\beta_0$ , see Hausman (1983, equation (4.39)). More generally, equation (13) is similar to the form of the LIML objective function and reaches a minimum at the LIML estimate. Thus, the confidence interval will be centered at the LIML estimate. As  $\beta_0$  varies confidence intervals for  $\beta$  are generated. As Kleibergen point out the confidence intervals can sometimes have “peculiar shapes” that need not be convex and they can be

infinite. Given the close relationship of the Kleibergen statistic to the LIML estimator and our experience with the “no moments” problem, we wonder how well the statistic will perform when LIML displays poor performance. This is a topic for future research.

#### IV. An Application

Estimating the return to education has been a well-researched problem over the past 25 years. The usual result is that researchers find the OLS estimate to be smaller than the 2SLS estimate by approximately 25%-50%, e.g. David Card (2001). Joshua Angrist and Alan Krueger (1991) used a sample of  $n = 329,509$  observations to estimate the returns to education where the 2SLS estimator is considerably closer to the OLS estimator than usual as demonstrated in Table 1.

#### **\*\*Table 1 goes here\*\***

For the  $K=30$  case the difference of OLS 0.071 and 2SLS of 0.089 is 25%, and the Hausman (1978) specification test does not reject the OLS estimator.<sup>6</sup> Similarly, the LIML, Fuller and JN2SLS, while slightly higher than the 2SLS estimator also do not reject. However, the reverse 2SLS estimator (R2SLS) estimates the return to education to be 0.163, a large difference although the Hahn-Hausman (2002a) test does not reject the 2SLS estimator. When quarterly interactions are not used to form the instruments, and  $K=3$ , the 2SLS estimator in Table 1 is now 0.105, a sizeable increase of 47.9% over the OLS estimator as would be expected from equation (4), the bias expression for 2SLS.<sup>7</sup> The LIML, Fuller, and JN2SLS estimators all increase and now all of the estimates reject the OLS estimate using a Hausman specification test. Given the approximate 14% increase in the LIML and Fuller estimators, a question may arise of how good the asymptotic formulae are because these estimators are second order unbiased so the estimate should not depend on  $K$  contrary to our empirical finding. We conclude that the results can depend on the number of instruments used. Using too many instruments can create bias towards the OLS estimator, which is one of our definitions of the WI problem.

The Angrist-Krueger sample is larger than usual for empirical research, although other studies sometimes have similar sized samples. We consider a random 1% subsample which has  $n=3293$ , more in keeping with the typical situation. We now find



the LIML estimator to be 0.855, a non-believable number that may well arise from a “no moments” violation. We also find the Fuller and reverse Fuller estimates to differ by a large amount and to give non-believable results, which causes us to question whether in finite samples the Fuller estimator always solves the LIML “no moments” problem. We similarly find that the forward and reverse 2SLS estimates differ considerably, and a Hahn-Hausman (2002a) test rejects the use of the 2SLS estimators. Thus, we have a strong indication of a weak instruments problem. The JN2SLS estimator give a “reasonable estimate” with an increase over the 2SLS estimate, as expected by equation (3). The JN2SLS also leads to a rejection of the OLS estimator, contrary to the other IV estimators (except R2SLS).

Kleibergen (2002) estimates the AK model with similar results to ours found in Table 1, although LIML is somewhat larger at 0.108. As expected, his confidence interval is centered at the LIML estimate but is somewhat larger than the LIML confidence interval. The Kleibergen confidence interval is about 50% larger than the confidence estimator that would arise from the first order asymptotic LIML confidence interval. It would be interesting to see how the Kleibergen procedure works when the sample size  $n = 3293$  and the LIML estimator does not perform well.

#### V. A Cautionary Note

All of our analysis, and indeed all of the analysis of the WI situation in the literature, assumes valid instruments so that  $z$  is orthogonal to  $\varepsilon$ . Suppose you do OLS and 2SLS along with the other IV estimators we have discussed and the result is that the IV estimators are reasonable close to each other and exceed the OLS estimator by a substantial amount. Are the results ready for acceptance? Not necessarily. Suppose that the instruments are only slightly correlated with the stochastic disturbance; indeed, much less correlated than  $y_2$  (or  $v_2$ ) is correlated with  $\varepsilon_1$ . IV estimation may lead to very poor results.

Hahn-Hausman (2002c) have analyzed this problem. They consider the “large sample bias” of 2SLS with invalid instruments:

$$(15) \quad \text{plim}[b_{2SLS}] - \beta \approx \frac{\sigma_{W'\varepsilon}}{\Theta} = \sigma_{W'\varepsilon} \frac{1 - R^2}{R^2}$$

where  $W = z\pi_2$  and the last equality follows from the normalization. Thus, when  $R^2$  is small, e.g. 0.01, a large amount of bias results which does not decrease with increasing sample size. To further analyze the problem they use a local specification similar to the approach in Hausman (1978, Theorem 2.1):

$$(16) \quad \varepsilon_1 = z(\gamma / \sqrt{n}) + e \text{ for } \gamma \neq 0$$

where  $(e, v_2)$  is homoscedastic and zero mean normally distributed with covariance matrix, as before. Hahn-Hausman derive the asymptotic distribution of the 2SLS estimator with locally invalid instruments to be

$$(17) \quad \sqrt{n}(b_{2SLS} - \beta) \Rightarrow N\left(\frac{\Xi + \mu\sigma_{ev}}{\Theta}, V_{2SLS}\right) \approx N\left(\frac{[\sqrt{n}\sigma_{W'\varepsilon} + (K/\sqrt{n})\sigma_{ev}](1-R^2)}{R^2}, V_{2SLS}\right)$$

where  $W = z\pi_2$  is the instrument and  $\Xi = \pi'z'z\gamma/n$ , which is assumed to be fixed. The first term in the numerator of the mean  $\Xi$  arises from failure of the orthogonality condition. The second term is the usual finite sample bias term and it decreases with the sample size. The variance continues to be  $V_{2SLS}$  under instrument invalidity because of the local departure in equation (16). Hahn-Hausman (2002c) compare this result to the OLS estimator under the same local departure

$$(18) \quad \sqrt{n}\left(b_{OLS} - \left(\beta + \frac{\sigma_{ev}}{\Theta + \sigma_{vv}}\right)\right) \Rightarrow N\left(\frac{\Xi}{\Theta + \sigma_{vv}}, V_{OLS}\right) \approx N\left(\sqrt{n}\sigma_{W'\varepsilon}(1-R^2), V_{OLS}\right)$$

The distribution is centered around the usual OLS bias, and the numerator of the mean of the distribution arises from the instrument invalidity. The variance  $V_{OLS}$  under instrument invalidity with the local departure in equation (16)

$$(19) \quad V_{OLS} = \frac{\sigma_{\varepsilon\varepsilon}}{\Theta + \sigma_{vv}} - \frac{\sigma_{ev}^2}{(\Theta + \sigma_{vv})^2} - \frac{2\sigma_{ev}^2\Theta^2}{(\Theta + \sigma_{vv})^4} = [1 - \rho^2(1-R^2)(1+2R^4)](1-R^2).$$

Hahn-Hausman (2002c) explore situations where the OLS MSE from equation (18) may be less than the 2SLS MSE from equation (17). Their findings highlight the result that when  $R^2$  is low (below 0.1) OLS may do better than 2SLS. This finding emphasizes the importance of the test of overidentifying restrictions, if the size problems can be corrected, or the Hahn-Hausman (2002a) specification test, both of which should be sensitive to instrument invalidity.

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<sup>1</sup> Another approach to the definition of the WI problem is Douglas Staiger and James Stock (1997) and Stock et. al. (2002). References to the literature can be found in the latter paper and in Hahn-Hausman (2002a).

<sup>2</sup> These parameters are theoretical values from the underlying model specifications for given parameter values.

<sup>3</sup> The Monte-Carlo design is the same as in Hahn-Hausman (2002a).

<sup>4</sup> Hahn-Hausman (2002b) demonstrate that equation (4) can be used to solve for a second order unbiased estimator of  $\beta$ , which turns out to be the Nagar estimator.

<sup>5</sup> The root mean square error of LIML and Nagar are again often considerably higher than 2SLS.

<sup>6</sup> We estimate  $\rho$  to be -0.11, not particularly high. We estimate  $R^2$  to be .00045 and the F statistic to be 4.9 with 30 degrees of freedom in the numerator where the 5% critical value is 1.46.

<sup>7</sup> We estimate  $R^2$  to be .00029 and the F statistic to be 32.3 with 3 degrees of freedom in the numerator where the 5% critical value is 2.60.

**Table 1: Estimation with the Angrist-Krueger (1991) Model**

<u>n and K</u>	<u>OLS</u>	<u>2SLS</u>	<u>R2SLS</u>	<u>LIML</u>	<u>Fuller</u>	<u>RFuller</u>	<u>JN2SLS</u>
n=329509 K=30	.071 (.0003)	.089 (.016)	.163* (.024)	.093 (.018)	.093 (.018)	.096 (.018)	.093 (.016)
K=3	---	.105* (.020)	.118* (.021)	.106* (.020)	.107* (.020)	.110* (.021)	.106* (.020)
N=3293 K=30	.073 (.003)	.098 (.043)	.694* (.369)	.855 (.963)	.396 (.291)	.831 (.865)	.159* (.041)

Note: \* denotes Hausman (1978) specification test rejects null hypothesis of OLS consistency